Tagungsbericht 25/1996

Dynamical Systems Methods in Fluid Mechanics

June 30 - July 6, 1996

This conference was organized by G. Iooss (Nice), K. Kirchgässner (Stuttgart), J. E. Marsden (Pasadena) and J. Scheurle (München). There were sixteen plenary sessions with talks about the following topics:

- Solitary Waves
- Homoclinic and Bifurcation Phenomena
- Shallow Water Dynamics
- Geometric Methods in Fluid Dynamics
- Singular Perturbations
- Stability and Instability in Fluids
- Nonlinear Waves
- 3D Euler and Navier Stokes Equations
- Euler Equations on Thin Domains
- Nonlinear Evolution Equations
- Computational Methods
- Quasiperiodic Solutions
- Systems with Symmetries
- Cosymmetry, Integrability and Diffusion Limits
- Stability Results for Plasmas and Gravitational Matter

In addition, evening lectures were presented by V. I. Arnold (Moscow and Paris), P. J. Holmes (Princeton) and A. Mielke (Hannover).

The organizers and all the participants of this conference are greatly indebted to the Oberwolfach Institute for providing a stimulating atmosphere for discussions and the exchange of ideas concerning common research interests.

Abstracts

M. ALBER :

Complex Billiards on Riemann surfaces and Nonlinear PDE's (joint work with R. Camassa, D. Holm and J. Marsden)

We investigate the class of peakons introduced by Camassa and Holm for a shallow water equation. We put this equation in the framework of complex integrable Hamiltonian systems on Riemann surfaces. Deformations of these Riemann surfaces yield n-solitons, solitons with quasiperiodic background, billiards, and n-Peakon solutions and complex angle representations for them. Also, explicit formulas for phase shifts of interacting peakons are obtained using the method of asymptotic reduction of the angle representation.

V. ARNOLD :

Dynamical Systems Methods in Fluid Mechanics

1. Helicity and asymptotical Hopf invariant: from knots and links to divergence-free vector fields.
2. Particles stretching and short-waves asymptotics: Anosov systems hydrodynamical applications.
3. Eulerian hydrodynamics as infinite-dimensional rigid body dynamics: geodesics of the groups of diffeomorphisms and of their extensions.
4. Polyintegrable flows: pseudoperiodical topology of flows with several multivalued integrals.
5. Nonintegrability: not only absence of the integrals, but also of preserved geometrical objects of other types.

C. BARDOS :

Diffusion Limits of Deterministic Reversible Systems

Some examples of diffusion limits of kinetic equations are described.

The difference between linear (interaction with obstacle) and nonlinear interaction (self interaction) is discussed in relation with the appearence of decay of entropy.

These properties are illustrated by an explicit example constructed with the Arnold cat map.

J. BATT :

Stability Results for Plasmas and Gravitational Matter

Three late results on the stability for the Vlasov-Poisson system are described: one which is obtained by a rearrangement argument in [1], one which is based on the energy-Casimir method [2] described in [3], and one which uses the existence of a conserved quantity, called the free energy, in [4]. The existence theory of the Vlasov-Poisson system is reviewed and the construction of stationary solutions is described. A recent result of obtaining the asymptotic profile for the distribution function, the local density (charge) and the Newtonian (electric) field by using a rescaling method [5] is also presented.

[1] J. Batt, G. Rein. A rigorous stability result for the Vlasov-Poisson system in 3 dimensions. Annali di Mat. Pura et. Appl. 164 (1993), 133-154.

[2] G. Rein. Nonlinear Stability for the Vlasov-Poisson System - the Energy-Casimir Method. Math. Meth. Appl. Sci. 17 (1994), 1129-1140.

[3] D.D. Holm, J.E. Marsden, T. Ratiu, A. Weinstein. Nonlinear Stability of Fluid and Plasma Equilibria. Phys. Reports 123 (1985), 1-116.

[4] J. Batt, Ph. Morrison, G. Rein. Linear Stability of Stationary Solutions of the Vlasov-Poisson System in three Dimensions, Arch. Rat. Mech. Anal. 130 (1995), 163-182.

[5] J. Batt, M. Kunze, G. Rein. Preprint 1996.

Y. BRENIER :

On the Geometric Description of Incompressible Inviscid Ideal Fluids

The motion of an incompressible inviscid fluid moving in a three-dimensional vessel D can be related (following Arnold 1966, Ebin and Marsden 1970) to a geodesic curve along the group G of all orientation and volume preserving diffeomorphisms of D, for the metric inherited from the natural embedding of G into $L^2(\mathcal{D}, \mathbb{R}^3)$. We address the problem of finding a shortest path along G between two elements g_0 and g_1 of G. Shnirelman (1987) showed that such a shortest path may not exist in G. He also showed that the completion of G for the geodesic distance is as large as the semi-group S of all measure preserving maps of D in the measure theory sense.

