MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 7/1997

Adaptive Methoden für partielle Differentialgleichungen

16.02. – 22.02.1997

The conference was organized by R.E. Bank (San Diego), G. Wittum (Stuttgart) and H. Yserentant (Tübingen).

Of the 39 participants, 27 came from Germany and the rest from Austria, Bulgaria, Great Britain, the Netherlands and the United States of America.

In contrast to the situation some years ago, adaptive methods have become an indispensable tool in the numerical solution of partial differential equation and have found a broad acceptance both in the mathematical and in the engineering community. The twenty-eight talks given at the conference (ranging from one-hour surveys to short contributions of fifteen minutes) reflected this fact and have shown that in the meantime the interest shifted from simple mathematical model problems to real applications like fluid mechanics.

E. Bänsch, Freiburg, Germany

Adaptive finite elements for exterior domain problems

We present an adaptive finite element method for solving elliptic problems in exterior domains, that is for problems in $\Omega = \mathbb{R}^d \backslash \omega$, where $\omega \subset \mathbb{R}^d$ is bounded ($d = 2,3$). A residual based error estimator is derived, giving a reliable bound for the error e_h in the energy norm:

$$\begin{aligned} ||e_h||^2 &= ||u - u_h||^2_{\Omega_h} + ||u||^2_{\Omega\backslash\Omega_h} \\ &\leq C\eta^2 + \text{data approximation terms.} \end{aligned}$$

The procedure generates a sequence of finer and larger grids until the desired accuracy of the solution is reached. (Joint work with W. Dörfler)

P. Bastian, Stuttgart, Germany

Fully coupled multigrid solution of two-phase flow in porous media

The flow of two inviscid fluids in a porous medium is described by two coupled nonlinear time-dependent partial equations. In the incompressible case one of these equations is of elliptic type and the other one is of parabolic or (nearly) hyperbolic type depending on the solution.

For the numerical solution these equations are discretized in space with a finite-volume scheme and in time with BDF(1) (implicit Euler) or BDF(2). The resulting system of nonlinear algebraic equations is solved by a Newton-multigrid method. A closer look at the Jacobian matrices shows highly variable coefficients that are not aligned with grid lines and depend on the solution. A modified restriction is proposed to handle these difficulties.

Numerical simulations in two and three dimensions for various applications are presented. The experiments show that multigrid performance can be achieved for these cases.

J. Bey, Tübingen, Germany

Finite volume methods for elliptic boundary value prolems in N space dimensions

We consider finite volume schemes for N-dimensional elliptic boundary value problems of the form

$$\left.\begin{array}{lll} \nabla\cdot(-A\nabla u+bu)+cu & =f & in\ \Omega \\ u=0 & & on\ \Gamma \end{array}\right\}$$

The method is derived from a generalized weak formulation of the problem, which is related to a given box partition B_h of the domain and can be shown to be equivalent to the usual weak formulation. Assuming that B_h is in fact a dual boxmesh for a standard simplicial finite element triangulation, and using a canonical isomorphism from the Finite Element Ansatz space to the Finite Volume testfunction space, we are able to interprete the method as a conforming generalized Galerkin approach. Applying a technique presented in a paper of W. Hackbusch for the twodimensional case (Computing 1989), we then prove that under usual finite element assumptions similar first and second order error estimates hold, provided the dual boxmesh B_h satisfies certain balance and regularity conditions. After a brief discussion of certain variants of the method (mass lumping, use of quadrature formulas) we then show, how boxmeshes satisfying these conditions can be constructed in the N-dimensional case. This construction generalizes the well-known center-of-mass method to N dimensions. Finally, we present a simple first order upwind scheme, which can be interpreted as symmetric perturbation of the diffusion matrix A.

W. Dahmen, Aachen, Germany

Adaptive techniques for integral equations

This talk is concerned with adapted wavelet based discretizations of operator equations with special emphasis on operators with global Schwarz kernel. The basic assumptions are: (i) The operator is an isomorphism from a Sobolev space into its dual. (ii) The kernel is smooth except on the diagonal and has certain asymptotic decay properties. (iii) The underlying domain is a union of essentially disjoint smooth parametric images of a cube. The core ingredient of the approach is an isomorphism between the relevant Sobolev spaces on the global domain and a product space whose components are Sobolev spaces on the individual patches subject to certain boundary conditions. This gives rise to a domain decomposition where the coupling conditions are reflected by modifications of the kernel. The solution of the original operator equation reduces to a convergent Schwarz type iteration requiring the solution of elliptic problems on the individual patches. The local problems, in turn, can be solved by fast wavelet methods on the unit cube. A corresponding adaptive scheme can be shown to converge without assuming the saturation property.

