Tagungsbericht 24/1996

Geometrische Analysis auf singulären Räumen

23. bis 29. Juni 1996

Die Tagung fand unter Leitung der Herren J.-M. Bismut (Paris), J. Brüning (Berlin) und R. Melrose (Cambridge) statt.

Die Veranstaltung schloß sich thematisch an die vorangegangenen Tagungen zum Themenkreis "Geometrische Analysis auf singulären und nichtkompakten Mannigfaltigkeiten" aus den Jahren 1987, 1991 und 1994 an.

Schwerpunkte der Vorträge und Diskussionen lagen bei Indextheorie und Spektraltheorie sowie verschiedenen (spektralen) Invarianten von Mannigfaltigkeiten.

Bei den Invarianten ist die analytische Torsion und ihr Zusammenhang mit der kombinatorischen Torsion von zentralem Interesse. Dabei wurden auch wichtige Verallgemeinerungen im Hinblick auf nichtkompakte Überlagerungen kompakter Mannigfaltigkeiten besprochen (Dai, Kappeler, Lück). Ein weiterer Beitrag befaßte sich mit einem Indexsatz für solche Überlagerungen (Leichtnam). Die η-Invariante, eine weitere wichtige spektrale Invariante, wurde in einem informellen Vortrag (Lesch) behandelt.

Ein weiterer zentraler Themenkreis der Tagung waren symplektische und Kählersche Mannigfaltigkeiten. Hier wurden Beweise der Guillemin–Sternberg–Vermutung (Meinrenken, Vergne, Zhang), Beiträge zum Calabi–Problem (Tian) sowie kanonische Metriken auf Kählermannigkeiten (Leung) besprochen.

Sehr präzise Resultate lassen sich für spezielle Klassen von Mannigfaltigkeiten erzielen, wobei Riemannsche Flächen oder allgemeiner symmetrische Räume immer wieder im Zentrum des Interesses stehen (Bunke, Jeffrey, Lott, Müller, Zworski).

Weitere Beiträge behandelten nichtlineare Probleme (Mazzeo, Müller), Methoden der nichtkommutativen Differentialgeometrie (Wu, Melrose (informell), Kordyukov (informell)), Determinantenbündel (Piazza), Novikov–Ungleichungen (Braverman) sowie Mannigfaltigkeiten mit Krümmungsschranken (Colding).

Die Teilnehmer waren sich darin einig, daß dies eine sehr fruchbare Tagung auf mathematisch anspruchsvollem Niveau war. Es bestand einhelliges Interesse an einer Fortsetzungstagung in etwa zwei Jahren.

Scattering for Riemann surfaces

M. ZWORSKI (joint work with L. GUILLOPÉ)

Let X be a Riemannian surface of finite geometric type and with hyperbolic ends. The resolvent, $(D - s(1-s))^{-1}, \mathrm{Re}\, s > 1$, of the Laplacian, D, on X extends to a meromorphic family of operators on $\mathbb{C}$ and its poles are called resonances. If $N(r)$ is the number of resonances in a disc of radius r, then we prove the optimal polynomial bounds:

$$r^2/C \leq N(r) \leq Cr^2, \quad r > C.$$

We define the relative scattering matrix of the surface with respect to the infinite volume ends and we show that the corresponding scattering phase, $\sigma_X(s)$, enjoys Weyl type asymptotics, which in the constant curvature infinite volume case take the form

$$\sigma_X(s) = \frac{\mathrm{vol(N)}}{4\pi}|s|^2 + O(|s|), \quad \mathrm{Re}\, s = \frac{1}{2},$$

where N is the Fenchel–Nilsen region. The results are a consequence of a trace formula involving the relative scattering matrix and a Poisson formula for resonances.

