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Mathematisches Forschungsinstitut
Oberwolfach

Tagungsbericht

Analytical Problems of Branching Process Theory

4. bis 10. Juni 1967

Leitung: Prof. D. G. Kendall
Prof. Dr. H. Dinges

The meeting was attended by 23 invited contributors drawn from a wide range of countries. It was felt by the organisers that, with the rapid increase in the number of persons working in probability theory, the time had come to follow the example set in other branches of mathematics by planning a specialist conference for a small group of participants working in a relatively narrow field. The small size of the conference made it possible for the participants to give their lectures without being pressed for time while allowing also plenty of time for discussion, both formally (immediately after the lectures) and informally on afternoon excursions.

It is the opinion of the organisers that their objectives were achieved, and the holding of further probability conferences on this scale is strongly recommended, although it is appreciated that the need for much larger and relatively open conferences on probability and statistics will still be needed.

Teilnehmer:

Blumenthal, Prof. Dr. R. M., Erlangen
Bühler, Dr. W., Heidelberg
Dietz, Dr. K., Freiburg
Dinges, Prof. Dr. H., Frankfurt
Föllmer, H., Erlangen
Harris, Prof. Dr. T. E., Los Angeles
Jagers, P., Göteborg
Jiřina, Prof. Dr. M., Prag
Joffe, Prof. Dr. A., Straßburg
Kendall, Prof. Dr. D. G., Cambridge
Kolmogoroff, Prof. Dr. A. N., Moskau
Krengel, Dr. U., Erlangen
Krickeberg, Prof. Dr. K., Heidelberg
Lamperti, Prof. Dr. J., Aarhus
Mandl, Prof. Dr. P., Prag
Ney, Prof. Dr. P., Madison
Papangelou, F., Heidelberg
Pedersen, K., Aarhus
Prokhorov, Prof. Dr. J., Moskau
Rényi, Prof. Dr. A., Budapest
Sevastyanov, Prof. Dr. B., Moskau
Szász, D., Budapest
Yaglom, Prof. Dr. A., Moskau

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In addition to the above the following, invited to contribute papers, did so but were unable to attend in person:

Čistyakov Seneta Vere-Jones

Vortragsauszüge:

BÜHLER, W.: Unendliche Teilbarkeit bei Grenzverteilungen von Verzweigungsprozessen

Die Frage nach unendlicher Teilbarkeit der Grenzverteilungen von Verzweigungsprozessen (ganzzahlige Werte, Markovsch, zeithomogen) wird teilweise beantwortet für den superkritischen Fall: ($Z_o = 1$, $Z_t/EZ_t \to W$)

(i) Z_{t_o} unendlich teilbar $\Rightarrow Z_t$ unendlich teilbar für $t \geq t_o$;

(ii) Alle Z_t ($t \geq 0$) unendlich teilbar $\Leftrightarrow p_0 = 0$;

(iii) $p_o = 0 \Rightarrow W$ unendlich teilbar;

(iv) $\mathfrak{L}(W \mid W > 0)$ unendlich teilbar.

(i) gilt auch im Fall diskreter Zeit.

(ii) wird mit Hilfe der Prozesse mit Werten in R^+ bewiesen, für die automatisch alle Z_t unendlich teilbar sind.

(iii) folgt aus (ii), (iv) folgt durch Anwendung von (iii) auf einen Prozeß $\hat{Z}_t$, für den $\hat{p}_o = 0$ und $\mathfrak{L}(\hat{W}) = \mathfrak{L}((1-q)W \mid W > 0)$.

