Tagungsbericht 22/1998
Systems with Multiple Scales
24.5. - 30.5.1998

Die Tagung fand unter Leitung von W. Jäger (Heidelberg) und A. Mikelić (Lyon) statt. In Vorträgen wurden an fünf Vormittagen und drei Nachmittagen Resultate zu folgenden Bereichen von Mehrskalenproblemen präsentiert: Homogenisierung, Variationsprobleme, verschwindende Viskosität in hyperbolischen Gleichungen, stochastische Differentialgleichungen, Hamilton-Jacobi Gleichungen, Bildverarbeitung, Modellierung von Strömungsprozessen und Kristallisation von Polymeren sowie numerische Verfahren. In drei Abendsitzungen wurden offene Fragen und mögliche Entwicklungen diskutiert.

Vortragsauszüge

G. BOUCHITTE
Homogenization of elliptic problems in a fiber reinforced structure. Nonlocal effects.
We present recent results on the homogenization of the stationnary heat equation

$$-div(a_\varepsilon \nabla u) = f \text{ on } \Omega$$

boundary conditions

in two cases:
1) $\phi_\varepsilon \to +\infty$ on subsets $T_\varepsilon(\Omega)$ such that $| T_\varepsilon | \to 0$.
2) $\phi_\varepsilon \to 0$ on connected subsets $\Omega_\varepsilon = \Omega\ T_\varepsilon$ and $| T_\varepsilon | \to \theta \,| \Omega |$ $(0 < \theta < 1)$.
In both cases the limit is nonlocal and the associated energy can be written as a Dirichlet form on $L^2(\Omega)$.
Then we consider some extensions of these results to nonlinear elliptic equations and also to the system of elasticity. The latter case leads to a lot of open problems.

A. BOURGEAT

Convergence in law of the heterogeneous solution to the homogenized one for a $1-D$ second order operator

We give under various conditions of mixing and under moderate deviation conditions the rate of convergence in law for the solution of a second order operator with randomly oscillating coefficient towards the homogenized solution.

M. BRIANE

Increase of dimension by homogenization in a weakly connected domain

The paper deals with the homogenization of a Neumann's problem in a thin periodic "weakly connected" domain of R^3. The domain Ω_n is composed of a large number n of disjoint periodic connected components linked by a periodic lattice ω_n of very thin bridges. The measures of Ω_n and ω_n are such that $|\omega_n| << |\Omega_n| = O(1/n)$. The scale of periodicity ε_n satisfies $\varepsilon_n = o(1/n)$. According to the distribution and to the size of the linking bridges, the limit problem as $n \to +\infty$ is either a $4d$ Neumann's problem or a $4d$ non-local problem. The additional term corresponding to the increase of dimension is due to the capacitary effect of the bridges.

V. CAPASSO

From stochastic individuals to integrodifferntial mean field models in population dynamics

A model for a spatially structured population of N biological individuals is presented, subject to a density dependent social behaviour. The Lagrangian description is based on a system of Stochastic Differential Equations driven by Brownian motions and "long ranged" aggregation with "short ranged" repulsion. The biological motivation comes from the analysis of field experiments on the species of ants Polyergus Rufescens , which exhibit a clear tendency to aggregate, still avoiding overcrowding. For the Eulerian description we refer to the time evolution of the empirical measure associated with the system of N particles. For a finite and small number N of individuals the empirical measure suffers significant stochastic fluctuations. But a "law of large numbers" proves that for N tending to infinity the stochastic fluctuations tend to disappear, so that the Eulerian description may be based on the evolution of a spatial density subject to an integrodifferential equation, in which local and nonlocal gradients of the density compete. The mathematical theory is in the framework of "moderate limit" of interacting particles.
(in collaboration with Silvia Boi, Daniela Morale, Karl Oelschlager).

T. CLOPEAU

On the vanishing viscosity limit for the $2D$ incompressible Navier-Stokes equations with the friction type boundary conditions.

The vanishing viscosity limit is considered for the incompressible $2D$ Navier-Stokes equation in a bounded domain. Motivated by studies of turbulent flow, we suppose Navier's friction condition in the tangential direction, i.e. creation of vorticity proportional to the tangential velocity. We prove existence of the regular solutions for the Navier-Stokes equations with smooth compatible data and of the solution with bounded vorticity for the initial velocity being only bounded. Finally, we establish a uniform L^∞ bound for the vorticity and convergence to the incompressible $2D$ Euler equations in the inviscid limit.

