MATHEMATISCHES FORSCHUNGSINSTITUT
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Harmonische Analyse und Darstellungstheorie topologischer Gruppen

04.05–10.05.1997

Die Tagung wurde organisiert durch R. Howe (New Haven), E. Kaniuth (Paderborn) und G. Schiffmann (Strasbourg). Ein wesentliches Ziel der Tagung war die Zusammenführung von Mathematikern, deren Arbeitgebiete die ganze Breite der modernen Harmonischen Analyse und der Darstellungstheorie umfassen. Dies spiegelt sich in den Vortragsthemen der Teilnehmer wider. So wurden unter anderem Vorträge aus den folgenden Bereichen gehalten: Darstellungstheorie diskreter Gruppen und ihre C^*-Gruppenalgebren, Harmonische Analyse symmetrischer Räume und Darstellungstheorie halbeinfacher Gruppen, Gelfandpaare und Harmonische Analyse auf nilpotenten und auflösbaren Liegruppen, Darstellungen p-adischer Gruppen, C^*-Algebren von Transformationsgruppen und ihre Anwendungen auf die Darstellungstheorie. Neben den Vorträgen blieb ausreichend Zeit für Diskussionen und Kurzvorträgen in kleineren Kreisen.

Vortragsauszüge

M.B. Bekka

On the characters of $SL(n, \mathbb{Z}), n \geq 3$

Let Γ be an almost periodic countable group (that is, the finite dimensional unitary representations of Γ separate the points of Γ). Let $C^*(\Gamma)$ be the full C^*-algebra of Γ. A natural question is whether the finite dimensional representations separate the points of $C^*(\Gamma)$. This is indeed the case if Γ is amenable or if Γ is a non-abelian free group. This is also true for $SL(2, \mathbb{Z})$. In contrast, we show that the finite dimensional representations do not separate the points of $C^*(SL(n, \mathbb{Z}))$ for $n \geq 3$. This also holds for other arithmetic groups like $Sp(n, \mathbb{Z}), n \geq 2$ or $SL(2, \mathbb{Z}\sqrt{\delta}), \delta > 0$.

A character of Γ is an indecomposable central positive definite function on Γ. Using information on the restriction of such functions to appropriate subgroups of Γ, we show that there is no faithful tracial state on $C^*(\Gamma)$ where $\Gamma = SL(n, \mathbb{Z}), n \geq 3$. This answers a question of E. Kirchberg.

Moreover, we give a description of all characters of $SL(\infty,\mathbb{Z}) = \varinjlim SL(n,\mathbb{Z})$. An essential tool in our proofs is the congruence subgroup property of $SL(n,\mathbb{Z}), n \geq 3$.

C. Benson

Combinatorics and spherical functions on the Heisenberg group

Let V be a Hermitian vector space and K a compact Lie subgroup of $U(V)$ which acts in a multiplicity free fashion on $\mathbb{C}[V]$. One obtains a canonical basis $\{p_\alpha | \alpha \in \Lambda\}$ for $\mathbb{C}[V_\mathbb{R}]^K$, consisting of homogeneous polynomials, and also a basis $\{q_\alpha | \alpha \in \Lambda\}$ by orthogonalization of the p_α's. The coefficients that appear in the expression for q_α in terms of the p_β's are called generalized binomial coefficients by Z. Yan. We describe some new combinatorial identities that involve these coefficients. These have applications to analysis on the Heisenberg group $H = V \times \mathbb{R}$ obtained from V. Indeed, the polynomials q_α determine most of the bounded spherical functions for a Gelfand pair obtained from the action of K on H.

T.P. Branson

Spectra of intertwinors

In a recent paper [JFA, 1996], Olafsson, Ørsted and I presented a new way of computing intertwining operators

$$\mathrm{Ind}_{MAN}^G \sigma \otimes \nu \otimes 1 \xrightarrow{J} \mathrm{Ind}_{MAN}^G \sigma \otimes (-\nu) \otimes 1$$

when such exist. Here G is a semisimple Lie group, MAN a maximal parabolic subgroup, and we assume that K-types occur with multiplicity one. The idea is to compute the spectra of the operators J.

