MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 24/1998

Algebraic Geometry

14.06-20.06.1998

The conference was organized by David Eisenbud, Joe Harris and Frank-Olaf Schreyer with an emphasis on topics around the moduli space of curves, quantum cohomology, and Gromov-Witten invariants. In selecting the speakers (as well as the participants) precedence was given to the bright young people in the field, for example Kai Behrend, Carel Faber, Barbara Fantechi, Lothar Goettsche, Brendan Hassett, Rahul Pandharipande, Michael Thaddeus, and Ravi Vakil.

The number of talks was kept to four per day, each of 50 minutes to allow plenty of time for discussion and to encourage questions at the end of the talks. Perhaps partly because of these policies, the attendance at the talks was very high. There were also many lively discussions among the members between the talks, and several research projects moved forward in this time.

The enthusiasm of the participants, the level of activity in discussions among them, and the quality of the talks, made us feel that this was a highly successful conference.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Abstracts of talks

MICHAEL THADDEUS

On the quantum cohomology of a symmetric product of a curve

The dth symmetric product of a curve of genus g is a smooth projective variety. The lecture is concerned with the little quantum cohomology ring of this variety, that is, the ring having its 3-point Gromov-Witten invariants as structure constants. This is of considerable interest, for example as the base ring of the quantum category in Seiberg-Witten theory. The main results give an explicit, general formula for the quantum product unless d is in the narrow intervall $[\frac{3}{4}g, g-1)$. Otherwise they still give a formula modulo third order terms. Explicit generators and relations can also be given unless d is in $[\frac{4}{5}g - \frac{3}{5}, g-1)$.

(The talk is a report on a joint paper with Aaron Bertram).

WILLIAM FULTON

Eigenvalues of Hermitian matrices and Schubert calculus (after Klyachko)

A. Klyachko has shown that if A and B are Hermitian $n \times n$ matrices, and $C = A + B$, the eigenvalues $\alpha: \alpha_1 \geq \ldots \geq \alpha_n$ of A, $\beta: \beta_1 \geq \ldots \geq \beta_n$ and $\gamma: \gamma_1 \geq \ldots \geq \gamma_n$ of C satisfy the inequalities

$$\sum_{k \in K} \gamma_k \leq \sum_{i \in I} \alpha_i + \sum_{j \in J} \beta_j$$

for subsets (I, J, K) of $\{1, \ldots, n\}$ of cardinality r such that the Schubert class σ^K appears in $\sigma^I \cdot \sigma^J$ in the cohomology of the Grassmannian of r-planes in $\mathbf{C}^n$; here σ^I corresponds to the partition $(i_r - r, \ldots, i_1 - 1)$, where $I = \{i_1 < i_2 < \ldots < i_n\}$. Together with the equation $\sum \gamma_k = \sum \alpha_i + \sum \beta_j$, these equations completely characterize which (α, β, γ) can arise as eigenvalues.

If (α, β, γ) are integral, there are corresponding representations $V^\alpha, V^\beta, V^\gamma$ of $GL_n(\mathbf{C})$ with these highest weights. Klyachko shows that $V^{N\gamma} \subset V^{N\alpha} \otimes V^{N\beta}$ for some positive N exactly, when the above inequalities are valid.

The saturation problem asks if $V^{N\gamma} \subset V^{N\alpha} \otimes V^{N\beta}$ implies $V^\gamma \subset V^\alpha \otimes V^\beta$. If this is true, it raises the challenge to algebraic geometers: to show that Schubert calculus for n-planes in m-space – at least as far as the question of whether products are zero or not – should be controlled by Schubert calculus for r-planes in n-space, for $1 \leq r < n$.

CAREL FABER

Identities for integrals on $\overline{M}_g$

Consider the following tautological classes on $\overline{M}_{g,n}$: ψ_n $(1 \leq i \leq n)$, the first chern classes of the n cotangent line bundles; $\kappa_j = \pi_{n+1,n*}(\psi_{n+1}^{j+1})$ (with $\pi_{n+1,n}: \overline{M}_{g,n+1} \to \overline{M}_{g,n}$ forgetting the last point); $\lambda_k = c_k(\mathbf{E})$, the k th Chern class of the Hodge bundle, pulled back from $\overline{M}_g$.

