MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Gewöhnliche Differentialgleichungen

16.03.–22.03.1997

The conference was organized by J. Mawhin (Louvain–la–Neuve), K. Schmitt (Salt Lake City), and H.O. Walther (Gießen). As a main theme the organizers had chosen

Ordinary differential equations and reduction of high–dimensional systems.

Forty–three scientists from thirteen countries (10 European, 3 American) took part. Twenty–eight lectures were presented. Participants organized three additional informal seminars. An evening lecture was devoted to the work of the late M.A. Krasnoselskii who had died several weeks before the conference.
Directly concerned with the main theme were twelve lectures. A singular perturbation approach provided new insight into the dynamics of systems of ODE's describing the firing patterns of coupled nerve cells, with an emphasis on mechanisms for *synchronization and desynchronization patterns*. For a basic delay differential equation (DDE), also related to neural networks, an attracting invariant set was shown to be a three–dimensional *stratification* of invariant smooth submanifolds in the infinite–dimensional ambient phase space; the ODE–dynamics on this set were obtained in detail. Another singular perturbation type approach, now for DDE's with state–dependent delay, led to the study of *limit profiles* for possible shapes of periodic solutions. These profiles are invariant under difference equations. Their existence, uniqueness and geometry are related to solutions of *fixed point problems with little compactness*, which also occur in statistical mechanics and in applied areas such as machine scheduling. — The largest subset of lectures focussed on the reduction of the dynamics generated by Partial Differential Equations (PDE's). For a class of parabolic equations over a one–dimensional domain, with a suitable *integer–valued Liapunov–functional*, a smooth finite–dimensional invariant manifold containing the global attractor was obtained.

For parabolic equations on spatially large domains, *modulation equations* (like the Ginzburg–Landau equation) were discussed, and the invariant *inertial manifolds* of these simpler equations were compared to those of the original system. The problem of transmission of encoded information through long optical fibres motivated the reduction of a PDE to an ODE on an invariant manifold composed of travelling wave solutions of the *pulse packet* type; their stability and dynamics were analyzed. A combination of local invariant manifold theory with singular perturbation helped to understand better the geometry in phase space which underlies observed *transient behaviour* in the Cahn–Hilliard equations for rapid cooling in composite materials. This theory was supplemented by an interesting estimate involving eigenfunctions of the Laplacian. Reduction by symmetry and *normal forms* of ODE's were used to analyze the scenarios for the motion of the tip of a meandering spiral, as it is observed in various chemical reactions. Symmetries and reduction to center manifolds led to results on the persistence of complicated stable *heteroclinic cycles* under perturbations, for a PDE modelling a fluid in a rotating sphere. These results may help to understand pole reversal of the earth's magnetic field. ODE's permitted to find all periodic solutions of special reaction–diffusion–systems of the Ginzburg–Landau type, under Dirichlet boundary conditions. The motion towards the global attractor of this PDE and on it can be described by means of a new invariant, the *torsion number*. Stationary solutions and limit sets for *degenerate parabolic equations*, which are not accessible by established techniques, were analyzed with the help of associated ODE's. In particular it was shown that bounded solutions of porous media type equations settle down at single stationary solutions. A variety of new existence results on positive *radial solutions* for elliptic boundary value problems were presented, including equations with a *p–Laplacian*.

A next group of lectures concentrated on reversible and Hamiltonian dynamics. New upper bounds for *Arnol'd diffusion* in one of the basic examples were found. A result on boundedness of all solutions was derived using *twist dynamics* close to infinity. *Reversible and symplectic iteration schemes* were proven to provide better numerical results, in particular on attracting invariant tori of perturbed integrable systems. Bifurcation of *subharmonic periodic orbits* was explained in terms of suitable reduced problems.

The third group of lectures dealt with a variety of topics of current interest: Existence and description of *chaotic motion via isolating blocks*, degenerate

bifurcation from homoclinic solutions under nonautonomous perturbations, *coexistence* in periodic environments, the role of invariant *Lagrangian manifolds* in a control problem and the use of two–point boundary problems to find suboptimal feedback strategies. A fourth group of lectures were devoted to topics of current interest in neighbouring fields: Variational methods for *nonlinear Schr"odinger equations* in situations without a Palais–Smale condition, results on linear stability for travelling waves by means of the characteristic *Evans function* (which is defined in terms of an ODE), parabolic functional differential equations with deviating *spatial* argument, results on the fixed point index for maps with *asymptotically homogeneous nonlinearities*, and on the dynamics given by certain orientation preserving homeomorphisms of the disk.
In addition participants organized informal seminars with lectures and discussions on DDE's (global bifurcation of periodic orbits, chaotic motion), on PDE's (variational methods and elliptic regularization, location of extrema of solutions, conjugacy of elliptic dynamics with an ODE), and on positive radial solutions of boundary value problems for ODE's and PDE's. Each of these seminars attracted a large and engaged audience.
With special gratitude the friendliness, assistance, and extra efforts of the staff of the institute must be mentioned who helped to make the conference a success.
The meeting ended on Friday, March 21, at 3:30 p.m.

