MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 02/1998

Global and Geometric Theory of Delay Differential Equations

11.01.–17.01.1998

The conference was organized by J. Mallet–Paret (Providence), R.D. Nussbaum (New Brunswick), and H.O. Walther (Gießen). Thirty–six participants from 11 countries (8 European including Russia; Israel; Canada; USA) took part. It was regretted that due to severe weather conditions and in other cases, lack of financial support several colleagues had to cancel their participation shortly before the meeting. Twenty–eight lectures were presented. One additional evening discussion was organized.
The largest group of lectures (8) focussed on the global dynamics of (ordinary) delay differential equations (DDEs). The topics included new existence results for periodic orbits and chaos, the structure of global attractors, singular perturbation, invariant manifold theory for state–dependent delays, and infinite frequency solutions.
A next group of 6 lectures discussed equations used as models in the life sciences, namely equations for the dynamics of blood cell populations and blood diseases, for neural networks, for competing species in a chemostat, and for mating problems.
Four lectures presented results on initial value problems for partial differential equations with time delays. One lecture discussed elliptic equations with transformed arguments, which arise in elasticity and nonlinear optics.
Another group of 4 lectures was devoted to linear DDEs and addressed, among others, problems of control theory (subtle effects of small delays, hybrid control) and in number theory.
In a set of 5 lectures DDEs in neighbouring fields related problems were discussed, notably stochastic DDEs, shadowing and numerical aspects, evaluating experiments by means of Conley index methods, and last not least, the embedding of given flows into center manifolds of parabolic partial differential equations and DDEs.

The meeting brought together a relatively large number of colleagues who had never met before. The research summaries posted close to the lecture hall seemed to help considerably to initialize an exchange of ideas.
We are grateful to the staff of the institute whose friendly and patient assistance contributed much to make the stay at the institute very pleasant.
The meeting ended on Friday, January 17, at 12:30 p.m.

Vortragsauszge

O. ARINO

Slowly oscillating periodic solutions of a state–dependent delay equation arising from a fishery problem

Joint work with K.P. Hadeler and My.L. Hbid

A possible motivation: a model describing the dynamics of a population of fish, divided into several stages, the passage from a stage to the next one being conditioned by a threshold mechanism. Simplified assumptions lead to the system

$$\frac{dx}{dt} = -f(x(t-\tau(t)), \frac{d\tau}{dt} = h(x(t), \tau(t)),$$

with $f : \mathbb{R} \to \mathbb{R}\ C^1, f(x)x > 0$ for $x \neq 0, 0 < \tau_1 \leq \tau(t) \leq \tau_2 < +\infty, h\ C^1, h(x,\tau_1) > 0$ for all $x, h(x,\tau_2) < 0$ for all $x, h(x,\tau) \leq 1$ for all x,τ. Two special cases lead in the singular limit to $\tau(t) = \tau^*$ (pure delay) and $\tau(t) = k_2(x(t))$ (usual state–dependent delay). We adapt the ejective fixed point method developed in the 70's for the logistic delay equation and some analogues to determine nontrivial slowly oscillating periodic solutions. We highlight the main ingredients:
1) the choice of a suitable notion of slowly oscillating solutions; 2) the determination of a suitable closed convex subset in the space of the (φ,τ)'s in which the trivial solution $(\tau^*, 0)(h(\tau^*, 0) = 0)$ is ejective.

H.J. DIAMOND

Analysis of a pair of linear difference differential equations

This is a report on work with H. Halberstam and H.-E. Richert on difference differential equations (DDE's) arising in sieve theory. Sieve theory is a branch of number theory that estimates the number of elements remaining in sequences of integers from which certain residue classes have been removed. Upper and lower bound estimates are expressed in terms of Cauchy–Euler type delay DDE's of the form $uf'(u) = af(u) + bf(u-1)$. Solutions are found with the aid of advanced argument adjoint DDE's. Specific problems discussed were properties of the AOS function family (DDE: $uj'(u) = kj(u) - kj(u-1), k$ a parameter) and the coupled system of DDE's for the author's sieve procedure.

T. FARIA

Stability and bifurcation for a delay Lotka–Volterra system without and with diffusion

A well–known Lotka–Volterra predator–prey system with one delay and a unique positive equilibrium E^* is considered, as well as a corresponding delayed reaction–diffusion system with Neumann conditions. The latter illustrates the use of the 'adjoint' theory for FDEs with diffusion when studying stability problems. However, the theory described in [Lin, So & Wu, Proc. Roy. Soc. Ed., 1992] was improved and addressed here in a more general framework, in order to include situations where the operator giving the linear terms of an FDE with diffusion mixes the modes of the Laplacian. In fact, this is the case of the proposed model with diffusion. In this setting, stability results for E^* as an equilibrium of the reaction–diffusion system are deduced from the case without diffusion, already known. Namely, it is shown that a Hopf bifurcation occurs at E^* as the delay τ passes through a critical value τ_0.

