# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 23/1998

## Freiformkurven und Freiformflächen

07.06.-13.06.98

Die Tagung fand unter der Leitung von R.E. Barnhill (Kansas), C. de Boor (Madison, Wisconsin) und J. Hoschek (Darmstadt) statt. Im Mittelpunkt der Vorträge standen Fragen der Konstruktion und Darstellung von Kurven und Flächen im Bereich des Computer Aided Geometric Design (CAGD). Dabei wurden unter anderem folgende Schwerpunkte gesetzt:

- Verfahren zur Rekonstruktion von Kurven und Flächen aus Meßdaten,
- Anwendungen der Differentialgeometrie im CAGD,
- Erzeugung von Kurven und Flächen durch Unterteilungsalgorithmen,
- Gestalterhaltende Verfahren zur Kurven- und Flächenkonstruktion,
- Effiziente Visualisierung umfangreicher Datenmengen,
- Simulation physikalischer, technischer und medizinischer Phänomene.

Neben Mathematikern nahmen an der Tagung auch Informatiker, Ingenieure und Industrievertreter teil, dadurch kam es zu einem rege geführten interdisziplinären Austausch. Dank des aufgelockerten Vortragsprogramms entstand für die Tagungsteilnehmer die Möglichkeit zu vielen individuellen Gesprächen.

Dank einer Unterstützung im Rahmen des EU-Programmes TMR (Training and Mobility of Researchers) konnten zusätzlich einige jüngere Mathematiker zu der Tagung eingeladen werden. Dies ist einerseits eine hervorragende Förderung des wissenschaftlichen Nachwuchses und gibt andererseits den etablierten Kollegen die Gelegenheit, besonders begabte junge Mathematiker kennenzulernen.

# Vortragsauszüge

G. Albrecht

**Quadric triangles**

The topic of this talk are rational triangular Bézier patches of degree two. In the first part of the talk we are concerned with analyzing a given patch with respect to quadric surfaces. First, by means of the related Veronese surface in five-dimensional projective space, we determine whether a given rational triangular Bézier patch of degree two lies on a quadric surface. Then we establish the quadric's affine type by means of the quadric's intersection with the plane at infinity after having determined the quadric's projective type.

In the second part we construct rational quadratic quadric triangles in threedimensional projective space without prescribing an underlying quadric. First we prescribe the three corner points of the patch which determine a plane $E$, then we specify a conic $k$ through them and finally we choose a point $p$ outside the plane $E$. The point $p$ and the conic $k$ determine a cone $K$, and the cone $K$ together with the double plane $E$ span a pencil of quadric surfaces which all touch each other along $k$. The remaining three inner control points on the intersection lines of the tangent planes in the corner points of the patch result to depend on three parameters, e.g. their affine positions on these lines. The rest of the original unknowns, e.g. the three inner weights, are then determined such that the patch is guaranteed to lie on a quadric surface. Every such patch induces seven complementary patches on the same quadric which are connected to each other by certain harmonic cross ratios.

C. Bajaj

**Scalar (and Vector) Topology for Modelling and Visualization**

Scalar and vector fields arise in several scientific applications. Existing visualization techniques require that the user infer the global structure from what is frequently an insufficient display of information. We present a combination of robust numeric and symbolic techniques to detect the affine invariant structure at all scales, removing from the user the responsibility of extracting information implicit in the data, and presenting the structure explicitly for analysis. Topology preserving, finite element approximations form a crucial and necessary step in this structure determination, as does the accurate solution of systems of polynomial equations. We further demonstrate how scalar and vector topology computation prove useful for several multi-scale modelling and visualization applications.

R.E. Barnhill

**Free Form Curves and Surfaces – Introduction to the meeting**

The participants were welcomed to the 7th Oberwolfach meeting on Free Form Curves and Surfaces, also known as Computer Aided Geometric Design (CAGD). The first such meeting was held in April, 1982, and the meetings have continued in two or three year intervals since.

These meetings are examples of international collaborations which advance science and engineering and, in turn, everyday life. A statistic often cited is that during the past 50 years, approximately half the increase in human productivity is due to scientific research. Much of this research is performed in universities such as those represented at these meetings. In 1898 examples of significant inventions included sewing machines, farm equipment, and bicycles. In 1998 primary growth areas include biotechnology, computers, and new materials. A future list will include such fields as nanoelectronics, nano-scale biological computing devices and materials for space exploration.

