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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 14/1998

Algebraic Groups

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Die diesjährige Tagung über Algebraische Gruppen stand erneut, und in dieser Besetzung zum letzten Male, unter der Leitung von T. A. Springer (Utrecht), P. Slodowy (Hamburg) und J. Tits (Paris). An ihr nahmen 39 Mathematiker aus 13 Ländern teil. Einige der jüngeren Teilnehmer wurden dabei durch EU-Mittel unterstützt.

In 25 Vorträgen wurde über Fortschritte auf dem sich weit verzweigenden Gebiet der Theorie der algebraischen Gruppen berichtet. Neben den auch auf den letzten Tagungenim mittelpunkt stehenden Schwerpunkten

- Struktur- und Darstellungstheorie
- Algebraische Transformationsgruppen
- Schubertvarietäten
- Quantengruppen und Heckealgebren

galt diesmal auch den neuen Entwicklungen in den Bereichen

- Theorie der Gebäude
- Galoiskohomologie

besondere Beachtung (Einzelheiten entnehme man den folgenden Vortragsauszügen).

Zu Ende der Tagung sprachen die Teilnehmer den sheidenden Organisatoren, T. A. Springer und J. Tits, ihren Dank für die langjährige, vorbildliche Tätigkeit im Dienste der mathematischen Gemeinschaft und des mathematischen Forschungsinstitutes Oberwolfach aus.

Dank einer Unterstützung im Rahmen des EU-Programmes TMR (Training and Mobility of Researchers) konnten zusätzlich-einige jüngere Mathematiker zu der Tagung eingeladen werden. Dies ist einerseits eine hervorragende Förderung des wissenschaftlichen Nachwuchses und gibt andererseits den etablierten Kollegen die Gelegenheit, besonders begabte junge Mathematiker kennenzulernen.



Vortragsauszüge

M. BRION

Criteria for smoothness and rational smoothness

A complex algebraic variety X of dimension d is rationally smooth if

$$H_x^n(X) = \{ \begin{array}{c} \mathbb{Q} \quad for \ n = 2d \\ 0 \quad otherwise \end{array}, for all \ x \in X,$$

where $H_x^*(X)$ denotes the cohomology with support in x, and rational coefficients (clearly, smooth varieties are rationally smooth). For Schubert varieties, criteria for smoothness and rational smoothness have been obtained by Carrell Peterson, Kumar and Arabia. In this talk I presented generalizations of these criteria to a variety with an action of an algebraic torus T and an "attractive" fixed point x (i.e. all weights of T in the Zariski tangent space of X at x are contained in an open half space). I gave applications of these criteria to closures of double classes BwB in a "wonderful" compactification of a connected semisimple groupe G (where B is a Borel subgroup of G), and to closures of orbits of a symmetric subgroup of G in the flag manifold G/B.

A. BROER

Deutsche Forschungsgemeinschat

Semisimple Lie algebras and hyperplane arrangements

Let g be a semisimple Lie algebra over \mathbb{C} , t a Cartan subalgebra, R^+ positive roots, $\mathcal{A} = \{H_{\alpha}, \alpha \in R^+\}$, where $H_{\alpha} := \ker (\alpha : t \to \mathbb{C})$. Fix a subset $S \subset R^+$ (arbitrarily), define $\vartheta := \bigcap_{\alpha \in S} H_{\alpha}, \mathcal{A}_{\beta} := \{H_{\alpha} \cap \partial, \alpha \in R^+, h \not\supset \vartheta\}$ and let Q be a defining polynomial in $\mathbb{C}[\vartheta]$ of $\bigcup_{H \in \mathcal{A}_{\beta}} H \subset \vartheta$. Put $\vartheta^0 := \vartheta - \bigcup_{H \in \mathcal{A}_{\beta}} H$. Let L be the Levi subgroup of the adjoint group G with the Lie algebra $\mathfrak{z}_{\mathfrak{g}}(\vartheta)$. Choose a parabolic subgroup P with the Levi decomposition $P = P^u$. L. Put $n = \operatorname{Lie} P^u$, then $\vartheta + n$ is the solvable radical of \mathfrak{p} . Define:

$$\vartheta \leftarrow Y := G \times^{\mathfrak{p}} (\vartheta \ast \mathfrak{n}) \leftrightarrow Y^{L} \leftrightarrow 1 \ast \vartheta \cong \vartheta$$

Restriction to \mathfrak{d} gives gives a graded map of $S := \mathbb{Q}[\mathfrak{d}]$ modules

$$\rho : \operatorname{Mor}(Y, \mathfrak{g}) \to \operatorname{Mor}(\mathfrak{d}, \vartheta)$$

Identify $Mor(\mathfrak{d}, \vartheta)$ with $Der_{\mathbb{C}}(S)$.

Theorem 1. $Mor_G(Y, \mathfrak{g})$ is a free graded S-module, independent of the choice of P.

2. ρ is injective with image $\{D \in \text{Der}_{\mathbb{C}}(S), DQ \subset (Q)\}$.

Corollary [Orlik-Terao] The hyperplane arrangement $A_{\mathfrak{d}}$ in \mathfrak{d} is free. Corollary [Broer, Sommers-Trapa] We have

$$\sum_{i=0} \dim H^i(\mathfrak{d}^0, \mathbb{C}) t^i = \prod_{i=0}^s (1+e_i t),$$

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where $s = \dim \mathfrak{d}$ and e_1, e_2, \dots, e_s are the degrees of a homogeneous basis of $Mor_G(T^*(G/P), \mathfrak{g})$.

