MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 6/1997

Automorphic Forms and Geometry

09.02. – 15.02.1997

The meeting was organized by S. Kudla (College Park, Maryland) and J. Schwermer (Eichstätt). The program of 20 lectures emphasized new developments in the theory of automorphic forms, particularly those involving interactions with geometry, topology and arithmetic algebraic geometry. The topics included:

1. The topological, relative and Lefschetz trace formulas, in particular:
   - geometric aspects in the use of these trace formulas for constructing automorphic forms and the interplay between these methods
   - Orbital integrals, stabilization, fundamental lemma.
2. The conjectures of Langlands (local and global) for GL_n: recent progress
3. Cycles on Shimura varieties
4. Integrals of automorphic forms over certain subgroups, the Rankin–Selberg method.
5. Recent developments in the cohomology theory of arithmetic groups; in particular:
   - interactions with the theory of automorphic forms
   - l–adic representations attached to cohomological automorphic representations
   - Hodge theory and semisimplicity under the Hecke–algebra action.

The variety of these topics indicates the vigorous activity and diversity of current research in automorphic forms, and stimulated much fruitful discussion. The deposit of recent pre– and reprints in front of the lecture hall was accepted.

Vortragsauszüge

M. J. Tilouine

p–adic families of Siegel–Hilbert cusp eigensystems and their Galois representations.

We study a big p–adic Hecke algebra as algebra over an Iwasawa algebra by following Hida's cohomological approach. For $G = GSp_4/F$, we construct the Hecke algebra h by its faithful representation on the interior cohomology $H_!^{3d}(S_G(U_1(p^\infty)), \bar{L}^a(\chi))$, $d = [F : \mathbb{Q}]$. (χ very regular).

Then, one takes its nearly ordinary part h^{no} by considering the largest direct factor which the Hecke correspondences T_p^1, T_p^2 act invertibly. We explained in the talk consequences of our main result with E. Urban, that h^{no} is independent of the chosen weight χ, is finite, torsion–free over the Iwasawa algebra $O[[\hat{T}]](T$ = max. torus of $G)$ and is well–controlled at arithmetic primes (in the sense of Iwasawa theory). This gives rise to p–adic families of Siegel–Hilbert cusp eigensystems and of their Galois representations (constructed by Weissauer).

J. Rogawski

Integrals of automorphic forms

We define a general method for renormalizing the integral $\int_{G(F)\backslash G(\mathbb{A})^1} \varphi(g)dg$ where φ is an automorphic form, based on Arthur's truncation operators. This is used to give a formula for $\int \wedge^T \varphi(g)dg$ by inversion. A similar procedure is applied to the calculation of $\int_{H(F)\backslash H(\mathbb{A})^1} \wedge^T \varphi(g)dh$ where H is the fixed point set of an involution. In particular, if E/F is a quadratic extension, $G = H_{/E}$, we obtain a formula which can be made explicit in the case $G = GL(n)$. This problem is motivated by the wish to characterize "distinguished representations", i. e., cuspidal representations (π, V_π) such that $\int_{H(F)\backslash H(\mathbb{A})^d} \varphi(h)dh \neq 0$ for some $\varphi \in V_\pi$. The main tool is Jacquet's relative trace formula. In computing the contribution of the continuous spectrum, one is led to calculate $\int \wedge^T E(h, \varphi, \lambda)dh$ for E an Eisenstein series.

Guy Henniart

Recent progress on the Langlands conjecture for $GL(n)$ (a report).

Let F be a non–Archimedean local field of residue characteristic p. The conjecture predicts that for any integer $n \geq 1$ there exists a canonical bijection between the set $g_F^\circ(n)$ of (isomorphism classes of) irreducible continuous degree n representations of the absolute Weil group of F and the set $A_F^\circ(n)$ of (isomorphism classes of) smooth irreducible supercuspidal representations of $GL_n(F)$. Class field theory gives $n = 1$. There should also be a global version of that correspondence, compatible with the local ones. Besides the local correspondence should preserve ε–factors for primes: if $\sigma \in g_F^\circ(n)$ corresponds to $\pi, \sigma' \in g_F^\circ(n')$ to π' then $\varepsilon(\sigma \otimes \sigma') = \varepsilon(\pi \times \pi')$. When F has positive characteristic there are such bijections (Laumon, Rapoport, Stuhler). If F is a p–adic field, M. Harris