One may consider relaxing the restrictive assumptions above; namely

(1) $\sigma \to \sigma$ to $\sigma \to \lambda$ (changing M-types).
(2) Multiplicity one to higher multiplicity (of K-types).
(3) $\sigma \otimes \nu \otimes 1$ to $\sigma \otimes \nu \otimes \tau$ (N acts nontrivially before inducing).
(4) MAN maximal to arbitrary parabolics.

In this talk, we describe progress in direction (1), and applications to the solution of some old problems about Stein-Weiss operators (gradients) D. In particular, we determine:

- which linear combinations of operators D^*D are elliptic or of 0th order.
- the spectra of all D^*D on the standard sphere S^n.

J.L. Clerc

Compressions and contractions of hermitian symmetric spaces

Let D be a hermitian symmetric space, realized as a bounded domain in $\mathbb{C}^n$ (Harish-Chandra embedding). Denote by G the (connected component of) the group of holomorphic diffeomorphisms of D, and let K be the stabilizer of the origin $0 \in D$, so that $D \equiv G/K$. The action of G is by rational maps, and there is a corresponding action of its complexification $G^\mathbb{C}$. The compression semigroup

Γ is the set of elements in $G^{\mathbb{C}}$ such that $g(D) \subseteq D$. We show that Γ acts by contractions on D, when D is equipped with the Bergman metric (= G-invariant metric). This uses the realization of the semigroup Γ as $G\exp(C)$ (Olshanskii's theorem), where C is the maximal invariant cone (up to ± 1) in $i\mathfrak{g}$. There is strong indication that a stronger statement is true, as Neretin showed for classical hermitian symmetric spaces, and we formulate a conjecture for the general case.

M. Cowling

Intertwining operators and the Kunze-Stein phenomenon

(joint work with S.Meda)

Let G be a non-compact semisimple Lie group with finite centre. Although G is non-compact, it behaves in some respects as if it were. The Kunze-Stein phenomenon, that $L^p(G) * L^2(G) \subseteq L^2(G)$ if $1 \leq p < 2$, is one manifestation of this. A related phenomenon has been observed for split rank one groups, namely, that $|EF| \geq C|E||F|$, where $|E|$ denotes the Haar measure of a measurable subset E of G, C being a constant independent of E and F. This is deduced from the result that $L^{p,1}(G) * L^p(G) \subseteq L^p(G)$, where $L^{p,1}(G)$ is the usual Lorentz space on G. We give real variable proofs of an estimate on matrix coefficients related to the Kunze-Stein phenomenon and of the measure theoretic inequality, which rely on showing that the Knapp-Stein intertwining operators map $L^p(G/P)$ into $L^{p'}(G/P)$ for the class-one principal series representations which are naturally isometric on $L^p(G/P)$ and $L^{p'}(G/P)$ respectively.

H. Fujiwara

A conjecture of Corwin and Greenleaf

Let $G = \exp \mathfrak{g}$ be a connected, simply connected nilpotent Lie group with Lie algebra $\mathfrak{g}$. We consider a monomial representation $\tau = \mathrm{ind}_H^G \chi$ of G induced from a unitary character χ of an analytic subgroup $H = \exp \mathfrak{H}$. Let $D_\tau(G/H)$ be the algebra of smooth invariant differential operators on G which leave stable the space of functions satisfying the same H-covariance relation as those of τ-space. When τ is of finite multiplicity, Corwin and Greenleaf proved that $D_\tau(G/H)$ is commutative and conjectured that it should be isomorphic to the algebra of H-invariant polynomial functions on the affine space

$$\Gamma_\tau = \{l \in \mathfrak{g}^* : l|\mathfrak{h} = -\sqrt{-1}\, d\chi\}$$

of $\mathfrak{g}^*$. We study their conjecture by applying Penney's Plancherel formula for τ to get some partial affirmative results.

B. C. Hall

The Berezin-Toeplitz quantization for Lie groups of compact type

I describe a quantization scheme for the cotangent bundle of an arbitrary Lie group K of compact type. The cotangent bundle of K can be given a canonical complex structure which allows us to identify the cotangent bundle with the complexification of K. I describe a Hilbert space of holomorphic functions on the cotangent bundle which in the simplest case $K = \mathbb{R}^n$ reduces to the well-known Segal-Bargmann space of holomorphic functions on $\mathbb{C}^n = T^*(\mathbb{R}^n)$. Once we have this space, we may define for each function ϕ a Toeplitz operator T_ϕ ; the map $\phi \to T_\phi$ is the Berezin-Toeplitz quantization. I describe several results and conjectures about this quantization scheme.