Singular reduction and quantization

ECKHARD MEINRENKEN

Let (M,ω) be a compact symplectic manifold, and G a compact connected Lie group acting on M with moment map $\Phi : M \to \mathfrak{g}^*$. The symplectic quotient is defined as $M//G = \Phi^{-1}(0)/G$. If 0 is a regular value of Φ this is a symplectic orbifold, otherwise it is in general a singular space. Now suppose $L \to M$ is a G-equivariant Hermitian line bundle with G-connection ∇ which is "prequantum", i.e.

$$\mathrm{Vert}\,(\xi_L) =< \Phi, \xi > \left(\frac{\partial}{\partial\varphi}\right)_L \quad \text{and} \quad \frac{i}{2\pi}\mathrm{curv}(\nabla) = \omega.$$

Define $L//G = (L|\Phi^{-1}(0))/G \to M//G$. The equivariant index of the Spin^c-Dirac operator on L is called the Riemann-Roch-number; $\mathrm{ind}_G(\partial\!\!\!/) =: \mathrm{RR}(M,L) \in \mathrm{Rep}(G)$. I proved that if 0 is a regular value of Φ, the G-invariant part $\mathrm{RR}(M,L)^G \in \mathbb{Z}$ is equal to $\mathrm{RR}(M//G, L//G)$. This "Quantization commutes with reduction" theorem was conjectured by GUILLEMIN-STERNBERG in 1982; various special cases had been obtained by many different authors and methods. Using partial desingularization, SJAMAAR and I extended this result to the singular case. In joint work with WOODWARD, I applied the result to moduli spaces of flat connections on Riemann surfaces; in particular we use it to derive Verlinde's fusion rules.

Canonical metrics on Kähler manifolds and vector bundles

NAICHUNG CONAN LEUNG

We study Kähler metrics on manifolds and Hermitian metrics on holomorphic vector bundles from two different viewpoints:

1. minimization of the energy functional,
2. geometric invariant theory.

Relations with Futaki-Bando invarinats will be discussed. We also introduce a new obstruction for stability of vector bundles.

Determinant bundles and surgery

P. PIAZZA

Let $M \to B$ be a fibration with even dimensional compact fibre. Let $g_{M/B}$ be a metric on the vertical tangent bundle and let $E \to M$ be a hermitian vertical Clifford module endowed with a unitary Clifford connection.

These data determine a family $\partial\!\!\!/ = (\partial\!\!\!/_z)_{z\in B}$ of generalized Dirac operators. Associated to $(\partial\!\!\!/_z)$ there is a determinant bundle, $\mathcal{L}$, with its Quillen metric, $\|\cdot\|_Q$, and metric compatible Bismut–Freed connection, $\nabla^{\mathcal{L}}$.

Suppose now that the fibration M is the union along a fibering hypersurface H of two fibrations with boundary M_0, M_1. Thus each fibre $M_z, z \in B$, is the union along H_z of two manifolds with boundary: $M_z = M_z^0 \cup_{H_z} M_z^1$. In this talk I have described the behaviour of the Quillen metric and of the Bismut–Freed connection on $M \to B$ (and in particular its curvature and holonomy) under the operation of surgery. By surgery we mean the stretching of the collar neighborhood of H in M to infinity. If $H = \{x = 0\}$, this intuitive idea can be described as follows: introduce the metric $g(\epsilon) = \frac{dx^2}{x^2+\epsilon^2} + g_{M/B}$ and let $\epsilon \searrow 0$. We denote by $\partial\!\!\!/(\epsilon) = (\partial\!\!\!/(\epsilon)_z)_{z\in B}$ the Dirac family associated to the metric $g(\epsilon)$. The analysis of the surgery problem involves 3 fundamental tools:

1. The notion of spectral section P associated to the family of self–adjoint operators $D_H = (D_{H_z})_{z\in B}$: this notion (due to MELROSE and myself) is needed in order to define 2 smooth families of boundary value problems of gernalized APS type, $\partial\!\!\!/_P^{M_0}, \partial\!\!\!/_{1-P}^{M_1}$, on the 2 fibrations with boundary.
2. The b–calculus: this is a calculus of pseudodifferential operators needed in order to define and investigate the hermitian geometry of the determinant bundles associated to $\partial\!\!\!/_P^{M_0}, \partial\!\!\!/_{1-P}^{M_1}$.
3. The surgery calculus of MAZZEO and MELROSE which allows for a uniform study of the resolvent $(\partial\!\!\!/(\epsilon)^2 - \lambda)^{-1}$ and the heat-kernel $\exp(-t\partial\!\!\!/(\epsilon)^2)$ as $\epsilon \searrow 0$.