The intersection numbers of these classes can be computed: for those involving ψ_i and κ_j, this is Witten's conjecture = Kontsevich's theorem; to include the λ_k, use Mumfords's formula for $ch(\mathbf{E})$ and recursion (see alg-geom/9706006 for details).

In joint work with R. Pandharipande we prove the following explicit formulas for some of these numbers. Define $Z_k(t) \in \mathbf{Q}[k][t]$ by

$$Z_k(t) = 1 + \sum_{g \geq 1} t^{2g} \int_{\overline{M}_{g,1}} \frac{k^g + k^{g-1}\lambda_1 + \ldots + \lambda_g}{1 - \psi_1}.$$

Theorem. *1)* $Z_k(t) = Z_0^{k+1}(t)$, *2)* $Z_0(t) = \frac{t/2}{\sin(t/2)}$.

In the talk, we outlined proofs of $Z_{-1}(t) = 1$ resp. $Z_{-2}(t) = \frac{\sin(t/2)}{t/2}$ using global generation of the line bundle ψ_1 almost every where resp. a calculation on the hyperelliptic locus. 1) is proved using localization on $\overline{M}_{g,1}(\mathbf{P}^1, 1)$.

KRISTIAN RANESTAD

Abelian surfaces on Calabi-Yau 3-folds

(work in progress with Klaus Hulek). Abelian surfaces of small degree are often contained in Calabi-Yau 3-folds, similarly Calabi-Yau 3-folds of small degree often specialize to Calabi-Yau 3-folds with abelian surfaces on them. The first assertion is intimately connected with the fact that the moduli space of abelian surfaces of small degree is uniruled: An abelian surface on a CY 3-fold moves in a linear pencil, and therefore gives rise to a line in the moduli space of abelian surfaces. This idea was taken up and explored by Gross and Popescu starting with a very singular CY variety, the secant variety of an elliptic normal curve. The translation scrolls inside the secant variety are degenerate abelian surfaces and form a line on the boundary of the moduli of abelian surfaces. They show that the secant variety deforms to CY 3-folds with only isolated singularities and with a pencil of abelian surface as long as the degree of the elliptic curve is less than 11. This limit is related to the Del Pezzo bound for the possible smoothing of minimal elliptic surface singularities.

We explore a similar setting. In a $\mathbf{P}^2$ scroll over an elliptic curve any anticanonical divisor, if there is one, is a possibly degenerate abelian surface. If one can glue two $\mathbf{P}^2$ scrolls over elliptic curves along an anticanonical divisor, the union is a singular Calabi-Yau 3-fold. If furthermore the anticanonical divisor moves in a pencil on (at least) one of the two scrolls, then we are in a position like above.

We start by asking for smooth abelian surface with two pencils of plane cubic curves on it. The two pencils would then define $\mathbf{P}^2$-scrolls whose union is Calabi-Yau. It turns out that purely numerical considerations bounds the degree of these abelian surfaces to 18. The bound is obtained by abelian surfaces which form the complete intersection $((0,3),(3,0))$ in $\mathbf{P}^2 \times \mathbf{P}^2$ with its Segre embedding. For each degree d, $10 \leq d \leq 18$ there are numerical possibilities which are easily realized. For this talk we study the case $d = 12$, i.e. the case of abelian surfaces embedded linearly normal in $\mathbf{P}^5$.

We find and describe the abelian surfaces of degree 12 and the two scrolls defined by their pencils of plane cubic curves. The union of the two scrolls is a nonnormal Calabi Yau 3-fold of degree 12.

In a separate approach we construct via projected Del Pezzo 3-folds, nonnormal CY 3-folds in degrees 10,11,12 and 13. This construction gives a characterization of these CYs which we use in the final sections to identify the union of two ellipic scrolls considered earlier as a specialization of the nonnormal CY 3-folds of degree 12. The talk concluded with a proof using Reyes results on webs of quadrics and apolarity we describe the nonnormal locus of the Del Pezzo 3-folds.

BRENDON HASSETT

Limiting Plane Curves and the Minimal Model Program

Let $\mathcal{P}_d$ denote the smooth plane curves of degree d up to isomorphism. One natural question is to describe the closure of $\mathcal{P}_d$ in the moduli space $\overline{\mathcal{M}}_{g(d)}$ where $g(d) = \frac{1}{2}(d-1)(d-2)$. The corresponding curves are called **limiting plane curves**.