ČISTYAKOV, V.P. (read by K. Dietz): Transient phenomena in branching stochastic processes

Let $\mu_k(t) = \{\mu_{k1}(t), \ldots, \mu_{kn}(t)\}$ be a branching process with n types of particles and let

$$P\{\mu_{kj}(t) = \omega_j;\ j = 1, \ldots, n\} = \delta_k^\omega + P_k^\omega t + o(t) \quad (k = 1, \ldots, n)$$

when $t \to 0$. Here $\omega = \{\omega_1, \ldots, \omega_n\}$, $\delta_k^\omega = 1$ for $\omega_k = 1, \omega_j = 0$ ($j \neq k$) and $\delta_k^\omega = 0$ in other cases. We define the generating functions by $f_k(x_1, \ldots, x_n) = \Sigma P_k^w x_1^{w_1} \ldots x_n^{w_n}$ ($k = 1, \ldots, n$) and denote factorial moments by

$$a_{kj} = \left.\frac{\partial f_k}{\partial x_j}\right|_{x=1}, \quad b_{ij}^{(k)} = \left.\frac{\partial^2 f_k}{\partial x_i \partial x_j}\right|_{x=1}, \quad c_{ijl}^{(k)} = \left.\frac{\partial^2 f_k}{\partial x_i \partial x_j \partial x_l}\right|_{x=1}.$$

Let A be a compact set of undecomposable matrices $a = \|a_{kj}\|$

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(k, j = 1, ..., n); $\lambda = \max_{1 \le i \le n} (\mathrm{Re}\lambda_i)$ where the numbers λ_i satisfy the equality $|a - \lambda_i E| = 0$ (E being the unity matrix) and $v = \{v_i\}_{i=1}^n$, $u = \{u_i\}_{i=1}^n$ satisfy the equalities $au = \lambda u$, $va = \lambda v$, $\sum_{k=1}^n v_k^2 = \sum_{k=1}^n u_k v_k = 1$. Let K(A, d, B, c) be a class of $\{f_k(\bar{x})\}$ with $a \in A$, $0 < d < \sum_{i,j,k=1} b_{ij}^{(k)} < B < \infty$, $c_{ijl}^{(k)} < c < \infty$. The following asymptotic formula for $t \to \infty$, $\lambda \to 0$ holds true uniformly for all $\{f_k\} \in K$;

$$1 - P\{\mu_{ij}(t) = 0,\ j = 1, \ldots, n \mid \mu_i > 0\} \sim u_i \frac{e^{\lambda t}(\sum_{i=1}^n v_i(1-x_i))}{1 + \frac{b}{2} g(\lambda, t) \sum_{i=1}^n v_i (1-x_i)}$$

where

$$g(\lambda, t) = \left\{ \begin{array}{ll} \frac{e^{\lambda t} - 1}{\lambda} & ; \lambda \neq 0 \\ t & ; \lambda = 0 \end{array} \right\}, \quad \mu_i = \sum_{j=1}^n \mu_{ij}(t), \quad b = \sum b_{ij}^{(k)} v_k u_i u_j .$$

The probability distributions

$$S_k^{(t)}(y_1, \ldots, y_n) = P\left\{ \frac{\mu_{kj}(t)}{M\{\mu_{ki} \mid \mu_k > 0\}} < y_i,\ j = 1, \ldots, n \mid \mu_k > 0 \right\}$$

converge to an exponential distribution as $t \to \infty$, $\lambda \to 0$ uniformly for all $\{f_k\} \in K$.

DIETZ, K.: Grenzverteilungen von n-dimensionalen Verzweigungsprozessen

Wir betrachten einen kritischen Verzweigungsprozeß mit stetigem Zeitparameter. Seien $Z_k^{(1)}(t), \ldots, Z_k^{(n)}(t)$ die Anzahlen der Individuen der n Typen $T_1, \ldots, T_n$, die im Zeitintervall $[0, t]$ von einem Individuum des Types T_k erzeugt werden. Mit Hilfe der asymptotischen Formeln für $Q_k(t) = P\{\sum_{j=1}^n Z_k^{(j)}(t) > 0\}$ für $t \to \infty$ von Savin (1962) werden asymptotische Ausdrücke für die wahrscheinlichkeitserzeugenden Funktionen des Prozesses hergeleitet, die es gestatten, die Laplacetransformierten der Grenzverteilungen

$$\lim P\left\{ \frac{Z_k^{(j)}(t)}{\beta_i t} < y_j;\ j = 1, \ldots, n \mid \sum_{i=1}^n Z_k^{(j)}(t) > 0 \right\};\quad k = 1, \ldots, n$$

explizit anzugeben. (β_j sind geeignete Konstante). Es zeigt sich, daß die Grenzverteilung stark von den Beziehungen der einzelnen Typen unter-

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einander abhängt. Bei einigen Beispielen gelingt es, die Grenzverteilungen selbst explizit anzugeben.

