MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Aspects of Computational Fluid Dynamics

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The conference was organized by C. Johnson (Gothenburg), K.W. Morton (Oxford), and R. Rannacher (Heidelberg). The 30 participants came from 8 countries and represented research groups working in numerical mathematics and scientific computing as well as computational mechanics. The central theme of the 23 lectures was the theoretical background of numerical methods in fluid mechanics.

The majority of the lectures concerned new developments in finite element and finite volume methods for the Euler and Navier–Stokes equations. A topic of central interest was a posteriori mesh–size selection and error control in view of the particular needs in flow computations. Closely related to this were fast solution methods based on multigrid and other preconditioning techniques. Several new ideas were presented for robust discretizations which can handle multi–scale phenomena, transport behavior as well as various kinds of limiting processes, e.g., low Mach–number flows. A smaller group of lectures reported on progress in non–standard approaches like characteristic-Galerkin methods, sparse grid techniques, particle methods, and stochastic models. The topic of turbulence was addressed in the context of subgrid-scale models and sparse grid techniques. Besides the standard problems in classical CFD, also numerical methods for more complex applications like chemically reacting flows and crystal growth were presented. Finally, an online computer demonstration (by K. Kuwahara) showed the power of modern high–performance workstations for flow computation and visualization.

The lectures and the accompanying discussions represented the current state of theory in CFD. It became clear that particularly the growing input of mathematical analysis is leading to a major impuls which has still to be fully exploited. Most promising developments were seen under the key–words *adaptive methods*, *hierarchical models and discretizations*, and *fast solvers*. These issues inherit many challenging theoretical questions and are developing into major contributions of modern mathematics to real–world technology. Due to the limited number of talks, much time was left for group work and discussions. Particularly the expository morning lectures initiated intensive and sometimes even controversial discussions which were generally perceived as very fruitful. It was agreed that the subject of this conference is of growing practical importance and should be further pursued in the future within the mathematical community.

E. BÄNSCH

Simulation of Crystal Growth with Thermal Convection

We present finite element methods for the numerical simulation of dendritic crystal growth including convection effects. The problem is modelled by the Stefan problem with Gibbs–Thomson condition coupled with the Navier–Stokes equations in the liquid phase. Thus, the following system of equations has to be solved: Navier–Stokes equations,

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} - \frac{1}{\sqrt{Gr}} \Delta \vec{u} + \nabla p = \theta \vec{e}_2 , \quad \nabla \cdot \vec{u} = 0 \quad \text{in } \Omega_{liquid} ,$$

energy equation,

$$\frac{\partial \theta}{\partial t} + C_{conv} \, \vec{u} \cdot \nabla \theta - \frac{1}{Pr\sqrt{Gr}} \Delta \theta = 0 \quad \text{in } \Omega_{liquid}$$

$$\frac{\partial \theta}{\partial t} - \frac{1}{Pr\sqrt{Gr}} \Delta \theta = 0 \quad \text{in } \Omega_{solid} ,$$

Stefan condition $-LV_\Gamma = [cD\partial_\nu \theta]$ on Γ, and Gibbs–Thomson law $\epsilon_V V_\Gamma + \epsilon_C C_\Gamma + \theta = 0$ on Γ, together with initial and boundary conditions.

Here, $\vec{u}$ is the flow field, θ the temperature, p the pressure, V_Γ the velocity of the free boundary, and C_Γ the curvature of Γ. The numerical method consists of a parametric finite element method for the evolution of the interface, coupled with finite element Navier–Stokes and heat equation solvers.

R. BECKER

Weighted Residual-Based Error Estimation in Flow Computations

In many engineering applications of CFD the objective of the computation is the precise prediction of some local quantity such as the drag- or lift-coefficient of an object submerged in a gas or liquid flow. We present an approach which allows to computationally quantify the local errors in finite element computations. On this basis we derive an adaptive algorithm which leads to locally refined meshes which are adapted to the physical question under consideration. Our general approach is illustrated by a two-dimensional benchmark problem. It is the flow around a cylinder of an incompressible fluid. The quantities to be computed are the drag- and lift-coefficient as well as the pressure difference on the cylinder. In order to design a mesh which is optimal (in some appropriate sense) for the given problem, it is clearly not efficient to control the pointwise error over the whole domain. On the other hand, it is not sufficient to refine the grid near the cylinder. The important point is to measure the effect of the discretization error on the accuracy of the desired quantity for each cell. This sensitivity analysis is achieved via an appropriate dual problem which is solved together with the primal one.