The relationship of CAGD with these modern activities is that the mathematics of free form curves and surfaces, together with the availability of modern computer graphics equipment, enables one to "see" trends, solutions that would not be detected from stacks of numbers and words. CAGD is crucial to the present and future critical technology of high performance modelling and simulation. CAGD, modelling and simulation permit the scientific approximation of reality in complex situations such as the detection of difficult diseases and the design of complicated physical objects.

The history of CAGD as a named discipline began with the conference at the University of Utah in 1974. The new discipline required meetings and a scholarly journal at high levels of quality. The Oberwolfach meetings were the first systematic meetings in the new discipline and so the Mathematics Institute has a special place of recognition in the history of CAGD.

At the present meeting we celebrate the 70th birthday of Professor Wolfgang Boehm, one of the pioneers in CAGD, who has particularly emphasized the 'G', geometry, in the discipline.

G. Brunnett*, F. Isselhard, T. Schreiber, M. Vanco

**Triangulation based versus direct segmentation of discrete surface data**

In this talk recent results of two different approaches (triangulation based or direct) towards the segmentation of discrete surface data are presented.

In the first part, we report on a graph based method to obtain a polyhedron with triangular faces interpolating the given data. This method operates on the dual of the tetrahedrization of the point set. The polyhedron is constructed by an iterative procedure that successfully approximates a spanning Voronoi tree that is minimal with respect to a certain cost functional. Based on this triangulation we discuss different strategies to group the triangles into segments appropriate for the surface fitting phase.

In the second part, a procedure is presented that allows the segmentation of the surface data without use of a triangulation. Here, an efficient algorithm is needed to solve the problem of $k$-nearest neighbors computation. Our approach uses a recursive space partitioning together with a hashing method. Finally, we present different methods to estimate surface properties as normals and curvatures based on this structure.

M. Daniel* and E. Malgras

**Intersection Problems and Marching Techniques**

As our study of marching is based on a surface-surface intersection method, we should outline its main steps. A location stage is first available for B-spline or NURBS surfaces. We obtain on both surfaces, without subdivision, a set of corresponding regions where potential intersections can be found. For each couple of regions, checking for normal vector collinearity provides us information about closed intersection curves or any singularity like tangency or identical patches. For this test, normal bounding volumes have been constructed and optimized. Surfaces are subdivided until one of the following conditions occurs. 1) The bounding volumes do not overlap. Only open curve segments have to be found. 2) A singular point is found (intersection point having the same normal vector on both surfaces).

The open curves are determined by marching, which remains the longest step. It is why we carefully studied it. Both prediction and correction must be considered in order to define the most efficient and reliable global method: a prediction on straight lines in the parametric domains and a correction with a $3 \times 4$ Newton method.

We also developed a simultaneous control of intersection curves so that they are obtained with the best number of points. Estimating the curvatures of the curves in the parametric domains is required. These curvatures are evidently used for a more accurate prediction.

We finally proposed an algorithm which determines the common boundary of two partially identical patches, a first point being given.

W.L.F. Degen

**Geometric Continuity and Order of Contact**

The order of contact was introduced during the early period of differential geometry; later on a second version of this notion became familiar. While research in differential geometry did no longer persuade these questions a new interest arose with CAGD in order to smoothly join curve and surfaces segments.

After some more historical notes the relations between the order of contact and geometric continuity were pointed out and a proof of the equivalence of both versions of the former was given for hypersurfaces in $\mathbb{R}^d$ by means of local coordinate systems and tangent parameters. Furthermore, it was shown that the order of contact is invariant under parameter transformations as well as under differential mappings of the surrounding space, whereby the notions of jet (of order $k$) and the connection matrices arising from the iterated chain rule play a crucial role.

These methods were applied to the case where one hypersurface is given implicitly, the other in parametric representation yielding a simple criterion for $G^k$-continuity. Further applications concern among others rational curves and the control points of Bézier curves; explicit formulas for the connection matrices were given in both cases.

