Mathematisches Forxchungsinstitut
Oberwolfach

Tagungsbericht

Die Geometrie der Gruppen und die Gruppen der Geometrie

25. April bis 1. Mai 1965

Ziel und Zweck dieser Tagung wie schon ihrer vielen Vorgänger war es, die gegenseitige Beeinflussung der Gruppentheorie einerseits und der Theorie der geometrischen Strukturen andererseits durch Erfahrungsaustausch zwischen den Experten aus beiden Gebieten zu fördern. Hierbei war es bsonders nützlich, daß so viele transatlantische Experten an dieser Tagung teilnehmen konnten.

Viele Vorträge befassten sich mit Themen, die sichtbar diesem Zwischenreich, das zwischen Gruppentheorie und Geometrie schwebt, angehören. Hierbei entstanden Fragen, die entweder in der Diskussion während der vortragsfreien Zeit weiterbehandelt werden konnten oder zu weiteren Vorträgen Anlass gaben; diese entstanden sowohl aus den Wünschen der Tagungsteilnehmer als auch durch die angeregten Neuentwicklungen.

Die oben angedeuteten engen Zusammenhänge wurden vielfach deutlich So gab Wilbur Jonsson einen geometrischen Beweis eines gruppentheoretischen Satzes. Michio Suzuki und D.R. Hughes sprachen über transitive Erweiterungen einfacher Gruppen, ein Problem, das ursprünglich auf Zassenhaus'sche Untersuchungen geometrischer Gruppen zurückgeht. Die Resultate von Hughes wurden von Tits während der Tagung auf gewisse unendliche Gruppen erweitert und die neuen Resultate in einem der vielen nicht angekündigten Vorträge vorgelegt. Es gelang Tits auch, eine von John Thompson während der Tagung aufgeworfene Frage zu beantworten.

<u>Teilnehmer:</u>

André, Prof.Dr.J., Saarbrücken
Baer, Prof.Dr.R., Frankfurt
Brauer, Prof.Dr.R., Zürich
Cofman, Dr.J., Frankfurt
Dixmier, Madame S., Lille
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Foulser, Prof.Dr.D., Chicago
Heineken, Dr.H., Frankfurt
Held, Dr.D., Frankfurt
Higman, Prof.Dr.D.G., Ann Arbor
Huppert, Dr.B., Mainz
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Jonsson, Prof.Dr.W., Winnipeg
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Lenz, Prof.Dr.H., München
Lingenberg, Prof.Dr.R., Darmstadt
Livingstone, Prof.Dr., Urbana
Mäurer, Dr.H., Frankfurt
Melter, Prof.Dr., Amherst
Norman, Dr.C.W., London
Ostrom, Prof.Dr.T.I., Washington
Piper, Prof.Dr.C.F., London
Suzuki, Prof.Dr.M., Urbana
Thompson, Prof.Dr.J., Chicago
Tits, Prof.Dr.J., Bonn
Zassenhaus, Prof.Dr.H.,Columbus

Vortragsauszüge:

BRAUER, R.: On a Characterization of PSL (3,q).

Given a group G which contains an involution J satisfying the following conditions

(a) $C_G(J) \simeq GL(a,q)/L$ where L is a subgroup of Z(GL(2,q) and $q \equiv -1 \bmod 4$

(b) G does not contain a subgroup of index 2

Then we have one of the following cases

(1) $G \simeq PGL(3,q)$ ς $|L| = 1$.

(2) $q \equiv 1 \pmod 3$ ς $|Z(G)|$ $|L| = 3$ and $G/Z(G) \simeq PSL(3,q)$

(3) $G \sim M_{11}$ (the Mathieu group of order 7920), $q = 3$ $|L| = 1$.

COFMAN, J.: On the non-existence of finite projective planes of LENZ-BARLOTTI type I_6.

Let π be a finite projective plane of order n, satisfying the following condition:

(C) π contains an incident point-line pair (X,y) and a one-to-one mapping μ of the points of $y-\{X\}$ on to the lines passing through X and different from y such that the plane is (P, g)-transitive for $P \varepsilon y - \{X\}$ and $g = P^{\mu}$.

