MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 9/1997
Quantenfeldtheorie und Wellenfronten
02.03-08.03.1997

Recently it has turned out that microlocal techniques can be applied very fruitful to quantum field theory on curved spacetime. However microlocal analysis is still quite unknown to physicists while mathematicians are in general not aware of the new field of applications in physics. Therefore the idea of this workshop, which was organized by Klaus Fredenhagen (Hamburg), Bert-Wolfgang Schulze (Potsdam) and Eberhard Zeidler (Leipzig) was to bring physicists woking on quantum field theory and mathematicians working on microlocal analysis together and to give them the opportunity to learn something of the research of the other group. Some of the subjects corvered in the talks are

- Fundamentals of quantum field theory,
- Fundamentals of microlocal analysis,
- Applications of microlocal techniques to quantum field theories on curved spacetime,
- Pseudo differential operators on manifolds with conical singularity,
- Analytic microlocalization.

Quantum field theory – Introductory remarks for mathematicians
Arthur E. Wightman

Quantum electrodynamics of the free electromagnetic field (no charges or currents) is a useful example on which to display the operation of quantization: Maxwell's theory of the electromagnetic field + quantum mechanics $\Rightarrow$ quantum electrodynamics [Jordan and Pauli, Zeitschr. fr Physik 47, 151-173 (1928)]. The resulting electromagnetic field observable is an operator-valued distribution, which transforms under the unitary representation of the Poincar group like: $U(a,\Lambda)F_{\kappa,\lambda}(x)U(a,\lambda)^{-1} = \Lambda^{\mu}_{\kappa}\Lambda^{\nu}_{\lambda}F_{\mu\nu}(\Lambda x + a)$. The representation $U(a,\Lambda)$ expresses some of the basic properties of the theory: the characterization of the vacuum state $U(a,\Lambda)\Psi_0 = \Psi_0$ and the spectral condition $U(a,1) = \exp(iP^{\mu}a_{\mu})$, $\mathrm{sp}(P^{\mu}) \subset \bar{V}_+ = \{p \,|\, p{\cdot}p \geq 0,\ p^0 \geq 0\}$. A characterization of the theory in terms of vacuum expectation values of products of fields leads to the conclusion that such vacuum expectation values are boundary values of holomorphic functions. This leads to a formal solution of some Lagrangian theories by quadratures (constructive quantum field theory). When all this is

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

applied to gauge fields serious difficulties arise, even for the free field case. They can be got around by enlarging the state space and introducing an indefinite metric (Gupta-Bleuler formalism). The geometry of polarization vectors can be used to motivate this procedure.

An introduction to microlocal analysis and wave front sets
L. Rodino

We present a survey on wave front sets and applications. Attention is fixed, for sake of simplicity, on definition of Hrmander, 1971, in the C^∞-Schwartz distributions frame, though other equivalent definitions and variants appear to have some relevance.

The application to the problem of the product of two distributions is discussed first, and related elementary examples are presented. Concerning the general theory of the linear partial differential operators with smooth coefficients, the following two basic applications are reviewed:

The theorem of micro-ellipticity, asserting that solutions are micro-regular at a certain point, if the datum is micro-regular and the equation is micro-elliptic at the same point.

The theorem of propagation of singularities, showing that propagation of the wave front set for an operator of real principal type takes place along bicharacteristic strips.

Quantum fields in curved space-time
Bernard S. Kay

This talk was intended to introduce quantum field theory in curved spacetime to mathematicians familiar with PDEs (and microlocal analysis) but not necessarily with quantum field theory. It focussed on a linear model: the covariant Klein-Gordon equation, $(\Box_g + m^2)\phi = 0$ on a globally hyperbolic space-time (M, g). The main moral of the talk was that any problem concerning the corresponding quantum theory may be reduced to a question about the set of all distributions W_2 on $M \times M$ satisfying for all $F_1, F_2 \in C_0^\infty(M)$ the conditions ($E = E^+ - E^-$ denotes the advanced minus retarded Greens function):

1. $W_2(F_1 \otimes F_2) = W_2(F_2 \otimes F_1)$ (symmetry)
2. $(W_2(\Box_g + m^2)F_1 \otimes F_2) = W_2(F_1 \otimes (\Box_g + m^2)F_2) = 0$ (i.e. G is a bisolution of the Klein-Gordon equation)
3. $W_2(F_1 \otimes F_1)W_2(F_2 \otimes F_2) \geq |E(F_1 \otimes F_2)|^2$ (positivity)
4. A suitable generalization to curved space-time of the well known universal short-distance behaviour of the (unsmeared) symmetrized two-point function of physically interesting states in Minkowski space.

Item 4 used to be dealt with by the "Hadamard condition" but an alternative (the "microlocal spectrum condition") which promises many advantages has recently been proposed by Marek Radzikowski.

The physical significance of this set of $W_2's$ is that they represent the symmetrized two-point distributions of the set of physically interesting quantum states on the *-algebra of smeared quantum fields satisfying the commutation relations

$$[\hat{\phi}(F_1), \hat{\phi}(F_2)] = iE(F_1 \otimes F_2)\mathbf{1} \tag{1}$$

together with linearity and the (weak) Klein-Gordon equation.