Using the normal form theory for FDEs in [Faria & Magalhães, JDE, 1995] and for FDEs with diffusion in [Faria, to appear in Trans. AMS], the Hopf bifurcation occuring at $E^*, \tau = \tau_0$ is described for a particular example. By computing normal forms on the stable local centre–manifold, it is proven that the Hopf bifurcation is supercritical and that the non–trivial solutions are stable, in both cases with and without diffusion.

W.E. FITZGIBBON

Periodicity in diffusive epidemic models

Joint work with J.J. Morgan and M.E. Parrott

Our long range concern is developing a framework for analysing the effects of temporal variations on the spread of disease through a dispersing population. In the case at hand, we guarantee the existence of periodic solutions arising in response to periodic forcing. We assume that the population is dispersing by means of random Brownian motion and that this dispersion is approximated in the standard way using diffusion operators. The disease features a period of latency or incubation during which the individuals neither manifest the symptoms of the disease nor are capable of infecting other individuals. This period of latency is immediately followed by a fully infected period. Susceptible individuals contract the disease via interaction with fully infected individuals. This interaction is modelled by a 'mass action' term which is proportional to the product of the spatial densities of the susceptible and the fully infected individuals which appears as loss term for the susceptible class. The stages of the disease are specified via the introduction of an independent variable 'a' which represents the time elapsed since an individual contracted the disease. These considerations produce a distributed parameter system coupling a semilinear parabolic equation with a diffusive age transport equation.

We also obtain partial results describing how the periodic solutions drive the overall dynamics of the system.

F. GIANNAKOPOULOS

Generation of epileptiform activity in a neural network

Joint work with H. Luhmann, U. Bihler, Ch. Hauptmann

We were interested in modelling the generation of experimentally induced epileptiform signals in neocortical slices. Experimental data indicate that suppression of inhibition and enhancement of excitation leads to the expression of spontaneous and stimuli–induced signals. In our model we consider a network with m neurons. For each neuron the suggested mathematical model consists of the network equation and in intrinsic oscillator. The network equation describes the transmission of signals. As intrinsic oscillator we use the FitzHugh–Nagumo system which models the impulse generation at the axon–hillock. The resulting system consists of $3m$ coupled delay differential equations. Mathematical analysis and computer simulation indicate that both, intrinsic and synaptic mechanisms are involved in generating epileptiform activity.

K.P. HADELER

Delay equations and conservation laws

It is well known that certain hyperbolic systems of partial differential equations can be 'reduced' to systems of ordinary differential equations or delay equations. In the delay case the reduced system is valid only after a transient time interval. Although such reductions have been used frequently, there are several open problems:
i) What is the underlying general principle? ii) What is the proper relation between the state space of the partial differential equation and that of the delay equation? iii) If the partial differential equations preserves positivity, in what sense does the reduced system preserve positivity? To some of these questions preliminary answers can be given. The main applications are structured population models. In particular, an interpretation is given for population models in the form of neutral differential equations.

U. AN DER HEIDEN

Bifurcations and chaos in $d^2x(t)dt^2 + x(t) = f(x(t-\tau))$ with non-smooth f

Substantial analysis of the above non–linear differential–delay equation can be done if the nonlinearity f is assumed to be piecewise constant. Applications may be found in control systems with discrete control (e.g. machine, current, or light either 'on' or 'off'; or either 'paying attention' or 'not paying attention'). Trajectories of the associated system

$$dx(t)/dt = y(t), dy(t)/dt = f(x(t-\tau)) - x(t)$$

can then be represented in the $x-y$–plane by continuous curves $t \to (x(t), y(t))$ which are piecewise composed of arcs of circles. The circles have centers at $(c, 0), f$ discontinuous at c.

In case of negative feedback, where (without loss of generality) $f(\xi) = 1/2$ for $\xi \leq \Theta, f(\xi) = -1/2$ for $\xi > \Theta, \Theta$ some constant, the following theorems can be proved:

Theorem 1 *Let $\Theta \in [0, 1[2]$. For each $n \in \mathbb{N}$ and for each $\tau \in (0, 2n\pi)$ there exists a periodic orbit with minimal period τ/n.*

Theorem 2 *Let $\Theta \in [0, 1/2]$, For each $n \in \mathbb{N}, n$ odd, and for each $\tau \in (n\pi, 2n\pi]$ there exist periodic orbits with minimal period $2\tau/n$.*

Proofs of these results may be found in the paper of Wolf Bayer and Uwe an der Heiden, *Oscillation types and bifurcations of a nonlinear second order differential–difference equation*, appearing 1998 in the *J. Dynamics & Differential Eqs.*. There it is also shown how the solutions of these theorems bifurcate from each other and how they exhibit multistability.

