MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1997

Galois Groups and Fundamental Groups

15.06 - 21.06.1997

The meeting on Galois groups and fundamental groups was organized by D. Harbater (Philadelphia), Y. Ihara (Kyoto) and B. H. Matzat (Heidelberg). It was attended by 49 participants, coming from Austria, France, Germany, Israel, Japan, Netherlands, Russia, Spain and the USA.

The 26 talks during the week gave an excellent overview on the state of the art in inverse Galois theory including the Noether problem and on recent results on arithmetic fundamental groups including achievements concerning the anabelian conjecture of Grothendieck and the relationship between the absolute Galois group of $\mathbb{Q}$ and the Grothendieck-Teichmüller group. In a special session problems were presented that might be interesting to solve within the next time.

The meeting was rounded up by a hiking trip in the lovely surrounding of the institute. The team spirit of the Oberwolfach Institute also appeared in a music session given by several of the participants.

Vorträge:

S. Abhyankar: Arithmetic fundamental groups

Now that Harbater and Raynaud, as well as Pop, Tamagawa, and others, have thrown sufficient light on the algebraic fundamental group of an affine algebraic curve over an algebraically closed ground field, it is time to start speculating about the algebraic fundamental group in some other situations. Towards this end, I shall present some raw material and pose some conjectures. As usual, let me proceed in a historical manner.

During my Ph.D. work, my guru Zariski advised me to use Chevalley's local rings to algebracize Jung's surface desingularization of 1908 for carrying it over from the complex domain to the case of positive characteristic p. In my Amer. Jour. paper of 1955, I concluded that this cannot be done because in that case the algebraic local fundamental group above a normal crossing of the branch locus is not even solvable. A simplified version of the example I constructed for this purpose is the surface $F(Y) = Y^{\langle m-1 \rangle} + XY + Z$ over an algebraically closed ground field k of characteristic p, where $q > 1$ is any power of p and $m > 1$ is any integer, and I am using the abbreviation $\langle i \rangle = 1 + q + q^2 + \ldots + q^i$. Clearly the branch locus $Z = 0$ has a simple point at the origin, and it turns out that $\mathrm{Gal}(F, k(X,Z)) = \mathrm{Gal}(F, k((X,Z))) = \mathrm{PGL}(m,q)$. This supports the

Local Conjecture. For $d \geq 2$ and $t \geq 1$ we have $\pi_A^L(N_{k,t}^d) = P_t(p)$.

Here $N_{k,t}^d$ represents a neighbourhood of a simple point on a d-dimensional algebraic variety over k from which we have deleted a divisor having a t-fold normal crossing at the simple point. Moreover, $P_t(p)$ is the set of all (p,t)-groups, i.e., finite groups G for which $G/p(G)$ is an abelian group generated by t generators, where $p(G)$ is the subgroup of G generated by all of its p-Sylow subgroups. The above example also supports the

Global Conjecture. For $d \geq 2$ and $t \geq 0$ we have $\pi_A(L_{k,t}^d) = P_t(p)$.

Here $L_{k,t}^d$ represents the d-dimensional affine space L_k^d over k from which we have deleted t hyperplanes $H_1, \ldots, H_t$ which together with the hyperplane at infinity have only normal crossings. This can appropriately be generalized by replacing the hyperplanes by hypersurfaces. The above example supports the

Local Global Conjecture. For $d \geq 2$ we have $\pi_A^{LG}(L_{k,1}^d) = P_1(p)$.

Here the algebraic local-global fundamental group $\pi_A^{LG}(L_{k,1}^d)$ is defined to be the set of all Galois groups of finite unramified Galois coverings V of $L_{k,1}^d$ for which there exists an affine line L in L_k^d meeting H_1 in a point P such that the inverse images of P and H_1 on V are irreducible. This local-global conjecture is obviously stronger then the $t = 1$ cases of the above local and global conjectures as well as of the so called Abhyankar conjecture for a once punctured affine line proved by Harbater.

E. Bayer-Fluckiger: Multiples of trace forms and Galois cohomology

Let k be a field, G a finite group and let L be a G-Galois algebra over k. Let

$$q_L : L \times L \mapsto k, \quad (x,y) \mapsto Tr_{L/k}(xy)$$

be the trace form. This is a G-form.
Set $r \otimes q_L = \underbrace{q_L \oplus \ldots \oplus q_L}_{r}$ (orthogonal sum). (the G-Galois algebra corresponds to a homomorphism $\phi_L : \Gamma_k \to G$, which induces $\phi^* : H^1(G, \mathbb{Z}/2\mathbb{Z}) \to H^1(G, \mathbb{Z}/2\mathbb{Z}) = H^1(k, \mathbb{Z}/2\mathbb{Z})$. The image of $x \in H^1(G, \mathbb{Z}/2\mathbb{Z})$ is denoted by $x_L : x_L = \phi_L^*(x)$.).