A generalized framework was introduced in Brenier (JAMS 1989) where the concept of generalized flow plays a crucial rôle. It is closely related to Young's measures in homogenization theory. We can show that all sequences

$(g_n(t); 0 \leq t \leq 1)$ of approximate shortest paths connecting g_0 and g_1 have their acceleration field $\ddot{g}_n \circ g_n^{-1}$ converging to the same acceleration field $-\nabla_x p(t,x)$. So, the pressure field turns out to be the relevant unknown of the shortes path problem.

J. BRIDGES :

Instability of Spatially Quasiperiodic Patterns

Motivated by recent results on the rigorous existence of spatially quasiperiodic solutions of elliptic partial differential equations, and documented experimental observations of spatially quasiperiodic patterns, particular questions about the geometry and linear stability of quasiperiodic patterns in two space dimensions are considered. A class of model partial differential equations, for which the time-independent part is a toral equivariant elliptic PDE, is considered. First, the concept of relative equilibrium, where a solution corresponds to flow along a group orbit, is generalized to the case of elliptic operators in two space dimensions. In this setting the quasiperiodic pattern corresponds to geometric tori. This geometry is then used to prove sufficient conditions for spectral linear instability, by projecting the linear stability problem onto the tangent space to the torus.

R. CAMASSA :

Weak Solutions of Completely Integrable PDE's

The spectral problem for the equation $u_t - u_{xxt} + 3uu_x - 2u_x u_{xx} - uu_{xxx} = 0$ with boundary conditions $u(x,t) \to 0$ as $|x| \to \infty$ is considered. In this case the spectrum is purely discrete and a countable infinity of eigenvalues exists. Thus, an infinite number of solitons can be expected to emerge from general initial data. These solitons are weak solutions of the PDE and have a finite jump discontinuity of the first derivative at their peak.An infinite number of cnservation laws are generated by traces (iterated kernels) of the integral equation form of the spectral problem. The motion of the discontinuity of the first derivative from any of these conservation laws is the same, unlike the case of hyperbolic equations like Burger's, and dictates that the location of the discontinuity moves at a speed equal to the value of u at that location.

P. CHOSSAT :

Symmetry Breaking Bifurcations of Convective Dynamos in a Spherical Shell

The so-called "convective dynamo problem" in a spherical shell has been extensively studied in the past 40 years because of its geophysical and astrophysical implications. However most of this work was aimed to show that relatively simple flows could indeed provide a dynamo effect, specially in the case of a fast rotationg domain. Our approach is different: We consider the non-rotating case and exploit the spherical symmetry of the system to apply the formalism of symmetry-breaking bifurcation theory. A numerical code has been written, which allows to input realistic convective steady velocities into the induction equation and then to solve the critical eigenvalue problem for magnetic perturbations. This code is set up in a way which permits an "easy" determination of the action of the symmetries of the basic flow on critical eigenmodes. From this, qualitative informations are obtained by application of the theory of symmetry-breaking bifurcations.

This method has been applied to the case when the radius ratio of the spherical shell is equal to 0.1, 0.3, and 0.4. The results are qualitatively very different in these three cases, but show a steady-state bifurcation of a magnetic field sustained by thermal convection in any case.

M. DELLNITZ :

The Computation of Invariant Sets and Invariant Measures

Complicated dynamical behavior can be described by two mathematical objects: an invariant set and a (natural) invariant measure with support on this set. Hence it is of interest to develop numerical algorithms which allow to approximate these two objects. In this talk we propose such algorithms. The invariant set is approximated by a subdivision algorithm, for which convergence (in Hausdorff distance) is guaranteed in a very general setting. Once the invariant set is known - at least approximately - we determine the dynamical behavior on that set. For this we compute a Markov chain describing the dynamics up to a certain accuracy. The stationary distribution of this process gives an approximation of he variant measure, and, under certain hypotheses, convergence to an SBR-measure can be obtained. The results and properties of the algorithms are illustrated by several examples and a video presentation.

P. DEUFLHARD :

Dynamic Dimension Reduction in Reaction Diffusion Systems (joint work with J. Heroth and U. Maas)

Dynamic process simulation requires the numerical solution of nonlinear diffusion reaction systems in terms of physical quantities T, ρ, p (temperature, density, pressure) and n chemical species. Complexity reduction techniques for this challenging class of problems include adaptivity in time (time step control via the adaptive Rothe method) and space (adaptive multi level FEM for the treatment of complex 3D boundary and interface conditions). Recently, splitting techniques into slow and fast components have revived: they are based on a real block-Schur decomposition. The talk focusses on both mathematically sound and computationally cheap monitoring criteria to determine the dimension $d < n$ of slow components and the possible regeneration of the splitting. These criteria have first been worked out in the context of the ODE reaction models and are now being transferred into the PDE enviroment: first, into a large method of lines software package, and second, into a full adaptive Rothe method for the coupled PDE system.