W. Dörfler, Freiburg, Germany

Adaptive methods: Robustness, saturation assumption and convergence

We consider the Poisson equation as a model problem and show how to set up an initial discretization (from data information only) such that the error will be monotone decreasing on in a certain way refined grids. So in the first stage the main task is robust integration of the data error. Since this is not an efficient algorithm one has to switch to more efficient methods (now not robust) if 'saturation' of the problem is detected. This concept is then applied to the case of the nonlinear Poisson problem and the case of the linear problem with nonpolygonal boundary. (Joint work with M. Rumpf)

R. Ewing, College Station, USA

Application of adaptive refinement methods

Highly localized phenomena can often dominate important physical processes. In large-scale simulation processes, attempts to implement local grid refinement can often destroy the efficiency of existing codes through complex data structures and associated solution algorithms. Patch refinement methods arising from domain decomposition techniques are described which are accurate and can be incorporated relatively easily in existing simulation codes. Preconditioned iterative techniques are presented to allow a wide variety of applications of these adaptive refinement concepts. Computational results are described and compared with theoretical convergence rates and superconvergence error estimates for local refinement of mixed finite element methods, incomplete factorization preconditioners, and local time-stepping techniques. Applications to fluid flow in porous media are presented as examples of large-scale, time dependent applications.

J.E. Flaherty, Troy, USA

An adaptive and parallel framework for partial differential equations

Adaptive computational techniques are having a dramatic impact on the way that scientists and engineers solve problems involving partial differential equations. Like adaptivity, parallel computation is being used to an increasing degree. We have developed a framework for adaptive and parallel computation that is capable of (i) generating three-dimensional unstructured meshes of tetrahedral elements, (ii) automatically refining and coarsening these meshes, (iii) partitioning the computation into subdomains that may be processed in parallel, and (iv) maintaining a balanced parallel computation through element migration. Mesh generation is supported by a hierarchical database in which spatial regions are linked to their bounding faces, which are linked to their bounding edges, which are linked to their endpoints. The database is connected to geometrical modeling procedures and to a parallel library having capabilities for processor scheduling and reassignment. The adaptive finite element framework is being used to study several flow problems.

M. Griebel, Bonn, Germany

An adaptive multilevel method for sparse grid discretizations of PDEs based on the finite difference approach

For the efficient representation of discrete functions and for the solution of PDEs, the sparse grid technique has been developed in the last years. It is mainly based on the finite element approach using a specific tensor product of 1D hierarchical basis ansatz functions. The resulting linear system can be solved efficiently by multilevel methods. While usual discretization methods require basically $O(h^{-d})$ grid points, the sparse grid approach needs only $O(h^{-1}\log(h^{-1})^{d-1})$ grid points. Here, h denotes the mesh size employed and d denotes the dimension of the problem. The accuracy of the approximation, however, deteriorates pointwise and with respect to the L_2- and $L_{\max}$-norm only slightly form $O(h^2)$ to $O(h^2\log(h^{-1})^{d-1})$ provided that the function to be represented is sufficiently smooth. In the non-smooth case, adaptive refinement can be applied straightforwardly and helps to maintain the advantage of the sparse grid approach over an adaptive conventional h-version of the finite element mehtod. However, the setup and solution of the linear system is quite complicated, especially in the case of PDEs with non-constant coefficient functions. A sparse grid approach using the finite difference philosophy is in this respect much more simple and gives in some cases even a better performance, i.e. a $O(h^2)$ accuracy without the log-terms.

However, the resulting systems are not more symmetric in general. Furthermore, the solver employs the BiCG iteration and a multilevel preconditioner using so-called prewavelets. It converges independently of the mesh size of the problem. We report on our method and its application to 2D and 3D PDEs of 2nd oder with general coefficient functions, discuss the adaptive refinement approach and describe how we can deal with general complicated domains.

