MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Pseudodifferential Operators and Microlocal Analysis

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The conference was organized by M. Beals (New Brunswick), B. Gramsch (Mainz), B.-W. Schulze (Potsdam) and H. Widom (Santa Cruz). This meeting followed three conferences on pseudodifferential operators in 1986, 1989 and 1993. More than 40 mathematicians from 9 different countries used the excellent atmosphere of the Oberwolfach institute for fruitful discussions and stimulating exchange of ideas, in particular, the younger ones profited extremely from this opportunity.
The 29 talks covered large parts of pseudodifferential and microlocal analysis, and showed again that this topic is located in between pure and applied mathematics. The importance of the techniques of microlocal and pseudodifferential analysis for the treatment of (non-linear) partial differential equations from different points of view was emphasized during the whole conference.
Several talks treated the Schrödinger equation and other equations in mathematical physics, e.g. Klein-Gordon, KdV, multiparticle quantum mechanics. Heat trace expansions and boundary value problems also on singular manifolds have been considered by five speakers.
From the more functional analytic point of view seven talks dealt with index theory, C^*- and Ψ^*-algebras of pseudodifferential operators, and pseudodifferential operators on nilpotent groups.
Another focus of interest were Gårding's inequality, local solvability for multiple characteristics, caustics for semilinear equations, nonlinear geometric optics and trace formulas in scattering theory.
Finally, concrete applications of pseudodifferential analysis to thermoelasticity and magnetic resonance tomography were discussed.

Vortragsauszüge

High frequency oscillating waves for gauge invariant systems

M. Bezard (joint with G. Métivier)

For different semilinear systems of pseudodifferential equations arising in relativistic theory and having some gauge invariance, we develop a weakly nonlinear geometrical optics strategy to build asymptotic solutions, and to prove that these so-called asymptotic solutions are in fact asymptotic to a true solution. As a first example of such a strategy that can be carried out for other systems (e.g. Maxwell-Dirac, Young-Mills, ...) we consider the case of the Maxwell-Klein-Gordon system for a massive relativistic spinless particle in its own self-induced electromagnetic field. This system deals one complex valued field φ and a real valued L-potential A_μ in $\mathbb{R}^{1+3}$, Minkowski space, and writes:

$$\begin{cases} D_\mu D^\mu \varphi + m^2\varphi &= 0 \\ \partial_\nu F^{\mu\nu} &= Im(\overline{\varphi} D^\nu \varphi) \end{cases} \quad \text{where} \begin{cases} F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ D_\mu = \partial_\mu + iA_\mu \\ m = \text{mass of the particle} \end{cases}$$

For this system we show how to construct high order formal asymptotic solutions of the type

$$\varphi = \underset{0}{\varphi}(x^\mu) + \varepsilon \underset{1}{\varphi}\left(x^\mu, \frac{\vartheta(x)}{\varepsilon}\right) + \varepsilon^2 \underset{2}{\varphi}\left(x^\mu, \frac{\vartheta(x)}{\epsilon}\right) + \dots$$

and $A_\mu = \underset{0}{A_\mu}(x^\mu) + \varepsilon \underset{1}{A_\mu}(x^\mu, \vartheta/\varepsilon) + \dots$, where the functions

$$\underset{1}{A}(x,\vartheta),\ \underset{2}{A}(x,\vartheta), \dots, \underset{1}{\varphi}(x,\vartheta),\ \underset{2}{\varphi}(x,\vartheta), \dots$$

are periodic in $\vartheta \in S'$ and ϑ satisfies the eikonal equation.
Taking advantage of the gauge invariance at each step of the construction allows one to split up the systems in non oscillating and purely oscillating parts, the latter satisfying some kind of transport equation, despite the lack of hyperbolicity.
We then prove via an iteration scheme in algebras of stratified functions along the level surfaces of ϑ that these formal solutions are asymptotic to real ones.

Operator pencils in the spectral theory of periodic Hamiltonians

M. Sh. Birman (with T.A. Suslina)

In $L_2(\mathbb{R}^2)$ we consider the periodic magnetic Hamiltonian

$$H = \left(\frac{1}{i}\nabla - \vec{A}(x)\right)^2 + V(x),\ \ \vec{A}(x+n) = \vec{A}(x),\ \ V(x+n) = V(x),$$

with $n \in \mathbb{Z}^d$. It is shown that the spectrum of H is absolutely continuous. In the study we use the L. Thomas scheme. In the case of $V = \partial_1 A_2 - \partial_2 A_1$ the needed estimates for the operator pencil in the mentioned scheme are obtained. The passage to the general V does not supply new difficulties. It is established that for $d \geq 3$ the analogous estimates cannot be fulfilled in general. Same special algebra of pseudodifferential operator with parameter occurring in the study was discussed.