Together with Wolf Bayer a proof was also obtained for the existence of *chaos* in case that the feedback–function f has two discontinuities:

Theorem 3 *Let Θ be a positive constant. Let $f(\xi) = 1/2$ for $\xi \in [0, \Theta], f(\xi) = -1/2$ otherwise. Then there is an open set in the $\tau - \Theta$–parameter space such that for each element in this set System (*) has infinitely (countably) many periodic orbits of different minimal period and uncountably many asymptotically aperiodic orbits such that for any two of these aperiodic solutions (x_1, y_1) and (x_2, y_2),*

$$\lim_{t\to\infty} \inf ||(x_1(t), y_1(t)) - (x_2(t), y_2(t))|| = 0$$

and

$$\lim_{t\to\infty} \sup ||(x_1(t), y_1(t)) - (x_2(t), y_2(t))|| > 0.$$

The proof relies on constructing a Poincaré–map on a one–dimensional subset of the state space and reducing the result to a snap back repeller–property of a certain difference equation.

A.V.M. HERZ

Global theory of pulse–coupled oscillators without and with delays

Pulse–coupled oscillators exhibit a wide variety of collective non–equilibrium phenomena. Global synchronization with temporal periodicity is beautifully seen in the pulsed light patterns generated by colonies of flashing fireflies. More complex phenomena include periodic oscillations without global synchrony, spatial synchronization without temporal periodicity, and intermittent locking processes which in some models are accompanied by self–organized criticality.
Although the approach to a stationary state is slow in many of these systems, approximately periodic oscillations are often achieved within a few cycles of activity. This phenomenon might be important for rapid information processing in neural systems. For a class of idealized models with zero pulse width and no signal delays, it is proved that strictly periodic solutions are reached as soon as every element has been active once. For systems with delays and/or nonzero pulse width, a Lyapunov functional shows that periodic solutions are reached asymptotically. If there are discrete delays only, application of the same functional proves that transients have finite duration.

W. HUANG

Heteroclinic orbits of delay differential equations and applications to the singular perturbation problem

The existence of heteroclinic orbits of delay differential equations has long been a very interesting problem in the area of the functional differential equations. One of the important applications of heteroclinic orbits arises from the investigation of the existence of the square–wave–like periodic solution for a class of singularly perturbed delay differential equations that occur as models for some nonlinear optical problems and biological problems, where heteroclinic orbits serve as the transition layers for this type of periodic solutions.
In this report, we present a general existence and uniqueness result of heteroclinic orbits for a class of systems of delay differential equations. Our main approach is the homotopy method of [Chow, Lin and Mallet–Paret, 1989]. Monotone iteration and properties of positive operators in Banach space have been used to obtain detailed information about the heteroclinic orbits and the spectral properties of its linearization. The system under investigation takes the form

$$\dot{x}(t) = F(x(t), x(t-r))$$

where the time delay $r > 0$ is a constant and the nonlinear function $F = (F_1, ..., F_n) : \Omega \times \Omega \to \mathbb{R}^n (\Omega \subseteq \mathbb{R}^n)$ is assumed to satisfy the following type of monotonicity property: For $x = (x_1, ..., x_n) \in \Omega$ and $y = (y_1, ..., y_n) \in \Omega$,

$$
\begin{aligned}
\frac{\partial F_i(x,y)}{\partial x_j} &\leq 0, \quad \text{for} \quad i \neq j, i,j = 1,2,...,n, \\
\frac{\partial F_i(x,y)}{\partial y_j} &\leq 0 \quad \text{for} \quad i,j = 1,2,...,n.
\end{aligned}
$$

A.F. IVANOV

On the discretization of a delay differential equation

A delay difference equation (1) of the form

$$\mu[\Delta x_n + a\Delta x_{n-N}] = -x_{x+1} + f(x_{n-N})$$

where $\mu > 0, \Delta x_n := x_{n+1} + x_n, f : \mathbb{R} \to \mathbb{R}$ is a continuous function, is considered. Equation (1) can be looked upon as a discrete analogue of

$$\varepsilon[\dot{x}(t) + a\dot{x}(t-1)] = -x(t) + f(x(t-1));$$

it also appears as an Euler discretization of the latter (in a slightly different form). A relationship between dynamics of solutions of eq. (1) and that defined by the limiting case $\varepsilon = 0$, the interval map f, is established. It is shown in particular that hyperbolic attracting cycles of the map f give rise to asymptotically stable periodic solutions of equation (1) for sufficiently small μ.