R. CARTER

Canonical bases and Lusztig's PL-function

A report was given on joint work by R.W. Carter and R.J. Marsh. This concerns the canonical basis B of the negative part $U^- = \langle F_1, ..., F_l \rangle$ of a quantum group U of type A_l .

The longest element w_0 of the Weyl group has a reduced expression of form $\underline{j} = 135...246...135...(N \text{ terms})$ where N = l(l + 1)/2, and U^- has the corresponding PBW-type basis $B_{\underline{j}} = \{F_{\underline{j}}^{c}; \underline{c} = (c_1, ..., c_N)\}$ where $c_i \in \mathbb{Z}, c_i \geq 0$. For each $b \in B$ there exists a unique \underline{c} such that $b \equiv F_{\underline{j}}^{c} \mod v\mathcal{L}$ where \mathcal{L} is the lattice $\mathbb{Z}[v]B_{\underline{j}}$. In this way canonical basis elements in B can be parametrised by non-negative integral vectors $\underline{c} \in \mathbb{R}^N$. The behaviour of the canonical basis vector appears to depend upon the regions of linearity of a PL-function $R : \mathbb{R}^N \to \mathbb{R}^N$ defined by Lusztig.

Each reduced word <u>i</u> for w_0 gives rise to a set $\mathcal{P}(\underline{i})$ of N-l partial quivers, determined by the chambers in its braid diagram. Here a partial quiver is a Dynkin diagram in which certain edges are labelled by arrows, such that the set of edges with arrows is non-empty and connected. It is shown how to construct, for each such <u>i</u>, a set of N non-negative integral vectors \underline{c}_{α_i} , $i = 1, ..., l; \underline{c}_p$, $p \in \mathcal{P}(i)$ parametrized by the *l* simple roots and the N-l partial quivers obtained from <u>i</u>. These vectors do not depend on <u>i</u>, but only on α_i and P respectively. It is conjectured that the set of all non-negative combinations of these vectors form a region of linearity $\Lambda(\underline{i})$ of Lusztig's function R, and that the canonical basis vectors $b \in B$ corresponding to vectors <u>c</u> in the interior of $\Lambda(i)$ are given by unonomials in $F_1, ..., F_l$ of form $F_{in}^{\alpha_i}, ..., F_{in}^{\alpha_N}$ for certain non-negative integral vectors $\underline{a} = (a_1, ..., a_N)$ which were explicitly described.

E. BAYER-FLUCKIGER

Galois Cohomology of the Classical Groups

Let k be a field, k_s a separable closure of k and $\Gamma_k = \text{Gal}(k_s/k)$. Let G be a linear algebraic group over k, smooth. As usual, one defines $H^1(k, G) = H^1(\Gamma_k, G(k_s))$. The following conjectures were made by Serre in 1962:

Conjecture 1: If $cd(k) \leq 1$, G connected, then $H^1(k, G) = 0$

Conjecture 2: If $cd(k) \le 2$, G semisimple, simply connected, then $H^1(k, G) = 0$.

Conjecture 1 was proved by Steinberg in 1965. Conjecture 2 is still not proved in full generality. We have the following:

Theorem [E. B.-Parimala, 1995]: If G is of classical type (with the possible

exception of groups of type triality D_4), of type G_2 or F_4 , then conjecture 2 holds.

More recently, Colliot-Thélène and Scheiderer made the following "Hasse Principle Conjectures". One says that a field k has virtual cohomological dimension $\leq n$, written $vcd(k) \leq n$, if there exists a finite extension k'/k such that $cd(k') \leq n$. Let Ω be the set of all orderings of k. For $v \in \Omega$, let k_v be the real closure of k.

HP Conjecture 1: vcd $(k) \leq 1, G$ connected, then the natural map $H^1(k, G) \rightarrow \prod_{v \in \Omega} H^1(k_v, G)$ is injective.

HP Conjecture 2: $vcd(k) \leq 2, G$ semisimple, simply connected, then $H^1(k, G) \rightarrow \prod_{v \in \Omega} H^1(k_v, G)$ is injective.

HP Conjecture 1 was proved by Scheiderer in 1996 (after some partial results by Colliot-Thélène and Dueros). In the case of classical groups and groups of type G_2 and F_4 , HP Conjecture 2 was proved by Parimala and E. B. The proof makes extensive use of the theorem of Merkurjev-Suslin.

M. ROST

On algebraic cobordism and the common slot lemma for algebras

An important consequence of the recent work of V. Voevodsky is the following: **Degree formula**: Let X, Y be proper smooth varieties over a field k (Char(k) \neq 0) of dimension $d = p^n - 1$ (p a prime, $n \ge 1$). Then for any morphism $f : X \to Y$ one has

$$\left(\frac{S_d(X)}{p}\right) = (\deg f) \left(\frac{S_d(Y)}{p}\right) \mod I_Y$$

Here $I_Y \subset \mathbb{Z}$ is the ideal generated by the degrees of the closed points on Y. The characteristic number $S_d(X) \in \mathbb{Z}$ is given by $S_d(X) = Q_d(c_1(TX), ..., c_n(TX))$ where Q_d is the d-th Newton polynomial. It is known (Milnor) that $S_d(X) \in p\mathbb{Z}$. Corollary 1: $\frac{S_d}{p} \in \mathbb{Z}/I_X$ is a birational invariant of X.

