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Geometric Analysis and Singular Spaces

Oberwolfach, 21. bis 27. Juni 1998

The international conference on *Geometric Analysis and Singular Spaces* was held from June 21st to June 27th, 1998, in Oberwolfach. The organizing committee consisted of J.-M. Bismut (Orsay), J. Brüning (Berlin), and R.-B. Melrose (Boston).

Following an already established tradition, the conference presented a very limited number of scheduled talks, among them three survey presentations, and evening talks organized by the participants upon request. Most of the talks caused a very lively discussion and fruitful interaction among the participants showing, in particular, the very active state of the subject.

The survey talks (by Altschuler, Lejan, and Youssin) were directed at techniques which were not presented on former meetings of this kind and less familiar to most of the participants; they were very well received.

The main topics communicated in the other talks could be grouped into a few areas, naturally with some overlap. These were

Deformation, localization, and gluing techniques (Bismut, Köhler, Meinrenken, Porti, Rumin, Singer),

Computations or estimates for spectral invariants (Dai, Goette, Guillopé, Lott, Müller, Nicolaescu),

Index theorems (Bismut, Köhler, Lesch, Lott, Meinrenken, Moscovici, Salomonsen, Zhang),

Complete noncompact manifolds, compactifications, and harmonic analysis (Bunke, Farber, Ji, Meinrenken, Müller, Paradan, Shubin, Sjamaar).

All participants regarded the meeting as very informative and stimulating; unanimously, the participants asked the organizers to apply for a followup meeting in two years.

Dank einer Unterstützung im Rahmen des EU-Programmes TMR (Training and Mobility of Researchers) konnten zusätzlich einige jüngere Mathematiker zu der Tagung eingeladen werden. Dies ist einerseits eine hervorragende Förderung des wissenschaftlichen Nachwuchses und gibt andererseits den etablierten Kollegen die Gelegenheit, besonders begabte junge Mathematiker kennenzulernen.

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Scheduled talks

Name: DANIEL ALTSCHULER
Title: *Configuration spaces, Feynman diagrams, and knot invariants* (joint work with L. FREIDEL)
Abstract: This talk was based on a joint paper with L. Freidel, *Vassiliev knot invariants and Chern-Simons perturbation theory to all orders*, Commun. Math. Phys. **187**, 261 (1997).

To any trivalent graph Γ with a distinguished cycle there corresponds a function $I(\Gamma)$ on the space of smooth embeddings of S^1 in $\mathbb{R}^3$. This correspondence is due to Kontsevich and Bott-Taubes. It is obtained by integrating a topform over a compactified configuration space, which is a compact manifold with corners. Both the topform and the configuration space are specified by the graph. Define the degree of a graph to be (half) the number of vertices. We proved the following:

- Summing the contributions $I(\Gamma)$ over the graphs Γ of a given degree N leads to knot invariants for any N.
- These knots invariants are Vassiliev (or finite type) invariants of degree N.
- All Vassiliev knot invariants are linear combinations of these invariants.

Name: MIKHAEL FARBER
Title: *Witten deformation and partial differential forms* (joint work with E. SHUSTIN)
Abstract: We consider the Witten deformation on a noncompact manifold and restrict it to differential forms which behave polynomially near infinity. Such polynomial differential forms naturally appear on manifolds with cylindrical structure. We prove that the cohomology of the Witten deformation, d_h, acting on the complex of polynomially growing forms can be computed as the cohomology of the negative remote fiber of h.

Name: LAURENT GUILLOPÉ
Title: *The wave trace on Riemannian surfaces* (joint work with M. ZWORSKI)
Abstract: For a Riemannian surface $M = \mathbb{H}/\Gamma$, Γ torsion free and of finite type, the resonance spectrum $\mathcal{R}_M$ is defined as the set of poles of the meromorphic extension to $\mathbb{C}$ of the resolvent $(\Delta_M - \frac{1}{4} - \lambda^2)^{-1} : L^2_{\text{comp}}(M) \to L^2_{\text{loc}}(M)$, defined as an L^2-bounded operator for $\operatorname{Im} \lambda < 0$. Each resonance is given a convenient multiplicity $m(\rho)$ which is the standard one when the resonance is an L^2-eigenvalue. The unusual λ-parameter choice induces a formulation of the counting result similar to the Euclidean framework.