F. DIAS :

Multi-Packets of Capillary-Gravity Waves

It is now well-known that symmetric solitary waves in the form of single wave packets bifurcate at the minimum of the phase speed of capillary waves. This problem has been studied by methods of dynamical systems theory (center manifold reduction, normal form theory). The normal form near the minimum of the phase speed (1:1 resonance) admits a one-parameter family of homoclinic orbits. But only the two symmetric ones persist. In this talk, we provide evidence from asymptotics and from numerics that multi-packets exist. They appear at a finite value of the bifurcation parameter (phase velocity minus minimum phase velocity). The asymptotic analysis is carried beyond all orders (Yang and Akylas), and two-packet solitary waves are constructed. There is an infinity of such two-packet solitary waves (symmetric and asymmetric). Numerical computations are performed on the full Euler equations with free surface, and multi-packets are obtained.

A. DOELMAN :

Singularly Perturbed and Non-Local Modulation Equations for Systems with Interacting Instability Mechanisms (joint work with V. Rottschäfer)

In this talk, two systems of coupled modulation equations are studied and compared. The modulation equations are derived for a certain class of basic systems which are subject to two distinct, interacting, destabilizing mechanisms. We assume that the ratio of the widths of the neutral parabolas of the two (weakly) unstable modes is small - as for instance is the case in double layer convection - and use this ratio as a second bifurcation parameter. Based on these assumptions we derive a singularly perturbed modulation equation and a, simpler, modulation equation with a non-local term. This non-local equation can be interpreted as a limit of the singularly perturbed one. We study the behaviour of the stationary solutions of both systems and compare them. It is found that stationary periodic solutions of the non-local system exist under the same conditions as stationary periodic solutions for the singularly perturbed system. Moreover, these solutions can be interpreted as representing the same quasi-periodic patterns in the underlying basic system. However, a large variety of heteroclinic and homoclinic connections are found for the singularly perturbed system. These solutions correspond to patterns in the underlying system which approach (stable) periodic solutions at $\pm\infty$. It is shown that these solutions do not have a counterpart in the non-local system.

J.-P. ECKMANN :

Nonlinear Stability of Periodic Solutions for the Swift-Hohenberg Equation (joint work with C. E. Wayne)

Based on a paper by G. Schneider (Commun. Math. Phys., in print), we show that the stationary solutions $u_\varepsilon(x) \approx \varepsilon 2\cos(x)/\sqrt{3}$ of the equation

$$\partial_t u = (\varepsilon^2 - (1+\partial_{xx})^2)u - u^3$$

are stable for perturbations in $H^r_{2,q}$, when q and r are sufficiently large. Our methods comprise a new center-manifold reduction of the problem, after it has been rescaled from x, t to $\xi = x/t^{\frac{1}{2}}, \tau = \log t$. These methods show that if $u(t=0) = u_\varepsilon + v$, then $u(t) \approx u_\varepsilon + const.\, t^{-\frac{1}{2}} exp(-const.\, x^2/t) + o(t^{-\frac{1}{2}})$.

B. FIEDLER :

Coalescence of Reversible Homoclinics Causes Elliptic Resonances (joint work with D. Turaev)

Time reversible flows can possess robust reversible homoclinic orbits to a saddle equilibrium. They are limits of reversible periodic orbits, at infinite period. We consider coalescence of two such homoclinics in a one-parameter family of reversible flows. We show how intersection topology then produces a family of elliptic reversible periodics, i.e. with complex Floquet multipliers on the unit circle, even if all eigenvalues at the saddle equilibrium are real.

S. FRIEDLANDER :

Linear and Nonlinear Instability for the Euler Equations (joint work with M. Vishik)

We present a sufficient condition for linear instability of a smooth steady state solution of the Euler equations. This instability criterion is given by a Lyapunov exponent type quantity which provides a lower bound for the growth rate of the linearized Euler operator. The condition is effective: For example, it can be used to prove linear instability of any steady Euler flow with exponential stretching, even along one Lagrangian trajectory.

Also, we discuss an abstract theorem which states, under certain conditions, that spectral instability implies nonlinear instability. Concerning solutions of the Euler equations, this theorem applies to shear flow with a sinusoidal profile. It then follows that this Euler equlibrium is nonlinearly unstable.

Th. GALLEY :

Stability of Travelling Waves for a Damped Hyperbolic Equation (joint work with G. Raugel)

We consider a non-linear damped hyperbolic equation on the real line depending on a positive parameter ε : $\varepsilon u_{tt} + u_t - u_{xx} = u - u^2$. For all $\varepsilon > 0$, this equation has a one-parameter family of uniformly translating travelling waves (TW) indexed by the speed $v \in [2/\sqrt{1+4\varepsilon}, 1/\sqrt{\varepsilon})$. We show that these TW's are stable with respect to sufficiently small, sufficiently decaying perturbations. Moreover, if $\varepsilon \ll 1$, we show that the TW's are stable with respect to large perturbations satisfying some positivity conditions. In both cases, we obtain an estimate of the decay rate of the perturbations as $t \to +\infty$. These results are obtained using standard energy estimates combined with the maximum principle for hyperbolic equations.

