MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12/1997

Reelle algebraische Geometrie

23.3.1997 - 29.3.1997

The conference "Reelle algebraische Geometrie" was organized by Prof. Dr. L. Bröcker, Prof. Dr. M. Coste and Prof. Dr. M. Knebusch. The main topics of the talks and discussions were latest developments in Real Algebraic Geometry, Semialgebraic Geometry, Real Algebra, Stellensätze, Real Differential Algebra and topological and analytical aspects of Real Algebraic Geometry.

The atmosphere of the conference was very international since experts from a variety of different countries participated.

Thorsten Wörmann

Semialgebraic Multiplicities

Gilbert Stengle, Lehigh University

Let A be a ring containing Q. In A let S be a multiplicatively closed set, T a preordering and I an ideal. Define the <u>real hull</u> of I (relative to S,T) to be

$$\rho(I) = \{a | s \cdot a^{2m} + t \in I^{2m} \text{ for some integer } m, s \in S, t \in T\}.$$

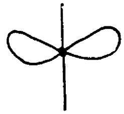
1. $\rho(I)$ is an ideal.
2. $\rho(\rho(I)) = p(I)$.
3. $x(t+t') \in \rho(I) \Rightarrow xt, xt' \in \rho(I)$ for $t,t' \in T$.
4. $\rho(I^m)\rho(I^n) \subset p(I^{n+m})$.

The evident inclusion $\rho(I) \subset \{s | sa^{2m} + t \in I\}$ shows, in the case that $A = R[x_1,\ldots,x_n]$, that $\rho(I)$ can be regarded as "improved" equations for the real zeros of I that satisfy the inequalities $S > 0$, $T \geq 0$.

Example: $I = ((x^2+y^2)y^2)$, $\rho(I) = (x^2y^2, xy^3, y^4)$. I has complex zeros whereas $\rho(I)$ has only real zeros.

Property 4 allows the graded ring $G = \bigoplus_{k=0}^{\infty} \rho(I^k)/\rho(I^{k+1})$ as a tool for defining multiplicities.

Example:

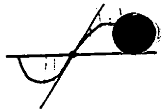


a) $A = R[x,y](y^2 - x^2 - x^4)$, $I = (\overline{x})$.

b) Same as a) but with T generated by $\overline{y}(\overline{x} - \overline{y})$. Here

$$G \cong R[v], \text{ deg } v = 1,$$

which is characteristic of a simple point.



Local separation of semialgebraic sets

Francesca Acquistapace, Università di Pisa
(with Carlos Andradas and Fabrizio Broglia)

Let $M \subseteq \mathbb{R}^n$ be a real algebraic variety and $A, B \subset M$ be two semialgebraic sets. Consider the following set

$$S(A,B) = \{p \in M \mid \forall U \text{ s.a. neigh. of } p,$$
$$A \cap U \text{ and } B \cap U \text{ are not polynomially separable}\}$$

We ask the following: Is $S(A,B)$ a semialgebraic set?

The answer is YES. The answer is given as an application of a geometric criterion of separation for semialgebraic sets (by the same authors). The proof works only for the field $\mathbb{R}$ of real numbers.

For non archimedean real closed fields the answer is NO, as shown by examples suggested to us during this meeting in conversations with Bröcker, Coste and Prestel.

For the proof we first give the answer in the case M is compact nonsingular and the Zariski boundary $\overline{\partial A}^Z \cup \overline{\partial B}^Z$ has nonsingular normal crossings components, in which case the criterion applies. In the general case we use a semialgebraic stratification of a desingularization M^ν of M and a compatible stratification of the map $\pi : M^\nu \to M$.

Rational points on certain rational varieties over the function field of a real curve

Antoine Ducros, Université Paris–Sud

Let C be a curve over the field $\mathbb{R}$ of real numbers. Let K denote the function field $\mathbb{R}(C)$, and for any P closed point of C let K_P denote the corresponding completion.

One studies the following problems: If X is a (proper, smooth) K–variety such that for every P $X(K_P) \neq \emptyset$ does it imply $X(K) \neq \emptyset$? The answer is of course negative in general. One studies especially the case of "Conic fibrations over $\mathbb{P}^1 K$". (It means that X is a surface with a proper morphism $X \to \mathbb{P}^1 K$ whose fibres are reduced conics.) One can define a "reciprocity obstruction" (see to Colliot–Thélène) to the existence of a K point on X if $X(K_P) \neq \emptyset$ for any P.

To do this, we define a subset $\mathcal{E}(X) \subset MX(K_p)$ by cohomological methods and show that necessarily $X(K) \subset \mathcal{E}(X)$. So if $\mathcal{E}(X) = \emptyset$ then $X(K)$ is empty. This is the "reciprocity obstruction".

The main result is:

> If X is a conic fibration over $\mathbb{P}^1 K$, if $X(K_p) \neq \emptyset$ for every P and if $\mathcal{E}(X) \neq \emptyset$ then $X(K) \neq \emptyset$.

Real and Complex Algebraic Cycles

Joost van Hamel, Vrije Universiteit Amsterdam

For the study of the structure of Chow groups of algebraic varieties defined over $\mathbb{R}$ the equivariant cycle map

$$cl : CH^k(X) \to H^{2k}(X(\mathbb{C}); G, \mathbf{Z}(k))$$

as defined by V. A. Krasnov is an important tool. In the talk this has been illustrated by giving necessary and sufficient conditions for the natural map

$$\pi^* : CH^k(X) \to CH^k(X_{\mathbb{C}})^G$$

to be surjective when X is a nonsingular projective geometrically irreducible surface over $\mathbb{R}$ with $H_1(X(\mathbb{C}), \mathbf{Z}) = 0$ and $X(\mathbb{R}) = \emptyset$.

In particular, if $H^2(X, \mathcal{O}_X) = 0$, then π^* is <u>not</u> surjective. When X is a real rational surface, X satisfies the hypothesis of the result, and the present proof is a significant simplification of the original proof by Parimala and Sujatha, without using the classification of real rational surfaces as Colliot–Thélène did in his proof.