Corollary 2: If $I_Y \subset p\mathbb{Z}$ and $S_d(X) \notin p^2\mathbb{Z}$, then deg f is prime to p.

We discussed an application of Corollary 2 to the common slot lemma for cyclic algebras of degree p.

A major problem is to compute the number $S_d(X)$ for certain X. Here one uses equivariant resolution of singularities and a theorem of Conner-Floyd on fixed point free $(\mathbb{Z}/p)^n$ -actions.

J.-P. SERRE

On a formula of Kac and a theorem of Burnside

Let G be a semisimple algebraic group over a field k of characteristic 0. Assume G is of adjoint type. Let $g \in G$ be an element of G of finite order m, and $Z_G(g)$ its centralizer.

Theorem: One has dim $Z_G(g) \ge l + 2\sum_{i=1}^l \left[\frac{d_{i-1}}{m}\right]$, where $l = \operatorname{rank}(G)$ and the



 d_i are the degrees of the invariant polynomials for the root system of G (e.g. $d_i = 2, 8, 12, 14, 18, 20, 24, 30$ if G is of type E_8). Moreover, there is equality if g is contained in a principal PSL_2 of G.

The proof uses a formula of V. Kac (LN 848) and H. Weyl. Another application of this formula is:

Theorem : Let χ be an irreducible character of a compact Lie group K. Assume $\chi(1) > 1$. Then, there exists $x \in K$ of finite order, such that $\chi(x) = 0$. When K is finite, this is a theorem of Burnside.

O. MATHIEU

Modular representations of GL_n

Let $k = \overline{\mathbb{F}}_p$ and let $L(\lambda)$ be the simple $GL_n(k)$ -module with hightest weight λ . We have the following facts:

1: For $n \leq p$, there is a conjecture (Lusztig) for $ch(L_{\lambda})$.

2: For $n \ll p$, the Lusztig conjecture is proved (Andersen, Jantzen, Soergel). However, for the stable modular theory, i.e. the modular theory of $GL_n(\overline{\mathbb{F}}_p)$ when $n \to \infty$ (p is fixed) there are very few results and no conjectures. Denote by h_1, h_2, \ldots the simple coroots, and set $h_{ij} := h_i + \ldots + h_j$. We will explain a formula of a joint work with G. Papadopoulo for the $ch(L(\lambda))$ for all λ of the form:

$$\lambda = \sum_{i \leq l \leq j} a_i \omega_i$$
, with $(\lambda + \rho)(h_{ij}) \leq p$

As a consequence we get an explicit character formula for $L(m\omega_i)$ for all n, p, m, i(ω_i is the *i*-th fundamental weight). We will also mention recent joint work with J. Jensen about modular representations of the symmetric group.

The proof of the results is based on a modular version of Verlinde's formula (G. Georgiev and O. M. 1992).

P. LITTELMANN

Frobenius splitting and the quantum Frobenius map

Let \mathfrak{g} be a semisimple complex Lie algebra, $\mathfrak{g} = \mathfrak{n}^- \oplus \mathfrak{h} \oplus \mathfrak{n}^+$ the triangular decomposition, $\mathbb{Z} = \mathbb{Z}[\text{all roots of unity}]$ and $U_{\hat{\mathbf{Z}}}(\mathfrak{g})$ a Kostant-form of the enveloping algebra over \mathbb{Z} . The pairing $U_{\hat{\mathbf{Z}}}(\mathfrak{b}^-) \times \oplus_{\lambda \in P} V_{\hat{\mathbf{Z}}}(\lambda)^* \to \mathbb{Z}$, $(u, \sum f^{\lambda}) \mapsto \sum f^{\lambda}(uv_{\lambda})$ is non degenerate, where v_{λ} is the highest weight vector. A similar pairing can be defined for quantum groups at *l*-th roots of unity. The Frobenius maps of Lusztig Fr: $U_{\hat{\mathbf{C}}}(\mathfrak{b}^-) \to U_{\hat{\mathbf{Z}}}(\mathfrak{b}^-)$, Fr': $U_{\hat{\mathbf{Z}}}(\mathfrak{b}^-) \to U_{\hat{\mathbf{C}}}(\mathfrak{b}^-)$ induce dual maps $\operatorname{Fr}^* : \oplus_{\lambda \in P^+} V(\lambda)^* \to \oplus_{\lambda \in P^+} V_{\hat{\mathbf{C}}}(\lambda)^*$.

Theorem [P. L., S. Kumar] Let \mathfrak{g} be simply laced and k an algebraically closed field of characteristic p = l. Then Fr^* specializes to a splitting of the Frobenius

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map:

$H^{0}(G/B, \mathcal{L}_{\lambda}) \to H^{0}(G/B, \mathcal{L}_{p\lambda}) \xrightarrow{\operatorname{Fr}^{**}} H^{0}(G/B, \mathcal{L}_{\lambda}).$

Further, the splitting is the same as the one induced by the section of $\mathcal{L}_{2\rho}$ corresponding to the divisor consisting of the Schubert varieties of codim1 and the opposite Schubert varieties of codim1.