Star products and Toeplitz operators

L. Boutet de Monvel

The title is slighlty misleading and this talk is meant in fact as a survey talk, with more open problems than new results, on the theory of star algebras $\mathcal{E}$ over a real or complex conic Poisson manifold and of coherent $\mathcal{E}-$modules (related to systems of pseudodifferential equations). Not much is known in the complex case, in contrast with the real one, in part because the obstruction groups, or moduli groups which classify such objects, are related to cohomology groups of holomorphic sheafs and tend to be huge. However, I believe the complex case is important for the theory of differential equations; e.g. one is typically confronted with the complex theory in the problem of deciding if a pseudodifferential system comes in fact from a differential system.

Semiclassical pseudodifferential operators and nonlinear scattering

V. Buslaev

It is well known that the solutions of some completely integrable nonlinear wave equations can be expressed in terms of solutions of some integral equations. As a result, investigation of the asymptotic behaviour of the solution of difference equation for large t can be reduced to the explicit asymptotic invertibility of some integral operators, in fact, semiclassical operators with symbols discontinuous with respect to both phase variables. Such integral operators constitute a special class in the theory of Ψdo's. We separate two subclasses of such operators and obtain for them explicit formulae for their inverses. Finally, we find the explicit formulae for the t-asymptotic solutions of NSE, MKdV and KdV equations.

Berezin-Toeplitz Quantization: an application to C^*-algebras

L.A. Coburn

I discuss a particular application of the Berezin-Toeplitz Quantization. The results are joint work with C.A. Berger. The main objective is to exhibit a non-separable unital C^*-subalgebra of the Calkin algebra for which "Voiculescu's Double Commutant Theorem" (VDCT) fails. (VDCT) is known to hold for all separable unital subalgebras and many non-separable unital subalgebras. Our candidate for failure is the "canonical commutation relation" (CRR) C^*-algebra.

Microlocal regularity for the Schrödinger equation

W. Craig

The subject of this talk is the Schrödinger equation

$$\begin{aligned} i\partial_t\psi &= -\frac{1}{2}\sum_{j,\ell=1}^{n} \partial_{x_j} a^{j\ell}(x)\partial_{x_\ell}\psi + V(x)\psi, \quad x \in I\!R^n, \\ \psi(x,0) &= \psi_0(x) \in L^2(I\!R^n). \end{aligned}$$

The microlocal smoothness of solutions of this equation is related to the localization properties of the initial data, and to the global behaviour of the bicharacteristics. In case $(a^{j\ell}(x))$ is asymptotically flat, the principal theorem in case $V(x) = 0$ is:

Theorem. Suppose that $(x_0, \xi^0) \in T^*(I\!R^n)\backslash\{0\}$ is not trapped backwards by the bicharacteristic flow, and denote

$$(x_{-\infty}, \xi_{-\infty}) = \lim_{s\to-\infty} \Big(X(s; x_0, \xi^0) \;/\; |X(s)|,\ \Xi(s; x_0, \xi^0) \Big).$$

If $(x_{-\infty}, \xi_{-\infty}) \notin \widehat{WF}(\psi_0)$, then for all $t > 0$,

$$\begin{aligned} &(1) \qquad (x_{-\infty}, \xi_{-\infty}) \notin \widehat{WF}(\psi(x,t)) \\ &(2) \qquad (x_0, \xi^0) \notin WF(\psi(x,t)). \end{aligned}$$

There are also regularity results for the Schrödinger kernel, and for the case in which there is a potential term.

Blow-up solutions of non-linear parabolic equations of second order

Y. Egorov

Let Ω be a bounded domain in $I\!R^n$ with a smooth boundary Γ. We consider two boundary value problems:

$$(1) \quad \frac{\partial u}{\partial t} = \Delta u - f(u) \text{ in } [0,T] \times \Omega, \quad \frac{\partial u}{\partial n} = +g(u) \text{ on } [0,T] \times \Gamma, \quad u(0,x) = \varphi(x).$$

$$(2) \quad \frac{\partial u}{\partial t} = \Delta u + f(u) \text{ in } [0,T] \times \Omega, \quad \frac{\partial u}{\partial n} + g(u) = 0 \text{ on } [0,T] \times \Gamma, \quad u(0,x) = \varphi(x),$$