An analogy with the harmonic oscillator (1+0 dimensional quantum field theory!) was used to explain how curved backgrounds lead to creation of particle pairs, to explain the origin of eq. (1). and last but not least to explain why the concepts "vacuum" and "particles" are inappropriate in general: All states (i.e. all distributions G) are on an equal footing and one *must* use the algebraic approach to QFT.

Meditation on spacetime singularities

A. Rendall

The intention of this talk was to present various physical and mathematical aspects of the notion of a spacetime singularity. Physically this can be thought of as a region where extreme conditions cast doubt on the applicability of known physical laws. The relevance of quantum gravity and quantum field theory on fixed background to this situation was discussed, but the main emphasis was on the classical Einstein equations. It was pointed out how the mathematical problem of the global properties of solutions of the Einstein equations can be situated within the more general context of nonlinear hyperbolic systems. The standard mathematical definition of a spacetime singularity used in the singularity theorems of Penrose and Hawking, is in terms of geodesic completeness. These theorems do not imply that curvature becomes large, as illustrated by the Misner spacetime. The question of the stability of geometries of this kind is related to the study of classical or quantum fields on such a geometry which can be attacked by using microlocal analysis. The question of the nature of spacetime singularities is largely open but those results which have been obtained, and conjectures which have been made, where summarized.

Fourier integral operators and Wightman functionals

Marek Radzikowski

We view the Wightman two-point distribution W_2 of a quasifree scalar Klein-Gordon field, satisfying the global Hadamard condition, on a globally hyperbolic curved spacetime as a Fourier integral operator, by identifying it as the difference of two Duistermaat-Hrmander distinguished parametrices. Physical interpretations of the wave front set of W_2, which is restricted to positive spectrum and the propagation of singularities theorem as applied to W_2 are discussed.

Construction of Hadamard states

Wolfgang Junker

The Hadamard states are the physical states of the linear Klein-Gordon quantum field $\hat{\phi}$ on a globally hyperbolic spacetime manifold (M, g). In this talk their existence was shown and it was discussed, how they can be constructed by microlocal techniques. In particular the two-point distribution $W_2(f_1, f_2) = \langle\psi, \hat{\phi}(f_1)\hat{\phi}(f_2)\psi\rangle \in \mathcal{D}'(M \times M)$ of a pure Hadamard state can be characterized by two pseudodifferential operators R and I (such that R is symmetric, I elliptic, selfadjoint, positive, invertible) on $L^2_{\mathbb{R}}(\Sigma, d^3\sigma)$ with respect to a Cauchy surface Σ in the following form:

$$W_2(f_1, f_2) = \langle (R - iI - n^a\nabla_a)Ef_1, I^{-1}(R - iI - n^a\nabla_a)Ef_2\rangle$$

where n^a is the future pointing normal vector field on Σ and $E := E^+ - E^-$ is the causal propagator). It was shown that R and I can be constructed by a factorization of the Klein-Gordon operator into first order factors with the help of asymptotic expansion.

Equations on singular spaces and pseudo-differential calculus with operator-valued symbols

B.-W. Schulze

The analysis of partial differential equations on manifolds with singularities gives rise to new classes of pseudo-differential operators, expressed in terms of hierarchies of operator-valued symbols and associated operator levels. The singularities may be defined by degenerate Riemannian metrics which correspond to (warped) cones, wedges corners or higher polyhedral singularities, cusps, non-compact exits to infinity, and many types of combinations of such configurations. The associated operators (in particular Laplace-Beltrami operators) are degenerate in a typical way, and there are cone (Fuchs-), edge-, corner-, cusp-degenerate operators. The program to construct parametrices in the elliptic case requires the interior elliptic symbols together with conormal symbols, edge symbols, corner symbols etc. The latter ones are operator-valued, and their ellipticity means bijectivity for every point in the parameter space. Parametrices are then obtained by inverting symbols and constructing corresponding operators. This yields, in particular, elliptic regularity with asymptotics in weighted Sobolev spaces and, globally, the Fredholm property. Also parabolicity can be treated in the context of Volterra operators. Many problems are sill an enormous challenge, in particular, concerning hyperbolic equations on singular spaces, though there exist interesting special results.

Interacting quantum field theory on curved spacetime

Klaus Fredenhagen

Wightman fields are operator valued distributions $\phi(f) : D \to D$ where f denotes a test function and D a dense domain in some Hilberspace. They are assumed to satisfy certain local and global conditions. If Minkowski space is replaced by a globally hyperbolic spacetime, the local properties (field equations, commutation relations) can easily be generalized, but the global properties which typically use the translation symmetry of Minkowski space have no obvious counter part. So neither the spectrum condition nor the concept of a vacuum or of particles are declared. But these properties describe a stability property of the quantum system which should be preserved in a curved spacetime.

