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Functional Analytic and Complex Analytic Methods in the Theory of Linear Partial Differential Equations

19. April 1998 bis 25. April 1998

This was the first Oberwolfach conference on Functional and Complex Analytic Methods in the Theory of Linear Partial Differential Equations. It was organized by R. Meise (Düsseldorf), B.A. Taylor (Ann Arbor) and D. Vogt (Wuppertal).

To the organizers as well as to the participants opinion this conference has been very successful. The 24 talks in which excellent recent research results were exhibited have been throughout on a high scientific level and of a very good quality of presentation. Main topics were:

- Problems of surjectivity and existence of continuous right inverses for linear partial differential operators, systems of partial differential equations, convolution operators and restriction operators under various regularity conditions.
- Linear and geometric aspects of the theory of analytic and plurisubharmonic functions with applications to surjectivity problems of linear partial differential equations, Phragmén-Lindelöf conditions on algebraic varieties.

Moreover, there were contributions to the theories of hyper– and microfunctions, interpolation of analytic functions, pseudoconcave manifolds, nonlinear Schrödinger equations, special functions, fundamental solutions, products of algebras and of weighted function spaces.

Due to the restricted number of talks there was enough time for critical and fruitful discussions, which was appreciated by the participants and led to further research progress. Moreover, the pleasant atmosphere provided by the staff of the Mathematisches Forschungszentrum Oberwolfach contributed a lot to the success of the conference.

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Vortragsauszüge

K.D. Bierstedt : (joint work with Silke Holtmanns)

An operator representation for weighted spaces of vector valued holomorphic functions

Let G denote an open subset of $\mathbb{C}^N$ and V a Nachbin family on G which induces a topology stronger than that of uniform convergence on the compact subsets of G. For the weighted space $HV(G)$ of holomorphic functions with O-growth conditions and for any quasibarrelled locally convex space E, we prove the topological isomorphism

$$HV(G, E'_b) = \mathcal{L}_b(E, HV(G)).$$

A similar, but technically more complicated isomorphism for weighted spaces $CV(X)$ of continuous functions is also derived. This generalizes some results in joint papers of the author with J. Bonet [*Results Math.* **14** (1988)], J. Bonet and A. Galbis [*Michigan Math. J.* **40** (1993)], and J. Bonet and J. Schmets [*Note Mat.* **10** (1990)], and it should be compared with the ε-product representations for the corresponding spaces $HV_0(G, E)$ (resp., $CV_0(X, E)$) of holomorphic (resp., continuous) functions with o-growth conditions. Finally, we show the topological isomorphism

$$HV_1(G_1, HV_2(G_2)) = H(V_1 \otimes V_2)(G_1 \times G_2).$$

J. Bonet : (joint work with A. Galbis and S. Momm)

Convolution operators on spaces of ultradifferentiable functions

Let $\mathcal{E}_{(\omega)} = \mathcal{E}_{(\omega)}(\mathbb{R}^n)$ be the space of ultradifferentiable function of Beurling type associated with a quasianalytic weight ω. Let $\mu \in \mathcal{E}'_{(\omega)}$. The Fourier–Laplace transform of μ is denoted by $\hat{\mu}$. The surjectivity of the convolution operator $T_\mu : \mathcal{E}_{(\omega)} \to \mathcal{E}_{(\omega)}$ associated with μ is characterized in terms of several equivalent slowly decreasing conditions on $\hat{\mu}$. This is applied to obtain the following results: Let $\omega \leq \sigma$ be weights.

(1) $T_\mu \mathcal{E}_{(\omega)} \supset \mathcal{E}_{(\sigma)}$ if and only if T_μ is surjective on $\mathcal{E}_{(\sigma)}$.

(2) $\mathcal{E}_{(\sigma)}$ is contained in the range of every ultradifferentiable operator on $\mathcal{E}_{(\omega)}$ if and only if there are $C > 0$, $R > 0$ such that for all $R \geq R_0$ we have

$$\inf_{S>R} \frac{\omega(S)}{\log(\frac{S}{R})} \leq C \frac{\sigma(R)}{\log(\frac{R}{\sigma(R)})}.$$

The corresponding results for the Roumieu classes $\mathcal{E}_{\{\sigma\}}$ are obtained.