After the talk, Colliot–Thélène pointed out that apart from real rational surfaces, the class of real Barlow surfaces satisfies the hypothesis of the result as well.

Integral Geometry of Semialgebraic Sets

Ludwig Bröcker, Universität Münster

First, let $S \subset \mathbb{R}^n$ be a compact differentiable submanifold. Let $\mathcal{N} = \{(N,x) | N \in S^{n-1}, x \in S, N \perp S_x\}$ be the normal unit frame and $\mathcal{N}_r = \{(N,x) | N \in B(0,r), x \in S, N \perp S_x\}$ the r–tube of S. Classical formulas in Integral Geometry are

1) Gauß–Bonnet: $\int\limits_{S^{n-1}} \left(\sum\limits_{(N,x)\in\mathcal{N}} i(N,x) \right) dN = \text{vol }(S^{n-1})\chi(S)$ where $i(N,x)$ is the orientation character of the Gauß–map $\mathcal{N} \to S^{n-1}$.

2) Volume of tubes (H. Weyl 1939): $\text{vol}(\mathcal{N}_r) = \sum\limits_{i=0}^{p} c_i l_i(S) r^{n-p+i}$ for small r. Here $p = \dim(S)$, $l_i(S)$ is the i^{th} Lipschitz–Killing invariant of S and the c_i are universal constants. For instance, one has $l_0 = \text{vol}(S)$, $l_p = \chi(S)$.

These formulas are generalized to the case that $S \subset \mathbb{R}^n$ is compact semialgebraic. This requires a suitable definition of the index $i(N,x)$ and also $\text{vol}(\mathcal{N}_r)$ is replaced by a modified volume $\widetilde{\text{vol}}(\mathcal{N}_r)$. Then the $l_i(S)$ appear in any dimension $i \leq \dim(S)$ and provide suitable constants for a kinematic formula for a pair of semialgebraic sets $S, T \subset \mathbb{R}^n$.

Classification of Dedekind cuts over real closed fields

Marcus Tressl, Universität Regensburg

A cut p over a real closed field R is a pair (p^L, p^R) with $p^L \cup p^R = R$ and $p^L < p^R$. If A is a convex valuation ring of a real closed field R, then the Cherlin-Dickmann theorem says that the structure (R, A) has quantifier elimination (where we assume that R is formulated in the language $\{+, -, \cdot, -1, \leq, 0, 1\}$ of ordered fields). If $p = A^+$ (this is the cut p with $p^R = \{a \in R\ a > A\}$) this theorem translates in "(R, p^L) has quantifier elimination".

We give model completeness and quantifier elimination results for reasonable definable expansions of the structures (R, p^L) for all cuts p, The expansion depends on the following cuts associated to the cut p:

(1) $W_0(p) := \{a \in R | a + p = p\}$ the (convex) invariance group of p.

(2) $A_p := A(W_0(p))$ where $A(G)$ denotes the convex valuation ring $\{b \in R | b \cdot G \subseteq G\}$ if G is a convex subgroup of R.

Thm I If p is not definable (i.e.: not of the form $a^{\pm}$ or $\pm\infty$) and $W_0(p) = \{0\}$ then $Th(R, p^L)$ has quantifier elimination.

Thm II If G is a proper convex subgroup of R and if G is not of the form $a \cdot A(G)$ or $a \cdot m_{A(G)}$ ($m_{A(G)}$ is the max. ideal of $A(G)$) then $Th(R, A(G), G)$ has quantifier elimination.

Thm III If G is as in theorem II and p is a cut of R such that $G = W_0(p)$ and such that p is not of the form $a \pm G^+$ then $Th(R, A(G), G, p^L)$ is model complete and does not have quantifier elimination.

Thm IV If G is a proper convex subgroup of R and G is of the form $a \cdot A(G)$ and if p is a cut of R such that p is not of the form $b \pm W_0(p)^+$ (where $G = W_0(p)$) then $Th(R, A(G), p^L, a)$ is model complete and does not have quantifier elimination.

On strictly positive polynomials

Thorsten Wörmann, Universität Dortmund

A technique is proposed to use the theory of partial archimedean orderings to apply the representation theorem of Kadison–Dubois to prove theorems of Pólya, Habicht and Schmüdgen and some variants.

I) $f \in \mathbb{R}[X_1, \ldots, X_n]$ homogeneous
$f_{|\{X_1 \geq 0, \ldots, X_n \geq 0, \sum X_i \neq 0\}} > 0 \Rightarrow \exists N \in \mathbb{N} : (\sum X_i)^N \cdot f \in < \mathbb{R}_+, X_1, \ldots, X_n >$.

II) $f \in \mathbb{R}[X_1, \ldots, X_n]$ homogeneous
$f_{|\mathbb{R}^n} > 0 \Rightarrow \exists N \in \mathbb{N} : (\sum X_i^2)^N \cdot f \in \sum \mathbb{R}[\underline{X}]^2$.

III) $f, g_1, \ldots g_m \in \mathbb{R}[\underline{X}]$, $S = \{g_1 \geq 0, \ldots, g_m \geq 0\}$ compact
$f_{|S} > 0 \Rightarrow f \in < \sum \mathbb{R}[\underline{X}]^2, g_1, \ldots, g_m >$.

(< ... > means the semiring generated by the mentioned elements.)