G. DAL MASO

Nonlocal approximations of the Mumford-Shah functional

Let Ω be a bounded open set in R^n with a Lipschitz boundary, let $g \in L^\infty(\Omega)$, and let $f : [0,+\infty[\to [0,+\infty[$ be a nondecreasing continuous function such that $f(0) = 0, f'(0) = 1, f(+\infty) < +\infty$. For every $\varepsilon > 0$ let u_ε be a minimum point of the nonlocal problem

$$\min_{u \in L^2(\Omega)} \left\{ 1/\varepsilon \int_\Omega f(\varepsilon ⨍_{B_\varepsilon(x)\cap\Omega} | \nabla u(y) |^2 dy) dx + \int_\Omega | u(x) - g(x) |^2 dx \right\},$$

where $⨍$ denotes the mean value (we set $\int_{B_\varepsilon(x)\cap\Omega} | \nabla u(y) |^2 dy = +\infty$ if $u \notin H^1(B_\varepsilon(x))$. Andrea Braides and I proved that there exists a sequence $\varepsilon_k \to 0$ such that (u_{ε_k}) converges in $L^1(\Omega)$ to a solution of the Mumford-Shah problem

$$\min_{u \in SBV(\Omega)} \left\{ \int_\Omega | \nabla u(x) |^2 dx + \frac{f(+\infty)}{2} \mathcal{H}^{n-1}(S_u) + \int_\Omega | u(x) - g(x) |^2 \, dx \right\},$$

where $SBV(\Omega)$ is the space of special functions of bounded variation, S_u is the set of essential discontinuity points of u, $\mathcal{H}^{n-1}$ is the $(n-1)$-dimensional Hausdorff measure, and ∇u is the approximate gradient of u. Another approximation of the Mumford-Shah problem is based on finite elem= ents. Assume, for simplicity, that $n = 2$ and that Ω is polygonal. Let $w(\varepsilon)$ be a continuous function with $w(0) = 0$ and $w(\varepsilon) \geq 10\varepsilon$, let $\theta_0 > 0$, and let $_{\varepsilon} = \tau_\varepsilon(w, \theta_0)$ be the set of all triangulations of Ω such that the sides of all triangles are between ε and $w(\varepsilon)$, while all angles are larger than or equal to θ_0. Let $\mathcal{V}_\varepsilon$ be the set of all piecewise affine continuous functions subordinate to one of the triangulations of τ_ε, and let v_ε be the minimum point of the discrete problem

$$\min_{v \in \mathcal{V}_\varepsilon} \left\{ 1/\varepsilon \int_\Omega f(\varepsilon | \nabla u(x) |^2) dx + \int_\Omega | u(x) - g(x) |^2 dx \right\}.$$

Antonin Chambolle and I proved that, if $0 < \theta_0 \leq 15°$, then there exists a sequence $\varepsilon_k \to 0$ such that (v_{ε_k}) converges in $L^2(\Omega)$ to a solution of the Mumford-Shah problem

$$\min_{u \in SBV(\Omega)} \left\{ \int_\Omega | \nabla u |^2 dx + f(+\infty) \sin\theta_0 \mathcal{H}^1(S_u) + \int_\Omega | u - g |^2 dx \right\}.$$

H. FREISTÜHLER

Non-uniformities of the vanishing-viscosity limit to hyperbolic systems of conservation laws

Let S^ε be the solution operator to the parabolic problem

$$u_t + f(u)_x = \varepsilon(B(u)u_x)_x, \quad \varepsilon > 0.$$

Assume, it converges to a solution operator S^0 of the limiting problem

$$u_t + f(u)_x = 0.$$

This limit may display the following non-uniformities:
(i) S^0 may depend on B.
(ii) S^0 may be discontinuous.
In this talk I present (partly other people's) examples for both effects, in particular examples with direct physical significance.