P. KLOEDEN

Bishadowing in neutral delay equations

Bishadowing combines direct shadowing, where every pseudo–trajectory is shadowed by a true trajectory, with indirect shadowing, where every true trajectory is shadowed by a pseudo–trajectory – but here the pseudo–trajectories are true trajectories of a family of functions, e.g. continuous functions. Hyperbolicity implies bishadowing, but so does a weaker version called semi–hyperbolicity in which the maps need only be Lipschitz, the tangent space splitting is not continuous or invariant, nor is the set under consideration invariant, e.g. consider an open set containing a saddle point. Both concepts were introduced and explored by P. Diamond, P. Kloeden, V. Kozyakin and A. Pokrovskii. Here work of A. A.-Nayef, Kloeden and Pokrovskii is reported - bishadowing results are applied to nonsmooth nonlinear perturbations of a linear neutral delay differential equation, where locally condensing mappings are used as the family of comparison mappings in bishadowing, which is applied to the respective shift operators in $C^1([-h, 0])$. These results have been published in the Journal of Differential Equations, July 1997.

H.P. KRISHNAN

A phase space formulation for an equation with a state–dependent delay

We provide a complete phase-space description for the equation (1) $\dot{x}(t) = f(x(t), x(t-r)), r = r(x(t))$. In particular, for any $\varphi \in W^{1,\infty}([-r^*, 0])$, where $r^* = \sup_\xi r(\xi)$, we show that there exists $\alpha > 0$, dependent on $\|\varphi\|^\infty$ but otherwise independent of φ, such that $x_t(\varphi, \cdot)$ exists and is unique on $[0, \alpha]$. We then show that the semiflow $T(t) : W^{1,\infty}([-r^*, 0]) \to W^{1,\infty}([-r^*, 0])$ is not smooth in φ by direct analysis. It is, however, possible to differentiate T in φ relative to certain manifolds. In particular, near the point 0, we consider the linear equation (2) $\dot{x}(t) = f_x(0,0)x(t) + f_y(0,0)x(t-r(0))$. If 0 is hyperbolic, we show that the unstable subspace U associated with (2) is locally diffeomorphic to the unstable set of (1). We conclude by establishing Lyapunov–type stability criteria for the equation (1) and as a specific example, prove that all solutions to the equation (3) $\dot{x}(t) = -\alpha x(t) - \beta x(t-r), r = r(x(t))$, are ultimately uniformly bounded whenever $r_+ r_- < \frac{1}{\beta^2}$. Here $r_+ = \lim_{\xi\to+\infty} r(\xi)$ and $r_- = \lim_{\xi\to+\infty} r(\xi]$.

T. KRISZTIN, J. WU

The structure of an attracting set defined by delayed and monotone positive feedback I-II

Joint work with H.O. Walther

The delay differential equation

$$\dot{x}(t) = -\mu x(t) + f(x(t-1))$$

with $\mu \geq 0$ and a real function f satisfying $f(0) = 0$ and $f' > 0$ models a system governed by delayed positive feedback with respect to the rest state given by $\xi = 0$, and instantaneous damping. Applications arise in neural networks. For certain μ and f unbounded solutions exclude a compact global attractor of the semiflow defined by the equation on the phase space C of continuous real functions on the initial interval $[-1, 0]$. Under mild additional assumptions we study the simplest nontrivial substitute of a compact global attractor, which is the closure of the forward extension W of a 3–dimensional C^1–submanifold of the local unstable manifold at the stationary point $0 \in C$. Among others we prove that $\overline{W}$ is a smooth solid spindle for $\mu > 0$, and a smooth solid cylinder for $\mu = 0$. A smooth disk bordered by a periodic orbit separates $\overline{W}$ into halves formed by solutions which converge to one of the tips of the spindle in case $\mu > 0$, and to ∞ or $-\infty$ if $\mu = 0$.
In case $\mu > 0$ and f bounded the semiflow has a global attractor A, and $\overline{W} \subset A$. We conjecture that under certain conditions on $\mu > 0$ and f, $\overline{W} = A$. The proof of $\overline{W} = A$ requires, among others, a new uniqueness result for periodic solutions. We present such a result for odd nonlinearities f.

Some of the aforementioned results have been recently extended, in joint work with Y. Chen, to the system

$$\dot{x}(t) = -\mu x(t) + f(y(t-1)), \quad \dot{y}(t) = -\mu y(t) + f(x(t-1))$$

with special emphasis on the existence and attractivity of a phase–locked oscillations.

E. LITSYN

Stabilization of linear differential systems via hybrid feedback controls

We study so–called 'hybrid feedback stabilizers' for an arbitrary general system of linear differential equations. We prove that under assumptions of controllability and observability there exist a hybrid feedback output control which makes the system asymptotically stable. The control is designed by making use of a discrete automaton implanted into the system's dynamics. In general, the automaton has infinitely many locations, but it gives rise to a 'uniform' (in some sense) feedback control. The approach we propose goes back to the classical feedback control technique combined with some ideas used in the stability theory for equations with time–delay.

