

Tagungsbericht 13/1998

## Arbeitsgemeinschaft über den Beweis der Milnor-Vermutung nach Voevodsky

29.03. - 04.04.98

Organizers: B. Kahn (Paris), M. Spieß (Heidelberg)

The aim of this Arbeitsgemeinschaft was to understand the proof of the following theorem, due to Vladimir Voevodsky:

**Theorem.** *Let  $k$  be a field of characteristic  $\neq 2$  and  $m, n$  be positive integers. Then the norm residue homomorphism*

$$K_n^M(k)/2^m \longrightarrow H_{\text{ét}}^n(k, \mu_{2^m}^{\otimes n})$$

*is an isomorphism.*

This had been conjectured by Milnor. More generally one expects:

**Conjecture (Kato).** *For any positive integers  $m, n$  with  $m$  prime to  $\text{char}(k)$  the norm residue homomorphism*

$$K_n^M(k)/m \longrightarrow H_{\text{ét}}^n(k, \mu_m^{\otimes n})$$

*is an isomorphism.*

As in the previous cases where this conjecture could be established (Merkurjev, Suslin, Rost), the proof is based on the computation of certain cohomology groups attached to norm varieties. However, contrary to the former approaches, Voevodsky works with motivic cohomology rather than algebraic  $K$ -theory: for this reason, talks 1 to 12 were devoted to the construction of this motivic cohomology. Two further key points in the proof are the existence of a pure motive associated by Markus Rost to certain quadrics, and Voevodsky's observation that these quadrics are related to the theory of complex cobordism (their  $\mathbb{C}$ -valued points are generators of the mod 2 complex cobordism ring). In order to exploit this connection he developed (partly together with F. Morel) analogues of (stable) homotopy theory, Steenrod-algebra, Thom spaces and the cobordism spectrum in the context of algebraic varieties: Talks 13-14 covered the Rost motive and talks 15 to 19 covered the homotopy part.

## 1. Introduction

In 1970, Milnor introduced a map

$$K_n^M(F)/2 \rightarrow H^n(F, \mathbb{Z}/2)$$

for every  $n \geq 0$  and conjectured that it is an isomorphism. Here  $F$  is a field of characteristic  $\neq 2$ ,  $K_n^M(F) = (F^\times)^{\otimes n} / \langle \dots \otimes a \otimes 1 - a \otimes \dots \rangle_{\mathbb{Z}}$  is its  $n$ -th Milnor  $K$ -group, and  $H^n(F, -)$  is the  $n$ -th Galois cohomology. This conjecture was recently proved by V. Voevodsky (after earlier work of Merkuriev, Suslin and Rost for  $n \leq 4$ ). The aim of this talk was to give an overview, especially on the background. A more general conjecture (stated by K. Kato in 1979) proposes that a similar map

$$K_n^M(F)/m \rightarrow H^n(F, \mu_m^{\otimes n})$$

(introduced by Tate in 1976) is an isomorphism for any field  $F$  and any  $m$  not divisible by  $\text{char}(F)$ . This was proved for  $n = 2$  by Merkuriev and Suslin in 1982. Around 1983 Beilinson and Lichtenbaum (partly independently and partly influenced by each other) proposed that this map should be regarded as one from "motivic cohomology" to étale cohomology. They postulated the existence of certain complexes (in the Zariski or the étale topology) whose cohomology would be the motivic cohomology, and several "axioms" which would among other things imply the Kato conjecture. For the latter especially an analogue of Hilbert's theorem 90 was shown to be crucial. After earlier attempts by Bloch in 1983, Voevodsky (partly in joint work with Suslin and Friedlander) defined such complexes with many good properties. By developing a whole arsenal of new techniques he could prove enough of the "axioms" to get the Milnor conjecture.