Further, let Δ be the group generated by all the (P, P^{μ})-perspectivities for $P \varepsilon y - \{X\}$. Then the following theorems are valid:

Theorem 1: If in a finite projective plane a condition (C) is satisfied and if in the permutation group Δ^* induced by Δ on y only the identity fixes 3 distinct points of $y - \{X\}$, then the plane is desarguesian.

Theorem 2: If a projective plane of order $n \equiv 4 \mod 8$ satisfies condition (C), then the plane is desarguesian i.e. $n = 4$.

From these theorems using LÜNEBURG's result about the non-existence of finite projective planes of LENZ-BARLOTTI type of odd order it follows

Theorem 3: If there exists a finite projective plane of LENZ-BARLOTTI typ I_6 it is of order $n \equiv 0 \mod 8$.

FISCHER, B.: Fixed-point-free automorphisms of order $2p$.

Let G be a finite group admitting a fixed-point-free automorphism f of order $2p$ for a prime p. Then G is solvable if G has one of the following properties: (I) $C_G(f^p)$ is a 2-group. (II) $C_G(F^p)$ contains a 2-Sylow subgroup of G. (III) If $q \neq 2$ is a prime then a q-Sylow subgroup of G is abelian.

FOULSER, D.A.: Solvable primitive permutation groups of low rank.

Let G be a primitive permutation group on a finite set S, and let G_0 be the stabilizer subgroup of a point in S. The rank of G, r(G), is the number of orbits of G_0 in S. If G is also a solvable group, then $|S| = p^k$, for some prime p, and G_0 is an irreducible group of linear transformations of degree k over $GF(p^k)$. An analysis of solvable linear groups (e.g., as by B. HUPPERT) enables the determination of these groups of low rank (e.g., $r(G) \leq 5$). In particular, the exceptional doubly transitive groups of HUPPERT are determined directly from the permutation properties of G_0.

HIGMAN, D.G.: Permutation groups of finite diameter.

Let G be a finite transitive permutation group on Ω, and for $a \in \Omega$ denote the G_a-orbits by $\Gamma_0(a) = \{a\}, \Gamma_1(a), \dots, \Gamma_{r-1}(a)$, with $\Gamma_i(a)^g = \Gamma_i(a^g)$ for all $a \in \Omega$, $g \in G$. The incidence matrix $B_\alpha = (\beta_{ah}^{(\alpha)})$ of Γ_α is defined by $\beta_{ah}^{(\alpha)} = 1$ if $a \in \Gamma_\alpha(h)$, o otherwise. The intersection matrix $M_\alpha = (\mu_{ij}^{\alpha})$ of α is defined by $\mu_{ij} = |\Gamma_\alpha(a) \cap \Gamma_i(b)|$, $a \in \Gamma_j(b)$. It is shown that M_α and B_α have the same minimum polynomial. Consequences fo this are

(1) $M_\alpha L = l_\alpha L$, $L = (1, l_1, \dots l_{r-1})'$, $l_\alpha = |\Gamma_\alpha(a)|$;

(2) if the minimum polynomial of M_α has degree r then the eigen-

values of M_α are simple and there is a 1-1 correspondence between these eigenvalues and the irreducible constituent of the permut.-representation D which preserves conjugacy. The multiplicity of an eigenvalue of M_α as an eigenvalue of $B_\alpha (\alpha \neq 0)$ is the degree of the corresponding irreducible constituent of D. This multiplicity can be computed from M_α alone.

HUGHES, D.R.: Transitive Extensions of classical groups.

By elementary means it is shown that no collineation group of a finite Desarguesian projective space containing a "classical" group acting in the "ordinary" manner has a transitive extension, except in the known cases (e.g. Mathieu group on 22 letters), and possibly excepting a unitary group on the plane of order 16. (cf. Theorem of Suzuki rejects this case, however).

JONSSON, W.: Geometrischer Beweis eines Satzes von JORDAN.