has constructed bijections $\pi \mapsto \sigma(\pi)$, $A_F^\circ(n) \xrightarrow{\sim} g_F^\circ(n)$; they preserve ε factors for $n < p$; in any case they preserve conductors $f(\pi \times \pi') = f(\sigma(\pi) \otimes \sigma(\pi'))$ [Bushnell–Henniart, Kutzko]. When $n < p$ the correspondence can be explicitly described, at least for odd n
Note. If F is a global function field, the global Langlands conjecture was proved for $GL(2)$ by Drinfeld, using his theory of shtukas. For $n > 2$, L. Lafforgue has investigated the moduli space of shtukas and has proved, at least for odd n, the Ramanujan–Petersson conjecture: if π is a cuspidal unitary automorphic representation of $GL_n(\mathbb{A}_F)$ then its unramified components π_v are tempered.

MARK GORESKY

Geometry behind Arthur's Lefschetz Formula

This is a report on the paper "Discrete Series Characters and the Lefschetz Formula For Hecke Operators" by Goresky, Kottwitz, MacPherson and "Weighted cohomology" by Goresky, Harder and MacPherson. We consider the reductive Borel–Serre compactification $\overline{X}^{RBS}$ of a locally symmetric space $X = \Gamma\backslash G/K$ and a Hecke correspondence $\overline{X}'^{RBS} \Longrightarrow \overline{X}^{RBS}$ which is defined by $g \in G(\mathbb{Q})$. We show this correspondence is weakly hyperbolic in the sense of "Local contribution to Lefschetz Fixed Point Formula" (Inv. Math. 111, 1993, 1 – 33). If $WC^\bullet$ denotes the weighted cohomology complex on $\overline{X}^{RBS}$ then the local contribution to L. F. P. formula from a fixed point $x \in \overline{X}^{RBS}$ is the trace on the relative cohomology group $WH^\bullet(N(x), R)$ where R denotes the "expanding part of the boundary of a neighborhood $N(x)$ of x". This group is a subgroup of $\mathfrak{n}$–cohomology $H^\bullet(\mathfrak{n}_p)$ where $x \in X_P$ lies in the stratum corresponding to P. By Kostant's theorem there is a roots and weights description of this $\mathfrak{n}$–cohomology. ⊛
If X is Hermitian then Goresky–Harder–MacPherson showed that the middle weighted cohomology $W^vH^\bullet(\overline{X}^{RBS})$ coincides with the L^2 cohomology $H^\bullet_{(2)}(X)$ and hence Arthur's formula gives another expression for the local contibutions to LFP formula in terms of characters of discrete series.Θ
It turns out that the expression ⊛ is, in fact, a new closed formula for the character Θ, thus giving a new proof of Arthur's formula.
See my homepage at http://www.math.ias.edu/~goresky/

DAVID SOUDRY

Towards explicit constructions of (backwards) functorial maps from GL_n to classical groups.