The real spectrum of a non-commutative ring

Murray Marshall, University of Saskatoon

This is a report on joint work with K. H. Lenug and Y. Zhang. This work was motivated, in part, by the work by Andradas, Bröcker and Ruiz and, more recently, by myself, on abstract real spectra (also called spaces of signs). A denotes a ring with 1. An ordering of A is a subset $P \subset A$ satisfying $P + P \subseteq P$, $P \cdot P \subseteq P$, $P \cup -P = A$ and $P \cap -P = a$ prime (called the support of P). A real prime is a prime $\mathfrak{p}$ of A which is the support of an ordering. If $\mathfrak{p}$ is a real prime then it is a strong prime, i.e. $ab \in \mathfrak{p} \Rightarrow a \in \mathfrak{p}$ or $b \in \mathfrak{p}$. Because of this one sees immediately that Sper A, the set of all orderings of A, is a spectral sp (exactly as in the commutative case). For $a \in A$, let $\bar{a}$: Sper $A \to \{-1, 0, 1\}$ defined by

$$\bar{a}(P) = \begin{cases} 1 & \text{if } a \notin -P \\ 0 & \text{if } a \in P \cap -P \\ -1 & \text{if } a \notin P \end{cases}$$

and let $GA = \{\bar{a} | a \in A\}$. The main result is that the pair (Sper A, GA) is an abstract real spectrum. In concrete terms this means that various local–global principles hold, connecting (Sper A, GA) with the residue spaces ($\text{Sper}_\mathfrak{p} A, G_\mathfrak{p} A$), $\mathfrak{p}$ a real prime of A, and each residue space is a space of orderings. Here, $\text{Sper}_\mathfrak{p} A = \{P \in \text{Sper } A | P \cap -P = \mathfrak{p}\}$ and $G_\mathfrak{p} A = \{\bar{a}|_{\text{Sper}_\mathfrak{p} A} | a \in A \backslash \mathfrak{p}\}$. In the commutative case, each residue space is identified with the space of orderings of the corresponding residue field. In the non-commutative case it is often not possible to embed the domain $A/_\mathfrak{p}$ into a skew field.

Differential equations from a real algebraic point of view

Thomas Grill, Universität Regensburg

An algebraic differential equation is an equation of the type

$$f(g_1, \ldots, g_k, y_1, y_1', \ldots, y_1^{(l_1)}, \ldots, y_n, \ldots, y_n^{(l_n)}) = 0$$

where $f \in \mathbb{R}[T_1, \ldots, T_k, y_1, \ldots, y_n^{(l_n)}]$ and $g_1, \ldots, g_k \in C^\infty(I)$. The g_i's are the coefficients. In case of a system of algebraic differential equations with constant coefficients every solution determines a differential ring homomorphism $\varphi : A \to C^\infty(I)$ where $A = \mathbb{R}\{a_1, \ldots, a_n\}$ is a differential $\mathbb{R}$–algebra of (differentially) finite type. Conversely every such homomorphism determines a solution of the system. In this setting constant solutions are corresponding to differential morphism $\varphi : A \to \mathbb{R}$. Hence every constant solution determines a real, maximal, differential ideal. The converse is also true (since A is not noetherian this is not trivial).

If $\alpha \in \text{Sper } A$ is an ordering there are associated in a natural way the direct generalizations α^+ and α^-. (For this supp α does not be a differential ideal.) The supports of α^+ and α^- are equal to $(\text{supp } \alpha)^\#$ the biggest differential ideal contained in supp α. If supp α is a maximal ideal, then α^+, α^- can be interpreted in terms of a differential ring hom. $\varphi : A \to C^\infty(I)$. (This leads to a initial value problem for a first order system.) There exists an interval $I \subset \mathbb{R}$, $t_0 \in I$ and a unique $\varphi : A \to \mathcal{A}(I)$ the analytic functions on I such that $a \in A$, $a(\alpha^-) > 0$ iff $\varphi(a)(+) > 0$, $\forall t \in]\tau, t_0[$ for some $\tau > 0$. Similarly for α^+.

In an example it is demonstrated that the equation $y'' + \sin t \cdot y = 0$ computes preimages of orderings.

Algebraically constructible functions and signs of polynomials

Zbigniew Szafraniec, University of Gdansk
(a joint work with Adam Parusinski)

Definition of an algebraically constructible function has been given by McCrory and Parusinski:

Let W be a real algebraic set. A function $\varphi : W \to \mathbf{Z}$ is called algebraically constructible if there is a proper regular morphism $h : X \to W$, where X is an algebraic set, such that for every $w \in W$:

$$\varphi(w) = \chi(h^{-1}(w)).$$

We have proved

Theorem. Let $\varphi : W \to \mathbf{Z}$ be an algebraically constructible function. Then there are polynomials $g_1, \dots, g_s : W \to \mathbb{R}$, such that

$$\varphi(w) = \text{sgn } g_1(w) + \ldots + \text{sgn } g_s(w)$$

This theorem immediately implies

Theorem. (Coste and Kurdyka) Suppose that W is irreducible. Then there are a proper algebraic subset $\sum \subset W$ and a polynomial $g : W \to \mathbb{R}$, such that for every $w \in W - \sum$

$$\varphi(w) \equiv (s-1) + \text{sgn } g(w) \pmod 4.$$

We have also proved

Theorem. For $w \in W$, let $L(w)$ denote the link of W at w. Then the mapping

$$W \ni w \mapsto \frac{1}{2}\chi(L(w))$$

is algebraically constructible.

As a corollary one gets

Theorem (Coste and Kurdyka) Suppose that W is irreducible. Then there are a proper algebraic subset $\sum \subset W$ and an integer s, such that for every $w \in W - \sum$

$$\chi(L(w)) \equiv s \pmod 4.$$

Empty real Enriques surfaces

V. Kharlamov, Université Strasbourg
(joint work with A. Degtyarev)

N. Mitchin proved that the Euler characteristic $\chi(E)$ and signature $\sigma(E)$ of a compact orientable 4–dimensional Einstein manifold E satisfy the inequality $|\sigma(E)| \leq \frac{2}{3}\chi(E)$, the equality holding only if either E is flat or the universal covering is a $K3$–surface. In the latter case E is either a $K3$–surface, or the quotient of a $K3$–surface by a free holomorphic involution (i. e., an Enriques surface), or the quotient of an Enriques surface by a free antiholomorphic involution.

The varieties of each of the extremal types, except the last one, are known to form connected families: two varieties of the same type can be deformed continuously into each other. To our knowledge, the same question for the quotients of Enriques surfaces, i. e., for empty real Enriques surfaces, was not known. We give the answer:

Theorem *All empty real Enriques surfaces are of the same deformation type.*

As an immediate corollary one gets: *All compact orientable Einstein manifolds are of the same deformation type.*

The proof is based on a systematic study of real elliptic pencils and gives explicit models of all empty real Enriques surfaces.

