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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 12/1998

Applications and Computation of Orthogonal Polynomials

22.03.–28.03.1998

The meeting was organized by Walter Gautschi (West Lafayette), Gene H. Golub (Stanford), and Gerhard Opfer (Hamburg). There were 46 participants from 13 countries, more than half coming from Germany and the United States, and a substantial number from Italy. A total of 23 plenary lectures were presented and 4 short informal lectures. An evening session was also held dealing with open problems. Problems were posed by Laura Gori, Arieh Iserles, Dirk Laurie, Hrushikesh Mhaskar, and Ed Saff. Wednesday afternoon was set aside for an excursion and a social gathering at the restaurant "Zum Ochsen" in Schapbach.

The principal scientific areas of attention were:

(a) Computational aspects, including numerical and symbolic computation, matrix interpretation of relevant algorithms, and convergence, perturbation, and stability analyses;

(b) Generalizations of ordinary orthogonal polynomials, such as s-orthogonal, matrix- and tensor-valued, Müntz-type, and complex orthogonal polynomials;

(c) Applications to problems in applied mathematics and engineering; prominent among the former were least squares approximation, Gaussian and related quadrature, iterative methods in linear algebra, the detection of singularities, and integral equations; the latter included the use of wavelets in medical diagnostics and the relevance of orthogonal polynomials in optimal control problems, dynamical systems, and gas dynamics.

	Monday	Tuesday	Wednesday	Thursday	Friday
Morning session	Monegato Peherstorfer Petras	Calvetti Li Suchy	Beckermann Fischer, H.-J. Fuchs	Ammar Freund Gutknecht	Van Barel Ehrich Reichel
Afternoon session	Gori Laurie	Foupouagnigni Milovanović		Mhaskar Skrzipek	Ripken Runckel
Evening session		Hanke Fischer, B. Gragg Weideman			Open problems session

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Abstracts

Algorithms and applications of eigenproblems for Hessenberg matrices related to Szegő polynomials

Gregory S. Ammar (DeKalb)

Szegő polynomials, i.e., polynomials orthogonal with respect to a measure on the unit circle in the complex plane, can be viewed as the characteristic polynomials of a structured Hessenberg matrix that is determined by the recurrence coefficients of the polynomials. This is in analogy with the relationship of polynomials orthogonal with respect to a measure on the real axis and Jacobi matrices. Similarly, computational problems involving Szegő polynomials can be developed by recasting the problems in terms of the structured Hessenberg matrix.

The structured Hessenberg matrices can be viewed as submatrices of a larger unitary Hessenberg matrix. The unitary Hessenberg matrix itself arises in the special case that the Szegő polynomials are orthogonal with respect to a singular measure. Unitary Hessenberg matrices have a structure that is quite amenable to exploitation in eigenvalue computations, and a variety of efficient algorithms have recently been developed for solving unitary Hessenberg eigenproblems. We will give an overview of these points, and give particular attention to Gragg's unitary Hessenberg QR algorithm. We'll outline a derivation of the UHQR algorithm using a device for describing the efficient implementation of single-bulge chasing procedures on unitary Hessenberg matrices.

We then show how our device for deriving the UHQR algorithm can be used to derive an efficient implementation of a Francis QR step on (real) orthogonal Hessenberg matrices. In particular, we will see that the double-bulge chasing sweep that arises from the Francis shift strategy can be implemented by interleaving three single-bulge chasing sweeps. The resulting OHQR algorithm avoids the additional storage and computation associated with the complex arithmetic that is required when the single-shift UHQR algorithm is applied to a real matrix.

We will also outline some current work on how the QR algorithm can be efficiently applied to a submatrix of a unitary Hessenberg matrix. The resulting algorithm, which is being developed in collaboration with William Gragg and Chunyang He, provides a new approach to computing the zeros of an arbitrary Szegő polynomial.

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Multiple orthogonal polynomials and simultaneous rational approximation

Walter van Assche (Leuven)

The usual notion of orthogonal polynomials on the real line associated with a positive measure can be extended to multiple orthogonality on r sets with respect to r positive measures. Usual orthogonal polynomials are connected with Padé approximation of one (Stieltjes) function; multiple orthogonal polynomials are connected with simultaneous Hermite-Padé approximation of r (Stieltjes) functions. Not every system of r Stieltjes functions gives a normal Hermite-Padé table, but there are two systems that appear to be natural: Angelesco systems where the orthogonality is on r disjoint intervals, and Nikishin systems where the orthogonality is on one interval but the weight functions are recursively defined as Stieltjes transforms of certain measures on disjoint intervals.