(3) Contrary to what is known to happen in the non-quasianalytic case, there exist a quasianalytic weight ω and $\mu \in \mathcal{E}'_{(\omega)}$ such that $T_\mu \mathcal{E}_{(\omega)}$ does not contain the space $\mathcal{A}(\mathbb{R}^n)$ of real analytic functions.

R.W. Braun : (joint work with R. Meise and B.A. Taylor)

Phragmén-Lindelöf conditions: A characterization for a class of graph varieties

For a homogeneous polynomial P_m of degree m in n variables the partial differential operator $P(D) = P_m(D) - i\partial/\partial x_{n+1}$ is investigated. It is shown that $P(D)$ can only have a continuous linear right inverse on $C^\infty(\mathbb{R}^{n+1})$ if the localization of P_m in each real root is square-free. In three variables, this leads to the following theorem: $P(D)$ has a continuous linear right inverse on $C^\infty(\mathbb{R}^4)$ if and only if P_m has real coefficients and no elliptic factors, and for each real $\theta \neq 0$ with $P_m(\theta) = 0$ the polynomial P_m is locally hyperbolic in θ and $(P_m)_\theta$ is square-free. From the point of view of Phragmén-Lindelöf conditions, this means that a characterization is given of those homogeneous polynomials P_m in three variables for which the variety of $P_m + 1$ satisfies a close analogue of the classical Phragmén-Lindelöf principle in the plane.

S. Dierolf : (joint work with Khin Aye Aye and K.H. Schröder)

Semidirect products of groups and algebras

Motivated by the concept of a (top.) group X, which is the (top.) semidirect product of a normal subgroup G and a subgroup H, which was introduced by W. Roelcke in the seventies, we call a (top.) algebra A the (top.) semidirect product of an ideal C and a subalgebra B, if the map $C \times B \to A$, $(c, b) \mapsto c + b$, is bijective (resp. top.).
We introduce a general method to construct such semidirect products, which includes the adjunction of a unit element and direct products as special cases. Another special case is the semidirect product $C \times_s C$ of an algebra C with itself; given algebra topologies $\mathfrak{T}$, $\mathfrak{S}$ on C, the semidirect product $(C \times_s C, \mathfrak{T} \times \mathfrak{S})$ is a topological algebra iff $\mathfrak{S} \supset \mathfrak{T}$ (provided C has a unit element).
From this observation we derive a short exact sequence of commutative algebras

$$0 \to (C, \mathfrak{T}) \to (C \times_s C, \mathfrak{T} \times \mathfrak{S}) \to (C, \mathfrak{S}) \to 0,$$

endowed with Banach space topologies, which is topologically exact as a sequence of Banach spaces, such that $(C, \mathfrak{T})$, $(C, \mathfrak{S})$ are Banach algebras but multiplication in $(C \times_s C, \mathfrak{T} \times \mathfrak{S})$ is not continuous.

P. Domański : (based on a joint paper with Mikael Lindström)

Interpolation of analytic functions with restricted growth

Let $\mathbb{D}$ be the unit disc on the complex plane and let $v : \mathbb{D} \to \mathbb{R}_+$ be a strictly positive continuous function tending to zero at the boundary which is *radial* (i.e., $v(z) = v(|z|)$) and of *moderate decay* (i.e., $\inf_{n\in\mathbb{N}} \frac{v(1-2^{-n-1})}{v(1-2^{-n})} > 0$). We consider a Banach space of analytic functions of the form:

$$H_v^\infty(\mathbb{D}) := \{f \in H(\mathbb{D}) : \|f\|_v := \sup |f(z)|v(z) < \infty\}.$$

A sequence $(z_n) \subseteq \mathbb{D}$ is called a set of interpolation (linear interpolation, sampling, resp.) iff the map $T : H_v^\infty(\mathbb{D}) \to l_v^\infty$, $T(f) = (f(z_n))_{n\in\mathbb{N}}$, is surjective (has a continuous linear right inverse, is an isomorphism into, resp.) where $l_v^\infty := \{x = (x_n) : \|f\|_v := \sup_{n\in\mathbb{N}} |x_n|v(z_n) < \infty\}$.