G. RÖHRLE

Recent results on the action of parabolic groups

Let k be an infinite field and let V be a finite-dimensional k-space. Further let GL(V) be the linear group of V and let P be a stabilizer of a flag F in V. By P_u we denote the unique maximal unipotent normal subgroup of P, the unipotent radical of P. Now P acts on P_u via conjugation and on \mathfrak{p}_u , the Lie algebra of P_u via the adjoint action. We describe some recent results classifying all instances when P acts on \mathfrak{p}_u or P_u with a finite number of orbits. Furthermore, in this instance we obtain a combinatorial formula for the number of orbits in the finite cases. This classification result for GL(V) involves a detailed study of the representation theory of a particular quiver with certain relations. For k algebraically closed, we shall provide a complete description of the partial order given by orbit closures on the set of P-orbits on \mathfrak{p}_u in the finite instances. It turns out that this partial order is equivalent to one given by purely combinatorial means and thus can be computed easily.

For k algebraically closed, we also present the classification of all parabolic subgroups P in any simple algebraic group of classical type with a finite number of orbits on \mathfrak{p}_u .

This is a report on various parts of joint work with T. Brüstle, L. Hille and G. Zwara.

L. HILLE

Actions of parabolic subgroups of GL_n

Let $P(d) \subseteq GL_n$ be a parabolic subgroup, which is the stabilizer of some flag $0 \subset V_1 \subset V_2 \subset ... \subset V_t$ of vector spaces of dimension vector $d = (\dim V_1, ..., \dim V_t)$. **Theorem** [H.-Röhrle] P acts on the unipotent radical with finitely manny orbits if and only if $t \leq 5$ for a proper flag as above.

More generally we consider the action of P on $P_u^{(l)}$, where $P_u^{(l+1)} := [P_u, P_u^{(l)}]$ and $P_u^{(0)} = P_u$.

Theorem [Brüstle-H.] P acts on $P_{u}^{(t)}$ with finitely manny orbits precisely if **1**. l = 1, 0 and $t \le 5 + 3l$.

2. l = 1, 2, ... and t < 6 + 2l.

Let Q be a directed biquiver, that is an oriented graph with two types of arrows:

Solid and dotted arrows. Assume that Q is a directed finite and connected biquiver. We define an algebraic variety $P(d) := \Pi GL(V_i) \times \bigoplus_w \operatorname{Hom}(V_i, V_j)$, where w runs over all dotted path (consisting of dotted arrows in Q) and dim $V_i = d_i$. This algebraic variety is a group with natural multiplication given by concatenation of paths. Let R(d) be an algebraic subvariety of

$$\overline{R}(d) := \bigoplus_{\overset{w}{\sim} \stackrel{a}{\rightarrow} \overset{w}{\sim}'} \operatorname{Hom}(V_i, V_j)$$

where w, w' are dotted paths and α is a solid arrow. Moreover, assume P(d) acts on R(d) via conjugation in the natural way given by the biquiver Q. Theorem [Bruüstle-H.] There exists a quasi-hereditary algebra A together with modules $\Delta(i)$ (called standard modules) such that the orbit of the action of G(d) on R(d) are in natural bijection with the modules over A having a Δ -filtration. In particular we can replace the action of P(d) on $\overline{R}(d)$ by an equivalent one of a reductive group.

H. KRAFT

Jordan's work on invariants and covariants of binary forms

In 1868 Paul Gordan proved that invariants and covariants of binary forms are finitely generated. His method was "constructive" and leads to explicit construction of these generators. Less known is a subsequent paper of C. Jordan (1876/79) where he gives explicit degree bounds for the generators.

Both results and in particular the technique called "symbolic method" were completely forgotten after Hilbert's famous paper (1890/93). But there are several reasons to look more closely at these old results:

1. Jordan's bounds are by far the best we have, and cannot be reproduced by our "modern" tools from representation theory.

2. The symbolic method leads to the explicit construction and description of covariants.

3. There is hope to generalize this to other groups than SL_2 .

In joint work with J. Weyman (Northeastern University) we have been able to understand Jordan's proof (and verify the bounds). Moreover, we we were able to work out the cases of binary cubics and quartics which were not completely known by the classics. We also developed some "straightening law" technique for the symbolic methods, for handling the symbolic expressions. The talk was a report on Jordan's result and a modern way how to understand his proof.

Theorem: Let W be a representation of $SL_2(\mathbb{C})$. Assume that all irreducible components of W have dimension $\leq N+1$. Then the covariants (=U-invariants) are generated by covariants of degree $\leq N^6$ and order $\leq 2N^2$.

Example: Let V_d denote the $(\overline{d} + 1)$ -dimensional irreducible representation of SL_2 . Consider the representation $M \otimes V_3 = V_3 \oplus ... \oplus V_3$. Then the ring of U-invariants of $M \otimes V_3$ is generated by 10 types of covariants corresonding to



the irreducible representations (of $GL(M) \times SL_2$) of the following list:

$$\begin{array}{l} \Lambda^2 M \otimes V_0 \qquad S^4 M \otimes V_0 \qquad S^{3,3} M \otimes V_0 \\ S^{2,1} M \otimes V_1 \qquad S^{4,1} M \otimes V_1 \\ S^2 M \otimes V_2 \qquad S^{3,1} M \otimes V_2 \\ M \otimes V_3 \qquad S^3 \otimes V_3 \qquad \Lambda^2 M \otimes V_4 \end{array}$$

