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Mathematisches Forschungsinstitut
Oberwolfach

B E R I C H T

Endliche Gruppen und Liesche Ringe

12. - 18.7.65

Die Theorie endlicher Gruppen und die Theorie Liescher Ringe stehen in einem natürlichen Zusammenhang, da die meisten bekannten endlichen einfachen Gruppen vom sogenannten Lie-Typ sind, d.h. mit Hilfe von Lie-Algebren konstruiert werden können. Dieser Zusammenhang wurde besonders in dem einleitenden Vortrag von Carter deutlich.
Besonderes Gewicht lag bei den gruppentheoretischen Vorträgen auf der Darstellungstheorie. Reiner gab einen Überblick über die Entwicklung der Theorie der ganzzahligen Darstellungen während der letzten Jahre. Reynolds berichtete über Beiträge von Dade und Thompson zur modularen Darstellungstheorie. Außerhalb der angekündigten Vorträge berichtete Wielandt über eine allgemeine Methode zur Untersuchung von Permutationsgruppen; als Beispiele behandelte er Gruppen der Grade p und p^2.
Ein großer Teil der Beiträge führte zu angeregten Diskussionen.

Teilnehmer:	Bender, H.	Frankfurt
	Blackburn, Prof.Dr.N.	Manchester
	Block, Prof.Dr.R.E.	Urbana / USA
	Brandis, Dr.A.	Tübingen
	Carter, Dr.R.W.	Newcastle
	Curtis, Prof.Dr.C.W.	Eugene / USA
	Fischer, Dr.B.	Frankfurt
	Gorenstein, Prof.Dr.D.	Boston
	Gruenberg, Prof.Dr.K.	London
	Heineken, Dr.H.	Frankfurt
	Hughes, Prof.Dr.D.R.	London
	Huppert, Prof.Dr.B.	Mainz
	Kappe, Prof.Dr.W.	Columbus / USA

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Kronstein, Dr.K.M.	Frankfurt
Livingstone, Prof.Dr.	London
Nagai, Prof.Dr.O.	z.Z. Frankfurt
Reiner, I.R.	Urbana / USA
Reynolds, Prof.Dr.W.F.	Medford / USA
Roggenkamp, K.	Gießen
Rose, J.S.	z.Z. Mainz
Seligman, Prof.Dr.G.	New Haven
Siebenthal, Prof.Dr.J.de	Lausanne
Sims, Prof.Dr.	New Brunswick / USA
Szép, Prof.Dr.J.	Budapest
Tamaschke, Dr.O.	Tübingen
Walter, Prof.Dr.J.H.	Urbana / USA
Wielandt, Prof.Dr.H.	Tübingen
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Es folgen die von den Vortragenden verfaßten Auszüge:

Blackburn, N.: <u>p-groups with a centraliser of type (p,p^r).</u>

Let G be a p-group having an element s of order p such that $C_G(s) = \{s\} \times \{t\}$ for some t. The lower central series of G is dicussed.

Block, R. E.: <u>Trace forms on Lie algebras.</u>

A discussion is given of the determination of the Lie algebras L for which $L_\Delta^\perp = 0$ (where Δ is some representation of L and $L_\Delta^\perp$ is the annihilator of the trace form $f(a,b) = tr(\Delta a \Delta b)$ of Δ on L) and of the proof in the algebraically closed case that such on algebra is a direct sum of algebras which are either 1-dimensional, simple of classical type, or total matrix algebras of degree a multiple of p. A construction from matrices is given of a class of algebras which are of the form $\bar{L} = L/L_\Delta^\perp$ with $\bar{L}$ indecomposable but with $\bar{L}^2/z\bar{L}$ a direct sum of any desired number of simple algebras. There follows a discussion of the theorem that in the algebraically closed case every algebra of the form $L/L_\Delta^\perp$ is a direct sum of algebras which are either isomorphic to algebras of this construction or one

of the types of summands allowed in the case $L_{\Delta}^{\perp} = 0$. Analogous results are discussed for algebras with bilinear forms arising from projective representations.