Vortragsauszüge

TH. BARTSCH:

On a nonlinear Schrödinger equation with periodic potential

I report on joint work with Yanhong Ding. We consider the nonlinear Schrödinger equation

$$(NS) \qquad \begin{cases} -\Delta n + V(x)n = \lambda n + g(x,n) & x \in \mathbb{R}^N \\ n(x) \to 0 & |x| \to \infty \end{cases}$$

with V, g periodic in the x-variables. The nonlinearity g is assumed to be superlinear and subcritical, e.g.: $g(x,u) = a(x)|u|^{p-2}u, 2 < p < \frac{2N}{N-2}, a > 0$. The spectrum of $-\Delta + V$ is purely continuous and consists of a union of closed intervals in $\mathbb{R}$, bounded below. We prove that (NS) has a nontrivial solution $u \in H^2_{loc}(\mathbb{R}^N) \cap L^p(\mathbb{R}^N), p$ depending on g, if $\lambda \in \sigma(-\Delta + V)$ is a right endpoint of $\sigma(-\Delta+V)$, i.e., $(\lambda, \lambda+\alpha) \cap \sigma(-\Delta+V) = \emptyset$ for some $\alpha > 0$. There are a number of existence results for $\lambda < \inf \sigma(-\Delta + V)$ or λ in a gap of the spectrum (Coti–Zelati & Rabinowitz, Heinz, Küpper & Stuart, Alamo & Li, Troestler & Willem, Kryszewski & Szulkin). However, this seems to be the first existence result for $\lambda \in \sigma(-\Delta+V)$. In the proof we consider the functional $\phi : H^1(\mathbb{R}^N) \to \mathbb{R}$ associated to (NS). Using a 'strongly indefinite linking' we obtain a generalized Palais–Smale sequence (u_n) of ϕ and show that (u_n) converges weakly in some completion of E towards a weak solution u of (NS). We do not know whether u lies in H^1 but can prove $u \in H^2_{loc} \cap L^p$ and $u(x) \to 0, |x| \to \infty$.

M. BÜGER:

Periodic solutions of a reaction–diffusion–system

Given the reaction–diffusions–system

$$\frac{d}{dt}\begin{pmatrix} u(t,x) \\ v(t,x) \end{pmatrix} = \begin{pmatrix} l_u & 0 \\ 0 & l_v \end{pmatrix} \Delta_x \begin{pmatrix} u(t,x) \\ v(t,x) \end{pmatrix} + f(u(t,x), v(t,x)) \text{ for } t > 0, 0 < x < 1,$$

$$u(t,0) = u(t,1) = v(t,0) = v(t,1) = 0 \text{ for all } t \geq 0,$$

with $u(0,\cdot) = u_0, v(0,\cdot) = v_0$ and positive constants l_u, l_v, we ask whether the semiflow of solutions $[0,\infty) \ni t \mapsto (u(t,\cdot), v(t,\cdot)) \in H^1_0(0,1) \times H^1_0(0,1)$ contains a periodic solution. We examine periodic solutions for the special vector field which is given by $\hat{f}(r,\varphi) = (r(1-r), 1)$ in polar coordinates. For $t > 0$ the solutions of the reaction–diffusion–system exist in the classical sense. If there are periodic solutions, we are interested in their properties. For $l_u = l_v$ we solve the problem by reducing it to the one–dimensional case. We give conditions for existence and stability of periodic solutions. The case $l_u \neq l_v$ is more difficult. We examine whether the results shown in the first case can be obtained in this case, too.

J. CAMPOS:

Homeomorphisms of the disk with trivial dynamics and extinction for competitive systems

In this talk we obtain some type of trivial dynamics for a class of homeomorphisms $f : \mathbb{D} \to \mathbb{D}$ where

$$\mathbb{D} = \{z \in \mathbb{C} : |z| \leq 1\}.$$

In particular we prove if f is orientation preserving and

$$Fix\{f\} \subset \partial\mathbb{D}$$

then the omega limit set of every point is always contained in $Fix\{f\}$. This result depends essentially on the topology of the plane and the proof follows from ideas developed by Brown (1995) and from a classical result developed by Brouwer on homeomorphisms of the plane without fixed points. We also give an application to the study of the three dimensional competition system and give a condition for extinction of some of the species. This proof follows from a reduction principle of the dynamics to a two dimensional disk and the application of the previous result.

M. FEČKAN:

Bifurcation from degenerate homoclinics in autonomous ordinary differential equations with periodic perturbations

Bifurcations of homoclinic solutions are investigated for ordinary differential equations with periodic perturbations possessing a degenerate homoclinic solution. It is shown that generically the set of bifurcation values is diffeomorphic to a surface of the Morin singularity. Also more degenerate Morin singularities are obtained.

E. FEIREISL:

On convergence for certain degenerate equations

We shall study the long–time behaviour of solutions to certain degenerate partial differential equations of parabolic type. Degenerate means that certain leading coefficients in the equation vanish at some points either due to their dependence on the solution itself or because of some external constraints. Consequently, the standard methods do not apply and a refined analysis is needed. As an example, we show that any bounded solution of a porous medium type equation stabilizes for large times to a unique stationary state. The solutions of the underlying ODE stationary problem are studied in detail.

