MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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MATHEMATISCHE STOCHASTIK

9.3. – 15.3.1997

Die Tagung fand statt unter der Leitung der Herren J. Gärtner (Berlin), R.D. Gill (Utrecht) und E. Mammen (Heidelberg).

Es trafen sich 50 Wissenschaftler mit Arbeitsgebieten in verschiedensten Teilen der mathematischen Stochastik. Die beiden am stärksten vertretenen Richtungen waren die Statistik und die statistische Mechanik. So wurden längere Überblicksvorträge gehalten einesteils über Anwendungen nicht- und halbparametrischer Schätzer (Comets, Spokoiny, van der Vaart, Dümbgen, Levit), Schätzer für verrauschte Signale (Beran, Tsybakov) und Regressionsmodelle (Hallin, Kreiß) und andererseits über interagierende Partikelsysteme (Giacomin, Georgii, Grunwald, Landim, Löwe), Systeme von Verzweigungsprozessen (Wakolbinger, Klenke) und Markoffsche Zufallsfelder (Comets, Steif).

Ferner wurden u.a. Resultate vorgestellt in Gebieten wie Spektra zufälliger Matrizen (Biane, Giné), zufällige Approximation (Dyer, Catoni, Grübel), Folgen schwach abhängiger Zufallsgrößen (Rio, Doukhan) und Große Abweichungen (Deuschel, Eichelsbacher).

Es wurden acht Haupt- und 25 Kurzvorträge gehalten. Die Athmosphäre war durchweg angenehm und entspannt. Es wurde als positiv empfunden, daß es neben den Vorträgen genügend Zeit gab für eine Vielzahl kleinerer Diskussions- und Arbeitsgruppen, in denen aktuelle Probleme diskutiert wurden.

HAUPTVORTRÄGE

Introduction to free probability

P. Biane

We gave an introductory talk to the theory developed in the last 15 years by Dan Voiculescu. We started from the problem of computing the spectrum of the sum of two Hermitian matrices knowing the spectrum of each one. When the matrices have a very large dimension, and are chosen independently and uniformly among matrices having a specified spectrum, the spectrum of the sum, with great probability, is determined by the spectra of each matrix. In order to compute this spectrum (or more correctly the empirical distribution associated to it) one develops a combinatorial method which is based on the concept of freeness, and free convolution. Some explicit formulas can be written, and we discussed some results of probabilistic nature related to these concepts, such as free central limit theorem and free Levy-Khintchine formula.

Two questions about statistics of Markov random fields

F. Comets

(joint work with M. Janzura)

Parametric and non-parametric estimation for Markov random fields meet the phenomenom of phase transition, and the breakdown in the central limit theorem. We develop here two questions:

1) Local estimates, as Besag's maximum pseudo-likelihood estimators (say, in the parametric case), are in fact asymptotically normal, regardless of phase transition, multiplicity of phases,... This was first shown by Guyon-Künsch in the 2-dimensional Ising model, but it was extended later to more general cases. In collaboration with Martin Janzura, we prove this result holds even if the underlying distribution is not translation invariant; this requires of course random norming, similar to Student-ization. The proof relies on a central limit theorem for local, conditionally centered random fields.

2) Detecting phase transition from a single sample (and thereby, checking if central limit holds for general functions...). In a 1994 paper I propose to estimate the rate function for large deviations using Erdös-Renyi statistics. This method has some important drawbacks, and in a 1997 paper I propose another method, estimating the rate function by computing the frequencies of deviations of moving averages in a "small" window moving inside the box of observations.

Randomized approximate counting

M.E. Dyer

Most interesting combinatorial counting problems are known to be $\#P$-complete. For these problems we seek only *approximations*. The correct notion appears to be randomized counting. We are then interested in studying problems possessing "fully polynomial randomized approximation schemes" (fpras). There are few general techniques, but the most common is based on proving rapid convergence of a Markov chain on the configurations. We will review these ideas and give some examples. Proving convergence occurs rapidly is usually not easy, and again there are few general methods. We will examine one approach: *coupling*, and a simple variant "path coupling" which makes the analysis more combinatorially tractable in many cases. We consider two applications of the method: low-degree graph colourings and linear extensions of a partial order.

On the heat content of planar regions with a fractal polygonal boundary

F. den Hollander

(joint work with M. van den Berg)

After a general introduction to heat conduction in planar regions, we described the following problem.

Fix $k \geq 3$ integer and $0 < s < 1$. Let $D \subset \mathbb{R}^2$ be the domain that is obtained by taking a regular k-gon with sides of unit length, attaching k-gons with sides of length s to the middles of the outer edges, and proceeding in this way: each time the new generation of k-gons is a factor s smaller and is attached to the middles of the outer edges of the previous generation. The set D thus obtained has a fractal polygonal boundary. There exists some $s_k > 0$ such that no k-gons overlap if and only if $0 < s \leq s_k$.

We study the heat content of D as a function of t, denoted by $E_D(t)$, when D initially has temperature 0 and the boundary is kept at temperature 1. We derive the complete short-time expansion of $E_D(t)$. It turns out that for $s \neq 1/(k-1)$ the expansion has the form

$$E_D(t) = p(\log t)t^{1-\frac{d}{2}} + At^{\frac{1}{2}} + Bt + O(e^{-\frac{r}{t}}),$$

where p is a $\log(1/s^2)$-periodic function, $d = \log(k-1)/\log(1/s)$ is a fractal dimension, A and B are constants, and r is an error exponent. For $s = 1/(k-1)$ the $t^{\frac{1}{2}}$-term carries an additional $\log(1/t)$. The proof is based on the Brownian motion representation of the heat kernel and makes use of techniques where D is locally approximated by regions of a simple form.