We give sufficient conditions and necessary conditions for a sequence to be a set of (linear) interpolation or of sampling. In some cases we obtain characterizations. In particular, we show that sets of interpolation are stable with respect to small (in the pseudohyperbolic metric) perturbations. They are also uniformly discrete with respect to the same metric.

The results strengthen some earlier results of K. Seip and they are based on an observation that in some cases the faster the weight v tends to zero at the boundary the more interpolation sequences exist.

U. Franken :

Extension of real analytic data on a characteristic hypersurface

We let $P(D)$ be a linear partial differential operator with constant coefficients in the variables $x_1, \ldots, x_n$, $H := \{x \in \mathbb{R}^n : x_n = 0\}$ and let P_m denote the principal part of the associated polynomial P. We will characterize an extension property for real analytic data on H. To be more precise we will show that there exists $R \geq 1$ such that for each $f \in \mathcal{A}(H \cap B_R)$ with $P(D)f = 0$ there exists $g \in \mathcal{A}_\omega(B_1)$ with $P(D)g = 0$ such that $f|_{B_1 \cap H} = g|_{B_1 \cap H}$ if and only if P_m satisfies the Petrowsky condition, i.e. for each $\xi' \in \mathbb{R}^{n-1}$ the polynomial $\tau \mapsto P_m(\xi', \tau)$ either vanishes identically or has only real roots. Here B_R denotes the unit ball of radius $R > 0$ and center 0, $\mathcal{A}$ denotes the class of real analytic functions and $\mathcal{A}_\omega$ denotes the class of partially real analytic functions in the variables $x_1, \ldots, x_{n-1}$ which are $\{\omega\}$–ultra–differential with respect to the variable x_n.

L. Frerick :

Extension operators for spaces of C^∞–functions

Let $K \subset \mathbb{R}^n$ be compact. We assume that K is C^∞–determining, i.e. whenever $f \in \mathcal{E}(\mathbb{R}^n) := \{g : \mathbb{R}^n \to \mathbb{C} : g \text{ is arbitrary often differentiable}\}$ vanishes on K, then also all its derivatives $\partial^\alpha f$, $\alpha \in \mathbb{N}_0^n$, vanishes on K.

We consider the question, when there exists an operator E from the space $\mathcal{E}(K)$ of all C^∞–functions on K into $\mathcal{E}(\mathbb{R}^n)$ such that $E(f)|_K = f$ for all $f \in \mathcal{E}(K)$.

We give a characterization for the existence of such an extension operator in the spirit of Pawłucki and Plésniak, which reads as follows: $\xi(K)$ admits an extension operator if only if there exist $\theta \in (0,1)$, $p \in \mathbb{N}_0$ such that for all $\alpha \in \mathbb{N}_0^n$ there exists $r > 1$ such that for all k there is $C_k > 0$ such that for all $P \in \mathbb{C}[x_1, \ldots, x_n]$, $\deg(P) \leq k$ and all $x_0 \in K$:

$$|\partial^\alpha P(x_0)| \leq C_k \sup_{x \in K} |P(x)|^\theta \inf_{1 \geq \varepsilon > 0} \tfrac{1}{\varepsilon^r} ||P||_{B_\varepsilon(x_0) \cap K, p}^{1-\theta}.$$

Here denotes $||\cdot||_{B_\varepsilon(x_0) \cap K, p}$ the p-th quotient norm w.r.t. the set $\{x \in K : ||x - x_0||_\infty \leq \varepsilon\}$.

H. Komatsu :

Suppleness of sheaves of microfunctions associated with ultradistributions

A sheaf J of vector spaces is said to be *supple* if

$$J_{A\cup B}(U) = J_A(U) + J_B(U)$$

holds for any closed sets A and B in U. Here $J_A(U)$ denotes the space of sections over U with support in A.

We sketch a proof of the following theorem (due to Bengel–Schapira (1979) for distributions and to Eida (1989) for ultradistributions) under the framework of our definition of microfunctions [*Lecture Notes in Math* **1495** (1991)].

Theorem *The sheaf C^* of microfunctions associated with ultradistributions of class $*$ is supple.*

An immediate consequence is Martineau's edge-of-the-wedge theorem in each class $*$.