B. FIEDLER:

Meandering Spirals

We derive normal forms for the dynamics near relative equilibria of finite–dimensional Lie groups G. The Lie group need not be compact and the action need not be differentiable originally. Isotropy may occur, due to non-free action. Based on earlier work by Wulff, and joint work with Sandstede, Scheel, Wulff, we have to study skew products

$$\dot{g} = ga(v)$$

$$\dot{v} = \varphi(v)$$

where $a(v) \in alg(G), \varphi(v) \in \mathbb{R}^n, v$ in a slice V to the group action, and $\varphi(0) = 0$. By transformations $g \to gg_0(v)$, we can eliminate non–resonant terms from $a(v)$; resonance is between spec $\varphi'(0)$ and spec $ad(a(0))$. For example, drift resonances and homoclinic tip shifts in Takens–Bogdanov bifurcations of v turn out to be small beyond any finite order. We also observe random tip motions, for $G = SE(2)$, if v converges to a nondegenerate (relative) homoclinic orbit.

D. FLOCKERZI:

Nonlinear L^2–Gain Analysis

As introduction we review the standard theory of dissipation inequalities for affine control systems $(*)\dot{x} = a(x) + b(x)u$ with to be controlled variable $z = c(x)[a(0) = 0, c(0) = 0]$. The problem then is: Find (small) $\gamma > 0$ and $k(x) \geq 0$ with

$$\int_0^T |c(x)|^2 dt \leq \gamma^2 \int_0^T |u|^2 dt + K(x_0)$$

along solutions of the initial value problem $(*)$, $x(D) = x_0$ (with internal stability). In the second part we generalize to the local state feedback H^∞–problem where disturbances $v(t)$ are included in the model $(*)$ so that one is interested in L^2–gain estimates like

$$\int_0^T [|c(x)|^2 + |u(x)|^2] dt \leq \gamma^2 \int_0^T |v|^2 dt + K(x_0)$$

for an appropriate feedback $u(x)$. The third and main part presents the game theoretic approach to the nonlocal H^∞–problem. We relate the Isaacs problem and the Hamilton–Jacobi–PDE to the existence of certain Lagrangian integral manifolds and indicate how two–point boundary value problems together with Riccati–type equations can be used in the computation of suboptimal feedback strategies.

M. GARCIA-HUIDOBRO:

Positive singular solutions for a class of non–homogeneous p–Laplacian–like equations

We present some results concerning the behavior of positive radially symmetric solutions to an equation of the form

$$-\mathrm{div}(A(|\nabla u|)\nabla u = f(u) \qquad (1)$$

near an isolated singularity at the origin. Here, the function $\phi(s) := sA(s)$ is assumed to be an odd increasing homeomorphism of $\mathbb{R}$ onto $\mathbb{R}$ and $f \in C(\mathbb{R}^+)$. We first give an a priori estimate of the order of such singularity without imposing any growth restriction on ϕ or f, and then classify the behavior of positive radial solutions to (1) under an asymptotic Serrin type condition, that is, a condition under which any positive solution to (1) is either regular or it behaves as a 'fundamental' positive singular solution to

$$-(r^{N-1}\phi(u'))' = 0, r \in (0, r_0), r_0 > 0.$$

We also prove existence of such solutions.

T. KAPITULA:

Locating eigenvalues with the Evans function

The Evans function $E(\lambda)$ is an analytic function whose zeros coincide with the eigenvalues of the operator, L, obtained by linearizing about a travelling wave. The order of the eigenvalue λ_0 is equal to the order of the zero of $E(\lambda)$. If p is the order of the eigenvalue, the term $\partial_\lambda^p E(\lambda_0)$ is shown to be proportional to the L^2 inner product of an eigenfunction of $L^* - \lambda_0^*$ and a generalized eigenfunction of $L - \lambda_0$. A consequence of this result is the ability to locate the small eigenvalues for the pulse solution to the parametrically forced NLS equation.

U. KIRCHGRABER:

On Stoffer's approach to symplectic and reversible integration

Numerical ODE–solvers are handy tools to explore the dynamical properties hidden behind systems of ode's. However, how reliable are the results they yield? A Runge–Kutta method generates an approximation to the time–h–map of the de under consideration. Do these two discrete dynamical systems (dds) share the basic geometric properties? In the talk we present the following result due to D. Stoffer. Given a perturbed integrable system, let us assume that it admits a weakly attractive invariant torus. Then the dds shares this property provided it is generated by a so–called symplectic Runge–Kutta scheme and if the step–size h satisfies the relation

$$h \leq \left(\frac{1}{ln(\frac{1}{\varepsilon})}\right)^{1/\beta} \qquad (*)$$

Here ε denotes the perturbation parameter and β is a suitable positive constant. Of course $(*)$ implies $h \to 0$ as $\varepsilon \to 0$. Yet $(*)$ is a 'good result' in the sense that the bound on h is large compared to ε, asymptotically speaking as $\varepsilon \to 0$. In the second part of the talk we present Stoffer's variable step size mechanism, which, if combined with a reversible Runge–Kutta scheme, respects the reversible structure of the underlying de. The efficency of this method is illustrated with the help of highly excentric Kepler orbits.