The constants A and B have an interesting geometric interpretation in terms of the edges respectively vertices of D. This interpretation is in the spirit of Mark Kac's paper 'Can one hear the shape of a drum?'. It remains an open problem to show that p is non-constant. This property may be visualized by thinking of a 'heat front' that moves in from the boundary and that has a period $\log(1/s^2)$ on a logarithmic time scale due to the discrete self-similar structure of the boundary of D in combination with the classical space-time scaling for heat conduction.

The Berry-Esseen theorem for weakly dependent sequences

E. Rio

We extend the proof method of the Berry-Esseen theorem proposed by Bergström to sequences of weakly dependent random variables. In particular, we show that, for stationary sequences of real, bounded random variables, the rate of convergence in the central limit theorem, in Lèvy distance, is of order $n^{-\frac{1}{2}}$ if the sequence $(\theta_p)_{p>0}$ of the uniform mixing coefficients satisfies the condition $\sum_{p>0} p\theta_p < \infty$.

Estimation of a function with jumps and image denoising via pointwise-adaptive window choice

V. Spokoiny

A new method of nonparametric estimation is discussed in connection with two specific problems: estimating a univariate function with discontinuities and image denoising. The method is based on an adaptive (data-driven) choice of an averaging window. It is shown that this method allows rate-optimal estimation both the underlying function and the location of jumps (in the first problem) or location of the edge (in the second problem).

Branching populations and their genealogy

A. Wakolbinger

(joint work with D. Dawson and L. Gorostiza)

Our starting point is the result of Cox and Griffeath (1985) on the occupation time fluctuations of branching Brownian particle systems in dimension $d \geq 3$. We show how the normings as well as the asymptotic covariance kernels can be understood from the space-time correlations of the measure of a "typical" genealogical tree, and generalize this to reversible Markovian dynamics. For example, if the Green operator of the motion has the property $G^2\varphi < \infty$, φ nonnegative, bounded with bounded support, then the norming is the classical $t^{1/2}$, and the covariance of the limiting Gaussian fluctuation field is given by $G + \frac{1}{2}G^2$, where the two summands come from the pairs of particles that are resp. are not related in direct line. The property $G^2\varphi < \infty$, which has a simple interpretation in terms of mass creation, guarantees a transient behavior of the equilibrium clans of related individuals. Corresponding results have been obtained for infinite variance branching as well as 2-level branching systems; in part, the proofs employ ideas used by Iscoe for 1-level superprocesses.

Likelihood methods in semiparametric models

A. van der Vaart

Statistical inference for parametric models is based on the log likelihood function. Its point of maximum is the maximum likelihood estimator, its second derivative at this point, the "observed information", serves as a measure of accuracy, and the difference between the log likelihood function at its maximum and a fixed point is the log likelihood ratio statistic. In semiparametric models there is a nuisance parameter next to a parameter of interest. The main result presented in this talk was that the "profile log likelihood function" now can be used in much the same way. This function is defined as follows. First, one carefully chooses a function of the observations and parameters that is called the "likelihood". This may take several forms, ordinary, empirical, penalized, sieved, mixtures of these, and still others. Next, the profile likelihood for the parameter of interest is the supremum of the likelihood over the nuisance parameter. Our main result is a quadratic expansion of the profile likelihood function. The coefficient of the linear term is the (random) sum of the efficient score function at the observations, while the quadratic terms is lead by the efficient information. This expansion implies all the desired results: the normality of mle, the chisquared distribution of the likelihood ratio statistic, and the consistency of the observed information. Rather than giving a detailed proof of the expansion, we have discussed two approaches to proving the asymptotic normality of mles directly. The first is based on the efficient score equation, and (possibly) a rate theorem for the nuisance parameter to verify a "no-bias" condition. The second is based on inverting an infinite system of likelihood equations. The first approach seems unable to handle empirical likelihoods. The second approach does not have this problem, but requires that the model is parametrized in a regular manner. Thus, they both appear to be useful.

KURZVORTRÄGE

Modulation versus model selection

R. Beran

For recovering a discrete-time signal from observed signal plus Gaussian white noise, we compare three procedures: C_p model selection; BIC-style model selection; and modulation. Each procedure is applied to orthogonally transformed data. The outcome, transformed back to the original coordinate system, is the signal estimator. In nested submodels, the BIC-style procedures have poor asymptotic maximum risk if the signal-to-noise ratio is large. By contrast, the asymptotic maximum risk of the C_p-procedure is never more than twice the Pinsker bound in each submodel. However, monotone modulation estimators achieve the asymptotic Pinsker bound in each nested submodel.

Stochastic optimization algorithms: speed-up methods

O. Catoni

Markov chains with exponential small transitions depending on a given irreducible rate function are a general model for stochastic optimization algorithms, known as the generalized Metropolis algorithm in the time homogeneous case and as the generalized simulated annealing algorithm in the case when the transitions are decreased as a function of time. We study the decay of the probability of failure with the length of the algorithm and point out the existence of uniformly nearly optimal temperature sequences for the simulated annealing algorithm. We present two other speed-up methods: the iterated energy transformation algorithm and the partial freezing method, with which it is possible to decrease further the probability of failure. Eventually, we report some experiments made on a resource assignment problem, which show that the predictions of this asymptotic study of the probability of failure are in good agreement with the practical efficiency of these various algorithms.