Carter, R.W.: Applications of Lie theory to the theory of finite groups.

An exposition was given of ideas from the theories of Lie algebras and algebraic groups which are of current interest in abstract group theory. Descriptions were given of the root systems of simple Lie algebras and the construction of the Chevalley groups L(K) of type L over a field K. The Borel subgroups and Cartan subgroups were described in the case when K is algebraically closed and the axioms for a (B,N) pair were introduced. The parabolic subgroups of a group with a (B,N) pair were described. The situation in the Chevalley groups over finite fields was discussed finally.

Curtis, C.W.: On the complex characters of finite groups of Lie type.

A method is given for extending the formulas for certain irreducible characters of the finite general linear groups (R. Steinberg, Trans. Amer. Math. Soc. 71 (1951), 274-282) to analogous formulas for certain irreducible characters of arbitrary finite groups with (B,N)-pairs in the sense of J. Tits.

Fischer, B.: p-Transpositionen in endlichen Gruppen.

Sei D eine Menge von Involutionen, die eine endliche Gruppe G erzeugt; sei $D = D^G$ und p eine Primzahl, so daß das Produkt von nicht vertauschbaren Elementen von D die Ordnung p hat; dann ist D eine Menge von p-Transpositionen von G. Ist G auflösbar, so ist G' eine p-Gruppe, oder G" eine 2-Gruppe und $o(G'/G") = p$. Sei S eine 2-Sylow Gruppe von G und $n = |D \cap S|$; ist n = 1, so ist G' eine p-Gruppe; ist n = 2 oder 3, so ist G zur symmetrischen Gruppe vom Grade 2n oder 2n + 1 isomorph.

Gorenstein, D.: A characterization of the finite simple groups $G_2(3^n)$.

In the course of his proof of the classification of N-groups, Thompson gives a purely group-theoretic characterization of the simple group $G_2(3)$. Starting with a more general set of conditions, which evolve fairly naturally from his, we are able to give a corresponding characterization of the family of simple groups $G_2(3^n)$. These conditions are themselves a particular case of the more general notion of an OL-group which we introduce. The conditions defining an OL-group are stated in terms of the prime 2 and an odd prime p. It appears that all the Chevalley and Steinberg groups defined over $GF(p^n)$, with the exception of certain ones of very low rank, are OL-groups.

Gruenberg, K.: Projective presentations of groups.

We consider the category of presentations with abelian kernels of a fixed group. The projective objects are identified. Applications include simple proofs of a theorem of Magnus and a theorem of Gaschütz.

Heineken, H.: Groups with periodic commutator operation.

Let $x^{(1)}oy = xoy = x^{-1}y^{-1}xy$, $x^{(k+1)}oy = xo(x^{(k)}oy)$. It is obvious that all finite groups satisfy an identity of the form $x^{(m+n)}oy = x^{(n)}oy$. On the other hand the problem arises: what can be said of a finite group G satisfying such an identity where m and n are given. If G is finite and soluble, the integer m limits the exponent of the Fitting factor group; for finite groups one finds p'-closedness for all but certain primes dependant on m. If G is finite and m is odd or twice a prime, then G is soluble.

Kegel, O.H.: Über den Normalisator von subnormalen und erreichbaren Untergruppen.