C. DE CONCINI

Cohomology of Coxeter groups and braid groups

(joint work with M. Salvetti)

In the talk we have explained the construction of certain algebraic complexes for computing the cohomology of a finite Coxeter group \mathcal{W} with coefficients in a $\mathbb{Z}[\mathcal{W}]$ -module M. Similarily: If $B_{\mathcal{W}}$ is the associated Artin-Tits braid group we have decribed similar complexes for computing the cohomology of $B_{\mathcal{W}}$ with coefficients in a $\mathbb{Z}[B_{\mathcal{W}}]$ -module N. In the case in which a $\mathbb{Z}[\mathcal{W}]$ -module M is considered as a $\mathbb{Z}[B_{\mathcal{W}}]$ -module λ , we have defined a map $\tilde{\gamma}$ of complexes inducing the map $\gamma^* : H^*(\mathcal{W}, M) \to H^*(B_{\mathcal{W}}, M)$ corresponding to the quotient $\gamma : B_{\mathcal{W}} \to \mathcal{W}$. Using this in the case $M = \mathbb{Z}(-1)$, the sign module, we have deduced that if (\mathcal{W}, S) is irreducible and not of type A_{n-1} , with n having two distinct prime divisors, then the genus of the fibration $K(P_{\mathcal{W}}, 1) \to K(B_{\mathcal{W}}, 1)$, $P_{\mathcal{W}} = \ker\gamma$, is equal to n + 1.

J. CARRELL

Singular loci of Schubert varieties

We decribe the singular locus of a Schubert variety X(w) in G/B. Let SingX(w) have irreducible decomposition $X(x_1) \cup ... \cup X(x_k)$ where $x_1, ..., x_k < w$ in W. The problem is to identify these x_i . The point is that there exists a degeneration of the tangent space (à la Zariski) $T_{rx}X(w)$ to a subspace of $T_r.X(w)$ along a curve running from rx to x and contained in X(w), for any reflection $r \in W$ so that $x < rx \le w$. The main tool is a theorem of Dale Peterson: Suppose X(w) is nonsingular at rx for each r so that $x < rx \le w$, and suppose also that the degenerations from $T_{rx}X(w)$ into $T_xX(w)$ give the same result for all such r. Then X(w) is nonsingular at x. Therefore x determines an irreducible component of Sing X(w) if and only if X(w) is nonsingular at each rx a above and there exist two degenerations that give different results. We can give an (interesting) description of when this occurs in terms of root strings.



P. POLO

Generic singularities of certain Schubert varieties

The results presented in this talk are from joint work with M. Brion. Let k be an algebraically closed field, G a connected semi-simple algebraic group over k, $T \subset B \subset Q$ a maximal torus inside a Borel subgroup inside a parabolic subgroup Q. Let W be the Weyl group, R the root system, R^+ the positive roots and Δ the simple roots. For $w \in W$ let $C_{wQ} = BwQ/Q$ and $X_{wQ} = \overline{C_{wQ}}$. It is known that when $C_{yQ} \subset X_{wQ}$, there exists a T-stable locally closed subvariety $N_{yQ,wQ}$ of X_{wQ} such that $X_{wQ} \simeq C_{yQ} \times N_{yQ,wQ}$. We study $N_{yQ,wQ}$ in certain cases.

First let P be a parabolic subgroup containing B, L the Levi subgroup of Pcontaining T and β a simple root not in L. Let $V_L(-\beta)$ be the Weyl module for L with highest weight $-\beta$ and $\mathcal{C}_L(-\beta)$ the orbit closure of the highest weight vector. Also, let U_P^- be the unipotent radical of the parabolic opposed to P. **Theorem 1:** $U_{P}^{-}\dot{P}/P \cap \overline{Ps_{\beta}P}/P$ is L-isomorphic to $C_{L}(-\beta)$.

We then extend this result to show that under certain hypotheses on y, w, $N_{uQ,wQ}$ is also L-isomorphic to a certain orbit closure of a highest weight vector. Finally, we show that the hypotheses are satisfied if G has only components of type A, D, E and Q is a maximal parabolic corresponding to a minuscule fundamental weight.

Theorem 2: Let G, Q be as above, $w \in W$. Let $P(w) = \operatorname{Stab}_G(X_{wQ})$, then: 1. The singular locus of X_{wQ} , $Sing(X_{wQ})$, equals $X_{wQ} \setminus P(w)e_{wQ}$ (hence is as large as possible).

2. For every irreducible component X_{yQ} of $Sing(X_{wq})$, the hypotheses mentioned above are satisfied. Therefore $N_{yQ,wQ}$ is isomorphic to a certain orbit closure of a highest weight vector.

This result can be extended to cover also the case of Schubert varieties in the Grassmannian of lagrangian subspaces in a symplectic vector space. ۰.,

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V. LAKSHMIBAI

Deutsche Forschungsgemeinschaf

Degeneracy schemes and Schubert varieties

Let G be a semisimple algebraic group over a field k. For a Schubert variety $X(w) \subset G/Q, Q$ being a parabolic subgroup of G, let $Y(w) := X(w) \cap O^{-}$, where O^- is the opposite big cell in G/Q. Note that Y(w) is normal, Cohen-Macauley and has rational singularities in all characteristics. We exhibit two classes of affine varieties:

1. Ladder determinantal varieties

2. Quiver varieties (orbit closures in the space of representations of equioriented quivers of type A),

for both of which the normality and Cohen-Macauley properties are concluded by identifying them with Y(w)'s for suitable X(w)'s in suitable SL_n/Q . As a consequence we obtain that the degeneracy schemes Ω_w constructed by Fulton (in the context of universal Schubert polynomials) are reduced, Cohen-Macauley and normal in all characteristics.