We limit ourselves here to the discussion of lower bounds which are based on the singularities of the wave trace, hence the title of this talk. The Poisson formula

$$0-\operatorname{Tr}\cos t\sqrt{\Delta_M - \frac{1}{4}} = \frac{1}{2}\sum_{\rho\in\mathcal{R}_M} m(\rho)\, e^{i|t|\rho}, \quad t \neq 0$$

gives, through the short time wave asymptotics, the lower bound

$$\#\{\rho \in \mathcal{R}_M, |\rho| \leq r\} \geq Cr^2.$$

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This is valid for any compact perturbation $\widetilde{M}$ of the constant curvature surface M, provided the $\mathrm{o}-\mathrm{vol}\,(\widetilde{M})$ is non-zero. That the lower bound is a strip

$$\#\{\rho \in \mathcal{R}_M | \operatorname{Im}\rho \leq \varepsilon^{-1}\} = \Omega(r^{1-\epsilon}),\ \varepsilon \in (0, 1/2),$$

is obtained by following an argument of Ihawa and using the Selberg wave trace formula.

Name: Lizhen Ji
Title: *Analytic structures of compactifications of symmetric and locally symmetric spaces* (joint work with Armand Borel)
Abstract: Given a semisimple algebraic group $\mathbf{G}$ defined over $\mathbb{Q}$, there are four natural spaces:

1. $G = \mathbf{G}(\mathbb{R})$, a pseudo-Riemannian symmetric space induced from the non-degenerate Killing form on $\mathfrak{g}$, the tangent space at the identity element.

2. $X = G/K$, a Riemannian symmetric space, where K is a maximal compact subgroup of G.

3. $\Gamma\backslash X$, a locally (Riemannian) symmetric space, where $\Gamma \subset \mathbf{G}(\mathbb{Q})$ is an arithmetic subgroup.

4. $\Gamma\backslash G$, a locally (pseudo-Riemannian) symmetric space.

These spaces arise naturally from many different situations. The spaces G, X are non-compact, and for many natural Γ, the quotient spaces $\Gamma\backslash X$, $\Gamma\backslash G$ are also non-compact.

Problems which were considered in the talk were how to construct compactifications of these spaces and how to understand their properties and relations. Compactifications of X, $\Gamma\backslash X$ have been studied for a long time, and we are interested in their analytic structures. Compactifications of G, $\Gamma\backslash G$ have not been studied in detail before, and we construct their compactifications and study their analytic structures. Results on compactifications are presented in the order: X, $\Gamma\backslash X$, G, $\Gamma\backslash G$.

Name: Kai Köhler
Title: *Localization of equivariant torsion: A Lefschetz fixed point theorem in Arakelov geometry* (joint work with Damian Roessler)
Abstract: We consider algebraic varieties (flat schemes over Spec $\mathbb{Z}$ with some regularity conditions) endowed with an action of the group scheme of n-th roots of unity and we define an equivariant arithmetic K_0-theory of equivariant bundles equipped with hermitian metrics for these varieties. The direct image in this K_0-theory is provided by the equivariant complex analytic torsion and equivariant Bott-Chern currents. We show that the direct image localizes on the fixed point scheme, which implies a version of Bismut's conjecture of an equivariant arithmetic Riemann-Roch theorem. As a corollary one gets the explicit calculation of the heights of certain arithmetic varieties.

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Name: YVES LEJAN
Title: *Geodesic integrals and stable processes* (joint work with N. ENRIQUEZ and J. FRANCHI)
Abstract: Let $\mathcal{M}$ be a geometrically finite hyperbolic surface with infinite volume, having at least one cusp. We obtain the limit law under the Patterson-Sullivan measure on $T^1\mathcal{M}$ of the windings of the geodesics of $\mathcal{M}$ around the cusps. This limit law is stable with parameter $2\delta - 1$, where δ is the Hausdorff dimension of the limit set of the subgroup Γ of Möbius isometries associated with $\mathcal{M}$. The normalization is $t^{\frac{-1}{2\delta-1}}$, for geodesics of length t. Our method relies on a precise comparison between geodesics and Brownian paths, for which we need to approach the Patterson-Sullivan measure mentioned above by measures that are regular along the stable leaves.