where f and g are increasing functions, $f(0) = g(0) = 0$. The positive solutions of (1) are blowing-up at a finite time if $f(z) \le (1-\delta)g(z)g'(z)$ for $z \ge z_0$, $\delta > 0$ and if $\int_{z_0}^{\infty} \frac{dz}{g(z)g'(z)} < \infty$. For the problem (2) it happens if $f(z) \ge (a_0 + \delta)g(z)$, where $a_0 = |\Gamma|/|\Omega|$ and if $\int_{z_0}^{\infty} \frac{ds}{f(s)} < \infty$. We show also the asymptotics of solutions near the points of blow-up. In particular, we prove that such points are localized on the boundary for the solutions of (1) and strongly in interior of Ω for the problem (2). There are given also the conditions sufficient for global existence and for tending to 0 as $t \to +\infty$. In particular, all positive solutions of (1) are exploding at a finite time if $f(u) \le (a_0 - \delta)g(u)$, $\delta > 0$ and $\int_{z_0}^{\infty} \frac{dz}{f(z)} < \infty$.

Oscillatory integrals applied to Feynman path integrals

D. Fujiwara

1) A new proof of Yajima's theorem.
   Yajima constructed the fundamental solution of Schrödinger equation with vector potential $A_j(t,x)$, $j = 1,2,3$ and scalar potential $V(t,x)$ under the assumptions:

$$\begin{array}{lcll} |\partial_x^\alpha A_j(t,x)| & + & |\partial_x^\alpha \partial_t A_j(t,x)| \le C_\alpha, & |\alpha| \ge 1,\ j = 1,2,3\,, \\ |\partial x^\alpha B(t,x)| & \le & C_\alpha(|t,x|)^{-1-\varepsilon} & |\alpha| \ge 1, \\ |\partial_x V(t,x)| & \le & C_\alpha & |\alpha| \ge 2. \end{array}$$

   (J. d'Analyses Math. 1991). His method is based on integral equation and not related to Feynman path integral. Tsuchida and Fujiwara proved that the time slicing approximations of the Feynman path integrals converge to the fundamental solution of Schrödinger equation under the same assumption as above. (To appear in J. Math. Soc. Japan). This gives another proof of Yajima's theorem. Our method is based on the stationary phase asymptotics proved by Tsuchida. (Nagoya Math. J. 1994).

2) In view of its importance in quantum field theory, Feynman path integrals over the space of manifolds are very important and interesting. Here I present a very simple example. Its lattice approximation leads us to a new type of oscillatory integrals. An estimate of Kumanogo-Taniguchi type for such an oscillatory integral is obtained.

Pseudodifferential and analytic pseudodifferential operators on nilpotent groups and contact manifolds

D. Geller

We explain to what extent one is able to generalize the theory of pseudodifferential operators to the context of nilpotent groups with dilations. Explicit product formulas, and analogues of elliptic operators, will be discussed. In the case of the Heisenberg group, we explain how one can obtain a calculus of analytic pseudodifferential operators, and how the calculi on the Heisenberg group can be used to study operators on contact manifolds. The analytic calculus is applied to obtain relative analytic parametrices for transversally elliptic left invariant differential operators on the Heisenberg group (the inhomogeneous case being joint work with Peter Heller), and also to obtain relative analytic parametrices for $\overline{\partial}_b$ on boundaries of real analytic strictly pseudoconvex domains in $\mathbb{C}^2$.

Heat trace asymptotics for generalized Dirac operators with well-posed boundary conditions

G. Grubb

Let P be a first-order elliptic operator on a compact manifold X with boundary X', provided with a boundary condition $B(u/_{X'}) = 0$ defining the realization P_B. Asymptotic expansions of the heat operator traces

$$(*) \quad Tr(\varphi e^{-t\Delta_i}) \sim \sum_{-n\leq k<0} c_k t^{\frac{k}{2}} + \sum_{k=0}^{\infty}(c_k \log t + c'_k)t^{\frac{k}{2}},\ t \to 0,$$

for $\Delta_1 = P_B^* P_B$, $\Delta_2 = P_B P_B^*$, φ a morphism, have been shown in a joint paper with R.T. Seeley in Inventiones [GS95], for the case where P equals $\sigma(\partial_{x_n} + A_1)$ on a collar neighbourhood of X', with σ being a unitary morphism and $A_1|_{x_n=0} = A + P_0$, A selfadjoint elliptic on X' and P_0 of order 0. Here B is the orthogonal projection $\prod_{\geq}$ onto the nonnegative eigenspace of A (plus a finite rank operator). In the present work we show that $(*)$ holds also for general P and with B just well-posed, as defined by Seeley in CIME notes 1969. This depends on a study of P_B in terms of parameter-dependent Calderón projectors, and an elaboration of the calculus of weakly polyhomogeneous ψdo's in [GS95], for which we establish spectral invariance.