M. GOLUBITSKY :

Meandering of the Spiral Tip: An Alternative Approach

Meandering of an one-armed spiral tip has been noted in chemical reactions and numerical simulations. Barkley, Kness and Tuckerman show that meandering can begin by Hopf bifurcation from a rigidly rotating spiral wave (a point that is verified in a B-Z reaction by Li, Ouyang, Petrov and Swinney). At the codimension two point where (in an appropriate sense) the frequency at a Hopf bifurcation equals the frequency of the spiral wave, Barkley notes that spiral tip meandering can turn to linearly translating spiral tip motion.

Barkley also presents a model showing that the linear motion of the spiral tip is a resonance phenomenon, and this point is proved rigorously by Fiedler and Wulff. In this paper we give an alternative formal development of Barkley's model extending the center bundle construction of Krupa from compact groups to Euclidean groups and from finite dimensions to function spaces. This approach allows to consider in one context various bifurcations from a rotating wave. In particular, we can analyze in a straightforward manner the codimension two Barkley bifurcation and the codimension two Takens-Bogdanov bifurcation from a rotating wave. We also discuss Hopf bifurcation from a many armed spiral showing that meandering and resonant linear motion of the spiral tip do not always occur. And when meandering does occur, Hopf bifurcation from a many armed spiral can lead to complicated multifrequency motion. Note, however, that we have not rigorously proved the extension of Krupa's center bundle results to PDE-systems.

F. GUYARD :

Forced Symmetry Breaking for Periodic Orbits

The modelling of physical phenomena often leads to take into account symmetries which are not present in the phenomena but in an "ideal" model of it. In order to understand the passage from the model to the physical phenomena in the context of bifurcation theory, we study the following problem:

Let (1) : $\dot{z} = f(z)$, be a G-equivariant system of ODE's with G a compact Lie group and let (2) : $\dot{z} = f(z) + \varepsilon h(z)$, be an L-equivariant perturbation of (1) with L a subgroup of G. How can we relate the bifurcation diagram of (1) to the one of (2)? A general framework to deal with this problem has been developed by R. Lauterbach and M. Roberts and applied to the study of symmetry breaking perturbations of relative equilibria. We use the same framework to study symmetry breaking perturbations of group orbits of periodic orbits. We show how to determine the periodic orbits (as well as their symmetry) which are forced by the geometry of the problem to persist to the perturbation.

D.D. HOLM :

Hamilton's Principle, Asymptotics and an Integrable Shallow Water Equation

We rederive the integrable one-dimensional shallow water equation of Camassa and Holm [PRL 71(1993), 1661] by using asymptotics in Hamilton's principle for Euler's equations of incompressible stratified fluid flow in the Boussinesq approximation. This derivation clarifies the equivariance of the "unidirectionalization" hypothesis in Camassa and Holm's original derivation.

Ph. HOLMES :

Low Dimensional Models of the Turbulent Boundary Layer

For turbulent flow one has a well-accepted mathematical model: the Navier-Stokes equations. Why, then, is the "problem of turbulence" so intractible? The difficulty is, of course, that the equations appear insoluble in any reasonable sense. (A direct numerical simulation certainly provides a "solution", but it provides little understanding of the process per se.) However, three recent developments offer some hope. (1) The discovery, by experimental fluid mechanics, of coherent structures in certain fully developed turbulent flows; (2) the suggestion, by Ruelle, Takens and others, that strange attractors and other ideas from dynamical systems theory might play a rôle in the analysis of the governing equations, and (3) the introduction of the statistical technique of Karhunen-Loeve or proper orthogonal decomposition (by Lumley in the case of turbulence).

Drawing on work on low dimensional models for the dynamics of coherent structures in turbulent flows done over the past ten years, I will describe how these three threads can be drawn together to weave low dimensional models which yield new understanding of turbulence generation. Most of this work is non-rigorous and I will emphasise open mathematical questions and problems.

J. HUNTER :

Singularity Formation and Integrability in a Nonlinear Wave Equation

Weakly nonlinear solutions of the variational nonlinear wave equation (1) : $u_{tt} = c^2(u)u_{xx} + c(u)c'(u)u_x^2$ are described by the following asymptotic equation (2) : $(u_t + uu_x)_x = \frac{1}{2}u_x^2$. Equation (2) is completely integrable and also arises as the high-frequency limit of the integrable Camassa-Holm equation in water waves. Smooth solutions of (2) break down in finite time since $u_x \to -\infty$. Equation (2) has global weak solutions, including conservative solutions, which have constant energy, and dissipative solutions, which loose energy at a maximal rate. The integrability structure of (2) remains valid for conservative weak solutions even after their derivative blows up.

The wave equation (1) does not have global smooth solutions, in general, even for small data. Global existence of weak solutions remains an open question.