Our main result can be stated as follows:

(i) For ϵ small enough there is an explicit natural isomorphism $\det(\partial\!\!\!/(\epsilon)) \equiv \mathcal{L}^\epsilon \longrightarrow \det(D_P^{M_0}) \otimes \det(D_{1-P}^{M_1})$.

(ii) The curvature of the Bismut–Freed connection $\nabla^{\mathcal{L}^\epsilon}$ converges to the sum of the curvatures $(\nabla^{M_0,P})^2 + (\nabla^{M_1,1-P})^2$.

(iii) The holonomy converges to the product of the holonomies.

On a problem of Calabi

GANG TIAN

Many years ago, E. CALABI asked when a compact Kähler manifold has a Kähler-Einstein metric. A necessary condition is that the first Chern class $c_1(M)$ has to be definite. In 1976, AUBIN & YAU proved that if $c_1(M) < 0$ there is always a Kähler-Einstein metric on M, and YAU proved the same thing for $c_1(M) = 0$ as a corollary of his solution of the Calabi conjecture. What remained was the case that $c_1(M) > 0$. In 1990, I proved that a complex surface M with $c_1(M) > 0$ has a Kähler-Einstein metric iff its Lie algebra of holomorphic vector fields is reductive. In this talk, I discussed recent progress in case of higher dimension. I introduced the notion of K-stability in terms of special degenerations of M and Futaki invariants on the central fiber. I proved that if M has a Kähler-Einstein metric, then M is K-stable. An example of a 3-fold which is not K-stable is constructed. This provides a counterexample to the folklore conjecture that if $c_1(M) > 0$ and M has no holomorphic vector fields, then M has a Kähler-Einstein metric.

Geometrically infinite hyperbolic 3-manifolds

JOHN LOTT

Let M be a topologically tame hyperbolic 3-manifold with nonabelian fundamental group and infinite volume. Let Δ_p be the Laplacian acting on square-integrable p-forms on M. We discuss the following questions :

1. What is $\ker(\Delta_1)$?

2. Is zero in the spectrum of Δ_1 acting on $\Lambda^1(M)/\ker(d)$?

We concentrate on the case when M is geometrically infinite. By work of THURSTON, BONAHON and CANARY, we know that M has "tubular" ends. If the injectivity radius of M is zero, we show that $0 \in \operatorname{spec}$ (Δ_1 acting on $\Lambda^1(M)/\ker(d)$). If M has positive injectivity radius, we use work of MINSKY to give a biLipschitz model of M. Using this biLipschitz model, we compute the reduced and unreduced L^2-cohomology groups of the ends of M. This allows us to answer question 2 above in terms of the surjectivity of differentiation on a certain weighted L^2-space. Finally, we compute $\ker(\Delta_1)$ in the case when $0 \notin \operatorname{spec}$ (Δ_1 acting on $\Lambda^1(M)/Ker(d)$).

Analytic and Reidemeister torsion for representations in finite type Hilbert modules

THOMAS KAPPELER

For a closed Riemannian manifold (M, g) we extend the definition of analytic and Reidemeister torsion associated to a unitary representation of $\pi_1(M)$ to a A-Hilbert-module $\mathcal{W}$ of finite type where $\mathcal{A}$ is a finite von Neumann algebra. If $(M, \mathcal{W})$ is of determinant class, we prove, generalizing the CHEEGER-MÜLLER theorem, that the analytic and Reidemeister torsion are equal. The main feature of the proof is to use the Witten deformation of the deRham complex and study the corresponding deformed analytic torsion.

A higher Atiyah-Patodi-Singer index theorem on Galois covering

ERIC LEICHTNAM (joint with PAOLO PIAZZA)

Let $\Gamma \to \tilde{M} \to M$ be a Galois covering with boundary and let $\tilde{D}$ be a generalized Dirac operator. Under the following two assumptions

1. the group Γ is virtually nilpotent,

2. the L^2-spectrum of the boundary operator $\tilde{D}_0$ has a gap at zero,

we prove a higher Atiyah-Patodi-Singer index formula, thus settling a conjecture of LOTT and extending work of LUSZTIG, CONNES-MOSCOVICI, and LOTT.