We have the following results: Let k_s be a separable closure of k, $\Gamma_k = Gal(k_s/k)$.
Theorem [E.B.-J.-P. Serre]: Suppose that $cd_L(\Gamma_k) \leq 1$. Let L and L' be two G-Galois algebras. Then

$$q_L \cong q_{L'} \iff x_L = x_{L'} \text{ for all } x \in H^1(G, \mathbb{Z}/2\mathbb{Z}).$$

Theorem [E.B.-M. Monsurre]: Suppose that $cd_L(\Gamma_k) \leq 2$. Let L and L' be two G-Galois algebras. Then

$$2 \otimes q_L \cong 2 \otimes q_{L'} \iff x_L \cdot (-1) = x_{L'} \cdot (-1) \text{ for all } x \in H^1(G, \mathbb{Z}/2\mathbb{Z}),$$

where $\cdot$ denotes cup product.

P. Dèbes: On the Beckmann-Black problem

S. Beckmann and E. Black proved in two different ways that if K is a number field and E/K is a Galois extension with abelian group G, then E/K is the specialization of a Galois branched cover of $\mathbb{P}^1$ defined over K with group G. We showed that this result can in fact be extended to any field K. E. Black conjectures the same holds even for non-abelian groups G. We have two further results about her conjecture. The first one is that it implies a positive answer to the Regular Inverse Galois Problem. The second one is a 'mere' form of the conjecture over ample fields. In this mere form, the realizing Galois cover is required to be defined over K only as mere cover; a field K is called ample if each smooth K-variety defined over K has infinitely many K-rational points provided there is at least one.

D. Haran: Regular embedding problems and patching of fields

We give an elementary proof, based on our method of 'patching of fields,' to the following theorem of Pop:

Theorem: Let K be an ample field, let x be transcendental over K, and let E be a finite separable extension of $K(x)$. Let F/E be a finite Galois extension and assume that

$G(F/E)$ acts on a finite group H. Then the corresponding split embedding problem has a regular solution. I.e., there exists a Galois extension $\hat{F}/E$ such that $\hat{F}$ is regular over the algebraic closure $\hat{K}$ of K in F and there is an isomorphism $\theta: H \rtimes G(F/E) \longrightarrow G(\hat{F}/E)$ such that $\text{res} \circ \theta = \text{pr}$, where $\text{res}: G(\hat{F}/E) \longrightarrow G(F/E)$ and $\text{pr}: H \rtimes G(F/E) \longrightarrow G(F/E)$.

Our proof essentially reduces the problem to the case $E = K(x), F = \hat{K}(x)$, with some special conditions.

(A joint work with M. Jarden.)

D. Harbater: Formal patching and embedding problems in finite characteristic

Patching in formal or rigid geometry has been used to obtain information about fundamental groups of curves X in characteristic p – e.g. the set $\pi_A(X)$ of finite quotients of π_1, and the solvability of embedding problems. This includes the solution of Abhyankar's Conjecture, the geometric case of the Shafarevich conjecture, and results on π_1 of projective curves of genus g. In formal geometry, the key result is Grothendieck's Existence Theorem, but sometimes constructions require 'smaller open sets' to be patched – e.g. Spec of complete local rings at points on closed fibres. Recent work with Kate Stevenson allows reducible k-curves to be thickened to curves over $k[[t]]$, and G-covers of reducible curves to be thickened to covers over $k[[t]]$, by giving the local behavior near the singular points. This leads to simpler proofs of results relating to Abhyankar's Conjecture for the affine line, and to information about π_1 of projective curves.

Concerning the latter, we know by SGA1 that each $G \in \pi_A(X)$ has the form $G = \langle a_1, b_1, \ldots, a_g, b_g \rangle$ such that $\Pi[a_i, b_i] = 1$, in arbitrary characteristic. The full converse is false in characteristic p, but our patching methods yield a partial converse. For example, a group G of this form will lie in $\pi_A(X)$ provided that $H = \langle a_1, \ldots, a_g \rangle \lhd G$ and that either $p \nmid |H|$ or $Out(H) = 1$; $p \nmid o(a_i)\ \forall i$.

The above methods can be used as well to obtain a simpler proof of Pop's result that quasi-p embedding problems have solutions, over affine curves in characteristic p. This also uses the p-embedding property, that over an affine variety X, any embedding problem with p-group kernel can be solved, and moreover can extend any given solution over a closed subset $X' \subset X$. A proof of Pop's result then proceeds by thickening the affine curve X over $k[[t]]$; blowing up a point ξ to get an exceptional divisor E; and using the p-embedding property to obtain an extension of the given cover whose behavior, over the generic point ξ^o of the thickening $\bar{\xi}$ of ξ, agrees with that of a quasi-p cover of $\mathbf{A}^1$ near ∞. The resulting cover is then patched to this quasi-p cover of $\mathbf{A}^1$ to obtain the desired conclusion. Moreover, a similar strategy can be used to show the analogous result in higher dimension.

M. Jarden: Large-small normal algebraic fields

Let K be a number field. Denote the absolute Galois group of K by $G(K)$. For each $\sigma \in G(K)$ let $\tilde{K}(\sigma)$ be the fixed field of σ in $\tilde{K}$. Denote the maximal Galois extension of K

which is contained in $\tilde{K}(\sigma)$ by $\tilde{K}[\sigma]$. Now choose $\sigma \in G(K)$ at random and let $N = \tilde{K}[\sigma]$. Then N has some properties of large algebraic fields:

- (1a) N is PAC;
- (1b) $G(N) \cong \hat{F}_\omega$.