On a new weak dependence condition

P. Doukhan

The purpose of this talk is to propose an unifying weak dependence condition.
Mixing sequences, functions of associated or Gaussian sequences, Bernoulli shifts as well as models with a Markovian representation are examples of the models considered. We establish Marcinkiewicz-Zygmund, Rosenthal and exponential inequalities for general sequences of centered random variables. Inequalities are stated in terms of the decay rate for the covariance of products of the initial random variables subject to the condition that the lag of time between both products tends to infinity.
As applications of those notions, we obtain a version of the functional CLT and an invariance principle for the empirical repartition process. The guidelines of this talk are Doukhan & Portal (1983) and Rio (1993).

Large deviations of the longest increasing subsequence

J.-D. Deuschel

(joint work with O. Zeitouni)

We study the fluctuations, in the large deviation regime, of the longest increasing subsequences of a random i.i.d. sample on the unit square. In particular, our results yield the precise upper and lower exponential tails for the length of the longest increasing subsequences of a random permutation.

Nonparametric confidence sets for shape-restricted regression functions

L. Dümbgen

The first part of the talk discusses goodness-of-fit tests for the white noise model. One observes a stochastic process V of the form $dV(t) = f_o(t)dt + n^{-1/2}dW(t)$, where f_o is an unknown function, $n \geq 1$ is a known scale parameter ("sample size") and W is standard Brownian motion on $[0, 1]$. For testing "$f_o = 0$" versus "$f_o \neq 0$" a simple Bayes model leads to a special test statistic $T(n^{1/2}V)$. The resulting test turns out to be rate-optimal within Hölder classes of functions. The corresponding confidence set $C(V, \alpha)$ of all functions f

such that $T(n^{1/2}(V - \int f)) \leq c(\alpha)$ has good adaptivity properties if we impose shape restrictions on f_o such as monotonicity or convexity.

This procedure can be imitated in several models such as density estimation or nonparametric regression. For the latter example we propose a confidence set based on signs of residuals. This approach requires minimal model assumptions but the computation is very difficult for large sample sizes. As a possible way out we propose and illustrate a stochastic approximation procedure.

Coupling and decoupling in the theory of large deviations

P. Eichelsbacher

(joint work with U. Schmock)

We study large and moderate deviation principles for m-fold products of empirical measures and for U-empirical measures, e.g.,

$$L_n^m = \frac{1}{\binom{n}{m}} \sum_{1 \leq i_1 < \cdots < i_m \leq n} \delta_{(X_{i_1}, \ldots, X_{i_m})},$$

m fixed, where the underlying i.i.d. random variables $(X_i)_i$ take values in a measurable, not necessarily Polish, state space S. The LDP can be formulated on a suitable subset of the set of probability measures endowed with a topology which is stronger than the usual τ-topology, and which make the map $\nu \mapsto \int \varphi \, d\nu$ continuous even for certain unbounded φ taking values in a real separable Banach space. A special feature of the LDP is the non-convexity of the rate function. Applications are improved versions of large and moderate deviation principles for Banach space valued U-statistics. Furthermore, we improve the Gibbs conditioning principle for interacting ensembles of particles. We can extend our results to certain weakly dependent sequences.

We use a partition dependent **coupling** to extend Sanov's theorem to triangular arrays, when S is measurable and the topology on the set of probability measures is the τ-topology. We deduce a LDP for the empirical measures of exchangeable sequences with a measurable state space.

For a moderate deviation result for products of empirical measures in a scaling $b_n/n \to 0$ and $n/b_n^2 \to 0$, we use an improved version of a Bernstein type inequality for type 2 Banach-space valued U-statistics, which uses a **decoupling** technique introduced by de la Peña 92. If μ denotes the common law of the $(X_i)_i$, we get

$$P\Big(\frac{n}{b_n}(L_n^m - \mu^{\otimes m}) \sim \varrho\Big) \approx \exp\Big(-\frac{b_n^2}{n} I_m(\varrho)\Big)$$

with a convex rate

$$I_m(\varrho) = \frac{1}{2} \int_S \left(\frac{d\varrho_1}{d\mu}\right)^2 d\mu,$$

if $\varrho(S^m) = 0$, $\varrho_1 \ll \mu$ and $\varrho = \sum_{i=1}^m \mu^{\otimes i-1} \otimes \varrho_1 \otimes \mu^{\otimes m-i}$.

SK-model with local deterministic interaction

G. Giacomin

(joint work with F. Comets and J.L. Lebowitz)

We consider an Ising spin model with both long range random and short range deterministic interactions. The long range part is the Sherrington–Kirkpatrick type, while the short range is chosen to be ferromagnetic. Our main result is the central limit theorem for the free energy at high temperature. More precisely, denoting by $\mathbb{E}_\kappa$ the expectation with respect to an infinite volume Ising measure at temperature $1/\kappa$ (κ_c is the critical value) and by H_N^{SK} the S–K Hamiltonian in a box $\Lambda_N \subset \mathbb{Z}^d$ ($d \geq 1$), we show that $\mathbb{E}_\kappa[\exp(-\beta H_N^{\mathrm{SK}})]$, divided by its expectation with respect to the disorder, converges weakly to a log-normal distribution for $\kappa < \kappa_c$ and $\beta < 1/\theta_\kappa$, where θ_κ is the sum of the square of the correlations in the Ising model. This result is established also at $\kappa = \kappa_c$, as long as uniqueness holds for the Ising measure at the critical point. As corollaries we derive some results on the free energy and on critical exponents. We use the stochastic analysis technique introduced in [1] and the main technical estimates arise in studying the overlaps of independent copies of the Ising model.