M. FEISTAUER

Numerical Techniques for the Solution of Viscous Compressible Flow: Analysis and Application

The lecture is concerned with the theory and numerical simulation of viscous compressible fluid flow problems. The numerical simulation of viscous compressible flow is one of the most difficult areas of Computational Fluid Dynamics because of several obstacles: mixed hyperbolic-parabolic character of governing equations, convection effects dominating over diffusion, existence of boundary layers and (in high speed flow) of shock waves, the interaction of boundary layers and shock waves. In addition, there is a significant lack of theory for the continuous problem. Since the viscosity and heat conductivity of gases are small, the viscous dissipative terms are often considered as perturbations of the inviscid hyperbolic Euler system. This leads to the idea of discretiszing the inviscid terms independently by an adequate technique based, e.g., on a flux vector splitting finite volume approach, whereas the parabolic character of viscous terms suggests their discretization by the finite element method. As a result we obtain a combined finite volume – finite element method (FVM – FEM) which can be applied on general unstructured meshes. Special attention is paid to the increase of accuracy of the method with the aid of higher order recovery (similar to ENO) and automatic adaptive mesh refinement based on suitable shock and error indicators. The applicability and robustness of the scheme is demonstrated by the numerical solution of technically relevant flow problems with complicated structure of shock waves, boundary layers and wakes.

PH. GRESHO

Issues and Problems Related to Rapid and Impulsive Start-Up of Incompressible Flow Past a Circular Cylinder

A penetrating numerical analysis of problems and solutions related to a mixed finite element application of early time behavior starting from rest via an inlet boundary condition of $U = a(1 - exp(-\lambda t))$ for large λ at $RE = 1000$ (also $RE = 0$) is described. Among the "issues and problems" are: (1) A truly impulsive start is mathematically impossible (the problem is ill-posed); (2) The 'next best thing' is very FAR from a fluid at rest: potential flow except at the cylinder, where the no-slip boundary condition is applied, thus generating a vortex sheet(which we also discuss and TRY to compute); (3) finite elements that are 'stable' for viscous flows (LBB, etc.) are shown to be at least somewhat 'unstable' for inviscid and nearly -inviscid flows (they fail to pass the first Brezzi stability condition), but that convergence will still occur; (4) the pressure near the cylinder, and thus the drag force, is non-convergent (P goes like $1/h$ as $h- > 0$, where h is the distance from the cylinder to the first node in the fluid) for small time (whose explanation we shall provide) ; (5) but beyond a mesh-dependent time which we call the 'Minimum Time of Believability, Viz. $h^2/4\nu$, where ν is the viscosity, convergence can and does occur. This behaviour is independent of the element chosen and must also occur for finite difference, finite volume, and even the (exalted) spectral methods. This failure to converge at small time is a direct consequence of the inability of ANY mesh to describe the viscous effects caused by the vortex sheet. We conclude by summarizing the sad state of affairs with respect to element 'stability' when an explicit time-marching method is employed, and implore the numerical analysis community to address both this issue and that mentioned above (inviscid or near-inviscid flows).

F.-K. HEBEKER

On Finite Elements, Error Control, and Adaptivity for Reactive Flows

In a joint work with R. Rannacher and Ch. Führer (Heidelberg) we investigate new numerical methods for systems of convection-dominated flows with stiff source terms. They are based on a conservative second-order finite element method enhanced with efficient shock-capturing features (artificial viscosity, streamline diffusion, and residual-based shock-capturing). This method provides for a unified approach to such kind of problems and in particular avoids Riemann solvers and operator splitting approximations. The 'Discontinuous-Galerkin' variant DG(1) is capable of an L-stable implicit treatment of the stiff source and diffusion terms and, moreover, paves the way to rigorous error control. The traditional way of residual-based a posteriori error estimation uses global stability constants which (if available at all) quite often greatly overestimate the actual discretization error for advanced nonlinear problems. Therefore, we employ a posteriori estimates that use local weights rather than global constants. As a test example (R. LeVeque), we compute detonation waves with moderate thickness of the reaction zone and low-order diffusion terms. Our method is able to accurately compute the Chapman-Jouguet velocity of detonations as well as the sharp ZND-profiles (Zel'dovich-VonNeumann-Döring) of pressure, fuel concentration, etc..