N. Dyn

**Hermite subdivision schemes on triangles for the evaluation of the PS-12 split element**

It is observed that the Powell-Sabin 12-split triangle is refinable, namely the same split of the 4 subtriangles of a triangle contains the lines of split of the original triangle. This property of the split is the key to the existence of a subdivision scheme, for the evaluation of the $C^1$ quadratic spline on the split which interpolates function and gradient values at the 3 vertices of the triangle, and normal derivatives at the midpoints of the edges. Explicit formulae for the Hermite subdivision step are given. For rendering the interpolant it is suggested to use the triangulation and the function values at the vertices obtained after a small number of subdivision iterations, and to use the known values of the gradient at the vertices to obtain the normals to the surface at the vertices of the triangulation. The shading of the 3D triangulation can then be done by Gouraud shading. It is further suggested to perturb the $C^1$-Hermite subdivision scheme which evaluates the above interpolant on the Powell-Sabin 12-split triangle, to obtain other $C^1$ schemes with a shape parameter.

G. Farin* and D. Hansford

**A Permanence Principle for Shape Control**

We analyze the discrete version of the bilinearly blended Coons patch which creates a rectangular control net from four input boundary polygons. We note two properties: a) the discrete Coons patch creates subquadrilaterals that are as close as possible (least squares sense) to parallelograms. b) consider a sub-control net. It has four control boundary polygons. If we apply the Coons patch to it, we create a control net which is identical to the one obtained from the original boundaries.

Applying b) (the permanence principle) to all 3x3 subnets, we see that each interior control point is a certain linear combinations of its eight neighbors. Writing out all these relationships (masks), we have a linear system the solution of which is the discrete Coons patch. By varying the coefficients of the masks we obtain significant improvements over the shape of the discrete Coons patch.

We then apply the same principle to Bézier triangular control nets, surface fairing, and to mesh optimization of irregular triangular meshes.

M.S. Floater

**Shape Properties of Tensor-Product Bernstein Polynomials and B-splines**

In this talk I will review recent work on conditions for monotonicity and convexity of tensor-product spline functions, including Bernstein polynomials as a special case.

A simple condition is to demand monotonicity or convexity, respectively, of the (piecewise bilinear) control net. While monotonicity of the control net could be a useful condition in practice, convexity of the control net can only be satisfied by trivial, so called 'translational', spline functions. The latter deficiency has motivated the development of weaker, more practical convexity conditions. Such conditions, both linear and nonlinear, have been used to effect in constrained least squares scattered data fitting.

I will also discuss some very recent progress towards characterizing tensor-product systems of functions which preserve monotonicity. It has been shown that a necessary condition is that both the univariate systems which form the tensor-product system must preserve linearity, unlike in the univariate case. Using this fact one can show that total positivity does not imply preservation of monotonicity, again in contrast to the univariate case. A set of sufficient and necessary conditions for monotonicity preservation of a tensor-product system has not yet been found and remains an interesting research problem.

R. Goldman

**Complex Contour Integration and B-splines of Negative Degree**

B-splines of negative degree have been introduced recently by the author, who has extended the de Boor recurrence to negative degrees. Although these negative degree B-splines are rational functions rather than piecewise polynomials, they share many properties with their polynomial kin: they are linearly independent, form a partition of unity, obey a de Boor recurrence, and satisfy a Marsden identity. There is also a novel theory of blossoming associated with B-splines of negative degree.

The standard B-spline have other interesting properties. For example, the divided difference of a differentiable function can be expressed as a real integral involving B-splines of positive degree. Here we show that for B-splines of negative degree and arbitrary analytic functions the analogous divided difference identity involves complex contour integration.

Divided difference expressions for the coefficients of an arbitrary analytic function relative to the B-splines of negative degree are then derived from the Marsden identity for B-splines of negative degree by invoking the complex contour integration formula for the divided difference. Similarly, divided difference expressions for the dual functionals for the rational basis functions that appear in the Marsden identity juxtaposed to the B-splines of negative degree are derived from the Marsden identity by invoking a contour integration identity for the B-splines of negative degree. These dual functionals are then applied to derive a divided difference formula for the B-splines of negative degree.

We close with some observations concerning the origins of the axioms for the multiaffine and multirational blossom and we explain how these axioms are related to complex contour integrations formulas and to axioms for the divided difference.

