Mathematisches Forschungsinstitut
Oberwolfach

Tagungsbericht
Kategorien und Funktoren
17. bis 23. Juli 1966

Unter der Leitung von A. Dold und S. MacLane wurde hier zum erstenmal eine Tagung ausschließlich der Kategorien-Theorie gewidmet. Die Anzahl und Länge der Vorträge wurde in vernünftigen Grenzen gehalten, sodaß noch viel lebendiges Interesse für allgemeine und private Diskussionen übrig blieb. Hoffentlich ist diese Veranstaltung der Anfang einer anregenden Tradition.

Teilnehmer:

André, M., Genève
Barr, M., Urbana
Beck, I., Ithaca
Benabou, I., Paris
Brinkmann, H.B., Saarbrücken
Bunge, Marta, Freiburg
Eilenberg, S., New York
Felscher, W., Freiburg
Fittler, R., Heidelberg
Freyd, P., Philadelphia
Gray, J.W., Zürich
Hilton, P., Heidelberg
Kleisli, H., Fribourg
Kock, A., Zürich
Laudal, O.A., Oslo
Lawvere, F.W., Zürich
Linton, F.E.J., Zürich
MacDonald, I.L., Frankfurt
Mostert, P.S., New Orleans
Pareigis, B., München
Rentschler, R., München
Röhrl, H., San Diego
Roos, J.E., Lund
Schmid, I., Fribourg
Schulte-Mönting, I., Freiburg
Schumacher, Freiburg
Tierney, München
Ulmer, F., Zürich
Volger, H., Freiburg

Vortragsauszüge: (in zeitlicher Reihenfolge)

MAC LANE, S.: Iterated Homotopies

Report on the investigations of Adams-MacLane on the construction of cohomology operation by means of PACTs (for def. see MacLane Bull. AMS 1965 Jan.)

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THEOREM: If C' is a chain complex, F(C) the cobar construction (assume a diagonal in C) and Ω a PACT, then there is another PACT Ω' such that each action of Ω' on C defines in a canonical way an action of Ω on F(C), such that each operation of Ω interchanges with the diagonal in F(C). (The "interchange" idea is like that in the interchange of product and coproduct in a Hopf algebra.)

THEOREM: For a topological space X the canonical map $F(C_r X) \to C_r$ (loops X) commutes with the action of a suitable "Steenrod" pact up to homotopy.

HILTON, P.: Interesting limits and colimits in an abelian category and completing filtrations

Two cases arise naturally where the natural map $\varinjlim_m \varprojlim_n \to \varprojlim_n \varinjlim_m$ is an isomorphism; one concerns spectral sequences ($E_\infty = \varinjlim_m \varprojlim_n E_{mn} = \varprojlim_n \varinjlim_m E_{mn}$) and the other a filtration. In the latter case the double limit is the completion. In certain applications we may be interested in a (weaker) quasi-completion which also involves two commuting limit operations.

LAWVERE, F.W.: Elementary Theories

ANDRÉ, M.: Simplicial homological algebra

A category $\underline{N}$, a full subcategory $\underline{M}$ used as model, a functor T used as coefficient, with value in an abelian category (with exact direct sums) that is enough for doing homological algebra. The definition of the homology objects H(N, T) involves long chains $M_n \to M_{n-1} \to \dots M_o \to N$. If $\underline{N}$ is an abelian category with sufficiently many projective objects and $\underline{M}$ is the full subcategory of those projectives, we refind the derived functions of an additive functor. There is the same result with cotriple derived functors. Singular homology appears also in this context. As application we study cohomology groups in commutative algebra. If R is a ring, S a commutative associative R-algebra and E a S-module, cohomology groups $H^n_R(S,E)$ are defined: H^o has to do with derivations,

H^1 with classifications of extensions, H^2 with regularity properties of local rings. For a triple of commutative rings $R \to S \to T$ and a T-module E, a long exact sequence is established

$$\ldots \to H_S^n(T,E) \to H_R^n(T,E) \to H_R^n(S,E) \to H_S^{n-1}(T,E) \to \ldots$$

KLEISLI, H.: Resolutions in tensored categories

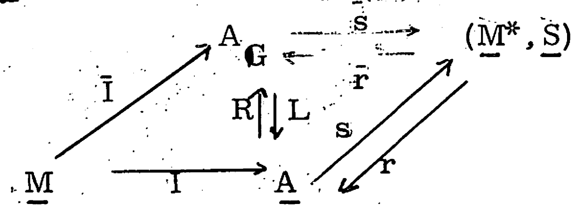
A definition of left and right acyclic and representable simplicial complexes relative to a pointed object is given, such that there is a comparison theorem. In addition a resolution relative to a monoid (in the sense of Benabou) is left and right acyclic and representable.

LINTON, F.: Triples and Theories

Algebra over triples are by now known. Taking a theory to be a minimal adjoint pair a la Kleisli, the Kleisli construction provides an equivalence between the categories T_h of theories (over a fixed cat. $\underline{A}$) and the category T_r of triples. Taking algebras over a theory to be more or less representable contavariant functors on the theory, there results a category over A. The observation then is that there Lawvere-like algebras and the triple algebras are the same, i.e. that the two semantic functors $T_h \to (\text{cat}, \underline{A})$ and $T_r \to (\text{cat}, \underline{A})$ are, mod. the equivalence $T_h \leftrightarrows T_r$, the same.