This is a joint work of D. Ginzburg, S. Rallis und D. Soudry. Given a self–dual representation τ of $GL_m(\mathbb{A})$, which is irreducible, automorphic and cuspidal, such that a certain pole condition is satisfied ($L^S(\tau, \Lambda^2, s)$ has a pole at $s = 1$, in case $m = 2n$, or $L^S(\tau, \mathrm{Symm}^2, s)$ has a pole at $s = 1$, in case $m = 2n$, or $m = 2n + 1$), we construct a generic, automorphic representation $\sigma(\tau)$ of a corresponding classical group G (SO_{2n+1} in the first case, SO_{2n} – in the second case, where $m = 2n$, Sp_{2n}– in the second case with $m = 2n+1$). In the first case, we distinguish two cases according to $L(\tau, \frac{1}{2})$ being zero, or non–zero. If $L(\tau, \frac{1}{2}) \neq 0$, the we construct $\sigma(\tau)$ on $\tilde{S}p_{2n}(\mathbb{A})$. We conjecture that $\sigma(\tau) \neq 0$ and $\sigma(\tau)$ is cuspidal. Moreover, τ should be the functorial lift of $\sigma(\tau)$. The construction is related to the corresponding Rankin–Selberg integrals for $G \times GL_m$. Let R be SO_{4n} in the first case and in the remaining cases $Sp_{4n}, SO_{4n+1}, \tilde{S}p_{4n+2}$. Consider the Siegel type parabolic subgroup P of R and the induced representation $\mathrm{Ind}_{P_{\mathbb{A}}}^{R_{\mathbb{A}}} \tau \otimes |\det|^{s-\frac{1}{2}}$. Consider the corresponding Eisenstein series $E_{\tau,s}(g)$. With the above pole assumption on τ, $\underset{s=1}{\mathrm{Res}}\, E_{\tau,s}(g) \neq 0$. We construct a certain Fourier coefficient or Fourier–Jacobi coefficient of this residue $(\underset{s=1}{\mathrm{Res}}\, E_{\tau,s}(g))^{\psi}$ which when restricted to $G_{\mathbb{A}}$ defines a representation $\sigma(\tau)$. $\sigma(\tau)$ fits into a sequence of representations $\sigma_k(\tau)$ of the form $(\underset{s=1}{\mathrm{Res}}\, E_{\tau,s}(g))^{\psi_k}$ of a tower of groups of type G where we dele[te] or add series of "hyperbolic planes" e.g. in the first case $SO_1(\mathbb{A}), SO_3(\mathbb{A}), ..., SO_{4n-1}(\mathbb{A})$. The sequence $\{\sigma_k(\tau)\}$ satisfies the "tower property", which means that the first index ln s. t. $\sigma_{ln}(\tau) \neq 0$ is s. t. $\sigma_{ln}(\tau)$ is cuspidal and $\sigma_i(\tau)$ not cuspidal for $i > ln$. We prove in case $G = SO_{\mathrm{odd}}$ or $G = \tilde{S}p$ that $ln \geq n$. This implies that if σ is a generic, cuspidal rep. of $SO_{2k+1}(\mathbb{A})$ (resp. $\tilde{S}p_{2k}(\mathbb{A})$) and $k < n$, $L^S(\sigma \otimes \tau, s)$ is holomorphic at $s = 1$. This has local analog, where we take τ supercuspidal of $GL_m(F), F - p$–adic, and the pole assumption is now at $s = 0$. We construct similar towers $\{\sigma_k(\tau)\}$ and prove that for σ supercuspidal, generic of $SO_{2k+1}(F)$ (resp. $\tilde{S}p_{2k}(F)$), $k < n$, $L(\sigma \otimes \tau, s)$ is holomorphic at $s = 0$. We also prove, at the local case, for $SO_{2n+1}(F)$ (resp. $\tilde{S}p_{2n}(F)$) that $\sigma_n(\tau) \neq 0$. Thus $\sigma_n(\tau)$ is supercuspidal, generic and $L(\sigma_n(\tau) \otimes \tau, s)$ has a pole at $s = 0$. Finally, we show, using relative trace formula that given τ_0–supercuspidal of $GL_{2n}(F_{v_0})$, with $\underset{s=0}{\mathrm{Res}}\, L(\tau_0, \wedge^2, s) \neq 0$ there is a global cuspidal τ of $GL_{2n}(\mathbb{A})$, with $\underset{s=1}{\mathrm{Res}}.\, L^S(\tau, \wedge^2, s) \neq 0$, $L(\tau, \frac{1}{2}) \neq 0$, $\tau_{v_o} \cong \tau_o$, such that $\sigma_n(\tau) \neq 0$. Thus $\sigma_n(\tau)$ is a generic, automorphic, cuspidal rep. af $\tilde{S}p_{2n}(\mathbb{A})$, such that $L^S(\sigma_n(\tau) \otimes \tau, s)$ has a pole at $s = 1$, and of course $L(\sigma_n(\tau_0) \otimes \tau_0, s)$ has a pole at $s = 0$.

LOUISE NYSSEN

Langland's correspondence between representations of GL_2 and Galois representations at extraordinary primes

This is a new proof of a result of Carayol. Let $\mathbb{A}$ be the adeles of $\mathbb{Q}$. The global Eichler–Shimura correspondence associates to an automorphic representation of $GL_2(\mathbb{A})$ wh[ose] weight is greater than 2, a system of l–adic Galois representations σ^l. It is now known that it induces the local Langland's correspondence, up to normalisation, between π_p (the local component of π at p) and σ_p^l (the restriction of σ^l to the Weil group at p). This was proved in several steps, first by Langlands, then Deligne, and finally Carayol who solved the very peculiar case of extraordinary places: he used Langland's base change and the theory of bad reduction for some Shimura curves. But this doesn't seem to be sufficient to apply to other groups than GL_2, like GL_n. It is possible to compute the correspondence at extraordinary places for GL_2, with a completely different method, based on congruence properties, between modular forms of weight one, and of weight greater than two.