Microlocal analysis and multiparticle quantum mechanics

V. Ivrii

We discuss the application of spectral asymptotics and microlocal analysis to problems linked with the ground state of operator

$$\sum(-\Delta_{x_i} - V(x_i)) + \sum_{i<j}(x_i - x_j)^{-1}$$

describing N electrons in the field of M nuclei, as $N \to \infty$ and Z (the total charge of the nuclei) $\to \infty$. Questions discussed: Ground State Energy, Ground state density, Excessive positive (for molecules) and negative charge, minimal distance between nuclei, ionization energy. Results, expressed in terms of Thomas-Fermi theory are justified.

Symbolic construction of the fundamental solution for the heat equation and curvature on Riemannian manifolds with boundary

Ch. Iwasaki

Let $\Delta_p = d_{p-1}\partial_p + \partial_{p+1}d_p$ be the Laplacian acting on differential p-forms on a n-dimensional compact Riemannian manifold M with boundary ∂M. Let $e_p(t,x,y)$ be the kernel of the fundamental solution $E_p(t)$ for the following heat equation:

$$\begin{cases} \left(\frac{\partial}{\partial t} + \Delta_p\right) E_p(t) = 0 \quad \text{in } (0,T) \times M, \\ E_p(0) = I \quad \text{in } M, \\ E_p(t) \in Dom(\delta_p) \quad , \quad d_p E_p(t) \in Dom(\delta_{p+1}) \end{cases}$$

I have a theorem which is an extension of both Günther and Schimming ('77) and Iwasaki ('95).

Theorem. For any integer ℓ such that $0 \leq \ell \leq n$ as t tends to 0, we have the precise expression of the following $C_\ell(x)$ and $D_\ell(x)$ with the second fundamental form and the Riemannian curvature tensors on M and ∂M:

$$\text{(I)} \quad \sum_{p=0}^{\ell}(-1)^p \binom{n-p}{n-\ell} \operatorname{tr} e_p(t,x,x) = \begin{cases} C_\ell(x) t^{-\frac{n}{2}+\frac{\ell}{2}} + O\left(t^{-\frac{n}{2}+\frac{\ell}{2}+\frac{1}{2}}\right), & x \in M, \\ 2D_\ell(x) t^{-\frac{n}{2}+\frac{\ell}{2}-\frac{1}{2}} + O\left(t^{-\frac{n}{2}+\frac{\ell}{2}}\right), & x \in \partial M \end{cases}$$

$$\text{(II)} \quad \int_M \sum_{p=0}^{\ell}(-1)^p \binom{n-p}{n-\ell} \operatorname{tr} e_p(t,x,x)\,dv = t^{-\frac{n}{2}+\frac{\ell}{2}} \left(\int_M C_\ell(x)\,dv + \int_{\partial M} D_\ell(x)\,d\sigma\right) + O\left(t^{-\frac{n}{2}+\frac{\ell}{2}+\frac{1}{2}}\right).$$

Nonexponential decay in linearized thermo-elasticity

H. Koch

In thermo-elasticity the heat is coupled to pressure waves. In a homogeneous isotropic material there are nontrivial shear waves which do not dissipate. The reflection at the boundary of a shear wave however leads in general to reflected shear and pressure waves, even if the reflection is orthogonal.

Suppose that there is a two-periodic orbit of the billiard in a domain with smooth boundary. We show that then there exist waves which have almost all their energy in the shear part of the waves up to a given large time T. The high frequency part is concentrated near a bicharacteristic corresponding to the two-periodic orbit. This implies that there are solutions which decay slower than exponentially.

Dirichlet and Neumann eigenvalue problems on domains in Euclidean spaces

A. Laptev

Let $\Omega \subset I\!R^d$ be an open domain whose Lebesgue measure is finite. Let $-\Delta^{\mathcal{D}}$ be the operator of the Dirichlet boundary problem in Ω. We give a simple proof of the inequality

$$(1) \qquad Tr(\lambda + \Delta^{\mathcal{D}})_+ \;\leq\; \lambda^{1+\frac{d}{2}}\, |\Omega|\, (2\pi)^{-d}\, v_d\, \frac{d+1}{d+2}, \quad \text{for all} \;\; \lambda > 0,$$

where v_d is the volume of the unit ball in $I\!R^d$. This inequality implies the well-known uniform eigenvalue estimate for the Dirichlet Laplacian due to Li and Yau. Besides, it allows to prove the Pòlya conjecture for the class of domains $\Omega = \Omega_1 \times \Omega_2$ assuming that Ω_1 is a "tiling" domain. Simple generalizations of the inequality (1) allow us to study the Dirichlet eigenvalue problem for operators $B(D)$, $D = -i\frac{\partial}{\partial x}$, where $B(\xi)$ is a positive definite $n \times n$ matrix. Finally, we discuss some inequalities for the eigenvalues of the Neumann boundary problem.