DINGES, H.: An elementary calculation for the pure death process

The problem to find all positive solutions $(\mu_1, \mu_2, \ldots)$ of $\mu_i = \sum_{k=i}^{\infty} \binom{k}{i} m^i (1-m)^{k-i} \mu_k$ $(0 < m < 1$ fixed) was solved by an elementary calculation. The extremal elements $(\mu_1^{(a)}, \mu_2^{(a)}, \ldots)$ are defined by

$$\sum_{i=1}^{\infty} \mu_i^{(a)} s^i = \sum_{-\infty}^{+\infty} \left(e^{(s-1)a m^k} - e^{-a m^k}\right)$$

in the unit circle where $0 < a < \infty$. Since $\mu^{(a \cdot m)} = \mu^{(a)}$, the set of extremals is homeomorphic to the 1-sphere. The set of all nonnegative solutions μ is in 1-1-correspondence with the set of finite positive measures on this circle.

HARRIS, T.E.: Random Measures and Point Distributions

Two problems are considered for infinite random sets of points.

I. Let $X_i = \{x_i(t), M_{i,t}, P_{i,x_i}\}$, $i = 1, 2, \ldots$ be standard Markov processes with compact metric state spaces (E_i, B_i). Let X be the process with sample paths $x(t) = (x_1(t), x_2(t), \ldots)$ corresponding to independence of the X_i, and $E = E_1 \times E_2 \times \ldots$. If A is a stochastically closed measurable set in the state space, and $x', x'' \in A$, conditions are given insuring that $A = A' \cup A''$, (disjoint union), $x' \in A'$, $x'' \in A''$, A' and A'' stochastically closed. In particular, if $E_1 = E_2 = \ldots = R_1 \cup \{\infty\}$ amd the $x_i(t)$ are temporally homogeneous additive, all with same law, then $\Sigma(x'_i - x''_i)^2 = \infty$ is sufficient.

II. Let Z be the quotient space for E above (taking $E_1 = E_2 = \ldots$), with $x', x'' \in E$ being equivalent if the coordinates of x' are a permutation of the coordinates of the other.

Let x_∞ be a fixed point of E_1 and let Z^* be the subset of Z corresponding to points x such that $\lim x_n = x_\infty$ but $x_n \neq x_\infty$ for any n. The quotient topology in Z^* is examined; it is very "bad", but by some enlargement, it becomes a metric topology induced by a metric analogous to that of Prokhorov for finite measures.

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JAGERS, P.: Migration in the Theory of Branching Processes

At first it was noted that an age-dependent branching process (agpr) where individuals may disappear before their deaths, i.e. "emigrate", is essentially the same thing as an agpr of the traditional type. Then a model was studied where at random moments of time random numbers of individuals arrive (immigrate) into some area, each immigrant initiating an agpr (with emigration) independently of his colleagues. For the case where intervals between successive immigrations are independent equally distributed random variables and the numbers of immigrants are also independent with the same distribution, it was shown under some moment conditions that the number of individuals alive in the area at time t has a limit distribution as $t \to \infty$ in the subcritical case. Otherwise the number of individuals tends to infinitiv in probability.