Sei G zweifach transitiv auf Ω ($|G|, |\Omega| < \infty$) und $\alpha, \beta \in \Omega$. Ist $G_{\alpha,\beta} = 1$, so bilden die Elemente von G, die alle oder kein Element von Ω festlassen einen regulären Normalteiler. Dieser Satz wird folgendermaßen bewiesen: Ein Netz N vom Defekt eins wird konstruiert, dessen Punkte die Elemente von $\Omega \times \Omega$ sind. G ist eine Permutationsgruppe auf $\Omega \times \Omega$ durch $(\alpha, \beta)^g = (\alpha^g, \beta^g)$. Die Bahn eines Punktes (α, β) unter $G_\gamma (\alpha, \beta \neq \gamma)$ zusammen mit (γ, γ) ist eine Gerade. Genau dann sind zwei Punkte nicht verbindbar, wenn aus $(\alpha_1, \beta_2)^g = (\alpha_2, \beta_2)$ entweder $g = 1$ folgt oder g kein Element von Ω festläßt. Ein Netz vom Defekt eins kann immer auf eine und nur eine Weise zu einer affinen Ebene erweitert werden. Dadurch beweist man, daß das Produkt zweier Elemente, die alle oder kein Element von Ω festlassen, wieder ein solches Element von G ist.

LIVINGSTONE, D.: On a permutation representation of JANKO's group.

It is possible to find explicitly permutations which generate JANKO's group in its primitive representation of degree 266. Subsequent verification of the existence can be obtained in a short time by exhibiting the group as the full group of symmetries of 266 sets of 11 objects each.

The determination of the generators is made by considering the construction as a problem of building a transitive extension of an appropriate intransitive group with few objects, $\cong PSL_2(11)$.

values of [illegible] correspondence between these [illegible] and the irreducible constituents of the permuta- [illegible] the multiplicity of [illegible] This multi- [illegible]

[illegible]

By [illegible] [illegible] However).

[illegible] von [illegible]

[illegible]

LIVINGSTONE [illegible] group.

The possibility [illegible]

The determination [illegible]

The solution of the extension problem is essentially unique.

LIVINGSTONE, D.: On set-transitive permutation groups.

The following theorem - contained in a joint paper with A. WAGNER - is proved:

Let G be a permutation group on a set Ω and k an integer $k \geq 2$, $2k \leq |\Omega|$. Then if G induces a transitive group on the unordered sets of k elements of Ω, G is (k-1)-transitive. If $k \geq 5$ then G is k-transitive.

OSTROM, T.J.: Finite planes of square order.

An attempt will be made to summarize the highlights of recently discovered facts about various finite planes of square order and their collineation groups.

PIPER, F.C.: Collineation groups containing perspectivities.

Notation $\Sigma_{x,e}$ = group of all (x,e) elations. If Π is a collineation group which fixes no point or line of a plane P, then Π is transitive on the centre-axis flags for elations of prime order p in Π. Thus $|\Sigma_{p,e}|$ is independent of the choice of P and e.

If $|\Sigma_{p,e}| > 2$, if a centre has more than one axis and dually, then the centres and axes of elations in Π form a desarguesian subplane of P and Π, restricted to this subplane, contains its little projective-group.

If $|\Sigma_{p,e}| = 2$, either the centres and axes form disjoint FANO subplanes or they are the points and lines of a plane of order four minus an oval and its dual. In the latter case Π, restricted to this subplane, is isomorphic to either A_6 or S_6.

SUZUKI, M.: Transitive extensions of a class of doubly transitive groups.

Let G be a transitive group on Ω, H the stabilizer of $a \in \Omega$. Assume that $H \triangleright Q$ such that Q is regular on $\Omega - \{a\}$, $|Q| = p^n$ is a power of a prime p and Q char H. Suppose furthermore G does not have a normal subgroup which is regular on Ω. Then G admits no transitive extension except when $|Q| \leq 9$.

Application: Let S be one of the groups $L_2(q)$, $U_3(q)$, $S_z(2^n)$, $R(3^n)$ and Aut $S \supseteq G \supseteq S$. Consider G as a permutation group on Sylow -p-groups of S when $q = p^n$, $p = 2$, $p = 3$. Then G does not admit a transitive extension except known exceptions.

THOMPSON, J.G.: Modular Representations.

Results of J.A. GREEN and R. BRAUER can be used to determine the structure of a block with cyclic defect group. This has been done by E.D. DADE. A special, but important case, was treated in the lecture. A recent result of G. GLAUBERMAN was presented, a special case of which is the corollary that Sylow-2-subgroups of simple groups are never direct products of generalized quaternion groups.

TITS, J.: On a conjecture of L. SOLOMON.