Pseudodifferential operators and boundary potentials

P. Laubin

An elliptic boundary value problem in a smooth domain can be reduced to a pseudodifferential equation on the boundary. For non-smooth boundaries the symbolic calculus has to be adapted. For example, a general cone calculus has been developed by Schulze for conical singularities. Here we are interested in special boundary reductions that lead to a bijective and strongly elliptic operator and that are suitable for numerical methods.

For the Laplace operator, it was shown by Verchota that in Lipschitz domains, the simple and double layer potentials which are involved still have good mapping properties in L^2 of the boundary. We investigate the high order regularity of the double layer potential in polygons. It turns out that it defines isomorphisms between spaces of Sobolev type and arbitrary high regularity taking into account the singular functions generated by the corners.

Still sharper singularities occur in domains with cuts. Moreover the double layer potential does not define a bijective boundary operator in this framework. We consider the classical Dirichlet problem

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega\backslash\Gamma \\ u_{|\partial\Omega} = f, & u_{|\Gamma\pm} = f\pm \end{cases}$$

where Ω is a bounded polygon of the plane with a connected boundary $\partial\Omega$ and $\Gamma = \Gamma_1 \cup \ldots \cup \Gamma_p$ is the union of a finite number of cuts. We allow different boundary values on both sides of the cuts. For this problem, we consider the potential

$$u(x) = \frac{1}{2\pi}\int_{\Gamma\cup\delta\Omega} \frac{(x-y).\nu_y}{|x-y|^2}\, g(x)d\sigma(y) + \frac{1}{2\pi}\int_{\Gamma} \frac{(x-y).t_y}{|x-y|^2}\, T_\Gamma h(y)d\sigma(y)$$

$$+\sum_{j=1}^{p} u_j(x)\int_{\Gamma_j} s_j(y)h(y)d\sigma(y).$$

Here T_Γ is defined in the following way. Let $\sigma : [a,b] \longrightarrow \Gamma_j$ be a parametrization of Γ_j by arc length. We have

$$(T_\Gamma h)(\sigma(s)) = -\frac{1}{\pi}\, p.v. \int_a^b \frac{\sqrt{(b-s)(s-a)}}{\sqrt{(b-t)(t-a)}}\, \frac{h(\sigma(t))}{s-t}\, dt.$$

It is an inverse of the Hilbert transform restricted to a bounded interval modulo an operator of finite rank. We show that this potential defines a bijectiv boundary operator. It also defines isomorphisms between spaces of Sobolev type and arbitrary high order.

The structure of a Ψ^*-algebra of totally characteristic operators on manifolds with boundary

R. Lauter

Ψ^*-algebras were introduced by Gramsch in 1981 in connection with a perturbation theory for Fréchet-algebras. Since then many algebras occurring naturally in the pseudodifferential analysis were shown to be Ψ^*-algebras. However, the algebra $\Psi^0_{b,cl}(X, {}^b\Omega^{\frac{1}{2}})$ of b-pseudodifferential operators on a compact manifold X with boundary Y introduced by Melrose in 1981 turns out to be not a Ψ^*-algebra; to circumvent these difficulties we constructed a Ψ^*-algebra $\mathcal{A}(X)$ containing the algebra $\Psi^0_{b,cl}(X, {}^b\Omega^{\frac{1}{2}})$ as a dense subalgebra, reflecting the C^∞-structure of X, and having the following additional properties:

- There is a symbol map $\tau : \mathcal{A}(X) \longrightarrow Q_\infty \subseteq Q$ characterizing the Fredholm property by means of the invertibility of $\tau(a)$; here Q_∞ is a Ψ^*-algebra of "smooth" symbols.
- $\mathcal{A}(X) \subseteq \bigcap \mathcal{L}\left((H^s(X, {}^b\Omega^{\frac{1}{2}})\right)$ where $H^s = H^s(X, {}^b\Omega^{\frac{1}{2}})$ is the scale of b-Sobolev-spaces.
- Separate from the boundary one recovers the original pseudodifferential calculus.

