MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Gruppentheorie: Strukturtheorie der endlichen einfachen Gruppen und Anwendungen

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The meeting was organized by M. Liebeck (London), B. Stellmacher (Kiel) and G. Stroth (Halle). The main topic of the conference was application of the theory of finite simple groups to various areas like geometry, galois theory, topology, computational group theory, invariant theory and division algebras. Here it was shown that the force of group theory, results and methods are having more and more impact in other areas of mathematics. Besides this also talks on the structure of the finite simple groups like subgroup pattern, generation results and new methods in the classification of the finite simple groups were presented.

The inspiring discussions and the great facilities of Oberwolfach will certainly be of importance for the future of this field of mathematics. At least two papers were written during the conference.

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Vortragsauszüge

S. Abhyankar

Attempts to use the power of modern group theory of finite simple groups for calculating Galois groups

The muse of poetry is responsible for the above poetic rendition of the original title of my talk which was "Recognition Theorems and Galois Theory". At any rate, various Recognition Theorems of Group Theory provide powerful tools for computing Galois groups. Examples of such Recognition Theorems are:

(1) CT = Classification Theorem of Finite Simple Groups,
(2) CDT = Classification of Doubly Transitive Permutation Groups (using CT),
(3) CR3 = Classification of Rank 3 Permutation Groups (again using CT),
(4) Jordan-Margraff Theorems on Limits of Transitivity,
(5) Burnside's Theorem (which is a special case of the O'Nan Scott Theorem),
(6) Zassenhaus-Feit-Suzuki Theorem,
(7) Kantor's Rank 3 Theorem (using Buekenhout-Shult's Polar Space Theorem),
(8) Cameron-Kantor's Theorems on Transitive Collineation Groups, and
(9) Liebeck's Orbit Size Theorems (which uses CT).

I shall illustrate how these Recognition Theorems can be used for discovering nice equations whose expected Galois groups are various preassigned nice groups and then for establishing that their Galois groups indeed have the desired values.

M. Aschbacher

Quasithin groups

The classification of the quasithin groups is one of the major steps in the classification of the finite simple groups. Unfortunately Mason never completed his work on quasithin groups, so the step remains to be done. Steve Smith, Ulrich Meierfrankenfeld, and I are working to correct this situation. In addition we take a more general definition of even characteristic, in line with the Gorenstein, Lyons, Solomon revision. I described how this work is going and outlined our proposed proof.

R. Baddeley

The congruence lattice problem

It is an open problem to decide whether or not every finite lattice is isomorphic to the congruence lattice of a finite algebra. Pálfy and Pudlák have reduced this problem to the question: is every finite lattice isomorphic to an interval in the subgroup lattice of a finite group? It is generally believed that this question has a negative answer, and to date most work has been directed towards finding a lattice of weight 2 that does not so arise. These efforts have culminated in the work of Lucchini & RB which reduces the problem to a series of questions concerning the finite simple groups. However, these questions are not readily resolvable. The talk proposes that the scope of investigation is extended to a much wider class of lattices in the hope of reducing to questions that can be resolved.

References: R. Baddeley & A. Lucchini, On representing finite lattices as intervals in subgroup lattices of finite groups (to appear in J. Algebra),
R. Baddeley, A new approach to the finite lattice representation problem (in preparation).

B. Baumeister

Transitive subgroups of primitive permutation groups

For geometric reasons I became interested in the transitive subgroups of primitive permutation groups (G, Ω) which are conjugate to G_ω in $Aut(G)$.
Notice, $A \leq G$ is transtive if and only if $G = G_\omega A$.
We prove:

Theorem 1: Let G be a primitve permutation group and $G = G_\omega G_\omega^\alpha$ for some $\alpha \in Aut(G)$. Then one of the following holds:
(i) G is affine: $G \cong E_{2^{3r}}(L_3(2) \wr X)$, $X \leq S_r$ transitive;
(ii) G is almost simple: $E(G) \cong A_6, Sp_4(2^n), n \geq 2, P\Omega_8^+(q)$ or M_{12};
(iii) G is of product action type: $G \leq H \wr S_r$, H of type (ii).