Our interest is in the computation of these multiple orthogonal polynomials. One can show that they satisfy a recurrence relation of $(r+1)$st order, extending the well-known three-term recurrence relation for usual orthogonal polynomials. Hence we are interested in computing the recurrence coefficients and the zeros of multiple orthogonal polynomials, which turn out to be eigenvalues of a banded Hessenberg matrix (extending the tridiagonal Jacobi matrix for usual orthogonal polynomials).

Orthogonal polynomial vectors and discrete least squares approximation

Marc Van Barel (Heverlee)

We give the solution of a discrete least squares approximation problem in terms of orthonormal polynomial vectors. The degrees of the polynomial elements of these vectors can be different. An algorithm is constructed computing the coefficients of recurrence relations for the orthonormal polynomial vectors. In case the function values are prescribed in points on the real axis or on the unit circle, variants of the original algorithm can be designed which are an order of magnitude more efficient. Although the recurrence relations require all previous vectors to compute the next orthonormal vector, in the real or in the unit-circle case only a fixed number of previous vectors are required.

The sensitivity of least squares polynomial approximation

Bernhard Beckermann (Villeneuve d'Ascq)
Joint work with Ed Saff (Tampa)

We consider the least squares problem of finding the coefficients with respect to a polynomial basis $\{p_0, p_1, \ldots, p_n\}$, $\partial p_j = j$, of a polynomial P, $\partial P \leq n$, such that, for a given

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function f, the weighted error $\sum_{j=0}^{N} |w_j(z_j)|^2 |f(z_j) - P(z_j)|^2$ is minimized. The maximal magnification κ_n of relative errors in the data $f(z_0), \ldots, f(z_N)$ equals the condition number of some rectangular weighted Vandermonde-like matrix. The aim of this talk is to study the nth root behaviour of κ_n for bases of orthogonal polynomials and for some triangular array of nodes in the case $N/n \to C > 1$. As tools we require complex potential theory, such as the constrained energy problem in the presence of an external field.

Estimation of the L-curve via Lanczos bidiagonalization

Daniela Calvetti (Cleveland)

The L-curve criterion is often applied to determine a suitable value of the regularization parameter when solving ill-conditioned linear systems of equations with a right-hand side contaminated by errors of unknown norm. However, the computation of the L-curve is quite costly for large problems; the determination of a point on the L-curve requires that both the norm of the regularized approximate solution and the norm of the corresponding residual vector be available. Therefore, usually only a few points on the L-curve are computed, and these values rather than the L-curve, are used to determine a value of the regularization parameter. We describe a new approach to determine a value of the regularization parameter based on computing an L-ribbon that contains the L-curve in its interior. An L-ribbon can be computed fairly inexpensively by partial Lanczos bidiagonalization of the matrix of the given linear system of equations. The connection between orthogonal polynomials and quadrature rules is an essential tool in the determination of the L-ribbon.

Stieltjes polynomials and interpolation

Sven Ehrich (München)

Stieltjes polynomials are defined by

$$\int_{-1}^{1} P_n(x) E_{n+1}(x) x^k \, dx = 0, \qquad k = 0, 1, \ldots, n,$$

where P_n are the Legendre polynomials. The zeros of the Stieltjes and the Legendre polynomials are used by the Gauss-Kronrod quadrature formula. We investigate the quality of the interpolation processes based on the zeros of the Stieltjes polynomials E_{n+1}, respectively on the zeros of the product $P_n E_{n+1}$. We present new inequalities for the Stieltjes polynomials and show that the Lebesgue constants of both interpolation processes have the optimal order $\mathcal{O}(\log n)$. Furthermore, we give weighted L^p error bounds as well as applications to product integration and to the numerical solution of integral equations.

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Polynomial wavelets with application to evoked EEG Oscillations

Bernd R. W. Fischer (Lübeck)

In this talk we present a unified approach for the construction of polynomial wavelets. Our main tool are orthogonal polynomials. More precisely, our derivations make use of the general theory of kernel polynomials. This allows us to treat not only weight functions which are supported on a compact interval (e. g., Jacobi weights) but also weight functions which are supported on the real line (e. g., Hermite weight) or on the real half line (e. g., Laguerre weight).

Several examples illustrate the new approach. In particular, we apply the polynomial wavelet scheme to signals obtained by visual cortex recordings of auditory and visual evoked potentials in the human brain. The obtained results strongly support the suggestion that alpha oscillations in the corresponding EEG are event-related oscillations.