A. KRASNOSELSKII:

Degenerate equations with asymptotically homogeneous nonlinearities

The talk concerns equations with degenerate principal linear parts and asymptotically homogeneous nonlinearities. Asymptotic homogeneity of non–linear operators allows to reduce the calculation of topological characteristics of such equations in infinite–dimensional Banach spaces to the calculation of analogous topological characteristics in the finite–dimensional spaces. In applications to concrete boundary value problems these spaces are often 1– or 2–dimensional and this makes it possible to calculate these characteristics exactly. For two–point BVP new theorems on existence and on asymptotic bifurcation points are presented. The problem of forced periodic oscillations is considered for higher order ODE with hysteresis nonlinearities.

R. LAUTERBACH:

Heteroclinic sets in a degenerate bifurcation problem

A heteroclinic cycle is a series of equlibria x_i and connecting orbits $y_i, i = 1, ..., n$, with $x = \alpha(y_i), x_{i+1} = \omega(y_i)$ and everything is taken mod n. If one (or more) of the connections is replaced by a higher dimensional set of connections then we call the resulting object a generalized heteroclinic cycle or a heteroclinic set. Heteroclinic cycles are not generic within general dynamics. However, they can occur generically in systems with symmetry. Heteroclinic sets may occur in degenerate bifurcation problems with symmetry.
One such degeneratic bifurcation problem comes up when one studies the existence of flows in the spherical Bénard problem which support magnetic pole reversals as they have taken place in the earth magnetic field every 10000–40000 years. The main problem in our mathematical model, which is based on convectively driven dynamo, is to show the existence and stability of a certain heteroclinic set.
From these results we infer the persistence of this set in the slowly rotating case. Applying this to the dynamo problem leads to heteroclinic behaviour in the motion of the fluid in a rotating sphere. This leads finally to pole reversals.

S. MAIER-PAAPE:

Spinodal decomposition for the Cahn–Hilliard equation in higher space dimensions

One of the pattern formation phenomena which are modeled by the Cahn-Hilliard equation will be addressed, namely the initial stage phase separation known as spinodal decomposition. It will be shown that in one, two, and three space dimensions most solutions of the Cahn–Hilliard equation which originate near certain spatially homogenous equilibria will develop certain patterns which are strongly related to a characteristic wave length. These results agree with numerical and physical experiments. (Joint work with Thomas Wanner, currently Georgia Tech, Atlanta).

J. MALLET-PARET:

A nonlinear eigenvalue problem arising from a state–dependent delay differential equation

We begin with a class of state–dependent delay differential equations of the form

$$\varepsilon\dot{x}(t) = f(x(t), x(t-r)), \quad r = r(x(t)),$$

with a singular perturbation parameter $\varepsilon \ll 1$, a time delay $r = r(x)$, and a nonlinearity f which enjoys a monotonicity (negative feedback) condition in the second argument. Our interest is in the limiting shape of slowly oscillating periodic solutions $x(t)$ as $\varepsilon \to 0$. Upon defining the limiting profile $\Omega \subset \mathbb{R}^2$ to be the limit $\Gamma_{\varepsilon_k} \to \Omega$, in the Hausdorff topology, of a sequence of graphs $\Gamma_{\varepsilon_k} = graph(x_{\varepsilon_k}) \subset \mathbb{R}^2$ of slowly oscillating periodic solutions $x_{\varepsilon_k}(t)$, we parameterize the increasing and decreasing portions of Ω as graphs $t = \psi_n(x)$. Here each $\psi_n : [-\nu, \mu] \to \mathbb{R}$ is a continuous function, where μ and $-\nu$ are the limiting maximum and minimum of the solution sequence, with $n \in \mathbb{Z}$. Using a singular perturbation analysis, we show that each $\psi = \psi_n$ satisfies a functional equation of the form

$$(*) \qquad p + \psi(x) = \max_{\gamma(x) \le \xi \le \mu} [a(x,\xi) + \psi(\xi)], \quad -\delta \le x \le \mu,$$

where $a(x,\xi)$ is a known kernel, $\gamma(x)$ is a continuous monotone function with $0 < \gamma(x) < x$ for $0 < x < \mu$, and $\gamma(0) = 0$, and where p is an unknown 'additive eigenvalue' representing the limiting period of the solution sequence. We then show, under various monotonicity conditions on r, that p can be explicitly given, that $a(x,\xi) = a(\xi)$ is independent of x, and that the general solution of $(*)$ has the form

$$\psi(x) = \max_{1 \le i \le M} \{\psi^{(i)}(x) + c_i\}$$

for a certain 'basis' $\{\psi^{(i)}\}_{i=1}^{M}$ of solutions, where $c_i \in \mathbb{R}$ are arbitrary constants. Although $M = 1$ generically (implying uniqueness of the limiting profile Ω), it is possible that $M > 1$, yielding nonuniqueness of solutions of $(*)$. This also shows that Ω can vary discontinuously with respect to other parameters in the system.

R. MANASEVICH:

Some existence results for positive solutions for p–Laplacian type equations

We consider systems of the form

$$(P) \begin{cases} -(r^{N-1}\phi(u'))' = r^{N-1}v^{\delta}, r \in (0,R), \\ -(r^{N-1}\theta(v'))' = r^{N-1}u^{\mu}, r \in (0,R), \\ u(R) = 0, u'(0) = 0, v(R) = 0, v'(0) = 0, \end{cases}$$

where $' = \frac{d}{dr}$. These systems arise when studying radial solutions to some non linear partial differential equations on a ball in $\mathbb{R}^N$.