J. TITS

Algebraic simple groups of rank two and Moufang polygons

Let (\mathcal{W}, S) be a Coxeter system. A building of type (\mathcal{W}, S) is a set Δ endowed with a "distance function" $d: \Delta \times \Delta \to \mathcal{W}$ satisfying certain axioms which will not be recalled here but which roughly mean that Δ contains "many" subsets isometric to \mathcal{W} itself (endowed with the metric $(w, w') \mapsto w^{-1}w'$). The building Δ is said to be thick if Card $\{x \in \Delta, d(x, x_0) = s\} \ge 2$ for all $x_0 \in \Delta$ and $s \in S$.

If k is a field, <u>G</u> a simple k-group, G the group <u>G(k)</u> of its k-rational points, <u>P</u> a minimal k-parabolic subgroup of <u>G</u>, $P = \underline{P}(k)$, W the relative Weyl group of G and S the generating set of W canonically associated to P, then the set $\Delta = G/P$ endowed with the W-metric d defined (via Bruhat decomposition) by $Pg^{-1}g'P = Pd(gP,g'P)P$ for $g,g' \in G$ is a building of type (W, S). Thus, to every k-group <u>G</u> as above there is naturally associated a building of spherical type (i.e. $Card(W) < \infty$). As is shown in the Springer LNM 386, the converse is true for thick buildings of irreducible type and rank ≥ 3 , provided that one suitably extends the notion of algebraic group: One must admit as such classical groups over arbitrary division rings and also the "mixed groups" of type $F_4(k, K)$ (see loc.cit.) to which are associated buildings of type F_4 .

The analogous result is definitely false in rank 2; indeed, for any integer $m \geq 3$, "generalized m-gons", i.e. buildings of type of the dihedral group of order 2m, are totally "unclassifiable". However, in 1974, the speaker conjectured that one could characterize the generalized polygons arising from algebraic simple groups (of relative rank 2) by imposing a certain geometric condition, the "Moufang property", which roughly means the existence of "sufficiently many transvections". This again supposes a suitable extension of the notion of algebraic groups; in particular, the Ree groups of type ${}^{2}F_{4}$ (corresponding to the diagram - or index -(()) are "responsible" for the existence of Moufang octagons. That conjecture covers a great variety of statements which were progressively established in the course of the recent years. An important breakthrough was achieved by the speaker and shortly after, with a shorter proof, by R. Weiss; namely, both of them showed in the early '80's that Moufang mgons exist only for m = 3, 4, 6, 8. The classification of Moufang quadrangles, the only remaining problem in 1997 was completed by R. Weiss (see below the summary of his lecture) who inexpectedly discovered in so doing, a new family of Moufang quadrangles, but it was shown by B. Mühlherr and H. van Maldeghem (see summary of the latter's lecture)that those "new" quadrangles were in fact associated with forms of mixed groups of type F_4 (forms corresponding to the index (, in trying to give an explicit formulation of his conjecture in 1974, the speaker had overlooked the possible existence of such forms.



R. WEISS

The classification of Moufang polygons

(joint work with J. Tits)

Moufang n-gons exist only for n = 3, 4, 6, 8. Moufang triangles are parametrized (in a precisely defined way) by alternative division rings (classified by Bruck and Kleinfeld). Moufang hexagons by anisotropic cubic norm structures (classified by Petersson and Racine) and Moufang octagons by pairs (K, σ) , where K is a field of characteriste 2 and σ an endomorphism of K whose square is the Frobenius map. Let U_1, U_2, U_3, U_4 be root groups of a Moufang quadrangle Γ . If $[U_1, U_3] = 1$ or $[U_2, U_4] = 1$, then Γ is parametrized by a pair (K, σ) , where K is a skew-field and σ an involution of K, or a triple (K, L, q), where L is a vector space over K, a commutative field, and q is an anisotropic quadratic form on L, or $[U_1, U_3] = [U_2, U_4] = 1$ and Γ is parametrized by a field K of characteristic two and two additive subgroups having certain properties. Suppose $[U_1, U_3] \neq 1$ and $[U_2, U_4] \neq 1$, then Γ has a certain canonical subquadrangle Γ_0 of involution type or of quadratic form type as just described. In the first case, Γ is parametrized by a pair (X,q), where X is a right vector space over the skew field K and q an anisotropic pseudo-quadratic form on X with respect to the involution σ . If Γ_0 is parametrized by a triple (K, L, q), then q must have certain properties; in particular, $\dim_K L_0 = 6, 8$ or 12 and in each of these three cases, Γ is uniquely determined, or char(K) = 2, then there exists a field F such that $K \supseteq F \supseteq K^2$, $L_0 \cong K^4 \oplus F$, $F = \operatorname{rad}(q)$ and again Γ is uniquely determined. These are the quadrangles of type E_6, E_7, E_8 or F_4 .

Theorem Every Moufang polygon is isomorphic to one of the polygons described above.