M. Langenbruch :

Surjective partial differential operators on spaces of real analytic functions

Let $\mathcal{A}(\Omega)$ be the space of real analytic functions defined on an open set $\Omega \subset \mathbb{R}^n$. Let $P(D)$ be a partial differential operator with constant coefficients and principal part P_m. The localizations of P_m at ∞ are defined by

$$L(P_m) := \left\{\lambda Q \not\equiv \text{const} \,|\, \lambda > 0, \exists\, x_j \to \infty, x_j \in \mathbb{R}^n : Q(x) = \lim_{j\to\infty} \frac{P_m(x+x_j)}{\tilde{P}_m(x_j)}\right\}$$

where $\tilde{P}_m(x) := (\sum_\alpha |P_m^{(\alpha)}(x)|^2)^{1/2}$.

Our main result is the following:

Theorem *Let $P(D) : \mathcal{A}(\Omega) \to \mathcal{A}(\Omega)$ be surjective, $\Omega := \{x \in \mathbb{R}^n | \langle x, N\rangle < 0\}$. Then we have for any $Q \in L(P_m) : \forall x \in \mathbb{R}^n, 0 \neq \tau \in \mathbb{R} : Q(x+i\tau N) \neq 0$ if N is noncharacteristic for Q.*

By a result of Andreotti/Nacinovich this can be extended to convex open sets Ω.
The proof relies on the existence of fundamental solutions for $P(D)$ which are real analytic on large compact sets and on extension of regularity for the solutions of $P(D)$.

P. Laubin :

Complex canonical transformations in partial differential equations

Representation of distributions or hyperfunctions in the complex domain as boundary values of holomorphic functions in a strictly pseudoconvex open subset of $\mathbb{C}^n$ is a classical tool in linear PDE. We review two particular representations which are global and almost explicit. We then describe the construction of local transformations of this kind which can be adapted to the geometry of a given linear PDE. We give an application to the lagrangian structure of the solution of a boundary value problem.

O. Liess :

Hyperfunctions, Fourier transforms and duality

Let $l : \mathbb{R}^n \to \mathbb{R}_+$ be sublinear and denote for $\varepsilon > 0$ by $L^2(\mathbb{C}^n, l, -\varepsilon)$ the L^2_{loc} functions f so that $f\, e^{-l(\mathrm{Re}\,\zeta)+\varepsilon|\mathrm{Im}\,\zeta|} \in L^2(\mathbb{C}^n)$. If $\mathcal{B}(U)$ denotes the hyperfunctions on $U \subset \mathbb{R}^n$, we denote by $F^{-1} : L^2(\mathbb{C}^n, l, -\varepsilon) \to \mathcal{B}(|x| < \varepsilon)$ the map (called "inverse Fourier transform") defined by the formal integral

$$u = F^{-1}(\mu) = (2\pi)^{-n} \int_{\mathbb{C}^n} e^{i\langle x,\zeta\rangle}\, \mu(\zeta)\, dx(\zeta). \tag{1}$$

For the regularization of (1) cf. [1].

If we fix $c > 0$ and denote by $V(c) = \{\zeta : |\mathrm{Im}\,\zeta| < c\,|\mathrm{Re}\,\zeta|\}$, then the contribution of the region $\mathbb{C}^n \setminus V(c)$ to u is real analytic. Consider next the space $\mathcal{A}(V(c), F\, l(\mathrm{Re}\,\zeta), \varepsilon\,|\mathrm{Im}\,\zeta|)$ of functions analytic on $V(c)$ so that $f\, e^{F\, l(\mathrm{Re}\,\zeta)-\varepsilon|\mathrm{Im}\,\zeta|} \in L^2(V(c))$. $F^{-1}(f|_{\mathbb{R}^n})$ can be given a meaning as in (1). If $f \in \mathcal{A}(V(c), l, -\varepsilon)$, then $F^{-1}f$ is real analytic for $|x| > \varepsilon$. Next fix $\varepsilon < B < A$. When $u \in \mathcal{B}(|x| < A)$ is real analytic for $|x| > \varepsilon$, we denote by $F_{loc,A,B}\, u(\zeta)$ the function

$$\int_{|y|<B} \int_{|x|\le A} e^{-i\langle x,\zeta\rangle - |x-y|^2 \sqrt{\zeta_1^2+\cdots+\zeta_n^2}/2}\, u(x)\, dx\, dy.$$