STEVE RALLIS

Trace formulae for some dual pairs

This is joint work with Z. Mao (to appear in Duke Math. Journal). We use the method of the relative trace formula to prove the existence of an automorphic functorial lift from SL_2 to a group G (which is the commutator group of the Levi factor of a parabolic P' of G' where $U_{P'}$ = unipotent radical of P' = Heisenberg group). The pair $SL_2 \times G$ is a dual pair in G'. The method introduces the use of coperiods (an idea originally from work of Jacquet, Lai and Rallis, Duke Math. Journal 1994).

JOACHIM MAHNKOPF

L-functions of twists of automorphic representations

Let $\mathbb{A}$ denote the ring of adeles of $\mathbb{Q}$, π a cuspidal representation of $GL_n(\mathbb{A})$ and $\chi : \mathbb{Q}^*\backslash\mathbb{A}^* \to \mathbb{C}^*$ a character of finite order. Using Rankin–Selberg–convolutions on $GL_n \times GL_{n-1}$ and their zeta-integrals we derive a formula for the values $L(\pi \otimes \chi, s)$ in dependence of the character χ, which runs over all characters with conductor $f\chi = p^e$ a p–power and fixed infinity component $\chi_\infty = 1$ or sgn. Interest in these values comes form algebraicity or p–adic interpolation of special values of L–functions. For the group GL_2 the formula coincides with A. Weil's formula for the twist of modular forms. We apply it in the case GL_3 and obtain the algebraicity of $L(\pi \otimes x, 1)/\Omega(\pi), \Omega(\pi) \in \mathbb{C}^*$ a certain period, for representations π of $GL_3(\mathbb{A})$ which are of cohomological type at infinity.

CHRISTIAN KAISER

A twisted fundmental lemma for GSp_4 : a local proof

The Frobenius twisted fundamental lemma (f. l.) for $G = GSp_4 = GS_p(V, <, >)$ (and the elliptic endoscopic group $H = GL_2 \times_{\mathbb{G}_m} GL_2$) was proved by R. Weissauer (unpublished) by global means: reduction to the unit element, then applying Kottwitz's f. l. for base change and the ordinary f. l. for the unit element. We proved one special case again purely locally by computing both sides (after reducing to the elliptic regular case). The interest in this special case comes from the fact, that you need it for the stabilization of Lefschetz's trace formula in char p for the Siegel modular 3–fold. The hard part of the computation was to calculate the twisted orbital integrals $TO_S(\Phi_m), \Phi_m := \text{char}\, G(\mathbb{Z}_{p^m}) \begin{pmatrix} p & & & \\ & p & & \\ & & 1 & \\ & & & 1 \end{pmatrix} G(\mathbb{Z}_{p^m})$.

T. N. VENKATARAMANA

Restriction maps in cohomologies of Shimura Varieties.

We obtain a criterion–purely linear–algebraic – which determines when for holomorphic forms of a certain representation type $A_\mathfrak{q}$ on a Shimura variety S restrict to non-zero forms on a given Sub–Shimura variety. This is shown to imply that the Mumford–Tate groups in degree = real rank of the Shimura variety S are abelian, in many cases. Applications include a counter–example to a question of A. Borel, concerning occurrence of cohomological representations in $L^2(\Gamma\backslash G)$ for a fixed family (Γ) of commensurable arithmetic lattices of G.

JEAN-LOUP WALDSPURGER

Intégrales orbitales unipotentes et stabilité pour les groupes unitaires

Soient F un corps local non archimédien de caractéristique nulle, de caractéristique résiduelle p, G un groupe unitaire non ramifié défini sur F, g son algèbre de Lie. On suppose p grand relativement à la dimension de G. On détermine alors explicitement les distributions sur $g(F)$ qui sont stablement invariantes et à support nilpotent. Soient G_1, G_2 deux groupes unitaires non ramifiés définis sur F, supposons que $G_1 \times G_2$ soit un groupe endoscopique (elliptique) de G. On a conjecturalement une application de transfert qui associe une distribution invariante par $G(F)$ sur $g(F)$ à une distribution stablement invariante sur $(g_1 \times g_2)(F)$. On détermine cette application conjecturale restreinte aux distributions à support nilpotent.