On the other hand N has some properties of small algebraic fields (i.e., of number fields). They are related to elliptic curves E without complex multiplication which are defined over N. To this end let l range over the set of prime numbers and let E_l (resp., E_{l^∞}) be the group of points of E annihilated by l (resp., a power of l). Then the following statements hold:

- (2a) $\mathcal{G}(N(E_{l^\infty})/N)$ is an open subgroup of $\mathrm{GL}(2, Z_l)$;
- (2b) For almost all l, $\mathrm{SL}(2, Z_l) \leq \mathcal{G}(N(E_{l^\infty})/N)$;
- (2c) E has only finitely many cyclic isogenies which are defined over N;
- (2d) Analog of Tate's Conjecture: There is a natural isomorphism
$$\mathrm{End}_N(E) \otimes Z_l \cong \mathrm{End}_{Z_l[G(K)]} T_l(E).$$
- (2e) Isogeny theorem: Let E' be another elliptic curve over N without complex multiplication. If $E_l \cong_{G(K)} E'_l$ for a set of prime numbers l of a positive Dirichlet density, then E and E' are isogenous.

Property (2) is proved in a joint work with Gerhard Frey.

G. Kemper: Invariant fields of finite irreducible reflection groups

We prove the following
Theorem: If G is a finite irreducible reflection group defined over a field k; then the invariant field $k(V)^G$ of G is purely transcendental over k.

If $char(k)$ does not divide $\mid G \mid$, then already the invariant ring $k[V]^G$ is isomorphic to a polynomial ring. Hence we only have to consider the modular case.
The main ingredients of the proof are:

- The classification of the finite irreducible reflection groups by Kantor, Wagner, Zalesskiĭ and Serežkin
- The following **proposition:** Let $f_1, \ldots, f_n \in k[V]^G$ be homogeneous and algebraic over $k(f_1, \ldots, f_n)$ and
$$\prod_{i=1}^{n} deg(f_i) < 2 \mid G \mid .$$
Then $k(V)^G = k(f_1, \ldots f_n)$.

(Joint work with Gunter Malle)

Question: Is the above theorem still true if the irreducibility hypothesis is dropped?

M. Matignon: Order p automorphisms of p-adic open discs

This lecture is a report on a common work with B. Green (Stellenbosch) and based on two preprints:

- On Liftings of Galois covers of Curves, to appear in Compositio
- Order p automorphisms of the open disc of a p-adic field, Laboratoire de Mathematiques Pures de Bordeaux, series preprint.

Let R be a complete discrete valuation ring ($char(R) = 0$), π a uniformising parameter with $k := R/\pi R$ assumed to be algebraically closed ($char(k) = p > 0$). We are interested in the description of finite order R-automorphisms of $R[\![Z]\!]$ and more particularly those of order p (the residue characteristic). In the sequel σ is an order p automorphism with at least one fixed point. Denote by $F_\sigma := \{Z_0, \ldots, Z_m \in K^{alg} : |Z_i| < 1\}$ the set of fix points which we assume K-rational (i.e. $Z_i \in \pi R$.) Denote by (D^o, F_σ) the marked open disc ($D^o := SpecR[[Z]]$) we can define the minimal semi-stable model $\mathcal{D}^o$ which spreads the specialization of F_σ to smooth distinct points; σ is an R-automorphism of $\mathcal{D}^o$. We describe the geometry of the special fiber $\mathcal{D}^o_s$ and in particular one shows that F_σ specializes to only terminal components. In doing so we prove
Theorem 1: If $m = 0$ then σ is linearizable i.e. after change of a parameter $\sigma(Z) = \zeta Z$ for ζ a suitable primitive p-th root of unity.
Theorem 2: If $m < p$, then $\mathcal{D}^o_s$ has only one component which is a projective line: i.e. the fixed points F_σ are equidistant.
Theorem 3: If $m < p$, there are only a finite number of conjugacy classes in $Aut_R(R[[z]])$ of order p automorphisms with no inertia at (π) (i.e. the order of σ mod π is p) and such that $| F_\sigma |= m+1$. Moreover, we give a set of representatives which are induced by p-cyclic covers of $\mathbb{P}^1$ which have good reduction.

Note: We proved Theorem 2 for $m = 2, 3, 4$ and M. Raynaud indicated the general case in a letter to the authors.

M. Matsumoto: Topological methods in studying Galois actions

Let V be a geometrically connected variety defined over a number field $K \subset \mathbb{C}$. Then we have an outer representation

$$\rho_V : Gal(\bar{K}/K) \to Out\pi_1(\bar{V}).$$

Belyi proved that this is injective for V being a projective line minus three points. It is generalized in [M, Crelle, 96] to any affine curve X with non abelian fundamental group.

The proof goes as follows.