It is inverse to F^{-1} in the sense that $(F_{loc,A,B}\, F^{-1} f - f)(\zeta)$ is exponentially decreasing and $F^{-1} F_{loc,A,B}\, u - u$ is real analytic. Starting from all this, one can show that $\mathcal{B}/\mathcal{A}$ (the germs of hyperfunctions, modulo the real analytic functions at 0) can be identified with the set of linear continuous functionals on the spaces

$$\mathcal{E}_{-l,\varepsilon,c} = \{f : F_{loc,A,B}\, f \in \mathcal{A}(V(c), -l, \varepsilon)\}.$$

[1] O. Liess: Higher microlocalization and propagation of singularities. Proc. N.A.S.I on "Microlocal Analysis and Spectral Theory". 1996, *Kluver Acad. Publ.* 1997, 61–91.

R. Meise : (joint work with R.W. Braun and B.A. Taylor)

Homogeneous polynomials P for which $(P+Q)(D)$ admits a continuous linear right inverse for all perturbations Q

The proof of the following result was presented:

Theorem *For a homogeneous polynomial P_m of degree $m \geq 2$ in $n \geq 2$ complex variables, the following assertions are equivalent:*

(1) $(P_m + Q)(D) : C^\infty(\mathbb{R}^n) \to C^\infty(\mathbb{R}^n)$ *admits a continuous linear right inverse for each polynomial Q of degree less than m.*

(2) $\operatorname{grad} P_m(x) \neq 0$ *for each $x \in \mathbb{R}^n \setminus \{0\}$, P_m is real up to a complex factor and no irreducible factor of P_m is elliptic.*

The theorem extends an earlier sufficient condition of Meise, Taylor and Vogt (see *J. AMS* **11** (1998), 1–39).

S. Momm :

Elliptic partial differential equations for real analytic functions

For $K \subset \mathbb{R}^N$ convex, compact with $\mathring{K} \neq \emptyset$, $\mathcal{A}(K)$ denotes the space of all real analytic functions on K. For a given constant coefficients linear partial differential operator $P(D)$, we ask whether $P(D) : \mathcal{A}(K) \to \mathcal{A}(K)$ admits a continuous linear right inverse. Contrary to the case of $C^\infty(K)$ (instead of $\mathcal{A}(K)$), it happens that for certain K the operator $P(D) : \mathcal{A}(K) \to \mathcal{A}(K)$ does not have a continuous linear right inverse. For example, $\Delta : \mathcal{A}(K) \to \mathcal{A}(K)$ has a continuous linear right inverse if and only if $\partial K \in C^{1,1}$.

To prove this, we evaluate — applying Lundin's description of the pluricomplex Green function of K — an abstract criterion which is given in terms of extremal plurisubharmonic function and which is derived from results of Vogt, Zahariuta, Kiselman, Lempert.

M. Nacinovich :

Weak unique continuation in abstract 1–pseudoconcave CR manifolds

Let M be an abstract, i.e. not necessarily locally embeddable CR manifold, which is assumed to be strictly 1–pseudoconcave. The CR structure is defined by a formally integrable distribution $\tau^{0,1}(M)$ of smooth complex valued vector fields, of rank n, such that $2n < \dim_{\mathbb{R}} M$, $\tau^{0,1}(M) \cap \overline{\tau^{0,1}(M)} = \{0\}$. Strict one–pseudoconcavity is a condition on the commutators $[L, \overline{L}]$ with $L \in \tau^{0,1}(M)$ which ensures the subelliptic $\frac{1}{2}$–estimate for functions: $\sum \|L_i u\|^2 \geq c\|u\|^2_{\frac{1}{2}} - C\|u\|^2_0 \; \forall u \in C_0^\infty(\Omega)$ if Ω is a relatively compact open domain in M.

Then the weak unique continuation principle holds for solutions u of the differential inequality:

$$(*) \quad \forall L \in \tau^{0,1}(M) \quad u, Lu \in L^2_{loc}(M),\ |u(x)| \leq k_L(x)\,|L\,u(x)| \text{ a.e. in } M \text{ with } k_L \in L^\infty_{loc}(M).$$

This means that, if $\Omega \subset M$ is connected and $u = 0$ a.e. on $\emptyset \neq \omega$ open $\subset \Omega$, then $u = 0$ a.e. in Ω. The proof requires first to reduce to uniqueness for a noncharacteristic Cauchy problem for $(*)$, next to settle this point by Carleman–type estimates. The result can be extended to higher degrees forms in the CR complex, by strenghtening the pseudoconcavity assumption.