The starting, but typical result is the following lemma:

Lemma *A real Enriques surface E admits a real elliptic pencil iff there is $x \in$ $H_2(E;\mathbf{Z})/\mathrm{Tors}$ with $\mathrm{conj}_* \ x = -x$ and $x^2 = 0$. If E is unnodal and x is a primitive element, either x or $-x$ is the class of a multiple fiber.*

Desingularization procedures for an family of analytic v–fields on S^2

Z. Denkowska, Université Angers/Gracorie

Finite Cyclicity Conjecture: (Roussarie, Francoise)

$X_\lambda = f(x,y,\lambda)\partial/_{\partial x} + g(x,y,\lambda)\partial/_{\partial y}$ f,g anal. on $S^2 \times \Lambda$. Prove that any elementary limit periodic set (L.P.S — see Francoise and Pugh) has finite cyclicity i.e. if Γ L.P.S. (an invariant set of X_{λ_0}, limit of periodic orbits) containing only elementary singular points (1 eigenvalue $\neq 0$, at least) then $\exists V_{\lambda_0}$; $\exists c, \varepsilon$ s.th. $\lambda \in V_{\lambda_0} \Rightarrow$ # limit cycles in $\Gamma_\varepsilon \leq C$.

THIS CONJECTURE, TOGETHER WITH A DESINGULARIZATION PROCEDURE, WILL IMPLY HILBERT 16th PROBLEM (NOT EXPLICIT BOUNDS).

What is known:

The case without parameter (so called Dulac thm)

$$X = f(x,y)\partial/_{\partial x} + g(x,y)\partial/_{\partial y}$$

is done by

1. desingularization (nice and elementary) by Seidenberg, van den Essen, Dumortier (independently),
2. the first return map does not oscillate (very difficult, Ilyashenko, Moussu, Martinet, Ramis, Ecalle).

What is not known:

Finite cyclicity conjecture (with parameter)

/ 1. How to desingularize families to get only elementary polycycles and keep track of limit cycles?

\ 2. Show that the return maps for elementary polycycles do not oscillate.

While 2. is being treated by many outstanding mathematicians (Ecalle, Moussu, Yomdin, Francoise, Risler), 1. was only touched by:

A) Roussarie, Denkowska, Mourtada, Panazzdo

B) Trifonov (remake by Jankowska Denkowska, in the real, without Hironaka full desingularization, to be published).

The work of Cano does not apply (unfortunately) because generic results don't save us, and for other reasons, too. Neither A) nor B) prove that the good desingularization exists for <u>all</u> analytic $\{X_\lambda\}$.

Singular perturbation get in the way. The two methods are totally different.

PLEASE, GIVE IT A TRY!

The method A) (Boll. Soc. Bras. 1991, 1995) consists in blowing up $\{X_\lambda\}$ as a field in $S^2 \times \Lambda$, along smooth centers contained in the set of singular points. Blowing up admits weights (an old idea), for instance $\varphi : S^N \times \mathbb{R}^+ \ni (\overline{x}_1, \ldots, \overline{x}_{N+1}, r) \to (r^{\alpha_1}\overline{x}_1, \ldots, r^{\alpha_{N+1}}\overline{x}_{N+1}) \in \mathbb{R}^{N+1}$. Centers are chosen in a coherent way along a given polycycle and we recuperate "generalized families of v-fields" with no less limit cycles than the original one. This method works for most analytic families (numerous applications — see Roussarie), but stucks at singular perturbations $\begin{cases} \dot{x} &= \varepsilon f(x,y) \\ \dot{y} &= g(x,y) \end{cases}$

The method B) (Functional Analysis 95) consists in blowing up sections (van den Essen parametrized) and then recuperating sections with the means of Hironaka's thm. Keeping track of limit cycles is impossible.

Bernstein inequality, division by an ideal, applications

J. P. Francoise, Université Paris VI

This is a report on 3 articles (joint works with Y. Yomdin, to appear in J. Funct. Analysis, proceedings of the birthday conference Lojasiewicz in Krakow, joint work with M. Briskin and Y. Yomdin, to appear).

1) Bernstein inequality

   Definition 1.1 $f \in B^1_{R,Q,K}$ $\max(|f(z), z \in \Delta_R)/\max(|f(z)|, z \in \Delta_{\alpha R}) \leq K$. Jensen's inequality $\Rightarrow \#f^{-1}(0) \cap \Delta_{\alpha R} \leq \log K/\log(1+\alpha^2)/2\alpha$ (M. Waldschmidt).

   Definition 1.2 $f = \sum f_k z^k \in B^2_{C,N,R} \Leftrightarrow |f_j|^{R^j} \leq C \cdot (\mathrm{Max}_{i=0,\ldots,N}(|f_i|R^i)$. The two classes are essentially equal.

2) Projection of analytic sets

   Definition 2.1 $f : \mathbb{C}^n \times \mathbb{C} \to \mathbb{C}$, $f : (\underline{\lambda}, z) \mapsto f_{\underline{\lambda}}(z) = f(\underline{\lambda}, t)$ defines a A_0-series if $f(\underline{\lambda}, z) = \sum_k : f_k(\underline{\lambda})z^k$, $f_k(\underline{\lambda}) \in \mathbb{C}[\underline{\lambda}]$ $\underline{0}$ $f_k(\underline{\lambda}) \leq \alpha k + \beta \sum |f_k(\underline{\lambda})|R^k = M \leq \infty$.

   Thm 1 if $f(\underline{\lambda}, z)$ is a A_0-series then $\sum |f_k(\underline{\lambda})|z^k \in B^2_{N,R',C}$ where N =min# of elements $(f_0(\underline{\lambda}), \ldots, f_N(\underline{\lambda}))$ which generates the ideal I generated by all the coefficients $f_0(\underline{\lambda}), \ldots, f_N(\underline{\lambda})), \ldots$, called the Bautin ideal.

   Let$(h_1, \ldots, h_d)$ be a Gröbner basis of I, $\rho = \max|h_i - x^{E_i}|$ where $E_i = \exp(h_i)$ then $R' = R/(1+\rho)^{n\alpha}$, C can be explicitly related to the norm of change of bases to standard basis, exponent β.