A question raised by J. THOMPSON during the "Tagung" led to the following result:

Let G be a group and H be a subgroup. A function $\varphi : G \to A$ is said to be <u>H-invariant</u> if $\varphi(gh) = \varphi(g)$ whenever $h \in H$. Consider the following property of a triple of subgroups $H_i (i = 1,2,0)$ of G.

(h) Let $\varphi_i : G \to A$ be three functions with values in an abelian group. Assume φ_i is H_i-invariant and $\Sigma \varphi_i = 0$. Then there exists three functions $\psi_i : G \to A$ such that ψ_i is H_{i+1}-and H_{i+2}-invariant, and $\varphi_i = \psi_{1+i} - \psi_{i+2}$ (indices are reduced mod 3).

Theorem: The property (H) holds in the following two cases:
(i) G is a group with BN-pair, and the H_i's contain B.
(ii) G is a Coxeter group, and the H_i's are generated by fundamental generators of G.

Consequence: Let G, H_i be finite groups satisfying (H), let M be a G-module, and let M_i be the set of fixed points of M under H_i. Then $M \cap (M_1+M_2) = M_0 \cap M_1 + M_0 \cap M_2$, as is easily seen by considering the regular representation of G. As a result, part (ii) of the Theorem settles a conjecture of L. SOLOMON on finite groups generated by reflections.

TITS, J.: Transitive extensions of classical groups.

Report on a result inspired by a conversation with D.R. HUGHES, during the "Tagung".

It is shown, without finiteness assumption, that the groups PO, PU and PS_p (and some related groups), acting on the coresponding "quadrics" (i.e. sets of isotropic lines; in the PS_p case, the "quadric" is the projective space itself), have no transitive extension, except in trivial cases. To that affect, two lemmas of more general nature are produced; they take care respectively of the oval and the non oval case.

ZASSENHAUS, H.: On the theorem of the primitive element (together with J. SONN).

An n-parallelotope over a field F, $|F| = 2^n$, is a subset $S = \{\Sigma_{i=1}^{n} \varepsilon_i u_i | \varepsilon_i = 0,1\}$ of a linear space L over F such that u, -, u_n are linearly independent over F. The inter-section of S with a d-dimensional linear manifold L' of L contains at most 2^d elements. Let L be a separable extension of L with basis u_1, -, u_n over F. There are n distinct isomorphisms σ_1, -, σ_n of L into a given minimal splitting field of L over F. and there are n-1 proper subfields $L_i = \{x | x \varepsilon L \text{ and } x = \sigma_i x\}$ $(1 < i \leq n)$ of L such that the dimension of the linear space L_i of L over F is at most n. Then there is an element of S with distinct conjugates over F, i.e. a primitive element.

ZASSENHAUS, H.: On a logarithmic map of a group into a Lie-Ring.

Given a group G and an epimorphism σ of a free group F into G. Interprete F as the group generated by the power series $\exp x = 1 + x + \frac{x^2}{2!} \ldots$ ($x \varepsilon X$) of the free associative power series ring in a set X of variables over the rational number field. The Lie-ring L(F) generated by the power series $\log y = \Sigma_{n=1}^{\infty} (-1)^{n+1} \frac{(y-1)}{n}$ log a - log b for which a, b ε F, σ a = σ b. It follows that the mapping $g \rightarrow \log_{\sigma} g = \log \sigma^{-1} g / A(\sigma, G)$ of G into the Lie ring $L(\sigma,G) = L(F)/A(\sigma,G)$ is unique, that $L(\sigma,G)$ is the smallest Lie subring of $L(\sigma,G)$ containing $\log_{\sigma} G$ and that $\log_{\sigma}$, $L(\sigma,G)$ are essentially independent of σ. If G is nilpotent of class c then the mapping $g \rightarrow \log g / L(G)^{c+1}$ of G into $L(G)/L(G)^{c+1}$ is one-to-one.

ZASSENHAUS, H.: On a theorem of ALPERIN.

ALPERIN proved that for a finite p-group ($p \neq 2$) in which for any two elements a, b, always $D(\langle a,b \rangle)$ is cyclic one has $D^2G = 1$. This theorem is equivalent to the following theorem on Lie rings L that are finite nilpotent of p-power order. If for any two elements a, b of L always $D(\langle a, b \rangle)$ is cyclic then $D^2L = 0$. For the groups ALPERIN considered it follows $D(\emptyset\, D\, G) = 1$.

H. Heineken (Frankfurt a.M.)