MICHAEL HARRIS

On the local Langlands conjecture for $GL_n, n < p$.

Let F be a p-adic field, and let $\mathcal{A}_n^o(F)$ denote the set of supercuspidal representations of $GL(n,F), G_n^o(F)$ the set of irreducible n-dimensional representations of the Weil group W_F, up to equivalence. In earlier work a bijection was defined

$$\begin{array}{ccc} \mathcal{A}_n^o(F) & \longleftrightarrow & G_n^o(F) \\ \pi & \longmapsto & \sigma(\pi) \end{array}$$

I prove that this bijection is compatible with ε–factors for pairs $\pi \in \mathcal{A}_n^o(F), \pi' \in \mathcal{A}_m^o(F)$, when n and m are prime to p. Compatibility with ε–factors for all m, n would follow from a generalization of Carayol's theorem on the local Galois representations associated to Hilbert modular forms.

The proof is global and uses the fact that, for $(n,p) = 1, G_n^o(F)$ consists of representations induced from characters. It is shown that a local π can be realized as a local component of a cohomological automorphic representation associated to an n–dimensional complex representation of an appropriate Weil group. The key step is to construct non–Galois automorphic induction in certain sufficiently general situations.

STEPHEN S. KUDLA

Central derivatives of Eisenstein series and height pairings.

Let $V, (\,,)$ be a quadratic space over $\mathbb{Q}$ of dimension $n+2$ and signature $(n,2)$. Let D be the space of oriented negative 2–planes in $V(\mathbb{R})$ and let $G = G\mathrm{Spin}(V)$. For a compact open subgroup $K \subset G(\mathbb{A}_f)$, $X_K = Sh(G,h)_K$, the associated Shimura variety over $\mathbb{Q}$

has dimension n.
We consider certain weighted sums $Z(d,\varphi;K)$ of algebraic cycles of codimension $r, 1 \leq r \leq n$, in X_K, parameterized by $d \in \mathrm{Sym}_r(\mathbb{Q})$ and $d > 0$, and $\varphi \in S(V(\mathbb{A}_f)^n)$. Then $Z(d,\varphi;K)$'s define elements of $CH^r(X_K)$.
In the special case $n = 1$, the $Z(d,\varphi;K)$'s are 0–cycles on the Shimura curve X_K. On the other hand, we consider incoherent Eisenstein series on $M_p(W)_{\mathbb{A}}$ where W is a symplectic vector space of dimension $2n+2$. These series have a naturally occuring zero at their center of symmetry, $s = 0$. First we restrict via $W = W_1 + W_2$,

$$E'(g,o,\Phi) \quad M_p(W)_{\mathbb{A}} \overset{i}{\leftarrow} M_p(W_1)_{\mathbb{A}} \times M_p(W_2)_{\mathbb{A}} \quad E'(i(g_1,g_2),o,\Phi)$$

with dim $W_i = r_i$, $r_1 + r_2 = n+1$. Then compute the Fourier coefficient

$$F_{d_1,d_2}(g_1,g_2,\Phi) \quad , \quad d_1 \in \mathrm{Sym}_{r_1}(\mathbb{Q}), d_2 \in \mathrm{Sym}_{r_2}(\mathbb{Q}),$$

of $E'(i(g_1,g_2),o,\Phi)$. In the case $n = 1$, we show that this coefficient is connected with the height pairing $\langle \hat{Z}(d_1,\varphi_1,K), \hat{Z}(d_2,\varphi_2,K)\rangle$, when Φ is determined by $\varphi_1 \otimes \varphi_2$. We speculate that there is an analogous relation in higher dimensional cases.