T.N.T. Goodman

**Variable degree splines**

Spline functions of degree $n$ are generalised to allow different degrees on different intervals. As the degree on an interval goes to infinity, the spline on that interval converges to a polynomial of degree $n-2$, thus providing a tension parameter. The theory of blossoms for splines whose pieces lie in Chebyshev systems is extended to cover the above case and applications to shape preserving interpolation by spline curves is mentioned.

G. Greiner

**Efficient Visualization of Volume Data Based on Finite Element Techniques**

When visualizing volume data, which are either generated by measurement (e.g. CT or MR) or resulting from a numerical simulation (e.g. CFD), one faces the problem to handle an enormous amount of data. In order to do this in an efficient way, data reduction is necessary, leading to an approximation problem. Moreover, a multiresolution representation of the data will allow a tradeoff between speed and accuracy.

Our method to generate multiresolution representations of volume data uses a hierarchical basis representation, based on adaptive multilevel finite elements. For a given data set a sequence of nested approximating spaces is supplied. The hierarchy of finite element spaces is generated iteratively by adaptive refinement of the underlying mesh with the help of an *a posteriori* error estimator. The error estimator and the approximation itself can be based on various norms. Besides the standard $L_2$ norm we also use Sobolev norms. The latter though only slightly more complicated than the standard $L^2$-norm allows for better approximation. This is in accordance with the error estimates we obtain. For example, for Sobolev norm approximation $L^\infty$-estimates are possible.

The triangulation sequence has to be *stable* with respect to some measure of degeneracy. Our algorithm which combines regular (red) and irregular (green) mesh refinement. We use a hybrid mesh which consists of tetrahedra and octahedra. There is only one regular refinement rule for tetrahedra and for octahedra respectively. Thus each cell of the initial mesh only produces two congruence classes. The local rules are combined and rearranged into a *global refinement algorithm* which guarantees for stability and conformity.

M. Gross

**Finite Element Procedures for Surgery Simulation**

Surgery simulation has become an attractive and challenging research area, which combines methods from various disciplines including robotics, graphics, vision, and CAGD. This talk addresses some of the modeling aspects in surgery simulation.

We present two Finite element models which can be used in the context of cranio- and maxillo-facial surgery. The first model constructs a globally $C^1$ continuous variational surface over unstructured triangle meshes describing the initial facial geometry. The underlying soft tissue is represented by springs attached to the facial surface and scull. The postsurgical facial surface is computed by solving the variational problem for given boundary conditions using Galerkin FE projections.

The second model computes the underlying governing equations for linear elasticity in volumetric settings. We start with a tetrahedral decomposition of the facial soft tissue and suggest $C^0$ tetrahedral Bézier elements as shape functions for the finite element approach. Nonlinear material behavior can be approximated by iteration of the linearized model.

We discuss some of the current problems and sketch future work comprising hierarchical solvers and $C^1$ continuous volume modeling. Further information can be found on: `http://www.inf.ethz.ch/department/IS/cg`.

H. Hagen

**Simulation Based Modeling**

In this talk, we present new approaches for the modeling, animation, and visualization with skeleton-based implicit surfaces. As a general base for our investigations, we provide a mathematical definition of implicit primitives, serving as a starting point for a comprehensible explanation of the terms skeleton-based implicit primitive and skeleton-based implicit surface. As an approach to solving the problem to efficiently model animated implicit models with possibly varying topology, we introduce a method for the dynamic polygonization of skeleton-based implicit surfaces, which combines the high performance of particle-based sampling with a closed surface representation in the form of an adaptive triangulation. Focusing on the application of skeleton-based implicit surfaces, we developed new techniques for flow simulations, and for modeling of virtual humans.

B. Jüttler* and C. Mäurer

**Rational Approximation of Rotation Minimizing Frames with Cubic Pythagorean Hodograph Splines**

The talk is devoted to spatial cubic Pythagorean Hodograph (PH) curves which enjoy a number of remarkable properties, such as polynomial arc-length function and existence of associated rational frames. Firstly, we derive a construction of such curves via interpolation of $G^1$ Hermite boundary data with PH cubics. Based on a thorough discussion of the existence of solutions we formulate an algorithm for approximately converting arbitrary space curves into cubic PH splines. Secondly we discuss applications to sweep surface modeling. With the help of the rational frames that are associated with PH curves we construct a rational approximation of the rotation minimizing frames of space curves.