T.J.R. HUGHES

The Variational Multiscale Method – A Paradigm for Subgrid-Scale Modeling and Computational Fluid Dynamics

Historically, the Galerkin finite element method has been the fundamental and ubiquitous constructive element in the development of numerical methods for mechanics. Despite this, it is argued that this method represents an inadequate paradigm for many practically important problems, in particular, those involving fine scale features that are numerically unresolvable due to the length scale of elements composing the mesh. It is observed that even if one is uninterested in resolving, or "seeing", the fine scale features, their effect on the coarse, or resolvable, scales must be accounted for in order to accurately calculate the coarse scales. The variational multiscale method represents a new and more robust paradigm. It consists of a two-step approach: The first is purely non-numerical – the original problem is decomposed into two subproblems. One involves solving for the fine scales in terms of the coarse scales. The result is substituted into the second subproblem which results in a modified problem involving only the coarse scales. This is sometimes referred to as a subgrid scale model. This problem turns out to be a suitable one for presentation to the standard Galerkin finite element method employing simple polynomial-based elements. Because unresolvable scales have been removed, a successful approximation follows. Application of the Galerkin method to the modified problem is the second and final step of the variational multiscale approach. Various practical applications can be made within the variational procedure. For example, one can assume that the fine scale phenomena exist only in the interior of element subdomains, before introduction of, and regardless of, the particular approximating finite element spaces to be introduced in the Galerkin step. This assumption leads to a framework which permits identification with stabilized methods. The methodology is described by way of examples. Outstanding issues which need to be overcome are addressed.

R. JELTSCH

Error Estimators for the Position of Discontinuities in Hyperbolic Conservation Laws with Source Terms which are Solved Using Operator Splitting

It is well known that solving nonlinear hyperbolic conservation laws with stiff source terms using operator splitting may create wrong shock speeds if the timestep is too large. One first introduces the numerical position of the discontinuity. Then an error estimator for this position is derived. The main idea is developed for the Riemann problem of a scalar hyperbolic conservation law with a source term. In that situation the solution has a Taylor expansion on both sides of the discontinuity if the source and the flux funtions are smooth enough. Hence the position of the discontinuity is a smooth function and one can compare the Taylor expansions of the exact and the numerical solution. The estimator is then applied to Burgers equation with a stiff source term, the combustion model of Majda and the reacting Euler equations in one and two space dimensions. In two space dimensions, the estimator is only used for discontinuities which are in space smooth curves. One simply applies the one–dimensional estimator in direction normal to this curve. The results are due to my Ph.D. student P. Klingenstein.

C. JOHNSON

Error Control in CFD and Hydrodynamic Stability

We gave an overview of our recent work with various collaborators on quantitative error control in computational fluid mechanics based on Galerkin orthogonality and computational evaluation of stability factors. We also discuss a model for transition to turbulence in parallel flow.

R. KLEIN

Asymptotic Analysis and Numerical Simulation of Flows at Low Mach Number

Typical explosion events begin with an extended phase of slow burning. At this stage, characteristic flow velocities are $0.1...1.0 m/s$ and the flow fields induced are essentially incompressible. It follows a phase of turbulent flame acceleration during which the combustion front can achieve supersonic speeds. A numerical simulation scheme suitable for this kind of event should be able to handle incompressible, variable density flows, weakly compressible flow including long wave acoustic effects at leading order in the velocity field and fully compressible flow with shocks. The goal here is to derive appropriate constraints on the numerical fluxes of a finite volume scheme – to be applied at small and zero Mach numbers – and to devise an associated modification of a Godunov-type compressible flow solver. The asymptotics involve a multiple length - single time scale analysis of the compressible Euler equations which reveals the compressible → incompressible transition as the Mach number vanishes, but also accounts for the influence of long wave acoustics. A key ingredient of the analysis is the pressure expansion which reads

$$P(\bar{x}, t; M) = P_o + Mp^{(i)}(\bar{\xi}, t) + M^2 p^{(l)}(\bar{x}, \bar{\xi}, t) + \dots$$

where $\bar{x}$ is a space coordinate resolving small scale convective phenomena and $\bar{\xi} = M\bar{x}$ resolves long wave acoustics. The knowledge gained is then transforced into a correction step for an explizcit higher–order upwind scheme, designed to enforce the effect of vanishing or small flow divergence in the numerical fluxes of mass, momentum and energy. Preliminary results demonstrate the capability of the scheme of dealing with incompressible constant density inviscid flow, the baroclinic vorticity generation due to interaction of long wave acoustics with transverse density gradients and weakly nonlinear acoustic effects.