TIERNEY.: Categories with models

Work of Appelgate and Tierney on a categorical description of global objects patched together from local ones by means of an atlas. This occurs as follows: Let $\underline{M}$ be a small category of models, $I : \underline{M} \to \underline{A}$ a functor where $\underline{A}$ has colimits. I determines a diagram



$S\,A(M) = \underline{A}(IM, A)$ $\qquad r \dashv s, \ \bar{r} \dashv \bar{s}.$

The global objects are $\underline{A}_G$, the category of coalgebras determined by the cotriple $G = rs$.

MOSTERT, P.S.: Torsion Theories and the Theory of Sheaves

Let X be a topological space, γ the category of presheaves, $\mathfrak{T} = \mathfrak{T}(X)$ the category of sheaves as a coreflective subcategory of $\gamma = \gamma(X)$. Let $f: X \to Y$ be a map of top. spaces and $f^*: \gamma(Y) \rightleftarrows \gamma(X): f_*$ the adjoint pair of presheaf functors, $f^*: \mathfrak{T}(Y) \rightleftarrows \mathfrak{T}(X): f_*$ the corresponding functors of sheaves. Let $\Gamma_X: \mathfrak{T}(X) \to \gamma(X)$ be the inclusion functor, $\Gamma_X: \mathfrak{T}(X) \to A$ (the value category) the functors $\Gamma'_X F \cong F(X)$. Let $L_X: \gamma(X) \to \mathfrak{T}(X)$ be the associated sheaf map (the coreflection). f_*, Γ, Γ' are left exact functors. The right derived functors give the Leray sheaf Rf_*F, the cohomology presheaf $R\,\Gamma\,F = \mathfrak{H}(X, F)$, and the cohomology object $H(X, F) = \mathfrak{H}(X, F)_X$. The stalk of the Leray sheaf is given by $(Rf_*F)_y = H(f^{-1}_{(y)}, F)$ when f is a closed map and Y is regular.

FREYD, P.: The Grothendieck Group for Stable Homotopy

The Grothendieck Group for Stable Homotopy is free. And for a basis take:

Spheres $\cup \{[X] - [R_X] \mid X$ indecomposable, B a bouquet of spheres, such that $f: X \to B$, $g: B \to X$ and a prime integer p with $fg = p^n \cdot 1_B$, $f = p^n \cdot 1_X$ for some $n\}$.

RÖHRL, H.: Foundations of category theory

Grundlagen der Kategorientheorie werden angegeben nebst einem relativen Widerspruchfreiheitsbeweis, so daß alle "großen" Konstruktionen ausgeführt werden können.

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BENABOU, I.: Algebraic structures on categories

Let I be a set, M(I) the free monoid generated by I. A 2-dimensional algebraic theory over I-objects is a 2-category Π, having M(I) as set of objects such that every $i_1 \dots i_n \in M(I)$ is equipped with arrows $pc_K: i_1 \dots i_K \to i_K$ defining it as the Π-product of the objects i_k $(K = 1, \dots, n)$. If $\mathbb{C}$ is a 2-category with finite products the Π-algebras of $\mathbb{C}$ are the 2-functors $\Pi \to \mathbb{C}$ which commute with finite products. Again this is made a 2-category $\underline{\mathrm{Alg}}\,(\Pi, \mathbb{C})$ with an underlying 2-functor $U: \underline{\mathrm{Alg}}\,(\Pi, \mathbb{C}) \to \mathbb{C}^I$

1) Every "2-presentation" determines a 2-theory.
2) Given any 2-theory Π, there exists an algebra A in $\underline{\underline{\mathrm{Cat}}}$ such that A is an injection on objects, arrows and 2-cells.
3) The "semantic" functor has an adjoint.

GRAY, J.W.: The Transpose

A pair of adjoint functors $A \overset{S}{\underset{r}{\rightleftarrows}} B$ determines "transpose" isomorphism $\mathbb{B}(S(A), B) \approx \mathbb{A}(A, T(B))$.

In the situation

$$\begin{array}{ccc} \mathbb{A}_o & \xrightarrow{F_o} & \mathbb{A}_1 \\ T_o \uparrow \downarrow S_o & & T_1 \uparrow \downarrow S_1 \\ \mathbb{B}_o & \xrightarrow[F_1]{} & \mathbb{B}_1 \end{array}$$

this spezializes to a transpose isomorphism

$$\mathbb{B}_1^{\mathbb{A}_o}(S_1F_o, F_1S_o) \approx \mathbb{A}_1^{\mathbb{B}_o}(T_1F_1, F_oT_o).$$

Let $\mathbb{F}\mathrm{un}_L$ (resp. $\mathbb{F}\mathrm{un}_R$) denote the category where objects are pairs of adjoint functors and where morphisms are triples

$(F_o, F_1, \gamma: S_1F_o \to F_1S_o)$ (resp. $F_1F_o, \gamma: T_1F_1 \to F_oT_o$) and where composition is $(G_o, G_1, \partial)\,(F_o, F_1, \gamma) = (G_oF_o, G_1F_1, G_1\gamma \circ \delta F_o)$.