UWE JANNSEN (Köln)

## 2. Closed Model Categories and triangulated categories

For every category and class of arrows there exists an extension called localization in which every arrow of the given class becomes invertible. A more precise description of the localization can be given when the class of arrows admits a calculus of left or right fractions. The notion of closed model category is introduced. In that case the localization is also the quotient of the subcategory of fibrant and cofibrant objects by an homotopy relation. Examples of closed model categories are given: topological spaces, chain complexes, simplicial complexes, spectra. If the closed model category is pointed then the localization can be endowed with a "triangulated structure". In the additive case it is the usual notion of triangulated category if the suspension functor is an equivalence. The notion of total derived functor is introduced.

GEORGES MALTSINIOTIS (Paris)

### 3. Pretheories

In this talk the definition and basic properties of pretheories are presented: Let  $S$  denote a smooth scheme over a field  $k$  and let  $p : X \rightarrow S$  be a smooth curve over  $S$ , then the group of relative cycles  $c(X/S)$  of relative dimension 0 is defined; base change and push forward morphisms are described. A pretheory  $(F, \phi)$  is a presheaf on the category of smooth schemes over  $k$  with values in the category of abelian groups, together with homomorphisms of abelian groups

$$\phi_{X/S} : c(X/S) \rightarrow \text{Hom}(F(X), F(S))$$

for each smooth curve  $p$  as above such that 1. the behavior of  $\phi_{X/S}$  for  $S$ -points of  $p$  is induced directly by this  $S$ -point, 2. the structure is compatible with pullbacks and 3. the functor  $F$  commutes with finite sums.

Each pretheory induces a sheaf  $F_{\text{ZAR}}$  in Zariski-topology on the category of noetherian schemes over  $k$ . When  $F_{\text{ZAR}}$  is restricted to the category of smooth schemes over  $k$  it has a unique structure of a pretheory, if  $(F, \phi)$  is supposed to be homotopy invariant, i. e.  $F(X) \cong F(X \times \mathbb{A}_k^1)$  for all smooth  $X$  over  $k$ . Furthermore,  $F_{\text{ZAR}}$  is homotopy invariant, too, and even of homological type, i. e. the push forward on  $c(X/S)$  is compatible with the transfer maps.

OTMAR VENJAKOB (Heidelberg)

### 4. Pretheories II

This talk continues the technical line of the former talk giving a connection between the notions of [pretheory] and [presheaves with transfers].

§1. The Nisnevich topology was introduced and illustrated in the cases of Spec (field) and Spec (noetherian ring of dim 1).

§2. The category of smooth correspondences; presheaves with transfers are contravariant functors from it to  $\mathbb{Z}$ -modules.

§3. Presheaves with transfer “are” (induce) pretheories of homological type. (This is the technical translation of structure for further investigations of the HOMOTOPICAL INVARIANCE of presheaves with transfers.)

§4. Main Theorem: For a presheaf with transfers, which is *homotopical invariant*, the associated sheafed versions with respect to the Zariski and Nisnevich topology ( - and also the corresponding cohomologies of these - ) still are

- *homotopical invariant*
- are isomorphic and
- in the case of a *perfect* base field so are also the corresponding cohomologies.

A proof was sketchy given in §5, §6 investigating separately the Zariski resp. Nisnevich associated sheaves. The accent was put on the geometrical and functorial character of the needed (technical) constructions, isolating explicitly the use of the hypothesis “ $k$  perfect”. Main reference of this talk: *Cohomological theory of presheaves with transfers*, Vladimir Voevodsky, preprint 1995.

DAN FULEA (Mannheim)

## 5. The triangulated category of geometrical motives

The category of smooth schemes over a field  $k$  can be turned into an additive category by taking finite correspondences as morphisms. Voevodsky took the quotient of the homotopy category of bounded complexes over this category by a certain thick subcategory (to impose homotopy invariance and Mayer-Vietoris exact triangles), and defined the triangulated category of effective geometric motives as the pseudo-abelian completion of this quotient category. By comparing this category to the category of effective Chow motives it can be shown that the category of geometric motives, obtained by formally inverting the Tate object is a tensor triangulated category.