An analytic proof of the Guillemin-Sternberg conjecture

WEIPING ZHANG

We explain an analytic approach to the GUILLEMIN-STERNBERG geometric quantization conjecture, which was proved recently by MEINRENKEN and VERGNE in the abelian case and by MEINRENKEN in the general case. Besides providing a new proof of the full non-abelian case, our method also leads immediately to certain new results including refined Morse type inequalities in the holomorphic situation. (The above is a joint work with YOULIANG TIAN.)

Group cohomology and the singularities of the Selberg zeta function

ULRICH BUNKE (joint with M. OLBRICH)

Let $Z(s)$ be the Selberg zeta function associated to a geometrically finite hyperbolic manifold M^n without cusps. We give a description of the singularities (PATTERSON'S conjecture) of $Z(s)$ in terms of $H^*(\Gamma, \mathcal{O}_\lambda C^{-\omega}(\Lambda))$. Here $\Gamma = \pi_1 M$ and $\mathcal{O}_\lambda \mathcal{O}^{-\omega}(\Lambda)$ is the representation of Γ on the space of germs of holomorphic families of hyperfunction sections of the family $\left[(\Lambda^{n-1}T^*S^{n-1})^{\frac{1}{n-1}}\right]^{\lambda-(n-1)/2}$ of bundles with support in the limit set Λ of Γ. The idea of the proof is to compute the cohomology groups and to compare the results with that of PATTERSON/PERRY on $Z(s)$. To compute $H^*(\Gamma, \mathcal{O}_\lambda C^{-\omega}(\Lambda))$ we consider a particularly nice acyclic resolution. The result is expressed in terms of spectral/scattering data associated to the Laplacian of M.

Analysis and geometry of spaces with a lower Ricci curvature bound

TOBIAS H. COLDING

In this survey talk I described some new ideas and techniques introduced to study spaces with a given lower Ricci curvature bound. Further, we will explain some of their consequences.

In studying spaces with a given lower sectional curvature bound we have a very powerful tool in the Toponogov triangle comparison theorem. This allows us to study metric and topological properties of such spaces.

In the case where we only assume a lower Ricci curvature bound no such estimate is available. Classically, the only general estimates that are known of this type for Ricci curvature are the Bishop-Gromov volume comparison theorem and the Abresch-Gromoll inequality.

In order to study manifolds with a given lower Ricci curvature bound there are at least two obstacles to overcome. First, many results from the sectional curvature case do not remain true for Ricci curvature. Second, due to the lack of a good estimate on the distance function we do not have good control on the local geometry in this case.

I discussed in this survey that in some sense the second obstacle is the most serious.

Namely, I discussed a new estimate of the distance function and later saw that this type of estimate has a large number of consequences.

My main focus in the talk was the Geometry and Topology of manifolds with a lower Ricci curvature bound.

I also mentioned regularity properties of general metric spaces that are (Gromov-Hausdorff) limits of n-dimensional manifolds with a given lower Ricci curvature bound. This is in part motivated by Gromov's compactness theorem.

The results that I described represented some of my own work, joint work with Jeff Cheeger and joint work with Jeff Cheeger and Gang Tian.

Analytic and Reidemeister torsion for spaces with conical singularities

XIANZHE DAI

We describe some joint work in progress with RAFE MAZZEO. For compact smooth manifolds with boundary, it is a well-known theorem of CHEEGER and MÜLLER (also known as the Ray-Singer conjecture) that the analytic torsion and the Reidemeister torsion are the same. For manifolds with isolated conical singularity, the analytic torsion and the Reidemeister torsion can be similarly defined; the analytic torsion using CHEEGER's theory of manifolds with conical singularity and the Reidemeister torsion using the intersection homology of Goresky-MacPherson. It is a natural question as to whether the analogue of CHEEGER-MÜLLER's theorem still holds in this category of singular spaces.

We approach this question from the view point of conic degeneration. By this we mean a family of Riemannian metrics on a closed manifold M which gradually pinches an embedded hypersurface to a point. Our starting point is that any manifold with isolated conical singularity can be "embedded" into a conic degeneration. Therefore we can start with CHEEGER-MÜLLER's theorem on the smooth manifold and pass to the conic limit.