V. Palamodov :

Special functions of several variables

A general approach to the theory of higher special functions will be discussed.

A series of special singular functions of several variables is constructed by means of improper integrals over versal families of algebraic hypersurfaces. This gives a natural generalization of Gauss hypergeometric functions as well as Airy–type integrals.

M. Poppenberg : (joint work with H. Lange and H. Teismann)

Smooth solutions for a class of nonlinear Schrödinger equations

In some domains of classical and quantum physics a set of evolutionary nonlinear Schrödinger equations of type

$$i\partial_t u = -\Delta u + V(x)u + f(|u|^2)u + \kappa\Delta h(|u|^2)h'(|u|^2)u \tag{1}$$

(where V is a given space dependent potential, f, h are real functions and κ is a real parameter) play an important rôle. In this lecture the simplest case of an equation of type (1) is considered, namely the whole space Cauchy problem

$$i\partial_t u = -\Delta u + \kappa(\Delta|u|^2)u, \quad u(0,x) = \phi(x). \tag{2}$$

Equation (2) may be called the 'superfluid film equation' of fluid mechanics and plasma physics (cf. S. Kurihara, Large–amplitude quasi–solitons in superfluid films, *J. Phys. Soc. Japan* **50**, 1981, 3262-3267, or E.W. Laedke, K.H. Spatschek, L. Stenflo, Evolution theorem for a class of perturbed envelope soliton solutions, *J. Math. Phys.* **24**, 1983, 2764-2769). Equations of type (1), (2) are also considered in the theory of Heisenberg ferromagnets and in dissipative quantum mechanics.

The mathematical difficulties with (1), (2) are various. The nonlinearity appears in the highest order space derivatives. Hence classical energy methods (which can e.g. be applied in the case of the well studied semilinear Schrödinger equations) fail, and a problem called 'loss of derivatives' occurs. The existence of semigroups for the linearized problem is by no means obvious since the linearized equation is not dissipative. Up to now even the local well posedness of (1), (2) seemed not to be known.

The purpose of this lecture is to prove the local well posedness of (2) for smooth solutions. The proof is based on new techniques on Nash–Moser type implicit function theorems for Fréchet spaces combined with linear semigroup theory. The smoothness is a result of using the space H^∞ defined as the intersection of all Sobolev spaces H^k. A crucial part of the proof consists in showing the necessary Nash–Moser estimates for the solutions of the corresponding linearized inhomogeneous equation.

M.S. Ramanujan : (joint work with S. Buckley and D. Vukotic)

Bounded and compact coefficient multipliers between Bergman and Hardy spaces

We investigate the boundedness and compactness of the coefficient multiplier operators between various Bergman spaces A^p and Hardy spaces H^q, thus extending and complementing some recent works by various authors.

We study the coefficients of A^1 functions and some new characterizations of the multipliers between the Hardy and Bergman spaces with exponents 1 or 2 are also derived. We characterize the compact multipliers from H^1 to H^2 and from A^1 to A^2, and compute the essential norm of certain multiplier operators. We show that if $p > 1$, then there are bounded non-compact multiplier operators from A^p to A^q if and only if $p \leq q$.

J. Schmets : (based on a joint research with M. Valdivia)

About analytic extension of Whitney jets

Let F be a closed subset of $\mathbb{R}^n$ and $\mathcal{E}(F)$ designate the Fréchet space of the Whitney jets on F. Then the Whitney theorem says that the continuous linear restriction map

$$R: C^\infty(\mathbb{R}^n) \to \mathcal{E}(F) \qquad f \mapsto (D^\alpha f|_F)_{\alpha \in \mathbb{N}_0^n}$$

is surjective. In 1961, Mityagin has proved that

(a) if $n = 1$ and $F = \{0\}$, R has no continuous linear right inverse;

(b) if $n = 1$ and $F = [0,1]$, R has a continuous linear right inverse.