W. Hackbusch, Kiel, Germany
Error estimators for inexact solutions

The standard error estimators for adaptive refinement require an exact discrete solution. The lecture shows how the influence from unknown iteration errors can be measured. It turns out that there are three norms which are almost equivalent. The critical equivalence constant is discussed. Knowing this number, one can guarantee error bounds for the inexact solution as soon as a computationally available error norm is small enough.

R. Hiptmair, Augsburg, Germany
Multigrid method for Maxwell's equations

We consider Maxwell's curl-equations in a 3D cavity Ω with perfectly conducting walls. In the framework of time domain discretization an implicit timestepping is highly desirable by virtue of its unconditional stability. In a finite element setting each timestep involves the solution of a discrete variational problem for the bilinear form $(\cdot,\cdot)_{L^2} + (\mathrm{curl}\cdot, \mathrm{curl}\cdot)_{L^2}$ posed over $H(\mathrm{curl};\Omega)$. We rely on Nédélec's curl-conforming finite elements (edge elements) which properly reflect the continuity properties of the electric field. A multigrid method is employed as a fast iterative solution method. We observe that on the orthogonal complement of the kernel of the curl-operator the bilinear form resembles that of the Laplacian. On the kernel of curl we get a mass operator which can be converted into a H^1-elliptic operator using discrete potentials. Thus, guided by the construction of conventional multigrid schemes, we opt for a nodal (BPX-type) multilevel decomposition to treat both kern (curl) and its orthogonal complement.

Provided that material properties are homogeneous and an additional regularity assumption holds, we can prove that the multigrid method converges independently of the depth of refinement.

R.H.W. Hoppe, Augsburg, Germany
Residual based a posteriori error estimators for curl-conforming finite element approximations

Curl-conforming finite element approximations by means of Nédélec's edge elements are an appropriate tool in the computation of the electromagnetic fields or electromagnetic vector potentials in conducting media. They are based on a weak formulation of the underlying boundary value problem involving the Hilbert space $H(\text{curl};\Omega)$.

Given some iterative solution of the Nédélec approximation, for the purpose of local adaptive refinement of the triangulations we are interested in an efficient and reliable a posteriori error estimator for the total error which consists of the discretization error and the iterative error and is measured in the natural norm of $H(\text{curl};\Omega)$.

The basic tool in the construction of such an error estimator is a Helmholtz type decomposition of the total error into a curl-free part and a weakly divergence-free part which allows to establish upper and lower bounds for these parts separately.

W.F. Mitchell, Gaithersburg, USA

Overwiev of a parallel hierarchical adaptive multilevel method

In this talk we present an overview of PHAML, the parallel hierarchical adaptive multilevel method for elliptic partial differential equations. The foundation of the method is built upon the hierarchical finite element basis for a triangulation obtained by adaptive refinement using newest node bisection. Recent developments in parallelizing the method in a SPMD distributed memory environment via overlapping subdomains on each refinement level will be discussed. The approach produces a full domain partition, in which the usual subdomain on each processor is extended to cover the full domain.

P. Oswald, St. Augustin, Germany

Modified solenoidal 2D-Stokes discretization: Multilevel preconditioning (and adaptivity)

We present a modified $P1$ velocity element (combined with discontinous $P0$ preserve elements) for which the space Z_h of discretely divergence-free elements admits a stable multilevel splitting. It is based on unusual coarse-to-fine intergrid operators which preserve the (discrete) divergence-free condition. $\mathcal{O}(I)$-condition number behavior is proved for uniformly and adaptively refined 2D-triangulations. The multilevel preconditioner can (and should primarily) be used for other Stokes elements. Open problems: 3D case and performance optimization (numerical tests performed so far indicate the need for this!).