1. Consider the configuration space of n points on X. Call it V.

2. Construct $n-1$ (tangential) morphisms from projective line minus three points to V.

3. Construct a (tangential) morphism from X to V.

4. The pullback of these morphisms becomes a $\mathbb{Q}$-rational point, which serves as a (tangential) base point.

5. Use a real two-0 topological relation between the image of $\pi_1(\bar{X})$ and the image of the fundamental groups of projective lines minus three points, to show that if an element of the Galois group acts trivially on $\pi_1(\bar{X})$, so does on the fundamental group of the projective line minus three points.

It seems that the varieties whose outer representation extends to a Grothendieck-Teichmüller group action are special. The above V seems to be a mixture of such a special variety and a general curve X.

S. Mochizuki: The anabelian geometry of curves over local fields

Let K be a field of characteristic zero. Let $\overline{K}$ be an algebraic closure of K. Write $\Gamma_K \stackrel{\text{def}}{=} \text{Gal}(\overline{K}/K)$. Next, let X be a variety over K, and fix a prime number p. Then we introduce notation as follows:

$\Pi_X^{\text{prf}} \stackrel{\text{def}}{=} \pi_1(X)$ (for some choice of base-point)

$\Delta_X^{\text{prf}} \stackrel{\text{def}}{=} \pi_1(X \otimes_K \overline{K})$

$\Delta_X \stackrel{\text{def}}{=}$ the maximal pro-p quotient of Δ_X^{prf}.

Since the kernel of the quotient $\Delta_X^{\text{prf}} \to \Delta_X$ is normal not only in Δ_X^{prf}, but also in Π_X^{prf}, we may form the quotient Π_X of Π_X^{prf} by this kernel. Then we get an exact sequence of topological groups

$$1 \to \Delta_X \to \Pi_X \to \Gamma_K \to 1$$

Of course, it follows from the construction of this exact sequence that this exact sequence is determined by the K-variety X. On the other hand, the *anabelian philosophy of Grothendieck leads one to conjecture that, for certain ("anabelian") X and certain types of K, the converse also holds*, i.e., that X is determined as a K-variety by this exact sequence. This anabelian philosophy is partially realized by the following result:

Theorem. *Let p be a prime number. Let K be a subfield of a finitely generated field extension of $\mathbf{Q}_p$. Let X_K be a smooth variety over K, and Y_K be a hyperbolic curve over K. Write $\mathrm{Hom}_K^{\mathrm{dom}}(X_K, Y_K)$ for the set of dominant K-morphisms from X_K to Y_K, and $\mathrm{Hom}_{\Gamma_K}^{\mathrm{open}}(\Pi_X, \Pi_Y)$ for the set of open, continuous group homomorphisms $\Pi_X \to \Pi_Y$ over Γ_K, considered up to composition with an inner automorphism arising from Δ_Y. Then the natural map*

$$\mathrm{Hom}_K^{\mathrm{dom}}(X_K, Y_K) \to \mathrm{Hom}_{\Gamma_K}^{\mathrm{open}}(\Pi_X, \Pi_Y)$$

is bijective.

The proof of this Theorem may be regarded as an application of the p-adic Hodge theory of Faltings and the theory of the p-adic exponential map due to Bloch-Kato and builds on techniques introduced by Tamagawa over finite fields.

P. Müller: Finiteness results for Hilbert's Irreducibility Theorem

Let $f(X,t) \in \mathbb{Q}[X,t]$ be an irreducible polynomial. Set $\mathcal{R} := \{t_0 \in \mathbb{Z} | \, f(X,t_0)$ is reducible $\}$. It is well known that $|\mathcal{R} \cap [-n,n]| < Cn^{1/2}$ with C depending on f. In view of $f(X,t) = X^2 - t$, this bound is optimal without further assumptions on f.

We use a Galois theoretic translation to a question about finite permutation groups in order to show that $|\mathcal{R}| < \infty$ under various assumptions on the Galois group of $f(X,t)$ over $\mathbb{Q}(t)$. A simple sample result is

> **Theorem:** *Let $f(X,t) \in \mathbb{Q}[X,t]$ be an irreducible polynomial of prime degree. Assume that the curve given by $f = 0$ has positive genus. Then $\mathcal{R}$ is finite.*

Stronger versions of this theorem for non–prime degree and analogs over number fields require the classification of the finite simple groups. Work on this is in progress.

H. Nakamura: Galois representations in Teichmüller modular groups

The Galois representation in $\pi_1(\mathbf{P}^1 - \{0,1,\infty\})$ gives a parametrization of $G_{\mathbf{Q}} = Gal(\bar{\mathbf{Q}}/\mathbf{Q})$ by a standard parameter $f : G_{\mathbf{Q}} \to \hat{F}_2$ $(\sigma \mapsto f_\sigma(x,y))$ valued in the free profinite group with free generators x, y together with the cyclotomic character $\chi : G_{\mathbf{Q}} \to \hat{\mathbf{Z}}^\times$.

Th.1: Let $\hat{B}_3$ be the profinite braid group generated by τ_1, τ_2 with the braid relation $\tau_1\tau_2\tau_1 = \tau_2\tau_1\tau_2$. Then the following relation holds:

$$(IV): \quad f_\sigma(\tau_1, \tau_2^4) = \tau_2^{8\rho_2(\sigma)} f_\sigma(\tau_1^2, \tau_2^2) \tau_1^{4\rho_2(\sigma)} (\tau_1\tau_2)^{-6\rho_2(\sigma)} \qquad (\sigma \in G_{\mathbf{Q}}).$$

Here, $\rho_2 : G_{\mathbf{Q}} \to \hat{\mathbf{Z}}(1)$ is the Kummer 1-cocycle w.r.t. the roots of 2.

