MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/1997

Schubert Varieties; Geometry, Algebra and Combinatorics

31.03. bis 05.04.97

The conference was held under the leadership of Professors W. Fulton (Chicago), A. Lascoux (Paris) and P. Pragacz (Toruń). 41 mathematicians from all around the world attended the meeting. The organizers' goal was to bring together mathematicians working with Schubert varieties though representing the different fields of geometry, algebra and combinatorics. The aim of the meeting was to create an opportunity for an exchange of ideas among such mathematicians representing different areas and points of view. One should stress that up to the conference, the contacts between different groups of mathematicians working on Schubert varieties were rather loose; it was only after the conference when the interaction between these groups started to be stronger. In the organizers' opinion, the conference fulfilled this main goal; it has begun a serions exchange of information between different people working on Schubert varieties. Visible fruits of some of the discussions during the meeting are some recent papers whose ideas were born during the conference.

The seventeen talks covered almost all subjects of those chapters of geometry, algebra and combinatorics which are related to Schubert varieties. One of the central subjects of the meeting was the Schubert calculus, different aspects of which were discussed in the talks by S. Fomin, M. Brion, M. Haiman, S. Billey, F. Sottile and W. Graham. New results about singularities of Schubert varieties and the related Kazdhan-Lusztig polynomials were presented in the talks by S. Kumar and F. Brenti. The rich algebro-combinatorial theory of Bott-Samelson schemes which are desingularizations of Schubert varieties was the subject of the talks by M. Shimozono, P. Magyar and A. Zelevinsky. The standard-monomial-theoretic approach to Schubert varieties was represented by the talks of K. N. Raghavan, P. Magyar and M. Shimozono. Schubert varieties for classical groups were discussed in the talks by S. Billey, M. Brion and W. Graham The related Schubert polynomials (simple and double) and Kostant polynomials arose in the talks by M. Haiman, S. Billey and W. Graham. New results about symmetric functions related to the Schubert calculus were presented in the talk by B. Leclerc. The topology of real Schubert varieties was discussed in the talk by A. Vainshtein. Total positivity in

Schubert varieties was the subject of the talk by A. Zelevinsky. Plactic algebra and noncommutative methods were represented by the talk by P. Littelmann. The appearance of modern cohomology theories (quantum cohomology and arithmetic intersection theory) in the theory of Schubert varieties was shown in the talks by I. Ciocan-Fontanine and H. Tamvakis.

During the meeting a demonstration of two computer systems: ACE and SYMMETRICA took place. These computer systems are particularly suited to helping in research about Schubert varieties.

According to the common opinion of the participants of the meeting, the conference was very fruitful and the choice of the talks reflected in a competent way the proportions of recent trends in the research in the theory of Schubert varieties. The conference confirmed a recent, quick progress of this theory.

It would be very desirable to repeat, in a few years, a similar meeting about Schubert varieties.

Vortragsauszüge

(following "Vortragsbuch-Oberwolfach")

Kostant Polynomials and the Cohomology ring for G/B

Sara Billey

The Schubert Calculus for G/B (G =Kac-Moody group) can be completely determined by a certain matrix related to the Kostant polynomials. These polynomials are defined by vanishing properties on the orbit of a regular point under the action of the Weyl group. For each element w in Weyl group the polynomials also have nonzero values on the orbit points corresponding to elements which are longer then w in the Bruhat order. Our main theorem is an explicit formula for these values. We then generalize this formula to give explicit formulas for the functions ξ^u defined by Kostant and Kumar in [The Nil Hecke ring and cohomology of G/P for a Kac-Moody group G^*, Adv. Math. 62 (1986), 187-237].