For free quantum fields Radzikowski found an appropriate local version of the spectrum condition in terms of the wave front set of the Wightman functions $W_n(x_1, \ldots, x_:n) = \langle\psi, \phi(x_1)\ldots\phi(x_n)\psi\rangle$, $\psi \in D$ and proved that this condition is equivalent to the so called Hadamard condition. Using this property Wick polynomials of the free field could be constructed and the wave front set of their Wightman functions was determined. This led to a conjecture on the wave front set of Wightman functions of interacting fields. In order to test this conjecture interacting quantum fields where constructed in the sense of formal power series (see the contribution of Romeo Brunetti). As an intermediate step an ansatz for timeordered products of Wick polynomials was made: $TA_1(x)\ldots A_n(x_n) = \sum t_l(x_1, \ldots, x_n)A_1^{l_1}(x_1)\ldots A_n^{l_n}(x_n)$ with Wick polynomials $A_i, A_i^{l_i}$ and numerical distributions t_l. Provided the wave front sets of t_l are in a certain set Γ_n^{to}, the product above defines an operator valued distribution. An inductive construction of the t_l with the required properties was described in the talk of R. Brunetti.

Renormalization in curved space-time

Romeo Brunetti

This talk was a continuation of the talk given by K. Fredenhagen. The inductive procedure with which it is possible to define the perturbation theory for scalar fields with polynomial interaction on globally hyperbolic manifolds is sketched. The main point was to construct inductively the time ordered distributions t_l mentioned in the talk of Fredenhagen. They where first constructed on the manifold $M_l \setminus \Delta_l$, where Δ_l is the full diagonal, and then it was shown that the problem of extending them to the full manifold M^l is equivalent to renormalising the theory. The tools for this task are the singular degree of t_l at Δ_l (which is connected to the superficial degree of divergence for Feynman diagrams) and its microlocal extension. The latter is crucial for proving that tensor products of distributions have of distributions have singular degree that is the sum of the terms of the product.

Finally it was shown that the procedure of extending t_l to the full manifold M^l is consistent with the basic requirements and that moreover, one obtains the Weinbergs power counting theorem classifying theories as renormalizable or not as much as in the Minkowski case.

Maslov's canonical operator and Fourier integral operators
Boris Sternin, Victor Shatalov

The aim of this talk was the presentation of the theory of Fourier integral operators. At present there are several versions of constructing this theory both from the operational viewpoint and from the point of view of applications (smoothness or expansions in a small parameter).

For the presentation the construction of Fourier integral operators with the help of the semi-classical version of Maslov's canonical operator was chosen. The presentation was inductive and uses the example of the Cauchy problem for Schrdinger operators. Starting with rapidly oscillating initial data interpreted as an initial Lagrangian manifold it was shown that with the help of the phase flow of this manifold and of the corresponding transport equation one can find the semi-classical asymptotics of the Cauchy problem "in the large". The solution of the Cauchy problem with arbitrary initial data can be reduced to this case. Its solution has the form $u(x,t) = \int K(t,x,y)u_0(y)dy$ where $K(t,x,y)$ is an application of Maslov's canonical operator to a special function.

The right-hand side of the obtained expression is exactly the (family of) Fourier integral operators applied to the function $u_0(x)$. So Fourier integral operators are defined as integral operators $f \mapsto \int K(x,y)f(y)dy$ with "canonically represented" kernel $K(x,y)$ (i.e. K is a function in the image of the canonical operator).

Quantum field theory on de Sitter spacetime
Jacques Bros

De Sitter spacetime, which was one of the earliest known solutions of Einstein's equation (with cosmological constant R) may be represented as the one sheeted (d-dimensional) hyperboloid X_d $(x_0^2 - \cdots - x_d^2 = -R^2)$ embedded in $d+1$-dimensional Minkowski space $\mathbb{R}^{d+1}$. This spacetime manifold is globally hyperbolic and it presents so close similarities with the flat Minkowski spacetime (existence of global symmetry group $SO(1,d)$, existence of a complexified manifold $X_d^{(c)}$ equipped with tuboid domains and a "euclidean submanifold" $S_d = (i\mathbb{R} \times \mathbb{R}^{d-1}) \cap X_d^{(c)}$) that a general Wightman-type approach for quantum field theory on this universe is made possible. One of the major problems of quantum field theory on curved spacetime, namely "giving a substitute to the spectral condition" is solved here by prescribing global analyticity domains for the n-point functions of the theory, which closely reproduce those implied by the spectral condition in the Minkowskian case. The presentation is of the same nature as those which have been formulated in terms of wave front sets by previous lectures, but in this special case it can be made "global" instead of "microlocal". The method allows one to give a detailed treatment of (generalized) free fields on this space and a proof of a Kllen-Lehmann type representation for two-point functions of a general class of interacting fields [J. Bros, U. Moschella, Rev. Math. Phys 8 (1996) 327-391]. A few steps in the general study of interacting fields have also been taken [J. Bros, H. Epstein, U, Moschella, in preparation] they include the thermal interpretation of these theories, the Reeh-Schlieder property and preliminary results of a perturbation theory.

Analytic wave front of N-particle scattering functions in Minkowskian quantum field theory

Daniel Iagolnitzer

Several talks in this meeting have presented a wave front condition in curved spacetime that may replace the usual spectrum condition in Minkowskian spacetime. Here we present a different domain in which wave fronts have played a crucial role already in Minkowskian spacetime.