G. IOOSS :

Solitary Waves on a Free Surface in the Presence of Surface Tension and with an Infinite Depth Layer

We consider a 2-dimensional potential flow with surface tension at the free surface. We show that the two symmetric solitary waves, with damped oscillations at infinity, still exist in the infinite depth case. However, the exponential decay of oscillations for the finite depth case is replaced here by polynomial decay. To prove this, we first show that the spectrum of the linear part has 4 eigenvalues close to the imaginary axis, while the remaining part of the spectrum is the full real line (but with no eigenvalue). Splitting the space into a 4-dimensional part and an infinite dimensional "hyperbolic" part, we use Fourier transform for solving the infinite dimensional part and come back to the 4-dimensional problem as in the finite depth case. Polynomial decay occurs while we solve the infinite part.

Reference: G. Iooss and P. Kirrmann. Capillary gravity waves on the free surface of an inviscid fluid of infinite depth. Existence of solitary waves. To appear in Arch. Rat. Mech. Anal., 1996.

E. KNOBLOCH :

Mode Interaction in Large Aspect Ratio Convection

Mode interaction between odd and even modes in two-dimensional Boussinesq convection in a box is revisited. It is noted that in a large aspect ratio limit the structure of the amplitude equations depends on the boundary conditions applied at the sidewalls, however distant. With no-slip sidewall boundary conditions the equations approach those for an unbounded layer with periodic boundary conditions; this is not the case for free-slip boundary conditions. Thus only in the former case can the large aspect ratio system be considered a small perturbation of the unbounded system. The reasons for the different large aspect ratio limits are traced to the presence of "hidden" symmetries in the stress-free case. Homotopic continuation is used to extend these results to other types of boundary conditions.

H.-P. KRUSE :

On the Bifurcation and Stability of Rigidly Rotationg Inviscid Liquid Bridges (joint work with J. Scheurle)

We consider a model describing the motion of a drop of an ideal incompressible fluid which is trapped between two parallel plates. The drop moves under the influence of surface tension at the free surface and adhesion forces along the contact surfaces between the drop and the two plates. We present a Hamiltonian structure for this model. This Hamiltonian structure and the symmetry group of the system is used to study the stability of rigidly rotating drops. Explicit stability criteria are given for rigidly rotating cylindrical drops, which are solutions to the equations of motion for arbitrary angular velocities in the absence of adhesion effects. We study bifurcations from this family of solutions using the angular velocity and also the angular momentum as a bifurcation parameter.

R. LAUTERBACH :

Mode Coupling and Forced Symmetry Breaking

Heteroclinic cycles can serve as an explanation for nonperiodic behaviour in certain systems. A prominent example is the occurence of reversals in the earth magnetic field. Creation of such cycles can occur in dynamical systems with symmetry through several scenarios. One possibility is the occurence of invariant planes containing invariant lines with equilibria. Another way to get heteroclinic cycles is forced symmetry breaking, i.e. explicit perturbation of an equivariant system by terms with less symmetry.

We investigate the $\ell = 1, \ell = 2$ mode coupling in problems with sperical symmetry. It is known that this system can contain a heteroclinic cycle which is created by the first scenario. If the underlying sphere rotates we have an example of forced symmetry breaking leading to new dynamical behaviour. We investigate these dynamics using methods from invariant theory.

E. LOMBARDI :

Non-Persistence of Homoclinic Connections for Reversible Perturbed Integrable Systems

We are interested in the flow of an inviscid fluid layer under the influence of gravity and low surface tension (Froude number, F, close to 1, Bond number $b < \frac{1}{3}$). In this case, the following bifurcation of the spectrum of the differential occurs near the imaginary axis: "oscillatory"dynamics of order 1, induced by two simple opposite eigenvalues lying on the imaginary axis, is superposed on slow dynamics induced by a pair of eigenvalues moving from the hyperbolic case ($\pm\sqrt{|F-1|}$) to the oscillatory case ($\pm i\sqrt{|F-1|}$). The existence of reversible homoclinic connections to exponentially small periodic orbits is known, but the existence of homoclinic connections to 0 is still an open problem. In this lecture, we prove that vector fields obtained by perturbation of the normal form system of order 2 by small analytic perturbations, do not admit any homoclinic connection to 0, although the normal form system at any order does admit one.

A. MAHALOV :

Global Splitting, Integrability and Regularity of 3D Euler and Navier-Stokes Equations for Uniformly Rotating Fluids (joint work with A. Babin and B. Nicolaenko)

We consider 3D Euler and Navier-Stokes equations describing dynamics of uniformly rotating fluids. Periodic boundary conditions are imposed, the ratio of domain periods is assumed to be generic (non-resonant). We show that the solutions can be decomposed as $U(t,x_1,x_3,x_3) = \overline{U}(t,x_1,x_2)+V(t,x_1,x_2,x_3)+r$ where $\overline{U}$ is a solution of the 2D Euler/Navier-Stokes system with vertically averaged initial data (axis of rotation is taken along the vertical unit vector e_3). Here r is a remainder of order $Ro_a^{\frac{1}{2}}$ where Ro_a is the anisotropic Rossby number. The vector field $V(t,x_1,x_2,x_3)$ is exactly solved in terms of 2D dynamics of vertically averaged fields. We show that 3D rotating turbulence decouples into phase turbulence for $V(t,x_1,x_2,x_3)$ and 2D turbulence for vertically averaged fields $\overline{U}(t,x_1,x_2)$ if Ro_a is small. The mathematically rigorous control of the error r is used to prove existence on a long time interval T^* of regular solutions to 3D Euler equations ($T^* \to \infty$, as $Ro_a \to 0$) and global existence of regular solutions for 3D Navier-Stokes equations in the small anisotropic Rossby number case.