In order to generalize the theorem in the affine case we determine all the groups G which have a faithful and irreducible $GF(p)$-module of dimension d and a subgroup K such that $|G : K| = p^a$, $a \geq d$. As a corollary we obtain that a maximal transitive subgroup of a primitive permutation group G either contains $O_p(G)$ or $G \cong E_{2^{3r}}(L_3(2) \wr X)$, $X \leq S_r$ transtive and $A \cong G_\omega \cong L_3(2) \wr X$.

A. Chermak

Groups of rank 2

We present an outline of one possible approach to classify amalgams arising from a rather general class of "rank-2" groups.
For p a prime and S a p-group, let $\mathfrak{M}(S)$ denote the class of all finite groups L such that

(i) $S \in Syl_p(L)$ and $S \ntrianglelefteq L$, and
(ii) S lies in a unique maximal subgroup of L.

We consider groups L in $\mathfrak{M}(S)$ to be of "rank one". The goal is to classify amalgams (L_1, L_2, S) satisfying the following:

Main Hypothesis: There is a group $G = \langle L_1, L_2 \rangle$ with $L_i \in \mathfrak{M}(S)$ and with $S \cap O_p(G) = 1$. Further, we have, for both $i = 1$ and 2,
(1) $[O_p(L_i), O^p(L_i)] \neq 1$, and
(2) $O^p(L_i) \trianglelefteq \trianglelefteq \langle L_i, M_{3-i} \rangle$ where, for any j, M_j is the unique maximal subgroup of L_j containing S.

The approach to the classification suggested here makes strong use of the Delgado-Stellmacher work on Weak BN-pairs, and of the Meierfrankenfeld-Stellmacher work on Pushing Up Weak BN-pairs.

R.T. CURTIS

Symmetric generation of finite groups.

The main aim of this work is to produce a standard procedure for obtaining the sporadic finite simple groups.
Groups generated by a specified, highly symmetric set of elements are sought — the asssumption being that these symmetries are achieved by inner automorphisms of the group obtained. It turns out that many of the sporadic groups emerge quite naturally from this point of view, and those of moderate size are readily constructed by hand using these methods.

P. FLAVELL

Generation Problems for Finite Groups

The starting point for this work is the following result of J. Thompson, which he obtained as a corollary to his classification of minimal simple groups.

Theorem: A finite group G is soluble if and only if $\langle x, y\rangle$ is soluble for all $x, y \in G$.

An elementary proof of this result was obtained by the author a few years ago. We make the following conjecture.

Conjecture: Let G be a group and $x \in G$. Then $\langle x^G\rangle$ is soluble if and only if $\langle x, y\rangle$ is soluble for all $y \in G$.

As a first step towards proving this, we have

Theorem: Let G be a group, p a prime and $P \in Syl_p(G)$. Then G is p-soluble if and only if $\langle P, g\rangle$ is p-soluble for all $g \in G$.

The next step would be to establish the following conjecture:

Conjecture: Let G be a group, p a prime and $P \in Syl_p(G)$. Then $\langle P^G\rangle$ is soluble if and only if $\langle P, g\rangle$ is soluble for all $g \in G$.

Progress has been made on this problem. The following result is used to construct a signalizer functor.

Lemma: Let $P \cong \mathbb{Z}_p$, $p \neq 2$ act on a soluble p'-group G. Then $C_{[G,P]}(P) = \langle C_{[g,P]}(P) | g \in G\rangle$.

Conjecture: The above Lemma is true for all p'-groups G.

P. FLEISCHMANN

Linear groups with special fixed point free elements

This is joint work with W. Lempken and A.E. Zalesskii:
Let $G \leq GL(V)$ finite linear group, char $V = p$, $g \in G$ non-central, $o(g) = r$ prime, $C := g^G$.