P. Kaklis* and G.D. Koras

**Convexity-Preserving Fairing of Parametric Tensor-Product B-spline Surfaces**

A frequently occurring shape constraint in the Computer-Aided Geometric Design (CAGD) of surfaces is that of the local convexity of a surface. It is thus desirable to possess conditions, which secure the convexity of the surface under processing and, moreover, are handy from the computational point of view. We would also prefer these conditions to be of discrete character, depending only on the control points of the surface, the knot-vectors being kept fixed. In the first part of the talk we have presented four alternative sets of conditions for local convexity. The first condition-set is obtained by adapting the technique of Floater (1994) to parametric tensor-product B-spline surfaces of degree $m \times n$. The resulting condition-set consists of $2m(m+1)n(n+1)(mn-1)$ cubic and $16(m-1)m(m+1)(n-1)n\ (n+1)$ sextic, with respect to the coordinates of the associated control points, inequalities. In order to reduce the size of this condition-set, we develop two variations of the generalization of Floater's technique. The first variation appeals to an identity due to Mørken (1991), while the second one uses the Bézier representation of the quadratic and bilinear polynomials encountered in the course of Floater's technique. Combining these two variations, we get a considerably smaller condition-set, consisting of $6(3m-2)(3n-2)$ cubic and $9(3m-2)(3n-2)$ sextic inequalities. We discuss the relative weakness of the derived condition sets and the possibility of improving their weakness by knot insertion. The second part of the talk summarized our numerical experience with fairing of parametric tensor-product B-spline surfaces under convexity and tolerance constraints. Fairness is measured in terms of the so-called "thin-plate energy" functional, convexity is imposed via any of the four condition-sets, derived in the first part of the talk, while tolerance constraints are expressed as bounds on the deviation from the control points of the patch that has to be faired. The talk ended with presenting and discussing the numerical performance of the so-formed fairing scheme for three industrial surfaces, two of automotive and one of ship-building interest.

L. Kobbelt

**Free form modeling based on polygonal meshes**

During the last years the concept of multi-resolution modeling has gained special attention in many fields of computer graphics and geometric modeling. In this paper we generalize powerful multi-resolution techniques to arbitrary triangle meshes without requiring subdivision connectivity. Our major observation is that the hierarchy of nested spaces which is the structural core element of most multi-resolution algorithms can be replaced by the sequence of intermediate meshes emerging from the application of incremental mesh decimation. Performing such schemes with local frame coding of the detail coefficients already provides effective and efficient algorithms to extract multi-resolution information from unstructured meshes. In combination with discrete fairing techniques, i.e., the constrained minimization of discrete energy functionals, we obtain very fast mesh smoothing algorithms which are able to reduce noise from a geometrically specified frequency band in a multi-resolution decomposition. Putting mesh hierarchies, local frame coding and multi-level smoothing together allows us to propose a flexible and intuitive paradigm for interactive detail-preserving mesh modification. We show examples generated by our mesh modeling tool implementation to demonstrate its functionality.

R. Krasauskas

**New applications of real toric varieties in CAGD**

We propose the extended definition of toric varieties with non-standard real structures. It appears that the class of real toric surfaces includes Bézier surfaces (with control points in general position) and also other important low-degree rational surfaces in 3D space: all quadrics, cones over rational curves, quartic Dupin (also generalized) cyclides etc. New control point nets for latter surfaces are constructed.

We explain how the recent concept of global coordinates for toric surfaces is related to the universal parameterization concept (earlier introduced by the author). Also this leads to implicit equations of curves on toric surfaces and simple formulas of their intersection indices. One application of splines on toric surfaces is presented: $G^1$-blend of two natural quadrics using patches of rational canal surfaces.

Finally, in order to define blossoming we introduce a general concept of toric maps – common generalization of Bézier constructions and toric varieties.

Main conclusion: Toric Geometry is a natural field for free-form modeling, since it includes most classical constructions and is based on elementary convex geometry. The additional advantage is flexibility: product, cone, linear join, toric blowing up/down of toric varieties are also toric.