This reduces the problem to the study of the behavior of the torsions under conic degeneration. We examine the uniform behavior of the heat kernel under the conic degeneration and prove the convergence of the analytic torsion under certain acyclicity conditions.

Noncommutative spectral flow and the APS index theorem

FANGBING WU

The notion of noncommutative spectral flow is studied in the K-theoretic setting, generalizing the classical cases. A formula for computing the noncommutative spectral flow in terms of loops of unitaries is then presented, leading to the construction of the cyclic Chern character of the spectral flow.

Equivariant Novikov inequalities

MAXIM BRAVERMAN (joint work with M. FARBER)

We establish an equivariant generalization of the Novikov inequalities which allows to estimate the topology of the set of critical points of a closed basic invariant form by means of twisted equivariant cohomology of the manifold. The proof is based on Novikov type inequalities for differential forms with non-isolated zeros obtained in our previous work. We apply the equivariant Novikov inequalities to obtain a Novikov type inequality for a manifold with boundary. As another application we obtain a bound on the cohomology of the fixed point set of a symplectic torus action.

Hilbert modules and modules over finite von Neumann algebras, with applications to L^2–invariants

WOLFGANG LÜCK

Given a Riemannian manifold M with isometric Γ–action such that $M \to M/\Gamma$ is a covering over a closed manifold, Atiyah defined the p-th L^2–Betti number

$$b_p^{(2)}(M;\Gamma) = \int_{\mathcal{F}} \mathrm{tr}(e^{-t\Delta_p}(x,x)\ \mathrm{dvol}$$

where $\mathcal{F}$ is a fundamental domain and $e^{-t\Delta_p}$ the heat kernel. Let $\mathcal{N}(\Gamma)$ be the von Neumann algebra of the group. We define for any module M over the ring $\mathcal{N}(\Gamma)$ a dimension $\dim(M) \in \mathbb{R}$ with the following properties:

i) If M is finitely generated projective, there is an associated Hilbert $\mathcal{N}(\Gamma)$–module V and $\dim M$ is the von Neumann dimension of V.

ii) $0 \to M_1 \to M_2 \to M_3 \to 0$ exact $\Longrightarrow \dim M_2 = \dim M_1 + \dim M_3$.

iii) $M = \cup_{i\in I} M_i$ for a directed system of submodules $\{M_i \,|\, i \in I\} \Longrightarrow \dim M = \sup\{\dim M_i \,|\, i \in I\}$.

Given a topological space X with Γ–action, we define

$$b_p^{(2)}(X;\Gamma) = \dim(H_p^{\Gamma}(X;\mathcal{N}(\Gamma))).$$

This agrees with the old definition for $X = M$ as above. We use the properties of this dimension and the generality of this definition to reprove in an easy way the following result.

Theorem 1 [CHEEGER-GROMOV 86] *If Γ contains an infinite normal amenable subgroup then $b_p^{(2)}(\Gamma) = 0$ for all p.*

Furthermore, we prove the following results.

Theorem 2 *If $1 \to \Delta \to \Gamma \to \Pi \to 1$ is an extension of infinite groups, Δ is finitely generated, Γ finitely presented, and $\mathbb{Z}$ a subgroup of π, then*

1) $b_1^{(2)}(\Gamma) = 0$

2) $\mathrm{deficiency}(\Gamma) = \max\{g - r \,|\, < s_1 \cdots s_g | R_1 \cdots R_g >= \Gamma\} \leq 1$

3) *M closed oriented 4-manifold with $\pi_1(M) = \Gamma \Longrightarrow |\mathrm{sign}(M)| \leq \chi(M)$.*

Theorem 3 *Let F be Thompson's group. Then $b_p^{(2)}(F) = 0$ for all p.*

Residue formulae and lattice points in convex polytopes

MICHELE VERGNE (joint with Michel Brion)

Let $D = \{x \in \mathbb{R}^n \mid \langle u_i, x \rangle + \lambda_i \geq 0, i = 1, 2, \ldots, N\}$ be a rational convex polytope, with $u_i \in \mathbb{Z}^n, \lambda_i \in \mathbb{Z}$.