Since then several authors have given examples of sets F for which R has or has not a continuous linear right inverse. On the other hand, by use of the Vogt–Wagner splitting theorem, Tidten has proved that R has a continuous linear right inverse if and only if $\mathcal{E}(K)$ is isomorphic to a subspace of s, i.e. if and only if $\mathcal{E}(K)$ has the property (DN).

In fact the Whitney theorem is more precise since it says that every Whitney jet on F is the image by R of a $C^\infty(\mathbb{R}^n)$-function which is analytic on $\mathbb{R}^n \setminus F$. The main result of *Bull. Polish Ac. Sc. Math.* **45** (1997), 359-367 states that

Theorem *Let K be a compact subset of $\mathbb{R}^n$.*

(a) Every Whitney jet on K is the image by R of a $BC^\infty(\mathbb{R}^n)$-function which is analytic on $\mathbb{R}^n \setminus K$.

(b) If there is a continuous linear extension map from $\mathcal{E}(K)$ into $C^\infty(\mathbb{R}^n)$, then there also is a continuous linear extension map E from $\mathcal{E}(K)$ into $BC^\infty(\mathbb{R}^n)$ such that $E\varphi$ is analytic on $\mathbb{R}^n \setminus K$ for every $\varphi \in \mathcal{E}(K)$.

H.S. Shapiro :

Fock space techniques for linear holomorphic partial differential equations

The Fock space $\mathcal{F}_n$ is the Hilbert space of entire functions f on $\mathbb{C}^n$ such that $|f|^2 e^{-|z|^2}$ is integrable (w.r.t. volume measure), and has been studied for a variety of reasons. Our motivation is that one can rather easily derive some a priori estimates for differential operators using this norm, which enable one to prove solvability of boundary value problems having holomorphic (Fock space) data, within Fock space. Such results are rare, but of some interest in that they exhibit situations where the solution does not pick up singularities. It is hoped this may shed some light on the mechanism by which singularities are generated.

B.A. Taylor : (joint work with R. Braun and R. Meise)

Estimates for extremal plurisubharmonic functions

It is an open problem to characterize the algebraic varieties V on $\mathbb{C}^n$ with the property (SRPL):
The extremal plurisubharmonic function

$$U_{\mathbb{R}^n}(z, V) = \sup\{u(z) : u \text{ psh on } V,\ u(z) \leq |z| + o(|z|),\ u(z) \leq 0, \text{ for } z \in V \cap \mathbb{R}^n\}$$

satisfies an estimate

$$U_{\mathbb{R}^n}(z, V) \leq A\,|z| + B.$$

We give some new results for this problem, including the

Theorem *Let $V = \{P(z) = 0\}$ where $P(z) = P_m(z) +$ lower order terms, and P_m is a homogeneous polynomial on $\mathbb{C}^n$ such that*

(1) P_m has real coefficients and the zero set of each $Q_j(z)$ in the irreducible factorization $P_m(z) = \prod_{j=1}^q Q_j(z)$ satisfies $\dim_{\mathbb{R}}\{Q_j(z) = 0\} \cap \mathbb{R}^n = n - 1$.

(2) There are no repeated factors in the irreducible factorization of P_m.

(3) The lower degree terms in $P(z)$ also have real coefficients.

Then V satisfies (SRPL).

The conditions of the Theorem are not necessary as shown by the example, due to D. Bainbrigde,

$$V = \{(s, w_1, w_2) : (s^2 - w_1^2)^2 = w_2(w_1^2 - w_2^2)\}$$

which satisfies (SRPL).

D. Vogt : (joint work with P. Domański)

Splitting of Distributional Complexes

For topological exact complexes

$$0 \to E \hookrightarrow E_0 \xrightarrow{T_0} E_1 \xrightarrow{T_1} E_2 \to \cdots,$$

where $E_k \cong \mathcal{D}' \cong (s')^{\mathbb{N}}$ for every k, the following theorem was presented

Theorem 1 *Every such complex splits for $k \geq 1$, i.e. T_k has a right inverse* $\operatorname{im} T_k \to E$ *for every $k \geq 1$.*

If the complex is finite (i.e. $E_k = 0$ for $k \geq k_0$) and algebraically exact with continuous T_k then it is topologically exact, hence splits for $k \geq 1$.