A. McEntee and H. McLaughlin*

**The shape of noisy discrete data**

The shape of noisy discrete ordered planar data is defined by counting the number of inflections in an associated polygonal curve. Except for the first and last data points, each point of the data is grown into a chord-gate. The process ensures that no two consecutive chords intersect. The first and last points are point gates. Among all polygonal curves that interpolate the gates in order there is exactly one with minimal length. It is computed and is called the optimal path. The number of inflections in the optimal path is assigned to the original noisy discrete data as a shape descriptor.

For noisy data, taken from a smooth curve that crosses its normal line everywhere, the assignment of inflections is consistent with human perception provided the noise is "reasonable".

Computing the number of inflections in the optimal path is considered a preprocessing step in fairing algorithms. The number is used as a constraint in several existing fairing algorithms in order to control the number of inflections in the faired curves. Experiments show that this also works well.

B. Mulansky

**Constrained Interpolation with Boundary Conditions**

Direct methods for shape preserving interpolation frequently require the solution of weakly coupled systems of inequalities. It is advantageous to interpret such systems of bidiagonal structure as a chain of relations. An algorithm based on the composition of these relations is presented and applied to the problem of convex interpolation by cubic $C^1$ splines with boundary conditions.

S. Morigi and M. Neamtu*

**Some Results for a Class of Generalized Polynomials**

A class of generalized polynomials is considered consisting of the null spaces of certain differential operators with constant coefficients. This class strictly contains algebraic polynomials and appropriately scaled trigonometric polynomials. An analog of the classical Bernstein operator is introduced and it is shown that generalized Bernstein polynomials of a continuous function converge to this function. A convergence result is also proved for degree elevation of the generalized polynomials. Moreover, the geometric nature of these functions is discussed and a connection with certain rational parametric curves is established.

J. Peters

**Curvature Continuous Surfacing**

At the Oberwolfach meeting 1995, U. Reif and H. Prautzsch each presented a novel approach to building $C^k$ surfaces of unrestricted connectivity based on what I call regional reparametrization. The presentation reviews regional parametrization and contrasts it with local and global parametrization by relating it to the analytic chart-and-atlas based definition of surfaces in a 4-level diagram that reconciles the classical mathematics view with the CAGD view of surfaces and their continuity. Based on theory and experiments, the talk points out limitations of the regional reparametrization approach to curvature continuous surfacing, and presents two modifications that (a) increase the flexibility to avoid flat spots and other shape restrictions (b) decrease the polynomial degree of the surface construction to bi-4.

H. Pottmann

**Approximation with kinematic surfaces for reverse engineering of geometric models**

New results on the reconstruction of special kinematic surfaces from scattered points and on the approximation of surfaces by kinematic surfaces are presented. Basis of the work is an algorithm for reconstruction of rotational and helical surfaces using line geometry. There, the following well-known result is used: Cylinders, surfaces of revolution and helical surfaces are characterized by the property that their normals lie in a linear line complex. Thus, a set of estimated normals at the data (or a subset) is approximated by a linear complex, which yields the generating motion of the surface and, in the second step, the reconstructed surface itself. This technique is useful for segmentation-based reverse engineering of geometric models. Combining the rotational surface reconstruction with region growing and modification algorithms, we can approximate or reconstruct general moulding surfaces. A moulding surface is traced out by a planar curve, whose plane is rolling on a developable surface. With a straight line or a circle as profile curve, one obtains a developable surface or a pipe surface, respectively. Of special interest is the approximation of doubly-curved surfaces by developable surfaces, since this problem arises in the design and manufacturing of ship hulls. Finally, we briefly address a generalization to the reconstruction of spiral surfaces and the approximation by ruled surfaces.

M.J. Pratt

**On a Class of Pythagorean-normal Surfaces with Planar Lines of Curvature**

Surfaces are sought that have planar lines of curvature and rational parametrizations. The approach used is a classical one, based on the use of the Gauss map and tangential coordinates. Two such classes of surfaces arise; only one of them is examined in detail here. This is shown to include two subclasses:

1.) rational parametric surfaces with denominator $1 + u^2 + v^2$ and numerator of arbitrary degree $\geq 2$ (with the exception of 3, which is excluded);

2.) integral surfaces of arbitrary odd degree $\geq 3$.