We approach this question locally. Let C_0 be a germ of an isolated plane curve singularity. For each smoothing $C \to \Delta$ of C_0, we can consider the local stable reduction $C^c \to \tilde{\Delta}$. The central fiber C_0^c takes the form

$$C_0^{\nu} \cup_{p_1,\ldots,p_b} C_T.$$

where C_0^{ν} is the normalization of C_0 and $(C_T, p_1, \ldots, p_b)$ is called the **tail** of the stable reduction. The locus of such tails is denoted $\mathcal{T}_{C_0}$, and is a closed subvariety of the moduli space of pointed stable curves.

We replace the family of curves with a family of log surfaces $(\mathcal{S}, \mathcal{C})$, where $\mathcal{S} = \mathrm{Spec}\mathbb{C}[[x,y]] \times \Delta$ is a trivial family of surface germs. We then apply local stable reduction for log surfaces to obtain $(\mathcal{S}^c, \mathcal{C}^c) \to \tilde{\Delta}$. This stable reduction is computed using the log minimal model program. Thus our limiting curve C_0^c sits naturally in some (singular) surface S_0^c.

These surfaces may be used to describe components of $\mathcal{T}_{C_0}$ for certain types of plane curve singularities. The singularities we consider are of **toric type**, i.e. topologically equivalent to $x^p = y^q$. We show that certain weighted plane curves $C_T \subset \mathbf{P}(p,q,1)$ naturally occur as elements of $\mathcal{T}_{C_0}$, and conjecture they are dense. We apply these results to describe certain boundary divisors of $\overline{\mathcal{P}}_d$.

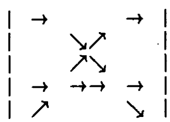
DUCO VAN STRATEN

Grassmann-like varieties and mirror symmetry

(Report on joint work with Klaus Altmann.) A flag-like quiver Q is an oriented graph with one source, l sinks and at other nodes as many arrows coming in as going out. The toric variety $\mathbf{P}(Q)$ associated to the flow polytope $\nabla(Q)$, which is always reflexive (Altmann-Hille), has many structural properties in common with the toric degenerations of Grassmannians ($l = 1$) and partial flagmanifolds that were considered by Sturmfels, Lakshmibai-Gonciulea, which in fact can be realized by very special quivers. The Picard group of $\mathbf{P}(Q)$ is $\mathbf{Z}^l$, the singularities sit in codimension ≥ 3 and the codimension 3 strata corresponds to (certain) nodes with two incoming and two outgoing arrows. The spaces of infinitesimal deformations and obstructions T^1 and T^2 can be understood in terms of Q, and although $T^2 \neq 0$ in general, it can be shown that $\mathbf{P}(Q)$ is smoothable in codimension 3. This means that the 3-dimensional nodal sections of $\mathbf{P}(Q)$ can be smoothed and in this way one gets new Calabi-Yau threefolds, for which one can perform (conjectural) mirror symmetric calculations for the rational curves using the hypergeometric series of the quiver Q. E.g.

$$\Phi = \sum_{m=0}^{\infty} \frac{(2m)!m!}{(m!)^3} \Big(\sum_{a,b,c\geq 0} \binom{b}{a}\binom{c}{b}\binom{c}{a}\binom{m}{b}\binom{m}{c}^2 \Big) q^m$$

is period of the mirror of the $(1,2)$ section of the deformed $\mathbf{P}(Q)$, where

$Q =$ 

LOTHAR GÖTSCHE

Counting curves on surfaces

I present a conjectural generating function for the numbers of curves of given genus in linear systems on surfaces. Let S be an algebraic surface, L a sufficiently ample line bundle on S, and $\delta \geq 0$. Let $t_\delta^S(L)$ be the number of curves in a δ-dimensional sub-linear system of $|L|$ which have δ nodes as only singularities. I give a conjectural generating function for the $t_\delta^S(L)$ as polynomials in the intersection numbers L^2, LK_S, K_S^2, and $c_2(S)$.

$$\sum_{\delta\geq 0} t_\delta^S(DG_2(\tau))^\delta = \frac{(DG_2(\tau)/q)^{\chi(L)} B_1(q)^{K_S^2} B_2(q)^{LK_S}}{(\Delta(\tau)D^2G_2(\tau)/q^2)^{\chi(O_S)/2}}.$$

Here $\chi(L)$ is the holomorphic Euler characteristic, G_2 and Δ are the well-known (quasi)-modular forms and $D = q\frac{d}{dq}$. B_1, B_2 are (unknown) universal power series. There is an algorithm for determining their coefficients. The conjecture is checked in many cases: it gives the formulas of Vainsencher and Kleiman-Piene for $\delta \leq 8$; it has been partially proven by Bryan-Leung for $K3$-surfaces and abelian surfaces; it is checked against the recursions of Caporaso-Harris and Vakil for $\mathbf{P}_2$ and rational ruled surfaces.