R. Rannacher, Heidelberg, Germany

Residual-based error estimation via duality arguments

The conventional strategy for controling the error in finite element (FE) methods is based on a posteriori estimates for the error in the global *energy* or L^2-norm involving *local* residuals of the computed solution. Such estimates contain constants describing the local approximation properties of the finite element spaces and the stability properties of a linearized dual problem. The mesh refinement then aims at the equilibration of the local error indicators. However, meshes generated in this way via controling the error in a global norm may not be appropiate for local error quantities like point values or contour integrals. This deficiency may be overcome by introducing certain weight-factors in the a posteriori error estimates which depend on the dual solution and contain information about the relevant error propagation. In this way *optimal* meshes may be generated for all kinds of error measures. This general approach is discussed for simple model situations. More complicated applications can be found, for example, in fluid mechanics (computation of drag and lift coefficients), in elasticity (limit loads in elasto-plastic deformation), and in radiative transfer (surface mean-radiation of stars).

S. Sauter, Kiel, Germany

Composite finite elements for coarse-level discretizations of PDEs with essential boundary conditions

Composite finite elements allow coarse-level discretizations of PDEs where the minimal number of degrees of freedom is independent of the number of microstructures of the problem (geometric details, oscillating coefficients, etc.). We can prove that, for this new finite element space, the approximation property is valid also for the very coarse discretizations. In our talk, we will focus on the treatment of essential boundary conditions. For this purpose, we will use prolongation operators which are stable in H^1, are defined locally, and preserve the essential boundary conditions.

L.R. Scott, Houston, USA

Error estimators and mesh optimization for high-order finite element simulation of Newtonian flows

This was a report on joint work done by the FLACS project at the University of Houston, in particular, Babak Bagheri, Andrew Ilin, Hector Tuarey and Ralph Metcalfe. Further information on FLACS (which stands for Flow Around Cylinders and Spheres, or Flow Around Complex Surfaces) may be found at *http://www.hpc.uh.edu/flacs.*

Our objective was a study of residual-based error estimators for high-order finite element methods for approximating solutions of the Navier-Stokes equations. We focused on the spatial correlation of the actual error (computed by comparison with numerical solutions on finer meshes) with the residual-based error estimator. This led us to reconsider the definition of mesh size, h_K, of an element K. We found that the correlation can depend strongly on possible different definitions (maximal edge length, diameter, etc.). We introduced an element-level averaged error estimator and showed that it yields a significantly improved correlation compared with the pointwise correlation. We also introduced a temporally averaged error indicator and demonstrated computationally its efficacy in determining better meshes for time-dependent problems. It should be noted that these results were all computational but they have suggested many open problems that should be addressed theoretically.

E. Süli, Oxford, U.K.

Finite element methods for hyperbolic problems: A posteriori error analysis and adaptivity

The aim of this talk is to present a critical review of recent results that concern the a posteriori error analysis of finite element approximation of initial & initial/boundary value problems for linear hyperbolic systems. Global & local residual-based error bounds (with the error measured in negative-order Sobolev norms) are considered. The implementation of these bounds into adaptive finite element algorithms is discussed.

A.H. Schatz, Ithaca, USA

A study of some averaging operators as local a posterior estimators for the maximum norm of the gradient on each element

A study is made of a class of simple local averaging operators for use as a posteriori error estimators for elliptic problems when using the finite element method. Included in this class of averaging operators are the L_2-projections of either the approcimate solution U_h or its gradients onto a space of polynomials of higher degree on a local patch of elements of size H (slightly larger than h). The methods include as a special case the averaging method proposed by Zienkiewicz and Zhu (1989). They also include difference quotients on a mesh of size H. Two model problems are analyzed. The first is a smooth Neumann problem for which conditions on the solution are derived in order that the maximum norm of the gradient $u_h - Au_h$ is an asymptotically exact estimator for the maximum norm of the gradient of $u - u_h$ on each triangle. Here Au_h is the local average of u_h. Roughly speaking the method is shown to work under some reasonable conditions which prevents pollution error from dominating the local error. The main analytical difficulty occurs in trying to compare the two errors on each triangle and this is overcome by using some very local error estimates for the finite element method. The same situation does not occur for the second model problem which is Dirichlet's problem with a nonconvex corner. If a quasi-uniform mesh of size h is used then these estimators are not asymptotically exact because of the effects of pollution from the nonconvex corner which precent averaging from producing a better approximation. Optimally refined grids are then considered and a method for obtaining asymptotically exact estimators via averaging is proposed. (Joint work with W. Hoffmann and G. Wittum)

M. Schemann, Berlin, Germany

An adaptive Rothe method for the wave equation

The adaptive Rothe method approaches a time-dependent PDE as an ODE in function space. This ODE is solved virtually using an adaptive state-of-the-art integrator. The actual realization of each time-step requires the numerical solution of an elliptic boundary value problem, thus perturbing the virtual function space method.