(Proof: Hironaka division theorem)

Thm 2 if the coefficients $f_k(\underline{\lambda})$ are homogeneous of degree $\alpha k + \beta$ then for all $\underline{\lambda}$, $\sum f_k(\underline{\lambda}) z^k \in B^2_{N,R'/|\underline{\lambda}|^\alpha, C|\lambda|^{n\beta}}$.

3) Proposition:

(1) $\frac{dy}{dx} = p(x)y^2 + q(x)y^3$, $p(x) = \lambda_2 x^2 + \lambda_1 x + \lambda_0, q(x) = \mu_2 x^2 + \mu_1 x + \mu_0$
(1) is called center if for all initial values y_0 $y(1) = y_1 = y_0$.

(3) An isolated solution of $y_0 = y_i$ is called a limit cycle of (1)

Thm 1 (1) is a centre if and only if

$$\frac{\lambda_2}{3} + \frac{\lambda_1}{2} + \lambda_0 = 0 \quad \frac{\mu_2}{3} + \frac{\mu_1}{2} + \mu_0 = 0 \quad \lambda_1\mu_2 - \mu_1\lambda_2 = 0$$

Thm 2 The number of limit cycle of (1) close to $y_0 = 0$ is less than 3

Ramification of orderings and involutions in Galois extensions

M.E. Alonso, University Madrid
(joint work with M.P. Vélez)

We consider a formally real field K with a real valuation ring V and a finite Galois extension L which is not formally real. An involution $\sigma \in G(L/K)$ is said to be real if the fixed field L_σ of σ in L is formally real. In this situation, real involutions have a distinguished property: if B is a valuation ring of L lying over V, every lifting of a real involutions of the residue field extension is an involution in $G(L/K)$. We interpret also the 2–characters of the ramification group of V with respect to L in terms of how real involutions collapse in the residue field of B.

Furthermore, for $\alpha \in \mathrm{Sper}(K)$ making V convex and having some extension to L_σ, we estimate the extensions of α to the different real valuation rings of L_σ lying over V.

Globally sub–Pfaffian sets

Zbigniew Hajto, University Barcelona

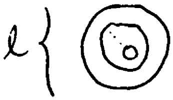
In the talk we survey recent results related with sub–Pfaffian sets generated by algebraic foliations. Considering only foliations with singularities of Kupka type we are able to define the boundary of a Pfaffian leaf Γ using the theory of o-minimal structures. The following elementary lemma (due to Pillay and Shiota) is a key point in the proof that the boundary $\partial\Gamma$ is sub-Pfaffian:

Let $(\mathbb{R}, \{S_n\})$ be an o-minimal structure such that "<" and "+" are definable. If there exists a definable non-semilinear C^1 function f on $[-1,1]$ such that f' is definable, then "·" is locally definable.

The topology of real algebraic symmetric M-curves

Th. Fiedler, Université Toulouse

Let $X_{\mathbb{R}}$ be a real algebraic projective plane curve of degree $2k$ and assume that $X_{\mathbb{R}}$ has the maximal number of components (<u>M-curve</u>). Let $\mathbb{R}P^2_+$ denote the union of the regions of $\mathbb{R}P^2 \backslash X_{\mathbb{R}}$, which are bounded by $X_{\mathbb{R}}$ from the outside. If the curve $X_{\mathbb{R}}$ has a subconfiguration of components like this



, then we say that it has a <u>nest</u> of depth l.

<u>Rokhlin's congruence.</u> <u>The Euler characteristics</u>

$$\chi(\mathbb{R}P^2_+) \equiv k^2 \bmod 8.$$

<u>(revised) Ragsdale's conjecture.</u> $\chi(\mathbb{R}P^2_+) \leq k^2$.

Up to isotopy there is a unique non-trivial involution on $\mathbb{R}P^2$, which can be given by

$$(-1) : \mathbb{R}P^2 \to \mathbb{R}P^2$$

$$[x_0 : x_1 : x_2] \mapsto [-x_0 : x_1 : x_2].$$

<u>Definition.</u> We call $X_{\mathbb{R}}$ <u>symmetric</u> if $(-1)X_{\mathbb{R}} = X_{\mathbb{R}}$.

We prove the following:

<u>Th. 1.</u> <u>If</u> $X_{\mathbb{R}}$ <u>is a symmetric M-curve then</u>

$$\chi(\mathbb{R}P^2_+) \equiv k^2 \bmod 16.$$

<u>Th. 2.</u> <u>If</u> $X_{\mathbb{R}}$ <u>is a symmetric M-curve which has a nest of depth</u> $\geq k-2$ <u>then</u>

$$\chi(\mathbb{R}P^2_+) \leq k^2.$$

Algebra of Taylor coefficients.

Y. Yomdin, The Weizmann Institute, Rehovot

This talk is closely related to the talks of J.–P. Francoise and J.–J. Risler (this conference).

We define a certain subclass of the power series $f_\lambda(x) = \sum_{k=0}^{\infty} a_k(\lambda)x^k$, $a_k \in \mathbb{C}[\lambda]$, which present "finiteness" properties more or less close to those of algebraic functions. The main tool is an algebra of the ideals $I_q = \{a_q, a_{q+1}, \ldots\}$, generalizing the classical Bautin ideal $I = I_0$.

The conditions are rather complicated, but in most of important cases they are (at least, partially) satisfied. Of course, the case of the first return mapping presents the main difficulties.

An archimedean Positivstellensatz

L. Mahé, Université de Rennes

It is a report on a work of my student J.–P. Monnier (Rennes). We prove an extension of Schmüdgen's theorem in the following sense:

Thm 1. Let $\text{Trd}_k A = d$ whith k a totally archimedean field, such that $A = H(A) = \{a \in A \mid \exists n \in \mathbb{N}, n \pm a > 0 \text{ on } \text{Spec}_r A\}$
$= \{\text{bounded elements of } A\}$.
Then for $f \in A$: $f > 0$ on Sper$A \Leftrightarrow f = r^2 + \sum x_i^2$ for $r \in \mathbb{Q}^*$, $x_i \in A$.