It is believed (though not yet fully proved) that no other such subclasses exist. Subclass 1 includes as its simplest case the cubic Dupin cyclide, which has a rational biquadratic parametrization and algebraic degree four. The simplest surface of Subclass 2 is Enepper's surface, with a bicubic parametrization and algebraic degree nine. An explicit representation is given for surfaces in the first subclass, and a method outlined for generating surfaces in the second subclass. These surfaces have a combination of properties making them suitable for use in computer-aided design.

H. Prautzsch

**Control net based modeling**

In 1995 I presented a simple method to construct regular $G^k$-spline surfaces of bidegree $2k+2$. In the meantime I proved the validity of this method for all $k$ and improved it to obtain "fair-shaped" surfaces. These splines form a linear space and minimize certain quadratic fairness functionals. The code is published under netlib (`http://netlib.bell-labs.com/netlib/a/index.html`, file: `FreeFormSplines`).

In the talk I showed several pictures of these splines and comparisons with other methods. The main result is that this and other low degree constructions are useful to represent "well-behaved" free form surfaces but are too stiff for "wild" forms. I also presented results of a method that works well in all cases.It is similar to Jörg Hahn's approach giving $G^k$ surfaces whose bidegree is $2k^2 + 2k + 1$.

The $G$-splines I presented are useful in geometric modelling applications. For example, a blend between two such spline surfaces can be obtained by blending their control nets and computing the spline defined by the blended control net. I showed examples of such blends computed by a program package developed by Lars Linsen.

U. Reif

**Analysis of subdivision algorithms for meshes of arbitrary topology**

A survey on the analysis of of subdivision algorithms for meshes of arbitrary topology is given. Among others, we present the following results:

- There exist five classes of subdivision matrices, characterized by their Jordan form, which admit the construction of linear stationary $C^1$-subdivision algorithms.

- If the subdivision matrix belongs to one of theses five classes, and if the characteristic map of the algorithm is regular and injective, then the subdivision surfaces generated by the algorithm are $C^1$-manifolds for almost all initial data.

- Using special sufficient conditions for symmetric subdivision schemes, the classical algorithms of Doo-Sabin and Catmull-Clark are verified to be $C^1$.

– Necessary and sufficient conditions for $C^k$-algorithms are given. In particular, if the algorithm generates $C^k$-limit surfaces of polynomial degree $d$, then it can be $2r$-flexible only if the degree estimate $d \geq 2r(k+1)$ holds.

– Topologically unrestricted rational B-splines (TURBS) provide a simple tool for constructing subdivision algorithms for arbitrary topology, flexibility and smoothness.

R. Sarraga

**A Variational Method for Fitting a $G^1$ Surface to Scattered Data Triangulated in $\mathbb{R}^3$ with Arbitrary Topology**

This talk presents a method for generating $G^1$ surface fits to input data points triangulated in $\mathbb{R}^3$ with arbitrary topology. The shape of a surface fit is obtained by minimizing an integral defined globally over the entire surface to be created. The theory of manifolds is used to express this global integral as a sum of integrals evaluated independently over each individual patch. The integral over each patch is computed by locally approximating the integrand with a function that is quadratic in the patch-defining function. Standard $G^1$ smoothness conditions are imposed between adjacent patches. The method is applied to data points taken from a sphere, a ball-nosed corner, a hexagonal corner, and a tube-like surface. In all cases, the method yields moderately good results, whose quality seems to be largely independent of the arrangement of input data points.

H.-P. Seidel

**Spline Approximation and Data Reduction for CNC Programs**

Conventional CNC machines use linear interpolation for geometry description. This kind of geometry representation exhibits several problems. For example, to describe a curved shape accurately, a huge amount of data is necessary. Another drawback is that the speed of the milling process is strongly restricted because the single segments are very short.

Since the latest generation of CNC machines can directly deal with spline data, there is a need for conversion of linear path descriptions to splines. Hereby the aim is to generate splines with relatively long polynomial segments and to reduce the data as much as possible. User specified tolerances have to be met, and sharp edges have to be detected and preserved automatically.

This talk presents the design and implementation of the spline translator **reduce** which has been developed jointly with Siemens AUT in the context of the SINUMERIK 840D control unit and has meanwhile been patented. The program is based on knot removal and realizes substantial gains over previous methods. A series of real data examples demonstrates the effectiveness of the method.