Fast solution of confluent Vandermonde-like matrices using polynomial arithmetic

Hans-Jürgen Fischer (Chemnitz)

If we want to calculate the weights σ_i and nodes τ_i of Gaussian quadrature for some measure σ directly from modified moments $\mu_k = \int p_k(x)\, d\sigma(x)$ with some given system of orthogonal polynomials p_k, we have a non-linear system of equations. The implementation of a Newton method leads to the solution of a confluent Vandermonde-like system with matrix

$$\begin{pmatrix} p_0(\tau_1) & \dots & p_0(\tau_n) & p_0'(\tau_1) & \dots & p_0'(\tau_n) \\ \vdots & & \vdots & \vdots & & \vdots \\ p_{2n-1}(\tau_1) & \dots & p_{2n-1}(\tau_n) & p_{2n-1}'(\tau_1) & \dots & p_{2n-1}'(\tau_n) \end{pmatrix}.$$

We propose a method of solution using only operations like addition, multiplication and division of polynomials and the evaluation at some points. These operations can be efficiently performed in any orthogonal base within $O(n^2)$ operations. For an appropriate base (Chebyshev of first or second kind) the algorithm can be speeded up to $O(n \log^2 n)$ operations.

Some remarks on the estimation of linear functionals

Klaus-Jürgen Förster (Hildesheim)

In this lecture, we consider estimates of linear functionals on $C[-1,1]$ using Peano kernel theory. We discuss several applications of an expansion of Peano kemels with ultraspherical polynomials, which has been proved by H. Brass and the author. Some examples concerning Gaussian quadrature are given.

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Laguerre-Freud equations applied to generalized Meixner and generalized Charlier orthogonal polynomials

Mama Foupouagnigni (Berlin)

Let $\mathbb{C}$ (respectively $\mathbb{C}[x]$) be the set of complex numbers (respectively the vector space of polynomials with complex coefficients). Let $\mathcal{L}$ be a regular linear form from $\mathbb{C}[x]$ to $\mathbb{C}$ satisfying $\Delta(\phi\mathcal{L}) = \psi\mathcal{L}$ (with $\langle\Delta\,\mathcal{L}, P\rangle = -\langle\mathcal{L}, \nabla\, P\rangle$, $P \in \mathbb{C}[x]$) where ϕ and ψ are given polynomials. The polynomial family $(P_n)_n$ orthogonal with respect to $\mathcal{L}$ satisfies a three term recurrence relation: $P_{n+1} = (x - \beta_n)P_n - \gamma_n P_{n-1}$, $P_0 = 1$, $P_{-1} = 0$. When the degree of ϕ is at most 2 and the degree of ψ is exactly one, the linear form and the corresponding polynomials are said classical and coefficients β_n and γ_n are well-known. But if the degree of ϕ is at least 3 or the degree of ψ is at least 2, the linear form and the corresponding polynomials are said semi-classical and the formula giving β_n and γ_n in terms of ϕ and ψ is not valid. In this work, we show that coefficients β_n and γ_n are solutions of two non linear equations

$$\gamma_{n+1} = F_1(\gamma_1, \ldots, \gamma_n, \beta_0, \ldots, \beta_n),$$
$$\beta_{n+1} = F_2(\gamma_1, \ldots, \gamma_{n+1}, \beta_1, \ldots, \beta_n).$$

Application of this result to generalized Charlier and generalized Meixner of class one and use of symbolic and numerical computation with Maple V.4 permit us to have information about the asymptotic behaviour of coefficients β_n and γ_n.

Matrix-valued formally orthogonal polynomials and their application in reduced-order modeling of MIMO systems

Roland W. Freund (Murray Hill)

Matrix Padeé approximation of the Laplace-domain transfer function can be used to construct reduced-order models of large-scale time-invariant linear dynamical systems with multiple inputs and multiple outputs. It is well known that for the case of a single input and a single output, the resulting Padé approximants are intimately connected to formally orthogonal polynomials associated with a scalar moment sequence. In this talk, we present an extension of this connection to the case of multiple inputs and multiple outputs. In this case, the corresponding polynomials are matrix-valued. We derive recurrences for these matrix-valued formally orthogonal polynomials, and we describe some of their properties. Numerical examples from circuit simulation are presented.

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Discrete polynomial least-squares approximation in moving time windows

Erich Fuchs (Passau)

With signal processing in moving time windows one tries to compute characteristic values of the current time window iteratively taking advantage of already computed values in previous time windows. It can be shown that this computation method applied in least-squares approximation on discrete data using orthogonal polynomials leads to fast algorithms for several discrete weights. Therefore these iterative algorithms providing signal trend information and estimators in the time domain are suitable for processing under real-time conditions.