In the case of finitely generated k–algebra, it is a reformulation of Schmüdgen's theorem, that can be proved in a completely algebraic manner using the representation theorem of Kadison–Dubois and a trick of Wörmann.

The extension to the finite transendence degree comes from the theorem of structure of "Totally archimedean rings".

Thm 2. Let $Trd_k A = d$, k a totally archimedean field, $A = H(A)$. Then $A = \varinjlim B$ with $B \subsetneq A$ finitely generated and $B = H(B)$.

As for $k = \mathbb{R}$ and $B = \mathbb{R}[V]$ finitely generated, we have $B = H(B) \Leftrightarrow V(\mathbb{R})$ compact, we may interpret Thm 2 as follows:

"A formal variety which is formally compact is a pro–compact object".

The proof of this theorem 2 comes from an improvement of the proof of a theorem of Becker and Powers (Crelle 1996):

Thm [Becker–Powers]: If $\mathrm{Trd}_k A = d$, then $H^{d+1}(A) = H^d(A)$.

Combinatorial Patchworking of M–Hypersurfaces

I. Itenberg, Université Rennes

Let X be a nonsingular hypersurface of degree m in $\mathbb{R}P^n$.

Smith–Thom inequality: $b_*(\mathbb{R}X) \leq b_*(\mathbb{C}X)$, where $\mathbb{R}X$ (resp. $\mathbb{C}X$) is the real (resp. complex) point set of X, and $b_*(.) = \dim M_*(.; \mathbf{Z}/2)$. If $b_*(\mathbb{R}X) = b_*(\mathbb{C}X)$, then X is called an M–hypersurface.

Theorem. For any $m, n > 0$, there exists an M–hypersurface of degree m in $\mathbb{R}P^n$.

In 1980 O. Viro proposed a construction which is supposed to give M–hypersurfaces. However, it is still unknown if Viro's construction really produces M–hypersurfaces.

We prove the existence of M–hypersurfaces using a construction based on the combinatorial patchworking. In fact, this construction is in a sense a "combinatorial version" of Viro's initial construction.

In the combinatorial construction it is rather easy to verify that the result is an M–hypersurface: a basis of $M_*(\mathbb{R}X; \mathbf{Z}/2)$ can be presented explicitly.

Uniformization of real algebraic curves

R. Silhol, Université Montpellier

The uniformization theorem of Riemann states that if C is an algebraic curve of genus $g \geq 2$ then C is conformally isomorphic to $\mathbb{H}/_G$ where $\mathbb{H}$ is the Poincaré upper half plane and G is a Fuchsian group. The classical uniformization problem is to relate equations for C and a Fuchsian group G.

Since $\mathbb{H}$ has a natural hyperbolic metric and this induces one on C one can reformulate the problem by asking to relate equations for C and the hyperbolic metric on C (considered as a Riemann surface). This is the approach we have taken in a joint work with P. Buser. We show how to compute the period matrix of a Riemann surface in terms of conformal capacities of certain geodesic polygons and how to compute numerically these capacities. Using Theta characteristic in the hyperelliptic case and other methods for some genus 3 curves, this yields a method to compute equations for Riemann surfaces in terms of the Fenchel–Nielsen coordinates.

In the other direction we have observed that the existence of coverings by "special" higher genus curves allow to deduce certain relations among the hyperbolic length of the closed geodesics of a part decomposition. This leads to a sort of uniformization "in families" i.e. curves with a certain type of equation (depending algebraically on some parameters) will have a part decomposition with certain relations between the Fenchel–Nielsen coordinates. For certain particular elements in the families it is even possible to compute both the algebraic and differential geometric moduli. This provides many new where the uniformization can be computed explicitly.

Real quotient singularities and nonsingular real algebraic curves in the boundary of the moduli space

J. Huisman, Université de Rennes 1

Let M be a connected real analytic manifold and G a group acting faithfully and properly discontinuously on M. Then the quotient $M/G =: N$ is a semianalytic variety. Let $\pi : M \to N$ be the quotient map. Define the boundary ∂N of the semianalytic variety N by

$$\partial N = \{x \in M | \text{the germ } (M, x) \text{ is not analytic}\}.$$

Then, for $x \in M$, $\pi(x)$ is in ∂N if and only if the stabilizer G_x of x is of even order. We apply this to moduli of real algebraic curves: Let $g \geq 2$ and let X be a nonsingular real algebraic curve of genus g. Let $R(X)$ be the set of (real) isomorphism classes of real algebraic curves Y such that there is an equivariant homeomorphism from $X(\mathbb{C})$ onto $Y(\mathbb{C})$. By Teichmüller theory and the preceding observations, $R(X)$ has a natural structure of a semianalytic variety. This variety $R(X)$ is the moduli space of real algebraic curves of genus g having the same topological type as X. It is known that $R(X)$ is connected and of dimension $3g - 3$ (Seppälä–Silhol, Math. Z. 201 (1989), 151–165). We determined the dimension of the boundary $\partial R(X)$ of $R(X)$.

<u>Thm.</u> Let $g > 2$ and let X be a nonsingular real algebraic curve of genus g. Let c be the number of connected components of $X(\mathbb{R})$. If $X(\mathbb{C})\backslash X(\mathbb{R})$ is connected then

$$\dim \partial R(X) = 2g - 1.$$

If $X(\mathbb{C})\backslash X(\mathbb{R})$ is not connected then

$$\dim \partial R(X) = \max\left\{\frac{1}{2}(3g - 3 + c), 2g - \left[\frac{1}{2}(c+1)\right]\right\}.$$

In particular, the boundary $\partial R(X)$ of $R(X)$ is <u>not</u> empty, i.e. $R(X)$ is <u>not</u> a real analytic variety.

The last assertion refutes a statement of Seppälä–Silhol (loc. cit.) to the effect that $R(X)$ would be analytic.