Name: JOHN LOTT
Title: *Signatures and higher signatures of S^1-quotients*
Abstract: Let M be a smooth compact oriented manifold of dimension $4k+1$, with an S^1-action. We define signatures and higher signatures of the quotient space $S^1\backslash M$. The signature $\sigma_{S^1}(M) \in \mathbf{Z}$ is an oriented S^1-homotopy invariant of M. Let M^{S^1} be the fixed-point set of the S^1-action. Add an S^1-invariant Riemannian metric to M. If the S^1-action is semifree, we prove

$$\sigma_{S^1}(M) = \int_{S^1\backslash M} L(T(S^1\backslash M)) + \eta(M^{S^1}). \qquad (1)$$

There is an extension of (1) to general S^1-actions.

To define higher signatures of $S^1\backslash M$, we use a noncommutative eta-form to define an analog of the right-hand-side of (1), with value in the noncommutative de Rham homology of an appropriate algebra. We show the metric invariance of the object that we construct and conjecture that it is an oriented S^1-homotopy invariant.

Name: ECKHARD MEINRENKEN
Title: *Localization for Hamiltonian loop group actions* (joint work with A. ALEKSEEV and C. WOODWARD)
Abstract: Let $(\mathcal{M}, \omega, \psi)$ be a Hamiltonian LG-space with proper moment map $\psi : \mathcal{M} \to L\mathfrak{g}^*$, equivariant with respect to the gauge action on $L\mathfrak{g}^* = \Omega^1(S^1, \mathfrak{g})$. If $\mathcal{O}$ is a regular value of ψ, the symplectic quotient $\mathcal{M}_{\text{red}} = \psi^{-1}(\mathcal{O})/G$ is a compact symplectic orbifold. In this talk we describe the symplectic volume, intersection pairings and (in the prequantizable case) the Riemann-Roch number of $\mathcal{M}_{\text{red}}$ in terms of fixed point formulas. In our excample $\mathcal{M}$ is the moduli space of flat G-conections on a surface Σ with boundary, possibly with a finite number of marked points. In this case $\mathcal{M}_{\text{red}}$ is the moduli space for the surface $\bar{\Sigma}$ obtained by capping off the boundary, and our theorem recovers (and extends) Witten's formulas for intersection pairings.

Name: HENRI MOSCOVICI
Title: *The transverse index theorem* (joint work with A. CONNES)
Abstract: The talk was devoted to the solution of a longstanding internal problem of noncommutative geometry, namely the computation of the index of transversally elliptic operators on foliations.

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In previous work of the authors a general solution was given to the construction of transversal elliptic operators for foliations. The first step consists in passing by a Thom isomorphism to the total space of the bundle of transversal metrics. The second step consisted in realizing that while the standard theory of *elliptic* pseudodifferential operators is too restrictive to allow the construction of the desired K-homology cycle, it suffices to replace it by its refinement to *hypoelliptic* operators. This was used in order to construct a *differential* (hypoelliptic) operator Q, solving the general construction of the K-cycle. One then arrives at a well posed general index problem. The index defines a map: $K(V/F) \to \mathbb{Z}$ which is simple to compute for those elements of $K(V/F)$ in the range of the assembly map. The problem is to provide a general formula for the cyclic cocycle $\mathrm{ch}_*(D)$, which computes the index by the equality

$$\langle \mathrm{ch}_*(D), \mathrm{ch}^*(E)\rangle = \mathrm{Index}\, D_E \qquad \forall E \in K(V/F),$$

where the Chern character $\mathrm{ch}^*(E)$ belongs to the cyclic homology of V/F. We showed in previous work that the spectral triple given by the algebra $\mathcal{A}$ of the foliation, together with the operator D in the Hilbert space $\mathcal{H}$ actually fulfills the hypothesis of a general abstract index theorem, holding at the operator theoretic level. It gives a "local" formula for the cyclic cocycle $\mathrm{ch}_*(D)$ in terms of certain residues that extend the ideas of the Wodzicki-Guillemin-Manin residue as well as of the Dixmier trace. However, although the general index formula easily reduces to the local form of the Atiyah-Singer index theorem when D is say a Dirac operator on a manifold, the actual explicit computation of all the terms involved in the cocycle $\mathrm{ch}_*(D)$ is a rather formidable task.