JAN NAGEL (Essen)

## 6. The triangulated category of effective motivic complexes

The main thrust in this talk is to shift emphasis from schemes towards sheaves, thus allowing for more flexibility by the use of the theory of cohomology of sheaves. The category  $DM_{-}^{\text{eff}}(k)$  of “effective motivic complexes over  $K$ ” is introduced; it is the subcategory of the derived category of bounded complexes of Nisnevich sheaves with transfers, consisting of complexes with homotopy invariant cohomology sheaves.

The “embedding theorem” asserts that there is a full dense embedding of tensor triangulated categories

$$DM_{gm}^{\text{eff}}(k) \hookrightarrow DM_{-}^{\text{eff}}(k)$$

from effective geometric motives to effective motivic complexes. This is extremely useful in understanding the first category: it allows one to “compute”, for instance, the geometric motive of a projective bundle or a blow-up. It also permits one to prove (under the assumption of resolution of singularities) that  $DM_{gm}^{\text{eff}}(k)$  is generated by smooth projective varieties.

ROBERT LATERVEER (Strasbourg)

## 7. The cdh topology

The cdh topology is the smallest Grothendieck topology on  $Sch/k$ , the category of  $k$ -schemes of finite type, which refines the Nisnevich topology and has  $Y \amalg Z \rightarrow X$  as a covering for each proper surjective  $f : Y \rightarrow X$  and each closed subscheme  $Z \subset X$  such that  $f : (Y - f^{-1}(Z))_{\text{red}} \xrightarrow{\sim} (X - Z)_{\text{red}}$ . If  $k$  has resolution of singularities, this topology is fine enough to admit coverings of any scheme by smooth schemes. On the other hand, it is still sufficiently close to the Nisnevich topology as to yield the same cohomology for the kind of coefficients one is interested in. This is the content of the main result of this talk:

*If  $k$  has resolution of singularities, and  $F$  is a pretheory, then*

$$H_{\text{cdh}}^i(U, F_{\text{cdh}}) = H_{\text{Zar}}^i(U, F_{\text{Zar}}) \quad \forall i, \text{ for } U \text{ smooth.}$$

*Also, if  $F_{\text{cdh}} = 0$  then  $\underline{C}_*(F)_{\text{Zar}}$  is acyclic.*

The talk presented the main steps in the proof of this theorem, due to Friedlander-Voevodsky. A key point in the proof is the study of sheaves associated with blowings-ups of smooth schemes, and of their Nisnevich cohomology. Another key technique is the description of cohomology by hypercoverings; by resolution of singularities, one can assume that they consist of smooth schemes, for cdh topology.

CLAUS SCHEIDERER (Regensburg)

## 8. Moving lemma and duality

We prove the following theorem:

**Theorem (Friedlander/Voevodsky)** *Let  $k$  be a field that admits resolution of singularities. If  $X$  is a scheme of finite type over  $k$  and  $U$  is a smooth quasiprojective equidimensional scheme of dimension  $n$ , then the canonical map*

$$\mathcal{D} : z_{\text{equi}}(U, X, r) \hookrightarrow z_{\text{equi}}(X \times U, r + n)$$

*induces a quasi-isomorphism of complexes of Nisnevich sheaves*

$$\mathcal{D} : \underline{C}_{*z_{\text{equi}}}(U, X, r) \rightarrow \underline{C}_{*z_{\text{equi}}}(X \times U, r + n)$$

Furthermore, if both  $X$  and  $U$  are smooth and projective over  $k$ , then the same statement holds without assuming resolution of singularities (in char  $p$ ) and even gives an isomorphism of homology presheaves.

This theorem implies in particular that motivic cohomology groups of smooth  $k$ -varieties agree with higher Chow groups.