Vector bundles, algebraic morphisms and realification of complex algebraic varieties

Wojciech Kucharz, University of Albuquerque

Let X be a real algebraic variety (that is a locally ringed space isomorphic to a Zariski locally, closed subset of $\mathbb{P}^n(\mathbb{R})$ for some n). Denote by $VB'_{\mathbb{C}}(X)$ the group of isomorphism classes of topological $\mathbb{C}$–line bundles on X. Let $VB'_{\mathbb{C}\text{-alg}}(X)$ be the subgroup of $VB'_{\mathbb{C}}(X)$ that consists of the isomorphism classes of $\mathbb{C}$–line bundles admitting an algebraic structure.

Problem. Compute $G(X) = VB'_{\mathbb{C}}(X)/VB'_{\mathbb{C}\text{-alg}}(X)$.

The problem is solved for a certain class of real algebraic varieties. Given a complex algebraic variety V, let $V_{\mathbb{R}}$ denote the underlying real algebraic structure of V.

Theorem 1. Given a complex abelian variety A, $\dim A \geq 2$, the following conditions are equivalent:

a) $G(A_{\mathbb{R}})$ is finite.

b) A is isomorphic to the product of pairwise isogenous complex elliptic curves with complex multiplication.

Theorem 2. Let E_1 and E_2 be isogenous complex elliptic curves with complex multiplication. Let $\mathrm{End}(E_j) = \mathbf{Z} + f_j\mathcal{O}(-d)$, where f_j, d are positive integers with d square free and $\mathcal{O}(-d)$ is the ring of integers in $\mathbb{Q}(\sqrt{-d})$. Let δ_j be the discriminant of E_j and let $f'_j = f_j/gcd(f_1, f_2)$. Then $G(E_{\mathbb{R}} \times E_{2\mathbb{R}})$ is isomorphic to

$\mathbf{Z}/f'_1\mathbf{Z} \oplus \mathbf{Z}/f'_2\mathbf{Z} \oplus \mathbf{Z}/\sqrt{\delta_1\delta_2}\mathbf{Z}$ if δ_1 and δ_2 are odd,
$\mathbf{Z}/2\mathbf{Z} \oplus \mathbf{Z}/2f'_1\mathbf{Z} \oplus \mathbf{Z}/2f'_2\mathbf{Z} \oplus \mathbf{Z}/\frac{1}{2}\sqrt{\delta_1\delta_2}\mathbf{Z}$ if δ_1 and δ_2 are even,
$(\mathbf{Z}/2f'_2\mathbf{Z})^2 \oplus (\mathbf{Z}/\sqrt{\delta_1\delta_2}\mathbf{Z})^2$ if δ_1 is even and δ_2 is odd.

Theorems 1 and 2 are just examples of results that can be obtained.

Positive polynomials and the Moment–Problem

Alexander Prestel, Universität Konstanz

Let $p_1, \ldots, p_s \in \mathbb{R}[X_1, \ldots, X_n]$ such that the set $K = \{p_1 \geq 0\} \wedge \ldots \wedge \{p_s \geq 0\}$ is a compact (semi–algebraic) subset of $\mathbb{R}^n$. Denote by $\sum^2(K)$ the semiring generated by squares from $\mathbb{R}[X_1, \ldots, X_n] = A$ and $p_1, \ldots, p_s$ and let $A^+(K) = \{p \in A | p \geq 0$ on $K\}$. By a theorem of Haviland a linear functional $L \in A^*$ is given by some positive Borel measure supported by K (i.e. $L(q) = \int_K q d\mu$) if and only if $L(p) \geq 0$ for all $p \in A^+(K)$.

In 1991 Schmüdgen showed that $L(\sum^2(K)) \geq 0$ already implies that L is given by some positive Borel measure on K. As a corollary be obtained that for every $p \in A$:

$$p > 0 \text{ on } K \Rightarrow p \in \Sigma^2(K).$$

Recently Wörmann has given an algebraic proof for this last implication. Obviously this implication gives also Schmüdgen's solution of the K–Moment Problem. In fact, $p \geq 0$ on K gives $p + \varepsilon > 0$ and hence $p + \varepsilon \in \sum^2(K)$. Thus every $L \in A^*$ with $L(\sum^2(K)) \geq 0$ satisfies (by continuity) $L(p) \geq 0$ for all $p \in A^+(K)$.

Real elliptic surfaces and the Ragsdale–Viro conjecture

Frédéric Mangolte, Université de Savoie (Chambéry)

Families of real elliptic surfaces are the last families of non general type surfaces for which the classification of topological type of the real part remains unknown.

V. Kharlamov informed me on an unpublished result:

Thm. Let X be a real elliptic surface with complex part $X(\mathbb{C})$ simply connected then $h^1(X(\mathbb{R})) \leq h^{1,1}(X)$.

The so-called Ragsdale–Viro conjecture was the same statement but without the restriction "elliptic". I. Itenberg constructed counter-examples for this conjecture wich are surface of general type.

The question I posed in my talk was: "Is there exist real elliptic surface such that $h_1(X(\mathbb{R})) = h^{1,1}(X)$ in each complex families (which are classified over $\mathbb{C}$ by $\chi(\mathcal{O}_x)$)". I give examples for $\chi(\mathcal{O}_x) \leq 5$ and $h^1 = h^{1,1} = 50$, and a theorem which gives restrictions on the type of singular fibers for all surfaces such that $h_1(X(\mathbb{R})) = h^{1,1}(X)$ and the hypothesis that $H_1(X(\mathbb{R}), \mathbf{Z})$ is generated by classes of real *algebraic* curves.

I proved in this way that the method used to produce examples for $\chi(\mathcal{O}_x) \leq 5$ can't be applied for $\chi > 5$.

Orderings of higher level in semialgebraic geometry

Ralph Berr, Universität Dortmund

It was the aim of the talk to explain the role of orderings of higher level in semi-algebraic geometry. The spectrum of higher level Sperh A of a commutative ring was introduced and its main properties and its geometric relevance was presented. For example it is a spectral space and the real spectrum Sper A is a dense subspace of Sperh A. On a constructible subset $X \subset$ Sperh A a sheaf $\mathcal{C}_X$ of s. a. functions has been defined. The restriction of $\mathcal{C}_X$ to $X_0 =$ Sper $A \cap X$ is then a subsheaf of the sheaf $\mathcal{C}_{X_0}$ of s. a. functions on X_0. This fact can be used to introduce the notion of the level of a s. a. morphism which allows to distinguish certain "types" of s. a. morphisms.