T. Varady

**Special Blending Surfaces**

Blending surfaces in this context are smooth transition surfaces, which connect adjacent primary surfaces and thus replace sharp edges of intersection. In this talk the interference of multiple blending surfaces is investigated in two situations.

The first problem is vertex blending, i.e., how to generate smooth transition surfaces between edge blends meeting at a vertex. The so-called setback-type vertex blends are analyzed. These are broadened patches with $2n$ sides, being capable of resolving difficult configurations, which would occur otherwise in the case of vertex blends without setbacks. A vertex blending concept, based on a few simple rules, is proposed, according to which it is possible to handle any number of convex, concave and smooth edges with uneven magnitude of radii in a uniform manner. Examples how to solve problems of terminating edge blends, shape imperfections, incomplete blends and independence of the blending sequence are also presented.

The second problem aims at constructing so-called overlapping blends, where adjacent blends may cover or intersect each other. Instead of this, such a construction is needed, which preserves the non-interfering parts of the blends, but at the same time it provides smooth transition between them. After investigating the topological structure of overlapping blends two solutions are shown. The first is based on double rolling balls, which touch each other during their sweeping motion. The second is the union of two free-form patches, which satisfy the initial boundary constraints.

M.G. Wagner

**Lossy Compression of Polygonal Meshes**

In this talk we present a simple and highly efficient approach to the continuous streaming of animated quadrilateral polygon meshes (morphs) in a low bandwidth environment. In order to reduce the number of frames for transmission we choose a sequence of I-frames which, along with velocity and acceleration vectors, allow to reconstruct an approximation of the morph on the client side. We then apply a lossy compression algorithm based on an energy minimizing subdivision scheme that enables us to compress the I-frame mesh data down to 5% of its original size without significant visual loss. The resulting multi-layered data structure provides robust and scalable transmission as well as real time encoding and decoding.

J. Wallner

**Global Results on Collision-Free Milling and General Offset Surfaces**

A surface $X$ is to be milled by a cutter $\Sigma$ which rotates around its axis while undergoing a 2-parameter translational motion. During the milling process the tool $\Sigma$ should not interfere which an already finished part of the surface. It is not an essential restriction if we reformulate the problem as follows: Given are a surface $X$ and a strictly convex body $\Sigma$, which undergoes a 2-parameter translational motion whose envelope is precisely $X$. Both surfaces are piecewise $C^2$, and convex edges are allowed.

It is easy to formulate conditions on the curvature of both $X$ and $\Sigma$ which imply that there are no collisions locally. By defining the general offset surface $\Gamma$ of $X$ with respect to $\Sigma$ and showing that the self-intersections of $\Gamma$ correspond to unwanted collisions in some cases, we are able to reduce the problems to showing that $\Gamma$ has no self-intersections. By a bit of global analysis this is done in several cases, including the case of a star-shaped surface $X$ and the case that $X$ is the graph surface of a function defined in a domain $D$ which has an outer parallel curve free of self-intersections.

A.J. Worsey

**Curve approximation and shaping using Greville abscissae**

We consider a problem related to the design and manufacture of turbine blades for aircraft engines, where the surfaces are constructed by lofting through planar sections curves. These section curves are designed as a composite Bézier curve with two segments of degree 7 or higher. After the initial design, the geometry must be analyzed and processed for manufacturing. This involves, for example, the operation of offsetting the planar section curves. Engineering and system constraints demand that the original design curves must in fact be approximated within a specified tolerance by cubic B-splines. Moreover, the approximation must not contain spurious inflection points that are not present in the original design. The problem arises, therefore, in trying to develop an approach for curve approximation that addresses the "competing" concepts of "shape fidelity" and "accuracy".

We present a method for approximating high degree Bézier curves with cubic B-splines that accommodates both issues. It is based on taking a convex combination of the methods of interpolation and the classical variation diminishing approximation. The combination is locally modifiable and hinges upon using the Greville abscissae as points of interpolation, rather than the knots of the B-spline approximation. We present examples showing how the method resolves the issues that other curve approximation schemes fail to address for the specific geometry related to turbine blades.

*Vortragender (bei mehreren Autoren).

Berichterstatter: B. Jüttler

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Gefördert durch DFG Deutsche Forschungsgemeinschaft

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