H. ISHII

Homogenization of Hamilton-Jacobi equations

I discussed periodic or almost periodic homogenization of Hamilton-Jacobi equations

$$u_t(x,t) + H(x,t,x/\varepsilon, Du(x,t)) = 0 \ in \ \Omega \times (0,\infty),$$

or

$$u(x) + H(x, x/\varepsilon, Du(x)) = 0 \ in \ \Omega.$$

Here u and H are real-valued functions and u_t and Du denote $\delta u/\delta t$ and $(u_{x_1}, \ldots, u_{x_N})$, respectively, Ω is an open subset of R^N, and $\varepsilon > 0$ is a constant to be sent to zero. I first made a quick review of recent developments concerning homogenization of these Hamilton-Jacobi equations. And then I discussed a little more on a few topics. These topics are: (1) Hamilton-Jacobi equations on domains with small scale periodic structure, (2) multiscale and almost periodic homogenization of Hamilton-Jacobi equations, and (3) some observations (or flat parts) of the effective Hamiltonians. These are taken from (1) 'Homogenization of Hamilton- Jacobi equations on domains with small scale periodic structure' by K. Horie and H. Ishii, (2) 'Multiscale homogenizations for first-order Hamilton-Jacobi equations' by M. Arisawa, and (3) 'Periodic homogenization of Hamilton-Jacobi equations II' by M. Concordel.

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S. KNAPEK

Upscaling techniques based on Multilevel subspace splittings

In this talk we present discrete homogenization methods for the pressure equation arising from operator adapted subspace splittings. We utilize ideas known from Multilevel-methods for the solution of elliptic operator equations, such as the Galerkin approximation to compute coarse grid operators and the construction of approximation spaces via operator dependent interpolation. We have to recompute an upscaled diffusion tensor from the coarse grid operator. We give results for the $2-D$ case. The method gives consistently better results as for example renormalization, and it is able to deal with nondiagonal diffusion tensors.

J. LIU

Numerical Methods for Oscillatory Solutions

In order to guarantee a good numerical approximation of an oscillatory solution of a differential equation, a fine computational grid is in general needed, but for many practically interesting cases, we can use large grids, but still be able to capture the homogenized solution. In this talk I will give a general sampling lemma for multiple scale periodic oscillation. Then I will apply this sampling lemma to show the upwind scheme, C-N scheme are strong convergence for linear convection equation with oscillatory coefficient in a probability $1-h^{1-\alpha}$.

E. MARUSIĆ-PALOKA

The effects of flexion and torsion on a fluid flow through a curved pipe

We consider a curved pipe P_ε with the smooth central curve γ and the cross-section εS, where $S \subset \mathbb{R}^2$ and $\varepsilon << 1$. The curve γ is parametrized by its arch length $y_1 \in [0, l]$. We denote by φ its natural parametrization, **n** and **b** its Frenet's basis, κ its curvature (flexion) and τ its torsion. We study a flow of a viscous incompressible fluid injected in the pipe P_ε by some prescribed velocity and governed by the Navier–Stokes system. Using the curvilinear coordinates on P_ε we find the first two terms in the asymptotic expansion for the flow in powers of the small parameter ε (the thickness of the pipe). The first term $\mathcal{U}^0$ in the expansion for the velocity has the direction of the tangent $\mathbf{t}$ on γ and depends only on the flux generated by the injection velocity and the geometry of the cross-section S. The first term in the expansion for the pressure drop is proportional to the mean value of $\mathcal{U}^0$. The second term in the velocity expansion contains the effects of flexion and of the torsion. In fact its tangential part is proportional to the torsion. The second term in the pressure expansion is proportional to the curvature. The expansion for the velocity gradient is also found. The boundary layers at the ends of the pipe were studied.

A. MEISTER

Asymptotic Multiple Scale Analysis and Numerical Methods

Recently, Klein proposed a single time scale, multiple space scale asymptotic analysis in order to gain a deeper insight into the limit behaviour of inviscid compressible flow fields. Motivated by these results, a numerical scheme for the simulation of inviscid compressible flows based on Godunov type solvers has been extended to weakly compressible fluid flows. The performance and applicability of such numerical schemes depends on the asymptotic sequence, which has to represent a measure of the compressiblitity of the fluid. Consequently, if an unsteady flow field is considered, a time-dependent adaptation of the asymptotic sequence is necessary in order to ensure high resolution of the flow phenomena. The talk is devoted to the extension of a numerical method to the low Mach number regime including a time-dependent asymptotic sequence.