Furthermore these algorithms can be extended to approximation problems of the type "find $p \in \mathcal{P}_n(\mathbb{R}, \mathbb{R})$ minimizing $\sum_{j=0}^{m}(l_j(p) - y_j)^2$", where l_j is a linear form like $l_j(p) = \int_j^{j+1} p(x)\,dx$. This allows the interpretation of a measured value as an evaluation of an integral.

The polynomials $\{p_0, \ldots, p_n\}$ orthogonal with respect to the inner product

$$(p|q) = \sum_{j=0}^{m} l_j(p) l_j(q) w_j$$

($w_j \in \mathbb{R}^+$ being discrete weights) can be determined by solving systems of linear equations based on other orthogonal polynomials, which can be computed easier. Due to the fact that such systems of orthogonal polynomials do normally not fulfill a three-term-recurrence relation, this way of computing this kind of orthogonal polynomials is a fast and easy method.

Some applications of s-orthogonal polynomials

Laura Gori (Roma)
Joint work with Elisabetta Santi (L'Aquila)

In this talk, after briefly surveying the main properties of s-orthogonal (or *power* orthogonal) polynomials, we present some recent results showing how a particular class of these polynomials, characterized by a peculiar invariance property of their zeros, can conveniently be chosen as a base for the construction of certain quadrature rules, which turn out to be particularly suitable for approximating singular integrals.

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Stabilization of the uhqr algorithm

William B. Gragg (Monterey)

The unitary Hessenberg QR algorithm is fundamental for statistical signal analysis, as is the closely related inverse algorithm (iuhqr). These algorithms are analogous with algorithms, tqr and itqr, for real symmetric tridiagonal matrices. No claim of numerical stability for uhqr was made when it was introduced in 1986. Indeed, an open problem was to make it perform as well as tqr. We introduce a device which appears, on the basis of a large number of experiments, to do the job. Indeed, M. Stewart has *proved* that a small variation of our algorithm *is* numerically stable.

A matrix interpretation of the Euclidean algorithm

Martin H. Gutknecht (Zürich)

We show that the classical Euclidean algorithm for polynomials (or power series) as well as its recently established forward-stable look-ahead version is equivalent to successively applying matrix multiplications to the Sylvester matrix of the given polynomials. The factors can be chosen such that the resulting matrix identity links directly the Sylvester matrix and the coefficients of the quotients in the Euclidean algorithm with six tridiagonal matrices containing the coefficients of the polynomials generated by the so-called extended Euclidean algorithm in its general look-ahead form.

Semiiterative regularization methods for ill-posed indefinite problems

Martin Hanke (Karlsruhe)
Joint work with Harald Frankenberger (Kaiserslautern)

We study semiiterative methods for approximating the solution f of linear operator equations $Kf = g$. We are primarily interested in the case where K is selfadjoint and indefinite, and where the spectrum of K clusters in the origin. Due to the latter property semiiterative methods can only have a sublinear rate of convergence, depending on smoothness properties of the solution line $f = |K|^\nu \omega$ where $|K| = (K^*K)^{1/2}$ and $\nu > 0$.

Our semiiterative methods construct iterates f_k with $g - Kf_k = p_k(K)g$ where p_k is a polynomial of degree k with $p_k(0) = 1$, a so-called residual polynomial. Given the information that the spectrum of K is contained in $[a, 1]$ with some specified $a < 0$, and that f is as above, we choose certain kernel polynomials as residual polynomials. More precisely, let $\{u_n\}$ be the orthonormal polynomials w. r. t. the weight function

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$|\lambda|^{2\nu-2}/\sqrt{(1-\lambda)(\lambda-a)}$, $a < \lambda < 1$, then we define

$$p_k(\lambda) = \sum_{n=0}^{k} u_n(\lambda)u_n(0) / \sum_{n=0}^{k} u_n^2(0).$$

Using asymptotic results by Badkov and Nevai we establish (optimal) convergence rates for the iteration. The semiiterative methods can be implemented using the short recurrences for the orthonormal polynomials $\{u_n\}$. For $\nu \in \mathbb{N}$ those recurrences can be derived from the recurrence relation of the Chebyshev polynomials (cf. Gautschi or Fischer/Golub) but for arbitrary $\nu > 1/2$ one can also use their explicit recurrence relations which are given by Magnus.