A.G. Helminck

On representations associated with p-adic symmetric spaces

(joint work with G.F. Helminck)

In this talk we generalize the concept of real reductive symmetric space to p-adic groups and analyse the multiplicities of the representations in the Plancherel decomposition of the left regular representation. These symmetric spaces are defined as follows. Let k be a p-adic field, G a reductive p-adic group, $\sigma \in \mathrm{Aut}(G)$ an involution and $H = G^\sigma$ the fixed point group of σ. The p-adic manifold $X = G/H$ is called a p-adic reductive symmetric space. There exists a G-invariant measure dx on X. Let L be the left regular representation of G into $L^2(X, dx)$. We first show that L decomposes multiplicity free, when H is compact. For H non-compact larger multiplicities can occur. Using induced representations and H-fixed distribution vectors we give an estimate of these multiplicities. We also discuss which of these representations occur in the Plancherel decomposition.

A. Hulanicki

Pluriharmonic functions on symmetric domains in $\mathbb{C}^n$

(joint work with E. Damek, D. Müller and M. Peloso)

Let D be a bounded homogeneous domain in $\mathbb{C}^n$. Then there exists a solvable Lie group S of biholomorphic maps of D onto D whose action is singly transitive. It contains a nilpotent subgroup $N(\Phi)$ which acts "parallel" to the Bergman-Shilov boundary of D and its action extends to this boundary on which it acts singly transitively. In a paper by E. Damek, A. Hulanicki and R. C. Penney an elliptic, real, second order operator $\mathbf{L}$ with the following properties has been constructed.

(1) $\mathbf{L}$ anihilates holomorphic functions
(2) the class of bounded $\mathbf{L}$-harmonic functions on D is equal to the "Poisson integrals" of L^∞-functions on the Bergman-Shilov boundary of D
(3) $\mathbf{L}$ commutes with the action of S.

Property (2) says that $\mathbf{L}$ defines "the smallest" class of bounded functions harmonic w.r. a real second-oreder elliptic operator which includes bounded pluriharmonic functions. A natural question arises whether one can exhibit additional second order operators which would characterize pluriharmonic functions. We are going to show how this can be done in the case of symmetric irreducible domains. Instead of bounded holomorphic functions we deal with H^2-functions. The Hardy space $H^2(D)$ is defined as the space of holomorphic functions F on D such that

$$(H^2) \qquad \sup_{z\in D} \int_{N(\Phi)} |F(u\cdot z)|^2 du < \infty.$$

It follows from recent work of E. Damek, A. Hulanicki and R. Penney that for the functions F on D which satisfy (H^2) we have

$\mathbf{L}F = 0$, iff F is the Poisson integral of a function $f \in L^2(N(\Phi))$.

We exhibit a number of second order degenerate elliptic S-invariant operators Δ_j on D which characterize pluriharomonic functions F, the real parts of holomorphic functions $F + i\tilde{F}$, among the ones which are $\mathbf{L}$-harmonic and satisfy (H^2). Then also $\tilde{F}$ satisfies (H^2), so $F + i\tilde{F} \in H^2(D)$. We restrict our considerations to irreducible symmetric domains, only. For those the number of operators Δ_j is small, e.g., for the Siegel domain biholomorphic to the unit ball in $\mathbb{C}^n$ the operator $\mathbf{L}$ is the Laplace-Beltrami operator w.r. to the Bergman metric and we have to add only one more operator $\Delta = T^2 + H^2 - H$, where T is the central element in the Lie algebra of $N(\Phi)$ and H is the invariant differentiation in the direction of A. We need r and $2r$ additional operators on the symmetric irreducible tube domain and for irreducible symmetric type II Siegel domains, respectively, r being the rank of the domain. Irreducibility of D is nesessary for this type of results. For the product of n copies of the upper-half plane the number of second order real operators needed to characterize pluriharmonic functions is of the order of n^2. Our methods yield a characterization of the functions $f \in L^2(N(\Phi))$ such that the Poisson integral of f is pluriharmonic. For the tube domains this is nothing new: our condition reduces to Bochner's theorem: $\hat{f}(\chi) = 0$ for $\chi \notin \Omega^* \cup -\Omega^*$. For type II Siegel domains our result generalizes a result of G. Laville.