J. MICHEL

Large deviations estimates for epigraphical superadditive processes in stochastic homogenization

Under mild growth hypothesis, we establish an exponential estimate for the law of $\mathcal{S}_A \mid A \mid$, where $\mathcal{S}$ is a random superadditive process in $I\!R$. We apply this result to various problems stemming from stochastic homogenization, like the estimations of the homogenized conductivity. Next we give an exponential estimate for the convergence towards the limit in the context of the law of large numbers for epigraphical processes studied by Attouch and Wets. Finally we give some numerical computations of the action in the large deviations property.

This work is a joint work with G. Michaille and L. Piccinini - University Montpellier II.

S. MÜLLER

Variational problems with multiple scales

I discuss joint work with Giovanni Alberti (Pisa) to develop a rigorous variational framework for problems with multiple small scales. As an illustration consider the problem of minimizing

$$I^\varepsilon(u) = \int_0^1 \varepsilon^2(u'')^2 + ((u')^2 - 1)^2 + a(s)u^2 ds$$

among one-periodic functions. It is known that for $a \equiv a_0 > 0$ and sufficiently small ε minimizers are periodic with minimal period $p^\varepsilon = L_0\varepsilon^{1/3}a_0^{-1/3} + O(\varepsilon^{2/3})$. Minimizers are very close to sawtooth functions with slope ± 1 but the corners are rounded off at scale ε. Our goal is to develop methods that allow one to eliminate the fastest scale ε, while keeping the $\varepsilon^{1/3}$ scale. We first define the blow-up $R^\varepsilon u^\varepsilon$, for one-periodic functions $u^\varepsilon : I\!R \to I\!R$, by $R^\varepsilon_s u^\varepsilon(t) = \varepsilon^{-1/3}u(s + \varepsilon^{1/3}t)$ and view $s \mapsto R^\varepsilon_s u^\varepsilon$ as maps from $(0, 1)$ into a (compact, metric) function space $K = \{v : R \to R\}$ measurable $= (L^\infty(R, [-1, 1]), weak*)$. Then we consider the Young measure $\nu \in L^\infty(R; \mathcal{M}(K))$ generated by (a subsequence of) the maps $R^\varepsilon u^\varepsilon$ and derive a variational principle for ν. A typical application is the following

Theorem: Suppose $a \in L^1(0,1), a > 0$ a.e. and let u^ε be a sequence of minimizers of I^ε. Then $\{R^\varepsilon u^\varepsilon\}_{\varepsilon\downarrow 0}$ generate a unique Young measure ν and for a.e. $s \in (0,1)$ the probability measure $\nu(s)$ is supported on periodic sawtooth functions (with slope ± 1) with period $L_0(a(s))^{-1/3}$.
Generalizations to higher dimensions and concentration effects are sketched briefly.

N. NEUSS
On the computation of constants in the Beavers-Joseph law
Recently, a rigorous derivation of the Beavers-Joseph law was given by Jäger and Mikelic. The computation of the constants appearing in this law involves solving a boundary layer problem of Stokes type in an infinite strip. In my talk I present a method for solving this problem, and show an error estimate. Numerical results indicate, that in general the effective pressure is not continous across the interface.

B. NIETHAMMER
Derivation of the Lifshitz–Slyozov–Wagner Theory of Ostwald Ripening
The theory of Lifshitz, Slyozov and Wagner describes Coarsening of many particles of solid phase in undercooled liquid by a mean–field model for the particle size distribution.
We present a rigorous derivation of this model by homogenization of a Stefan problem with surface tension. We solve the quasi–static case as well as the full parabolic problem and construct approximate solutions by means of comparison principles. The framework of Young measures is used to identify the limit equation.
We obtain that the crucial quantity is the capacity of the particles. In the case that the limit capacity of particles is zero one gets the LSW mean–field model whereas positive capacity yields a coupled system of equations for the particle size distribution and the temperature of the liquid.