We considered the adaptive Rothe method for hyperbolic equations in the model situation of the wave equation. All steps of the construction were given and an numerical example (diffraction at a corner) was provided for the 2D wave equation.

J. Schöberl, Linz, Austria

Efficient solvers for 3D contact problems on adaptive meshes

The boundary value problem of linear elasticity with unlateral boundary conditions is considered. We give two preconditioning techniques to separate boundary inequalities from inner equations, approximatively. One is based on Dirichlet domain decomposition, the other uses the augmented Lagrangian method. Both lead to level-independent iteration numbers. Each iterative step requires the approximative solution of a constrained minimization problem on the boundary. On adaptive meshes these CMPs are solved by CG-like quadratic programming algorithms. Both algorithms require only standard linear multi-level components. Numerical examples for 2D and 3D contact problems showing optimal time on adaptive meshes are presented.

E. Stein, Hannover, Germany

Adaptive hierarchical modeling of plates and shells with the finite-element-method

Given a sequence of reduced models of a continuous master model by homomorphic mapping, e.g. from the 3D-elastoplasticity down to 2D-elastic shell theory of lowest order, an expansion strategy is outlined on the basis of error-controlled adaptive finite-element-method (FEM). Within the approximation process the solution(discretization)-error and the model-error are both controlled. Model adaptivity is essential for stiffened plates and shells with layer disturbances and other singularities (e.g. in the vicinity of single columns).

Local error analysis using Dirichlet-problems with higher hierarchical test spaces (Lagrange- or Legendre-Polynomials)only admit an error analysis of the discretization- and of the dimensional errors but not for the model-error.

Local error analysis of Neumann problems, in which the outer fractions on patches are computed in a posteriori equilibration process, admit local error estimates with upper bounds and a split of the discretization and the model-error by computing local problems.

The strategy is realized for complex structures. Some open problems – like the saturation condition and the implementation of the pollution error – are topics of current research.

P.S. Vassilevski, Sofia, Bulgaria

Multilevel methods as block-factorization preconditioners

In this talk a unified block matrix presentation of some known multilevel methods will be given. The multilevel methods are treated as block-factorization preconditioners exploiting in general overlapping blocks. Some specific application are the classical HB (hierarchical basis), MG (multigrid) and a wavelet-like modified HB method. Also, multilevel methods that exploit certain algebraic coarsening strategies may fit into the scheme. One particular example of the algebraic coarsening strategy, i.e., based on matrix dependent intergrid transfer operators for non-conforming elements will be given. Some numerical experiments in 3D for the performance of various multilevel methods will also be presented.

R. Verfürth, Bochum, Germany

Robust a posteriori error estimates for singularly perturbed problems

As a model problem consider the singularly perturbed reaction-diffusion equation $-\varepsilon\Delta u + u = f$ in Ω with Dirichlet boundary conditions. Standard a posteriori error estimates applied to this problem yield upper and lower bounds the ratio of which behaves like $\varepsilon^{-1/2}$ or even ε^{-1} as $\varepsilon \to 0$. We overcame this drawback by modifying the weights of the different residual contributions. The resulting error estimator yields upper and lower bounds which are uniform in ε. The main tools are a trace theorem in deriving upper bounds and judiciously chosen local test function in deriving lower bounds. The techniques extend in particular to convection-diffusion equations with dominant convection. The lower bound then incorporates a term $\varepsilon^{-1/2}$ Peclet, where Peclet is the local mesh Peclet number.

C. Wagner, San Diego, USA

Adaptive methods for diffusion-reaction-transport processes in unsaturated porous media

In the first part, the multilevel ILU decomposition is introduced. Allowing new nonzero matrix entries only for connections to so-called parents nodes, a sparse approximation of the arising Schur-complements is guaranteed. A special construction scheme for those nonzero matrix entries makes sure that the decomposition exists and the corresponding iterative method converges for symmetric and positive definite matrices. Numerical experiments show h-independent convergence rates at least for model problems.