Combinatorial formulas for Kazdhan-Lusztig polynomials

Francesco Brenti

In this talk a new nonrecursive formula for the computation of the Kazdhan-Lusztig polynomials of a Coxeter group is presented and a comparision of this one with other previously known ones is discussed. Let (W, S) be a Coxeter system, T the set of reflections of W, $\varphi : T \to \{0, 1, 2, \ldots\}$ a total reflection ordering (as defined by M. Dyer), and $B(W)$ the Bruhat graph of W (i.e., the directed graph having W as the vertex set, and directed edges $\mu \to \nu$ iff $l(\mu) < l(\nu)$ and $\mu\nu^{-1} \in T$). If $\mu \to \nu$ in $B(W)$, then set $\Lambda(\mu, \nu) = \varphi(\mu\nu^{-1})$, and for a directed path $\Delta : a_0 \to a_1 \to \ldots \to a_r$, set $D(\Delta) = \{i \in [r-1], \Lambda(a_{i-1}, a_i) > \Lambda(a_i, a_{i+1})\}$. Given a lattice path $\Gamma : [0, \mu] \to \mathbb{Z}$ (i.e., $\Gamma(0) = (0)$, and $|\Gamma(i+1) - \Gamma(i)| = 1$ for $i = 1, \ldots, \mu - 1$) let $l(\Gamma) = \mu$, $d_+(\Gamma) = \frac{\Gamma(\mu)+\mu}{2}$, $N(\Gamma) = \{i \in [\mu - 1] : \Gamma(i) < 0\}$ and $\Gamma_{\geq 0} = \mu - 1 - |N(\Gamma)|$. Then the Kazdhan-Lusztig polynomial of $\mu, \nu \in W$, $\mu < \nu$, is given by

$$P_{\mu,\nu}(q) = \sum_{(\Gamma,\Delta)} (-1)^{\Gamma_{\geq 0} + d_+(\Gamma)} q^{\frac{l(\nu) - l(\mu) + \Gamma(l(\Gamma))}{2}}$$

where the sum is over all pairs (Γ, Δ) when Γ is a lattice path, Δ is a directed path in $B(W)$ from μ to ν, $l(\Gamma) = l(\Delta)$, $N(\Gamma) = D(\Delta)$, and $\Gamma(l(\Gamma)) < 0$.

Schubert varieties in group completions

Michel Brion

Let G be a connected complex algebraic group which is reductive, and let $B \subset G$ be a Borel subgroup. We consider generalizations of Schubert calculus in varieties X where G acts, such that B has only finitely many orbits in X (such varieties are called *spherical*). In particular, we consider the following questions.

1) (X smooth and projective) Describe the B-orbit closures in X, and the cohomology $H^*(X)$ in terms of their classes.
2) ($X = G/H$ homogeneous; then H has only finitely many orbits in the flag variety G/B). Describe the classes of H-orbit closures in G/B, in the cohomology $H^*(G/B)$.

To any B-invariant subvariety Y of a spherical variety X, we associate

- a subset $W(Y)$ of the Weyl group W, and
- a function $d(Y, -) : W(Y) \to \mathbb{N}$ (in fact the values of $d(Y, -)$ are powers of 2)

such that question 2 can be answered as follows:

Theorem 1. *Let G/H be a spherical homogeneous space, let $V \subset G/B$ be a H-invariant subvariety corresponding to a B-invariant subvariety $Y \subset G/H$. Then*

$$[V] = \sum_{w \in W(Y)} d(Y, w)[\Omega_{w_0 w}] \quad \textit{in} \quad H^*(G/B) \quad \textit{where} \quad \Omega_{w_0 w} = \overline{B w_0 w B}/B\,.$$

A similar statement describes the intersection of the closure of Y (in any equivariant completion X of G/B which is regular in the sense of De Concini and Procesi) with a closed G-orbit in X. In the case where X is a $(G \times G)$-equivariant completion of the group G itself, we obtain

Theorem 2. *Any "Schubert variety" $\overline{BwB} \subset \overline{G} = X$ is singular in codimension 2 exactly unless $w = w_0$ or G contains a direct factor of type A_1.*

Finally, we describe the classes of the $\overline{BwB}$ in the $(T \times T)$ - equivariant cohomology of $\overline{G}$, similarly to results of Kostant-Kumar and Arabia for G/B.