Y. LIU

Functional differential equations with proportional delays

Functional differential equations with proportional delays arise in applications such as collection of current by the pantograph head of an electric locomotive, wave propagation problems, probability theory on algebraic structures, absorption of light by interstellar matter, spectral theory of the Schrdinger operator, coherent states of the deformed oscillator algebra in quantum mechanics, and many others.
In general, initial value problems for neutral functional differential equations with a single proportional delay can be written as

$$y'(t) = f(t, y(t), y(qt), y'(t)), t \geq 0, \quad y(0) = y_0, y'(0) = y_1,$$

where f is a continuous function, $q \in (0,1)$ is a rescaling parameter, and $y_0, y_1 \in \mathbf{R}$ are given initial values that satisfy the consistence condition $y_1 = f(0, y_0, y_0, y_1)$. In order to guarantee the existence and uniqueness of a differentiable solution, it is often assumed, among others, that the Lipschitz constant of $f(t, y, z, \cdot)$ is less than one in a neighbourhood of $(0, y_0, y_0, y_1)$ (see, e.g. Nussbaum (1972)). A recent result of Feldstein and Liu (1989) shows that the following initial value problem

$$y'(t) = ay(t) + by(qt) + cy'(qt), t \geq 0, \quad y(0) = y_0$$

has infinitely many linearly independent solutions if $|c| > 1$.
Equations with proportional delays can differ vastly from equations with constant delays. For instance, Iserles and Liu (1997) proved that if $\sum_{i=1}^{M} |c_i| < 1$, $\text{Re } a \le 0$ and $a \ne 0$ then the asymptotic behaviour of the solution of

$$y'(t) = ay(t) + \sum_{i=1}^{M}[b_i y(q_i t) + c_i y'(p_i t)], t > 0, \quad y(0) = y_0,$$

depends mainly on the distribution of zeros of the algebraic equation $a + \sum_{i=1}^{M} b_i q_i^z = 0, z \in \mathbb{C}$. The neutral terms (or the coefficients $c_1, ..., c_M$) have little influence on the stability of the trivial solution.
There are many open problems related to equations with proportional delays. For instance, it is conjectured by Morris, Feldstein, and Bowen (1972) that

$$\lim_{n\to\infty} t_{n+1}/t_n = 1/q,$$

where $q \in (0,1)$ is a parameter, $0 < t_1 < t_2 < ...$ are zeros of the entire function $\sum_{n=0}^{\infty}(-t)^n q^{n(n-1)/2}/n!$, the only continuous solution of (†). One of the conjectures made in the preprint *On a functional Riccati equation*, Cambridge University Tech. Rep. DAMTP, 1996/NA19, states that if $\mu > 0$ then the solution of the functional–Riccati equation

$$y'(t) + q^2 y'(qt) + y^2(t) - q^2 y^2(qt) = \mu, \quad y(0) = y_0$$

obeys

$$\lim_{t\to\infty} y(t) = \sqrt{\mu/(1-q^2)} \quad \text{if} \quad y_0 > -\sqrt{\mu/(1-q^2)},$$

$$\lim_{t\to T-0} y(t) = -\infty \quad \text{for some} \quad T > 0 \quad \text{if} \quad y_0 < -\sqrt{\mu/(1-q^2)}.$$

M. MACKEY

Delay equations and the control of cell replication

Dynamic hematological diseases in which circulating numbers of white blood cells, red blood cells, and/or platelets show a periodic variation in time offer an ideal situation in which to study the dynamics of cellular replication. Cellular replication and its control involve significant delays due to cell cycle and maturation times typically on the order of days. Cyclical neutropenia (cn) is a periodic disease in which all three major celltypes display periodicities of abour 19-25 days in humans and 9-14 days in the grey collie. A model study framed as a nonlinear integro–differential equation demonstrates that this disease cannot be due to a loss of stability in the control of neutrophil production and release. Rather, it is shown that an elevation of cellular death rates in the HSC (which provides differentiating cells from all three cell types) is sufficient to explain the major characteristics of cn including the difference in periods between humans and the grey collies, and the response of both to treatment with granulocyte colony stimulating factor.

J. MAHAFFY

A mathematical model for erythropoiesis

An age–structured model for erythropoiesis is developed and compared to data following a blood donation. The model examines the negative feedback of the hormone erythropoietin on production of new erythrocytes. The age–structured model for the erythrocytes includes a precursor population and a mature population. The latter are actively destroyed, which results in a moving boundary condition. A plasma function is included to simulate blood loss, and the resulting system is compared to experimental data following a phlebotomy. Certain assumptions allow the model to be reduced to a state–dependent delay system, for which a bifurcation analysis is performed.