In the second part, an almost real life example for diffusion-reaction-transport processes in unsaturated porous media is discussed. The hierarchical movement algorithm for the dynamic grid adaption to the current solution is introduced. The solution process and simulation results are presented for an example case described by a coupled system of three partial differential equations.

W.L. Wendland, Stuttgart, Germany

On localized error estimators and adaptivity for boundary integral equations

We consider an integral equation

$$Au = f \tag{1}$$

on a boundary Γ of a domain $\Omega \in \mathbb{R}^{2,3}$ with given right hand side f and a strongly elliptic integral operator A of order 2α. For solving equation (1) we use the spline Galerkin method. This yields the Galerkin error $e_h = u - u_h$ and the computable Galerkin residue $r_h = f - Au_h$. Using the error equation, the global a posteriori error estimation

$$c_l \|r_h\|_{H^{s-2\alpha}(\Gamma)} \leq \|e_h\|_{H^{\alpha}(\Gamma)} \leq c_r \|r_h\|_{H^{s-2\alpha}(\Gamma)} \tag{2}$$

is obvious. We begin with some localized error estimates based on a localization of the error equation and the commutator property for pseudodifferential operators and C^∞-truncation functions which are independent of the Galerkin discretization. One obtains an error estimation on fixed parts (independent of the discretization) of the boundary with some perturbations which are of smaller order of the meshsize than the error itself. To prove mesh-dependent localized error estimates we use mesh-dependent truncation functions. Here, the main difficulty is the generalization of the commutator property of pseudodifferential operators for nonsmooth truncation functions. We obtain a mesh-dependent localized error estimation with an explicit relation between the smoothness of the truncation function (which is related to the size of the local supports) and the size of the perturbation terms.

In addition we present a method to compute the norm of the localized residue by solving local problems. For the localized equations we can use the same method as for the global equation (1); the only difference is the numerical realization of the commutators which, however, is only of technical nature. The efficiency of these methods is shown with some two-dimensional numerical examples. (Joint work with H. Schulz)

C. Wieners, Stuttgart, Germany

Adaptive multigrid methods for finite elements

We presented a general concept for the implementation of adaptive multigrid methods for finite elements. We considered conforming P_1 and P_2 elements, linear nonconforming elements and mixed RT_0, RT_1 and BDM elements. The adaptive solution process and various aspects of the parallelization were discussed in detail. Examples for the diffusion problem and in linear elasticity on locally refined grids of mixed type consisting of triangles and quadrilaterals in two dimensions and tetrahedrons, pyramids, prisms and hexahedrons were presented. Finally, we demonstrated the application of the parallel multigrid method on a problem in elastoplasticity.

B. Wohlmuth, Augsburg, Germany

A posteriori error estimators for Mortar finite element methods

In this talk, we are concerned with Mortar finite element methods for linear second order elliptic boundary value problems. We restrict us to the geometrical conforming situation where the intersection between the boundary of the different nonoverlapping subdomains is either empty, a vortex or a common face. In the first part of this talk, we will focus on the coupling between standard P1 conforming and P1 conforming finite elements. Based on a priori estimates for the error, we will investigate a residual based as well as a hierarchical error estimator. At the interfaces we have to take into account the Lagrange multiplier which provides an approximation of the normal derivative of the weak solution and the jump of the finite element solution. In the case of the hierarchical error estimator we have to use an adequate saturation assumption to obtain reliable and efficient error estimators. In principle, there are two different possibilities for the construction. The first one is based on the solution of Neumann boundary value problems on the subdomains and the second one on higher order ansatz functions for the Lagrange multipliers. Finally, we consider the coupling between standard conforming finite elements and mixed finite discretizations. This coupling can be realized without the use of Lagrange multipliers at the interfaces.

H. Yserentant, Tübingen, Germany

Coarse grid spaces

It has been shown that, with homogeneous Dirichlet boundary conditions, the condition number of finite element discretization matrices remains uniformly bounded independent of the size of the boundary elements, provided that the size of the elements increases with their distance to the boundary. This fact allows the construction of simple multigrid methods of optimal complexity for domains of nearly arbitrary shape with Lipschitz-continuous boundary.