The main thrust of the talk was to present a Hopf algebra that provides exactly the missing organizing principle to perform the computation for the index formula. This Hopf algebra $\mathcal{H}(n)$ acts on the C^*-algebra of the transverse frame bundle of any codimension n foliation (V, F) and the index computation takes place within the cyclic cohomology of $\mathcal{H}(n)$. We compute this cyclic cohomology explicitly as Gelfand-Fuchs cohomology. While the link between cyclic cohomology and Gelfand-Fuchs cohomology was already known, the novelty consists in the fact that the entire differentiable transverse structure is now captured by the action of the Hopf algebra $\mathcal{H}(n)$.

Name: WERNER MÜLLER
Title: *On the spectrum of laplacians on locally symmetric spaces of finite volume*
Abstract: Let G be a connected semisimple (reductive) Lie group. Assume that G has no compact factors. We fix a maximal compact subgroup K of G. Let Γ be a lattice in G, i.e. Γ is a discrete subgroup of G with $\mathrm{Vol}(\Gamma\backslash G) < \infty$. By R_Γ we denote the right regular representation of G on $L^2(\Gamma\backslash G)$ which is defined by

$$(R_\Gamma(g)f)(g') = f(g'g), \quad f \in L^2(\Gamma\backslash G).$$

One of the main problems in the theory of automorphic forms is to study the spectral resolution of R_Γ. It is known that $L^2(\Gamma\backslash G)$ decomposes in the direct sum of two invariant subspaces

$$L^2(\Gamma\backslash G) = L^2_d(\Gamma\backslash G) \oplus L^2_c(\Gamma\backslash G).$$

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where $L_d^2(\Gamma\backslash G)$ is the closed subspace spanned by all irreducible subspaces of R_Γ, i.e.

$$L_d^2(\Gamma\backslash G) = \bigoplus_{\pi\in\hat{G}} m_\Gamma(\pi)\mathcal{H}_\pi.$$

By Langlands' theory of Eisenstein series, the orthogonal complement $L_c^2(\Gamma\backslash G)$ of $L_d^2(\Gamma\backslash G)$ can be described in terms of Eisenstein series associated to the discrete spectrum of Levi components L_P of cuspidal parabolic subgroups P of G. This implies that the knowledge of the multiplicities $m_\Gamma(\pi)$ for all Levi components L_P, including G, determines the spectral decomposition completely. Therefore, the main issue is to study the multiplicities $m_\Gamma(\pi)$.

One of the basic tools to gain information about the multiplicities is the Arthur-Selberg trace formula. Given $f \in C_0^\infty(G)$, let

$$R_\Gamma(f) = \int_G f(g)R_\Gamma(g)\,dg.$$

If the real rank of G is equal to one, it was proved by Selberg that the restriction $R_\Gamma^d(f)$ of $R_\Gamma(f)$ to the discrete subspace is a trace class operator. Then

$$\mathrm{Tr}R_\Gamma^d(f) = \sum_{\pi\in\hat{G}} m_\Gamma(\pi)\Theta_\pi(f),$$

where Θ_π denotes the generalized character of π, and the trace formula is the evaluation of this sum in a different manner. The formula is a sum of distributions attached to the conjugacy classes of Γ plus a contribution coming from the intertwining operators associated to the Eisenstein series.

In 1989 it was proved by the author that, for general G, $R_\Gamma^d(f)$ is a trace class operator for all K-finite $f \in \mathcal{C}^1(G)$, where the latter space denotes Harish-Chandra's Schwartz space of integrable rapidly decreasing functions on G. The main result discussed in the talk is the elimination of the K-finiteness assumption.

Theorem 1. For every $f \in \mathcal{C}^1(G)$, the operator $R_\Gamma^d(f)$ is of the trace class.