Sind in dem perfekten Normalteiler N der Gruppe G alle subnormalen (oder alle aufsteigend erreichbaren) Untergruppen bereits normal, so normalisiert N jede subnormale (aufsteigend erreichbare) Untergruppe von G. (Dieses Resultat gilt analog

auch für Liesche Ringe). - Ist der Normalteiler N von G eine freie abelsche Gruppe derart, daß jeder Normalteiler $\neq 1$ von G, der in N enthalten ist, in N endlichen Index hat, so gibt es zu jeder aufsteigend erreichbaren Untergruppe S von G eine ganze Zahl $n = n(S)$ so, daß S von N^n normalisiert wird. - Hat jedes epimorphe Bild $\neq 1$ der Gruppe G einen Normalteiler $\neq 1$, der entweder endlich ist, oder endlich erzeugt und abelsch, oder in dem jede aufsteigend erreichbare Untergruppe bereits normal ist, so ist in G die Menge der aufsteigend erreichbaren Untergruppen ein Verband.

Kronstein, K.: Monomial representations of p-groups.

Let $p \neq 2$ and G be a p-group. Let F_o be the order p part of the cyclic sections T/K, for which the induced characters are irreducible and the character field is Q(e), e being a primitive $|T:K|$-th root of unity.
Let F be the family of sections U/K such that $|U:K| = p$ and $\eta(U/K)^G$ is Q-irreducible, for $\eta(U/K)$ a Q-irreducible on U with kernel K. Then F is the family of sections U/K such that for no section W/L we have $U \cap L = W \cap K$ and $|G:U| > |G:W|$.
If U/K is in F_o, N(K)/K is cyclic. Furthermore, let U/K and W/L belong to F, and N(K)/K, as well as N(L)/L, be cyclic. Then U/K belongs to F_o if, and only if, for every such W/L, with $U \cap L = W \cap K$, we have $|N(K):K| > |N(L):L|$.
If G and H are p-groups which are related by a projectivity that preserves the number of conjugate subgroups, and k is characteristic 0 field, G and H have the same number of k-irreducible representations of degree d.

Seligman, G.B.: Simple groups associated with Lie algebras.

Lie algebras have provided constructions of many simple groups, some of them new (Chevalley, Steinberg, Tits, Ree). A further example of a construction is given, which yields the Janko group, and which admits a general form. It is not known whether any other new simple groups result from this process. Over fields of characteristic different from 2,3, one may also construct groups from Lie algebras L which are forms of classical simple Lie algebras, and which contain

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elements $x \neq 0$ with $(adx)^3 = 0$, as follows: Let G be the group generated by all $\exp(adx)$, $(adx)^3 = 0$, adx of minimal rank with this property. Over finite fields, these are the simple groups of Chevalley and Steinberg; over perfect fields, it follows from results of Tits that they are simple. In all cases tested, they are always simple.

Reiner, I.: Integral group representations.

Two survey lectures were given, outlining recent developments in the theory of integral representations of finite groups. The following topics were considered: irreducible and indecomposable modules; relations between global, local, and modular representations; additive and multiplicative structure of the Grothendieck ring $K^0(ZG)$; the Whitehead group $K'(QG)$; existence of nilpotent elements in the representation ring of ZG-modules. Areas for further research were suggested.

Reynolds, W.F.: Blocks and normal subgroups.

There are well-known relationships, due to Clifford, between the irreducible representations of a finite group G and of a normal subgroup H; these hold in both the ordinary and modular cases. In the latter case, analogous questions arise concerning the indecomposable representations, the principal indecomposable characters, and the blocks of G and H. We discussed known results, methods and conjectures on these questions, as well as some applications.

Rose, J.S.: On the abnormal structure of finite soluble groups.

An alternative approach to results of R.W. Carter (Proc. London Math. Soc. (3) 12, 535-563 (1962)) on the relationship between system normalizers and Carter subgroups in an A-group, based on the following proposition: Let G be a finite soluble group, with abelian Sylow p-subgroups for some prime p. Let D be a system normalizer of G. Then D_p is a Sylow subgroup of some normal subgroup of G.

Siebenthal, J. de: Über gewisse Moduln in halbeinfachen Lieschen Algebren.