EDUARD LOOIJENGA

Stable cohomology of M_g

The results discussed reflect an approach to the conjecture of Mumford that says that the stable cohomology of M_g is generated by the tautological classes $\kappa_1, \kappa_2, \ldots$.

Given a compact connected oriented surface of genus g, S, then the mapping class group $\Gamma_0 := \pi_0 Homeo^+(S_g)$ admits a natural arithmetic quotient J_g that is obtained as an extension of the symplectic group $Sp(V_g)$ (here $V_g := H_1(S_g;\mathbf{Z})$) by the lattice $\Lambda^3 V_g/\omega_g \wedge V_g$ (here $\omega_g \in \Lambda^2 V_g$ is dual to the intersection form). The projection $\Gamma_g \to J_g$ induces $H^k(J_g;\mathbf{Q}) \to H^k(\Gamma_g;\mathbf{Q})$ and the latter map is known to stabilize as

$g \to \infty$. Kawarumi and Morita have shown that in the stable range the image of this map is just the subalgebra generated by the κ_i's. So Mumfords conjecture asserts the stable surjectivity of $H^k(J_g;\mathbf{Q}) \to H^k(\Gamma_g;\mathbf{Q})$.

Our approach is to introduce a moduli space $\check{M}_g$ (in the analytic setting) of what we call **irrational trees** of genus g; these are weakly normal curves C with only smooth irrational components of **positive** genus such that the normalization $\tilde{C} \to C$ induces an isomorphism on H_1. This is a quotient of the moduli space $\hat{M}_g$ of **good** curves of genus g (these are stable curves C such that the normalization $\tilde{C} \to C$ induces an isomorphism on H_1. One can show that the orbifold fundamental group of $\check{M}_g$ is isomrphic to J_g and that it maps isomorphically onto the 'stacky' fundamental group of $\hat{M}_g$. It is known that $H^k(\check{M}_g;\mathbf{Q}) \to H^k(\hat{M}_g;\mathbf{Q})$ is onto in the stable range and a recent theorem of U. Tillman (that says that the stable mapping class group is an infinite loop space) suggests that even:

Conjecture 1. *$H^k(\check{M}_g;\mathbf{Q}) \to H^k(M_g;\mathbf{Q})$ is onto in the stable range.*

The period map $M_g \to A_g$ extends to a proper morphism $\tilde{M}_g \to A_g$ which in fact factorizes through a morphism $\check{M}_g \to A_g$. Now over A_g we have a natural bundle $J_{g,0} \to A_g$ (in the orbifold sense) by Kummer varieties of tori such that $J_{g,0}$ is a virtual classifying space for J_g. Hain's universal normal function is a lift $\nu_g: M_g \to J_{g,0}$ of the period map. We prove

Theorem. *ν_g extends to a morphism $\check{\nu}_g: \check{M}_g \to J_{g,0}$ and $\check{\nu}_g$ is injective and 1-connected.*

We offer the following

Conjecture 2. *Given $k \in \mathbf{N}$, then $\check{\nu}_g$ is k-connected for g sufficiently large.*

This conjecture is strong and much harder to prove (hence easier to disprove) than conjecture 1. In fact conjecture 1 and 2 together imply Mumfords conjecture.

RAHUL PANDHARIPANDE

Methods of computing Gromov-Witten invariants

Gromov-Witten invariants are integrals of natural classes over the moduli space of stable maps. Already in 1990 three methods of determining GW invariants were known to the physicists: via the splitting axiom (WDVV), mirror symmetry, and Virasora constraints. I talked about the conjectural Virasora constraints of Hori and Xiang (and Katz) and explained joint work with E. Getzler in which we derived consequences of these formulas for certain chern class integrals over the moduli space of curves.