Quantum Cohomology of Flag Varieties

Ionut Ciocan-Fontanine

The small quantum cohomology ring of a (partial) flag variety F is a deformation of the usual cohomology ring, with parameter space $H_2(F, \mathbb{Z})$, obtained by using the 3-point, genus 0 Gromov-Witten invariants as structure constants for the basis of Schubert varieties. Presentations for this rings have been computed (for every G/P), and for the type A case there are analogues for the classical formulas of Schubert Calculus. The talk surveyed these results, due (in increasing generality) to Bertram, Fomin-Gelfand-Postnikov, and the speaker.

Quadratic algebras, Dunkl elements, and Schubert calculus

Sergey Fomin (joint with A.N. Kirillov)

Let $\mathcal{E}_n$ be the associative algebra with generators $[ij]$, $1 \le i < j \le n$, subject to relations

(i) $[ij]^2 = 0$ for $i < j$;

(ii) $[ij][jk] = [jk][ik] + [ik][ij]$

$[jk][ij] = [ik][jk] + [ij][ik]$ for $i < j < k$;

(iii) $[ij][kl] = [kl][ij]$ for distinct i, j, k, l.

(These are exactly the quadratic relations satisfied by the divided difference operators of type A.) In this algebra, the "Dunkl elements"

$$\theta_j = -\sum_{i<j}[ij] + \sum_{j<k}[jk]$$

generate a commutative subalgebra. We show that this subalgebra is canonically isomorphic to the cohomology ring of the flag manifold. Our main conjecture states that the evaluations of Schubert polynomials at the Dunkl elements θ_j belong to the cone spanned (over $\mathbb{Z}_+$) by noncommutative monomials in the $[ij]$. A proof of this conjecture would lead to a combinatorial proof of the nonnegativity of structure constants of the cohomology ring.

A generalization of the main construction to quantum cohomology is also suggested.

The paper (same title) is available from http://www-math.mit.edu/~fomin

The class of the diagonal in flag bundles

William Graham

This talk is on the class of the diagonal in $BB \times_{BG} BB$, where G is a complex reductive group, B a Borel subgroup, and BB, BG are the classifying spaces of these groups. The motivation for studying this class comes from degeneracy loci defined by flags of vector bundles in classical groups; the space $BB \times_{BG} BB$ is the universal space for studying such loci. If $H \subset B$ is a maximal torus and $\mathfrak{h}$ its Lie algebra, then $H^*(BB \times_{BG} BB) = R \otimes_S R$ where $R = S^\bullet(\mathfrak{h}^*)$, $S = S^\bullet(\mathfrak{h}^*)^W$. Let $[\Delta]$ denote the cohomology class defined by the diagonal. We discuss 2 problems:

1. Characterize $[\Delta] \in R \otimes_S R$
2. Find a lift of $[\Delta]$ to $R \otimes_{\mathbb{C}} R$

(note: H^* means cohomology with complex coefficients, so $R \otimes_{\mathbb{C}} R \twoheadrightarrow R \otimes_S R$).

Nilpotent Schubert Varieties

Mark Haiman (joint with Will Brockman).

Let $\mathcal{N}$ denote the "nullcone" of nilpotent $n \times n$ matrices. We study the subvariety $\mathcal{N}_w \subseteq \mathcal{N}$ consisting of those matrices which are compatible with some flag belonging to the Schubert variety $\mathfrak{X}_w$ (defined with respect to a fixed standard flag). These *nilpotent Schubert varieties*, as we propose to call them, were recently considered by J. Carrell, who showed using analytic methods that the scheme-theoretic intersection of $\mathcal{N}_w$ with the diagonal matrices h has coordinate ring isomorphic to the cohomology ring of $\mathfrak{X}_w$ (for GL_n). We give a purely algebraic proof of Carrell's theorem. The key to this is a lifting of the double Schubert polynomials $\mathfrak{S}_v(X, A)$ to "matrix" double Schubert polynomials $\underline{\mathfrak{S}}_v(\underline{X}, A)$ in which $\underline{X}$ is a matrix and $A = (a_1, \ldots, a_n)$ is a sequence. The polynomial $\underline{\mathfrak{S}}_v(\underline{X}, A)$ vanishes when there is a flag $F \in \mathfrak{X}_w$, for $w \not\geq v$, such that $\underline{X}$ is compatible with F, and $a_1, \ldots, a_n$ are the eigenvalues of $\underline{X}$, in the order imposed upon them by the flag F.