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$(G, C, V) \in FFG(p, r)$ iff $G = \langle C \rangle$ and $C_V(g) = 0$
$(G, C, V) \in FFG(p, r)^*$ iff moreover V is irreducible G-module.

Goal: Classifying $FFG(p, r)^*$, starting with $FFG(2, 3)^*$.
(This extends work of Mullineux ($G = Alt_n$) and R.L. Wilson ($G \in Chev(2^k)$).)

$(G, C, V) \in FFG(2, 3)^*$ implies
a) $F^*(G) = F(G) = Z(G) * E$, $E := \Omega_1(O_3(G)) \cong 3^{1+2m}$
or b) $F^*(G) = Z(G) * E(G)$, $E(G)$ quasisimple.

Our result at the present stage gives a full description of all groups G with $(G, C, V) \in FFG(2, 3)^*$, up to some ambiguities in case b) and $E(G)$ of type $Chev(2^k)$. Except this case and the case $G \cong \mathbb{Z}_3 \times PSp_{2m}(3)$ we get a full classification of all triples $(G, C, V) \in FFG(2, 3)^*$.

R.M. Guralnick

Rational Functions Which Are Bijective Modulo p

Let $\varphi \in \wp(x)$. Let φ_p denote the reduction of $\varphi \pmod p$ viewed in $\mathbb{F}_p(x)$. When is φ_p bijective on $\mathbb{P}^1_{\mathbb{F}_p}$ for infinitely many p? If $\varphi \in \wp[x]$, this was originally studied by Schur (1920's) who classified those of prime degree and conjectured that any such φ is a composition of cyclic polynomials (essentially x^r) and Dickson or Chebyshev polynomials. This was solved by Fried (1970) translating the problem into group theory. The polynomial condition is quite strong. If φ is rational, then we use the fact that the genus zero condition yields together the exceptionality that if φ is indecomposable (so Aschbacher-O'Nan-Scott applies) then up to a finite number of possibilities, φ is related to a polynomial or comes from an elliptic curve and is of degree r or r^2 for some odd prime r. Using the theory of elliptic curves, we can decide which degrees actually occur. This is joint work with Müller and Saxl.

A.A. Ivanov

On locally projective graphs of girth 5

Let Γ be a graph and G a 2-arc transitive automorphism group of Γ. For a vertex $x \in \Gamma$ let $G(x)^{\Gamma(x)}$ denote the permutation group induced by the stabilizer $G(x)$ of x in G on the set $\Gamma(x)$ of vertices adjacent to x in Γ. Then Γ is said to be a locally projective graph of type (n, q) if $G(x)^{\Gamma(x)}$ contains $PSL_n(q)$ as a normal subgroup in its natural doubly transitive action. Suppose that Γ is a locally projective graph of type (n, q), for some $n \geq 3$, whose girth (that is, the length of a shortest cycle) is 5 and suppose that $G(x)$ acts faithfully on $\Gamma(x)$. (The case of unfaithful action was completely settled earlier.) We show that under these conditions either $n = 4$, $q = 2$, Γ has 506 vertices and $G \cong M_{23}$, or $q = 4$, $PSL_n(4) \leq G(x) \leq PGL_n(4)$, and Γ contains the Wells graph on 32 vertices as a subgraph. In the latter case if, for a given n, at least one graph satisfying the conditions exists then there is a universal graph $W(n)$ of which all other graphs for this n are quotients. The graph $W(3)$ satisfies the conditions and has 2^{20} vertices.

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G. MALLE

The finite irreducible linear groups with polynomial ring of invariants

This is a report on a joint work with Gregor Kemper. In the talk I presented two recent results on the invariant theory of finite linear groups in arbitrary characteristic. The first one gives a characterization of those irreducible groups whose ring of invariants is a polynomial ring.

Theorem A: Let $G \leq GL(V)$ be finite irreducible. Then the ring of invariants $S(V)^G$ is polynomial if and only if G is generated by pseudo-reflections and the pointwise stabilizer of any proper subspace of V has polynomial invariants.