P.E.T. Jorgensen

Reflection symmetry for unitary representations of Lie groups

(joint work with G. Olafsson)

Let G be a Lie group, $\mathfrak{g}$ the Lie algebra of G, $\tau \in \mathrm{Aut}(G)$ an automorphism of period 2, $\pi \in \mathrm{Rep}(G, \mathcal{H})$ a unitary representation of G, $\mathcal{K}$ a closed subspace of $\mathcal{H}$, and $J : \mathcal{H} \to \mathcal{H}$ a unitary operator. We say that the system has reflection symmetry (RS) if $J\pi = (\pi \circ \tau)J$, $\mathcal{K}$ is invariant under the action of $H := G^\tau$ and $(v|Jv) \geq 0, \forall v \in \mathcal{K}$. We characterize the possibilities, including a classification if G is non-compact semisimple, and show that only trivial possibilities occur if G is the $ax + b$ group or the Heisenberg group. Let $\mathfrak{h}, \mathfrak{q}$ be the ± 1 eigenspaces of τ on $\mathfrak{g}$, and let $\mathfrak{g}^c = \mathfrak{h} + i\mathfrak{q}$. Let G^c be the corresponding simply connected Lie group. We show that if (RS) holds, then there is a unitary representation π^c of G^c on

$$\mathcal{H}^c = (\mathcal{K}/\{v : (v|Jv) = 0\})^{\sim}$$

such that π^c and π agree on H and $\pi^c(iy) = i\pi(y)$ for all $y \in Q$.

V.F. Molchanov

Tensor products and canonical representations

(joint work with G. van Dijk)

Quantization on para-Hermitian symmetric spaces G/H is closely connected with tensor products of representations induced by characters of maximal parabolic subgroups $P^{\pm}$ associated with G/H (or maximal degenerate series representations). These tensor products are connected with the so-called canonical representations which were introduced for Hermitian symmetric spaces by Berezin. In this talk we give the decomposition of the tensor products $\pi^{+}_{\mu,\nu} \otimes \pi^{-}_{\mu,\nu}, \mu \in \mathbb{R}, \nu = 0,1$, of maximal degenerate series representations of G for the case $G = SL(n,\mathbb{R}), H = GL(n-1,\mathbb{R})$. We use heavily the decomposition of the Berezin form for the space G/H obtained by the author earlier. It turns out that the Schwartz space $\mathfrak{D}_\nu(S \times S)$, where the tensor product initially acts according to its definition (here S is the sphere in $\mathbb{R}^n$), needs some "completion" to contain an orthogonal decomposition with respect to the Berezin form. If μ belongs to the interval $(\frac{-n-1}{2} + k, \frac{-n+1}{2} + k)$, $k \in \mathbb{Z}$, $k \neq 0$, then this completion includes $|k|$ irreducible subspaces in addition to the case $k = 0$ (when the decomposition includes irreducible unitary representations of continuous and discrete series). For $k > 0$ these spaces consist of distributions concentrated at the boundary Γ of G/H. The action of G on these distributions is diagonalizable. We give an explicit construction for this diagonalization.

K.-H. Neeb

Unitary highest weight representations and Riesz distributions

Let Ω be an irreducible symmetric cone and $L = \mathrm{Aut}(\Omega)_0$ the connected automorphism group of Ω. Then the dual cone Ω^* carries a distinguished family of L-semiinvariant tempered distributions $R_s, s \in \mathbb{C}$, called Riesz distributions. The positivity of these distributions corresponds to the unitarizability of the associated highest weight representation of $G = \mathrm{Aut}(T_\Omega)_0$, where T_Ω is the tube domain with basis Ω. We generalize this correspondence to the setting of vector valued highest weight representations and explain how their properties can be analyzed in terms of operator valued Riesz distributions.