Berichterstatter: Peter Leinen, Tübingen, Germany

e-mail and www-addresses:

Eberhard Bänsch	eberhard@mathematik.uni-freiburg.de
Randy Bank	rbank@ucsd.edu
Peter Bastian	peter @ica3.uni-stuttgart.de
Jürgen Bey	juergen@na.uni-tuebingen.de
Dietrich Braess	braess@num.ruhr-uni-bochum.de
Wolfgang Dahmen	dahmen@igpm.rwth-aachen.de
	http://www.igpm.rwth-aachen.de/˜dahmen
Willy Dörfler	willy@mathematik.uni-freiburg.de
Todd Dupont	dupont@cs.uchicago.edu
	http://www.cs.uchicago.edu/˜dupont
Richard E. Ewing	ewing@isc.tamu.edu
Joseph E. Flaherty	flaherje@cs.rpi.edu
	http://www.cs.rpi.edu/˜flaherje.html
Michael Griebel	griebel@iam.uni-bonn.de
	http://www.wissrech.iam.uni-bonn.de
Wolfgang Hackbusch	wh@numerik.uni-kiel.de
	http://www.numerik.uni-kiel.de/wh/
Pieter W. Hemker	pieth@cwi.nl
	http://www.cwi.nl/˜pieth
Ralf Hiptmair	hiptmair@math.uni-augsburg.de
	http://www.hoppe.math.uni-augsburg.de/˜hiptmair
Ronald H.W. Hoppe	hoppe@math.uni-augsburg.de
Helmut Jarausch	jarausch@igpm.rwth-aachen.de
Ralf Kornhuber	kornhuber@mathematik.uni-stuttgart.de
Rolf Krause	krause@mathematik.uni-stuttgart.de
Dietmar Kröner	dietmar@mathematik.uni-freiburg.de
Peter Leinen	leinen@na.uni-tuebingen.de
	http://na.uni-tuebingen.de/˜peter
Bill Mitchell	william.mitchell@nist.gov
	http://math.nist.gov/acmd/staff/wmitchell/
Hans D. Mittelmann	mittelmann@asu.edu
	http://plato.la.asu.edu/
Peter Oswald	peter.oswald@gmd.de
Rolf Rannacher	rannacher@iwr.uni-heidelberg.de
Stefan Sauter	sas@numerik.uni-kiel.de
	http://www.numerik.uni-kiel.de/sas/
Ridgway Scott	scott@uh.edu
	http://www.hpe.uh.edu

Endre Süli	endre.suli@comlab.ox.ac.uk
	http://www.comlab.ox.ac.uk
Alfred H. Schatz	schatz@math.cornell.edu
Martin Schemann	schemann@zib.de
Joachim Schöberl	joachim@numa.uni-linz.ac.at
Erwin Stein	stein@leibniz.ibnm.uni-hannover.de/
Christian Wagner	cwagner@sdna2.ucsd.edu
	http://www.ica3.uni-stuttgart.de/~chris
Wolfgang Wendland	wendland@mathematik.uni-stuttgart.de
Christian Wieners	wieners@ica3.uni-stuttgart.de
Gabriel Wittum	gabriel@ica3.uni-stuttgart.de
Barbara Wohlmuth	wohlmuth@math.uni-augsburg.de
Harry Yserentant	yserentant@na.uni-tuebingen.de
	http://www.na.uni-tuebingen.de/~harry/
Christoph Zenger	zenger@informatik.tu-muenchen.de

Tagungsteilnehmer

Prof.Dr. Randolph E. Bank
Dept. of Mathematics
University of California, San Diego

La Jolla , CA 92093-0112
USA

Prof.Dr. Wolfgang Dahmen
Institut für Geometrie und
Praktische Mathematik
RWTH Aachen
Templergraben 55

52056 Aachen

Dr. Eberhard Bänsch
Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Str. 10

79104 Freiburg

Willy Dörfler
Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Str. 10

79104 Freiburg

Dr. Peter Bastian
Institut für Computeranwendungen
Numerik für Höchstleistungsrechner
Universität Stuttgart

70550 Stuttgart

Prof.Dr. Todd F. Dupont
Dept. of Math. and Comp. Science
The University of Chicago
Ryerson Hall
1100 East 58th St.