K. MISCHAIKOW

From time series to symbolic dynamics

A new approach to constructing a dynamical systems model from experimental time series is presented. Using the ideas of delay reconstruction a multivalued dynamical system is constructed. The multivalued approach is taken to account for bounded experimental error. This constructed system is then analysed and algebraic invariants based on the Conley Index theory are computed. These invariants can be lifted back to the unknown physical system and have implications concerning the dynamics which must occur, e.g. symbolic dynamics. It is also argued that these methods lend themselves to potentially rigorous numerical analysis of high dimensional dynamical systems such as delay equations.

H. PETZELKOVA

Compactness and long time behaviour of solutions to conservation laws with memory

Compactness of a set of bounded entropy solutions to the equation

$$\frac{\partial}{\partial t}\int_{-\infty}^{t} K(t-s)u(s)ds + \sum_{i=1}^{n} a_i(u)\frac{\partial u}{\partial x_i} = 0$$

is discussed for the following choices of K:
(i) $K = \delta_0$ – standard conservation law
(ii) $K = \delta_0 + k, k \in L^1$, nonincreasing – conservation law with memory
(iii) $K \sim t^{-\alpha}$ at $0, 0 < \alpha < 1$ – fractional conservation law
A nondegeneracy condition introduced by Lions, Perthame and Tadmov for (i) gives

compactness also for the case (ii) and the compact set depends on L^∞–bounds for solutions and L^1–bounds for k only. This condition can be weakened in the case (iii). Compactness of bounded solutions is used in the proof of convergence of spatially periodic solutions to mean values of their initial data. The same result holds for the linear case (a_i constant) when the memory effect comes into play (i.e. in (ii) and (iii)) provided that the coefficients are rationally independent).

S. RUAN

Periodic solutions of planar systems with two delays

Consider a planar system with two delays

$$\begin{aligned} \dot{x}_1(t) &= a_0 x_1(t) + a_1 F_1(x_1(t-\tau_1), x_2(t-\tau_2)) \\ \dot{x}_2(t) &= -b_0 x_2(t) + b_1 F_2(x_1(t-\tau_1), x_2(t-\tau_2)) \end{aligned}$$

where $a_0 > 0, b_0 > 0, \tau_1 > 0, \tau_2 > 0, a_1$ and b_1 are constants, F_1 and F_2 satisfy

$$F_j \in C^3(R^2), F_j(0,0) = 0, \frac{\partial F_j}{\partial x_j}(0,0) = 0 \text{ for } j \in \{1,2\}, \frac{\partial F_1}{\partial x_2}(0,0) \neq 0, \frac{\partial F_2}{\partial x_1}(0,0) \neq 0,$$

$$x_2 F_1(x_1, x_2) \neq 0 \quad \text{for} \quad x_2 \neq 0, x_1 F_2(x_1, x_2) \neq 0 \quad \text{for} \quad x_1 \neq 0$$

When $a_0 = b_0 = 1, a_1 = b_1 = \alpha, \tau_1 = \tau_2 = 1$, the global existence of periodic solutions to the system has been studied by Táboas (Proc. Royal Soc. Edinburgh, 1990) and Baptistini and Táboas (J. Diff. Eqns., 1996). The method used by Táboas came from a well-known idea due to Jones (J. Math. Anal. Appl., 1962) together with a theorem of Nussbaum (Ann. Math. Pura Appl., 1974) on the ejectivity of fixed points. Another approach of studying the global existence of periodic solutions to delay systems is the degree theory method, c.f., Chow and Mallet–Paret (J. Diff. Eqns., 1978) and Erbe et al (J. Diff. Eqns., 1992).
We first carry out the local Hopf bifurcation analysis of the above system. By choosing one of the coefficients as parameter, the local stability domain is found and Hopf bifurcation values are determined. Then by using a global Hopf bifurcation theorem of Wu (1996), we show that the system has global nonconstant periodic solutions. Finally, as an example, we analyze a neural network model with two delays.

W.M. RUESS

Linearized stability for partial delay differential equations

The object of the talk is a principle of linearized stability for partial functional differential equations with delay of the form

$$(FDE) \quad \begin{cases} \dot{x}(t) + Bx(t) = F(x_t), t \geq 0 \\ x_{|I} = \varphi \in \hat{E}, \end{cases}$$

with B a generator of a strongly continuous semigroup in a Banach (state) space X, and, for given $I = [-r, 0], r > 0$ (finite delay), or $I = \mathbb{R}^-$ (infinite delay), and, for $t \geq 0, x_t : I \to X, x_t(s) = x(t+s), s \in I$, the history of x up to t, and $\varphi : I \to X$ a given initial history out of a subset $\hat{E}$ of a space E of functions from I to X.
While in previous works the results were restricted to
(a) the history–responsive operator F to be (globally defined and) continuously Fréchet–differentiable with
(b) locally Lipschitzian derivatives,
we shall remove these restrictions and show that the corresponding results hold in the general context of
(c) B possibly nonlinear and F allowed to be defined on 'thin' subsets $\hat{E}$ of initial histories (adapted to the respective problem – such as nonnegative functions for population and biochemical models), and
(d) Fréchet–differentiability for both B and F required only at the equilibrium point.
Applications of the general principle to reaction–diffusion equations with memory and age–dependent population dynamics will be given.