The geometric fundamental group of the moduli stack of 1-pointed projective smooth curves of genus g is called the (profinite) Teichmüller modular group $\hat{\Gamma}_{g,1}$. It is generated by a standard system of Dehn-Lickorish-Humphries twist generators $a_1, \ldots, a_{2g}, d$ satisfying certain braid-relations, $(a_1 a_2 a_3)^4 = dd'$ and the lantern relation.

Th.2: There exists a suitable tangential base point on the moduli stack which gives the Galois action $G_{\mathbf{Q}} \to Aut\hat{\Gamma}_{g,1}$ such that

$$\sigma(a_i) = f_\sigma(w_i, a_i^2)^{-1} a_i^{\chi(\sigma)} f_\sigma(w_i, a_i^2), \quad \sigma(d) = d^{\chi(\sigma)}.$$

where $w_i = (a_1 \cdots a_{i-1})^i$.

Using Th.1, one can rewrite the Galois action of Th.2 in a more compatible way with pants-decomposition of the Riemann surface, leading also to a 'half' of Drinfeld's pentagon relation for the Grothendieck-Teichmüller group. (Cf. also talk by L.Schneps).

M. Jarden: Large-small normal algebraic fields

Let K be a number field. Denote the absolute Galois group of K by $G(K)$. For each $\sigma \in G(K)$ let $\tilde{K}(\sigma)$ be the fixed field of σ in $\tilde{K}$. Denote the maximal Galois extension of K which is contained in $\tilde{K}(\sigma)$ by $\tilde{K}[\sigma]$. Now choose $\sigma \in G(K)$ at random and let $N = \tilde{K}[\sigma]$. Then N has some properties of large algebraic fields:

- (1a) N is PAC;
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On the other hand N has some properties of small algebraic fields (i.e., of number fields). They are related to elliptic curves E without complex multiplication which are defined over N. To this end let l range over the set of prime numbers and let E_l (resp., E_{l^∞}) be the group of points of E annihilated by l (resp., a power of l). Then the following statements hold:

- (2a) $\mathcal{G}(N(E_{l^\infty})/N)$ is an open subgroup of $\mathrm{GL}(2, Z_l)$;
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- (2e) Isogeny theorem: Let E' be another elliptic curve ovoer N without complex multiplication. If $E_l \cong_{G(K)} E'_l$ for a set of prime numbers l of a positive Dirichlet density, then E and E' are isogenous.

Property (2) is proved in a joint work with Gerhard Frey.

F.Pop: Recent and new results in anabelian geometry

After presenting the philosophy of the so called 'anabelian geometry', the following results were discussed:
A) Birational type results: For a field k, and rational prime number $l \neq char(k)$, let k' be the maximal abelian pro-l-extension of $k[\mu_\infty]$. Set further $k'' = (k')'$ and $G''_k = Gal(k''/k)$.
Theorem [Pop]:

1. Global type results: Let K be a finitely generated field of absolute transcendence degree ≥ 2. Then K is functorially encoded in G''_K.
2. Local type results: Let k be a locally compact field, and K/k a function field of transcendence degree ≥ 1. Then K is functorially encoded in $G''_K \to G''_k$.

B) Anabelian curves:
Theorem [Tamagawa]: Every affine, hyperbolic curve X over a finitely generated field k is functorially encoded in $\pi_1(X)$ up to pure inseparable covers.

For very recent and very strong **results by Mochizuki** see these proceedings.
Finally, the section conjecture was mentioned, and some possible consequences for the arithmetic of rational points were discussed.

M. Raynaud: Covers of curves and moduli

Let P be the projective line over a field k. Pick four rational points a_i, $i = 1, \dots, 4$. Let $G = A_5$ be the alternate group acting on 5 elements.

How many curves of P, Galois with group G, are there with signature $(3, 3, 3, 3)$?
When k has $char$ 0, we can count them - there are 9 solutions.
Now suppose, $char(k) = 5$. Let E be the elliptic curve which is the double cover of P ramified at the a_i. If E is ordinary, there are 4 solutions. If E is supersingular, there are 3 solutions. Those numbers are obtained by a close study of bad reduction from $char$ 0 to $char$ 5.

S. Reiter: GAR-Realizations of classical groups of Lie type

We call a triple $(A, B, C), A, B, C \in GL_n(q), ABC = id, rk(A - id) = 1$ with $\langle A, B, C\rangle \leq GL_n(q)$ irreducible a Belyi-triple.
Theorem: For all $f, g \in \mathbb{F}_q[X]$ monic, $deg(f) = deg(g) = n$, $f(0)g(0) \neq 0$, $(f, g) = 1$, exits up to conjugation in $GL_n(q)$ one Belyi-triple s.t.

$$minpol(B) =: f_B = f \text{ and } f_{AB} = g.$$

(This was independently proved by H. Völklein.)