Characterization of analytic surface germs with $\overline{s} = 2$

Carlos Andradas, University Madrid

Let X_0 be an analytic set germ, say irreducible, $\dim X_0 = d$, and let $\mathcal{O}(X_0)$ be the ring of analytic function germs on X_0. A basic open semianalytic set germ is one of the form

$$\{f_1 > 0, f_2 > 0, \ldots, f_r > 0\}, \ f_i \in \mathcal{O}(X_0)$$

and a basic closed semianalytic set germ is one of the form

$$\{f_1 \geq 0, f_2 \geq 0, \ldots, f_r \geq 0\}, \ f_i \in \mathcal{O}(X_0).$$

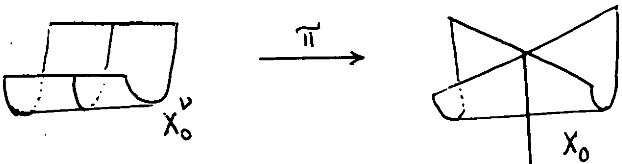
We denote by $s(X_0)$ the minimum integer k such that any open basic semianalytic germ can be described with k inequalities. Similarly $\overline{s}(X_0)$ is the corresponding integer for closed basic semianalytic germs. It was known that

$$s(X_0) = d$$

and

$$\frac{1}{2}d(d+1) - 1 \leq \overline{s}(X_0) \leq \frac{1}{2}d(d+1).$$

So, for surfaces germs we have $2 \leq \overline{s} \leq 3$. We prove that $\overline{s} = 2$ iff the morphism $\pi : X_0^\nu \to X_0$ from the normalization X_0^ν to X_0 maps irreducible curve germs onto irreducible curve germs. Thus,



$\overline{s}(X_0) = 3$ where X_0 is the Whitney's umbrella.

From that we get that $\overline{s}(\mathbb{R}_0^d) = \frac{1}{2}d(d+1)$ for $d \geq 3$.

On gradients of functions definable in an o–minimal structure

Krzystof Kurdyka, Université Grenoble)

Let $\mathcal{M}$ be an o–minimal structure on $(\mathbb{R}, +, \cdot)$ (see van den Dries, C. Miller, Duke Math. J. 84 (1996) for examples, definitions). Let $A \subset \mathbb{R}^n$, we say that $f : A \to \mathbb{R}^k$ is an $\mathcal{M}$–functions if its graph is definable in $\mathcal{M}$.

Th 1. Let $f : A \to \mathbb{R}$ be a $C^1\mathcal{M}$–function, where A is an open, bounded subset of $\mathbb{R}^n$. Suppose that $f(x) > 0$ for all $x \in A$. Then there exists $c > 0$, $\rho > 0$ and an $\mathcal{M}$–function $\psi : (0, \rho) \to \mathbb{R}$ which is C^1 strictly increasing positive, such that

$$\| \operatorname{grad}(\psi \circ f)(x) \| \geq c > 0$$

for each $x \in A$, $f(x) \in (0, \rho)$.

This generalizes a well known Lojasiewicz inequality $\|\operatorname{grad} f \| \geq c|f|^{\alpha}$, $0 \leq \underline{\alpha < 1}$, for an analytic function f, in a neighbourhood of $0 \in \mathbb{R}^n$, $f(0) = 0$.

Th 1 implies, as in the real analytic case, the following:

Th 2. Let $f : A \to \mathbb{R}$ be a C^1 function, where A is an open bounded subset of $\mathbb{R}^n$. Suppose that f is definable in some o–minimal structure $\mathcal{M}$. Then there exists $D > 0$ such that all trajectories of –grad f are of length bounded by D.

More precisely there exists $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ a continuous, strictly increasing $\mathcal{M}$–function, $\lim\limits_{t \to 0^+} \sigma(t) = 0$, such that

$$|\gamma(a,b)| \leq \sigma(|f(a) - f(b)|)$$

where $|\gamma(a,b)|$ is a length of γ between $a, b \in \gamma$.

Observe that f may by infinitely flat, discontinuous on ∂A, unbounded etc. For example (by result of A. Wilkie on $\mathbb{R}_{\exp}$) f may be of the form $f(x,y) = x^2 \exp\left(-\frac{y^3}{x^4+y^2}\right)$ on $(-1,1) \times (0,1)$.

Equivalence of analytic and rational functions on real varieties

Michael A. Buchner, University of New Mexico

Let M be a compact nonsingular algebraic subset of $\mathbb{R}^n$. A regular function on M is a rational function p/q where $q \neq 0$ on M. We give a criterion for an analytic function on M to be analytically equivalent to a regular function.

Let φ be a C^∞ function germ $(M, x) \to \mathbb{R}$. Let $C_x^\infty(M)$ denote the set of all such function germs. Suppose x is a critical point of φ. We say it is a critical point for f algebraically isolated in M if there exists a regular function $r : M \to \mathbb{R}$, such that $r^{-1}(0) = \{x\}$ and the germ $r_x \in \left\{\frac{\partial\varphi}{\partial x_1}, \ldots, \frac{\partial\varphi}{\partial x_n}\right\}$, where $(x_1, \ldots, x_n)$ is a C^∞ coordinate system around x and $\left\{\frac{\partial\varphi}{\partial x_1}, \ldots, \frac{\partial\varphi}{\partial x_n}\right\}$ is the ideal in $C_x^\infty(M)$ generated by $\frac{\partial\varphi}{\partial x_1}, \ldots, \frac{\partial\varphi}{\partial x_n}$.

Thm. Let $f : M \to \mathbb{R}$ be an analytic function with isolated critical points. Assume that, for each critical point a, there exists a local orientation preserving analytic diffeomorphism $\sigma_a : (M, a) \to (M, a)$, such that $f_a \circ \sigma_a$ is the germ at a of a regular function defined in a neighborhood of a, and a is a critical point of $f_a \circ \sigma_a$ algebraically isolated in M. Then there is an analytic diffeomorphism $\tau : M \to M$ such that $f \circ \tau$ is regular.

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