Let $P(h) = \{x \in \mathbb{R}^n \mid \langle u_i, x \rangle + h_i \geq 0\}$, and let $\mathcal{F} = \{(\gamma_1, \gamma_2, \ldots, \gamma_N) \mid \gamma_i \in [0, 1[\}$ and assume that

a) $\gamma_i = 0$ except for n indices σ, such that $u_i, i \in \sigma$, are linearly independent,

b) $\sum \gamma_i u_i \in \mathbb{Z}^n$.

Then for any polynomial function φ on $\mathbb{R}^n$

$$\sum_{m \in \mathbb{Z}^n \cap P(\lambda)} \varphi(m) = \sum_{\gamma \in \mathcal{F}} e^{2i\pi(\sum \lambda_k \gamma_k)} \prod_{k=1}^{N} \frac{\frac{\partial}{\partial h_k}}{1 - e^{2i\pi\gamma_k} e^{-\frac{\partial}{\partial h_k}}} \cdot \int_{P(h)} \varphi(x) dx \Big|_{h=\lambda}.$$

This formula generalizes formulae of KHOVANSKII–PUKHLIKHOV and CAPPELL–SHANESON. Proofs are elementary, based on the generating partition function.

Riemann surfaces of infinite genus and solutions of the KdV-equation

WERNER MÜLLER

We report on joint work with M. SCHMIDT and R. SCHRADER (FU Berlin). Since the work of GARDNER, GREEN, KRUSKAL, NOVIKOV,... it is well-known that the integration of the KdV-equation $u_t = 6u u_x - u_{xxx}$, $u(x, 0) = q(x)$ is closely related to the inverse spectral theory for the Sturm-Liouville operator $H = -\frac{d^2}{dx^2} + q$. In particular, for finite gap potentials, the KdV-equation can be solved using Riemann surfaces and the theory of the Riemann theta-function. This has been extended by MCKEAN and TRUBOWITZ to the case of periodic infinite gap potentials. The spectral surfaces are then infinite genus hyperelliptic surfaces, and a great deal of the function theory including the theory of the theta-functions can be extended to these surfaces. We describe extensions of this theory by constructing renormalized theta functions associated to a certain class of infinite genus hyperelliptic surfaces. Using these theta functions we are able to construct new solutions of the KdV-equation which in some cases are quasi-periodic.

Intersection numbers on moduli spaces of vector bundles on Riemann surfaces

LISA JEFFREY

If $G = U(n)$, the space M of flat G connections on a Riemann surface Σ has been the subject of intense interest for the past thirty years in algebraic and symplectic geometry as well as through its applications to topology. It appears in two additional guises: in topology as the space of representations of the fundamental group of the Riemann surface into G, and in algebraic geometry as the moduli space of semistable holomorphic vector bundles of rank n (and degree 0) on Σ. Of associated interest are the spaces $M(n,d)$ which appear in algebraic geometry as the moduli spaces of semistable holomorphic vector bundles of rank n, degree d and fixed determinant: if n and d are coprime these spaces are smooth Kähler manifolds.

In a fundamental 1982 paper studying the Morse theory of the Yang-Mills functional, ATIYAH and BOTT found formulas for the Betti numbers of the spaces $M(n,d)$, so that the characterization of the cohomology as a vector space is complete: its ring structure (or equivalently the value of the intersection pairings in the cohomology ring) has however remained obscure. In 1992, WITTEN used physical methods to find formulas for these intersection pairings: his work involved a two dimensional quantum field theory for which the Lagrangian was the Yang-Mills functional.

In recent joint work with FRANCES KIRWAN, we have proved Witten's formulas using methods from symplectic geometry (notably the technique of localization in equivariant cohomology).