Moreover, the C^*-closure $\mathcal{B}(X)$ of $\mathcal{A}(X)$ is shown to be solvable of length 2, the relationship between certain closed ideals is clarified, the spectrum and Jacobson topology of $\mathcal{B}(X)$ resp. $\mathcal{A}(X)$ are computed explicitly in terms of the underlying manifold; in fact, the spectra of $\mathcal{B}(X)$ and $\mathcal{A}(X)$ turned out to be homeomorphic. Parts of the results are related to the work of Mantlik, Melrose-Nistor, Plamenevskij and Schulze.

Edge operators with prescribed index

F. Mantlik

Let $W = X^\Delta \times Y$ denote a wedge, i.e. a product of a cone X^Δ with an edge Y. We denote by $\mathcal{Y}^m(W,g)$ an algebra of Ψdo's of order $m > 0$ over w which was introduced by B.-W. Schulze. If $A \in \mathcal{Y}^m(W,g)$ is elliptic then the Fredholm index of A only depends on the stable homotopy class of its principal symbol. It is our aim to construct for each such class c at least one operator $A_c \in \mathcal{Y}^m(W,g)$ whose index is known. This is mainly accomplished by means of a product construction and approximation. The technical difficulties stem from the complexity of the symbolic structure of the algebra $\mathcal{Y}^m(W,g)$. An essential point here is to extend the symbolic calculus to a certain completion of $\mathcal{Y}^m(W,g)$.

Smooth operators under the action of some compact Lie groups

S.T. Melo

A class of pseudodifferential operators on the circle can be characterized as the bounded operators on $L^2(S^1)$ which are smooth for the regular representation of S^1. This fact is proven using a local-global characterization of pseudos on the circle and H.O. Cordes' characterization of the operators on $L^2(\mathbb{R})$ which are smooth for the usual representation of the Heisenberg group.
In a joint work with H.O. Cordes, we also investigate the question of characterizing the bounded operators on $L^2(S^2)$ which are smooth for the left regular representation of $SO(3)$. Analogously as in the case of S^1, we consider operators defined by $Au = \sum_\ell a_\ell P_\ell u$, where (a_ℓ) is a bounded sequence in $C^\infty(S^2)$ and P_ℓ denotes the orthogonal projection onto the ℓ-th eigenspace of the Laplacian on the sphere. We prove that these are well-defined, $SO(3)$-smooth operators. We also exhibit a necessary and sufficient condition for smoothness.

L_p and Besov maximal estimates for solutions to Schrödinger equation

T. Muramatu

By making use of Besov norm we get the following results:
Theorem 2. Let $h(t,\xi)$ be real-valued, measurable and C^∞ in t. Assume that, with $\gamma > 0$,

$$|\partial_t^k h(t,\xi)| \le C_k(1+|\xi|)^{k\gamma} \ (k = 0,1,\ldots)$$

holds. Then

$$\left\| \left\{ \sup_{0<t<1} \left| \int\int e^{i(x-y)\xi + ih(t,\xi)} f(y) d\xi dy \right| \right\} \right\|_{L_2(\mathbb{R}^n)} \leq C\|f\|_{B^{\frac{\gamma}{2}}_{2,1}(\mathbb{R}^n)}.$$

Theorem 3. $a > 1$.

$$\left\| \left\{ \sup_{0<t<1} \left| \int\int e^{i(x-y)\xi + it|\xi|^a} f(y) d\xi dy \right| \right\} \right\|_{L_2(\mathbb{R}^n)} \leq C\|f\|_{B^{\frac{an}{4}}_{2,1}(\mathbb{R}^n)}.$$

Theorem 4. $a > 1$. $1 \leq p \leq \infty$. Then

$$\left\| \left\{ \sup_{0<t<1} \left| \int\int e^{i(x-y)\xi + it|\xi|^a} f(y) d\xi dy \right| \right\} \right\|_{L_p(\mathbb{R}^n)} \leq C\|f\|_{B^{a\sigma}_{p,1}(\mathbb{R}^n)}$$

where

$$\sigma = \begin{cases} \frac{1}{4} + \frac{1}{2}\left|\frac{1}{p} - \frac{1}{2}\right| & \text{if } n = 1 \\ \frac{1}{2} + (n-1)\left|\frac{1}{p} - \frac{1}{2}\right| & \text{if } n \geq 2 \end{cases}.$$

The estimate given in Theorem 3 is sharp for the case $n = 1$. Theorem 3 is an improvement of Sjölin's result which is given by Sobolev norm.