Under certain conditions for the functions $\phi, \theta : \mathbb{R} \to \mathbb{R}$ and for the powers δ and μ we show the existence of positive (componen-twise) solutions to (P). We will discuss generalizations of these results.

A. MIELKE:

Inertial manifolds for modulational problems

We consider parabolic problems on spatially large domains. If the problem is above but close to the threshold of instability of the natural spatially homogeneous state then the theory of modulation equation allows us formally to study the dynamics in the associated modulation equation. We show that in this situation both systems possess an intertial manifold and that the flow on them is closely related.
As application we consider the Swift–Hohenberg equation and – hopefully – the Navier–Stokes equation.

R.D. NUSSBAUM:

Limiting profiles for the differential–delay equation
$\varepsilon x'(t) = f(x(t), x(t-r)), r = r(x(t))$

In joint work with John Mallet–Paret we have proved the existence of slowly oscillating periodic solutions of the equation

$$(1)_\varepsilon \qquad \varepsilon x'(t) = f(x,t), x(t-r)), r = r(x(t)), \varepsilon > 0,$$

under natural assumptions on f and r. We suppose that $\varepsilon_k \to 0^+$ as $k \to \infty$ and x_{ε_k} is a slowly oscillating periodic solution of $(1)_{\varepsilon_k}$, and we consider Γ_{ε_k}, the graph in $\mathbb{R}^2$ of x_{ε_k}. By taking a subsequence of (ε_k) we can assume that $\Gamma_{\varepsilon_k} \to \Gamma$ in the Hausdorff metric on compact subsets of $\mathbb{R}^2$. If we define $\mu = \sup\{\xi | (\xi, \tau) \in \Gamma$ for some $\tau \in \mathbb{R}\}$ and $-\nu = \inf\{\xi | (\xi, \tau) \in \Gamma$ for some $\tau \in \mathbb{R}\}$, we have proved in great generality that $\mu > 0$ and $\nu > 0$. Our general program is to describe the set Γ as precisely as possible, and we have exactly determined Γ for large classes of f and r. In studying $(1)_\epsilon$ we are led to a class of 'max-type' equations:

$$(2) \qquad x(s) + p = \max\{a(s,t) + x(t) | \alpha(s) \le t \le \beta(s)\}, 0 \le s \le \mu.$$

Here $\mu > 0$ is viewed as given and $\alpha : [0,\mu] \to [0,\mu]$ and $\beta : [0,\mu] \to [0,\mu]$ are given continuous functions with $\alpha(s) \le \beta(s)$ for $s \in [0,\mu]$; $a(s,t)$ is a given continuous, real-valued function defined on $\{(s,t)|\alpha(s) \le t \le \beta(s), 0 \le s \le \mu\}$. One seeks $x \in C[0,\mu]$ and $p \in \mathbb{R}$ such that (2) is satisfied. In the discrete, finite-dimensional case ($p + x_i = \max_j(a_{ij} + x_j), x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$) there is an extensive literature (for example, in operations research) concerning such maps. If, in our notation, $\alpha(s) := 0$ and $\beta(s) := \mu$, R.B. Griffiths has considered (2) in connection with the Frenkel-Kontorova models of statistical mechanics. In our case, new problems are introduced by the fact that usually, when α and β are not constant, the right hand side of (2) defines a nonlinear operator which is <u>not</u> compact. Nevertheless, we develop a general abstract theory sufficient to handle (2). A very special case of our results is the following theorem.

<u>Theorem</u>: Assume that α is increasing on $[0,\mu]$ and β is decreasing on $[0,\mu]$; we allow α or β to be constant on subintervals of $[0,\mu]$. Assume that for all $s \in [0,\mu], \lim_{i\to\infty} \alpha^i(s) = 0$, where α^i denotes composition of α with itself i times. Assume either (a) α is constant on a neighbourhood of 0 or (b) there exists $s_0 > 0$ with $a(s_0,s_0) > 0$ and $s_0 \le \beta(\mu)$. Then eq. (2) has a solution (x,p) with $x \in C([0,\mu])$ and $p \ge a(s_0,s_0)$. The number p in eq. (2) is uniquely determined.

R. ORTEGA:

Boundedness of a piecewise linear oscillator and a variant of the small twist theorem

Consider the differential equation

$$\ddot{x} + n^2 x + h(x) = p(t)$$

where $n \geq 1$ is an integer, p is 2π-periodic and $h : \mathbb{R} \to \mathbb{R}$ is a nonlinear bounded function. In 1969, Lazer and Leach obtained conditions for the existence of 2π-periodic solutions that sometimes become necessary and sufficient. Namely, if h has limits at infinity, denoted by $h(+\infty)$ and $h(-\infty)$, and

$$h(-\infty) \leq h(x) \leq h(+\infty) \forall x \in \mathbb{R},$$

then there exists a 2π-periodic solution if and only if

$$\pi|\widehat{p_n}| < h(+\infty) - h(-\infty),$$

where $\widehat{p_n} = \frac{1}{2\pi} \int_0^{2\pi} p(t)e^{-int}dt$.