Singular locus of Schubert varieties

Shrawan Kumar

Let G be a semisimple complex algebraic group, $B \subset G$ a Borel subgroup and W the associated Weyl group. For any $w \in W$, let X_w be the Schubert variety $\overline{BwB/B} \subset G/B$. We determine the singular locus of X_w in terms of the Nil Hecke ring introduced by Kostant and Kumar. There is a similar criterion for the rational smoothness.

Schur functions and affine Lie algebras

Bernard Leclerc (joint with Séverine Leidwanger)

The Schur functions s_λ are symmetric functions which can be regarded as generating functions for irreducible characters of the symmetric groups:

$$s_\lambda = \sum_{\mu \vdash n} \chi_\lambda(\mu) p_\mu / z_\mu$$

Schur defined another family of symmetric functions P_λ which play the same role for irreducible spin characters of the spin symmetric groups.

It was discovered by Sato, Date, Jimbo, Kashiwara, Miwa in 1981 that Schur polynomials naturally arise in the representation theory of the infinite rank affine Lie algebras $\widehat{gl_\infty}$ and $\widehat{so_\infty}$. We exploit this interpretation to obtain algebraic identities relating S- and P-functions.

A plactic algebra for semisimple Lie algebras

P. Littelmann

Let $\mathfrak{g}$ be a complex semisimple Lie algebra. The plactic algebra can be viewed as a "noncommutative" model for the representation ring of $\mathfrak{g}$. Such a model was first constructed by Lascoux and Schützenberger for the Lie algebra sl_n. It is the word algebra on an alphabet (ordered) on n letters, modulo the two sided ideal generated by the Knuth relations. This algebra has a basis which is "naturally" given by the Young tableaux (semi-standard). As a byproduct of the construction they obtained the first correct proof of the Littlewood-Richardson rule to decompose tensor products of $sl_n(\mathbb{C})$-representations. In the talk we presented a generalization of the construction to arbitrary semisimple Lie algebras. The word algebra is replaced by the algebra of formal linear combinations of piecewise linear parts in $X_{\mathbb{R}}$ ending in an integral weight (X = weight lattice), and the Knuth relations are replaced by a sort of homotopy of paths. The resulting algebra is noncommutative; it is naturally equipped with a basis, every basis element has a "shape" and a weight (the endpoint of the path), such that the character of the irreducible representation V_λ is the sum $\sum e^{\eta(1)}$, where the sum runs over all basis elements of shape λ. The sums $(\sum_{\eta \text{ shape } \lambda} \eta)$ form a commutative subalgebra which is isomorphic to the representation ring of $\mathfrak{g}$. One of the consequences is a generalization of the Littlewood-Richardson rule in the setting of semisimple Lie algebras. It also provides a combinatorial model for standard monomial theory. An interpretation in terms of quantum groups is that the algebra above is a combinatorial construction of the algebra of crystal bases.

Standard Monomial Theory for Bott-Samelson Varieties

Peter Magyar

Let G be a reductive algebraic group, $W = \langle s_1, \ldots, s_r \rangle$ its Weyl group (with simple reflection generators), and $P_i \supset B$ the minimal standard parabolic subgroup generated by B and s_i. Let $\underline{i} = (i_1, \ldots, i_N)$ be a word with $i_1, \ldots, i_N \in \{1, \ldots, r\}$. Then the Bott-Samelson variety of $\underline{i}$ can be defined as a quotient

$$Z_{\underline{i}} = P_{i_1} \times P_{i_2} \times \ldots \times P_{i_N} / B^N$$

where $(p_1, \ldots, p_N) \cdot (b_1, \ldots, b_N) = (p_1 b_1, b_1^{-1} p_2 b_2, \ldots, b_{N-1}^{-1} p_N b_N)$.