In terms of the spectrum of the Casimir operator, this means that we can consider the spectrum in all K-types simultaneously. Theorem 1 follows from an estimation of the growth of the number of eigenvalues of the Casimir operator in a fixed K-type where the dependence on the K-type is made explicit. Given $\sigma \in \hat{K}$, let λ_σ be the Casimir eigenvalue of σ. Similarly, for $\pi \in \hat{G}$, let λ_π be the Casimir eigenvalue of π. Denote by $N_\sigma(\lambda)$ the number of eigenvalues less than λ of the Bochner-Laplace operator acting on sections of the locally homogeneous vector bundle E_σ associated to a given $\sigma \in \hat{K}$. Then Theorem 1 is a consequence of

Theorem 2. There exist constants $N \in \mathbb{N}$ and $C > 0$ such that

$$N_\sigma(\lambda) \leq C(1 + \lambda_\sigma^{2N} + \lambda_\pi^{2N}).$$

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Name: LIVIU I. NICOLAESCU

Title: *Eta invariants, spectral flows and Seiberg-Witten equations*

Abstract: The talk is devoted to finite energy Seiberg-Witten moduli spaces on 4-manifolds bounding Seifert fibrations. More precisely, I describe how to compute the virtual dimensions of these spaces. The concrete computation relies on the Atiyah-Patodi-Singer index theorem which requires the explicit knowledge of the eta invariants of some Dirac type operators and some spectral flows. Using these computations and a certain adiabatic process we derive explicit and often optimal upper bounds for the Froyshov invariants of Brieskorn homology spheres with three singular fibers.

Name: PAUL-EMILE PARADAN

Title: *The Fourier transform of semi-simple coadjoint orbits*

Abstract: Let M be a closed coadjoint orbit of a real connected semi-simple Lie group G, and let $F_M \in C^\infty(\mathfrak{g}^G)$ be it's Fourier transform. In our work we compute the restriction of F_M to the Lie algebra $\mathfrak{k}$ of a maximal compact subgroup K of G. Using a technique of localization in equivariant cohomology, we extend previous results by DUFLO, HECKMANN, VERGNE and SENGUPTA.

Name: JOAN PORTI

Title: *Geometrization of 3-orbifolds of cyclic type* (joint work with M. BOILEAU)

Abstract:

Theorem (Thurston 81). Let M^3 be a closed, orientable, irreducible and atoroidal 3-manifold. Assume there is a diffeomorphism $\phi : M^3 \to M^3$ of finite order that preserves the orientation and has nonempty fixed point set. Then M^3 has a ϕ-invariant Seifert fibration or a ϕ-invariant hyperbolic structure.

This is called the orbifold theorem, because a ϕ-invariant structure on M^3 is equivalent to a structure on the orbifold M^3/ϕ. Thurston's idea consists in analyzing singular structures on the quotient M^3/ϕ, called hyperbolic cone structures. Our main contribution is to introduce simplicial volume to study the collapses of such singular structures.

Name: MICHEL RUMIN

Title: *Riemannian limit of the spectrum of contact manifolds*

Abstract: We study the behavior of the spectrum of Laplacians on forms, $d+\delta$, and the signature operator, on contact manifolds endowed with a family of metrics that blows up along a transverse Reeb field. These spaces converge in the Gromov-Hausdorff sense towards a sub-Riemannian metric space. This limit is in some sense opposite to the classical adiabatic one where the unexploded directions need to be integrable. Anyway, a contact intrinsic spectral sequence occurs here in a similar way the Leray's one on fibrations does in the adiabatic case. This spectral sequence degenerates in fact in such a way that it leads to the so-called de Rham-contact complex (JDG 94). This algebraic structure predicts the different natures and behaviors of parts of the spectrum that diverge or collapse at different speeds. We show that the resolvents of the Riemannian operators considered here actually converge to their contact-counterparts. In particular, an interesting infinite dimensional collapsing of eigenvalues (to 0) occurs in "middle" degrees, which is precisely described with the help of the second order differential D of the contact complex. Aside from the algebraic structure, the main analytic tool we used to obtain this is some a

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priori estimate coming from a Bochner technique adapted to our situation (the Riemannian curvature blows up here). These resolvent convergences can then be improved (by a commutator technique) to obtain global and local convergences of heat kernels or eta functions towards the contact ones, for non small times.