The main questions in this talk are: Do the conditions of Lazer and Leach contain some information on the dynamics? Are all solutions bounded? A positive answer to these questions is given when h is the piecewise linear function

$$h(x) = \begin{cases} L & \text{if} \quad x \geq 1 \\ Lx & \text{if} \quad |x| \leq 1 \\ -L & \text{if} \quad x \leq -1 \end{cases}$$

and p is of class C^5. In this case all solutions are bounded and the dynamics (for solutions of large amplitude) can be described my means of a twist mapping.

The proof is based on a variant of Moser's invariant curve theorem that applies to certain mappings of the cylinder that have the intersection property and can be described as

$$\begin{cases} \theta_1 = \theta + 2\pi + \delta l_1(\theta, r) + \dots \\ r_1 = r + \delta l_2(\theta, r) + \dots \\ \qquad (\delta \text{ small}) \end{cases}$$

Among other conditions, it is required that the differential system

$$\dot{\theta} = l_1(\theta, r), \quad \dot{r} = l_2(\theta, r)$$

has a first integral. The invariant curves of the mapping are located in neighbourhoods of the closed orbits of the continuous dynamical system.

B. SANDSTEDE:

Finite–dimensional reduction of the dynamics of pulse packets

We consider dissipative nonlinear partial differential equations on the real line $\mathbb{R}$ which admit stable equilibria u_*. A pulse packet is on initial value resembling N copies of u_* widely separated in space. The issue is then the temporal behavior of the associated solution of the PDE. We prove the existence of N–dimensional, locally invariant manifold consisting entirely of pulse packets. The manifold is exponentially attracting with respect to the semiflow generated by the PDE. Moreover, it is parametrized by the translation of the pulse packet and the $N-1$ distances between consecutive copies of u_*. Therefore, as time varies, only the relative position of the different copies of u_* changes. Finally, we give an explicit description of the vector field governing the flow on the invariant manifold in terms of the equilibrium u_* alone.

R. SHIVAJI:

Semipositone Problems

In this presentation we discuss the critical developments in the theory of semipositone problems. A typical example is the study of non–negative solutions to the Dirichlet–problem

$$\begin{aligned} -\Delta u &= \lambda f(u) \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega \end{aligned}$$

where $\lambda \in (0,\infty)$ is a parameter, Ω is a smooth bounded region in $\mathbb{R}^n$, Δ is the Laplacian operator and $f : \mathbb{R} \to \mathbb{R}$ is a smooth function such that $f(0) < 0$. We further suggest important open problems.

C. SIMÓ:

The classical Arnold example of diffusion with two equal parameters

We consider the time dependent Hamiltonian with two degrees of freedom, which can be written as the autonomous Hamiltonian with three degrees of freedom given by

$$H(I,\phi) = \frac{1}{2}(I_1^2 + I_2^2) + I_3 + \varepsilon(\cos\phi_1 - 1)(1 + \mu(\cos\phi_2 + \cos\phi_3)).$$

This system (in a slightly different, equivalent, form) was introduced by Arnold to illustrate the transition chain mechanism. It is a popular example which displays a (very slow) drift of the actions for any value of ε, provided μ is small enough. This model has two difficulties to be considered as a typical example of diffusion. On one side, how small μ must be depends on ε. This is a consequence of a naive use of Melnikov's formula to measure the splitting of separatrices. On the other side, it has a strong degeneracy because all the 'normally' hyperbolic tori which appear for $\mu = 0$ subsist when introducing the perturbation. This communication deals with the first difficulty. To this end the two parameters are taken as equal. μ could also be taken as a power of ε. From now on, and for scaling purposes, we set $\mu = \varepsilon = \eta^2$.
The manifold T_ω given by $I_1 = \phi_1 = 0, I_2 = \omega, I_3 = 0$ is an invariant three dimensional whiskered torus for any value of ω. Our purpose is to show that, under a suitable diophantine condition on ω, the separatrices of these tori have a transversal intersection. This is enough to guarantee diffusion in the present case.
First we shall obtain a rigorous upper bound of the splitting of the separatrices of the whiskered tori. Then a combination of symbolic and numerical methods is used to compute the 'real' splitting on a suitable section and display the main features of the splitting. It is seen experimentally that the order of magnitude of the 'true' splitting is the same as the one given by the theoretical result. Then a method is sketched to obtain analytically the dominant term. The key idea is that, for a given value of η, one has to do some steps of a normal form procedure *before applying Melnikov's method.* The number of steps is constant for η on some interval, but when η goes to zero it increases to infinity. In this way the *most relevant resonances* for this

range of values of η are put in evidence. The dominant terms in the splitting are, of course, related to the approximants of ω.