The necessity of the two conditions has been shown by Serre. The case of characteristic 0 had been solved by Shephard-Todd and Chevalley. Our proof uses the classification of finite irreducible groups generated by pseudo-reflections by Wagner, Zalesskii-Sereskin, Kantor.
The second result concerns fields of invariants.

Theorem B: Let $G \leq GL(V)$ be finite irreducible generated by pseudo-reflections. Then the field of fractions of $S(V)^G$ is purely transcendental over the ground field.
Partial results in this direction had been obtained by Carlisle and Kropholler.

A. MANN

Counting finite groups and their defining relations

We discuss the following

Conjecture A. The number of finite groups of order n and d generators is at most $n^{cd \log n}$, for some constant c.

We show that the conjecture holds for soluble groups. For that end, we first show that a soluble group of order n and d generators can be defined by $(d+1)\lambda(n)$ relations. Here $\lambda(n)$ is the number of primes dividing n, including multiplicities. This leads to the following

Conjecture B. Let G be as in Conjecture A. Then G can be defined by $cd \log n$ relations.

Conjecture B implies Conjecture A. Moreover, to establish Conjecture B, it suffices to show it for simple groups. For most families of simple groups it is known to hold.
Finally, by considering p-groups, we show that the bounds of the conjectures are of the right order of magnitude.

C. PARKER

Extremal subgroups in an ultraspecial situation

Extremal subgroups in groups of Lie type were introduced. Applications to the symplectic amalgam ($b = 2$) problem were given.

L. Pyber

Cartesian products of nonabelian simple groups

Consider finitely generated subgroups Γ of the group $G = \prod_{i=1}^{\infty} PSL(n_i, q)$ where q is a fixed prime-power and $5 \leq n_1 < n_2 < \ldots$.
In joint work with Lubotsky and Shalev it is shown that for certain choices of the series $\{n_i\}$ the group G has subgroups Γ with subgroup growth of type $n^{\log n}$.
In joint work with Babai it is shown that such groups Γ have polynomial index growth but
(i) they are not linear
(ii) they are not boundedly generated.
These results answer questions of Segal and Platonov-Rapinchuk respectively.

M. Rassy

Vertex-transitive automorphism groups of graphs and "pushing up"

Given a vertex-transitive automorphism group of a connected graph such that the stabilizer of each vertex x is finite and acts primitively on the set of neighbours of x, the problem of determining the structure of the vertex-stabilizers is closely related to a certain kind of "pushing up"-problem.

G. Röhrle

Parabolic subgroups of classical groups and quivers

This is a report on recent joint work with L. Hille (TH Chemnitz). We consider the action of a parabolic subgroup P of a classical group G on the unipotent radical P_u respectively on its Lie algebra $\mathfrak{p}_u$. We get the following classification result. Here $\ell(P_u)$ denotes the nilpotency class of P_u.
Theorem 1. *Let G be a classical algebraic group and P a parabolic subgroup of G (defined over the algebraically closed field k). Suppose that* char k *is either zero or a "good" prime for G. The number of P-orbits on P_u is finite if and only if either $\ell(P_u) \leq 4$, or $G = SO_{2n}(k)$, $\ell(P_u) = 5$ and P satisfies some additional conditions.*
This theorem can essentially be reduced to the case of general linear groups $GL(V)$.
A Levi subgroup of a parabolic subgroup of $GL(V)$ is isomorphic to a product of linear groups $GL_{d_i}(k)$, for $1 \leq i \leq t$, with $\dim V = \sum d_i$ and $d = (d_1, \ldots, d_t)$ determines the conjugacy class of P. Thus we write $P = P(d)$.
Theorem 2. *Let k be field and V a finite-dimensional k-vector space. Let $P = P(d)$ be a parabolic subgroup of $GL(V)$. Write $d = (d_1, \ldots, d_t)$. Then the number of P-orbits on $\mathfrak{p}_u$ is independent of k if and only if $t \leq 5$. In particular, in the case that k is infinite, the number of P-orbits on $\mathfrak{p}_u$ is finite if and only if $t \leq 5$. There is a purely combinatorial formula for the number of orbits only*

depending on d.