STEFAN MÜLLER-STACH (Essen)

## 9. Main properties at the triangulated category of motives

After extending the functors  $L(-), \underline{C}_*(-)$  to the categories of schemes of finite type over  $k$  and introducing the functors  $L^c, \underline{C}_*^c$ , one can show that these are the expected distinguished triangles in the derived category  $DM_{gm}^{eff}$  (e.g. Mayer-Vietoris, blow-up, localization). This implies that  $\underline{C}_*^c(X), \underline{C}_*(X) \in DM_{gm}^{eff}$  for any  $X$  of finite type. Introducing the bivariant cycle cohomology one can use the groups  $A_{r,i}(X, Y)$  to describe certain Hom groups in  $DM_{gm}^{eff}$ . In particular for  $X, Y$  of finite type

$$\text{Hom}_{DM_{gm}^{eff}}(\underline{C}_*(Y)(r)[2r+i], \underline{C}_*^c(X)) = A_{r,i}(Y, X).$$

Furthermore, the embedding of  $DM_{gm}^{eff}$  in  $DM_{gm}$  is effective and the latter category contains all internal  $\underline{\text{Hom}}(A, B)$ . Moreover it is even a rigid tensor category.

RALPH KAUFMANN (MPI, Bonn)

## 10. Bloch-Kato Conjecture<sup>?</sup> and the Beilinson-Lichtenbaum Conjecture

The Milnor Conjectures are part of the general Bloch-Kato Conjecture which says the following:

**Bloch-Kato Conjecture:** For any field  $F$  over a field  $k$  (ground field), the natural maps

$$\eta_n : K_n^M(F)/\ell \rightarrow H_{\text{ét}}^n(F, \mu_\ell^{\otimes n})$$

are isomorphisms for  $\ell$  a prime different than  $\text{char}(k)$ .

The Beilinson-Lichtenbaum conjecture is the following:

**Beilinson-Lichtenbaum conjecture:** For any field  $F$  over  $k$ , the natural morphism in  $DM_{gm}^{eff}(k)$ ,

$$\mathbb{Z}/\ell(n) \rightarrow B/\ell(n) := \tau_{\leq n} R\pi_* \mu_\ell^{\otimes n} \quad (\text{here } \pi : (Sm/k)_{\text{ét}} \rightarrow (Sm/k)_{Zar})$$

is a quasi-isomorphism of complexes of sheaves.

Suslin-Voevodsky first show that the motivic cohomology groups  $H^n(F, \mathbb{Z}/\ell(n)) \simeq K_n^M(F)/\ell$ ; this result has been proved in various guises by others, e.g. Bloch. The main theorem proved by Suslin-Voevodsky is the following:

Bloch-Kato conjecture for  $k \implies$  Beilinson-Lichtenbaum conjecture for  $k$ .

Talk # 10 proved the isomorphism  $H^n(F, \mathbb{Z}/\ell(n)) \simeq K_n^M(F)/\ell$  and a few reduction steps needed for the next talk.

RAMDORAI SUJATHA (TIFR, Bombay, India)

## 11. BK conj. & BL conj. II (following Suslin-Voevodsky)

The talk was devoted to proving the following:

*Prop:*  $F$  a field of char 0. Assume that

- BK holds over  $F$  in weight  $n$
- BL holds over  $F$  in weights  $< n$

*Then:*  $H^j(F, \mathbb{Z}/\ell(n)) \xrightarrow{\eta} H_{\text{ét}}^j(F, \mu_\ell^{\otimes n})$  is injective for  $j \leq n$ .

Using the reduction of the previous talk this assertion suffices to prove the main theorem of talk # 10 by induction.