If $E = \Gamma(\Omega, \mathcal{E})$, $\Omega \subset \mathbb{R}^n$ open, $\mathcal{E}$ a translation–invariant sheaf on $\mathbb{R}^n$, e.g. if T_0 is a constant coefficient differential map, or a convolution map then we have

Theorem 2 *The complex splits at $k = 0$ iff E is strict.*

These results extend results of Palamodov on differential complexes and are closely related to works of Meise, Taylor and Vogt on right inverses of partial differential operators.

P. Wagner :

Representation of a fundamental solution of N. Zeilon's operator by elliptic functions

The Herglotz-Petrowsky formulae yield an expression by an $(n-1)$–fold integral for a fundamental solution of a homogeneous linear partial differential operator with constant coefficients in n variables. For $n = 3$ and elliptic operators, these formulae where derived by I. Fredholm and applied to construct explicitly a fundamental solution of the operator $\partial_1^4 + \partial_2^4 + \partial_3^4$ in terms of elliptic integrals. N. Zeilon applied Fredholm's theory in 1913 to the non-elliptic operator $\partial_1^3 + \partial_2^3 + \partial_3^3$, but without obtaining an explicit final result. Though not being an "evolution operator", this last operator admits fundamental solutions with conical lacunae, and as such it has been recently considered by R. Meise et al. in their investigations of continuous linear right inverses of linear partial differential operators.

In my talk, I would first discuss the analytic wave front set and the existence of lacunae for fundamental solutions of homogeneous operators, and then derive an explicit representation of a fundamental solution of $\partial_1^3 + \partial_2^3 + \partial_3^3$ by elliptic functions, thereby completing N. Zeilon's result.

J. Wengenroth :

Projective spectra of weighted (LB)–function spaces

In the first part of the talk, the definition of the projective limit functor Proj and its derivative Proj^1 as well as notion of being of strong (P)–type are explained. For projective spectra of (LB)–spaces, strong (P)–type and vanishing of Proj^1 are characterized by theorems due to Palamodov, Retakh, Vogt, Frerick and Wengenroth.

In the second part, these characterizations are evaluated for spectra of weighted Köthe function spaces Proj ind $L_p(\sigma_{nN})$ in terms of the weight functions. Finally, a relation to the projective description for weighted (LF)–spaces of holomorphic functions is explained.

V. Zahariuta : (joint work with P. Chalov and M. Dragilev)

Linear topological invariants and isomorphism of pairs of Köthe spaces

In the frame of the study of the isomorphic classification problem for pairs of Köthe spaces, the following special class of pairs is considered thoroughly: $F = F(\lambda, a) := (K(\exp(-\frac{1}{p}a_i)), K(\exp(-\frac{1}{p}a_i + \lambda_i)))$, $a = (a_i)$, $\lambda = (\lambda_i)$, $a_i \uparrow \infty$, $0 < \lambda_i < \infty$. This class contains some interesting concrete pairs, for example, pairs of spaces of analytic functions $(\mathcal{A}(D_0), \mathcal{A}(D_1))$, $D_0 \subset D_1 \subset \mathbb{C}^n$. The main tool is so-called m–rectangle characteristics $\mu_m^{(\lambda,a)}(\delta, \varepsilon; \tau, t)$, which calculate how many points (λ_i, a_i) are in the union of m rectangles $P_k = (\delta_k, \varepsilon_k] \times (\tau_k, t]$, $k = 1, \dots, m$, $\delta = (\delta_k)$, $\varepsilon = (\varepsilon_k)$, $\tau = (\tau_k)$, $t = (t_k)$.

The system of these characteristics $(\mu_m^{(\lambda,a)})_{m \in \mathbb{N}}$ is shown to be a complete invariant with respect to quasidiagonal isomorphisms (with an appropriate definition of the equivalency $(\mu_m^{(\lambda,a)}) \approx (\mu_m^{(\tilde{\lambda},\tilde{a})})$).

By the use of compound invariants it is shown also that any individual characteristic $\mu_m^{(\lambda,a)}$ is a linear topological invariant on this class.

As an application it is proved that there are continuum pairwise non-isomorphic pairs $(\mathcal{A}(D_0), \mathcal{A}(D_1))$, if $D_0 \subset D_1$ runs the set of pairs of bounded complete n–circular domains in $\mathbb{C}^n$.

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