We may also realize $Z_{\underline{i}}$ as a fiber product inside a product of flag varieties $(G/B)^{N+1}$:

$$Z_{\underline{i}} \cong eB \times_{G/P_{i_1}} G/B \times_{G/P_{i_2}} \cdots \times_{G/P_{i_N}} G/B \subset (G/B)^{N+1}$$

Let $\widehat{P}_i$ be the maximal parabolic associated to s_i, and $\mathcal{O}(1) \to G/\widehat{P}_i$ the minimal-degree ample line bundle. Consider the pullback $\pi_j^* \mathcal{O}(1)$ under $\pi_j : G/B \to G/\widehat{P}_i$

and for $\underline{m} = (m_1, \dots, m_N)$ with $m_j \geq 0$, define $\mathcal{L}_{\underline{m}} = \mathcal{O} \otimes \pi_{i_1}^* \mathcal{O}(m_1) \otimes \dots \otimes \pi_{i_N}^* \mathcal{O}(m_N)$, a line bundle over $(G/B)^{N+1}$, and denote by the same symbol $\mathcal{L}_{\underline{m}}$ this line bundle restricted to $Z_{\underline{i}}$. We have a surjection

$$H^0(G/\widehat{P}_{i_1}, \mathcal{O}(1))^{\otimes m_1} \otimes \dots \otimes H^0(G/\widehat{P}_{i_N}, \mathcal{O}(1))^{\otimes m_N} \twoheadrightarrow H^0(Z_{\underline{i}}, \mathcal{L}_{\underline{m}})$$

Problem: Find an explicit set of elements in the above tensor product which restrict to a basis of $H^0(Z_{\underline{i}}, \mathcal{L}_{\underline{m}})$.

Solution: Lakshmibai (1995) has given an explicit basis $\{p_\pi\}$ of $H^0(G/\widehat{P}_i, \mathcal{O}(1))$ indexed by the LS paths $\pi \in \mathbb{B}(\overline{\omega}_i)$ defined by Littelmann. [See Littelmann's talk at this conference.] (Here $\overline{\omega}_i$ is a fundamental weight.) Consider the set of paths $\mathbb{B}(\underline{i}, \underline{m})$ defined by

$$\mathbb{B}(\underline{i}, \underline{m}) = \left\{ \begin{matrix} f_{i_1}^{a_1}\left(\pi_{i_1}^{*m_1} * f_{i_2}^{a_2}\left(\pi_{i_2}^{*m_2} * f_{i_3}^{a_3}(\dots)\right)\right) \\ a_1, a_2, a_3, \dots \geq 0 \end{matrix} \right\}$$

where $\pi_i =$ the straight line path from 0 to $\overline{\omega}_i$, $\pi * \pi'$ denotes Littelmann's concatenation of paths, and f_i denotes Littelmann's lowering root operator.

Denote the piece of a path $\sigma \in \mathbb{B}(\underline{i}, \underline{m})$ by $\sigma = \sigma_{11} * \dots * \sigma_{1m_1} * \dots * \sigma_{Nm_1} * \dots * \sigma_{Nm_N}$. Then we have the "refined Demazure character formula"

Theorem. *The set*

$$\{p_{\sigma_{11}} \otimes \dots \otimes p_{\sigma_{1m_1}} \otimes \dots \otimes p_{\sigma_{N1}} \otimes \dots \otimes p_{\sigma_{Nm_N}}\}_{\sigma \in \mathbb{B}(\underline{i}, \underline{m})}$$

restricts to a basis of $H^0(Z_{\underline{i}}, \mathcal{L}_{\underline{m}})$.

Monomial Bases for Representations of classical complex semisimple Lie algebras

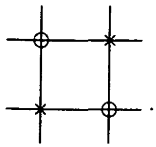
K. N. Raghavan (joint with P. Sankaran)

A procedure is given that associates monomial basis elements of an irreducible representation to standard tableaux (Lakshmibai-Seshadri paths) of the corresponding shape. As a main application one obtains an easy proof of standard monomial theory for classical groups.