Gegenbauer weight functions admitting L_2 Duffin and Schaeffer type inequalities

David B. Hunter (Bradford)
Joint work with Geno Nikolov (Sofia)

Denote by Π_n the set of all algebraic polynomials of degree n or less, and let $w_\lambda(x) = (1-x^2)^{\lambda-1/2}$ $(\lambda > -1/2)$. Suppose $Q_n \in \Pi_n$ has distinct real zeros which interlace with the extrema t_j $(j = 0, 1, \ldots, n)$ in $[-1, 1]$ of the ultraspherical polynomial $P_n^{(\lambda)}$ $(-1/2 < \lambda \leq 1/2)$. It is shown that if $p \in \Pi_n$ satisfies $|p(t_j)| \leq |Q_n(t_j)|$ $(j = 0, 1, \ldots, n)$, then

$$\int_{-1}^{1} w_{\lambda+k-1}(t)|p^{(k)}(t)|^2\,dt \leq \int_{-1}^{1} w_{\lambda+k-1}(t)|Q_n^{(k)}(t)|^2\,dt, \quad (k = 1, 2, \ldots, n),$$

and

$$\int_{-1}^{1} w_{\lambda+k-2}(t)|p^{(k)}(t)|^2\,dt \leq \int_{-1}^{1} w_{\lambda+k-2}(t)|Q_n^{(k)}(t)|^2\,dt, \quad (k = 1, 2, \ldots, n).$$

A number of related results are also obtained.

Two-term recurrences and problems related to Gaussian quadrature formulas

Dirk Laurie (Vanderbijlpark)

The formulation of orthogonal polynomials as two-term rater than three-term recursions has theoretical and computational advantages.

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Theoretical: the classical polynomials have slightly simpler coefficients over $[0,2]$ than the three-term coefficients over $[-1,1]$; questions relating to whether the leftmost zero of anti-Gaussian and Kronrod formulas is internal, are easily answered.

Computational: the finding of zeros becomes a singular value rather than an eigenvalue problem; recent advantages yield high relative accuracy of even the smallest zeros; no smearing of the zeros near the left end-point.

In particular, I will discuss what happens to mixed moments in two-term recursion, and how new light is shed on the problem of recovering the recursion coefficients from the Gaussian formula.

Construction and computation of a new set of orthogonal polynomials

Shikang Li (Hammond)

In this talk we will discuss how to construct and compute a new set of orthogonal polynomials from an existing one. For a given pair of positive integers (n, r) and a given positive measure $d\sigma(t)$, we will construct a set of orthogonal polynomials corresponding to the modified measure $d\sigma^*(t) = (\pi_n(t))^4 d\sigma(t)$. For $r = 2$ and the first kind Chebyshev measure we are able to find explicit formulas for the recurrence coefficients for any positive integer r and the first kind Chebyshev measure. For $r = 2$ and other measures, a computational method is proposed. Some other results are also stated.

Fourier transforms of orthogonal polynomials of singular measures and wave propagation in almost-periodic systems

Giorgio Mantica (Como)

I shall discuss the numerical and theoretical problems which have arisen in the description of quantum and classical motion in almost-periodic systems. I show that the common mathematical nature of these problems lies in the asymptotic properties of the Fourier transforms of the orthogonal polynomials associated with the spectral measure of the problem, which is typically singular. I describe some results that can be obtained in this approach.

Lagrange interpolation on the real line

Guiseppe Mastroianni (Potenza)

I prove that the Lagrange polynomials mainly based on the Laguerre or Hermite zeros are convergent in weighted uniform norm and easy to implement for a wide class of functions. I show also some results if simultaneous interpolation.

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Polynomial frames and detection of singularities

Hrushikesh N. Mhaskar (Los Angeles)

Let V_n be the class of all polynomials of degree at most 2^n, w be a weight function, and $W_n := \{p \in V_{n+1} : \int P(t)R(t)w(t)\,dt = 0\ \forall R \in V_n\}$. We construct a variety of kernels $K_n(x,t)$ (in W_n as a function of each variable) such that any $P \in W_n$ admits a representation $P = \sum_{k=1}^{2n+1} c_{k,n}(P)K_n(x, x_{k,2n+1})$ with the stability condition

$$A_n \int P(t)^2 w(t)\,dt \le \sum_{k=1}^{2n+1} c_{k,n}^2(P)\lambda_{k,2n+1} \le B_n \int P(t)^2 w(t)\,dt, \tag{1}$$