[1] F. Comets and J. Neveu (1995) *The Sherrington–Kirkpatrick model and stochastic calculus: the high temperature case*, Comm. Math. Phys. **166**:549–564.

Random matrix approximation of spectra of compact integral operators

E. Giné

Let $h : S^2 \to \mathbb{R}$ be a symmetric measurable function, where $(S, \mathcal{S}, P)$ is a probability space, such that the operator $H : L_2(P) \to L_2(P)$ given by $Hg(x) = \int h(x,y)g(y)\,dP(y)$ is compact. Let X_i, $i \in \mathbb{N}$, be i.i.d. (P) random variables, and let H_n be the matrix whose entries are $h(X_i, X_j)$ if $i \neq j$, zero if $i = j$, $i, j \leq n$. (H_n is a discretization of H obtained by random sampling.) We show that the ℓ_2 distance between the ordered spectra of H and H_n tends to zero a.s. iff $Eh^2(X_1, X_2) < \infty$ and obtain rates of convergence and even a central limit theorem for the difference between the ordered spectra of H_n and H, viewed as points of ℓ_2 or c_0, under additional assumptions on h.

Phase transitions in continuum Potts models

H.-O. Georgii

(joint work with O. Häggström)

The continuum Potts model is a system of point particles in $\mathbb{R}^d$, $d \geq 2$, having $q \geq 2$ distinct types and interacting via an (arbitrarily weak) interspecies repulsion. In addition, a type-independent background interaction is allowed. For a given density parameter $z > 0$ and inverse temperature $\beta > 0$, the class of all possible equilibrium distributions consists of the associated Gibbs measures.

It is shown that for any $\beta > 0$ and all $z > z_0(\beta)$ a phase transition occurs, in that there exist q translation invariant Gibbs measures which can be distinguished by their type densities. This follows from the occurrence of percolation in a random cluster representation of the model, which in turn can be established by stochastic comparison with Bernoulli site–bond percolation on $\mathbb{Z}^d$.

For $d = 2$ and smooth interaction, all (tempered) Gibbs measures are translation invariant. It is unknown whether non-translation-invariant Gibbs measures can exist in higher dimensions (as in lattice systems for $d \geq 3$).

Richardson extrapolation in computational probability

R. Grübel

Numerical treatment of stochastic models often involves a discretization parameter $h > 0$; let μ_h be the corresponding approximation to the quantity μ_0 of interest. If μ_h can be expanded about $h = 0$ then an extrapolation to the limit can be used to accelerate convergence. Examples from queueing and risk theory show that this technique can often be used to eliminate most of the discretization error.

Glauber spin-glass dynamics

M. Grunwald

Spin glass models (i.e., a type of disordered lattice spin systems) were introduced in the theoretical physicists literature to explain the behaviour of certain diluted magnetic alloys. The Sherrington-Kirkpatrick (SK) model is a mean-field version of a spin glass. In this talk we study the rate function of the large deviations upper-bound for a Glauber dynamics for the SK-model (see M.G., PTRF **106**, 187-232 (1996)). We show that the rate function has a unique minimum, which can be described in terms of a self-consistent equation for the correlation- and the response-function. The uniqueness implies for almost all realizations of the couplings the weak convergence of the law of the empirical measure to the measure concentrated at the minimum. Following an idea of G. Ben Arous and A. Guionnet (PTRF 1995) we prove – with the help of a replicated system – a random propagation of chaos result, i.e., a weak convergence result of the law of the distribution of the first k-coordinates to the law of a product of random measures.

Local asymptotic normality for long-memory processes with applications

M. Hallin

(joint work with A. Serroukh and M. Taniguchi)

The local asymptotic normality property is established for a regression model with fractional $ARIMA(p, d, q)$ errors. This result allows for solving, in an asymptotically optimal way, a variety of inference problems in the long-memory context: hypothesis testing, discriminant analysis, rank-based testing, locally asymptotically minimax and adaptive estimation, etc. The problem of estimating the lag parameter d in the simple long-memory model $(1 - L)^d X_t \varepsilon_t$ is treated in some detail. Various estimators have been proposed in the literature. Such estimators, at best, are asymptotically efficient (in the traditional BAN sense), under Gaussian assumptions. Our adaptive estimate is uniformly (with respect to the underlying innovation density) *locally asymptotically minimax* (LAM) in the sense of Fabian and Hannan (1982). The most striking result of the study is perhaps that the long memory aspects of fractional models do not appear in the local asymptotic structure, which is a classical LAN one, with traditional root-n rates.

Catalytic branching random walks

A. Klenke

(joint work with A. Greven and A. Wakolbinger)

Let $(X_t)_{t\geq 0}$ and $(Y_t)_{t\geq 0}$ be symmetric random walks on $\mathbb{Z}^d$. We consider critical binary branching random walks $(\eta_t)_{t\geq 0}$ and $(\xi_t)_{t\geq 0}$ with underlying motion processes (X_t) and (Y_t) respectively. While the branching rate of the η-particles is identically equal to 1 the branching rate of the ξ-particles located at $x \in \mathbb{Z}^d$ at time t is equal to $\eta_t(\{x\})$ (one-way interaction).