K. KUWAHARA

Results of Flow Computations

An online demonstration of flow computations and visualization is given.

M. LARSSON

Adaptive Error Control for FE Approximations of the Lift and Drag Coefficients in Viscous Flow

We derive estimates for the error in a variational approximation of the lift and drag coefficients of a body immersed into a viscous flow governed by the Navier-Stokes equations. The variational approximation is based on computing a certain weighted average of a finite element approximation to the solution of the Navier-Stokes equations. Our main result is an a posteriori estimate that gives a bound for the error in the lift and drag coefficients in terms of the local mesh size, a local residual quantity, and a local weight describing the local stability properties of an associated linear dual problem. The weight may be approximated by solving the dual problem numerically. The error bound is thus computable and can be used for quantitative error estimation. We apply it to design an adaptive finite element algorithm specifically for the approximation of the lift and drag coefficients and show some numerical results.

Ph. Le Floch

Non-Classical Shock Waves and Kinetic Reactions

We consider hyperbolic systems of conservation laws whose characteristic fields are not genuinely nonlinear (g.n.l.), and introduce a framework for the non-classical shocks that arise as limits of certain continuous or discrete, diffusive or diffusive-dispersive, approximations. A *non-classical entropy solution* is defined as a solution that satisfies a *single* entropy inequality. A *non-classical shock*, by definition, is a non-classical entropy solution that does not fulfill the Liu entropy criterion. Such a shock turns out to be under-compressive. We show that the Riemann problem admits a multi-parameter family of non-classical entropy solutions: each non-g.n.l. characteristic field generates a *two-dimensional wave set* instead of the classical one-dimensional wave curve. The Riemann problem can be solved uniquely with classical waves and non-classical shocks, provided an additional constraint is imposed: we stipulate that the entropy dissipation across any non-classical shock be a given constitutive function. In particular, the entropy dissipation may be sought as a function of the propagation speed. We call this admissibility criterion a *kinetic relation*, by analogy with similar laws introduced in material science for propagating phase boundaries. The kinetic relation may be derived from limits of traveling wave solutions to an augmented system and, typically, depends on the ratio of the diffusion and dispersion parameters. The theory applies to the propagation of shock waves in complex fluids or materials in which the effects of diffusion, capillarity... are in balance.

K.W. Morton

Evolution-Galerkin Methods for Unsteady Hyperbolic Systems

At the prompting of my co-ordinators, I devoted the first part of my talk to a brief review of these methods: the variety of formulations, the wide range applications, typical convergence results. In particular, the Godunov formulation in a finite volume framework has several advantages over the direct formulation common in characteristic Galerkin methods; and the basic piecewise constant approximation space leads naturally to the use of higher order recovering procedures which can be solution-adaptive. Secondly, I reported on recent progress in the error analysis of these methods. Again, this was due to adopting a Godunov formulation. Introducing an intermediate target approximation, specially selected for each scheme, this allows a natural decomposition of the error and leads to much sharper results for a much wider range of schemes on nonuniform meshes than was previously possible.

S. Noelle

Finite Volume Methods on Hexagonal Grids

Hexagonal grids have two attractive features for computations:

1) The neighbors across the edges from a full neighborhood of the cell, i.e., there are no corner cells like for cartesian or triangular grids.

2) The six neighbors of a hexagon make it possible to construct a 2D quadratic approxi-

mation using only direct neighbors of the cell.
We try to exploit these features in the context of the 2D–Euler equations. We compare several second– and third–order finite volume schemes on hexagonal and cartesian grids. In many applications there are no large differences for the computations on the two types of grids. A particular example, where differences can be seen is a radially symmetric pointblast, used as a test example by LeVeque and Walder in the context of astrophysical calculations. In this case we observe strong grid effects for the cartesian grid calculations, and no grid effects on the hexagonal grids.