BARBARA FANTECHI

Obstruction calculus for functors of Artin rings

This is joint work with Marco Manetti. Let K be a field, $\underline{art}$ the category of local artinian K-algebras with residue field K. We study good deformation functors, i.e. covariant functors

$$F: \underline{art} \to \underline{sets}$$

s.t. $\sharp F(K) = 1$ and Schlessinger's $(H1), (H2)$ hold. We introduce the notion of obstruction for such functor as a pointed set. We prove that universal obstruction exists and is unique and complete, i.e. an elemant $a \in F(A)$ lifts to $F(B)$ (for $B \to A \to 0$ in $\underline{art}$) iff it's unobstructed. We give explicit conditions for the obstruction to be linear, i.e. to have a natural vector space structure. As applications, we generalize Ran-Kawamata's T^1-lifting criterion and prove that group good deformation functors in char 0 are smooth.

NICK SHEPHARD-BARON

Long extremal rays and symplectic resolutions

A symplectic singularity is a complex space X with a resolution $f: \tilde{X} \to X$, where $\tilde{X}$ possesses a nowhere degenerate holomorphic 2-form. The motivating question is whether, locally on X, this can be modelled in an algebraic group, as was shown by Steinberg, Springer and Brieskorn when $dimX = 2$. It turns out that, under some technical assumptions, this is true for isolated singularities: if $dimX \geq 4$, then the only possibillity is that $\tilde{X} \to X$ is the collapsing of the cotangent bundle $T^*\mathbf{P}^n$ of projective space. A main tool in the proof is the following characterization of projective space (sharpening an earlier result of Cho and Miyaoka): if Z is a normal projective variety such that every maximal family of rational curves through a fixed general point covers Z, then Z is projective space.

KAI BEHREND

Modifying Gromov-Witten invariants for varieties with $h^{2,0} \neq 0$

(joint work with Barbara Fantechi). If you try to apply Gromov-Witten theory to count rational curves on a K3 surface X, you discover that the expected number of $\overline{M}_{0,0}(X,\beta)$ is -1. According to GW, rational curves should not exist. But, in fact, they do. If one deforms $X,\beta)$ in such a way that β does not remain of type (p,p) then one gets obstructions to deforming stable maps of class β. If one restricts to deformations such that β stays of type (p,p), all obstructions that actually occur are contained in a subspace of the GW-obstruction space $H^1(C, f^*T_X)$. This subspace is the kernel of the natural map

$$H^1(C,f^*T_X) \to H^0(X,\Omega^2_X)^*$$

Thus we may modify the usual GW-obstruction theory $E = (R\pi_* f^*T_X)^*$ by $H^0(X,\Omega^2_X)$ to get a modified obstruction theory $\tilde{E}$. If, for all stable maps $f: C \to X$ in $\overline{M}_{g,n}(X,\beta)$ the map $H^0(X,\Omega^2_X) \to H^0(C, f^*\Omega_X \otimes \omega)$ is injective, $\tilde{E}$ is 1-perfect and gives rise to modified GW-invariants. Examples of X for which this theory applies are irreducible complex symplectic varieties. In case X is K3 the modified expected dimension of $\overline{M}_{0,0}(X,\beta)$ is zero. If β is primitive, then $deg[\overline{M}_{0,0}(X,\beta)]^{vir} = n(h)$ where $\sum_{h=0}^{\infty} n(h)q^h = \frac{q}{\Delta(q)}$ And $2h-2 = \beta^2$, by arguments of Fantechi, Göttsche, van Straten, Beauville, Yau and Zaslow. Note that there is an alternative approach (in certain cases) by Bryan and Loung.

ANDRÉ HIRSCHOWITZ

Moduli of perfect complexes

Joint work with Carlos Simpson (Toulouse). Let us fix a natural integer $e \in \mathbf{N}$ and consider, for $a \in \mathbf{N}$, the affine (k-)scheme M_a of matrices of type $(a, a+e)$. It is nicely stratified by the rank of the matrix. For b larger than a, a large open subset M_b^o in M_b has (locally) smooth maps to M_a compatible to the stratifications. So what about glueing these M_a together into a single (locally?) algebraic stack? Just as the stack of vector bundles on $\mathbf{P}^1$ glues together the versal deformation spaces of these bundles.

After inspection, it turns out that the right object is a 2-stack, and not an ordinary stack. And the similar problem for complexes of length r leads to a $(r+1)$-stack.