Chicago , IL 60637
USA

Jürgen Bey
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

72076 Tübingen

Prof.Dr. Richard E. Ewing
Institute for Scient. Computation
Texas A&M University

College Station , TX 77843-3404
USA

Prof.Dr. Dietrich Braess
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Prof.Dr. Joseph E. Flaherty
Department of Computer Science
Rensselaer Polytechnic Institute
104 Amos Eaton Hall

Troy , NY 12180-3590
USA

Prof.Dr. Michael Griebel
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

53115 Bonn

Dr. Helmut Jarausch
Institut für Geometrie und
Praktische Mathematik
RWTH Aachen
Templergraben 55

52062 Aachen

Prof.Dr. Wolfgang Hackbusch
Mathematisches Seminar
Universität Kiel

24098 Kiel

Prof.Dr. Ralf Kornhuber
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57

70569 Stuttgart

Prof.Dr. Pieter W. Hemker
Centrum voor Wiskunde en
Informatica
Kruislaan 413

NL-1098 SJ Amsterdam

Rolf Krause
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57

70569 Stuttgart

Dr. Ralf Hiptmair
Lehrstuhl für Angewandte
Mathematik I
Universität Augsburg
Universitätsstr. 14

86159 Augsburg

Prof.Dr. Dietmar Kröner
Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Str. 10

79104 Freiburg

Prof.Dr. Ronald H.W. Hoppe
Lehrstuhl für Angewandte
Mathematik I
Universität Augsburg
Universitätsstr. 14

86159 Augsburg

Dr. Peter Leinen
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

72076 Tübingen

Dr. William F. Mitchell
Applied and Computational Math.
Division
N.I.S.T.
Building 820

Gaithersburg , MD 20899
USA

Prof.Dr. Alfred H. Schatz
Dept. of Mathematics
Cornell University
White Hall

Ithaca , NY 14853-7901
USA

Prof.Dr. Hans D. Mittelmann
Department of Mathematics
Arizona State University
Box 87

Tempe , AZ 85287-1804
USA

Martin Schemann
Konrad-Zuse-Zentrum für
Informationstechnik Berlin (ZIB)
Takustr. 7

14195 Berlin

Prof.Dr. Peter Oswald
SCAI, GMD
Schloss Birlinghoven

53754 Sankt Augustin

Joachim Schöberl
Institut für Mathematik
Universität Linz
Altenbergerstr. 69

A-4040 Linz

Prof.Dr. Rolf C. Rannacher
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 293

69120 Heidelberg

Heiko Schulz
Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57

70569 Stuttgart

Dr. Stefan A. Sauter
Institut für Informatik und
Praktische Mathematik, Haus I
Universität Kiel

24098 Kiel

Prof.Dr. Ridgway Scott
Department of Mathematics
University of Houston

Houston , TX 77204-3476
USA

Prof.Dr. Erwin Stein
Institut für Baumechanik und
Numerische Mechanik
Universität Hannover
Appelstr. 9A

30167 Hannover

Prof.Dr.-Ing. Wolfgang L. Wendland
Mathematisches Institut A
Universität Stuttgart

70550 Stuttgart

Dr. Endre Süli
Computing Laboratory
Oxford University
Wolfson Building
Parks Road

GB-Oxford OX1 3QD

Dr. Christian Wieners
Institut für Computeranwendungen
Numerik für Höchstleistungsrechner
Universität Stuttgart
Pfaffenwaldring 27

70569 Stuttgart

Prof.Dr. Panayot S. Vassilevski
Centr.of Computer Science + Techn.
Bulgarian Academy of Sciences
"Acad. G. Bontchev" str., block 25A

1113 Sofia
BULGARIA

Prof.Dr. Gabriel Wittum
Institut für Computeranwendungen
Numerik für Höchstleistungsrechner
Universität Stuttgart

70550 Stuttgart

Prof.Dr. Rüdiger Verfürth
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Dr. Barbara Wohlmuth
Lehrstuhl für Angewandte
Mathematik I
Universität Augsburg
Universitätsstr. 14

86159 Augsburg

Christian Wagner
Dept. of Mathematics
University of California, San Diego
Mail Code 0142
9500 Gilman Drive

La Jolla , CA 92093-0112
USA

Prof.Dr. Harry Yserentant
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

72076 Tübingen

Prof.Dr. Christoph Zenger
Institut für Informatik
TU München

80290 München