O. OLEINIK
On Homogenization in a nonperiodic structure
Last years many papers and books about homogenization in periodic structures appeared. In this lecture we consider homogenization in domain with nonperiodic structures. Let $\Omega \subset R^n$ be a bounded domain with smooth boundary $\partial\Omega$, $G_\varepsilon^j \subset \Omega, j = 1,\dots,N(\varepsilon); G_\varepsilon^i \cap G_\varepsilon^j \neq \phi, i \neq j$, G_ε^i has a piecewise-smooth boundary ∂G_ε^i; ε is a small parameter, $\varepsilon > 0$.
We set $G_\varepsilon = \cup_{j=1}^{N(\varepsilon)} G_\varepsilon^j$, $\Omega_\varepsilon = \Omega - \bar{G}_\varepsilon$,, $S_\varepsilon^* = \partial G_\varepsilon$, $S_\varepsilon = S_\varepsilon^* \cap \Omega$, $\Gamma_\varepsilon = \partial\Omega_\varepsilon - S_\varepsilon$, $M_\varepsilon = \partial\Omega \cap S_\varepsilon^*$.
We suppose that there exist some extensional operators $P_\varepsilon : H_1(\Omega_\varepsilon, \Gamma_\varepsilon) \to H_1(\Omega)$ such that the following conditions are valid:

$$\|P_\varepsilon u_\varepsilon\|_{H^1(\Omega)} \leq k_0 \|u_\varepsilon\|_{H^1(\Omega)}, \|\nabla(P_\varepsilon u_\varepsilon)\|_{L^2(\Omega)} \leq k_1 \|\nabla u_\varepsilon\|_{L^2(\Omega)}.$$

1. The Neumann condition: Consider the problem: $-\Delta u_\varepsilon = f$ in Ω_ε, $\frac{\partial u_\varepsilon}{\partial \nu} = 0$ on S_ε, $u_\varepsilon = 0$ on Γ_ε. Let v_0 be a smooth solution of the problem: $-\Delta v_0 = f$ in $\Omega, v_0 = 0$ on $\partial\Omega$. One

can prove that $(u_\varepsilon - v_0)$ satisfies the inequality

$$\|u_\varepsilon - v_0\|_{H^1(\Omega_\varepsilon)} \leq k_2 \left\{ max_{\bar\Omega}|f| \cdot |G_\varepsilon|^{1/2} + max_{\bar\Omega}|\nabla v_0| \cdot |G_\varepsilon|^{1/2} \right\} \leq k_3 |G_\varepsilon|^{1/2},$$

if $S_\varepsilon^* \cap \partial\Omega = \phi$, and

$$\|u_\varepsilon - v_0\|_{H^1(\Omega_\varepsilon)} \leq k_4(|G_\varepsilon|^{1/2} + |M_\varepsilon|^{1/2})$$

if $S_\varepsilon^* \cap \partial\Omega \neq \phi$, where $\mid G \mid$ is the Lebesgue measure of G.
2. The mixed boundary condition: Consider the problem: $-\Delta u_\varepsilon = f$ in Ω_ε, $\frac{\partial u_\varepsilon}{\partial \nu} + \beta(x) u_\varepsilon = 0$ on S_ε, $u_\varepsilon = 0$ on Γ_ε, $\beta(x) \geq \beta_0 > 0$. We apply the same method which we used to study the Neumann problem. Then the estimate

$$\|u_\varepsilon - v_0\|_{H^1(\Omega_\varepsilon)} \leq k_5 |S_\varepsilon|^{1/2}$$

holds.

R. RICCI

Some thwo phase problems in polymers

There are problems in polymer science showing a deep physical interaction between the space scale of macroscopical phenomena, like heat exchange or fluid motion, and the microscopical (molecular) scale.

In the crystallization process, the heat equation has to take into account the latent heat released from the crystallizing polymer macromolecules, and in turn the temperature effects the rate of growth of the crystall as well as the rate of appearence of crystall nuclei on a wide temperature range.

A possible description of the nucleation can be described by means of the solution of an appropriate Fokker-Plank type equation for the distribution function of virtual nuclei in a parameter space (typically the radius). Coefficients of this equation are dependent upon temperature. Then the complete mathematical model couples parabolic equation with the same time variable but different "space" variable, the ordinary space variable for the macroscopic phenomena (the heat equation) and a "space" variable in a parameter space for the Fokker-Plank equation.

A similar mathematical structure appears in the model for fluid motion of solution with relatively high concentration of polymer macromolecules. Here the average distribution of the orientation of the molecules (in a simple rod like model) obeys to a Fokker-Plank type equation (called Smoluchowski equation) whose coefficents depend on the velocity fields and its gradient at the macroscopical point in space. The fluid motion is then described in term of mass and momentum conservations. The coupling appears in the consitutive law for the stress tensor which involves, together with the velocity gradient, the averged orientation and also higher momenta of the distribution function.