Gluing and moduli for some noncompact geometric problems

RAFE MAZZEO

In this talk I described two seemingly different geometric situations: the study of complete, properly immersed minimal or constant mean curvature surfaces in $\mathbb{R}^3$, and of solutions of the singular Yamabe problem. In the past several years, new analytic techniques showed that solutions to these problems are much more flexible than previously thought. Much of the talk was spent describing the singular Yamabe problem on the sphere. Here one seeks a metric g on S^n which is conformal to the standard flat metric, and which is complete on the complement of a closed subset $\Lambda \subset S^n$. I concentrated on the case where Λ is a finite point set, in which case it is known that solutions must have positive scalar curvature. An analytic nondegeneracy condition for solutions is described. Then there are three different results:

Theorem 1 (with D. POLLACK AND K. UHLENBECK): *The moduli space of solutions with singularities at either a fixed set Λ or at an arbitrary set of k points in S^n is a real analytic set of virtual dimension k and $k(n+1)$, respectively. This virtual dimension is attained for the 'unmarked' moduli space, and is also attained for the solutions with specified singular set, for generic Λ.*

Theorem 2 (with D. POLLACK AND K. UHLENBECK) (soft gluing theorem): *Given any two nondegenerate complete manifolds of constant positive scalar curvature* (M_1, g_1) *and* (M_2, g_2)*, there exists a complete metric of constant positive scalar curvature on the connected sum* $M_1 \# M_2$.

Theorem 3 (with F. PACARD) (hard gluing theorem): *Given any set* Λ *of* k *points in* S^n, $k \geq 2$*, there exists a solution of the singular Yamabe problem with singularities and specified asymptotic geometry at the points of* Λ.

There are analogous results for constant mean curvature surfaces, obtained jointly, in various combinations, with KUSNER, POLLACK, PACARD and KAPOULEAS; the corresponding results for minimal surfaces were obtained by LOPEZ and ROS, and KAPOULEAS.

In all cases, the main point is a suitably detailed understanding of the linearization of the scalar curvature (or mean curvature) operator.

Berichterstatter: PD Dr. M. Lesch, Dr. M. Pflaum

E–Mail Addresses

Ammann, B.	ammann@mathematik.uni-freiburg.de
Bär, C.	baer@mathematik.uni-freiburg.de
Berline, N.	berline@cmat.polytechnique.fr
Brüning, J.	bruning@mathematik.hu-berlin.de
Colding, T.	colding@cims.nyu.edu
Christiansen, T.	tjc@math.missouri.edu
Felshtyn, A.	felshtyn@math-inf.uni-greifswald.d400.de
Friedlander, L.	friedlan@math.arizona.edu
Goette, S.	goette@mathematik.uni-freiburg.de
Grieser, D.	grieser@mathematik.hu-berlin.de
Jeffrey, L.	jeffrey@gauss.math.mcgill.ca
Kappeler, T.	kappeler@math.ohio-state.edu
Kordyukov, Y.	yurikor@math.ugatu.ac.ru
Leichtnam, E.	leicht@dmi.ens.fr
Lesch, M.	lesch@mathematik.hu-berlin.de
Leung, N.	leung@math.umn.edu
Lott, J.	lott@math.isa.umich.edu
Lück, W.	lueck@topologie.mathematik.uni-mainz.de
Mazzeo, R.	mazzeo@math.stanford.edu
Meinrenken, E.	mein@math.mit.edu
Melrose, R.	rbm@math.mit.edu
Mooers, E.	mooers@math.ucla.edu
Müller, W.	mueller@rhein.iam.uni-bonn.de
Pflaum, M.	pflaum@mathematik.hu-berlin.de
Schick, T.	thomas.schick@topologie.mathematik.uni-mainz.de
Schrohe, E.	schrohe@mpg-ana.uni-potsdam.de
Seeley, R.	rts@cs.umb.edu
Vasy, A.	andras@math.mit.edu
Wendland, K.	wendland@rhein.iam.uni-bonn.de
Wu, F.	fangbing@math.ksu.edu
Wunsch, J.	jwunsch@math.harvard.edu
Zworski, M.	zworski@math.toronto.edu

Tagungsteilnehmer

Bernd Ammann
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

79104 Freiburg

Prof.Dr. Jochen Brüning
Institut für Mathematik
Humboldt-Universität

10099 Berlin

Prof.Dr. Christian Bär
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

79104 Freiburg

Prof.Dr. Ulrich Bunke
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

37073 Göttingen

Prof.Dr. Nicole Berline
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

Prof.Dr. Tanya Christiansen
Dept. of Mathematics
University of Missouri-Columbia

Columbia , MO 65211-0001
USA

Prof.Dr. Jean-Michel Bismut
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Batiment 425