Sei Σ die Menge der Wurzeln einer halbeinfachen Lieschen Algebra g über $\mathbb{C}$; sei dann $\Sigma_0 \subset \Sigma$ mit: $\Sigma_0 = -\Sigma_0$, $\alpha, \beta \in \Sigma_0$, $\alpha+\beta \in \Sigma \Rightarrow \alpha+\beta \in \Sigma_0$. Die Partition

$\Sigma = \Sigma_0 \cup \Sigma_1 \cup \dots \cup \Sigma_s$ von Σ in Klassen mod Σ_0 hat die Eigenschaften:

$(\Sigma_0+\Sigma_i)\cap\Sigma = \Sigma_i$; $(\Sigma_i+\Sigma_j)\cap\Sigma$ leer, oder

$\exists k: (\Sigma_i+\Sigma_j)\cap\Sigma \subset \Sigma_k$; wenn $k \neq 0$ ist: $(\Sigma_i+\Sigma_j)\cap\Sigma = \Sigma_k$;

$\forall i, \exists i': (\Sigma_i+\Sigma_{i'})\cap\Sigma \subset \Sigma_0$.

In der Cartanschen direkten Summe $g = \mathfrak{h} \oplus \sum_{\alpha\in\Sigma} \mathbb{C} e_\alpha$ ist $g_0 = \mathfrak{h} \oplus \sum_{\alpha\in\Sigma_0} \mathbb{C} e_\alpha$ eine Unteralgebra, deren adjungierten Darstellung in g zerfällt:

$$g = g_0 \oplus g_1 \oplus \dots \oplus g_s, \qquad g_i = \sum_{\alpha\in\Sigma_i} \mathbb{C} e_\alpha :$$

$$[g_0, g_i] = g_i, \quad [g_i, g_j] = 0, \quad \text{oder } \exists k \quad [g_i, g_j] \subset g_k;$$

wenn $\quad k \neq 0 : [g_i, g_j] = g_k; \quad \forall i, \exists i': [g_i, g_{i'}] \subset g_0.$

Szép, J.: Halbgruppen mit Einselement.

Ist H eine Halbgruppe mit Einselement, dann gilt die Zerlegung $H = \Gamma + G$, wo G die maximale Gruppe in H. Es gilt dann $g\Gamma = \Gamma = \Gamma g$ $(g \in G)$. Mit den Permutationen

$$g \longrightarrow \begin{pmatrix} \Gamma \\ \Gamma g \end{pmatrix} = \pi_g, \quad g \longrightarrow \begin{pmatrix} \Gamma \\ g\Gamma \end{pmatrix} = {}_g\pi$$

kann man die Gruppen π_G und ${}_G\pi$ bilden, für die der Homomorphismus bzw. Antihomomorphismus

$$_G\pi \sim G, \qquad \pi_G \underset{a}{\sim} G$$

gelten. Die Relationen $\pi_{g'}\,{}_g\pi = {}_g\pi\,\pi_{g'}$ und andere ähnliche Relationen bieten die Möglichkeit, die innere Struktur von H zu untersuchen. So gilt z.B. der Satz: hat Γ ein Element mit Links- (Rechts-)inversem, dann ist die Abbildung $\pi_g \longleftrightarrow {}_g\pi$ ein Antiisomorphismus $\pi_G \underset{a}{\approx} {}_G\pi$ und ${}_G\pi \approx G$ ist.

Es ist möglich, auch ein Erweiterungsproblem zu erledigen: Man kann zu gegebener Halbgruppe $H = \Gamma + G$ sämtliche Halbgruppen $\bar{H} = \Gamma + \bar{G}$ mit $G \subset \bar{G}$ herstellen (in Schreier'schem Sinn).