JIŘINA, M.: On the Feller's Limit Procedure for Branching Processes

In the paper "Diffusion processes in genetics" published in the Proceedings of the 2nd Berkeley Symposium, W. Feller considers the following limit procedure: Let P_N be a sequence of one-dimensional branching processes with discrete states and discrete time parameter, $P_N(t, a_x)$ the probability distribution of the number of particles produced by one particle after t time units in the process P_N and $\phi_N(t,x) = \int_0^\infty e^{xa} P_n(t, da)$ the corresponding Laplace transform ($x \leq 0$); further, let us suppose that $\int_0^\infty a\, P_N(1, da) = 1 + \frac{\alpha}{N}$, $\int_0^\infty (a - 1 - \frac{\alpha}{N})^2 P_N(1, da) = \beta$ and that the third moments of $P_N(1, a)$ are uniformly bounded (with respect to N). Feller asserts in his paper that the transformed functions $\hat{\phi}_N(t,x) = [\phi_N([tN], \frac{x}{N})]^N$ converge to a limit $\phi(t,x)$ satisfying the partial differential equation

$$\frac{\partial \phi(t,x)}{\partial t} = \frac{\partial \phi(t,x)}{\partial x} (\alpha x + \frac{\beta}{2} x^2).$$

However, his proof is not complete, since it does not prove the existence of the limit $\phi(t,x)$. In the paper presented at the conference, it was shown, how the existence of the limit

$$\lim_{N \to \infty} \hat{\phi}_N(t,x) = \phi(t,x) = \frac{x e^{\alpha t}}{1 - \frac{\beta x}{2\alpha}(1 - e^{\alpha t})}$$

may be proved.

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JOFFE, A.: On multitype branching processes with $\rho \leq 1$

For the single type branching process or Galton-Watson process several basic probabilistic phenomena have only recently been studied under natural hypotheses. Here in treating the processes with k-types ($k \geq 1$) we obtain a description of known phenomena under their weakest possible conditions. For example Jiřina's theorem is obtained without second moment assumptions (Harris, T.E. The theory of Branching processes. Springer-Verlag 1963 p. 44.) These results have been obtained in a joint work with F. Spitzer.

KOLMOGOROV, A.: Theory of branching processes as a part of the general theory of populations

The formal mathematical theory of populations may be defined as the theory of Markov processes whose states are "counting measures" (measures which take only a finite number of values and are concentrated on finite sets). But there is a strong interest in more special schemes. It is shown by several examples, in which way the methods of the theory of branching processes, of diffusion processes and of the generalized Jiřina branching processes interlace in the investigation of the evolution of populations. The considered examples are generalizations of the model for the evolution of the numbers N_{aa}, N_{aA}, N_{AA} of individuals which differ with respect to gene A.

With reference to the model which is studied in the contribution of Yaglom the conjecture is proposed that the optimal control is attained by non-differentiable or even non-continuous functions p(N).

LAMPERTI, J.: Continuous state branching processes

We consider right-continuous Markov processes on $[0,\infty]$ whose transition function has the property that $P_t(x+y,\cdot) = P_t(x,\cdot) * P_t(y,\cdot)$ for all $t, x, y \geq 0$, where "*" means convolution. Such processes were introduced by M. Jiřina, and have recently been shown to be the possible limits in a certain natural sense of a sequence of simple branching processes. It is proved that all these "continuous state branching processes" can be

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derived from processes on R^1 with independent increments, unable to jump to the left, by means of a "random time change" using the function $v(x) = x^{-1}$. The processes which are possible limits for a single Galton-Watson process correspond to stable processes of index between 1 and 2, or to the Brownian motion process.

MANDL, P: Some optimization problems in branching processes

Ein geregelter Galton-Watsonscher Prozeß mit r Typen $\{1, 2, \ldots, r\} = I_0$, wird durch ein System von Wahrscheinlichkeitsverteilungen $\{p(i, n_1, n_2, \ldots, n_r; z); n_j = 0, 1, \ldots, i \in I_0\}$ definiert. $p(i, n_1, n_2, \ldots, n_r; z)$ ist die Wahrscheinlichkeit, daß die unmittelbare Nachkommenschaft eines Individuums vom Typus i genau aus n_1 Individuen vom Typus 1, n_2 vom Typus 2, usw. bestehen wird. Diese Wahrscheinlichkeit hängt vom Regelungsparameter $z \in J_0 = \{1, 2, \ldots, s\}$ ab. Jede Funktion $\omega(i)$ von I_0 nach J_0 stellt eine Regelung dar und definiert die Verteilung $\{p(i, n_1, \ldots, n_r; \omega(i)), \ n_j = 0, 1, \ldots, \ i \in I_0\}$ eines G.W. Prozesses im üblichen Sinne. Der Hauptteil des Vortraged wird der Bestimmung der Regelungen, für welche die Population am schnellsten wächst, gewidmet.