A.L. SKUBACHEVSKII:

Elliptic and parabolic functional differential equations and applications

A field rotation in a two–dimensional feedback loop of a nonlinear optical system leads to generation of multi–petal waves. A mathematical model of such a system is described by bifurcation of periodic solutions for the nonlinear parabolic functional differential equation

$$u_t + u = D\Delta u + K(1 + \gamma \cos u_g)(x \in Q, t \in \mathbb{R}) \tag{1}$$

with Neumann boundary condition. Here $Q \subset \mathbb{R}^2$ is a bounded domain with boundary $\partial Q \in C^\infty$; Δ is the Laplace operator; $D, \gamma, K \in \mathbb{R}$; $D > 0$ is the diffusion coefficient, $\gamma > 0$ is the visibility of the interference pattern, $K \neq 0$ is the nonlinearity coefficient, $u = u(x,t)$ is the nonlinear phase modulation, $u_g = u(g(x), t)$, g is a nondegenerate transformation, $g(x) \not\equiv x$. Let $\hat{w}$ be a solution of the equation

$$w = K(1 + \gamma \cos w) \tag{2}$$

for $K = \hat{K}$, and let

$$1 + \hat{K}\gamma \sin \hat{w} \neq 0. \tag{3}$$

We put $K = \hat{K} + \kappa$, and denote by $w(\kappa)$ a solution of equation (2) for sufficiently small κ. We define the unbounded linear elliptic functional differential operator $L(\kappa) : L_2(Q) \to L_2(Q)$ given by

$$L(\kappa)v = D\Delta v - v(\hat{K} + \kappa)\gamma v_g \sin w(\kappa),$$

$$D(L(\kappa)) = \{v \in W_2^2(Q) : \frac{\partial v}{\partial \nu}|_{\partial Q} = 0\},$$

where $W_2^2(Q)$ is a Sobolev space of functions in $L_2(Q)$ having all generalized derivatives up to the second order in $L_2(Q)$, ν is the unit normal vector to

∂Q. The operator $L(\kappa)$ has a discrete spectrum consisting of eigenvalues $\lambda_s(\kappa) = \delta_s(\kappa) + i\omega_s(\kappa)(s = 1, 2, \ldots)$. Assume that the following conditions hold:

(4) $$\lambda_1(0) = i\hat{\omega} \text{ is a simple eigenvalue of } L(0),$$

where $\hat{\omega} > 0$;

(5) $$n\hat{\omega}i \notin \sigma(L(0)) \quad (n = 0, 2, 3, \ldots);$$

(6) $$\delta_1'(0) \neq 0.$$

Denote by $W_2^{2,1}(\Omega_{2\pi})$ the Sobolev space of functions in $L_2(\Omega_{2\pi})$ having all generalized derivatives with respect to x up to the second order in $L_2(\Omega_{2\pi})$ and the first generalized derivative with respect to t in $L_2(\Omega_{2\pi})$, where $\Omega_{2\pi} = Q \times (0, 2\pi = 0)$. Let

$$W_{2,N}^{2,1}(\Omega_{2\pi}) = \{u \in W_2^{2,1}(\Omega_{2\pi}) : \frac{\partial u}{\partial \tilde{\nu}}|_{\partial Q \times (0,2\pi)} = 0, u|_{t=0} = u|_{t=2\pi}\},$$

where $\tilde{\nu} = (\nu, 0)$.

Theorem. Let conditions (3) - (6) hold, and let $g(Q) = Q, g(x) = Kx + b(x \in Q)$, where K is an orthogonal matrix such that $K^2 \neq E, b \in \mathbb{R}^2$. Then there is $\varepsilon_0 > 0$ such that there exists an analytical vector-valued function $\varepsilon \mapsto (v(\varepsilon), \omega(\varepsilon), \kappa(\varepsilon))$ from $(-\varepsilon_0, \varepsilon_0)$ to $W_{2,N}^{2,1}(\Omega_{2\pi}) \times \mathbb{R}^2$ such that

(7) $$v(0) = 0, \quad \omega(0) = 1, \quad \kappa(0) = 0.$$

Furthermore, the function $u(x,t,\varepsilon) = w(\kappa(\varepsilon)) + v(x, \tau, \varepsilon)$ is a $2\pi(\hat{\omega}\omega(\varepsilon))^{-1}$-periodic in t solution of equation (1) with the Neumann boundary condition, where $\tau = \omega(\varepsilon)\hat{\omega}t$.

R. SRZEDNICKI:

On chaos inside isolating blocks

By chaos for the flow generated by an ordinary differential equation we mean the existence of a local section such that its Poincaré map is semiconjugated to a subshift of finite type and the counterimage of any periodic point of the subshift contains at least one periodic point of the Poincaré map. We describe suitable forms of Conley's isolating blocks which guarantee the existence of chaos.

I. TEREŠČÀK:

Inertial manifolds for infinite dimensional dynamical systems with discrete Lyapunov functionals

We consider the following parabolic equation

$$u_t = u_{xx} + f(x, u, u_x) \qquad (1)$$

on an interval I with Dirichlet, Neumann or periodic boundary conditions. The nonlinearity F is assumed to be C^2 in all variables. Then this equation generates a C^1 semiflow on an appropriate Banach space $X \hookrightarrow C^1(I)$. If the semiflow is point dissipative then it admits a global compact attractor $\mathcal{A}$. We establish that there is a finite dimensional C^1 submanifold $\mathcal{M}$ of X containing the attractor $\mathcal{A}$ and being positively invariant under the semiflow. The construction is based on choosing a dimension n of $\mathcal{M}$ and finding an appropriate C^1 submanifold $\mathcal{S}$ of X diffeomorphic to the $(n-1)$-dimensional sphere such that flowing of $\mathcal{S}$ for all positive times together with the attractor $\mathcal{A}$ form a desired manifold $\mathcal{M}$. The structure of the semiflow enabling this construction is given by the fact that the difference of two solutions of our original equation satisfies a linear parabolic equation of type (1) and by two properties of this equation. The first property is a compactness of the solution operator for positive differences of times. The second one is crucial saying that the numbers of zeros on I of a nonzero solution of such a linear equation evolves monotonically with time and is finite for all positive time instances. This property actually gives a discrete Lyapunov functional for the semiflow.