H. VAN MALDEGHEM

Quadrangles of type F_4

(joint work with B. Mühlherr)

In this talk, I explained, how the recently discovered Moufang quadrangles were proved to be, after all, of "algebraic origin" by showing that they arise as certain "forms" (or equivalently as structure of fixed points of an involution σ) in a mixed group of type F_4 (or building of that type). Over the field with one element, the situation can be explained with the following figure (I drew an apartment, or Coxeter complex of type F_4 , the 24-cell, explaining the involution σ . There are exactly four fixed points and four fixed octahedra, forming a quadrangle, there are no fixed edges, nor fixed triangles). Certain lines form a hypercube (a \bigcirc subdiagram of \bigcirc and a subquadrangle of the new Moufang quadrangle is formed by the fixed squares together with four cubes.

B. MÜHLHERR

Quadrangles of type E_N

The classification of Moufang quadrangles due to Tits and Weiss provides as a by-product the commutation relations for all Moufang quadrangles. It was observed by Weiss that each quadrangle of type E_8 (resp. E_7) contains a quadrangle of type E_7 (resp. E_6) as a subquadrangle. Using the fact that all these quadrangles arise as sets of fixed points of involutions acting on the appropriate buildings, we have a geometric proof of this fact. As a further result we have a geometric proof for the existence of the quadrangles of type E_n . This proof is based on the "Local to Global Theorem" for twin buildings. We illustrate the idea at an example: Given an involution of $E_7(k)$ we construct the twin building $\tilde{E}_8(k)$. Now we obtain an involution of $E_8(k(t))$ by looking at the building at infinity.

P. ABRAMENKO

Finiteness properties of Kac-Moody groups over finfite fields

Let \underline{G} be a Kac-Moody group functor in the sense of Tits ("minimal version", split), (W, S) the associated Coxeter system and diag(\underline{G}) := diag(W, S) its Coxeter diagram. Suppose card $(S) < \infty$ and card $(W) = \infty$. We say that diag(\underline{G}) is n-spherical if $W_J := \langle J \rangle$ is finite for all $J \subseteq S$ with card $(J) \leq n$. Given a finite field \mathbb{F}_q , we set $G = \underline{G}(\mathbb{F}_q)$ and denote by (G, B_+, B_-, N, S) the standard twin BN-pair associated to G. The main result discussed in the talk was the following:

Theorem 1: Assume that diag(\underline{G}) is *n*-spherical, diag(\underline{G}) does not contain any subdiagram of rank $\leq n$ of type F_4 , E_6 , E_7 , E_8 , and $q \geq 2^{2n-1}$, then the parabolic subgroup $P_e^J := B_e W_J B_e$ of $G = \underline{G}(\mathbb{F}_q)$ is of type F_{n-1} for any $J \subseteq S$, $\epsilon \in \{+, -\}$.

If additionally card(W_J) < ∞ and diag(\underline{G}) is not (n + 1)-spherical, then P_e^J is not of type F_n .

The proof of this result uses decisively the action of G (and its parabolic subgroups) on the twin building associated to the twin BN-pair (G, B_+, B_-, N, S). For some other groups admitting twin BN-pairs, analogous results can be derived in a similar way, e.g. the following:

Theorem 2: Let \underline{H} be a simple \mathbb{F}_q -group of classical type, $n := \operatorname{rk}_{\mathbb{F}_q}(\underline{H}) \ge 1$ and $q \ge 2^{2n-1}$. Then $\underline{H}(\mathbb{F}_q[t]), \underline{H}(\mathbb{F}_q[t, t^{-1}])$ are of type F_{n-1} and $\underline{H}(\mathbb{F}_q[t])$ is not of type F_n .

Note that Theorem 2 is just a specialization of Theorem 1 in the case \underline{H} splits over \mathbb{F}_q since then $\underline{H}(\mathbb{F}_q[t, t^{-1}]) = \underline{G}(\mathbb{F}_q)$ for some Kac-Moody group \underline{G} of affine type.



E. SOMMERS

A new approach to computing the fundamental group of a nilpotent orbit

Let G be a connected, simple algebraic group over \mathbb{C} with Lie algebra g. Let N be a nilpotent element in g and let $Z_G(N)$ be the centralizer of N inG. When G is adjoint, we give a unified description of the conjugacy classes in the (finite) group $Z_G(N)/Z_G^O(N)$, generalizing the Bala-Carter classification of nilpotent orbits in g. Our result turns out to be enough to determine these groups.

We also state a conjecture for the G-module structure of the global functions on the universal cover of the orbit through N.

V.L. POPOV

Reductive subgroups of $Aut(\mathbb{A}^3)$ and $Aut(\mathbb{A}^4)$

Let k be an algebraically closed field of characteristic 0. All varieties, morphisms etc. below are defined over k.

Theorem 1: Let G be a connected reductive subgroup of $Aut(\mathbb{A}^3)$, then G is conjugate to a subgroup of GL_3 .

Theorem 2: Let G be connected reductive subgroup of $Aut(\mathbb{A}^4)$ which is not a one or two dimensional torus, then G is conjugate to a subgroup of GL_4 .

Remarks: 1. It is an open problem whether one can drop the assumption that G is connected in Theorem 1.