A. MIELKE :

Sideband Instability in Dissipative and Convective Systems

We present a simple method for proving instability of periodic patterns under nonperiodic perturbations. The method is called Principle of Reduced Instability and was developed jointly with Tom Bridges. It consists in two local reduction steps (either Lyapunov-Schmidt or center manifold reduction): The first treats the nonlinear steady system in order to show the bifurcation of the periodic pattern while the second reduction is applied to the spectral problem for linearization around the given periodic pattern. In the latter case, the spectral parameter and the sideband vector are considered as small parameters.

We consider the Rayleigh-Bénard convection problem and establish in a mathematical rigorous way the Eckhaus, zigzag, and skew varicose instability which where obtained earlier by formal approaches involving multiple scalings. Another application studies the instability of Stokes waves which were found in 1967 by Benjamin and Feir. We provide a spatial Hamiltonian formulation and use symplectic center manifold theory.

G. MISSIOLEK :

Conjugate Points in $D_\mu(T^2)$

The problem of existence of conjugate points in the group of volume-preserving diffeomorphisms of a flat torus ($D_\mu(T^2)$) was suggested by V.I. Arnold. An example of a geodesic in $D_\mu(T^2)$ containing conjugate points is presented in this talk.

R.L. PEGO :

Encapsulated Vortex Solutions to Nonlinear Schrödinger Equations
(joint work with H. Warchall)

Nonlinear Schrödinger equations and nonlinear Klein-Gordon equations in 2+1 dimensions admit nonradial standing wave solutions in the separated form $e^{i\nu t+im\theta}w(r)$. Neu found that the effective interaction of such structures resemble the interactions of point vortices in fluids. For two types of nonlinear terms, Iaia, Warchall and Weissler have shown there exist exponentially localized solutions of the form above, which we term encapsulated vortices. Solutions exist for any value of the "winding number" m, with any number of positive nodes $n \geq 0$.

We study the linearized stability of these waves, counting unstable eigenvalues by evaluating Evans functions numerically. We find a new class of apparently stable, nonradial, localized solutions, which occur for the cubic-quintic nonlinearity and others with similar shape. Stable solutions are found for any m, in a regime where the wave structure has a vortex core, "encapsulated" by a weakly curved membrane which localizes the wave.

J. PRÜSS :

Mathematical Modelling of Chemical Two-Phase Reactors

A model for the dynamical behavior of a two-phase chemical reactor with a single irreversible reaction and mass transfer is presented. The model consists of three parts. The macroscopic part contains the given hydrodynamics due to convection and dispersion, reaction in bulk, and the mass transfer rates. To obtain the latter, a microscopic submodel is formulated which consists of reaction and diffusion in a two-phase interface boundary layer element with thermodynamical equilibrium at the interface. From the solution of the microscopic part, local mass transfer rates follow which in the macroscopic part of the model are averaged against an assumed age distribution induced by turbulence.

Qualitative and quantitative mathematical features of this model are then discussed. In particular, the fast reaction limit, i.e. instantaneous reaction, is considered in detail. Its dynamical behavior is shown to be qualitatively the same as that for finite reaction speed. Also, the reaction plane in the microscopic model is examined.

G. RAUGEL :

Euler Equations on Thin Domains (joint work with J.E. Marsden and T. Ratiu)

We study the Euler equations in thin domains Q_ε in $\mathbb{R}^3$ with certain boundary conditions. The model thin domains in this talk are a thin cylinder $Q_\varepsilon \equiv \Omega \times]o, \varepsilon[$ where Ω is a rectangle, and a spherical shell of thickness ε. The limiting equations, as $\varepsilon \to 0$, are the Euler equations on Ω (resp. on the sphere S^2) coupled with some transport equations ("direction field " in the language of elasticity). After rescaling the domain Q_ε to the reference domain $Q = \Omega \times]0, 1[$ (resp. $Q = (r, \theta, \phi)|1 < r < 2, -\pi < \theta < \pi, 0 \leq \phi < 2\pi$) and defining the corresponding Sobolev spaces $H^s_\varepsilon(Q), s \geq 0$, we show the following properties: Given a positive constant K, there exists $\varepsilon_0 \equiv \varepsilon_0(K) > 0$ and, for $0 < \varepsilon \leq \varepsilon_0$, a time $T_\varepsilon \equiv T_\varepsilon(K) > 0$ such that, if $\|u_0\|_{(H^3_\varepsilon(Q))^3} \leq K$, then the rescaled Euler equations have a unique classical solution $(u(t), p(t)) \in (H^3_\varepsilon(Q))^3 \times (H^4(Q)/\mathbb{R})$ for the velocity and the pressure field in the time interval $[0, T_\varepsilon]$, where $u(0) = u_0$ and $\|u(t)\|_{(H^3_\varepsilon(Q))^3} \leq K\varepsilon^{-\alpha}$ with $0 < \alpha < 1$. The time T_ε tends to $+\infty$ as $\varepsilon \to 0$.