Y. Neretin

Boundary values of holomorphic functions and constructions of discrete spectra

Denote by $B_{p,q}$ (resp. $T_{p,q}$) the space of complex (resp. real) $p \times q$-matrices Z such that $\|Z\| < 1$. Obviously, one has $B_{p,q} = U(p,q)/(U(p) \times U(q))$, $T_{p,q} = O(p,q)/(O(p) \times O(q))$. Consider Hilbert spaces $H_s(B_{p,q})$ and $H_s(T_{p,q})$ defined by the reproducing kernel

$$K_s(z,u) = \det(1 - zu^*)^{-s},$$

where $s = 0,1,2,\ldots,p-1$ or $s > p-1$. It is easy to see that the restriction operator $H_s(B_{p,q}) \to H_s(T_{p,q})$ is a unitary isomorphism, equivariant with respect to $O(p,q)$. It is known that $\lim_{s\to\infty} H_s(T_{p,q})$ is $L^2\big(O(p,q)/(O(p) \times O(q))\big)$.
Problem Decompose the representation of $O(p,q)$ in $H_s(T_{p,q}) = H_s(B_{p,q})$.

Denote by M the set of real $p \times q$-matrices such that $Z^*Z = 1_p$. Then we obtained

Theorem (Neretin-Olshanskii) If $s < (q-2p-1)/2$, then there exists a correctly defined restriction operator $H_s(B_{p,q}) \to L^1(M)$.

Hence we obtain an action of $O(p,q)$ in some Hilbert space of functions on M. Constructions of such type allow to construct discrete increments to spectra in various problems of harmonic analysis.

T. Nomura

Berezin transforms related to multiplicity free actions

(joint work with E. Fujita)

Let V be a finite-dimensional complex vector space on which a compact Lie group K acts linearly. The action is said to be multiplicity-free if the space $\mathcal{P}(V)$ of holomorphic polynomial functions on V has a multiplicity-free K-irreducible decomposition $\mathcal{P}(V) = \sum_{\alpha\in A} \mathcal{P}_\alpha(V)$, where A is an index set. Fixing a K-invariant hermitian inner product on V, we consider the normalized Gaussian measure μ and the corresponding L^2-space $L^2(V, d\mu)$. Then each $\mathcal{P}_\alpha(V)$ is a subspace of $L^2(V, d\mu)$ with reproducing kernel κ_α. The *Berezin transform* B_α associated to $\mathcal{P}_\alpha(V)$ is, by definition, an integral operator on $L^2(V, \kappa_\alpha(z,z)d\mu)$ with integral kernel

$$\frac{|\kappa_\alpha(z,\omega)|^2}{\kappa_\alpha(z,z)\kappa_\alpha(\omega,\omega)}.$$

Here $\kappa_\alpha(z,z)$ is strictly positive on a dense open subset $\mathcal{O}$ of V. My major interest consists in the spectral decomposition of the bounded positive selfadjoint K-invariant operator B_α. Some general facts and two case-studies are presented.

G. Olafsson

Spherical Laplace Transform for ordered symmetric spaces

Let G/M be an irreducible, globally hyperbolic symmetric space, and let $\varphi_\lambda(s) = \int_{M/H\cap Z} P_{-\lambda}(sh)\,dh$ be a M-spherical function on the semigroup $S^0 = M\exp C^0, C^0$ an open cone in $\mathfrak{g}$. We derive a Harish-Chandra type formula for φ_λ:

$$\varphi_\lambda(a) = c_\Omega(\lambda) \sum_{w\in W_0} c_0(w\lambda)\Phi_{w\lambda}(a), a \in A^+ \subset S^0 \cap A.$$

This gives an analytic continuation of the spherical Laplace transform

$$\begin{aligned} V(S)^\# \ni F \mapsto \mathcal{L}F(\lambda) &= \frac{1}{c(x)} \int F(0,x)P_{-\lambda}(x)\,dx \\ &= \frac{1}{c(\lambda)} \int_{A^+} F(0,a)\varphi_\lambda(a)\delta(a)\,da \end{aligned}$$

where $V(S)^\#$ is the commutative algebra of G-invariant Volterra kernels. The expansion formula also shows that $\varphi_\lambda(s)$ is analytic on S^0 for all parameters, where φ_λ is defined, by using the work of Heckmann and Opdam. Let $c_{MC}(\lambda)$ be the Harish-Chandra c-function for the Riemannian symmetric space G/K. Define E_λ by

$$c_{MC}(\lambda)E_\lambda(s) = \sum_{w\in W_0\backslash W} \frac{c_{MC}(w\lambda)}{c(w\lambda)}\cdot\varphi_{w\lambda}(s)$$
$$= \sum_{w\in W_0\backslash W} \frac{c_+(w\lambda)}{c_\Omega(w\lambda)}\varphi_{w\lambda}(s).$$