DIHNA JIANG

Generalization of Kudla–Rallis' regularized Siegel-Weil Formula
Let $\mathbb{A}$ be the ring of adeles of a number field F. Let $G_n = S_p(2n)$ be the symplectic group of rank n and $P_r^n = M_r^n \cdot N_r^n$ the standard maximal parabolic subgroup of G_n with $M_r^n = GL(r) \times G_{n-r}$ $(m \in M_r^n \; m = (a_r, h_{n-r}))$. Then the modulus character is $\delta P_r^n = |\det a_r|^{p_r^n}$, $p_r^n = \frac{2n-r+1}{2}$. Let $I_r^n(s) = \mathrm{Ind}_{P_r^n(\mathbb{A})}^{G_n(\mathbb{A})}(|\det a_r|_{\mathbb{A}}^S)$ be the degenerate principal series representation of $G_n(\mathbb{A})$. As usual, we have Eisenstein series $E_r^n(g,s,f_s)$ for any section $f_s \in I_r^n(s)$. It is well kown that $E_r^n(g,s,f_s)$ converges absolutely for $Re(s) > p_r^n$, continues analytically to the whole s–plane, and has a functional equation relating s to $-s$. It is easy to check that for $Re(s) > 0$, $E_r^n(g,s,f_s)$ has possible poles of finite order in $X_r^n = (o, p_r^n] \cap \{p_r^n - i \mid i = 0,1,2,...\}$. For a given $s_o \in X_r^n$ one can consider the Laurent expansion of $E_r^n(g.s.f_s)$ at $s = s_o$. and deal with the problem to characterize the first term (nonvanishing at least for one holomophic section $f_{s_o} \in I_r^n(s_o)$) as an automorphic representation of $G_n(\mathbb{A})$). There is a conjectural First Term Identity for Eisenstein series of G_n whose precise form is given. It is shown that the conjecture holds in the following cases:

(1) f_s is a spherical section in $I_r^n(s)$

(2) When $n = l$, f_s is a Schwartz–Bruhat section in $I_r^n(s)$ [Kudla–Rallis]

(3) When $l = 1$, f_s is a general holomorphic section in $I_r^n(s)$

(4) The second part of the conjecture holds for archimedean spherical sections in $I_r^n(s)$.

Remark

(i) Statement (2) in the result is Kudla–Rallis' regularized Siegel–Weil formula (Ann. Math. 140 (1994))

(ii) Similar identities are expected to hold for Eisenstein series of orthogonal groups.

(iii) These identities can be used to study the special value of Langlands L–functions via the Rankin–Selberg method.

((a) Kudla–Rallis. Ann. Math. 140 (1994), (b) D. Jiang Mem. AMS 588 (1996)).

((c) Harris–Kudla. Ann. Math. 133 (1991), (d) Kudla–Rallis–Soudry, Invent. Math. 107 (1992))

JÜRGEN ROHLFS

Strong Poincaré duality and arithmetic groups

Let $G/\mathbb{Q}$ be a semi–simple algebraic group defined over $\mathbb{Q}$. Let K be a maximal compact subgroup of $G(\mathbb{R})$ and let $\Gamma \subset G(\mathbb{Q})$ be a congruence subgroup. Assume that E is a representation of $G(\mathbb{R})$, put $S_\Gamma := \Gamma\backslash G(\mathbb{R})/K$ and let ε be the sheaf of locally constant sections of the vector boundle $\mathbb{E}$ on S_Γ given by the K–action an E. Then the cohomology groups $H_c^r(S_\Gamma, \varepsilon)$, $H^r(S_\Gamma, \varepsilon)$ and $H^r(S_\Gamma, \varepsilon) := \mathrm{im}(H_c^r(S_\Gamma, \varepsilon))$ are defined. There is a natural action of the Hecke algebra $\mathcal{H} = \mathbb{C}[\Gamma\backslash G(\mathbb{Q})/\Gamma]$ on the cohomology groups.

It is shown that there exists hermitian metrics on the cohomology groups such that the orthogonal complement of a $\mathcal{H}$–stable subspace is also $\mathcal{H}$–stable (provided G admits a Cartan–like involution defined over $\mathbb{Q}$).

For the proof one constructs a semi–linear isomorphism $P : H_c^{m-r}(S_\Gamma, \varepsilon^v \otimes or) \xrightarrow{\sim} H^v(S_\Gamma, \varepsilon)$, $m = \dim S_\Gamma, \varepsilon^v$ the dual sheaf, or the orientation sheaf. Then the hermitian scalar product is given by $(\alpha \cup P^{-1}\beta)[S_\Gamma]$, evaluation of the cup product. Hence we see an isomorphism, where the general theory gives the Poincaré duality. For the construction of P one takes an increasing exhausting sequence $\{M_i\}_{i\in\mathbb{N}}$ of compact manifolds with boundary $\dot{M}_i$. Then P ist constructed with the help of maps $P_i : H_c^{m-r}(M_i, \varepsilon^v \otimes or) \xrightarrow{\sim} H^v(M_i, \varepsilon)$. The maps P_i are given by the $*$–operator; applied to harmonic representatives of cohomology classes. The harmonic representatives are tangential resp. normal to the boundary. Most of this argument works on general Riemann manifolds.