M. SAMMARTINO

Zero Viscosity Limit for Stokes and Navier-Stokes Equations

We consider the problem of the zero viscosity limit for incompressible fluids in presence of boundaries. In this limit the fluid shows two different regimes. The inviscid regime, away from boundaries, where the effect of viscosity can be neglected, and convective (e.g. Euler) equations can be used. The viscous regime, in a boundary layer, where the viscosity plays an essential role, and convection-diffusion (e.g. Prandtl) equations must be used. Some of the most recent results concerning Stokes, Oseen and Navier-Stokes equations are presented. In all cases we show how the solution can be constructed as a superposition of an inviscid solution, a boundary layer solution, and a correction term. The main differences between the linear and the nonlinear case are:

- the necessity of imposing, in the nonlinear case, analyticity on the initial data;
- the fact that, in the nonlinear case, the time for which the construction is valid is small.

The possibility of relaxing the analyticity requirement is discussed.

D. SERRE

When a shock profile meets a boundary layer, in parabolic systems of conservation laws

The vanishing viscosity method, for hyperbolic conservation laws, displays a major difficulty, due to the change of the number of scalar boundary conditions. This yields to the formation of boundary layers. On the other hand, the nonlinearity is responsible of the shock formation in the interior of the domain, the counterpart of which being the formation of viscous shock profiles. As a shock meets the boundary, a complex interaction occurs between the profile and the layer. Its description requires the construction of a solution in the half-space $x > 0$, but for all time $t \in R$.

T. SHAPOSHNIKOVA

On homogenization problems for polyharmonic equations in domains which are perforated along manifolds.

In this lecture we consider homogenization of the Dirichlet problem for polyharmonic equation. Let Ω be a smooth bounded domain in R^n; M_{n-s} is a smooth $(n-s)$-dimensional manifold, $s \geq 2$ and let P^j_{n-s} be a point of M_{n-s}. Suppose that points $P^j_{n-s}(j = 1, \ldots, N(\varepsilon, n, s))$ are situated in such a way that the balls $B_j(j = 1, \ldots, N(\varepsilon, n, s))$ of radius $c_0\varepsilon$ with center at P^j_{n-s} cover M_{n-s}. Let $T^{j,n-s}$ be a ball with radius $a_{\varepsilon,s}$ and center $P^j_{n-s}, a_{\varepsilon,s} \leq c_0\varepsilon$. We set $G^{n-s}_\varepsilon = \cup_{j=1}^{N(\varepsilon,n,s)} T^{j,n-s}_{a_{\varepsilon,s}}$. Let us define a partially perforated domain $\Omega_\varepsilon = \Omega - \sum_s \bar{G}^{n-s}_\varepsilon$. In Ω_ε consider the boundary value problem

$$(1) \qquad \Delta^m u_\varepsilon = f,\ x \in \Omega_\varepsilon,\ D^j u_\varepsilon = 0 \quad \text{on} \quad \partial\Omega_\varepsilon,\ 0 \leq j \leq m-1.$$

We suppose that $u_\varepsilon \in H_m(\Omega_\varepsilon, \partial\Omega_\varepsilon)$. We describe here some results of the behaviour of solutions $\{u_\varepsilon\}$ as $\varepsilon \to 0$.

1. Let $n < 2m, n = 2k+1$ and $s < 2k, s = 2l$ or $s = 2l+1$. Assume that $a_{\varepsilon,s}$ satisfies the following conditions:

$$\lim_{\varepsilon\to 0} a_{\varepsilon,s}^{2(l-k)+1}\varepsilon^{2(k-l)+1} = 0, \text{ if } s = 2l,$$

$$\lim_{\varepsilon\to 0} a_{\varepsilon,s}^{2(l-k)}\varepsilon^{2(k-l)} = 0, \text{ if } s = 2l+1.$$

Then $u_\varepsilon \to v$ as $\varepsilon \to 0$ weakly in $H_m(\Omega)$, where v is a weak solution of the problem: $\Delta^m v = f$ in $\Omega - M_{n-s}, \mathcal{D}^j v = 0$ on $M_{n-s}, 0 \le j \le m-l-1; \mathcal{D}^i v = 0$ on $\partial\Omega, 0 \le i \le m-1$.