If (X_t) is transient, (η_t) is known to be persistent (no loss of mass in the longtime limit) and ergodic. Hence the ergodic theorem yields (together with a well known dichotomy for spatial branching processes) that (ξ_t) is persistent if (Y_t) is transient, and $\xi_t \xrightarrow{\mathcal{D}} 0$, fdd, if (Y_t) is recurrent.

We consider the case where (X_t) is recurrent only for symmetric Bernoulli random walks on $\mathbb{Z}^d$.

In this case $\eta_t \xrightarrow{\mathcal{D}} 0$, fdd, while (ξ_t) is persistent and converges to

(i) a mean 1 Poisson field if $d = 1$.

(ii) a mixed Poisson field with spatially constant random intensity ζ with $E[\zeta] = 1$ and $\mathrm{Var}[\zeta] = \infty$.

One-dimensional polymer measures

W. König

(joint work with R. van der Hofstad and F. den Hollander)

We introduce polymer measures for one-dimensional simple random walks and Brownian motion which suppress or even forbid the path's self-intersections until a fixed time. We describe the long-time behavior of the endpoint as the length gets large in terms of a central limit theorem. The asymptotic behavior of the limiting drift and variance as the self-repellence strength gets small is described, and a central limit theorem for the coupled limit (long time and small repulsion) for simple random walk is stated. Depending on the precise manner of the coupling of the two parameters, the endpoint admits a scaling which perfectly interpolates between diffusive and ballistic behavior.

Resampling for nonlinear autoregression

J.-P. Kreiß

(joint work with J. Franke, E. Mammen and M.H. Neumann)

Kernel smoothing in nonparametric autoregressive schemes offers a powerful tool in modelling time series. In the talk it is shown that the bootstrap can be used for estimating the distribution of kernel smoothers. This can be done by mimicking the stochastic nature of the whole process in the bootstrap resampling or by generating a simple regression model. Consistency of these bootstrap procedures is shown.

For bootstrapping the sup-distance over a compact interval of the local polynomial estimator (LPE), we derive a strong approximation of LPE in nonparametric autoregression and nonparametric regression. This result suggests to use regression type arguments also for the situation of nonparametric autoregression. As an example we consider a simple wild bootstrap to construct simultaneous confidence bands for the conditional mean function in first order nonlinear autoregression.

Relaxation to equilibrium of interacting particle systems in infinite volume

C. Landim

Under mild assumptions we prove that the decay rate to equilibrium in L^2 of zero range dynamics on the d dimensional integer lattice is $t^{-d/2}$ with logarithmic corrections.

Adaptive estimation of infinitely smooth functions

B. Levit

The problem of efficient adaptive non-parametric estimation of an unknown signal is considered in a white noise model, under the assumption that the level of noise is small.

A scale of classes of infinitely differentiable functions is introduced based on the notion of fast decreasing Fourier transforms. For any such given class an efficient non-parametric estimator can be constructed. The problem of adaptive estimation arises when it is a priori unknown to which of the classes, within a given scale, the observed signal actually belongs.

Notion of *quasi-parametric* and *essentially non-parametric* subscales is introduced. An adaptive estimator will be described having the property of performing optimally for arbitrary non-parametric scale, subject to the condition of doing well for any quasi-parametric scale.

Non-Gaussian fluctuations in the critical Hopfield model

M. Löwe

(joint work with B. Gentz)

We consider the Hopfield model, i.e., the Gibbs measure $\varrho_{N,\beta,\xi}(\sigma) = \exp(-\beta H_N(\sigma))/Z_{N,\beta}$, where $H_N(\sigma) = -\frac{1}{\alpha N}\sum_{i,j}\sum_{\mu=1}^{M}\sigma_i\sigma_j\xi_i^\mu\xi_j^\mu$ and σ_i, ξ_i^μ are i.i.d. random variables on $\{-1,1\}$ with $P(\sigma_i = 1) = P(\xi_i^\mu = 1) = \frac{1}{2}$. It is well known that $\beta = 1$ is the critical temperature of the model.

We show that at $\beta = 1$ or for β_N converging to 1 faster than $\frac{1}{\sqrt{N}}$, the measures $\varrho_{N,\beta,\xi}(m_N(\sigma))^{-1}$, where $m_N(\sigma) = \left(\frac{1}{N}\sum_{i=1}^{N}\xi_i^\mu\sigma_i\right)_{i\mu=1,\dots,M}$ converge in P_ξ-distribution to a non-Gaussian random measure provided that M does not depend on N. More precisely, the limiting measure has got a density w.r.t. M-dimensional Lebesgue measure of the form $\exp(\frac{1}{2}\sum_{\mu<\nu}\eta_{\mu,\nu}x_\mu x_\nu - \frac{1}{4}\sum_{\mu<\nu}x_\mu^2x_\nu^2 - \frac{1}{12}\sum_\mu x_\mu^4)/\Gamma_\eta$ where $\eta_{\mu,\nu}$ are i.i.d. Gaussian random variables.