Tableaux Combinatories of Standard Monomial Bases of line Bundles over Bott-Samelson Varieties in type A

Mark Shimozono (joint with Vic Reiner)

The flagged Schur module $S_D^B(E)$ for the diagram D can be realized as $H^0(Z_{\underline{i}}, \mathcal{J}_{\underline{m}})$, the space of global sections of a line bundle $\mathcal{J}_{\underline{m}}$ over the Bott-Samelson variety $Z_{\underline{i}}$ given by the reduced word $\underline{i}$, when D does not contain the subdiagram



We give an explicit combinatorial decomposition of the character of $H^0(Z_{\underline{i}}, \mathcal{J}_{\underline{m}})$ into Demazure characters.

Pieri-type formulas for the Classical Groups

Frank Sottile (joint with N. Bergeron)

A Pieri-type formula in the Chow ring of a flag variety is a formula expressing the product of a Schubert class by a special Schubert class in terms of the Schubert basis. Recently, Pragacz and Ratajski have given such formulas in the Chow rings of all Grassmannians (isotropic subspaces of dim k ($\leq n$) in a $2n$ or $2n+1$ dimensional vector space equipped with a nondegenerate symplectic or othogonal form). We seek such formulas for the flag varieties, expressed in terms of chains in the Bruhat order.

This talk will discuss joint work with Nantel Bergeron towards extending and unifying known Pieri-type formulas. We first describe a geometric motivation behind this "chain-theoretic" expectation, and then express the Pieri-type formula for the classical SL_n/B flag variety in this form. We then present a new Pieri-type formula for Sp_{2m}/B, where the special Schubert class is the pullback of a class from the Lagrangian Grassmannian. Time permitting, we discuss how the results used in its proof shed light on the more general Littlewood-Richardson problem for Chow rings.

Arithmetic Intersection Theory on flag varieties

Harry Tamvakis

Let F be the complete flag variety over $\operatorname{Spec}\mathbb{Z}$ with the tautological filtration $0 \subset E_1 \subset \ldots \subset E_n = E$ of the trivial bundle E over F. The trivial hermitian metric on $E(\mathbb{C})$ induces metrics on the quotient line bundles $L_i(\mathbb{C})$. Let $\widehat{c}_1(\overline{L}_i)$ be the first Chern class of $\overline{L}_i$ in the arithmetic Chow ring $\widehat{CH}(F)$ and $\widehat{x}_i = -\widehat{c}_i(\overline{L}_i)$. Let $h \in \mathbb{Z}[x_1, \ldots, x_n]$ be a polynomial in the ideal $I_n =< e_1, \ldots, e_n >$. We give an effective aigorithm for computing $h(\widehat{x}_1, \ldots, \widehat{x}_n)$ in $\widehat{CH}(F)$; as the class of an invariant form on $F(\mathbb{C})$. All arithmetic Chern numbers are rational. An "arithmetic Schubert calculus" is established using the theory of Sl_n-Schubert polynomials.

On the number of connected components in the intersection of two open opposite Schubert cells in the real flag manifold

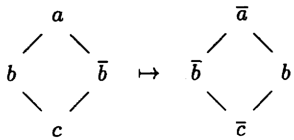
Alek Vainshtein (joint with B. Shapiro and M. Shapiro)

Problem: Given two opposite flags g, f in $\mathbb{R}^n$, find the number $\#_n$ of connected components in the set $U^n_{f,g}$ of all flags transversal to both f and g (actually, U^n does not depend on the choice of f and g).
Conjecture: $\#_n = 3 \cdot 2^{n-1}$ for $n > 5$.

Theorem 1. *(First combinatorial reduction) $\#_n$ equals the number of connected components of a graph $\tilde{G}^n$ whose vertices are pseudoline arrangements with signs assigned to all intersections and edges are defined by transition rules of Berenstein-Fomin-Zelevinsky.*

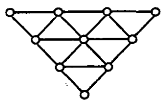
This graph is tremendously large and difficult to work with.