We introduce a hopefully convincing notion of Segal stacks suited for such purposes, and obtain, for bounded perfect complexes, the natural descent and algebraicity results.

RAVI VAKIL

Stable maps and characteristic numbers of plane quartics

A **characteristic number problem** is an enumerative problem of the following form: Given a dimension D family of plane curves (or curves in a larger projective space), how many pass through a general points and are tangent to $D-a$ general lines? The algebraic geometers of the last century were able to compute the characteristic numbers of curves of degree $d \leq 4$ (and genus $\leq \binom{d-1}{2}$), culminating in Zeuthen's determination of characteristic numbers of smooth quartics. The arguments were not rigorous, and one of Hilbert's problems was thus to put the computations of Schubert, Zeuthen et al. on a solid foundation. We complete this verification by computing the characteristic numbers of plane quartics in a manner somewhat reminiscent of Zeuthen, and incidentally give a quick method for conics and cubics as well.

These are really problems in intersection theory. Classically, the philosophy was to construct a good (hopefully smooth) compactification of the space of smooth curves, where the divisors A (corresponding to curves through a fixed general point) and TL (corresponding to curves tangent to a fixed general line) are hopefully Cartier, and $A^a TL^{D-a}$ gives the characteristic numbers of the family. The space of complete conics work for $d=2$, and Aluffi's "space of complete cubics" works for $d=3$. We use the space $\overline{M}_g(\mathbf{P}^2, d)^*$

($g = \binom{d-1}{2}$, $d = 2,3,4$), which is the component of the (Deligne-Mumford) moduli stack of stable maps generically corresponding to smooth plane curves. Although the machinery is heavy, the method itself is naive and straightforward. After defining A and TL correctly,

$$2(d-1)A = TL + \text{boundary}.$$

We find the boundary divisors and multiplicities, and intersect this relation with one-parameter families $A^a TL^{D-1-a}$ $(0 \le a \le D-1)$.

The fact that such a naive approach works shows the power of Kontsevich's moduli space of stable maps.

HUBERT FLENNER

Atiyah class and semiregularity map for modules

Let X be a complex algebraic manifold and $Y \hookrightarrow X$ locally a complet intersection. Then the tangent space of **H**, the Hilbert moduli space of X at $[Y]$ is given by $H^0(N_{Y/X})$ whereas the obstructions are given by $H^1(N_{Y/X})$. In particular, if $H^1(N_{Y/X}) = 0$ then **H** is smooth at $[Y]$. There are many situations, where **H** is smooth although $H^1(N_{Y/X}) \neq 0$. In the 70's S. Bloch introduced a map $\sigma: H^1(N_{Y/X}) \to H^{k+1}(\Omega_X^{k-1})$ and showed that the injectivity of this map already implies that **H** is smooth at $[Y]$. Another typical example where the 'natural' obstruction module does not vanish is given by the trace map of Artamkin-Mukai. In this talk, which described a joint work with R.O. Buchweitz (Toronto), we propose to introduce a whole bunch of semiregularity maps for deformation of modules, say $\mathcal{E} \in Coh(X)$, as follows. First one has the Atiyah class of $\mathcal{E}$ which is an element in $Ext^1(\mathcal{E}, \mathcal{E} \otimes \Omega_X^1)$. The obstructions for deformation of modules are just $Ext^2(\mathcal{E}, \mathcal{E})$. Multiplying by the power $at^k(\mathcal{E})/k! \in Ext^k(\mathcal{E}, \mathcal{E} \otimes \Omega_X^k)$ of the Atiyah class $at(\mathcal{E})$ we obtain a map

$$\sigma_k: Ext^2(\mathcal{E}, \mathcal{E}) \to Ext^{k+2}(\mathcal{E}, \mathcal{E} \otimes \Omega_X^k).$$

Again one has the result, that the injectivity of $\sigma = \oplus_{k \ge 0} \sigma_k$ implies that $\mathcal{E}$ has a smooth versal deformation. More generally, if $(S, 0)$ is the base space of the semiuniversal deformation, then $dimS \ge dimExt^1(\mathcal{E}, \mathcal{E}) - dimKer\sigma$. The map σ_0 is just Artamkin's map. Moreover, Bloch's semiregularity map is the composition

$$T^2_{Y/X} \to Ext^2(\mathcal{O}_Y, \mathcal{O}_Y) \xrightarrow{\sigma_{k-1}} H^{k+1}(\Omega_X^{k-1})$$

($k := codimY$), and so one can define such map more generally for the case of subspaces $Y \hookrightarrow X$ (without the assumption, that Y is a locally complete intersection). Finally, a similar construction using Atiyah classes leads to a new description of the infinitesimal Abel-Jacobi map for deformation of modules and give a description of its tangent map.