Asymptotic equivalence of density estimation and white noise experiments

M. Low

(joint work with L.D. Brown)

Nussbaum (1996) showed that the density estimation model $X_1,\dots,X_n$ i.i.d. $f \in \mathcal{F}$ is asymptotically equivalent to $dX(t) = \sqrt{f(t)}\,dt + \frac{1}{\sqrt{n}}\,dW(t)$, $0 \le t \le 1$, $f \in \mathcal{F}$ as long as $\mathcal{F} \subset \{f : [0,1] \to \mathbb{R} \,|\, f \ge \varepsilon, |f(y) - f(x)| \le C|y-x|^{\frac{1}{2}+\delta}\}$ where $\delta > 0$. The proof given by Nussbaum relies heavily on empirical process theory and in particular the Hungarian construction.

A simpler proof of the equivalence is given which is more constructive and which helps make clear why the equivalence holds.

Structure exploration in smoothing and scale space theory from computer vision

S. Marron

(joint work with P. Chaudhuri)

A useful statistical viewpoint for mode testing, and other types of feature identification, has been to study families of smooths indexed by the smoothing parameter. This same family has been well developed as an important model for "computer vision" by computer scientists, who have carried the mathematics very far. An overview of these ideas is given, with an emphasis on the relation to statistics. An application of these ideas, SiZer, is proposed, which studies SIgnificant ZERo crossings of the derivative smooths, from a scale space viewpoint. This gives visual insight into features that are statistically significant at various levels of resolution.

ANCOVA under shape restrictions

D. W. Müller

For real-valued $x_1, \dots, x_n$ ("covariables"), $y_1, \dots, y_n$ ("responses"), $s_1, \dots, s_n$ ("scores"), we consider the problem of computing the maximum of

$$(*) \qquad C(k) = \sum_{i=1}^{n} s_i \, 1_{[y_i \geq k(x_i)]}$$

over all convex functions k on $\mathbb{R}^1$. A recursive relation is given, and an algorithm based on it, to compute this value and an optimal k in $O(n^3)$ steps. For a special choice of scores ($s_i \in \{\frac{1}{n_1}, -\frac{1}{n_2}\}$) where $n_1 + n_2 = n, C^* \equiv \max C(k)$ can be interpreted as a generalized (one-sided) Kolmogorov-Smirnov statistic to test for treatment effect in nonparametric analysis of covariance ("ANCOVA"). There it is assumed that response and covariable are related by an equation

$$(**) \qquad y_i = k_0(x_i) + \varepsilon_i$$

where $x_1, \dots, x_n, \varepsilon_1, \dots, \varepsilon_n$ are independent r.v.'s with $\varepsilon_i \sim G$ (if $s_i = \frac{1}{n_1}$), $\varepsilon_i \sim H$ (if $s_i = -\frac{1}{n_2}$). The problem is to test the hypothesis $G = H$. The influence function k_0 is assumed to be convex and unknown to the statistician. The relation between the two shape assumptions (in $(*)$ and $(**)$) is explained in the large sample situation ($n_1 \wedge n_2 \to \infty$).

On a spatial transformation for a correction of earthquake location

Y. Ogata

An array of regularly spaced several seismic stations can estimate the location of a distant earthquake using arrival times at the various stations of seismic waves generated by the earthquakes. However, the accuracy decreases as the distance to the epicenter of the earthquake from the array increases. This talk is concerned with the modification of the estimated location by removing its bias which is locally systematic but globally complex, reflecting the structure of the earth. Spline surfaces are used to model such biases. Then a Bayesian procedure is carried out not only to tune the smoothness constraints but also to select the best combination among various sums of squares of differently weighted residuals and various roughness penalties for the smoothing. Using the estimated splines of the posterior mode, the newly determined epicenter locations are transformed to confirm its practical utility. Residual distributions show that our procedure improves the modification by the conventional procedure. A spatial pattern of the residuals reveals some geophysical characteristics.

Phase transitions for Markov random fields: some results and some questions

J. Steif

Phase transitions for Markov random fields can arise from purely combinatorial considerations in the context of so-called subshifts of finite type (soft) which are finite sets with a rule as to who can sit next to whom. In certain cases, analogous to the Ising Model, the presence of a certain symmetry in the system is necessary for this to occur. The characterization of which soft's possess a weighting on the elements which produces a phase transition when the underlying graph is a tree has been obtained (in a certain sense) by G. Brightwell and P. Winkler. Extending this to lattices seems to be an interesting problem. It is also of interest to know when one random field can be coded (as a translation invariant function of) from another random field. If one considers codings which are continuous a.e. (called finitary), then there does not exist a finitary code from any i.i.d. process to a Markov random field when the latter has a phase transition (meaning it is not the unique Markov random field with its conditional probabilities). It is not known if there exists a coding from an i.i.d. process to a Markov random field in the uniqueness regime when it is the unique Markov random field with its conditional probabilities (e.g., the high temperature Ising Model).

On signal-to-noise ratio in stochastic regression

H. Tong

(joint work with Q. Yao)

In this talk, we study three different types of estimates for the noise-to-signal ratios in a general stochastic regression setup. The locally linear and locally quadratic regression estimators serve as the building blocks in our approach. Under the assumption that the observations are strictly stationary and absolutely regular, we establish the asymptotic normality of the estimates, which indicates that the residual-based estimates based on the locally quadratic regression are to be preferred. The asymptotic theory also paves the way for a fully data-driven undersmoothing scheme to reduce the biases in estimation. Numerical examples with both simulated and real data sets are used as illustration.