COLETTE MŒGLIN

About the cohomology of $GL(n)$ over a totally imaginary field

Dans cet exposé, on commence par rappeler la décomposition des formes automorphes suivant leur support cuspidal, due indépendamment à Franke et Waldspurger. Puis on rappelle les conséquences de ce théorème profond sur la cohomologie (cela a été remarqué par Franke et Schwermer). Ensuite on montre comment ces idées appliquées à gl_n sur un corps totalement imaginaire permettent de calculer la cohomologie uniquement à l'aide de vraies (i.e. pas de dérivées) séries d'Eisenstein holomorphes. La fin de l'exposé est consacrée à décrire l'image de la cohomologie des formes automorphes de carré intégrable d'une part et des formes globalement temprérées d'autre part dans la cohomologie totale. Dans le premier cas seuls les pôles (bien connus) des fonctions L de paires jouent un rôle tandis que dans le deuxième les zéros de ces fonctions L interviennent de façon déterminante.

Pour GL_3 cela avait déjà été remarqué par Harder.

Erez M. Lapid

Multiplicities for $SL(n)$

Let F be a number field and G be a reductive group defined over it. The cuspidal spectrum of $G(F)\backslash G(\mathbb{A}_F)$ (with a given central character) decomposes discretely into a sum of irreducible representations, each occurring with a finite multiplicity. In the case where $G = GL(n)$ all multiplicities are one in this decomposition. Moreover, two cuspidal representations with the same Hecke eigenvalues almost everywhere are equivalent. Cuspidal representations of $SL(n)$ are intimately related to those of $GL(n)$. However the situation for $SL(n)$ changes dramatically. For example, it is known that L–packets of $SL(2)$ can be infinite, at least in the unstable case, hence naive strong multiplicity one cannot hold. More recently, Blasius showed that multiplicity can be bigger than 1 for $n > 2$, and also that strong multiplicity one does not hold for L–packets, so that two representations which are a.e. the same do not have to belong to the same L–packet. He also gave quantitative results for these, namely he constructed cuspidal representations, which are of Galois type, with multiplicity $\geq \phi(n)$. Here we will be interested in these two phenomena which tie up in the definition of *global multiplicity* (see below). The high multiplicities for $SL(n)$ are not surprising, since the cuspidal spectrum of $SL(n)$ has a natural action of $GL(n, F)$ on it by conjugation. If one takes into account those additional symmetries then the multiplicity is one. From a different point of view, high multiplicity has to do with the fact that two non–equivalent projective representations of a group may become equivalent when restricted to any cyclic subgroup. After defining the global multiplicity of a L–packet and giving some heuristics we will focus on a particularly handy case of endoscopic L–packets induced from Hecke characters. It turns out that in this case the global multiplicity is given naturally by an order of an Abelian group. The problem of computing the global multiplicity reduces to a completely algebraic question in finite groups representations. We can completely solve it in case where n is prime, giving an explicit computation of the global multiplicity in terms of the character we induce from. The essential tool in proving, and even stating, the results is the *base change* lift proved first by Arthur and Clozel.

Gérard Laumon

The Jacquet–Ye fundamental lemma in equal characteristic (following B.C. Ngô)

The Jacquet–Ye fundamental lemma is a family of relations between generalized Kloosterman sums associated to $GL(n, F)$ and $GL(n, F')$, where F'/F is an unramified quadratic extension of non archimedean local fields. A special case of those relations is the classical identity

$$\sum_{\substack{x_1, x_2 \in \mathbb{F}_p \\ x_1 \cdot x_2 + 1 = 0}} e^{\frac{2\pi i}{p}(x_1+x_2)} = - \sum_{\substack{x' \in \mathbb{F}_p^2 \\ x'^{p+1} + 1 = 0}} e^{\frac{2\pi i}{p}(x'+x'^p)}.$$

In the lecture I have presented Ngô's proof of this fundamental lemma when char $(F) =$

$p > 0$. It uses Grothendieck's fixed point formula, l-adic perverse sheaves and Fourier–Deligne transformation.