where the constants A_n, B_n do not depend upon w. In the case when w is a Jacobi weight, we discuss the L^p-versions of (1), $1 \le p \le \infty$. In this case, we are also able to demonstrate the localization of the kernels in the following sense. Let $f : [-1,1] \to \mathbb{R}$, $k \ge 0$ be an integer, and the derivative $f^{(k)}$ have a jump discontinuity at $x_0 \in (-1,1)$. Then the function $\tau_n(f,x) := \int f(t)K_n(x,t)w(t)\,dt$ is "large near x_0" and "small away from x_0". This behaviour can be described in a precise quantitative manner. Unlike classical compactly supported wavelets, our frames are able to detect the discontinuities in the derivatives of an arbitrarily high order.

Müntz orthogonal polynomials and applications

Gradimir V. Milovanović (Niš)

Let $\Lambda = \{\lambda_0, \lambda_1, \ldots\}$ be a complex sequence such that $\Re(\lambda_k) > -1/2$ for every $k \in \mathbb{N}_0$. A linear combination of the system $\{x^{\lambda_0}, x^{\lambda_1}, \ldots, x^{\lambda_n}\}$ is called a Müntz polynomial, or a Λ-polynomial. By $M_n(\Lambda)$ we denote span$\{x^{\lambda_0}, x^{\lambda_1}, \ldots, x^{\lambda_n}\}$ where the linear span is over the real (or complex) numbers. Such generalized polynomials can be orthogonalized and applied to quadrature problems. We investigate two Müntz systems which are orthogonal with respect to some inner products. Beside the general properties including some representations and recurrence relations, we consider a few interesting special cases of generalized systems. In particular, the systems with real Λ, as well as the case when some of the λ's are equal, are also considered. Zero distribution is also investigated.

A big problem is how to compute the values of orthogonal Müntz polynomials in a finite arithmetics. As a rule, such polynomials are ill-conditioned. An approach in numerical evaluation of these polynomials using complex integration is given. A numerical algorithm for the construction of generalized Gaussian quadratures was originally introduced over three decades ago by Karlin and Studden, and recently investigated by Ma, Rokhlin and Wandzura [SIAM J. Numer. Anal. 33 (1996)]. Using theory of orthogonality for Müntz systems, we present an alternatively numerical method for constructing

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generalized Gaussian quadrature rules

$$\int_0^1 f(x)\,dx = \sum_{k=1}^{n} A_k f(x_k) + R_n(f),$$

which are exact for each $f \in M_{2n-1}(\Lambda)$. Especially, we consider an important case of quadratures with a combined algebraic and logarithmic degree of precision. Our construction is quite different from the corresponding procedure for the classical Gaussian integration formulae with an algebraic degree of precision.

Numerical resolution of Symm's integral equation on polygons by means of orthogonal polynomials

Giovanni Monegato (Torino)

Several papers have been devoted to the numerical solution of Symm's integral equation on a polynomial domain. The main difficulty of this problem is caused by the singular behaviour that the solution shows at the corners of the polygon. All known numerical approaches are based on piecewise polynomial approximations on each side.

We show that by introducing first into the equation a proper smoothing change of variable we can always solve the transformed equation by a collocation method, which approximates the new unknown on each side of the polygon by a (global) polynomial and makes use of orthogonal polynomials.

Interpolatory quadrature formulae on special sets of abscissae

Sotirios E. Notaris (Athens)

We review the existing results, and present some new ones, regarding the interpolatory quadrature formulae on special sets of abscissae, particularly, Chebyshev and Bernstein-Szegő abscissae.

Orthogonal polynomials and dynamical systems

Franz Peherstorfer (Linz)

First the consequences of the Fermi-Pasta-Ulam recurrence phenomenon to nonlinear lattices and soliton equations are discussed. After a comparison of the chaotic Hénon-Heiles lattice with the integrable Toda lattice it is shown how to get in an easy way the complete integrability of the Toda lattice with the help of orthogonal polynomials. In addition it is proved that Stieltjes functions of the form $(f - \sqrt{H})/\rho$, where $H(\lambda) = \prod_{j=1}^{2N}(\lambda - \lambda_j)$ and f, ρ are polynomials of degree N and $N-1$ resp. which satisfy certain

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additional conditions with respect to time t, generate Toda lattices. In the second part first the connection between soliton equations and eigenvalue problems of systems of linear differential equations is discussed. More precisely, it is shown that the solutions of soliton equations (as the solutions of the KdV or of the generalized nonlinear Schrödinger equation) are potentials of linear systems of differential equations whose eigenvalues do not depend on time. Then we demonstrate how orthogonal polynomials with periodic recurrence coefficients can be used to get solutions of the nonlinear Schrödinger equation. Finally, results on orthogonal and L_q-minimal polynomials on Julia sets, in particular on dendrites, are presented.