Asymptotically exact testing of nonparametric hypotheses

A. B. Tsybakov

The signal-in-Gaussian-white-noise model is studied. We consider the problem of testing the hypothesis $H_0 : f \equiv 0$ (the signal is absent) against the nonparametric alternative $H_1 : f \in \Lambda_\varepsilon$, where Λ_ε is a set of functions on $\mathbb{R}^1$ of the form $\Lambda_\varepsilon = \{f \in \mathcal{F} : \varphi(f) \geq C\psi_\varepsilon\}$. Here $\mathcal{F}$ is a Hölder or Sobolev class of functions, $\varphi(f)$ is the minimax rate of testing; $\varepsilon \to 0$ is the asymptotic parameter of the model. The "exact separation constant" $C^* >$ is found, such that a test with given summarized asymptotic errors of first and second type is possible for $C > C^*$ and is not possible for $C < C^*$. Asymptotically minimax test statistics are constructed.

Berichterstatter: Wolfgang König

Die E-Mail-Adressen der Teilnehmer

Ben Arous, Gerard	benarous@dmi.ens.fr
Beran, Rudolf J.	beran@stat.berkeley.edu
Biane, Philippe	pbi@ccr.jussieu.fr
Bolthausen, Erwin	eb@amath.unizh.ch
Bovier, Anton	bovier@wias-berlin.de
Catoni, Oliver	catoni@dmi.ens.fr
Comets, Francis M.	comets@math.jussieu.fr
Deuschel, Jean-Dominique	deuschel@math.tu-berlin.de
Doukhan, Paul	paul.doukhan@math.u-psud.fr
Dümbgen, Lutz	lutz@statlab.uni-heidelberg.de
Dyer, Martin E.	dyer@scs.leeds.ac.uk
Eichelsbacher, Peter	peter@mathematik.uni-bielefeld.de
Franke, Jürgen	franke@mathematik.uni-kl.de
Gärtner, Jürgen	jg@math.tu-berlin.de
Gather, Ursula	gather@omega.statistik.uni-dortmund.de
Georgii, Hans-Otto	georgii@rz.mathematik.uni-muenchen.de
Giacomin, Giambattista	gbg@amath.unizh.ch
Gill, Richard D.	gill@math.ruu.nl
Giné, Evarist	gine@uconnvm.uconn.edu
Götze, Friedrich	goetze@mathematik.uni-bielefeld.de
Grübel, Rudolf	rgrubel@stochastik.uni-hannover.de
Grunwald, Malte	grunwald@math.tu-berlin.de
Hallin, Mark	mhallin@ulb.ac.be
Härdle, Wolfgang	haerdle@wiwi.hu-berlin.de
den Hollander, Frank	denholla@sci.kun.nl
Klenke, Achim	klenke@mi.uni-erlangen.de
König, Wolfgang	koenig@math.tu-berlin.de
Kreiß, Jens-Peter	j.kreiss@tu-bs.de
Landim, Claudio	landim@paris.polytechnique.fr
Levit, Boris	levit@math.ruu.nl
Linde, Werner	linde@minet.uni-jena.de
Löwe, Matthias	loewe@mathematik.uni-bielefeld.de
Low, Mark	lowm@stat.wharton.upenn.edu
Mammen, Enno	mammen@statlab.uni-heidelberg.de
Marron, James S.	marron@stat.unc.edu
Müller, Dietrich W.	dwm@statlab.uni-heidelberg.de
Nisiyama, Yoichi	nisiyama@ism.ac.jp
Nussbaum, Michael	nussbaum@wias-berlin.de
Ogata, Yosihiko	ogata@ism.ac.jp
Rio, Emmanuel	Emmanuel.Rio@math.u-psud.fr
Rost, Hermann	rost@math.uni-heidelberg.de
Spokoiny, Vladimir	spokoiny@wias-berlin.de

Steif, Jeffrey	steif@math.chalmers.se
Strasser, Helmut	helmut.strasser@wu-wien.ac.at
Stute, Winfried	winfried.stute@math.uni-giessen.de
Tong, Howell	ht1@ukc.ac.uk
Tsybakov, Alexander B.	tsybakov@ccr.jussieu.fr
Wakolbinger, Anton	wakolbin@math.uni-frankfurt.de
van der Vaart, Aad	aad@cs.vu.nl
van Zwet, Willem R.	vanzwet@wi.leidenuniv.nl

Tagungsteilnehmer

Prof.Dr. Gerard Ben Arous
Departement de Mathematiques et d'Informatiques
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05

Dr. Olivier Catoni
Departement de Mathematiques et d'Informatiques
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05

Prof.Dr. Rudolf J. Beran
Department of Statistics
University of California
367 Evans Hall

Berkeley , CA 94720-3860
USA

Prof.Dr. Francis M. Comets
U. F. R. de Mathematiques
Case 7012
Universite de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Philippe Biane
Laboratoire de Probabilites-Tour 56
Universite P. et M. Curie
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Jean Dominique Deuschel
Fachbereich Mathematik
Technische Universität Berlin
Straße des 17. Juni 136

10623 Berlin

Prof.Dr. Erwin Bolthausen
Institut für Angewandte Mathematik
Universität Zürich
Winterthurerstr. 190

CH-8057 Zürich

Prof.Dr. Paul Doukhan
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Batiment 425

F-91405 Orsay Cedex

Dr. Anton Bovier
Weierstraß-Institut für Angewandte Analysis und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39