A. Russo
Stabilization of Finite Element Methods Via Residual-free Bubbles
Stabilized finite element methods have been recently interpreted as standard Galerkin methods for classical spaces enlarged with specially constructed bubble functions ("residual-free" bubbles). In my talk I give an overview of this theory, showing its close relationship to the variational multiscale method recently developed by T.J.R. Hughes. Several applications to fluid mechanics equations are also be presented, including the role played by the bubble part of the solution as an a posteriori error indicator. At the end, limitations and perspectives of the residual-free bubbles approach are discussed. [1] F. Brezzi, A. Russo, Choosing bubbles for convection-diffusion problems, M3AS, 4, 1994, pp. 571-587 [2] A. Russo, Bubble stabilization of finite element methods for the linearized incompressible Navier-Stokes equations, CMAME, 132, 1996, pp. 335-343

D. Silvester
Fast and Robust Solvers for Time-Discretised Incompressible Navier-Stokes Equations
In this talk we consider the design of robust and efficient methods for solving the Navier-Stokes equations governing laminar flow of a viscous incompressible fluid. Two fundamental issues will be assessed in detail: the (weak-) enforcement of the incompressibility constraint in a mixed finite element setting, and the solution of the indefinite (Stokes-) systems arising at each time level. Our aim is to prescribe a framework for adaptive error control. The essential ingredients are: an unconditionally stable time discretisation; "natural" spatial discretisations which are (inf-sup) stable; and a fast iterative solution strategy generating iterates converging monotonically in an appropriate norm (which mimics the dissipation inherent in the continuous system). A distinguishing feature of our methodology is the use of multigrid preconditioning to accelerate the convergence of our Krylov subspace iterative solver. We motivate this with some analysis showing that the contraction rate is bounded away from unity independently of the choice of the mixed finite element method and the subdivision parameter. Analysis and implementation of "pure" multigrid methods seems to be relatively complicated and (discretisation-) method dependent by comparison.

I. Sofronov and W. Wendland

Domain decomposition and far-field boundary conditions for 2D compressible viscous flows

We consider a two- or three–dimensional, steady, compressible, viscous flow around a given profile. The steady state has to be obtained by solving instationary models of conservation laws through time stabilization. A domain decomposition couples different models, which are obtained by making appropriate simplifying assumptions of the flow. This procedure leads to a faster computation of the numerical solution, which has to accurately approximate the solution of the original problem. The coupling procedure which we propose takes into account the complex nature of the viscous, compressible flow in the closed neighbourhood of the profile. The compressibility of the irrotational flow in front and on the side of the profile is taken into account by the full potential equation there. Finally, the rotationality in the wake domain is not excluded by the linearized Euler equations. We consider the problem of imposing far-field conditions on the external boundary. The proposed method is based on the assumption that outside the computational domain the flow is governed by the Euler equations linearized around the free-stream uniform background. By using Green's formula, we write out the representation of general solution for our system outside the computational domain. We obtain the projector operator that maps arbitrary data given on the surface of the computational domain into the set of solutions of the linearized Euler equations. This operator is then used for constructing a numerical procedure for imposing far-field conditions for transonic flow problems.

E. Süli

Finite Element Methods for Hyperbolic problems:
A Posteriori Error Analysis and Adaptivity

Partial differential equations of hyperbolic and nearly-hyperbolic character are central in many fields, particularly fluid dynamics. From the computational viewpoint, the key difficulty is that solutions to these equations exhibit localised phenomena such as shocks and thin transition-layers; in order to resolve these features in an accurate an efficient manner it is desirable to use adaptively refined computational meshes whose construction is governed by sharp *a posteriori* error bounds. This talk presents a review of recent developments in the area of *a posteriori* error analysis for finite element and finite volume approximations to hyperbolic and nearly-hyperbolic problems. We derive residual-based *a posteriori* error bounds in $\|\cdot\|_{W_p^s}$, $s \leq 0$, and related norms, and illustrate the relevance of the theoretical results to the design of adaptive finite element and finite volume approximations to linear and nonlinear problems. The question of error propagation in numerical approximation of hyperbolic problems will be highlighted in the context of *a posteriori* error estimation and adaptivity. We also discuss the problem of *a posteriori* estimation for functionals of the solution to hyperbolic problems, such as the flux across part of the boundary of the computational domain.