THEOREM: The transpose determines functors $\mathbb{F}\mathrm{un}_L \rightleftarrows \mathbb{F}\mathrm{un}_R$ which are inverse to each other.

ULMER, F.: Darstellbare Funktoren mit Werten in beliebigen Kategorien

Es wird gezeigt, daß die Funktoren $\mathfrak{A} \xrightarrow{[A,-]} \text{Sets} \xrightarrow{B\otimes} \mathfrak{B}$, $A \in \mathfrak{A}$, $B \in \mathfrak{B}$ ähnliche Eigenschaften wie die mengenwertigen darstellbaren Funktoren haben; dabei ist $B\otimes : \text{Sets} \to \mathfrak{B}$ als der Linksadjungierte von $\mathfrak{B} \to \text{Sets}$, $(B \mapsto [B,-])$ definiert. Jeder Funktor $t: \mathfrak{A} \to \mathfrak{B}$ ist kanonisch direkter Limes von solchen Funktoren. Der verallgemeinerte Ioneda-Darstellungssatz lautet $[B\otimes[A,-],t] \cong [B,tA]$. Die dualen Funktoren $\mathfrak{A} \xrightarrow{[-,A]} \text{Sets} \xrightarrow{[-,B]} \mathfrak{B}$ haben analoge Eigenschaften ($[-,B]$ ist der symbolische Homfunktor).

LAUDAL, O.A.: Spectral sequences associated to $\varprojlim$ and $\varinjlim$.

Let $\ldots D_{p-1} \to D_p \to D_{p+1} \to \ldots$ be a projective system in a good category. There exists a set of exact couples S(D) associated to D, a cononical filtration of $\varprojlim D$ and a canonical cofiltration of $\varinjlim D$.

For any $E \in S(D)$

i) The limit term E^{∞} was related to the above filtration (resp. cofiltration)

(ii) necessary and sufficient conditions were given to the spectral sequence to converge

(iii) if the spectral sequence converges uniformly things become nice.

ROOS, J.E.: Structure theorems for some abelian categories having exact direct limits

Let $\underline{C}$ be an AB 5 category with a small set of generators. We can prove Theorem 1:

$\underline{C}$ satisfies AB 6 iff every $C \in \text{Ob}(\underline{C})$ can be written as ascending union $C = \coprod_{\alpha \in I} C_\alpha$ such that for any other such union $C = \coprod_{\gamma \in J} B_\gamma$ we have $C_\alpha \subset B_{\gamma(\alpha)}$ for some $\gamma(\alpha)$, $\forall\, \alpha$. Here is an application of this result:

Theorem 2: ("Wedderburn"). Let $\underline{C}$ be as above. Then the following two conditions are equivalent:

i) $\underline{C}$ satisfies AB 6 and the cohomological dimension of $\underline{C}$ is zero.

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(ii) $\underline{C} \simeq \Pi \operatorname{Mod}(K_\beta)$ where the $K_\beta : S$ are (skew-)fields.

These things are related to Comptes Rendus 261 (1965), p. 4954.

PAREIGIS, B.: Forgetful functors and ring homomorphisms

The functor assigning to each unitary ring R the category of unitary models together with the forgetful functor to the category of abelian groups, and to each ring homomorphism the functor "forget the operation of the second ring", is full, faithful and has an adjoint. This has applications to Frobenius-extensions of rings.

BARR, M.: Cotriple derived functors

If $\mathbb{G} = (G, \varepsilon, \delta)$ is a comonad in a category $\mathfrak{C}$ and $E : \mathfrak{C} \to \mathfrak{A}$ is a functor into an abelian category, homology functors $H_n(X, E)$ relative to $\mathbb{G}$ are defined as the homology objects of the complex associated with $\ldots \to EG^{n+1}X \to \ldots \to EG^2X \to EGX \to 0$ in $\mathfrak{A}$ with boundary $\partial_n = \Sigma(-1)^i G^i \epsilon G^{n-i}X$ for $0 \leq i \leq n$. Axioms for these functors, such as the exact homology sequence relative to a sequence $0 \to E' \to E \to E'' \to 0$ exact in the functor category $(\mathfrak{C}, \mathfrak{A})$ are considered. X is $\mathbb{G}$ projective iff a sequence $X \to GX \to X$ equal to the identity exists. Two comonads $\mathbb{G}$ and $\mathbb{K}$ give the same homology iff $\mathbb{G}$-projective $\rightleftarrows$ $\mathbb{K}$-projective. Certain computations on the relation of this cohomology theory to the one defined by André are given.

FREYD, P.: Elementary Theories and Maps

A laconic discussion of the probable necessity of considering other types of maps on models besides elementary imbeddings, of considering models other than normal, of considering maps other than functions.

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