Even for the finite groups $GL_n(q)$ this result is new.

In our proof of Theorem 2 the question concerning the finiteness of the number of orbits of $P = P(d)$ on $\mathfrak{p}_u$ with $d = (d_1, \ldots, d_t)$, $t \in \mathbb{N}$, is reformulated as one about the representation type of a particular category $\mathcal{M}(t)$ of modules of a quiver.

P. ROWLEY

A Baby graph

Suppose Γ is the rank 4 minimal parabolic geometry for the Baby Monster simple group. Let $\mathcal{G}$ denote the point-line collinearity graph of Γ (where the points are the objects of Γ whose stabilizer is $2^{1+22}Co_2$). Then $\mathcal{G}$ is a 11,707,448,673,375 vertex graph. This talk described joint work with Louise Walker which is concerned with determining local and global properties of $\mathcal{G}$.

J. SAXL

Linear groups of orders divisible by certain large primes

In a joint work with Guralnick, Penttila and Praeger, we classify the subgroups of $GL_d(q)$ of order divisible by a prime r which divides $q^l - 1$ for some $l > d/2$ but does not divide any $q^i - 1$ for $i < l$ (so-called Zsigmondy primes with $l > d/2$). This has application to problems concerning generation of almost simple groups, recognition theorems (as described in Abhyankar's talk) and computer recognition algorithms for classical groups.

Y. SEGEV

On conjectures of Margulis-Platonov and Potapchik-Rapinchuk

Our main result is

Main Theorem: *Let D be a finite dimensional division algebra over an arbitrary field. Then no quotiemt of D^* is a non-abelian finite simple group.*

The Main Theorem was conjectured by Potapchik and Rapinchuk. By their result it also makes a contribution to the Margulis-Platonov conjecture (see conjectures a.1 and a.2 in the book of Platonov-Rapinchuk).

Let X be a finite group. The commuting graph of X is the graph on the nontrivial elements of X whose edges are commuting pairs. It is denoted $\Delta(X)$. Denote by $diam(\Delta(X))$ the diameter of $\Delta(X)$, and let d be the usual distance function on $\Delta(X)$. We say that $\Delta(X)$ is balanced if there are elements $x, y \in \Delta(X)$ such that $d(x,y)$, $d(x,xy)$, $d(y,xy)$, $d(x,x^{-1}y)$, $d(y,x^{-1}y)$ are all bigger than 3. The Main Theorem is proved using the following two theorems:

Theorem A: *Let L be a non-abelian finite simple group. Suppose that either $diam(\Delta(L)) > 4$ or $\Delta(L)$ is balanced. Then no quotient of D^* is isomorphic to L.*

The proof of Thm. A does not require the classification of FSG.

Theorem B: Let L be a non-abelian finite simple group. Then either $diam(\Delta(L)) > 4$ or $\Delta(L)$ is balanced.

The proof of Thm. B is joint with Gary Seitz. It requires the classification of FSG.

A. SHALEV

Simple groups, permutation groups, and probability

We prove a technical result concerning the size of the intersection of some conjugacy classes in classical groups with certain maximal subgroups. We then show that this result can be used to settle several seemingly unrelated problems in finite simple groups and finite permutation groups. For example, we prove

Theorem 1: There exists $\epsilon > 0$ such that if G is an almost simple classical group in non subspace action, and G has rank l and field size q, then the fixed point ratio of G is $\leq q^{-\epsilon l}$.

For large l, this sharpens the cq^{-1} bound of Liebeck and Saxl.

Theorem 2: There exists a constant c such that a finite almost simple primitive group has a base of size $\leq c$, with known exceptions.