In addition, we can compare u with the solution v of the limiting equations with initial data equal to the average of u_0 in the thin direction of the domain. For example, for $0 \leq t \leq T_\varepsilon$ we have $\|u(t) - v(t)\|_{(L^2_\varepsilon(Q))^3} \leq C\varepsilon^\beta$, where C and $\beta \leq 1$ are positive constants. Similar estimates are true in $H^s_\varepsilon(Q), s = 1, 2$.

M. RENARDY :

Hopf Bifurcation with Small Frequency on the Hexagonal Lattice

In two-layer convection, there is a possibility of oscillatory onset when the Rayleigh numbers of the two layers are nearly equal. This oscillatory bifurcation arises from a near-crossover of real eigenvalues which form a complex pair only in a small region of parameter space. Hence the problem should properly be analysed as a perturbation of a double zero eigenvalue. The characterization of periodic orbits then leads to a very complicated problem, where Hilbert's 16th problem arises as an "easy" special case. A more manageable problem consists in finding solutions which are either steady or traveling waves. With the assumption of periodicity of the hexagonal lattice, such solutions are classified into symmetry types, and the algebraic systems of equations arising for various types of solutions are discussed.

Y.Y. RENARDY :

Topics in Double-Layer Convection

Two different immiscible liquids lie in layers at rest between horizontal walls and are heated from below. When the temperature difference between the walls reaches a critical value, new solutions bifurcate from the solution at rest. In the first part of this paper, we focus on instabilities that involve interfacial deformations and report on a particular critical situation with a pair of oscillatory modes at wave number α and a steady mode at wave number 2α (Fujimura and Renardy, Physica D 1995). In the second part, the focus is on the case where the interfacial mode is strongly stabilized by surface tension and a suitable density stratification. A mechanism for a Hopf bifurcation is the competition between the least stable of the bulk modes in each fluid. The well known criterion for balancing the effective Rayleigh numbers in both fluids is augmented with a criterion for non-selfadjointness of the system, yielding a heuristic method for picking suitable fluids when Hopf modes are desired. The pattern formation problem in three dimensions is addressed for the case of doubly periodic solutions on a hexagonal lattice. Of the solutions with maximal symmetry, the traveling rolls are found to be stable. This is in contrast with the results of the qualitatively different mechanism of interfacial instability (ZAMP 1996).

D. SATTINGER :

A Riemann-Hilbert Problem for an Energy Dependent Schrödinger Operator

We consider an inverse scattering problem for the Schrödinger Operator$D^2 + E^2 + (ikp(x) + q(x))$ where $E^2 = k^2 + 1$. The corresponding isospectral flows can be written, after a transformation,

$$k_t + (w + \frac{k^2}{2})_x = 0,\ w_t \pm u_{xxx} + (uw)_x = 0$$

where + and − are related to p real or p imaginary, and $p = u$, or iu, $q = \pm(\frac{u^2}{16} - \frac{w}{4})$. The inverse scattering problem is posed as a Riemann-Hilbert problem on the Riemann surface $E^2 = k^2 + 1$. We use the uniforinizing transformation

$$E = \frac{1}{2}(z + \frac{1}{z}),\quad k = \frac{1}{2}(z - \frac{1}{z}).$$

Global existence theorems are proved in the case + (p real) and − (p imaginary) under complementary conditions on the initial data, stated in terms of the scattering data.

G. SCHNEIDER :

Approximation of the Korteweg-de Vries Equation by the Nonlinear Schrödinger Equation

In 1968 Zakharov derived the Nonlinear Schrödinger equation as an approximation equation for the water wave problem. It is still an open problem if there are really solutions of the water wave problem which can be approximated by the solutions of the Nonlinear Schrödinger equation. In answering this question we made some progress for a model problem, the approximation of the Korteweg-de Vries equation by the Nonlinear Schrödinger equation.

H. SEGUR :

Waves in Shallow Water

The equation of Kadomtsev and Petviashili (1970), $(u_t + 6uu_x + u_{xxx})_x + 3u_{yy} = 0$, describes approximately the evolution of long, nearly one-dimensional waves of moderate amplitude as they propagate in shallow water of uniform depth. The equation admits exact solutions in the form $u(x,y,t) = 2\partial_x^2[\ln\theta_N]$, where θ_N is a Riemann theta function with N independent phase variables. $N = 1$ corresponds to the cnoidal wave first described by Kortewey and de Vries (1895). $N = 2$ provides an 8-parameter family of solutions that are spatially periodic in the x-y plane, and that propagate as waves of permanent form. $N = 3$ provides the simplest genuinely 2-dimensional solutions with nontrivial time-dependence.