Existence and non-existence of Gauss-Kronrod quadrature

Knut Petras (München)

In numerical software packages, we often find Gauss-Kronrod quadrature formulae. A classical problem concerning these formulae for weighted integration is "for which ultraspherical weight functions $w_\lambda(x) = \left(1 - x^2\right)^{\lambda-1/2}$ do Gauss-Kronrod quadrature formulae exist?". Szegő proved the existence for $\lambda \in [0, 2]$ and gave a counterexample for $\lambda < 0$. Gautschi and Notaris made numerical tests and conjectured that for given number $2n+1$ of nodes, a Gauss-Kronrod quadrature formula exists if and only if $0 \leq \lambda \leq \lambda_0(n)$. Assuming the existence of such $\lambda_0(n)$, they calculated some of these values numerically.

I present the results of a joint work with Franz Peherstorfer. We have proved that for $\lambda > 3$, Gauss-Kronrod quadrature formulae may exist only for sufficiently small n. Furthermore, for $\lambda = 3$ and sufficiently large n, Gauss-Kronrod quadrature exists. Generalizations to Jacobi weight functions are given.

The considerations are based on a new representation of Stieltjes polynomials, which also allows to derive an asymptotic expression for Stieltjes polynomials with respect to Jacobi weights.

Computation of Gauss-Kronrod quadrature rules

Lothar Reichel (Kent)

Recently Laurie presented a new algorithm for the computation of $(2n+1)$-point Gauss-Kronrod quadrature rules with real nodes and positive weights. The algorithm first determines a symmetric tridiagonal matrix of order $2n+1$ from certain mixed moments, and then computes a partial spectral factorization. We describe a new algorithm that does not require the entries of the tridiagonal matrix to be determined, and thereby avoids computations that can be sensitive to pertubations. Our algorithm uses the consolidation phase of a divide-and-conquer algorithm for the symmetric tridiagonal eigenproblem and is well suited for implementation on a parallel computer. Numerical examples illustrate the performance of the algorithm.

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Computation of complex orthogonal polynomials

Wolfgang Ripken (Hamburg)

For computing orthogonal polynomials with respect to a complex inner product (i. e. a positive definite Hermitean form) we can, in general, not use a three term recurrence relation. So we have to look for other methods. In the talk I describe an algorithm for obtaining the coefficients in an expansion with respect to the monomials. The algorithm proceeds in several steps. In each step orthogonal polynomials for a conveniently chosen auxiliary inner product are computed. The idea behind this procedure is to control the condition numbers of the matrices one has to deal with. However, the large number of steps may again lead to instability.

On neighbouring orthogonal polynomial sequences

Hans-Joachim Runckel (Ulm)

Given two difference equations

$$x_n = a_1(\lambda, n)x_{n-1} + a_2(n)x_{n-2}, \quad n \geq 1, \tag{2}$$
$$y_n = b_1(\lambda, n)y_{n-1} + b_2(n)y_{n-2}, \quad n \geq 1, \tag{3}$$

that define orthogonal polynomials, then for the continued fractions

$$\mathop{\mathrm{K}}_{\nu=1}^{n} \left(\frac{a_2(\nu)}{a_1(\lambda,\nu)}\right) = \frac{x_{n,2}}{x_{n,1}}, \quad \mathop{\mathrm{K}}_{\nu=1}^{n} \left(\frac{b_2(\nu)}{b_1(\lambda,\nu)}\right) = \frac{y_{n,2}}{y_{n,1}} \text{ hold for } n \geq 1,$$

where $x_{n,1}$, $x_{n,2}$ and $y_{n,1}$, $y_{n,2}$ are solutions to (2) and (3), respectively, with

$$\begin{pmatrix} x_{0,1} & x_{0,2} \\ x_{-1,1} & x_{-1,2} \end{pmatrix} = \begin{pmatrix} y_{0,1} & y_{0,2} \\ y_{-1,1} & y_{-1,2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(orthogonal polynomials of first and second kind, respectively).