This confirms a conjecture of Peter Cameron.
Other applications concern random generation, and the genus conjecture of Guralnick and Thompson , which is reduced to the case of subspace actions of classical groups.
This is a joint work with Martin Liebeck.

A. STEINBACH

Weak embeddings of generalized hexagons and groups of type G_2

(Joint work with Hans Cuypers, TU Eindhoven)
The group $G_2(L)$ is a subgroup of a 7-dimensional orthogonal group $\Omega(W,q)$. Each long root subgroup A is a Siegel transvection group, hence $[W,A]$ is 2-dimensional and $[[W,A],A] = 0$ (quadratic action).

We classify the embeddings of groups of type G_2 in linear groups satisfying these two assumptions.

Theorem. *Let L be a commutative field and G be a group generated by the class Σ of abstract root subgroups such that $\overline{G} := G/Z(G) \simeq G_2(L)$ and $\overline{\Sigma}$ is the class of long root subgroups of $\overline{G}$. We suppose that $G \leq \mathrm{GL}(V)$, where V is a vector space over the skew field K, such that $0 \neq \dim [V,A] \leq 2$ and $[[V,A],A] = 0$, for $A \in \Sigma$.*

Then L is isomorphic to a subfield of K and $[V,G]$ is a natural 7-dimensional or (in characteristic 2) 6- dimensional module for $G \simeq G_2(L)$ (over the bigger skew field K).

An important tool in the proof are *weak embeddings* of generalized hexagons and polar spaces $(\mathcal{P},\mathcal{L})$ (i.e. injective maps π from $\mathcal{P}$ into the point set of some projective space $\mathbf{P}$ generated by $\pi(\mathcal{P})$, such that the set $\{\pi(x) \mid x \text{ point of } l\}$

is contained in a line of **P**, for each line $l \in \mathcal{L}$, and for all points $p \in \mathcal{P}$, the image under π of $\{x \in \mathcal{P} \mid x$ not at maximal distance from $p\}$ is contained in a hyperplane of **P**).

In the proof of the theorem, we first construct a weak embedding of the generalized hexagon of type G_2 associated to G into the projective space $\mathbf{P} = \mathbf{P}([V,G])$. From this we obtain a weak embedding of the B_3-polar space containing the hexagon as a subgeometry into **P**. Now the results on weakly embedded classical polar spaces yield the theorem.

The method introduced above is very effective for the determination of subgroups of classical groups generated by transvections or long root elements.

F.G. TIMMESFELD

Presentations for certain Chevalley groups

Certain presentations for Chevalley groups of type A_n,D_n and E_n over arbitrary field K were given. For example:
$G = \langle A_i | i \in I, 2 < |I| < \infty \rangle$ such that:
(1) $A_i \neq 1 = (A_i)'$, $i \in I$
(2) For each $i \in I$ there exists a unique $i' \in I$ such that:
For each $a \in A_{i'}^{\#}$ there exists a $b \in A_i^{\#}$ with $A_i^b = A_{i'}^a$.
(3) If $j \notin \{i, i'\}$ then either $[A_i, A_j] = 1$ or
(i)$[A_i, A_j] = A_{f(i,j)}$, $f(i,j) \in I$
(ii)$[A_i, A_{f(i,j)}] = 1 = [A_j, A_{f(i,j)}]$
(iii)$[A_{i'}, A_{f(i,j)}] = A_j$
Then, if G is not a central product, G is a covering group of $PSL_n(K)$ or E_6^K, K a division ring or Cayley divison algebra or of a Chevalley group of type $D_n(K)$ or $E_n(K)$, K a field.