Uwe Weselmann

A twisted topological trace formula for Hecke–operators

For a connected reductive group $G/\mathbb{Q}$ of rank r and an automorphism η of finite order one wants to understand the cohomology $H^*(S_{K_f}, \mathcal{M})$ as an $\langle\eta\rangle \times \mathcal{H}(K_f)$-module. The alternating sum $H^\bullet = \sum(-1)^i H^i$ as an element of the Grothendieck group can be computed using a Lefschetz trace formula for η–twisted Hecke correspondences.
One can glue together 2^r copies of the Borel–Serre–compactification $\bar{S}_{K_f}$ along the boundary to obtain a compact manifold X_{K_f} (assuming K_f is sufficiently small) with an action of the group $\sum = \{\pm 1\}^r$ respecting the η–actions and the Hecke correspondences. The fixpoint components of the Hecke correspondences twisted by η and $d \in \sum$ can be computed to get a formula for the Lefschetz number of a Hecke operator times η acting on $H^\bullet(S_{K_f})$ as a sum over η–conjugacy classes of elements in $G(\mathbb{Q})$ of products of local orbital integrals, of $tr(\eta \circ \gamma | M)$, of some factors at ∞. The trace formula can be stabilized. One wants to compare it with trace formula for H, where $\hat{H} = \hat{G}^{\hat{\eta}}$ i.e. H is an endoscopic group for (G, η). One has $tr(\eta \circ \gamma / \mathcal{M}_\gamma^G) = tr(\tau(\gamma)/M_\gamma^H)$ where $\tau(\gamma) \in H(\bar{\mathbb{Q}})$ is the transfer of $\gamma \in G(\bar{\mathbb{Q}})$, and $\chi \in X^*(T_H) = X^*(T_G)^\eta$ is the highest weight of the representation $\mathcal{M}_\chi^G$ and M_χ^H. In the case $G = GL_4 \times \mathbb{G}_m, \eta(A, b) = (I \cdot {}^t g^{-1} \cdot I^{-1}, \det g \cdot r), H = GSp_4$ (where $I = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}$) one can use the fundamental lemma proved by Flicker to get a base change lift from $H^\bullet(S^H_{K_f}, \mathcal{M}^H_\chi)$ to $H^\bullet(S^G, \mathcal{M}^G_\chi)$.

Günter Harder

Mixed motives and the values of the Riemann ζ–function

A mixed motive $M/\mathbb{Q}$ is roughly a piece in the the cohomology $H^q(\mathcal{U}, \mathbb{Z})$ of a quasiprojective smooth variety $\mathcal{U}/\mathbb{Q}$ which also has some kind of nice compactification. The "piece" should be defined by some correspondences. Such a mixed motive has various cohomological realisations:

$$\begin{array}{rcl} M_B & = & \text{piece in} \quad H^q(\mathcal{U}(\mathbb{C}), \mathbb{Z}) \\ M_{DR} & = & \text{piece in} \quad H^q_{DR}(\mathcal{U}) \\ M_l & = & \text{piece in} \quad H^q_{\acute{e}t}(\mathcal{U} \times \bar{\mathbb{Q}}, \mathbb{Z}_l) \end{array}$$

where the pair $(M_B, M_{DR}) = M_{HdR}$ has various further structures, like filtrations, comparison etc. and where M_l is a module for the Galois group. We propose the construction of objects $\xi_n = M_n$ which sit in exact sequences

$$0 \to \mathbb{Q}(o) \to M_n \to \mathbb{Q}(-n-1) \to o$$

where $\mathbb{Q}(o) = H^\circ$ (point) and $\mathbb{Q}(-m) = H^{2m}(\mathbb{P}^m, \mathbb{Z})$. To such an object (mixed motive) we can attach a number $\xi_{n,HdR} \in Ext^1_{\text{mixed Hodge}/\mathbb{Q}}(\mathbb{Q}(-n-1), \mathbb{Q}) \simeq \mathbb{R}$. The objects which we construct will give for $n \geq 2$, even $\xi_{n,HdR} = \xi'(-n) \cdot \mathbb{Q}^*$ and is believed that any such an object will give the same result. (Beilinson's conjecture, slightly modified)

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