Using classification theorems (Wagner, Kantor and others) (see Kemper's talk) we can determine the group generated by a Belyi-triple. (E.g. let A be a transvection, $f_B = (X-1)^m(X-i)^m, f_{AB} = (X+1)^m(X+i)^m \in \mathbb{F}_p[X]$ for $p \equiv 5 \bmod 8, order(i) = 4$. Then $G/Z(G) \cong PCSp_{2m}(p)$ for $p \nmid m$.)

Using the rigidity criterion (Belyi, Matzat, Thompson) we find GA/GAR-realizations of classical groups of Lie-type (linear, unitary, orthogonal and symplectic groups) $G(p), p$ a prime, over $\mathbb{Q}$ under some conditions on the defining characteristic of these groups. (E.g: $PSp_{2m}(p)$ possesses GAR-realizations over $\mathbb{Q}$ for $(m,p) = 1$ and $p \not\equiv \pm 1 \bmod 24$.)

(The results for the linear groups have been already proved by Folkers and Malle.)

M. Saïdi: p-rank and semi-stable reduction of curves

Let R be a discrete complete valuation ring, with fraction field K, and algebraically closed residue field k of characteristic $p > 0$. Consider a germ $\mathcal{X} := Spec\, O_x$ of a stable R-curve at closed point x. Let $\mathcal{Y} \longrightarrow \mathcal{X}$ be a finite Galois cover of group G, a p-group, such that $\mathcal{Y}_K \longrightarrow \mathcal{X}_K = \mathcal{X} \times_R K$ is étale. Assume that $\mathcal{Y}$ admits a semi-stable model $\tilde{\phi} : \tilde{\mathcal{Y}} \longrightarrow \mathcal{Y}$ over R. If the point x is smooth, Raynaud proved that the p-rank of the fibre of a closed point of $\mathcal{Y}$ in $\tilde{\mathcal{Y}}$ equals zero. We consider the case where x is an ordinary double point. If the inertia subgroup H at a closed point y of $\mathcal{Y}$ is cyclic, we compute the p-rank of the fibre of y in $\tilde{\mathcal{Y}}$, in particular it is less than $\mid H \mid -1$.

A. Schmidt: Extensions of number fields with prescribed local behaviour

We investigate the existence of extensions of number fields which are are unramified outside a finite given set of primes S and with prescribed local behaviour at one or several primes in S. We resume several older and newer results, conjectures and counterexamples. Then we investigate the following situation:

Assume that K is a number field, S a finite set of places of K containing all archimedian primes and all primes dividing a given prime number p. We denote the maximal extension of K, which is unramified outside S, by K_S.

Theorem: The local fields $(K_S)_\mathfrak{p}$ associated to primes $\mathfrak{p}$ of K_S lying over a prime $p \in S$ are p-closed local fields.

Theorem: If S contains all primes dividing p then $G_S(K) = Gal(K_S/K)$ is a profinite duality group at p of dimension 2.

L. Schneps: The new version of the Grothendieck-Teichmüller group

We introduced a certain subgroup $\mathbb{I}\Gamma$ of the well-known Grothendieck-Teichmüller group $\widehat{GT}$ defined by Drinfeld. $\mathbb{I}\Gamma$ is obtained by adding two new relations to the definition of $\widehat{GT}$. It is known that $\widehat{GT}$ acts on the (algebraic) fundamental groups of moduli spaces of genus 0 curves with n marked points, respecting some natural homomorphisms between these groups. Furthermore, the absolute Galois group is included in $\widehat{GT}$ and its natural action on the fundamental groups of moduli spaces is compatible with this inclusion. Our purpose was to generalize these results to higher genus. Indeed, we can show that $\mathbb{\Gamma}$ an automorphism group of the (algebraic) fundamental groups of all moduli spaces of genus g curves with n marked points, and that the natural homomorphisms between these fundamental groups are respected by the action of $\mathbb{\Gamma}$; furthermore we again have an inclusion of the absolute Galois group into $\mathbb{\Gamma}$.

J-P. Serre: The Euler-Poincaré distribution of a profinite group

Let G be a profinite group and let p be a prime number. Let C_G be the category of finite dimensional $\mathbb{F}_p$-vector spaces with continuous action of G. Make the following finiteness assumptions:

- $\dim H^i(G, A) < \infty$ for every $A \in ob(C_G)$ and every $i \in \mathbb{Z}$.
- $cd_p(G) < \infty$.

Then, for every $A \in ob(C_G)$, the E-P characteristic $e(G, A)$ of A is defined by:

$$e(G, A) = \sum (-1)^i \dim H^i(G, A).$$

Let ϕ_A be the Brauer character of A. It is a locally constant map

$$\phi_A : G_{\text{reg}} \to \mathbb{Z}_p,$$

where G_{reg} is the subspace of G made up of the elements of (profinite) order prime to p.

The main result of the lecture is:

Theorem: There is a unique $\mathbb{Q}_p$-valued 'distribution' μ_G on G_{reg} such that:

1. For every $A \in ob(C_G)$, one has $e(G, A) = \langle \phi_A, \mu_G \rangle$.
2. μ_G is invariant by conjugation and by $s \mapsto s^p$.