D. TERMAN:

Networks of neural oscillators

Inhibition in oscillatory networks of neurons can have apparently paradoxical effects, sometimes creating dispersion of phases, sometimes fostering synchrony in the network. We analyze a pair of biophysically modeled neurons and show how the rates of onset and decay of inhibition interact with the time scales of the intrinsic oscillators to determine when stable synchrony is possible. We show that there are two different regimes in parameter space in which different combinations of the time constants and other parameters regulate whether the synchronous state is stable. We also discuss the construction and stability on non–synchronous solutions, and the implications of the analysis for larger networks. The analysis uses geometric techniques of singular perturbation theory that allow one to combine estimates from slow flows and fast jumps.

A. VANDERBAUWHEDE:

Subharmonic branching in reversible and Hamiltonian systems

In reversible or Hamiltonian systems periodic orbits typically appear in one–parameter families (parametrized for example by the energy in the Hamiltonian case), and branches of subharmonic solutions may bifurcate from such 'primary branches' of periodic orbits. The study of this subharmonic branching reduces, via the Poincaré map, to the study of the bifurcation of periodic points from a fixed point in diffeomorphisms which are either reversible (i.e. conjugate to their inverse via a linear involution) or symplectic. We present a general reduction method for this kind of bifurcation problem. Fix some integer $q \geq 1$; then there is a 1–1 relation between the bifurcating q–periodic points of the original diffeomorphism and the bifurcating q–periodic points of some reduced diffeomorphism which is defined on some low–dimensional 'reduced phase space' and which has a $\mathbb{Z}_q$–symmetry; this reduced diffeomorphism inherits the reversibility or the symplectic structure of the original diffeomorphism; moreover, q–periodic orbits of the reduced diffeomorphism must also be $\mathbb{Z}_q$–orbits. We show how this $\mathbb{Z}_q$–symmetry of the reduced problem explains the elementary subharmonic branching found in reversible and Hamiltonian systems.

Z.-Q. WANG:

Nonautonomous singularly perturbed elliptic BVPs

In this talk, we present some multiplicity results of positive solutions for a class of nonautonomous semilinear boundary value problems including both Dirichlet and Neumann boundary value problems. Our results show that with the spatial dependence the solution structure is affected by the shape of the graph of the potential function. This is in contrast to the earlier results for autonomous problems where the shape of the domain plays the dominant role. The shape of the solutions is also considered and all solutions given are showed to be single–peaked solutions. More precise qualitative information is also given.

J. WU:

A 3–dimensional invariant manifold with boundary for a delay differential equation: Geometry, topology and dynamics

In this work with T. Krisztin (Szeged) and H.O. Walther (Gießen), we consider the scalar delay differential equation

$$\dot{x}(t) = -\mu x(t) + f(x(t-1)),$$

where $\mu \geq 0$ is a constant and $f : \mathbb{R} \to \mathbb{R}$ is a C^1–map satisfying a certain positive feedback and dissipativeness condition with $f(0) = 0$. The spectrum of the generator of the C_0–semigroup associated with the linearized system (at zero) is given by a real λ_0 and a sequence $\{\mu_k \pm i\nu_k\}$ with $\nu_k \in ((2k-1)\pi, 2k\pi)$ for $k \geq 1$. Assuming $\mu_1 > 0$, then there exists a 3–dimensional invariant manifold W tangent at 0 to the reellified eigenspace of the generator associated with $\{\lambda_0, \mu_1 \pm i v_1\}$. We give a complete description of the geometry and topology of clW and of the dynamics of the semiflow restricted to clW.

F. ZANOLIN:

Periodic solutions for population models

Consider the nonautonomous differential system of Kolmogorov type:

$$(1) \qquad \begin{cases} x_i' &= x_i h_i(t, x_1, ..., x_N) \\ i &= 1, ..., N \end{cases}$$

where $\forall i = 1, ..., N, h_i : \mathbb{R} \times (\mathbb{R}_+)^N \to \mathbb{R}$ is a continuous function which is T-periodic ($T > 0$) in the t-variable. We propose an existence theorem for positive T-periodic solutions of (1) (that is, coexistence states), which is based on a continuation theorem in [Capietto, Mawhin, Zanolin, Trans. AMS 192]. As an application, we consider a three dimensional Kolmogorov system generalizing the classical May–Leonard model of cycling competition with periodic coefficients. A necessary and sufficient condition for the existence of coexistence states is obtained for some classes of equations. This is a joint work with Anna Battauz (University of Udine).

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