A. SZEPESSY

Adaptive Methods for Stochastic Differential Equations

Adaptive time–stepping methods, based on a posteriori error estimates, are well developed for deterministic ordinary differential equations. In the talk I discuss extensions to stochastic equations

$$dX = a(t, X)dt + b(t, X)dW, \quad 0 < t < T \qquad (0.1)$$

taking the form

$$E[|\text{error}|(T)^2] \leq \|(E[|\Delta \tilde{X}|^n])^{1/2}\|_{L_\infty} C(T) + \text{higher order terms},$$

where W_z is a Brownian motion and $\tilde{X}$ is the forward Euler approximation

$$\tilde{X}_{n+1} - \tilde{X} \equiv \Delta \tilde{X} = a(t, \tilde{X}_n)\Delta t + b(t, \tilde{X}_n)(W_{t_{n+1}} - W_{t_n}).$$

The stability factor

$$C(T) = \left(\int_0^T (E[|a_X 1'T\varphi|^n])11/2dt\right)^2 + \int_0^T E[|b_X'^T\varphi|^2]dt$$

is defined by the function φ, which is a solution of the dual equation to the linearized version of (0.1).

L. TOBISKA

Aspects of Numerics of Incompressible Navier-Stokes Equations

We discuss several aspects of nonconforming finite element discretizations related to the numerical solution of the incompressible Navier-Stokes equations, in particular

- use of stabilized schemes of upwind or streamline diffusion type,
- the influence of the Reynolds number in a-priori error estimates,
- a-posteriori error control,
- mesh adaption for resolving layers and
- parallelization.

The main objective is to explain the modifications necessary in the nonconforming case.

S. TUREK

A General Solution Framework for Discretized Incompressible Navier-Stokes Problems

On the basis of MPSC techniques ("multilevel pressure Schur complement") we have recently derived a complete solution framework for discretized incompressible flow problems which includes most of the existing approaches as special cases. To recall some of them: projection schemes, fractional step methods, SIMPLE-like techniques, Vanka smoother, etc. We demonstrate in our talk some severe deficiencies of these classical schemes. Based on examples we motivate how "optimal" solution schemes can be characterized. They all are of multigrid type and their numerical behaviour can be described as follows:

- independent of the underlying mesh, particularly for large aspect ratios.
- efficient and robust for all ranges of viscosity parameters.
- achieving improved convergence rates for decreasing time-step sizes.

In fact, we show how existing methods (projection methods, Vanka smoother, coupled solvers) can be essentially improved by only slight modifications to satisfy the characteristics above; hereby even exploiting the high performance of modern computer platforms. Only with these solution schemes as basic components can the supplementary aspects of the beginning be treated numerically, even for practical applications.

R. VERFÜRTH

A Posteriori Error Estimates for Low Order Finite Elements

We prove that appropriately scaled edge-residuals yield upper and lower bounds for the error of linear finite element approximations both with respect to the H^1 and the L^2 norm. The proof relies on suitable approximation and stability properties of the L^2 projection. It requires a condition on the triangulation which is more stringent than the shape regularity assumption, but which nevertheless admits local refinement. This extra condition can be avoided by using a modified Clement-type quasi-interpolation operator. This approach, however, only works for H^1 norm estimates and introduces additional higher order perturbations at the boundary.

H. YSERENTANT

A Lagrangean Approach to the Numerical Treatment of Compressible Fluids

Fluid mechanics describes the motion of mass in space under the influence of internal and external forces. The method presented in this talk is based on this fact. The fluid is subdivided into small mass packets, the particles. These mass packets can move independently of each other and can overlap. They have a fixed internal structure but can contract, expand, and rotate. The forces acting upon the particles are basically derived from a variational principle; to incorporate entropy generation in shock fronts, frictional forces are added. The exact conservation of mass, momentum, angular momentum and energy is automatically guaranteed by the approach. It is analyzed what happens when the particles split and their size tends to zero. As it turns out, limits exist which satisfy the basic physical principles underlying the Euler equations and can, in this sense, be regarded as solutions of these equations. Viscous fluids can be treated similarly.

CHR. ZENGER

Advantage of Sparse Grids Approximation in CFD

The direct numerical simulation of turbulent flows needs very fine grids with step–lengths approximately of the order of the Kolmogorov length. In practical calculations, we observed that the sparse grid discretization implemented by the combination method allows a significant reduction of the number of degrees of freedom up to a factor of about one hundred. A more detailed analysis of the computed solution by a hierarchical basis representation indicates that indeed bigger stepsize produces an unacceptably large error whereas the reduction of the dimension via the sparse grid approach leads to smaller errors. This may be a justification of this approach and also an explanation that the observable statistical quantities (e.g. 1, 2. and 3. moments of the velocity distribution) are in quite good agreement with full grid computations and with real experimental observations.

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