In the latter, the analytic wave front, rather than the C^∞ wave front, is best adopted and plays a crucial role in the study of causality and/or analyticity properties of N-particle scattering functions. In the talk the relevant mathematical definition was recalled and a general result, derived from locality and the spectral condition, on the analytic wave front of N-particle chronological functions and scattering functions was presented. It corresponds to the idea that energy-momentum propagates from initial to final points only in future cones, modulo exponential fall-off, in an asymptotic (classical) limit.

More refined results, corresponding to the idea that energy-momentum propagates more precisely via real stable intermediate particles, are also mentioned. Partial results of this type have been obtained in axiomatic field theory from the further axiom of "asymptotic completeness", and further ones have been achieved in constructive field theory.

The situation in theories with charged massive particles is outlined. A Buchholz-Fredenhagen analysis indicates that fields creating charged physical states are no longer necessarily local (even though basic observables are) but "strings" may have to be attached to each point. This considerably weakens results that may be derived for chronological functions but results on scattering functions (which do not depend on choice of "strings") appear to be weakened in a more limited way.

The Dirac pseudodifferential analysis

Andr Unterberger

Solving the free Dirac equation with initial data on some spacelike hyperplane $\mathbb{R}^3$ permits to identify $\mathbb{C}^4$-valued functions on $\mathbb{R}^3$ with certain functions on the whole spacetime. One then benefits from the symmetries arising from the Poincar group representation, from which it is possible to derive the definition of a new symbolic calculus of operators on $L^2(\mathbb{R}^3, \mathbb{C}^4)$, covariant under the afore-mentioned Poincar symmetries together with the so-called discrete symmetries C,P and T as well.

This is part of a long-term program the aim of which is to associate symbolic calculi of operators with species of elementary particles characterized by free wave equations (the free Schrdinger equation would yield the Weyl calculus as a result).

The Klein-Gordon analysis has been fully developed as a pseudo-differential analysis, just as good as the Weyl analysis: it has been applied to the study of some generalizations of the hypergeometric equation and of a deformation of spherical-function theory on rank-one symmetric spaces.

Noncommutative residues for manifolds with conical singularities

E. Shrohe

In 1984, M. Wodzicki showed that on the quotient algebra $\Psi_{cl}/\Psi^{-\infty}$ of all classical pseudodifferential operators modulo the ideal of smoothing elements there is a unique trace which he called the noncommutative residue.

Nowadays, the noncommutative residue plays an important role both in mathematics and mathematical physics. There are relations to Connes' noncommutative geometry, spectral theory, heat kernel asymptotics, KdV-equations, central extensions in QFT, and to the theory of gravitation.

Here we considered manifolds with conical singularities and the associated "cone algebra with asymptotics" introduced by Schulze. It turns out that for each conical point there is a trace on the cone algebra. Each of them vanishes on operators supported on the interior and is therefore different from Wodzicki's noncommutative residue. On the ideal of operators with vanishing conormal symbol however, we find another trace which coincides with Wodzicki's on operators in the interior. Moreover it can be shown that all these traces are essentially unique on a slightly extended version of the cone algebra.

Analytic microlocalization and Fourier transforms

Otto Liess

The talk was meant to give an elementary discussion of a number of results in analytic microlocalization. Special emphasis was put on the simultaneous use of the geometrical, respectively analytical point of view. New results on the use of a local form of the Fourier transform to hyperfunctions which will be sketched in the following was also reviewed.

Let us start with a function $f : \mathbb{C}^n \to \mathbb{C}$ so that $f \exp(-\delta|\operatorname{Re}\zeta| + \epsilon|\operatorname{Im}\zeta|) \in L^2(\mathbb{C}^n)$ for any $\delta > 0$. If there is a convex cone $G \subset \mathbb{R}^n$ so that $\operatorname{supp} f \subset \{\zeta | \operatorname{Re}\zeta \in G\}$ than we can define $\mathcal{F}^{-1}f$ as the hyperfunctional boundary value in $B(|x| < \epsilon)$ of the function $h(z) = \int \exp(i\langle z, \zeta\rangle) f(\zeta) d\lambda(\zeta)$ ($d\lambda$ denotes here the Lebesgue measure on $\mathbb{C}^n$). The function $h(z)$ is of course analytic for $\{z = x + iy \,|\, |x| < \epsilon,\ y \in G^\circ\}$. If the above support condition is not satisfied, we decompose f into a sum $f = f_0 + \sum_{i=1}^{n+1} f_i$, where $\operatorname{supp} f_0 \subset \{\zeta \,|\, |\operatorname{Re}\zeta| \leq M\}$ with $M > 0$ and $\operatorname{supp} f_i \subset \{\zeta | \operatorname{Re}\zeta \in G_i\}$ for some convex open cones G_i with $\bigcup_{i=1}^{n+1} G_i = \dot{\mathbb{R}}^n$. We then define $u = \mathcal{F}^{-1}f = \sum_{i=1}^{n+1} \mathcal{F}^{-1}f_i$ and call f a Fourier transform of u. Any $u \in B(|x| < \epsilon)$ is of the form $u = \mathcal{F}^{-1}f$ for some suitable f, but f is not unique.