Theorem 2. *(Second combinatorial reduction) $\#_n$ equals the number of connected components of a graph Γ^n whose vertices are standard pseudoline arrangements with signs at all intersections and edges are defined by elementary transformations:*



Theorem 3. *(Algebraic reduction) $\#_n$ equals the number of orbits of the action on the space V^m of $m \times m$ upper triangular matrices over $\mathbb{F}_2$ generated by transformations $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} a+a+d & b+a+d \\ c+a+d & d+a+d \end{pmatrix}$.*

Let $\mathcal{F}$ be the set of fixed points of this action, then $V^m/\mathcal{F} \cong V^{m-1}$ and we get the induced action on V^{m-1}. It can be described as follows: consider arrays of 0's and 1's on an equilateral triangle on the hexagonal lattice:



The generators are just adding the entry to each of its neighbours. Define a quadratic form $q(x)$ as follows: $q(x) = \#\{x = 1\} + \#\{\text{edges between } x = 1\}$

Theorem 4. *The conjugates to the action on V^{m-1} are just all the transformations preserving $q(x)$.*

This almost implies the conjecture.

Total positivity in Schubert varieties

Andrei Zelevinsky (joint with Arkady Bernstein and Sergey Fomin)

An $n \times n$ matrix is *totally nonnegative* if all its minors are ≥ 0. These matrices play important part in several areas of mathematics, from differential equations to combinatorics. Recently G. Lusztig extended the notion of total positivity to Schubert varieties in arbitrary semisimple groups. He also introduced a family of natural parametrizations of the totally positive varieties, which are "algebraic" counterparts of his combinatorial parametrizations of the canonical bases for quantum groups. Our main result is an explicit formula for the inverses of Lusztig's parametrizations.

Applications include:

- a family of minimal criteria for testing total positivity, generalizing classical criteria by Fekete and Cryer,
- P. Magyar's geometric study of Bott-Samelson desingularizations,
- a beautiful description of connected components of the intersection of two opposite open Schubert cells, due to M. and B. Shapiro and A. Vainshtein.

Tagungsteilnehmer

Prof.Dr. Michael Schneider
Fakultät für Mathematik und Physik
Universität Bayreuth

95440 Bayreuth
(†29. Aug. 1997)

Dr. Sara C. Billey
Mathematics Department
MIT
2 - 334
77 Massachusetts Ave.

Cambridge , MA 02139
USA

Prof.Dr. Francesco Brenti
Dipartimento di Matematica
Universita degli Studi di Roma
Tor Vergata
Via della Ricerca Scientifica

I-00133 Roma

Prof.Dr. Michel Brion
Ecole Normale Superieure de Lyon
Laboratoire de Mathematiques
46 allee d'Italie

F-69364 Lyon Cedex 7

Prof.Dr. Jim B. Carrell
Dept. of Mathematics
University of British Columbia
1984 Math. Road

Vancouver , BC V6T 1Z2
CANADA

Prof.Dr. Ionut Ciocan-Fontanine
Institut Mittag-Leffler
Auravägen 17

S-182-62 Djursholm

Prof.Dr. Adam Coffman
Department of Mathematics
The University of Chicago
5734 South University Avenue

Chicago , IL 60637-1514
USA

Prof.Dr. Sergey Fomin
Dept. of Mathematics
RM 2 - 363 B
MIT

Cambridge , MA 02139
USA

Prof.Dr. William Fulton
Department of Mathematics
The University of Chicago
5734 South University Avenue

Chicago , IL 60637-1514
USA

Dr. Lothar Göttsche
International Centre for
Theoretical Physics, International
Atomic Energy Agency, UNESCO
P. O. B. 586 Miramare

I-34100 Trieste

Prof.Dr. William Graham
School of Mathematics
Institute for Advanced Study
Olden Lane

Princeton , NJ 08540
USA

Prof.Dr. Mark Haiman
Department of Mathematics
University of California San Diego
9500 Gilman Drive

La Jolla , CA 92093-0112
USA

Prof.Dr. Bumsig Kim
Institut Mittag-Leffler
Auravägen 17

S-182-62 Djursholm

Dr. Axel Kohnert
Fakultät für Mathematik und Physik
Universität Bayreuth

95440 Bayreuth

Prof.Dr. Shrawan Kumar
Dept. of Mathematics
University of North Carolina
Phillips Hall CB 3250

Chapel Hill , NC 27599-3250
USA

Prof.Dr. Venkatramani Lakshmibai
Dept. of Mathematics
Northeastern University
567 Lake Hall