For the proof of the proposition let  $\alpha \in H^{n-1+1}(F, \mathbb{Z}/\ell(n))$  with  $\eta(\alpha) = 0$ . Let  $S^1 =$  affine line with 0,1 identified. It suffices to show that the image of  $\alpha$  in  $H^{n+1}(\partial\Delta^i \times S^1, \mathbb{Z}/\ell(n))$  is zero. We first establish:

*Lemma:* There is  $U \subset \partial\Delta^i \times S^1$  containing  $\partial\Delta^i \times \{0, 1\}$  such that the image of  $\alpha$  in  $H^{n+1}(U, \mathbb{Z}/\ell(n))$  vanishes.

We consider the commutative diagram

$$\begin{array}{ccccccc}
 H^n(U, B/\ell(n)) & \rightarrow & H_{\partial\Delta^i \times S^1 \setminus U}^{n+1}(\partial\Delta^i \times S^1, \mathbb{Z}/\ell(n)) & \rightarrow & H_{\partial}^{n+1}(\partial\Delta^i \times S^1, \mathbb{Z}/\ell(n)) & \rightarrow & H^{n+1}(U, \mathbb{Z}/\ell(n)) \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 H^n(U, B/\ell(n)) & \rightarrow & H_{\partial\Delta^i \times S^1 \setminus U}^{n+1}(\partial\Delta^i \times S^1, B/\ell(n)) & \rightarrow & H^{n+1}(\partial\Delta^i \times S^1, B/\ell(n)) & \rightarrow & H^{n+1}(U, B/\ell(n))
 \end{array}$$

The middle map can be shown to be an isomorphism using the BL conj. for lower weights. The left map becomes surjective after passing to a limit of neighbourhoods of vertices  $x$  sing.  $pt \subset \partial\Delta^i \times S^1$ . This exploits the BK conj. for weight  $n$ . A diagram chase shows the vanishing of  $\alpha$ .

ANNETTE HUBER (Münster)

## 12. Norm varieties

Let  $\bar{a} = (a_1, \dots, a_n) \in (k^*)^n$  be a sequence of elements,  $Q_{\bar{a}} : \ll a_1, \dots, a_{n-1} \gg = a_n t^2$  a Pfister neighbour of  $\ll a_1, \dots, a_n \gg$ . Then  $Q_{\bar{a}}$  is called a norm-quadric associated with  $\bar{a}$ . It has the following property: if for some field extension  $F/k$  one has  $Q_{\bar{a}}(F) \neq \emptyset$ , then  $Q_{\bar{a}}$  is a 2-splitting variety for  $\bar{a}$ , i. e. the symbol  $\{a_1, \dots, a_n\} \in K_n^M(F)$  is 2-divisible. Denote by  $H90(n, \ell)$  the following statement:

$\forall$  field  $k$  of characteristic zero! and any  $i \leq n$  one has  $H_{\ell}^{i+1}(F, \mathbb{Z}_{(\ell)}(i)) = 0$ .

**Theorem:** Let  $B(i) := \tau_{\leq i+1} R\alpha_* \alpha^* \mathbb{Z}_{(\ell)}(i)$ ,  $\beta_i : \mathbb{Z}_{(\ell)}(i) \rightarrow B(i)$  the canonical morphism. Then

$$H90(n, \ell) \Leftrightarrow \beta_i \text{ quasi-isomorphisms for all } i \leq n.$$

**Cor:**  $H90(n, \ell)$  implies Bloch-Kato conjecture in weight  $n$ :

$$K_n^M(F)/\ell \xrightarrow{\eta_n} H_{\text{et}}^n(F, \mu_{\ell}^{\otimes n})$$

is an isomorphism.

### Main theorem (Voevodsky)

Assume that  $H90(n-1, \ell)$  holds in char=0 and  $\forall a = (a_1, \dots, a_n)$  there exists a  $\ell$ -splitting variety  $X_{\bar{a}}$  with the following properties:

(i)  $X_{\bar{a}} \times_{\text{Spec}(k)} \text{Spec } k(X_{\bar{a}})$  is rational over  $k(X_{\bar{a}})$

(ii)  $H_B^{n+1}(\tilde{C}(X_{\bar{a}}), \mathbb{Z}_{(\ell)}(n)) = 0$

Then  $H90(n, \ell)$  holds in characteristic zero.