Corresponding to the ($N = 2$) solutions of the KP equation, we present experimental evidence of spatially periodic, finite-amplitude waves that propagate as waves of nearly permanent form, in shallow water of uniform depth. The waves appear to be stable in the experiments, even to perturbations that are not small. The KP equation describes these waves with reasonable accuracy. The experiments include both laboratory experiments and field observations.

A. SHNIRELMAN :

On the Non-Uniqueness of Weak Solutions of the Incompressible Euler Equations

A vector function $u(x,t) \in L^2(\mathbb{R}^n \times \mathbb{R}, \mathbb{R}^n)$ is called a weak solution of the incompressible Euler equations, if for every pair of test functions $v(x,t) \in C_0^\infty(\mathbb{R}^n \times \mathbb{R}, \mathbb{R}^n), \varphi \in C_0^\infty(\mathbb{R}^n \times \mathbb{R}, \mathbb{R})$, s.t. $\nabla \cdot v = 0$, the following integral relations hold:

$$\iint \left[(u, \frac{\partial v}{\partial t}) + (u \otimes u, \nabla \cdot v) \right] dx dt = 0 \; ; \quad \iint (u, \nabla \varphi) dx dt = 0.$$

In 1993, V. Scheffer published an example of a weak solution $u(x,t) \in L^2(\mathbb{R}^2 \times \mathbb{R})$, s.t. $u(x,t) \equiv 0$ for $|x|^2 + |t|^2 > 1$. In the talk we explain a much simpler construction, disclosing the nature of this strange phenomenon. It turns out that the main rôle is played by the "inverse energy cascade", a very familiar thing in the 2-dimensional hydrodynamics.

M.I. WEINSTEIN :

Resonances, Radiation Damping and Instability in Hamiltonian Nonlinear Wave Equations (joint work with A. Soffer)

We consider a class of nonlinear Klein-Gordon equations which are Hamiltonian and are perturbations of linear dispersive equations. The unperturbed dynamical system has a bound state, a spatially localized and time-periodic solution. We show that, for generic nonlinear Hamiltonian perturbations, all small amplitude solutions decay to zero as time tends to infinity at anomalously slow rate. In particular, spatially localized and time-periodic solutions of the linear problem are destroyed by generic nonlinear Hamiltonian perturbations via slow radiation of energy to infinity. The main mechanism is a nonlinear resonant interaction of bound states (eigenfunctions)and radiation (continuous spectral modes), leading to energy transfer from the discrete spectrum to continuous modes. A hypothesis ensuring that such a resonance takes place is a nonlinear analogue of the Fermi Golden Rule, arising in the theory of resonances in quantum mechanics. The techniques used involve: (i) a time-dependent method developed by the authors for the treatment of the quantum resonance problem and perturbations of embedded eigenvalues, (ii) a generalization of the Hamiltonian normal form which is appropriate for dispersive infinite dimensional systems and (iii) ideas from scattering theory. The arguments are quite general and we expect them to apply to a large class of systems which can be viewed as the interaction of finite dimensional and infinite dimensional dispersive dynamical systems.

F. VERHULST:

Periodic Solutions and Invariant Tori in Normal Forms of Nonlinear Wave Equations

Considering wave equations of the form

$$\frac{\partial^2 \phi}{\partial t^2} + L\phi = \varepsilon N(\phi) \quad \& \quad \text{initial and boundary conditions}$$

with L a linear, elliptic, selfadjoint operator one can apply averaging and Galerkin projection followed by averaging / normalization. We review work by Krol, van der Au, Buitelaar and Pals. In particular we are interested in problems where Kuksin's extension of the KAM theorem to PDE's meets obstructions. We illustrate this for the equation $\phi_{tt} - \phi_{xx} = \varepsilon\phi^3$ which has a fully resonant spectrum and for the equation $\phi_{tt} - \phi_{xx} - \alpha\phi_{yy} + \beta\phi = \varepsilon\phi^3$ on a square. In the second case there are groups of resonances and this is a good candidate for extension of KAM theory.

V.I. YUDOVICH :

Cosymmetry and Bifurcations

Cosymmetry of a vector field F on a manifold is, by definition, a differential 1-form L which nullifies the given vector field at each point x : $(Fx, Lx) = 0$. More generally, cosymmetry of a given section to a vector bundle is a section of the dual vector bundle which is orthogonal to the given one at each point. The following topics will be considered:

1. Examples of cosymmetric dynamical systems: filtrational fluid convection; mechanical systems with cosymmetric potential energy and others. Trivial and nontrivial cosymmetries.
2. Cosymmetric version of implicit function theorem and existence of submanifolds of equilibria.
3. Poincare-Andronov-Hopf bifurcation in dynamical system with cosymmetry. Delay of branching off of the limit cycle.
4. Bifurcations connected with cosymmetry-breaking perturbations. Collapse of equilibrium cycle into a finite set of equilibria and creating of slow periodic motion.

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