H. VÖLKLEIN

Braid-abelian generators of classical groups

The rigidity criterion in the case of 3 generators has been used extensively by Belyi, Thompson, Matzat's Heidelberg school and others to realize various classes of almost simple groups as Galois groups over $\mathbb{Q}$ and $\mathbb{Q}_{ab}$. The largest body of simple groups — those of classical Lie type — admit a natural class of rigid triples, the Belyi triples.
Here I show that the groups $PGL_n(q)$ and $PU_n(q)$ admit another class of rigid generating systems, of length $n+1$, which I called Thompson tuples. Their theory is quite similar to that of the Belyi triples, which allowed me to get some new results on the Belyi triples as well. The Thompson tuples yield the only known rigid generating systems of lenght > 3 of any almost simple group. In joint work with J. Thompson we found a class of generating systems of $Sp_n(q)$ of length $\frac{n}{2}+2$ which are not quite rigid, but very close: The pure braid group induces an abelian group of permutations on inner classes of these tuples. We use this to realize $PSp_n(q)$ over $\mathbb{Q}$ for $n \geq 4q^2$ and q a square (and under various other conditions on n and q as well).

TH. WEIGEL

Generation problems of finite simple groups and applications

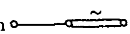
Various generation problems of finite simple groups have been solved recently. In the talk we present a proof of the Magnus Conjecture which asks whether a free group of rank at least 2 is residually X for every infinite family X of non-abelian finite simple groups. Using an estimate on the number of rational points on an affine variety of degree d and dimension k it is shown that a finite simple group of Lie-type G has a generation system $s, t \in G$ such that the length of a non-trivial relation R satisfies

$$l(R) \geq max\{\, log(q),\, f(l)\,\}$$

for almost all q, where $q = |F|$ denotes the cardinality of the field of definition, l denotes the Lie rank, and f is a certain isotonic unbounded function. From this one easily deduces the Magnus Conjecture.

C. WIEDORN

A Tilde geometry for $F_4(2)$

There exists a semiclassical parabolic system with diagram  and it is well known that only the sporadic simple groups M_{24} and He and a non-split extension $3^7Sp_6(2)$ can have such a system.
Now consider a parabolic system of rank 4 with diagram  and assume that the residues corresponding to the end nodes both belong to $3^7Sp_6(2)$. It can be shown that a group G with such a system always has a factor group isomorphic to $F_4(2)$ and that $G \cong 3^{833}F_4(2)$ (non-split), if the kernel of the homomorphism $G \to F_4(2)$ is abelian.
In the talk we construct an example of such a group as subgroup of the group $3^{4371}BM$, which is the automorphism group of the universal 2-cover of the Petersen-geometry for the Baby monster.

R.A. WILSON

Subgroups of the Baby Monster

The maximal subgroups of the Baby Monster are now (almost) completely determined. There are eight conjugacy classes of maximal subgroups (all non-local) not already listed in the Atlas. We describe how these were found, and how the final list is shown to be complete.

B. ZIMMERMANN

Finite groups and large groups of isometries of hyperbolic 3-manifolds

We study finite groups G, in particular linear fractional groups $PSL(2, q)$, which admit large actions by isometries on hyperbolic 3-manifolds $\mathcal{M}$. Here large means the volume (or the "Heegaard-genus") of the quotient $\mathcal{M}/G$ (which is a hyperbolic 3-orbifold) is small. Some of these small hyperbolic 3-orbifolds are uniformized (quotients of hyperbolic 3-space) by hyp. tetrahedral groups and some small Bianchi groups $PSL(2, \mathcal{O}_k)$. We classify the finite quotients of type $PSL(2, q)$ of these groups. For example, for the Bianchi groups $PGL(2, \mathcal{O}_1 = \mathbb{Z}[i])$, $PSL(2, \mathcal{O}_3)$ and $PGL(2, \mathcal{O}_3)$, all such quotients are obtained by reduction of coefficients mod p (and only few values of $q = p^n$ occur), whereas for the Picard group $PSL(2, \mathbb{Z}[i])$ almost all groups $PSL(2, q)$ are quotients (and are not obtained by reduction of coefficients). It would be interesting to consider other classes of finite (simple) groups. We discuss other types of hyperbolic 3-orbifolds of small volume resp. Heegaard-genus.

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