VICTOR BATYREV (Tübingen)

## 13. The Rost motive I

**Part I. Injectivity of the norm homomorphism  $A_0(X, K_1) \rightarrow F^*$  for  $X/F$  a norm quadric.**

**Theorem (Rost, 1988)** *Let  $X$  be a norm quadric.*

*Then the norm homomorphism  $N : A_0(X, K_1) \rightarrow F^*$  is injective.*

**Part II. Motivic decomposition of isotropic quadrics and the nilpotence theorem.**

We work in the category of Chow motives. The motive of a smooth projective variety  $X$  is denoted also by  $X$ . We write  $\mathbf{pt}$  for  $\text{Spec } F$ ,  $F$  our base field.

**Proposition** *Let  $\varphi = \mathbb{H} \perp \psi$  ( $\varphi, \psi$  quadratic forms,  $\mathbb{H}$  hyperbolic plane). Write  $X_{\varphi}$  ( $X_{\psi}$ ) for the projective quadric given by  $\varphi$  ( $\psi$ ). Then  $X_{\varphi} = \mathbf{pt} \oplus \mathbf{pt}(\dim X_{\varphi}) \oplus X_{\psi}(1)$ .*



**Theorem** Let  $X$  be a projective quadric,  $f \in \text{End}(X)$ ,  $E/F$  an extension. If  $f_E \in \text{End}(X_E)$  is nilpotent, then  $f$  itself is nilpotent.

NIKITA KARPENKO (Münster)

## 14. The Rost Motive II

### Construction of the Rost motive

We work in the category of the Chow motives. Let  $a_1, a_2, \dots \in k^*$ , ( $\text{char } k \neq 2$ ).  
 $\varphi_n := \ll a_1, \dots, a_n \gg = \langle 1 \rangle \perp \varphi'_n$ ,  $\psi_n := \varphi_{n-1} \perp \langle -a_n \rangle$   $X_n$  and  $Z_n$  denote the  $\varphi_n$   
 and  $\varphi'_n$  associated projective quadrics.

**Theorem:** (Rost)

On  $X_n$  there exists a special projector  $p_n \in \text{End}(X_n)$ :

$$(X_n, p_n) =: M_n$$

$$(i) \quad (X_n, id_{X_n} - p_n) \cong Z_{n-1} \otimes \mathbb{L}$$

$$(ii) \quad Z_n \cong M_n \otimes \bigoplus_{i=0}^{d_n-1} \mathbb{L}^{\otimes i}$$

where  $d_n := 2^{n-1} - 1$  and  $\mathbb{L}$  is the Tate-motive.

As an implication of the theorems of I and II we get

**Theorem:**

$$H_B^{2^n-1}(\check{C}(Q_{\underline{a}}), \mathbb{Z}(2^{n-1})) = 0,$$

where  $\underline{a} = (a_1, \dots, a_n)$ ,  $Q_{\underline{a}} = X_n$  and  $\check{C}(Q_{\underline{a}})$  the simplicial scheme.

WIELAND FISCHER (Regensburg)

## 15. Homotopy theory of schemes (I)

This talk gives a construction of an analogue to the (stable) homotopy category in algebraic topology replacing topological spaces by simplicial sheaves of sets on the Nisnevich site over the category of smooth schemes over a field  $k$ . This homotopy category should play the same role for representing (co)homology theories by spectra as in topology.

The construction is based on the language of closed model categories and consists of a four-step-process:

1. Construct the simplicial homotopy category  $\mathcal{H}^S(k)$  by using the points of the site to define weak equivalences and cofibrations "pointwise".