EMILIA MEZZETTI

On threefolds which are covered by a family of lines of dimension 2

This is a joint work with Dario Portelli. We study the problem of the classification of projective varieties X of dimension three, which are covered by a family of lines Σ of dimension 2, but not by a larger family. If P is a general point of X, then there is a finite number of lines of Σ passing through P: call μ this number. If Σ is reducible and $\Sigma_1, \ldots, \Sigma_s$ are its irreducible components, let μ_i denote the number of lines of Σ_i passing through P. In this case, we have $\mu = \mu_1 + \ldots + \mu_s$.

If $\mu = 1$, then X is birationally a scroll over a surface: the complete classification of scrolls seems to be hopeless, so from now on we assume $\mu \ge 2$.

Our point of view is the following: it is enough to classify hypersurfaces in $\mathbf{P}^4$ having a family of lines with the requested properties, because every threefold can be birationally projected into $\mathbf{P}^4$ to a hypersurface having the same degree.

We prove that such a hypersurface X either is birationally a quadric bundle or it belongs to one of the following families, which are characterized by the values of s and μ_i:

1. X is a cubic hypersurface, $\mu = 6$, Σ is irreducible if X is smooth;

2. X is a projection of a complete intersection of two quadrics in $\mathbf{P}^5$, $\mu = 4$ and in general Σ is irreducible;

3. X is a projection of a quintic threefold, general section of the Grassmannian $\mathbf{G}(1,4)$ with a $\mathbf{P}^6$, $\mu = 3$ and in general Σ is irreducible;

4. X is a projection of a threefold of degree 6 contained in $\mathbf{P}^7$, hyperplane section of $\mathbf{P}^2 \times \mathbf{P}^2$, Σ has two irreducible components with $\mu_1 = \mu_2 = 1$;

5. X is a projection of $\mathbf{P}^1 \times \mathbf{P}^1 \times \mathbf{P}^1$, Σ has three irreducible componens with $\mu_1 = \mu_2 = \mu_3 = 1$.

CHARKES WALTER

Non-pfaffian subcanonical subschemes

A subscheme $Z \subset X$ is subcanonical if it is locally Gorenstein, and its canonical bundle is the restriction of a line bundle on X.

One way of constructing subcanonical subschemes of codimension 3 in X is to take a vector bundle $\mathcal{E}$ of odd rank $2n+1$ on X, a line bundle L, and an alternating map $\phi : \mathcal{E}^* \to \mathcal{E}(L)$ such that the degeneracy locus $Z(\phi) := \{x \in X \mid \mathrm{rk}(\phi(x)) < 2n\}$ has the expected codimension 3. Such a $Z(\phi)$ is subcanonical and is called a *pfaffian subscheme* because its ideal sheaf is generated locally by the submaximal pfaffians of ϕ. This construction is due to Buchsbaum and Eisenbud.

In this talk I gave several examples of nonpfaffian subcanonical subschemes of codimension 3 in $\mathbf{P}^{n+3}$. They included nonclassical Enriques surfaces in $\mathbf{P}^5$ in characteristic 2, a union of ten 2-planes in $\mathbf{P}^5$ in characteristic 2, and a 4-fold of degree 336 in $\mathbf{P}^7$ in any characteristic. The basic method of construction was as a degeneracy locus where two lagrangian subbundles of a twisted orthogonal bundle have intersection of dimension more than 1.

Joint work with D. Eisenbud and S. Popescu.

ZIV RAN

On Nagata's problem

The problem states: Let $P_1, \ldots, P_r \in \mathbf{P}^2$ be generic points. Then any plane curve C satisfies

$$\sum_1^r mult_{P_i}(C) \leq \sqrt{r} deg(C), \quad r \geq 0.$$

The case when r is a square was done by Nagata himself. In this talk we discuss some recent progress on this problem culminating in a proof that the inequality holds provided the fractional part of $\sqrt{r}$ is not 'too close' to $\frac{1}{2}$.

Report by Frank-Olaf Schreyer

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Gefördert durch
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