NEY, P.: Branching processes and non-linear integral equations

The extinction probability F(t) of a continuous time branching process with particle production generating function f and lifetime distribution G is governed by the equation

$$F(t) = \int_0^t f\{F(t-y)\}\, dG(y). \tag{1}$$

Asymptotic properties of the solution of (1) and of some of its generalizations are studied as well as those of a related class of nonlinear renewal equations of the form

$$x = a + \mathfrak{M}x \tag{2}$$

where x is a real valued function and $\mathfrak{M}$ is a non-linear averaging operator. The work is joint with J. Chover, and will appear in the Journal D'Analyse.

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PAPANGELOU, F.: Mixing for branching processes

Let $F(x) = p_{1o} + p_{1r} x^r + \dots$ $(p_{1r} > 0)$ be the generation function of a Galton-Watson process.

LEMMA: For fixed j the sequence $\frac{p_{1j}^{(n)}}{p_{1r}^{(n)}}$, $n = 1, 2, \dots$ is non-decreasing.

Set $\pi(j) = \lim_{n\to\infty} \frac{p_{1j}^{(n)}}{p_{1r}^{(n)}}$ $(0 < \pi(j) < +\infty)$. It is immediate that $\gamma\pi(j) = \sum_i \pi(j)\, p_{ij}$.

The lemma furnishes very easy proofs of many (mostly known) limit theorems. For instance, if the mean m is < 1 then $\lim_{n\to\infty} P[Z_n = j \mid Z_n > 0]$ (whose existence was shown by Yaglom, Joffe, Heathcote-Seneta-Vere-Jones) is seen to be $\frac{\pi(j)}{\sum_k \pi(k)}$ where $\sum_k \pi(k) < +\infty$. If $m = 1$ then (Seneta)

$$u_j(k) = \lim_{n\to\infty} P[Z_n = j \mid \text{extinction at time } n+k] = \frac{\pi(j)\ (p_{jo}^{(k)} - p_{jo}^{(k-1)})}{\sum_k \pi(k)\, p_{ko}}$$

where $\sum_j u_j(k) = 1$ and $\{\pi(j)\}$ is an invariant measure. If the chain is irreducible and aperiodic (in which case $r = 1$) then the SRLP holds:

$$\lim_{n\to\infty} \frac{p_{ij}^{(n+m)}}{p_{kh}^{(n)}} = \gamma^m \frac{\tau(i)\pi(j)}{\tau(k)\pi(h)} .$$

The latter implies quasi-mixing properties such as those established for the stochastic case in "Strong ratio limits, R-recurrence and mixing properties of discrete parameter Markov processes", Zeitschr. für Wahr. verw. Geb., to appear in 1967.

PROKHOROV, Yu.: On characteristic functionals

In the first part the notions of characteristic functional and Laplace transform of a random measure on compact space are introduced.

Necessary and sufficient conditions for weak convergence of distributions of such measures in terms of characteristic functionals and Laplace transforms are given.

The second part concerns the rate of convergence. As a first step some multidimensional theorems are proved.

Results are partla published in Doklady ANSSSR and Theory of Prob. and its Appl. (1966).