If one defines $\mathbb{H}^i = \varinjlim H_c^i(U, \mathbb{Q}_p)$, where U runs through the open subgroups of G, and H_c^i is 'continuous cohomology', each $\mathbb{H}^i$ is in a natural way an admissible linear representation of G. Let μ_i denote the distribution-character of $\mathbb{H}^i$ (which is a distribution on G). Then:

Theorem: $\mu_G = \sum (-1)^i \mu_i$.

Various examples can be given, in which the μ_i, and hence μ_G, are computed explicitly:

- $G = Gal(\bar{K}/K), K$ finite extension of $\mathbb{Q}_p$. One finds that $\mu_G = -d \cdot \delta_1$, where $d = [K : \mathbb{Q}_p]$ and $\delta_1 =$ Dirac at 1.
- G is a p-adic Lie group without p-torsion. One finds $\mu_G = F\mu_{\text{Haar}}$, where $F(s) = det(1 - Ad(s^{-1}))$.

K. Stevenson: Fundamental groups of projective curves

Let D be a projective curve over a field $k = \bar{k}$ of characteristic $p > 0$. We would like to determine $\pi_1(D)$, the algebraic fundamental group, or at least its set $\pi_A(D)$ of finite quotients. Currently neither $\pi_1(D)$ nor $\pi_A(D)$ is known when the genus g of D is greater than or equal to 2. Moreover, there are no conjectures (even for a 'generic' curve D). We discuss the following two necessary conditions for a finite group G to lie in $\pi_A(D)$:

1. (Grothendieck) $G = \langle a_1, b_1, \ldots, a_g, b_g \rangle$ s.t. $\pi[a_i, b_i] = 1$
2. the p-rank of the maximal abelian p-quotient $\leq g$

For a prime to p group H, (1) is actually also sufficient, and for a p-group P, (2) is sufficient as long as D is a 'generic' curve of genus g. We consider groups of the form $G = P \rtimes H$ to show that (1) + (2) are not sufficient in general. Then we show that in the case that H can be generated by g elements, there is a necessary and sufficient condition for G to lie in π_A of a generic D. This condition involves comparing the H-module structure of P with that of $H^o(C, K_C)$ where $C \to D$ is an H- Galois cover. It is equivalent (in this case) to a condition of Nakajima (1984).

J. Swallow: Reduction of field of definition of Quaternion algebras and embedding problems

Let K be a field of characteristic not 2 and let $1 \to C_2 \to H \to G = Gal(L/K) \to 1$ be a Galois embedding problem. We consider the relationship of the subgroup embedding problems $1 \to C_2 \to M' \to M = Gal(L/K^M) \to 1$ and the full group embedding problem. More specifically, we examine the relationship between the class in $Br_2(K)$ which is the obstruction to the full embedding problem and the class of the obstruction to a subgroup embedding problem in $Br_2(K(\sqrt{d}\,))$.

We review results of Schneps and Kiming and outline a method for determining conditions over K for a Quaternion algebra over $K(\sqrt{d}\,)$ to be split. If the analogous problem for a tensor product of Quaternion algebras over $K(\sqrt{d}\,)$ could be solved, one would have a method for 'descent' of obstructions to embedding problems of 2-groups. We announce the result that, for $a, x, y, d, r \in F, x^2 + y^2 = d, A = (a, rd + ry\sqrt{d})_{F(\sqrt{d})}$ is split if and only if, first, $(a, d)_F$ is split, and if $a = c^2 - d$ modulo squares in F for $c \in F$ then additionally $(a, r(d - cy))(d, r)_F$ is split for some $r \in F$. The latter condition that $a \equiv c^2 - d \bmod F^{*2}$ must occur if a is not already a square in F.

A. Tamagawa: The fundamental groups of algebraic curves in characteristic > 0

- k : algebraically closed field of characteristic $p > 0$
- U : (smooth, connected) curve over k
- X : compactification of U, $g = \text{genus}(X), n = \sharp(X - U)$

Theorem: (g, n) is recovered group-theoretically from $\pi_1(U)$.
(i.e. $\pi_1(U) \simeq \pi_1(U') \Rightarrow (g, n) = (g', n')$)

Theorem: Assume $g = 0$ and either $k = \overline{\mathbb{F}_p}$ or $n \leq 4$. Then the isomorphism class of the profinite group $\pi_1(U)$ determines completely the isomorphism class of the scheme (not k-scheme) U.
(i.e. for such curves U, U', $\pi_1(U) \simeq \pi_1(U') \Leftrightarrow U \simeq U'$)