One can microlocalize this and can show that if $(0, \xi^0) \notin WF_A u$ then u can be represented near 0 in the form $u = \mathcal{F}^{-1}f$ where f satisfies in addition to the above condition also the condition $f \exp(-\delta|\operatorname{Re}\zeta| + \epsilon|\operatorname{Im}\zeta|) \in L^2(X)$ where $X = \{\zeta \in \mathbb{C}^n \mid \operatorname{Re}\zeta \in G\}$ for some suitable cone G which contains ξ^0. Applications to the theory of analytic pseudodifferential operators are given.

Geometrical aspects of gauge fixing

R. Stora

The perturbative regime of Quantum Electrodynamics and its non abelian analogs (Yang-Mills and generalization) involves in its very definition some arbitrariness referred to as gauge fixing. One of the basic problems is proving that the physics produced by such models does not depend on this arbitrariness which only affects intermediate calculations. The geometrical picture associated with gauge fixing can be summarized as expressing an integral over some $\mathcal{G}$-orbit space $P/\mathcal{G}$ ($\mathcal{G}$ denotes the gauge group) as integral over P. The "physics" lives on $P/\mathcal{G}$ and, when expressed in P is characterized in terms of a cohomology related to the cohomology of $\mathcal{G}$ with compact supports. This was an observation made some twenty years ago, and is now a geometrical fact. Of course, in the field theory setting all the corresponding arguments are completely formal and their correctness has to be checked with due care.

Local aspects of Tomita-Takesaki theory in quantum field theory

H.-J. Borchers

This talk has started with a review of the Reeh-Schlieder property of quantum field theory and with a review of Tomita-Takesaki theory. Both together imply for each open bounded region of spacetime the existence of a certain unitary group Δ^{it}, the modular group, which maps the observables located in this region to itself. The algebraic invariant of this group, the Connes spectrum, and its connection with the type of the local von Neumann algebra was dicussed. In addition the case of two regions $\mathcal{O}_1 \subset \mathcal{O}_2$ was treated and methods for the determination of the algebraic invariants was shown. The remainder of the lecture was dedicated to examples and applications to quantum field theory.

Superselection sectors in curved spacetime

J. E. Roberts

The essential part of the theory of superselection sectors involves neither global symmetries, such as translations, nor the concept of vacuum state. Instead only three properties of the vacuum representation are involved: irreducibility, the Borchers property and duality. Thus there is no obvious obstruction to generalizing from Minkowski space to a globally hyperbolic spacetime. Furthermore the basic question posed: analyze all representations of the observables net equivalent to the vacuum representation in restrictions to the spacelike complements of double cones, involves just the causal structure of our spacetime. R. Verch has shown that pure quasifree Hadamard states of Klein-Gordon fields provide representations with the properties needed to replace the vacuum representation.

If the globally hyperbolic spacetime has a non-compact Cauchy surface the analysis go through as in Minkowski space yielding the usual classification of statistics and the existence of a compact gauge group. The absence of braid statistics (for spacetime dimensions > 2) hinges on the fact that the set of pairs of spacelike separated points is a pathwise connected open set.

In the case of a compact Cauchy surface the analysis is incomplete. Permutation statistics can be established. However the construction of a left inverse has to be modified since charge can no longer be transferred to spacelike infinity. Instead it is transferred to a point. Similar obstacles have been met and overcome in treating quantum field theory on the circle.

Feynman-Kac and pseudodifferential operators

N. Jacob

The Feynman-Kac formula holds for a much larger class of pseudo differential operators, but these operators must necessarily be generators of a Markov process. In case of a Feller process the symbols $p(x,\xi)$ of the generators $-p(x,D)$ must have the property that $\xi \mapsto p(x,\xi)$ is continuous and negative definite. These symbols do in general not belong to some classical symbol classes. However it is possible to handle them in such a way that one can prove that (sometimes) the operator $-p(x,D)$ extends to a generator of a Feller semigroup, hence a Feller process, see N. Jacob, Pseudo differential operators and Markov processes, Akademie Verlag (1996) and the references therein. Once the Feller process is constructed, it is possible to write down a Feynman-Kac formula and to apply results of M. Demuth and J. van Casteren in order to study spectral properties of the operator $p(x,D)$.

Discussion on some lessons of quantum field theory

Rudolf Haag

Remarks on the question: what is a physical system, what is an event in the light of EPR-type experiments and interference experiments.

On wave front sets and scaling limits in quantum field theory

Rainer Verch

In many talks during the conference it has been emphasized that the notion of wavefront sets is a very useful tool in quantum field theory on curved spacetime which allows to impose stability conditions and thereby select the physical states. The previous works in this area have been focussing on the formulation of quantum field theory in curved space-time in terms of Wightman distributions. This contribution is a (tentative) proposal for an analog of the concept of wavefront set in the operator-algebraic setting of quantum field theory. This proposal has as its starting point that the wavefront set of a distribution $u \in \mathcal{D}'(\mathbb{R}^n)$ may be represented in a way which stresses the harmonic analysis aspect of the translation group on $\mathbb{R}^n$; namely, we claim that $(y,k) \in \mathbb{R}^n \times (\mathbb{R}^n \setminus \{0\})$ is not contained in $WF(u)$ iff there are $h \in \mathcal{D}(\mathbb{R}^n)$ with $h(0) = 1$, and an open neighbourhood V of k, so that

$$\int_{\mathbb{R}^n} e^{-i\lambda^{-1}k'\cdot x} h(x) \langle u, \tau_x(f_\lambda)\rangle dx = O(\lambda^N) \text{ as } \lambda \to 0$$

uniformly in $k' \in V$; for all $N \in \mathbb{N}$ and all families $(f_\lambda)_{\lambda>0}$ in

$$F_y(\mathcal{O}) := \{(f_\lambda)_{\lambda>0} \,|\, f_\lambda \in \mathcal{D}(\mathbb{R}^n), \mathrm{supp}(f_\lambda) \subset y + \lambda\mathcal{O}, \sup_\lambda \lambda \|f_\lambda\|_{H_1} < \infty\}.$$