Then $E_\lambda|A\cap S^0$ agrees with the K-spherical function ψ_λ. We have the inversion formula:

$$F(0,s) = c\int \mathcal{L}F(\lambda)E_{-\lambda}(s)d\lambda.$$

J. A. Packer

The equivariant Brauer group of principal bundles

(based in part on joint works with I. Raeburn, D. Williams and S.T. Lee)

We use the recently developed equivariant Brauer group of D. Crocker, A. Kumjian, I. Raeburn and D. Williams to establish conditions under which certain twisted transformation group C^*-algebras are strongly Morita equivalent to one another, in part generalizing a theorem of P. Green and M. Rieffel. This result can be applied to study certain twisted group C^*-algebras associated to discrete, finitely generated, torsion-free, two-step nilpotent groups.

R.C. Penney

The Riesz-Fischer theorem for the Hua system on non-symmetric domains in $\mathbb{C}^n$

In this talk we stated a version of the Helgason conjecture for any Kähler manifold, which relates to describing the boundary behaviour of the C^∞-functions that are harmonic with respect to the scalar valued universal differential operators. Our main results, however, were for a system (the Hua system) of differential operators on a homogeneous domain in $\mathbb{C}^n$ which generates the algebra of universal operators. Our main result states that a function that is harmonic with respect to this system has boundary values on the Shilov boundary and an L^2-function on the Shilov boundary is a boundary value of a Hua harmonic function, if and only if its Fourier transform is supported on a particular set of orbits in $(\mathbb{R}^n)^*$ under the adjoint action of the cone group. This set of orbits equals $(\mathbb{R}^n)^*$ a.e. if and only if the domain is symmetric.

G. Ratcliff

The spherical transform of a Schwartz function on the Heisenberg group

(joint with C. Benson and J. Jenkins)

Let H_n be the $(2n+1)$-dimensional Heisenberg group, on which the unitary group $U(n)$ acts by automorphisms. Let $K \subseteq U(n)$ be such that $K \subseteq K \ltimes H_n$ is a Gelfand pair, and let Δ_K be the Gelfand space. Then the spherical transform from $L^2_K(H_n)$ to $L^2(\Delta_K)$ is an isometry.

Question: What is the image of the space of K-invariant Schwartz functions under the spherical transform?

We give a complete answer in terms of the decay of operators applied to functions on Δ_K. These operators are combinations of differential and difference operators.

W. Rossmann

Action-angle variables and weight multiplicity

Let K be a compact classical group, $T \approx \mathbb{T}^r$ a maximal torus, and $\Lambda \subset L(\mathbb{T}^r)^* = \mathbb{R}^r$ the weight diagram of an irreducible representation of K, viewed as a set of points with multiplicities. It is known that Λ can be relized in a natural way as the image under a linear projection $\mathbb{R}^m \to \mathbb{R}^r$ of the integral points in a polytope Π in a Euclidian space of dimension equal ot the number N of positive roots (Gelfand-Tsetlin tables). This suggests that there should exist an N-torus $\mathbb{T}^m$ containing $\mathbb{T}^r$, which acts naturally in the representation space and with multplicities $= 1$, so that the map $\Pi \to \Lambda$ is induced by the projection $L(\mathbb{T}^m)^* \to L(\mathbb{T}^r)^*$. According to the method of geometric quantization, such a torus $\mathbb{T}^m$ would be expected to act symplectically on the orbit of the hightest weight $K \cdot \lambda \subset L(K)^*$. (In classical language, such a torus action is equivalent to action angle variables for the elements of $H \in L(\mathbb{T}^r)$, considered as Hamiltonian functions on $K \cdot \lambda$). The case $K = U(n)$ is well-known and amounts to a version of Jacobi's elliptic coordinates. We present a construction of action angle variables which applies to classical groups and produces the desired map $\Pi \to \Lambda$.