Name: Mikhail Shubin
Title: *Schrödinger operators on non-compact manifolds: qualitative results*
Abstract: Connections between classical and quantum completeness for the Schrödinger operator on a Riemannian manifold were discussed. In particular, a short proof of I. Oleinik's theorem was given which provides the most general sufficient condition of essential selfadjointness (in absence of singularities). A necessary and sufficient condition for the spectrum to be discrete was formulated.

Name: Michael Singer
Title: *Gluing theorems for anti-self-dual metrics*
Abstract: This talk, which was based mainly on joint work with Alexei Kovalev, was about the problem of finding an anti-self-dual (ASD) metric on a connected sum of ASD four-manifolds. Our approach follows a strategy suggested by Floer, in that all the analysis is carried out on manifolds with cylindrical ends (and in weighted Sobolev spaces). As special cases of our main theorem one obtains rather general gluing theorems for compact ASD orbifolds. This gluing problem has been considered by other authors (Donaldson and Friedman, and LeBrun and co-authors), using complex (twistor) methods. But our results are certainly more general than those obtained by twistor methods.

Name: Reyer Sjamaar
Title: *Linear inequalities and Schubert cycles*
Abstract: This is joint work with Arkady Berenstein. Consider a compact connected Lie group and a closed connected subgroup. Generalizing a result of Klyachko, we give a necessary and sufficient criterion for a coadjoint orbit of the subgroup to be contained in the projection of a given coadjoint orbit of the ambient group. The criterion is couched in terms of the "relative" Schubert calculus of the flag varieties of the two groups.

Name: Boris Youssin
Title: *Monomial resolutions of singularities*
Abstract: Let $\pi : \tilde{X} \to X$ be a resolution of singularities of an algebraic variety X of dimension n, and let E be its exceptional divisor. The resolution π is called a *monomial resolution* if the divisor $E = \bigcup_s E_s$ is normal crossing, and π has the following additional property. Consider the pullback cotangent sheaf $\pi^* T^* X$ (it is a coherent subsheaf of the cotangent sheaf $T^* \tilde{X}$): then we require that at any point $P \in \tilde{X}$ the sheaf $\pi^* T^* X$ has the following system of "monomial" generators:

$$d(x^{\alpha_i}) \text{ for } i = 1, \dots, k, \text{ and } d(x^{\beta_j} u_j) \text{ for } j = 1, \dots, n-k,$$

where we have:

$x_1, \dots, x_k, u_1, \dots, u_{n-k}$ is a local analytic (or étale) coordinate system on $\tilde{X}$ at P.

the coordinates $x_1, \dots, x_k$ are local equations at P of those components E_s that pass through P.

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$x^{\alpha_i} = x_1^{\alpha_{i1}} \cdots x_k^{\alpha_{ik}}$ and x^{β_j} are monomials in the coordinates $x_1, \ldots, x_k$.

In addition, the exponents $\alpha_1, \ldots, \alpha_k$ are required to be linearly independent and the set of monomials $x^{\alpha_1}, \ldots, x^{\alpha_k}, x^{\beta_1}, \ldots, x^{\beta_{n-k}}$ is required to be totally ordered by divisibility (the order may possibly be different from the one written).

Note that the exponents $\alpha_1, \ldots, \alpha_k \in \mathbf{Z}_+^k$ (where $\mathbf{Z}_+$ is the set of nonnegative integers) are required to be linearly independent, and hence, they generate the entire vector space $\mathbf{Q}^k$; we shall consider an additional requirement that each β_j is a linear combination over $\mathbf{Q}$ of only those α_i that satisfy $\alpha_i \leq \beta_j$ (i.e., of such α_i that x^{α_i} divides x^{β_j}).

Such resolutions are known to exist for any algebraic variety X over a field of characteristic zero in the following low-dimensional cases: $n = 2$ and $n = 3$.

The general argument I seem to have, appears to be very difficult, as it uses invariants of singularities more delicate than the ones used in the proof of the usual (non-monomial) resolution of singularities. For this reason I hesitate to claim it until it is written up in full detail.