Boston , MA 02115
USA

Prof.Dr. T. Lam
Department of Mathematics
National University of Singapore
Lower Kent Ridge Road

Singapore 119260
SINGAPORE

Professor Dr. Alain Lascoux
L. I. T. P.
Aile 45 - 55
Universite Paris 7
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Fatima Laytimi
U. F. R. Mathematiques
Universite de Lille 1

F-59655 Villeneuve d'Ascq Cedex

Dr. Bernard Leclerc
Dept. de Mathematiques
Universite de Caen

F-14032 Caen Cedex

Svante Linusson
Mathematical Sciences Research
Institute
2223 Fulton Street
1000 Centennial Drive

Berkeley , CA 94720
USA

Prof.Dr. Peter Littelmann
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. Peter Magyar
Dept. of Mathematics
Northeastern University
567 Lake Hall

Boston , MA 02115
USA

Prof.Dr. Vincent Maillot
Departement de Mathematiques et
d'Informatiques
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05

Prof.Dr. Laurent Manivel
Laboratoire de Mathematiques
Universite de Grenoble I
Institut Fourier
B.P. 74

F-38402 Saint-Martin-d'Heres Cedex

Prof.Dr. Olivier Mathieu
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. Patrick Polo
Institut de Mathematiques, T. 46
UMR 9994 du CNRS, 3eme etage,
Universite Pierre et Marie Curie
4 place Jussieu, B.P. 191

F-75252 Paris Cedex 05

Prof.Dr. Piotr Pragacz
Math. Inst. of the
Polish Academy of Sciences
ul. Reymonta 9m3

87100 Torun
POLAND

Prof.Dr. Claudio Procesi
Dipartimento di Matematica
Istituto "Guido Castelnuovo"
Universita di Roma "La Sapienza"
Piazzale Aldo Moro 2

I-00185 Roma

Prof.Dr. K.N. Raghavan
SPIC Mathematical Institute
92, G.N. Chetty Rd., T. Nagar

Madras 600 017
INDIA

Prof.Dr. Jan Ratajski
Institute of Mathematics of the
Polish Academy of Sciences
P.O. Box 137
ul. Sniadeckich 8

00-950 Warszawa
POLAND

Prof.Dr. Parameswaran Sankaran
SPIC Mathematical Institute
92, G.N. Chetty Rd., T. Nagar

Madras 600 017
INDIA

Dr. Thomas Scharf
Fakultät für Mathematik und Physik
Universität Bayreuth

95440 Bayreuth

Dr. Michael Shapiro
Dept. of Mathematics
Royal Institute of Technology

S-100 44 Stockholm

Prof.Dr. Mark Shimozono
Department of Mathematics
Massachusetts Institute of
Technology

Cambridge , MA 02139-4307
USA

Prof.Dr. Frank Sottile
Mathematical Sciences Research
Institute
2223 Fulton Street
1000 Centennial Drive

Berkeley , CA 94720
USA

Prof.Dr. Harry Tamvakis
Department of Mathematics
The University of Chicago
5734 South University Avenue

Chicago , IL 60637-1514
USA

Prof.Dr. Jean-Yves Thibon
Institut Gaspard Monge
Universite de Marne-la-Vallee
2, rue de la Butte-Verte

F-93166 Noisy-le-Grand Cedex

Dr. Alek Vainshtein
Dept. of Mathematics and Computer
Sciences
University of Haifa
Mount Carmel

Haifa 31905
ISRAEL

Prof.Dr. P. Vanchinathan
Mehta Research Institute
of Math. and Math. Physics
10 Kasturba Gandhi Marg

Allahabad 211 002
INDIA

Prof.Dr. Eric Vasserot
Departement de Mathematiques
Universite de Cergy-Pontoise
Batiment Saint-Martin
2, avenue Adolphe Chauvin

F-95302 Cergy-Pontoise Cedex

Dr. Volkmar Welker
FB 6 - Mathematik und Informatik
Universität-GH Essen

45117 Essen

Prof.Dr. Andrei Zelevinsky
Dept. of Mathematics
Northeastern University
567 Lake Hall

Boston , MA 02115
USA