K. RYBAKOWSKI

Chaotic dynamics of parabolic PDEs and delay equations

We prove the following
Theorem (Prizzi & Rybakowski, JDE, to appear) *Suppose $N \geq 2$. Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with smooth boundary. There is an analytic function $a : \mathbb{R}^N \to \mathbb{R}$ satisfying the following property: For every $m \in \mathbb{N}$ there is an $\varepsilon_m > 0$ such that whenever $u : \mathbb{R}^{N+1} \to \mathbb{R}^{N+1}$ is a vector field of class C_b^m with*

$$|u|_{C_b^m} < \varepsilon_m$$

then u can be realized on the center manifold of the reaction–diffusion–convection equation

$$\begin{cases} \partial_t u = \Delta u + a(x)u + f(x, u, \nabla u) & , \quad x \in \Omega \\ u(x,t) = 0 & , \quad x \in \partial\Omega \end{cases}$$

for an appropriate choice of the nonlinearity $f : \overline{\Omega} \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ of class C^m.
This result generalizes (to arbitrary domains) an earlier result of Rybakowski (JDE, 1994)

and Poláčik & Rybakowski (Annali Scuola Norm. Sup. Pisa, 1995) proved for some special domains, like $\Omega = B_1(0) \subset \mathbb{R}^N$. We also report on some recent vector field realization result for scalar delay–equations.

Theorem (Rybakowski, JDE, 1994) *Let* $C := C([-r, 0], \mathbb{R})$ *and* $L : C \to \mathbb{R}$ *be linear and bounded. Suppose that the linear FDE*

$$\dot{y} = Ly_t$$

has exactly N *eigenvalues (counting multiplicity) with real part zero. Then for* $f : C \to \mathbb{R}$ *with* $f(0) = 0$ *and* $|Df|$ *globally small there exists a center–manifold embedding*

$$\wedge = \wedge_f : \mathbb{R}^N \to C$$

such that the FDE

$$\dot{y} = Ly_t + f(y_t)$$

restricted to the center manifold $M = \wedge(\mathbb{R}^N)$ *takes the form*

$$\dot{z}_1 = z_2, ..., \dot{z}_{N-1} = z_n, \dot{z}_N = -a_1 z_1 - ... - a_N z_N + f(\wedge_f(z)), \quad z \in \mathbb{R}^N$$

The function $v(z) = f(\wedge_f(z)), z \in \mathbb{R}^N$, *is arbitrary, in some sense: For every* $v : \mathbb{R}^N \to \mathbb{R}$ *sufficiently smooth and small there exists* $f : C \to \mathbb{R}$ *of the form*

$$f(\varphi) = g(\varphi(0), \varphi(-r_1), ..., \varphi(-r_{N-1})), \varphi \in C$$

such that

$$f(\wedge_f(z)) = v(z), \quad z \in \mathbb{R}^N.$$

Here, the constants $a_1, ..., a_N \in \mathbb{R}$ *and the delays* $0 < r_1 < ... < r_{N-1} < r$ *depend only on the linear map* L.

This vector field realization result strengthens a previous jet realization result of Hale (Proc. Roy. Soc. Edinb., 1985). An analogous result can also be proved for systems, strengthening a jet realization result of Faria & Magalhães (Proc. Roy. Soc. Edinb., 1995).

M. SCHEUTZOW

Stability properties of stochastic delay differential equations

We discuss a number of different stability concepts for various kinds of stochastic functional differential equations (SFDE's). For equations which possess a constant solution, say 0, stability can be defined as local or global almost sure asymptotic stability or in terms of stability of moments. For equations subjected to additive white noise – which can never have a constant solution – it seems reasonable to define stability in terms of the existence of a stationary solution. If an SFDE admits a continuous stochastic solution semiflow (which not all SFDE's do) and if the equation has a stationary solution, then one can call that solution 'stable' if the top Lyapunov exponent of the linearization of the semiflow around the stationary solution is negative. Finally we discuss the stability of the model equation $dX(t) = \sigma \cdot X(t-1) \cdot dW(t)$, where $W(t), t \geq 0$ is Brownian motion. It turns out that the zero solution is almost surely globally exponentially stable for small σ and unstable for large σ. Most of the results are joint work with Salah E. Mohammed (SIU).