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RÉNYI, A.: Some remarks on branching processes of virology

The following model of the multiplication of viruses has been proposed in this talk:

Suppose there are two types of objects: objects A, called "reproductive objects", and objects B, called "catalyzing objects". Suppose that each object of type A, present at time n ($n = 0, 1, 2, \ldots$) gives birth at time $n+1$ to j objects A and k objects of type B with conditional probability $p_{jk}(l)$ ($j, k = 0, 1, 2, \ldots$), where l denotes the number of objects of type B, which are present at time n. Let X_n denote the number of objects of type A and Y_n the number of objects of type B which are present at time n. We suppose that an object of type B once born, does not disappear. Thus $\{X_n, Y_n\}$ is a branching process with two types of interacting objects. Put

$$P_{jl}^{(n)} = P(X_n = j, Y_n = l), \quad G_n(u,v) = \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} P_{jl}^{(n)} u^j v^l, \quad f_l(u,v) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} P_{jk}^{(l)} u^j v^k.$$

Let us put further $G_{n,l}(v) = \sum_{j=0}^{\infty} P_{jl}^{(n)} u^j$, that is $G_n(u,v) = \sum_{l=0}^{\infty} G_{n,l}(u) v^l$.

Then we have the fundamental recursion formula

$$G_{n+1}(u,v) = \sum_{l=0}^{\infty} G_{n,l}(f_l(u,v)) v^l. \tag{1}$$

If $p_{jk}(l)$ does not depend on l, i.e. $f_l(u,v) = f(u,v)$ is independent of l, then (1) reduces to the well known recursion formula for branching processes.)

One can deduce from (1) asymptotic formulae for the expectations $E(X_n)$ and $E(Y_n)$.

To describe the reproduction of polio-virus the following form of the probabilities $p_{jk}(l)$ was proposed:

$$p_{20}(l) = (1-a-b)(1-e^{-l\tau}); \quad p_{11}(l) = a + (1-a)e^{-l\tau},$$
$$p_{00}(l) = b(1-e^{-l\tau}),$$

for $l = 0, 1, 2, \ldots$, where $\tau > 0$, $a > 0$, $b > 0$ and $a+2b < 1$. The model is interpreted as follows: objects A are vegetative viruses, objects B enzymes catalysing the reproduction of the virus, and if an object A disappears this means that it is transformed into a mature virus covered

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with a protein coat which is no longer active. The number Z_n of such viruses produced up to time n has also been considered. As regards $E(X_n)$ the following recursion formula is obtained from (1): Putting $M_n(v) = \sum_{l=o}^{\infty} G'_{n,l}(1) v^l$ one has $M_n(1) = E(X_n)$ and thus

(2) $$E(X_{n+1}) = mE(X_n) - (m-1) M_n(e^{-T}); \quad m = 2-a-2b.$$

By supposition $m > 1$; this expresses that the process is supercritical.

SENETA, E. and D. VERE-JONES: Infinitely divisible laws connected with subcritical discrete-time continuous state branching processes
(presented by D.G. Kendall)

At time n (= 0, 1, 2, ...) the state of the system = $X_n \geq 0$. The basic assumption is that

$$E\{e^{-sX_{n+1}} \mid X_n, X_{n-1}, \ldots X_o)\} = e^{-X_n H(s)}$$

where $H(0+) = 0$, $H \in C^{\infty}(0, \infty)$, $H \geq 0$, $H' \geq /$, $H'' \geq 0$, etc., or equivalently

$$H(s) = \int_{[0,\infty)} \frac{1-e^{-s\theta}}{1-e^{-\theta}} \mu(d\theta) \qquad (*)$$

μ being totally finite. That is, $e^{-H(s)}$ is the Laplace transform of an infinitely divisible non-negative random variable. In this summary, for brevity, we suppose $m = H'(0) < 1$. H_n will denote the n' th functional iterate of H. Using a result of Kuczma (J. Austral. Math. Soc. 4 (1964) 149-51) it is shown that

$$H_n(s) \sim H_n(1)\, \psi(s) \qquad (n \to \infty)$$

where ψ is the (unique) solution to

$$\psi(H(s)) = m\psi(s), \quad \psi > 0, \quad \psi(1) = 1, \quad \psi(s)/s \downarrow.$$

It can now be shown, that $\psi(s)$ admits a representation of the form (*) with (probability) measure λ in place of μ. Thus $e^{-\psi(s)}$ is the Laplace transform of an infinitely divisible distribution $d\Psi$.