My original interest in such resolutions was due to its possible applications to L^2 cohomology: if the pullback cotangent sheaf is generated by the monomials as described above, then the pullback to $\widetilde{X}$ of the Fubini-Studi metric on X (assuming X projective) is quasiisometric in a neighborhood of any point $P \in \widetilde{X}$ to the metric

$$\sum_{i=1}^{k} d(x^{\alpha_i})\overline{d(x^{\alpha_i})} + \sum_{j=1}^{n-k} d(x^{\beta_j}u_j)\overline{d(x^{\beta_j}u_j)} .$$

Later I realized that these resolutions were not helpful where I needed them, and I postponed the completion of this project. I hope that monomial resolutions will be helpful in proving that any birational isomorphism can be decomposed into a composition of blowups and blowdowns.

In addition, I recently heard from Richard Melrose that the monomial resolutions may be helpful to extend his b-calculus to singular algebraic varieties.

Name: Wei-Ping Zhang

Title: *Sub-signature operators and sub-Dirac operators*

Abstract: In this talk, we described our construction of what we call sub-signature operators associated to sub-vector bundles of tangent bundles of manifolds. In some sense, these operators unify the Hirzebruch signature operator and the de Rham-Hodge operator without refering to spin-structures which might not exist. When the sub-vector bundle mentioned above does carry a spin structure, then we can construct the associated sub-Dirac operator (this construction was done jointly with K. Liu).

As for applications, we state a Riemann-Roch type theorem for certain generalized Atiyah-Patodi-Singer invariants for flat bundles. We also indicate that the Bismut-Lott analytic torsion form can be seen from the adiabatic limit of certain spectral invariants associated to sub-signature operators on fibered manifolds.

As another application, we described our joint work with K. Liu, in which we gave a direct geometric proof of Connes's vanishing theorem, which extends the classical Lichnerowicz vanishing theorem to manifolds with spin integrable sub-tangent bundles, for what we call the almost Riemannian foliations.

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Evening talks

Name: JEAN-MICHEL BISMUT
Title: *Symplectic geometry and the Verlinde formulas* (joint work with F. LABOURIE)
Abstract: The purpose of the talk was to review joint work with F. Labourie (*Formules de Verlinde pour les groupes simplement connexes et géométrie symplectique*. C.R. Acad. Sci. Paris, **325**, Série I, 1009-1014 (1997)). Let G be a compact connected simply connected simple Lie group. Let Σ be a Riemann surface with marked points. Let $\mathcal{M}$ be the moduli space of $G_{\mathbf{C}}$ semistable bundles on Σ (with parabolic conditions at the marked points). For a given $p \in \mathbf{N}$, $\mathcal{M}$ carries a line bundle λ^p. The Verlinde formula is a formula for $\dim H^0(\mathcal{M}, \lambda^p)$.

By Narasimhan-Sheshadri, it is legitimate to replace $\mathcal{M}$ by the set M/G of flat G-bundles on Σ. For generic choices of holonomies, M/G is an orbifold, to which the Theorem of Riemann-Roch-Kawasaki can be applied. This application was already given by Szenes (LMS Lecture Notes Series **208**, Cambridge University Press 1995) for $G = SU(3)$ and by Jeffrey-Kirwan (alg-geom/9608029 (1996)) in the case $G = SU(n)$: in such cases M/G is generically smooth.

We show how to treat the case of a general group G. Due possibly to the lack of vanishing of higher cohomology, we prove that only for large p, the Riemann-Roch number of M/G is given by the Verlinde formulas. The equality is also valid for any p in cases where vanishing of higher cohomology holds. We also treat the non generic case by a perturbation argument.

Name: ULRICH BUNKE
Title: *Computation of higher analytic torsion*
Abstract: The equivariant higher analytic torsion (defined by Lott) of a closed G-manifold M is an invariant formal power series $T(M) \in I(G)$ on the Lie algebra of G. We give a formula for $T(M)$ in terms of the equivariant Euler characteristic of M. Using this result we compute the Bismut-Lott higher analytic torsion form of fibre bundles with compact structure groups.