Commutator estimates and a sharp form of Gårding's inequality

M. Nagase

Let $\lambda(\xi)$ be a smooth weight function such that

$$(i) \quad \lambda(\xi) \geq 1 \text{ and } |\partial_\xi^\alpha \lambda(\xi)| \leq C_\alpha \lambda(\xi)^{1-|\alpha|} \text{ for all } \alpha.$$

We define a symbol class $S^m_{\varrho,\delta,\alpha}$ as $p(x,\xi) \in S^m_{\varrho,\delta,\alpha}$ if and only if

$$\left|p^{(\alpha)}_{(\beta)}(x,\xi)\right| \leq C_{\alpha,\beta}\lambda(\xi)^{m-\varrho|\alpha|+\delta|\beta|} \text{ for any } \alpha \text{ and } \beta.$$

The main theorem is

Theorem 1. Let $a_j(\xi)$, $c_j(\xi)$ be in $S^m_{1,0,\lambda}$ and $b_j(x) \in B^2$ for $j = 1, \ldots, N$, and assume that

$$Re \sum_{j=1}^{N} b_j(x) a_j(\xi) b_j(\xi) \geq c_0 \lambda(\xi)^{2m} \quad (|\xi| \geq R).$$

Then we have the following inequality:

$$Re \sum_{j=1}^{N} \left(b_j(x) a_j(D_x)u,\ c_j(D_x)u\right) \geq c_0\|u\|^2_{m,\lambda} - C\|u\|^2_{m-\frac{1}{2},\lambda}$$

for any $u \in \mathcal{S}$.
In order to prove Theorem 1 we use the following commutator estimate.
Theorem 2. If $a(\xi)$ is in $S^s_{1,0,\lambda}$ and $b(x)$ is in B^2 where $0 < s \leq 1$, then we have

$$||[a(D_x),\ b(x)]u||_{L^2(\mathbb{R}^n)} \ \leq\ C||u||_{s-1,\lambda}$$

for any $u \in \mathcal{S}$.

Distribution of scattering poles for several strictly convex obstacles

V. Petkov

Let $K = \bigcup_{j=1}^{Q} K_j$, $\overline{K}_i \cap \overline{K}_j = \emptyset$, $i \neq j, K_j$ being strictly convex bounded obstacles in $\mathbb{R}^3$. One considers the Neumann problem for the wave equation in $\mathbb{R} \times \Omega$, where $= \mathbb{R}^3 \backslash \overline{K}$.
Let $S(\lambda) : L^2(S^2) \longrightarrow L^2(S^2)$ be the scattering operator and let $\lambda_j \in \mathbb{C}$, $Im\lambda_j > 0$ be the scattering poles of the meromorphic continuation of $S(\lambda)$. Our goal is to obtain lower bounds for the counting function

$$N_{0,\delta}(r) = \{\lambda_j : 0 < Im\lambda_j \leq \delta,\ |Re\lambda_j| \leq r\}.$$

If the obstacle K satisfies the condition (H) of M. Ikawa we show that

$$N_{0,\delta}(r)\ \geq\ C_0 r^{\theta\left(\frac{1}{2}-\frac{b_2}{\beta}\right)} - C_1,\ \ r \geq C_2.$$

Here $\frac{1}{2} \leq \theta < 1$, $\delta = \frac{x}{\theta}\beta$, $x > 4$, while $b_2 > 0$ and $\beta > 2b_2$ are related to the geometric characteristics of K. In the case when (H) is not satisfied we obtain the same result in the generic case (P) assuming that some condition (D) is fulfilled. The condition (D) reflects the influence of rays with tangent segments to the behaviour of the reflecting rays, while (P) guarantees that there are periodic rays with tangent segments.

On the wave equation in a cylinder with edges

B.A. Plamenevskij

The wave equation with Dirichlet boundary condition in a finite cylinder with edges is considered. The existence and uniqueness of solutions in special, weighted spaces are proven. The asymptotics of solutions near edges are described.

Local solvability for nonlinear equations with multiple characteristics

L. Rodino

Consider a nonlinear partial differential equation of order m, defined by $F(x, v_\alpha)$, $|\alpha| \le m$, where F is an entire function with respect to v_α and C^∞ with respect to x in a neighbourhood of the origin in $\mathbb{R}^n$. Assume $F(x,0) = 0$. We say that F is locally solvable (at the origin and at the zero solution) for a given $f \in C^\infty$, if there exists a neighbourhood V of the origin and a constant $\varepsilon > 0$ such that

$$F(x, \partial^\alpha v) = \varepsilon f(x)$$

admits solution $v \in H^s(V)$, $s > \frac{n}{2} + m$.
Some results are reviewed concerning equations of principal type, and new results are presented for semilinear equations with multiple characteristics. In particular (work in collaboration with T. Gramtchev) in $\mathbb{R}^2$

$$(\partial_1 + i x_1^{2k} \partial_2)^m v + F(x, \partial^\alpha v)_{|\alpha| \le m-1} = \varepsilon f(x)$$

with F analytic with respect to $x = (x_1, x_2)$, is proved to be locally solvable for every $f \in G^s$, Gevrey class of order $s < m/(m-1)$.