Here, $\tau_x(f)(y') = f(y'-x)$ and $\mathcal{O}$ is any neighbourhood of $0 \in \mathbb{R}^n$. The class $F_y(\mathcal{O})$ of testing families has a counterpart in the scaling-algebra approach to the scaling-limit/renormalization group analysis of the short distance behaviour of algebraic quantum field theories [cf. D. Buchholz, R. Verch, Rev. Math. Phys. 7 (1995)] as an algebra of families, parametrized by $\lambda \in \mathbb{R}^+$, of operators in the observables algebra. Motivated by the stated result, we suggest a notion of wavefront set of a state on a quantum field theory in the operator algebraic formulation.

Non-commutative residue, current algebra, and renormalization

J. Mickelsson

This talk discusses the renormalization problems of fermion fields coupled to external Yang-Mills potentials from the point of view of representations of infinite-dimensional Lie algebras. The relevant Lie algebra (which includes renormalized currents) is an algebra of pseudodifferential operators (or even Fourier integral operators) on a manifold. Not all pseudodifferential operators are allowed: the restriction comes from the requirement that the operators should be canonically quantizable in the fermionic Fock space. The canonical quantization process defines a central extension of the original Lie algebra. The central term is most conveniently written using the Wodzicki-Guillemin residue; it is a twisted version of the Radul cocycle.

The phase of the fermionic quantum scattering matrix is determined as a parallel transport along the path of time-evolution operators [see J. Mickelsson and E. Langmann, J. Math. Phys (1996)]. In recent work briefly explained in this talk, it is shown that for Dirac fermions the phase can be defined in a gauge independent manner using a suitable representative for the connection in the cohomology class of quantum curvature determined by the canonical process.

Wave equations in domains with conical points

Ingo Witt

The talk leads to a proof of a local-in-time existence result for quasi-linear hyperbolic evolution equations of second order in domains with conical points. In the first part, the existence of solutions to the corresponding linear equations is discussed including the asymptotics of solutions near conical points. Using this information, the quasi-linear equations are then solved by the standard iteration procedure.

The exposition is based on Kato's semigroup-theoretic approach for solving abstract linear hyperbolic equations and Schulze's theory of pseudo-differential operators on manifolds with conical singularities. The former method provides the general framework, whereas the latter is the basic tool in treating the specific difficulties in the non-smooth situation. Significantly, Schulze's theory admits a parameter-dependent version, which allows the description of the branching behaviour in time of discrete asymptotics of solutions near conical points. The calculus is presented in a form in which the operators are permitted to have symbols with limited smoothness, as arises in non-linear problems.

On the use of modular theory in quantum field theory

H.-W. Wiesbrock

In this talk an elementary introduction to some algebraic structures and constructions arising naturally in quantum field theory are given. They were exemplified in the free case.

The well posedness of the Cauchy problem on a globally hyperbolic spacetime gives us a symplectic structure on the vectorspace of initial data. To it in a standard way we can associate a C*-algebra, the CCR-algebra. One might think of the elements as bounded fuctions of smeared free quantum fields. After specifying a distinguished physical vacua state on that algebra, (for example a quasifree Hadamard state), we can pass via the GNS-construction to a Hilbert space representation. Furthermore this construction naturally enables us to apply the modular theory in the sense of Tomita-Takesaki. In general this is a rather abstract structure but in quantum field theory one can find a nice geometrical interpretation of it due to Bisognano and Wichman. Namely in the case of observable algebras on the Minkowski space associated with wedge regions the modular group w.r.t. the vacuum gives the representation of a Lorentzboost. These results have a converse in the sense that if they hold the theory have to fulfill the spectrum condition. Even more, using modular theory one can characterize for a large class of quantum field theories on Minkowski space the models by distinguishing a finite set of algebras lying in a specified position relative to their modular theory. (This was successfully carried out in d< 4 and for d=4 it is under work.)

But also algebras associated to double cones allow a modular theory. I reviewed known results and end the talk with a conjecture concerning the geometrical content of these modular objects. Positive answers might open a new way to characterize the physical vacua states on quantum field theories on curved spacetimes by purely algebraic methods.