S. Sahi

The binomial formula for nonsymmetric MacDonald polynomials

The q-binomial theorem is "essentially" the expansion of

$$(x-1)(x-q)\dots(x-q^{d-1})$$

in terms of the monomials x^k for $k \leq d$. We describe a multivariable generalization of this, where the "x^k's" are replaced by "MacDonald's nonsymmetric polynomials" $E_\beta(\chi; q, t)$, and the q-shifted powers "$(x-1), \dots (x-q^{d-1})$" are replaced by the inhomogeneous polynomials $G_\beta(\chi; q, t)$ introduced by F. Knop and myself. The binomial coefficients in the expansion can themselves be expressed in terms of the G_β's.

G. Savin

Lifting automorphic forms from G_2 to $PGSp_6$ using the minimal representation of E_7

The lift is the first step in pressing that automorphic forms on G_2 are motivic. Gross and I have succesfully developed both local and global aspects of the theory. In addition, G_2 admits a canonical form (like the Ramanujan Δ in GL_2-case) wich lifts to a nice form on $PGSp_6$: holomorphic discrete series at infinity, Steinberg at $p = 2$, and unramified for $p > 2$.

T. Steger

Monotony of free group representations

(joint work with G. Kuhn)

Let Γ be a nonabelian free group on finitely many generators. Let Ω be the boundary of Γ, $C(\Omega)$ the C^*-algebra of continuous functions, and $\lambda : \Gamma \to \mathrm{Aut}(C(\Omega))$ the left regular action. A representation π' of the crossed product $\Gamma \ltimes_\lambda C(\Omega)$ on $\mathcal{H}'$ is given by a unitary representation $\pi' : \Gamma \to \mathcal{U}(\mathcal{H}')$ and a $*$-representation $\pi' : C(\Omega) \to \mathcal{L}(\mathcal{H}')$ satisfying, for $x \in \Gamma, g \in C(\Omega)$

$$\pi'(x)\pi'(g)\pi'(x)^{-1} = \pi'(\lambda(x)g).$$

For a fixed unitary representation $\pi : \Gamma \to \mathcal{U}(\mathcal{H})$, a *boundary realization* of π is a pair (ι, π') where

- π' is a $\Gamma \ltimes_\lambda C(\Omega)$-representation on $\mathcal{H}'$
- $\iota : \mathcal{H} \to \mathcal{H}'$ is an isometric Γ-map
- $\iota(\mathcal{H})$ is cyclic for π'.

The realization is *perfect* if ι is bijective. We say that *monotony* holds for π if, up to obvious equivalence, π admits a unique realization (ι, π) and that realization is perfect.

The author and G. Kuhn show that the representations they introduced in "More irreducible ... " Duke J. (82) 1996, are monotonous and they provide a new, easier, and more sophisticated proof of irreducibility.

P. Torasso

Minimal representations of simple Lie groups over a local field of zero characteristic

By means of Duflo's orbit method we attach unitary irreducible representations to the admissible minimal nilpotent orbits of simple Lie groups of relative rank at least three over a local field. Using Duflo's method we construct irreducible unitary representations of the standard parabolic subgroups which fit together on their pairwise intersections.

When the field of definition is real we prove that the infinitesimal annihilator of any one of our representations is a completely prime ideal, the Gelfand-Kirillov dimension of which is minimal: in particular when the absolute type of the group in hand is not A_n, this ideal is the Joseph ideal. If moreover the group possesses a maximal parabolic with abelian nilradical we give a realization of the infinitesimal version of our representation through the Gencharov homomorphism.

A. Valette

Richard Thompson's group F

F is the group of orientation-preserving homeomorphisms of [0,1] which are piecewise-linear, whose derivatives have finitely many discontinuities – all of them at rational dyadics, and whose slopes are powers of 2. By results of Brown-Geoghegan and Brin-Squier, it is known that F is a finitely presented group that does not contain the free group $\mathbb{F}_2$ as a subgroup. Geoghegan conjectured in 1979 that F is non-amenable (if true, F would be the first counterexample of finite presentation to von Neumann's question: is non-amenability of a group due to the presence of $\mathbb{F}_2$?).

We present results by Paul Jolissaint (1997): F and its commutator subgroup F' are inner amenable (better: the von Neumann factors $W^*(F)$ and $W^*(F')$ have property (Γ) of Murray and von Neumann). We give some speculations on how to prove Geoghegan's conjecture; in particular, we construct a holomorphic family of uniformly bounded representations of F which are not obviously similar to unitary representations.

Berichterstatter: Siegfried Echterhoff

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