If $a_1(\lambda, n) - b_1(\lambda, n)$ and $a_2(n) - b_2(n)$ are "sufficiently small" as $n \to \infty$, then using

$$f(\lambda) = \mathop{\mathrm{K}}_{\nu=1}^{\infty} \left(\frac{a_2(\nu)}{a_1(\lambda,\nu)}\right), \; g(\lambda) = \mathop{\mathrm{K}}_{\nu=1}^{\infty} \left(\frac{b_2(\nu)}{b_1(\lambda,\nu)}\right), \; \lambda \in \mathbb{C} \setminus J, \text{ where } J \text{ is an interval } \subset \mathbb{R}, \tag{4}$$

the density functions of $x_{n,1}$ ($x_{n,2}$) can be expressed in terms of the density functions of $y_{n,1}$ ($y_{n,2}$). Furthermore, the sequences $x_{n,1}$, $y_{n,1}$ and $x_{n,2}$, $y_{n,2}$ have "similar" asymptotic behaviour for $\lambda \in \mathbb{C} \setminus J$ and "similar" asymptotic zero distribution as $n \to \infty$. This generalizes earlier results in the case where $y_{n,1}$, $y_{n,2}$ are Chebyshev polynomials of first and second kind.

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An algorithm for the evaluation of linear combinations of Szegő polynomials

Michael Skrzipek (Hagen)

Let $\{\Phi_\nu\}_{\nu \in \mathbb{N}}$ be a sequence of polynomials orthogonal on the unit circle with respect to an inner product. It is known that by using their Szegő recurrence a Clenshaw type algorithm for evaluating polynomials, expanded in terms of the Φ_ν can be obtained. We make another approach which uses a modification of associated Szegő polynomials (i. e. Szegő polynomials with shifted reflection coefficients). So we derive an alternative algorithm for evaluating linear combinations of Szegő polynomials. We emphasize that we can compute the values of the derivatives of these polynomials with this algorithm, too.

Asymptotic error estimates for rational best approximants in $H^2(D)$ and $H^\infty(D)$

Herbert Stahl (Berlin)

Rational approximants of functions in the real Hardy spaces $H^2(D)$ and $H^\infty(D)$ with $D = \{|z| < 1\}$ are of great interest in control theory, stochastic modeling, or signal processing. In the talk we concentrate on the approximation of Markov functions $f(z) = \int (t - z)^{-1}\, d\mu(t)$. Starting from H^2-best rational approximants, where the orthogonality of the denominators polynomials alows to calculate interpolation points for these approximants, we come to rational best approximants in the H^∞-norm. Exact asymptotic rates are proved, and it is shown that interpolation points can be constructed such that the rational interpolants have asymptotically minimal error in the H^∞-norm.

Applications of tensorial Hermite polynomials in the kinetic theory of gases

Kurt Suchy (Düsseldorf)

In the kinetic theory of gases expansions of the velocity distribution function are important. About one century ago expansion of the angular part with Legendre polynomials were introduced, later on generalized to spherical harmonics. Half a century later expansions in the whole velocity space were introduced with tensorial Hermite polynomials for three variables, since their Gaussian weight function corresponds with the (local) Maxwell distribution. A generalization of these polynomials with an additional parameter proved to be useful. Their properties and applications are presented.

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New examples of orthogonal polynomials with indeterminate moment problem and a conjecture on the growth properties of the Nevanlinna matrix

Galliano Valent (Paris)

We consider several examples of orthogonal polynomials related to birth and death processes with cubic and quartic transition rates. The corresponding moment problems being indeterminate we compute their Nevanlinna matrix. These results give further support to a conjecture on the growth properties of the entire functions involved in the Nevanlinna matrix.

Spectral methods based on non-classical orthogonal polynomials

Andre Weideman (Corvallis)

Spectral methods for solving boundary-value problems numerically have traditionally been based on classical orthogonal polynomials such as those associated with the names of Chebyshev, Legendre, Laguerre, and Hermite. In this talk we investigate the potential advantages of spectral methods based on non-classical orthogonal polynomials.

Numerical examples include:

1. The solution of the Schrödinger equation on the real line by a spectral method based on orthogonal polynomials generated by the logistic density weight function. We show that in the case of the Morse potential this method is superior to the Hermite spectral method.

2. The solution of a boundary-value problem with a steep boundary layer by a method based on orthogonal polynomials generated by a rational weight function. The method may be viewed as a spectral method based on a rational interpolant with pre-assigned poles. The poles are chosen to mimic the almost singular behavior of the boundary layer. We show that this method is superior to the standard Chebyshev spectral method.

3. The solution of a Sturm-Liouville problem by the same method as in 2. The approximate location of the poles are determined by WKB analysis. This method was shown to be superior to the Chebyshev method.

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