H. Tsunogai: The stable derivation algebra associated with genus one braid group

Let C be a once punctured elliptic curve over $\mathbb{Q}$ dand $\bar{C} = C \times_{\mathbb{Q}} \bar{\mathbb{Q}}$. We want to determine the image of $G_{\mathbb{Q}} = \text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ under the representation $\varphi_C^{\text{pro-}l} : G_{\mathbb{Q}} \rightarrow \text{Out}\pi_1(\bar{C})^{\text{pro-}l}$ and its Lie algebraization $\text{Lie}\varphi_C^{\text{pro-}l}$. For this purpose we consider the configuration space $C \times C \setminus \Delta$ of two points on C and its fundamental group, that is, the braid group $\pi_1(C \times C \setminus \Delta)$ of two strings on C. We have a natural homomorphism f : $\text{Out}\pi_1(C \times C \setminus \Delta)^{\text{pro-}l} \rightarrow \text{Out}\pi_1(\bar{C})^{\text{pro-}l}$, which commutes with the Galois representations $\varphi_C^{\text{pro-}l}$ and $\varphi_{C \times C \setminus \Delta}^{\text{pro-}l}$. By explicit Lie calculus about the Lie algebraization of f, we obtained the following:

Theorem: *The homomorphism f is NOT surjective.*

Since the image of $G_{\mathbb{Q}}$ under $\varphi_C^{\text{pro-}l}$ is contained in the image of f, this theorem gives a new constraint for the Galois image. Using this constraint, we determined the Galois image under $\text{Lie}\varphi_C^{\text{pro-}l}$ in degree ≤ 12 for a generic elliptic curve. This is the first step to the stability in the case of genus one.

N. Vila: Arithmetical-geometrical constructions of Galois groups

In this talk we consider constructions of Galois groups arising from Galois action of $G_{\mathbb{Q}}$ on some arithmetical-geometrical objects. I present joint work with A. Reverter. First, we summarize results concerning images of mod p Galois representations attached to elliptic curves. Our main concern is to compute the image of mod p Galois representation for each elliptic curve $E/\mathbb{Q}$ with conductor $N \leq 200$ and for each prime p. The images of mod p Galois representations attached to the product of two K-isogenous elliptic curves are also determined. Concerning Galois groups arising from mod p Galois representations attached to modular form we have obtained, using cusp forms of level 1, that the groups $PSL_2(p^r), r$ even, $PGL_2(p^r), r$ odd, $2 \leq r \leq 10$, are Galois groups over $\mathbb{Q}$, for infinitely many primes p. Explicit conditions on p are obtained. Using modular forms of level 23,29,31 and weight 2 eigenvector of the Hecke operators, and previous results we obtain that $PSL_2(\mathbb{F}_{p^2})$ are Galois groups over $\mathbb{Q}$, for all $p \leq 2069$.

H. Völklein: Cases of abelian braid group action and associated Hurwitz spaces

The rigidity criterion in the case of 3 branch points has been used extensively by Belyi, Thompson, Matzat's school and others to realize various classes of almost simple groups over $\mathbb{Q}$. The largest body of simple groups - those of classical Lie type- admit a quite natural class of rigid generators, the *Belyi triples.* They were used to obtain realizations over $\mathbb{Q}$ for various classical groups $G(p), p$ a prime, by Malle, Häfner, Folkers, Reiter and others.

Last year I introduced the *Thompson tuples*, which yield rigid generating systems of length $n+1$ of the groups $PGL_n(q)$ and $PU_n(q)$. This can be used to realize these groups over $\mathbb{Q}$ for n even, $n \geq q$. The Thompson tuples yield the only known rigid generating systems of length > 3 of any almost simple group.

In joint work with J. Thompson we found related generating systems of $Sp_n(q)$ of length $n/2+2$ which are not quite rigid, but very close: The pure braid group induces an abelian permutation group on inner classes of these tuples. The associated Hurwitz spaces are unirational over $\mathbb{Q}$ in may cases. This yields realizations of $PSp_n(q)$ over $\mathbb{Q}$ for q odd, $n \geq 4q^2$ and q a square. The latter condition can be replaced by various congruence conditions on q, too.

M. Zieve: The classification of non-affine exceptional polynomials

I discussed joint work with Bob Guralnick. For the first result, let f be a univariate polynomial over a perfect field k; we say that f is indecomposable over k if it is not the (functional) composition of two lower-degree polynomials over k. We examined when it could happen that f is indecomposable over k but decomposes over the algebraic closure of k. Building on work of Guralnick and Saxl, we showed this could only happen if the degree of f is either a power of char(k), or is 21 or 55. The latter two possibilities do occur, over fields of characteristic 7 and 11 respectively, and we classified all examples of these two degrees.

The bulk of the proof is the group-theoretic contribution of Guralnick and Saxl, but to classify the sporadic examples we needed a result showing that certain Galois extensions were determined by their ramification data. We also applied this latter result to another problem, namely the classification of non-affine indecomposable exceptional polynomials. These are indecomposable polynomials $f(x) \in k[x]$ for which $(f(x) - f(y))/(x - y)$ has no absolutely irreducible factors in $k[x, y]$, and such that $\mathrm{Gal}(f(x) - t, k(t))$ is not a group of affine permutations. Following work of Fried, Guralnick, and Saxl, such polynomials can only exist if k has characteristic 2 or 3; examples were produced by Mueller, Cohen and Matthews, and Lenstra and me. We have now exhibited further examples and shown that we have the complete list.

Berichterstattung: S. Reiter

The author of the report thanks all of the speakers for sending a LaTeXversion of their talk resp. revising their talk.

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