[illegible] Untergruppen [illegible]

[illegible] Σ [illegible] Σ [illegible] $\Sigma \supset \Sigma$ [illegible]

[illegible]

[illegible] $\Sigma \cup \dots \cup \Sigma$ [illegible]

[illegible] $\Sigma \cap \Sigma$ [illegible] $\Sigma \subset \Sigma$ [illegible]

[illegible] $\Sigma \subset \Sigma \cap \Sigma$ [illegible]

[illegible]

[illegible] dann gilt die [illegible] Gruppe in [illegible]

[illegible]

$$[illegible] \longleftarrow [illegible]$$

[illegible] Automorphismus [illegible]

[illegible] und aber [illegible]

[illegible] Struktur [illegible]

[illegible] Automorphismen [illegible]

[illegible]

[illegible]

[illegible] Zuordnung [illegible]

[illegible]

Tamaschke, O.: Relations between conjugate classes and double cosets in finite groups.

After a short outline of a generalized character theory on finite groups [see "S-rings and the irreducible representations of finite groups", Journal of Algebra 1, 215-232 (1964)] the special case of double coset S-rings has been considered. Let G be a finite group, and H be a subgroup of G. The double cosets $T_i = Hg_iH$ $(i = 1,\ldots,t)$ give rise to the double coset S-ring T_H which has the elements $\tau_i = \sum_{g \in T_i} g$ $(i = 1,\ldots,t)$ as a C-basis. Corresponding to the irreducible representations $D_1,\ldots,D_r$ of G occuring in the representation 1_H^G which is induced by the 1-representation 1_H of H, generalized characters φ_ϱ $(\varrho = 1,\ldots,r)$ are introduced. Among other relations they satisfy the following generalization of the orthogonality relations of the irreducible characters χ_ϱ of G

$$\frac{|K_\alpha||T_i|}{|G|} \sum_{\varrho=1}^{r} \chi_\varrho(g_\alpha)\,\varphi_\varrho(g_i) = |K_\alpha \cap T_i| ,$$

where K_α denotes any class of conjugate elements of G, and $g_\alpha \in K_\alpha$, $g_i \in T_i$.

Walter, J.H.: Finite groups with abelian Sylow 2-subgroups.

A proof of the following theorem was discussed.
THEOREM: Let G be a finite group with abelian Sylow 2-subgroups. Designate by O(G) the maximal normal subgroup of odd order and bz $O^{2'}(G)$ the minimal normal subgroup of odd index. The $O^{2'}(G/O(G))$ is the direct product of a 2-group and simple groups L from the following classes:

(1) $PSL(2,q)$, $q \equiv 3$ or $5 \pmod 8$, $q > 3$

(2) $PSL(2,2^n)$, $n > 1$

(3) A group L with involution J such that $C_L(J) = \langle J\rangle \times K$ where K is a simple group from class (1).

Among the groups of class (3) are Janko's group and the Ree groups. It is know that they are either Janko groups or that they have the same internal structure as Ree groups.

Zassenhaus, H.: <u>On a problem of K. Hirsch.</u>

When is a finite group isomorphic to the automorphism group of a torsion free group? K. Hirsch and H. Zassenhaus showed recently that a necessary and sufficient condition for the group to be isomorphic to the unitgroup of an order (i.e. a unital subring $\mathfrak{o}$ of a semisimple hypercomplex system H that has a basis over the rational integer ring).
A.L.S. Comer showed in 1963 thatevery countable torsion free reduced (unital) ring of rank n is isomorphic to the endomorphism ring of a torsion free abelian group of rank $\leq 2n$.
It was shown here that for any order $\mathfrak{o}$ even an $\mathfrak{o}$-leftmodule $\mathfrak{m}$ of H containing $\mathfrak{o}$ can be found such that the left multiplication by the elements of $\mathfrak{o}$ are the only endomorphisms of $\mathfrak{m}$. For any proper H-module M every endomorphisms σ of M that multiplies every element u of M by an element $\lambda(u)$ of $\mathfrak{o}$ (depending on u!) is the multiplication by some <u>fixed</u> element λ of H. It is conjectured that λ be contained in $\mathfrak{o}$.

Bernd Fischer