2. Consider the 'critical' case. Let $n > 2m, n = 2k+1, s < 2m$ and $p \in [0, m - [s/2] - 1]$. Suppose that $\lim_{\varepsilon\to 0} a_{\varepsilon,s}^{n-2m+2p}\varepsilon^{s-n} = A_0 > 0$.

Let us introduce the function $u \in H_m(\Omega, \partial\Omega)$ such that $\mathcal{D}^j u = 0$ on $M_{n-s}, 0 \le j \le p-1$ and the following integral identity is valid

$$\sum_{i_1,\dots,i_m=1}^{n} \int_\Omega D^m_{i_1,\dots,i_m} u D^m_{i_1,\dots,i_m}\phi dx + \sum_{|\alpha|=p} c_\alpha \int_{M_{n-s}} D^\alpha u D^\alpha \phi d\bar{x} = (-1)^m \int_\Omega f\phi dx$$

for any $\phi \in H_m(\Omega, \partial\Omega), \mathcal{D}^j\phi = 0$ on $M_{n-s}, 0 \le j \le p-1$. Then $u_\varepsilon \to u$ as $\varepsilon \to 0$ weakly in $H_m(\Omega)$.

G. WITTUM

Multiscale Numerics

In the numerical treatment of pde's multiscale approaches are of utmost importance. In addition to the different modelling scales, numerics introduce new scales like gridsize and parallelism. A numerical multi-scale approach is using ad·ptivity, multi-grid and parallelism. In the talk several aspects of such multi-scale problems are discussed and the simulation results for some characteristic problems are shown.

W. YONG

Boundary conditions for hyperbolic relaxation problems

This work is concerned with boundary conditions for multi-dimensional first-order hyperbolic systems with stiff source terms (also called relaxation). It is observed that usual relaxation stability conditions and the uniform Kreiss condition are not enough for the existence of the zero relaxation limit. To remedy this, we propose a so-called *generalized Kreiss condition* for initial-boundary value problems (henceforth, IBVPs) of the relaxation systems. By assuming that the relaxation system admits the quasi-stability condition and the prescribed boundary condition satisfies the generalized Kreiss condition, we derive a *reduced boundary condition*, for the corresponding equilibrium system, satisfying the uniform Kreiss condition and show the existence of boundary-layers. Moreover, a class of boundary conditions is defined to be *weakly reflective* with an easily checked inequality. These weakly reflective boundary conditions naturally induce energy estimates and are

shown to satisfy the generalized Kreiss condition if the relaxation systems admit a more restrictive relaxation stability condition.
The present results are expected to be used as theoretical criteria to construct relaxation approximations for IBVPs of conservation laws, which are of practical interest.

Berichterstatter: B. Schweizer

e-mail Adressen

G. Bouchitte	bouchitte@univ-tln.fr
A. Bourgeat	bourgeat@anumsun1.univ-st-etienne.fr
A. Braides	braides@sissa.it
M. Briane	briane@ann.jussieu.fr
V. Capasso	capasso@miriam.mat.unimi.it
T. Clopeau	clopeau@lan.univ-lyon1.fr
G. DalMaso	dalmaso@sissa.it
H. Freistühler	hf@instmath.rwth-aachen.de
S. Heinze	heinze@espresso.iwr.uni-heidelberg.de
H. Ishii	ishii@math.metro-u.ac.jp
W. Jäger	wissrech@iwr.uni-heidelberg.de
S. Knapek	knapek@iam.uni-bonn.de
J. Liu	jliu@math.umo.edu
S. Luckhaus	luckhaus@mis.mpg.de
E. Marusic-Paloka	emarusic@math.hr
A. Meister	meister@math.uni-hamburg.de
J. Michel	jmichel@umpa.ens-lyon.fr
A. Mikelic	andro@mobylette.univ-lyon1.fr
S. Müller	sm@mis.mpg.de
M. Neuss	maria.neuss-radu@iwr.uni-heidelberg.de
N. Neuss	nicolas.neuss@iwr.uni-heidelberg.de
B. Niethammer	barbara.niethammer@iam.uni-bonn.de
O. Oleinik	oleinik@glasnet.ru
R. Ricci	ricci@mat.unimi.it
M. Sammartino	marco@gremat.math.unipa.it
B. Schweizer	ben.schweizer@iwr.uni-heidelberg.de
T. Shaposhnikova	shaposh@glasnet.ru
D. Serre	serre@umpa.ens-lyon.fr
G. Wittum	sekretariat@ica3.uni-stuttgart.de
W. Yong	yong@oasis.iwr.uni-heidelberg.de
V. Zhikov	zhikov@vgpu.eleom.ru