2. Since there are nonlinearizable actions of $O_2 = k^* \rtimes \mathbb{Z}/2$ on \mathbb{A}^4 , one cannot drop this assumption in Theorem 2.

in the proofs some general statements are used, namely:

Theorem 3: Let G be a connected semisimple group and $V = (L_1 \oplus ... \oplus L_1) \oplus ... \oplus (L_i \oplus ... \oplus L_j)$, where each L_i appears m_i -times and where the L_i are simple G-modules, $L_i \neq L_j$ for $i \neq j$. Assume that $k[V]^G = k$. If for all i we have $m_i = \dim L_i^H$, where H is the generic stabilizer for G on V, then any G-equivariant automorphism of V (as algebraic variety) is linear.

Theorem 4: Let G be a reductive algebraic group, V a simple G-module and H a reductive group acting by G-automorphisms of the algebraic variety V. Then the natural action of $G \times H$ on V is linearizable.

Theorem 5: Let G be an algebraic group, V a G-module and X an irreducible algebraic variety. Let $Y = V \times X$ and H be an algebraic group acting on Y by G-automorphisms (G acts on Y via V). Assume that:

1. For all $v \in V$ we have $0 \in \overline{G.v}$.

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2. $k[X]^* \subset k[X]^H$, where $k[X]^*$ stands for the invertible functions in k[X].

3. The group of all G-equivariant automorphisms of V is $k^* id_V$.

Then there is a character $\chi: H \to k^*$ and an action of H on X such that the

natural action of $G \times H$ is given by $(g, h).(v, x) = (\chi(h).g.v, h.x)$ for all $g \in G$, $h \in H$, $v \in V$ and $x \in X$.

M. GRINBERG

A generalization of Springer theory using nearby cycles

We state a condition on a smooth subvariety of \mathbb{C}^n , called transversality at infinity. For $X \subset \mathbb{C}^n$ transverse to infinity, we show that the Fourier transform on the nearby cycles sheaf on the asymptotic cone as $(X) \subset \mathbb{C}^n$ is an intersection homology sheaf on $(\mathbb{C}^n)^*$. This result is applied to the following situations:

1. $X \subset \mathbb{C}^n$ is the general fibre of a product of linear forms.

2. $X \subset g$ is a closed adjoint orbit in a semi-simple Lie algebra (this is the Springer theory case).

3. $X \subset \mathfrak{p}$ is a closed K-orbit in a symmetric space.

4. $X \subset V$ is the general fibre of a quotient map $V \to G \setminus V$ for a polar representation of G on V. This example generalizes many aspects of Springer theory.

A. HELMINCK

Orbits and invariants associated with a pair of commuting involutions

(joint work with G. Schwarz)

Let σ , θ be commuting involutions of the connected reductive algebraic group Gwhere σ , θ , G are defined over a (usually algebraically closed) field k, char $(k) \neq 2$. We have fixed point groups $H = G^{\sigma}$ and $K = G^{\theta}$ and an action $(H \times K) \times G \rightarrow G$, where $((h, k), g) \mapsto hgk^{-1}$, $h \in H$, $k \in K$, $g \in G$. Let $G//(H \times K)$ denote $\operatorname{Spec}\mathcal{O}(G)^{H \times K}$ (the categorical quotient). Let A be maximal among subtori B of G such that $\theta(b) = \sigma(b) = b^{-1}$ for all $b \in B$. There is the associated Weyl group $\mathcal{W} := \mathcal{W}_{H \times K}(A)$. We show:

1. The inclusion $A \to G$ induces isomorphisms $A/W \to G//(H \times K)$. In particular, the closed $(H \times K)$ -orbits are precisely those which intersect A.

2. The fibres of $G \to G//(H \times K)$ are the same as those occuring in certain associated symmetric varieties. In particular, the fibres consist of finitely many orbits.

We investigate:

1. The structure of W and its relations to other naturally occuring Weyl groups and the action of $\sigma\theta$ on the A-weight space of g.

2. the relation of the orbit type stratifications of A/W and $G//(H \times K)$.

Along the way we simplify some of Richardson's proofs for the symmetric case $\theta = \sigma$, and at the end we quickly recover results of Berger, Flensted-Jensen, Hoogenboom and Matsuki for the case $k = \mathbb{R}$.



D. PANYUSHEV

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On commuting varieties associated with semi-simple Lie algebras

Let \mathfrak{g} be a semi-simple Lie algebra over $k = \mathbb{C}$, $\mathcal{C} := \{(x, y) \in \mathfrak{g} \times \mathfrak{g}, [x, y] = 0\}$ the commuting variety. Except for irreducibility very little is known about \mathcal{C} . However, in some special cases, more information can be obtained. Consider the following particular case:

g is simple and **p** is a parabolic subalgebra with abelian nilpotent radical $V = \mathbf{p}^u$; let V^* be the nilpotent radical of the opposite parabolic. Define $\mathcal{C} := \{(x, y), x \in V, y \in V^*, \{x, y\} = 0\}$. The main results are:

1. $C = \bigcup_{i=0}^{r} C_i$, where r + 1 is the number of G-orbits in V ($G = \operatorname{Aut}(\mathfrak{g})^{0}$). 2. Each C_i is particular with

2. Each C_i is normal with rational singularities and the algebra of covariants $k[C_i]^U$ is polynomial (U the unipotent subgroup of G corresponding to V). 3. There is an explicit construction of an equivariant resolution of singularities

of \mathcal{C} .

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