The generation of semilinear singularities by a swallowtail caustic

A. Sà Barreto (joint with M.S. Joshi)

We construct examples of bounded solutions to a semilinear system

$$\mathcal{P}u = f(z,u),\ z \in \Omega \subset \mathbb{R},\ f \text{ smooth},$$

$\mathcal{P} = P \cdot Id$, with P a strictly hyperbolic differential operator of second order with smooth coefficients, such that for a time function t of $\mathcal{P}, u$ satisfy the following properties:
1) u is conormal to a smooth characteristic hypersurface $\sum$ in $t < 0$ such that $\sum$ develops a swallowtail singularity at time $t = 0$.
2) For $t > 0$, u is singular, not only at $\sum$, but also on the forward characteristic cone over the swallowtail tip.

Geometric quantization and the symbol of pseudodifferential operators, applied to the localization in magnetic resonance tomography

W. Schempp

Magnetic resonance tomography represents one of the most significant advances in non-invasive clinical radiodiagnostics. It is based on the Fourier transform implicitly performed by spin isochromats which are excited in tomographic slices. Due to the coadjoint orbit picture of the unitary dual $\widehat{G}$ of the Heisenberg nilpotent Lie group G, the tomographic slices are embedded as affine symplectic strata into the foliated projective space $\mathbf{P}(\mathbf{R} \times Lie(G)^{*})$ with non-dispersive plane of foci at infinity. The paper deals with the read-out of nuclear magnetic resonance absorption-mode information from the quantum holograms in the coadjoint orbits by means of the symbolic calculus of pseudodifferential operators.

References:

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Boundary value problems for manifolds with edges

E. Schrohe

The construction of a pseudodifferential calculus for boundary value problems on manifolds with edges (joint work with B.-W. Schulze, Potsdam) was presented. Basic objects are

1) The construction of operator algebras with symbolic structure and a sufficiently small class of residual elements (compact, regularizing, carrying asymptotic information).

2) A notion of ellipticity that allows the construction of parametrices within the calculus.

3) An appropriate scale of Sobolev spaces on which the operators act naturally and which permits precise results on regularity and asymptotics of solutions to elliptic equations.

4) All constructions should extend to higher order singularities.

In this particular case, the construction relies on the general edge concept of Schulze, the pseudodifferential calculus for manifolds with conical singularities (Schrohe/Schulze 1994/95) and a new concept of asymptotics.

On the index of elliptic operators on manifolds with conical singularities

V. Shatolov (joint with B.-W. Schulze, B. Sternin)

In the talk, we established the index formula for elliptic operators of arbitrary order on a manifold with cone-like singularities. Under a symmetry condition on the conormal symbols of the operator in question this formula expresses the index as a sum of two-terms. First of them - the contribution from the smooth part of the manifold - is written as the integral from Atiyah-Singer form over the manifold. The second - the contribution from singular points of the manifold - is a sum of multiplicities of spectral points of the conormal symbol lying in a certain strip in the complex plane.

A trace formula for dilation analytic Hamiltonians

J. Sjöstrand

We express the formal difference of the traces of $f(P_1)$ and $f(P_2)$, where P_j are semiclassical operators, in terms of the corresponding differences of sums of $f(\lambda_j)$, where λ_j are the resonances, plus a remainder term. Compared to the classical Poisson type formula in Lax-Phillips theory, the advantages are that our formula works also in even dimensions and for long range perturbations of the Laplacian.
As an application, we get very general results about the existence of "clouds" of $\sim h^{-n}$ resonances for $-h^2 \triangle + V$ on $\mathbb{R}^n$.
For more details see J. Sjöstrand: - Preprint, (Ecole Polytechnique) October 1996. Séminaire équations aux dérivés partielles, (Ecole Pol.) November 1996.

Nonlinear geometric optics for multidimensional shocks

M. Williams

We construct geometric optics expansions of high order for oscillatory multidimensional shocks, and then show that the expansions are close to exact shock solutions for small wavelengths. Expansions are constructed both for the oscillatory function defining the shock surface S, and for $u+-$, the solutions on each side of S. The profile equations yield detailed information on the evolution of $(u+-, S)$, showing for example how new interior oscillations are produced by nonlinear interaction between $u+-$ and S.

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