F-91405 Orsay Cedex

Prof.Dr. Xianzhe Dai
Dept. of Mathematics
University of Southern California
1042 W 36th place

Los Angeles , Ca 90089-1113
USA

Prof.Dr. Maxim Braverman
Department of Mathematics
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv, P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof.Dr. Alexander L. Felshtyn
Fachrichtung Mathematik/Informatik
Universität Greifswald
Friedrich-Ludwig-Jahn-Str. 15a

17489 Greifswald

Prof.Dr. Leonid Friedlander
Dept. of Mathematics
University of Arizona

Tucson , AZ 85721
USA

Dr. Kai Köhler
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Sebastian Goette
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

79104 Freiburg

Prof.Dr. Yuri A. Kordyukov
Dept. of Mathematics
Ufa State Aviation Technical
University
K. Marx Str. 12

450025 Ufa
RUSSIA

Daniel Grieser
Institut für Mathematik
Humboldt-Universität
Unter den Linden 6

10117 Berlin

Prof.Dr. Dimitri Kountourogiannis
Department of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139-4307
USA

Prof.Dr. Lisa C. Jeffrey
Dept. of Mathematics and Statistics
McGill University
805, Sherbrooke Street West

Montreal , P.Q. H3A 2K6
CANADA

Prof.Dr. Eric Leichtnam
Departement de Mathematiques et
d'Informatiques
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05

Prof.Dr. Thomas Kappeler
Department of Mathematics
Ohio State University
231 West 18th Avenue

Columbus , OH 43210-1174
USA

Dr. Matthias Lesch
Institut für Mathematik
Humboldt-Universität

10099 Berlin

Prof.Dr. Nai-Chung Conan Leung
Department of Mathematics
University of Minnesota
206 Church Street S.E.
127 Vincent Hall

Minneapolis , MN 55455
USA

Prof.Dr. Richard B. Melrose
Department of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139-4307
USA

Prof.Dr. John Lott
Department of Mathematics
University of Michigan

Ann Arbor ,MI 48109-1003
USA

Edith Mooers
Department of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139-4307
USA

Prof.Dr. Wolfgang Lück
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Prof.Dr. Werner Müller
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Prof.Dr. Rafe R. Mazzeo
Department of Mathematics
Stanford University
Building 380

Stanford , CA 94305-2125
USA

Dr. Markus Pflaum
Institut für Mathematik
Humboldt-Universität
Unter den Linden 6

10117 Berlin

Dr. Eckhard Meinrenken
MIT
77 Massachusetts Avenue

Cambridge , MA 02139
USA

Prof.Dr. Paolo Piazza
Universita "La Sapienza"
Istituto Matematico "G.Castelnuovo
P.le Aldo Moro 2

I-00185 Roma

Thomas Schick
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Boris Vaillant
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 10

53115 Bonn

Dr. Elmar Schrohe
Max-Planck Arbeitsgruppe
Part. Differentialgleichungen
und komplexe Analysis
Postfach 601553

14415 Potsdam

Andras Vasy
Department of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139-4307
USA

Prof.Dr. Robert T. Seeley
Dept. of Mathematics
University of Massachusetts
at Boston
Harbor Campus

Boston , MA 02125
USA

Prof.Dr. Michele Vergne
Departement de Mathematiques et
d'Informatiques
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05

Prof.Dr. Reyer Sjamaar
Dept. of Mathematics
Cornell University
White Hall

Ithaca , NY 14853-7901
USA

Katrin Wendland
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Prof.Dr. Gang Tian
MIT
77, Massachusetts Avenue

Cambridge , MA 02139
USA

Prof.Dr. Fang-bing Wu
Department of Mathematics
Kansas State University

Manhattan , KS 66506-2602
USA

Prof.Dr. Maciej Zworski
Dept. of Mathematics
University of Toronto

Toronto ONT L4J 7W2
CANADA

Jared Wunsch
Dept. of Mathematics
Harvard University
1 Oxford Street

Cambridge , MA 02138
USA

Prof.Dr. Wei-Ping Zhang
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street

New York NY 10012-1110
USA