Heat kernel in quantum field theory

I. Avramidi

The heat kernel for an elliptic differential operator acting on sections of a vector bundle over a Riemannian manifold, plays a very important role in quantum field theory. General covariant systematic methods are developed for calculating the heat kernel diagonal for operators of Laplace type by introducing some deformations of the background fields (including the metric of the spacetime manifold) and studying various asymptotic expansions associated with these deformations. In this way it turns out to be possible to get much more information about the heat kernel, also in the case of general background. For example, one can determine explicitly the terms of some specific class (say with highest covariant derivatives, or without any covariant derivative, or all the terms having not more than two covariant derivatives etc.) in the coefficients of the usual short-time asymptotic expansion of the heat kernel, so called Hadamard-Minackshisundaram-De Witt-Seeley coefficients a_k, which are known in general only for $k = 1, 2, 3, 4$. In particular, if one restricts oneself to a finite number of low-order covariant derivatives of the background fields, then there exist a set of covariant differential operators that together with the background fields and their low-order derivatives generate a finite dimensional Lie algebra. This simplifies considerably the calculations and enables one to obtain closed manifestly covariant formulas for the heat kernel diagonal.

Algebraic construction of Wightman type fields

J. Yngvason

The talk was devoted to the question of existence and abundance of Wightman fields in spacetimes of arbitrary dimensions. Quantum fields may be regarded as representations of the Borchers-Uhlmann tensor algebra built over a space of test functions on spacetime. These representations correspond, in turn, via GNS construction to states on the algebra that are invariant under spacetime automorphisms and vanish on two prescribed ideals, the two-sided causality ideal and a left ideal that accounts for the spectrum condition for energy and momentum. General strategies for constructing functionals satisfying these conditions where presented. In particular, a method was discussed which is based on defining a C*-norm on the Borchers-Uhlmann algebra modulo the causality ideal. A criterion was presented which guarantees the existence of Wightman fields that satisfy Bose commutation relations but have bounded field operators, in contrast to all known examples in dimension ≥ 3. This criterion has been partially verified in all dimensions. In space-times with dimension 2 explicit examples have been given by D. Buchholz and K. H. Rehren. If the criterion can be generally verified this would lead to new examples of Wightman fields in dimension ≥ 4, a problem that have been open since more than 30 years.

Author of the report: Michael Keyl

List of Email-Adresses

Huzihiro Araki	araki@.ma.noda.sut.ac.jp
Ivan Avramidi	avramidi@rz.uni-greifswald.de
Hans-Jrgen Borchers	borchers@theorie.physik.uni-goettingen.de
J. Bros	bros@spht.saclay.cea.fr
Romeo Brunetti	brunetti@x4u2.desy.de
Klaus Fredenhagen	fredenha@x4u2.desy.de
Helmut Friedrich	hef@aei-potsdam.mpg.de
D. Iagolnitzer	iago@spht.saclay.cea.fr
Wolfgang Junker	junker@aei-potsdam.mpg.de
Bernard S. Kay	bsk2@york.ac.uk
Michael Keyl	key10433@itp0.physik.tu-berlin.de
O. Kounchev	kounchev@math.uni-duisburg.de
Otto Liess	liess@dm.unibo.it
Fernando Lled	lledo@rz.uni-potsdam.de
Jouku Mickelsson	jouko@theophys.kth.se
Alberto Parmeggiani	alberto@dm.unibo.it
Marek Radzikowski	radzikow@mail.desy.de
Vladimir J. Rabinovich	rabinov@us.unisd.ac.su
Alan Rendall	rendall@aei-potsdam.mpg.de
John Roberts	roberts@mat.utovrm.it
Luigi Rodino	rodino@dm.unito.it
Dirk Schlingemann	schlinge@x4u2.desy.de
Elmar Schrohe	schrohe@mpg-ana.uni-potsdam.de
Bert-Wolfgang Schulze	schulze@mpg-ana.uni-potsdam.de
Victor Shatalov	shatalov@mpg-ana.uni-potsdam.de
Boris Sternin	sternin@mpg-ana.uni-potsdam.de
Andr Unterberger	andre.unterberger@univ-reims.fr
Julianne Unterberger	julia.unterberger@univ-reims.fr
H. Upmeier	upmeier@mathematik.uni-marburg.de
Rainer Verch	verch@x4u.desy.de
Hans-Werner Wiesbrock	wiesbroc@physik.fu-berlin.de
A. S. Wightman	remillar@viper.princeton.edu
Ingo Witt	ingo@mpg-ana.uni-potsdam.de
Jakob Yngvason	yngvason@thor.thp.univie.ac.at
Eberhard Zeidler	ezeidler@mis.mpg.de

Tagungsteilnehmer

Prof.Dr. Huzihiro Araki
Dept. of Mathematics
Faculty of Science and Technology
Science University of Tokyo
Yamazaki 2641, Noda-shi

Chiba-ken 278
JAPAN

Dr. Ivan Avramidi
Fachrichtungen Mathem./Informatik
Universität Greifswald
Jahnstr. 15 A

17489 Greifswald

Prof.Dr. Hans-Jürgen Borchers
Institut für Theoretische Physik
Universität Göttingen
Bunsenstraße 9

37073 Göttingen

Prof.Dr. Jacques Bros
Service de Physique Theorique
CEA CEN Saclay

F-91 191 Gif-sur-Yvette Cedex

Dr. Romeo Brunetti
II. Institut für Theoretische
Physik
Universität Hamburg
Luruper Chaussee 149

22761 Hamburg

Prof.Dr. Klaus Fredenhagen
II. Institut für Theoretische
Physik
Universität Hamburg
Luruper Chaussee 149