10117 Berlin

Dr. Lutz Dümbgen
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Prof.Dr. Martin E. Dyer
School of Computer Science
University of Leeds

GB-Leeds LS2 9JT

Dr. Peter Eichelsbacher
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131

33501 Bielefeld

Prof.Dr. Jürgen Franke
Fachbereich Mathematik
Universität Kaiserslautern

67653 Kaiserslautern

Prof.Dr. Jürgen Gärtner
Fachbereich Mathematik / FB 3
Sekr. MA 7-5
Technische Universität Berlin
Straße des 17. Juni 136

10623 Berlin

Prof.Dr. Ursula Gather
Fachbereich Statistik
Universität Dortmund

44221 Dortmund

Prof.Dr. Hans-Otto Georgii
Mathematisches Institut
Universität München
Theresienstr. 39

80333 München

Dr. Giambatista Giacomin
Departement de Mathematiques
Ecole Polytechnique Fede
de Lausanne
MA-Ecublens

CH-1015 Lausanne

Prof.Dr. Richard D. Gill
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof.Dr. Evarist Gine
Department of Mathematics U-9
University of Connecticut
196 Auditorium Road

Storrs , CT 06269
USA

Prof.Dr. Friedrich Götze
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131

33501 Bielefeld

Prof.Dr. Rudolf Grübel
Institut für Mathematische
Stochastik
Universität Hannover
Postfach 6009

30060 Hannover

Dr. Achim Klenke
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2

91054 Erlangen

Malte Grunwald
Fachbereich Mathematik / FB 3
Sekr. MA 7-4
Technische Universität Berlin
Straße des 17. Juni 136

10623 Berlin

Dr. Wolfgang König
Fachbereich Mathematik / FB 3
Sekr. MA 7-5
Technische Universität Berlin
Straße des 17. Juni 136

10623 Berlin

Prof.Dr. Marc Hallin
Institut de Statistique
Universite Libre de Bruxelles
Campus Plaine, C.P. 210
Boulevard du Triomphe

B-1050 Bruxelles

Prof.Dr. Jens-Peter Kreiss
Institut für Mathematische
Stochastik der TU Braunschweig
Pockelsstr. 14

38106 Braunschweig

Prof.Dr. Wolfgang Härdle
Inst.für Statistik und Ökonometrie
Humboldt Universität zu Berlin
FB Wirtschaftswissenschaften

10099 Berlin

Prof.Dr. Claudio Landim
Instituto de Matematica Pura e
Aplicada - IMPA
Jardin Botanico
Estrada Dona Castorina, 110

22460 320 Rio de Janeiro , RJ
BRAZIL

Prof.Dr. Frank den Hollander
Mathematisch Instituut
Katholieke Universiteit Nijmegen
Toernooiveld 1

NL-6525 ED Nijmegen

Prof.Dr. Boris Y. Levit
Faculty of Mathematics and Computer
Science
University of Utrecht
P.O.Box 80.010

NL-3508 TA Utrecht

Prof.Dr. Mark Low
University of Pennsylvania
Department of Statistics
The Wharton School

Philadelphia PA 19104-6302
USA

Prof.Dr. Yoichi Nishiyama
c/o Prof.Dr. Richard Gill
Mathematical Institute
University of Utrecht
PO Box 80010

NL-3508 TA Utrecht

Matthias Löwe
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131

33501 Bielefeld

Dr. Michael Nussbaum
Weierstraß-Institut für
Angewandte Analysis und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39

10117 Berlin

Prof.Dr. Enno Mammen
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Prof.Dr. Yosihiko Ogata
Institute of Statistical
Mathematics
4-6-7 Minami Azabu, Minato-ku

Tokyo 106
JAPAN

Prof.Dr. James Stephen Marron
Statistics Department
University of North Carolina
Phillips Hall

Chapel Hill , NC 27599-3260
USA

Prof.Dr. Emmanuel Rio
Mathematiques
Universite de Paris Sud (Paris XI)
Centre d'Orsay, Batiment 425

F-91405 Orsay Cedex

Prof.Dr. Dietrich Werner Müller
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Prof.Dr. Hermann Rost
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Dr. Vladimir Spokoiny
Weierstraß-Institut für
Angewandte Analysis und Stochastik
im Forschungsverbund Berlin e.V.
Mohrenstr. 39

10117 Berlin

Prof.Dr. Alexander B. Tsybakov
Universite Paris VI
Lab. de Stat. Theorique et Appl.
4 Pl. Jussieu, B.P. 158

F-75252 Paris Cedex 5

Prof.Dr. Jeffrey Steif
Dept. of Mathematics
Chalmers University of Technology

S-41296 Gothenburg

Aad W. van der Vaart
Mathematisch Instituut
Vrije Universiteit
De Boelelaan 1081 a

NL-1081 HV Amsterdam

Prof.Dr. Helmut Strasser
Institut für Statistik
Wirtschaftsuniversität Wien
Augasse 2 - 6

A-1090 Wien

Prof.Dr. Anton Wakolbinger
Fachbereich Mathematik
Universität Frankfurt
Postfach 111932

60054 Frankfurt

Prof.Dr. Winfried Stute
Mathematisches Institut
Universität Gießen
Arndtstr. 2

35392 Gießen

Prof.Dr. Willem R. van Zwet
Mathematisch Instituut
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. Howell Tong
Institute of Mathematics and
Statistics
The University
Cornwallis Building

GB-Canterbury , Kent, CT2 7NF