E. SHUSTIN

Infinite frequency oscillations in a discontinuous dynamical system with time delay

Joint wirk with R.D. Nussbaum

We study the equation

$$\dot{x}(t) = -\text{ sign } x(t-1) + F(x(t)), \quad t \geq 0,$$

with a continuous function $F : \mathbb{R} \to \mathbb{R}$ satisfying $|F(x)| \leq p < 1$, which is a model of an autonomous system with a retarded relay control element. The Cauchy problem $x(t) = \varphi(t), t \in [-1, 0]$, has a unique continuous solution x_φ for any $\varphi \in C[-1, 0]$. All these solutions oscillate around the zero level, and the *frequency function*

$$\nu_\varphi(t) = \text{ card } (x_\varphi^{-1}(0) \cap (t^* - 1, t^*)), \quad t^* = \max\{\tau \leq t : x_\varphi(\tau) = 0\}.$$

is *non–increasing*. Hence there always exists the *limit frequency*

$$N_\varphi = \lim_{t\to\infty} \nu_\varphi(t) \in \mathbb{N} \cup \{0\} \cup \{\infty\}.$$

In particular, once the frequency becomes finite, it will be finite further.
Properties of solutions with a finite limit frequency are basically known. We answer the question on the existence of solutions with *infinite limit frequency*, and on a possible lenght of the interval with infinite frequency of oscillations.
Theorem 1 *There are no solutions with infinite limit frequency, except for the case* $F(0) = 0, x(t) \equiv 0, t \geq 0$.
Theorem 2 *For any* $\varepsilon > 0$ *there exists* $T_\varepsilon > 0$ *(depending on F) such that*

$$\nu_\varphi(t) < \infty, \quad t > T_\varepsilon,$$

as far as

$$\max\{\text{length}(I) : I \in \pi_0([-1, 0] \setminus \varphi^{-1}(0)\} \geq \varepsilon.$$

If, in addition, $F(0) = 0$ *and F is differentiable at zero, then*

$$T_\varepsilon \leq C(\delta) \cdot (\frac{1}{\varepsilon})^{2+\delta}$$

with arbitrary positive δ and $C(\delta)$ depending only on δ and F.

A.L. SKUBACHEVSKII

Global properties of elliptic functional differential equations

We consider boundary value problems for stronlgy elliptic differential–difference equations. An equation is said to be stronlgy elliptic if it satisfies the Gårding inequality. It were obtained the necessary and sufficient conditions for strong ellipticity in algebraic form. Unlike elliptic differential equations, smoothness of generalized solutions of elliptic differential–difference equations can be broken inside a domain $Q \subset \mathbf{R}^n$ and preserves only in some subdomains $Q_r(\bigcup_r \overline{Q}_r = \overline{Q})$. It were stated results on the smoothness of generalized solutions near the boundaries of subdomains Q_r and in the neighborhood of angular points of the boundaries ∂Q_r. We obtain the asymptotics formula for eigenvalues of the corresponding strongly elliptic differential–difference operator.

S. VERDUYN LUNEL

Effects of small delays on stability and control

Joint work with J.K. Hale

Stabilization and control of partial differential equations through the application of forces on the boundary turns out to be very important. When the boundary forces are applied with no delays there is a rather complete theory. In applications, however, it is very likely that time delays will occur when applying the boundary forces. So it is of vital importance to understand the sensitivity with respect to small delays. We shall present a unifying framework and we shall explain the mechanism behind the phenomena observed in the literature. To illustrate the results we shall present four examples.

G. WOLKOWICZ

Joint work with H. Xia, S. Ruan, J. Wu

Delayed response in growth in models of the chemostat

The predictions of various models of competition in the chemostat that involve time delay (discrete or distributed) to model the lag in the conversion of nutrient to biomass were discussed. All of the models considered predict that the principle of competitive exclusion holds and that it is possible to predict which population avoids extinction based on the relative values of a parameter that represents a generalization of the break–even concentration of the ODE model, but that is a function of parameters describing the delay and that neglecting delay can lead to incorrect predictions. Including the delay may also help to explain some of the discrepancies between experimental data (e.g. Hansen and Hubbell 1980) and the simulations of the ODE model. In particular, in work in progress with H. Xia, considering the single species discrete delay model, although the unique positive equilibrium is globally attracting with respect to positive initial data, using local and global Hopf bifurcation theorems, it follows that unstable (rapidly oscillating) periodic solutions that bifurcate from unstable nonnegative equilibria are possible. Although these periodic solutions change sign, their existence may help to explain the transient oscillations seen in experiments, but not in simulations of the ODE model.

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