22761 Hamburg

Dr. Helmut Friedrich
Max-Planck-Institut für
Gravitationsphysik
Schlaatzweg 1

14473 Potsdam

Prof.Dr. Bernhard Gramsch
Fachbereich Mathematik
Universität Mainz

55099 Mainz

Prof.Dr. Rudolf Haag
Waldschmidtstr. 4b

83727 Schliersee-Neuhaus

Prof.Dr. Daniel Iagolnitzer
Service de Physique Theorique
CEA CEN Saclay

F-91 191 Gif-sur-Yvette Cedex

Prof.Dr. Niels Jacob
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2

91054 Erlangen

Dr. Wolfgang Junker
II. Institut für Theoretische Physik
Universität Hamburg
Luruper Chaussee 149

22761 Hamburg

Prof.Dr. Bernard S. Kay
Dept. of Mathematics
University of York

GB-Heslington, York YO1 5DD

Dr. Michael Keyl
Institut für Theoretische Physik
Technische Universität Berlin
Hardenbergstr. 36

10623 Berlin

Prof.Dr. Ognyan I. Kounchev
Bulgarian Academy of Science
Institute of Mathematics
P.O. Box 373

1090 Sofia
BULGARIA

Prof.Dr. Otto Liess
Dipartimento di Matematica
Universita degli Studi di Bologna
Piazza Porta S. Donato, 5

I-40127 Bologna

Fernando Lledo
Institut für Mathematik
Universität Potsdam
Am Neuen Palais 10

14469 Potsdam

Prof.Dr. Jouko Mickelsson
Department of Theoretical Physics
Royal Institute of Technology

S-144 00 Stockholm

Prof.Dr. Alberto Parmeggiani
Dipartimento di Matematica
Universita degli Studi di Bologna
Piazza Porta S. Donato, 5

I-40127 Bologna

Prof.Dr. Vladimir S. Rabinovich
c/o Prof.Dr. B.-W. Schulze
Institut für Mathematik
Universität Potsdam
Am Neuen Palais 10

14469 Potsdam

Dr. Mark Radzikowski
2399 Cape Horn Avenue

Coquitlam, British Colum.
V3K 1J9 CANADA

Dr. Alan Rendall
Max-Planck-Institut für Gravitationsphysik
Schlaatzweg 1

14473 Potsdam

Prof.Dr. John E. Roberts
Dipartimento di Matematica
Universita degli Studi di Roma
Tor Vergata
Via della Ricerca Scientifica

I-00133 Roma

Prof.Dr. Luigi Rodino
Dipartimento di Matematica
Universita di Torino
Via Carlo Alberto, 10

I-10123 Torino

Priv.-Doz.Dr. Rainer Schimming
Institut f. Mathematik/Informatik
Universität Greifswald
Jahnstr. 15 A

17487 Greifswald

Dr. Dirk Schlingemann
II. Inst. für Theoretische Physik
Universität Hamburg
Luruper Chaussee 149

22761 Hamburg

Dr. Elmar Schrohe
Max-Planck Arbeitsgruppe
Part. Differentialgleichungen
und komplexe Analysis
Am Neuen Palais 10

14469 Potsdam

Roman Schubert
Institut für Theoretische Physik
Universität Ulm
Albert-Einstein-Allee 11

89081 Ulm

Prof.Dr. Bert-Wolfgang Schulze
Institut für Mathematik
Universität Potsdam
Am Neuen Palais 10

14469 Potsdam

Prof.Dr. Victor Shatalov
Dept. of Mathematics and Mechanics
Moscow State University
Lenin Hills

119899 Moscow
RUSSIA

Prof.Dr. Boris Sternin
Dept. of Mathematics and Mechanics
Moscow State University
Lenin Hills

119899 Moscow
RUSSIA

Prof.Dr. Raymond Stora
Laboratoire d'Annecy-le-Vieux de
Physique des Particules
BP 909

F-74019 Annecy-le-Vieux

Prof.Dr. Nikolai Tarkhanov
Institut für Mathematik
Universität Potsdam
Am Neuen Palais 10

14469 Potsdam

Prof.Dr. Andre Unterberger
Departement de Mathematiques
Universite de Reims
Moulin de la Housse
BP 1039

F-51687 Reims Cedex 2

Prof.Dr. Harald Upmeier
Fachbereich Mathematik
Universität Marburg
Hans-Meerwein-Str.

35043 Marburg

Rainer Verch
II. Institut für Theoretische Physik
Universität Hamburg
Luruper Chaussee 149

22761 Hamburg

Dr. Hans-Werner Wiesbrock
Institut für Theoretische Physik
Freie Universität Berlin
Arnimallee 14

14195 Berlin

Prof.Dr. Arthur S. Wightman
Joseph Henry Laboratories
Princeton University
Jadwin Hall P.O.Box 708

Princeton , NJ 08544
USA

Dr. Ingo Witt
Institut für Mathematik
Universität Potsdam
Am Neuen Palais 10

14469 Potsdam

Prof.Dr. Jakob Yngvason
Institut für Theoretische Physik
Universität Wien
Boltzmanngasse 5

A-1090 Wien

Prof.Dr. Eberhard Zeidler
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26

04103 Leipzig