Name: XIANZHE DAI
Title: *Analytic torsion and Reidemeister torsion on manifolds with boundary*
Abstract: We prove a formula relating the analytic torsion and Reidemeister torsion on manifolds with boundary in the general case when the metric is not necessarily a product near the boundary. The product case has been established by W. Lück and S. M. Vishik. We find that the extra term that comes in here in the nonproduct case is the transgression of the Euler class in the even dimensional case and a slightly more mysterious term involving the second fundamental form of the boundary and the curvature tensor of the manifold in the odd dimensional case.

Name: SEBASTIAN GOETTE
Title: *Equivariant η-invariants and η-forms*
Abstract: The equivariant η-invariant of a Dirac operator D on a manifold M appears as a boundary correction term in Donnely's fixed-point formula, while the η-form appears in

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Bismut-Cheeger's family index theorem. Both invariants generalize the APS η-invariant. In the case of a family with compact structure group G acting isometrically on the typical fibre M, we show that both invariants are related via Chern-Weil theory, up to correction terms that can be computed locally on M. This formula not only explains singularities of the equivariant η-invariant as a function on G, but also makes η-forms explicitly computable in certain cases.

Name: MATTHIAS LESCH
Title: *On boundary value problems for Dirac type operators* (joint work with J. BRÜNING)
Abstract: I reported on a recent joint project with Jochen Brüning. I presented a new approach to boundary value problems for Dirac type operators in a functional analytic framework. There are several applications we have in mind: in a series of papers we will give an elementary and unified treatment of a variety of results like the Cobordism Theorem for Dirac operators, the Ramachandran index theorem, a generalization of the Agranovich Dynin formula, and the spectral flow formula. Future topics are heat trace asymptotics and the study of the functional determinant as a function of the boundary condition.

Name: PAOLO PIAZZA
Title: *A higher APS index theorem for the signature operator* (joint work with E. LEICHTNAM)
Abstract: Let N be a closed compact manifold. We assume that $\pi_1(N)$ is of polynomial growth with respect to a word metric. We denote by $A \equiv C^*_r(\pi_1(N))$ the reduced C^*-algebra of $\pi_1(N)$ and by $\mathcal{B}$ the subalgebra of rapidly decreasing functions. We fix a metric g on N and we consider the universal cover $\tilde{N}$ with the lifted metric $\tilde{g}$. The signature operator on $\tilde{N}$ seldom admits a gap in its L^2-spectrum. For this reason the integral defining Lott's higher eta invariant is not known to be convergent. In this talk I described how using spectral sections which are symmetric with respect to the Hodge-$\star$ operator, it is possible to define the higher eta invariant under the following assumption: *the signature-Laplacian $\Delta_{\tilde{N}}$ is L^2-invertible in middle degree.* I have then explained how this higher eta invariant fits into a higher Atiyah-Patodi-Singer index theorem for the signature operator. More precisely, if M is a manifold with boundary and if $\Delta_{\partial \tilde{M}}$ satisfies the above assumption, then, using symmetric spectral sections, we define a *canonical* signature-index class in $K_0(A) \equiv K_0(\mathcal{B})$ and compute its Chern character in terms of a local integral and of the higher eta invariants defined above.

Name: GORM SALOMONSEN
Title: *A new proof of the splitting formula for the η-invariant*
Abstract: Let $Z = Z_1 \cup_Y Z_2$ be an odd-dimensional Riemannian manifold, which is split into two manifolds with boundary by a hypersurface Y and let $E \to Z$ be a Dirac bundle over Z. We assume that all structures have product structure in a neighbourhood of Y. Let $M := ((-\infty, 0] \times Z) \cup ([0, \infty) \times Z_1 \cup Z_2)$, and let $F \to M$ be the double of E. Take a superstructure $F = F_+ \oplus F_-$ compatible with the Dirac operator D acting on $C_0^\infty(M \setminus \partial M, F)$. Using the approach developed by the speaker for doing index-theory for manifolds with corners, boundary conditions for D are given, such that the index formula gives a splitting formula for a Dirac operator A_+ on $E_+ \to Z$.

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In the end index theorems for manifolds with corners developed by the speaker and Hassel-Mazzeo-Melrose are compared, proving that the undesirable integer-valued term in the splitting formula vanishes in the case of the signature operator. This last proof is independent of the first.

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