Tagungsteilnehmer

Prof.Dr. Guy Bouchitte
U.F.R. des Sc. et Techn.
Universite de Toulon et du Var
B.P. 132

F-83957 La Garde Cedex

Dr. Alain Bourgeat
Dept. de Mathematiques
Universite de Saint Etienne
23, rue du Dr. Paul Michelon

F-42023 Saint-Etienne Cedex 02

Prof.Dr. Andrea Braides
S.I.S.S.A.
Via Beirut 2-4

I-34013 Trieste

Dr. Marc Briane
Laboratoire d'Analyse Numerique,
Tour 55-65
Universite P. et M. Curie(Paris VI)
4, Place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Vincenzo Capasso
Universita di Milano
Dipartimento di Matematica
"Federigo Enriques"
Via C. Saldini, 50

I-20133 Milano

Prof.Dr. Thierry Clopeau
Departement de Mathematiques
Universite Claude Bernard de Lyon I
LAN Bat. 101
43, Bd. du 11 Novembre 1918

F-69622 Villeurbanne Cedex

Prof.Dr. Gianni Dal Maso
S.I.S.S.A.
Via Beirut 2 - 4

I-34013 Trieste

Dr. Heinrich Freistühler
Institut
für Mathematik
RWTH Aachen
Templergraben 55

52062 Aachen

Dr. Steffen Heinze
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Prof.Dr. Hitoshi Ishii
Dept. of Mathematics
Tokyo Metropolitan University
Minami-Ohsawa 1-1
Hachioji-shi

Tokyo 192-03
JAPAN

Prof.Dr. Eduard Marusic-Paloka
Department of Mathematics
University of Zagreb
Bienicka 30

Zagreb 10000
CROATIA

Prof.Dr. Willi Jäger
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Andreas Meister
Institut für Angewandte Mathematik
Universität Hamburg
Bundesstr. 55

20146 Hamburg

Stephan Knapek
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

53115 Bonn

Prof.Dr. Julien Michel
Dept. de Mathematiques, U.M.P.A.
Ecole Normale Superieure de Lyon
46, Allee d'Italie

F-69364 Lyon Cedex 07

Prof.Dr. Jian-Guo Liu
Department of Mathematics
University of Maryland

College Park , MD 20742
USA

Prof.Dr. Andro Mikelic
Lab.d'Analyse Numerique
Universite Lyon I
Batiment 101
43, bd. du 11 novembre

F-69622 Villeurbanne Cedex

Prof.Dr. Stephan Luckhaus
Fakultät für Mathematik/Informatik
Universität Leipzig
Augustusplatz 10

04109 Leipzig

Prof.Dr. Stefan Müller
Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26

04103 Leipzig

Maria Neuss
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Prof.Dr. Marco Sammartino
Dipartimento di Matematica e Applicazioni
Universita di Palermo
Via Archirafi 34

I-90123 Palermo

Nicolas Neuss
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Dr. Ben Schweizer
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Dr. Barbara Niethammer
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

53115 Bonn

Prof.Dr. Denis Serre
Mathematiques
Ecole Normale Superieure de Lyon
46, Allee d'Italie

F-69364 Lyon Cedex 07

Prof.Dr. Olga A. Oleinik
Dept. of Mathematics
Moscow State University

119899 Moscow
RUSSIA

Prof.Dr. Tanya A. Shaposhnikova
Moscow University
Korpus "K", app. 133

117234 Moscow B-234
RUSSIA

Prof.Dr. Ricardo Ricci
Dipartimento di Matematica
"Federigo Enriques"
Universita di Milano
Via Saldini 50

I-20133 Milano

Prof.Dr. Gabriel Wittum
Institut für Computeranwendungen
Numerik für Höchstleistungsrechner
Universität Stuttgart

70550 Stuttgart

Dr. Wen-an Yong
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

69120 Heidelberg

Prof.Dr. Vasily V. Zhikov
Vladimir State Pedagogical University
Department of Mathematics
Prospect Stroitelej 11
Vladimir 600 024
RUSSIA

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