2. The homotopy category  $\mathcal{H}^{\mathbb{A}^1}(k)$  one gets by making the structure morphism  $\mathbb{A}_k^1 \rightarrow \mathrm{Spec}(k)$  to an isomorphism.
- 3.+4. Imitate the construction of spectra in topology replacing the suspension functor by the smash product with  $\mathcal{T} = \mathbb{A}^1/(\mathbb{A}^1 \setminus \{0\})$  and get first the strict and finally the stable homotopy category (of spectra)  $\mathcal{SH}(k)$ .

TORSTEN FIMMEL (Köln)

## 16. Homotopy theory of schemes (II)

The aim of this talk was to give indications of a proof to the following theorem of Voevodsky: Let  $k$  be a field with resolution of singularities,  $\mathfrak{x}$  a simplicial smooth scheme  $/k$ . Then for all  $p, q$  we have an isomorphism

$$H_B^p(\mathfrak{x}, \mathbb{Z}(q)) = \mathrm{Hom}_{\mathcal{SH}}(\Sigma_T^\infty \mathfrak{x}, \mathbb{E}(q)[p])$$

where  $\mathbb{E} := (E_n, e_n : T \wedge E_n \rightarrow E_{n+1}) \in \mathcal{SH}$  is the “Eilenberg MacLane”-spectrum, where  $E_n = L(\mathbb{A}^n)/L(\mathbb{A}^n - \{0\})$ , viewed as an object in  $\mathcal{H}_*^{\mathbb{A}^1}(k)$  and where we have set  $\mathbb{E}(q)[p] := S_t^q \wedge S_S^{p-q} \wedge \mathbb{E}$ . I gave a proof for the corresponding statement in the setting of pointed simplicial sets and gave a construction of a pair of adjoint functors

$$DM_-^{\mathrm{eff}}(k) \rightleftarrows \mathcal{H}_*^{\mathbb{A}^1}(k)$$

which is analogous to the topological case.

IVAN KAUSZ (Köln)

## 17. Steenrod Operations

### §1 Topological construction of Steenrod operations

$$H^n(X, \mathbb{Z}/2) \rightarrow H^n(X, \mathbb{Z}/2) \quad X \text{ a topological space}$$

from a class  $P_2 \in H^{2n}(K_n)$ ,  $K_n = K(\mathbb{Z}/2, n)$ .

Working in the stable homotopy category, we define the Steenrod algebra as the endomorphism of the Eilenberg-MacLane spectrum. It is a Hopf algebra and we describe its dual and define the Milnor elements.

### §2 Motivic Steenrod operations

We state results listed in Voevodsky’s preprint “The Milnor conjecture”.

### §3 Margolis cohomology and the Milnor conjecture

We show how the vanishing of certain Margolis groups of the (reduced) simplicial scheme associated to the Rost motive implies the Milnor conjecture.

FLORENCE LECOMTE (Strasbourg)

## 18. Thom Spaces and Cobordism

This talk provided topological background for the proof of the Milnor conjecture in the last talks. The treated topics were:

- Thom spaces of vector bundles.
- Orientations of vector bundles in generalised cohomology theories and Thom isomorphism.
- Characteristic classes of vector bundles and classifying spaces.
- The Thom spectrum and complex cobordism.
- The action of Steenrod operations on the cohomology of Thom spectra.

MICHAEL PUSCHNIGG (Münster)

## 19. Thom Spaces and Cobordism II

We complete the proof of the Milnor conjecture by showing the vanishing of the algebraic Margolis cohomology groups of a certain simplicial sheaves  $\mathfrak{x}$  which is attached to a quadric.

In order to do this we have to develop two essential things:

- algebraic cobordism groups  $MGL_{*,*}(-)$  on  $\mathcal{H}(k)$ ,
- a geometric realization functor  $t_c : \mathcal{H}(k) \rightarrow \mathcal{H}$ , when  $\mathcal{H}$  is the classical homotopy category of spaces.

The essential point in the proof is to show that a certain map does not vanish, which can be seen after geometric realization.

ALEXANDER SCHMIDT (Heidelberg)

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