If we start the n' th system with $X_o = \frac{1}{H_n(1)}$ and let Y_n be the size of population in the n' th system after n generations, then Y_n converges in law to $d\Psi$.

It is further shown that

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$\frac{H_n(1)}{m^n} \to 1$ if $E(X_1 \log X_1 | X_o = 1)$ exists and that otherwise $\frac{H_n(1)}{m^n} \to 0$.
Another interpretation of $\psi(s)$ is as follows. Suppose $H(\infty) > 0$ (i.e. $Pr(X_1 = 0 | X_o = 1) > 0$). Then

$$E[e^{-sX_n} | X_o = 1, \; X_n > 0] \to 1 - \frac{\psi(s)}{\psi(\infty)}$$

so that $\psi(s)$ is also related to a second limit law, but in this case as a linear function of the Laplace transform rather than as (minus) the logarithm.

When $X_o \neq 1$, the situation as regards this last result is more complicated. There exists a family of limit laws depending on a parameter $\alpha (0 < \alpha \leq 1)$ and convergence to the α'th member of this family occurs iff $-\log E(e^{-sX_o})$ is of regular variation with index α.

SEVAST'YANOV, B. A.: Two problems of age-dependent branching processes

We consider age-dependent branching processes with distribution function (d.f.) $G(t)$ of life-length of particles. Let μ_t be a number of particles at time t. Let p_n denote the probability that a particle at the end of its life is replaced by n new particles. The generating function (g.f.)
(1) $h(s) = \sum_{n=o}^{\infty} p_n s^n$ is introduced. We shall call the branching process with d.f. $G(t)$ and g.f. $h(s)$ a (G,h)-process.

1). (G,h)-processes will be called *regular*, if $P\{\mu_t < \infty\} \equiv 1$, $t \geq 0$; otherwise (G,h)-processes will be called *irregular*.

HYPOTHESIS: A (G,h) is regular if f satisfies

(2) $$\int_o^{\epsilon} G_{-1}\left(\frac{y}{1-h(1-y)}\right) \frac{dy}{y} = \infty$$

for sufficiently small $\epsilon > 0$. (Here G_{-1} is the inverse function of G.) For some classes of $G(t)$ the necessity and sufficiency of condition (2) for the regularity of a (G,h)-process is proved.

2). The more general case when probabilities p_n and generating functions (1) depend on age of the parent of the particle is considered. For such critical branching processes a limit theorem is proved.

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SZÁSZ, D.: Branching processes with scattering points

Let us suppose that on the real line a random point distribution is given. Each point has a random life time and after it each point generates a random PD. The new points also have random life times and after it they generate new RPD's and so on. The life times are independent of each other and of the history of the process. The generation of the RPD's depends only on the generating point and is independent of the history of the process. So we obtain an "age-dependent-branching process in general sense". The expectations of the process were determined and it was proved that the limit of the process is stationary on the real line in a wide sense only if the expectation of the number of the offspring of a single point is equal to the unity. Some remarks on the characteristic functions of the process and the limit of the process were made.

YAGLOM, A.: Some generalizations of branching processes connected with biological problems

Let N_t be the number of particles existing at the time t, where $t = 0, 1, 2, \ldots$. In the usual Galton-Watson branching process N_t tends either to zero or to infinity as $t \to \infty$. However in many biological problems N_t is finite and differs from 0 for all t. A group of people at Moscow University (namely Prof. I. Pyatecky-Shapiro, A. Leontovič and some others) investigated the problem of the evolution of number of cells in the issues of living organisms and constructed a special stochastic model giving an example of a population-size-dependent branching process. Such processes can be defined as follows: there is a group of particles, and every particle produces at every integer moment of time n new particles with the probability p_n which depend upon the number N_t of the particles existing at this moment. It is interesting to study the conditions under which a limiting distribution exists for N_t, where N_t is the number of particles at the time t for population-size-dependent branching process. The special population-size-dependent branching process studied by Pyatecky-Shapiro and others is the process in which every particle can produce at the moment t either two particles (with probability $p(N_t)$) or zero particles (with probability

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$1-p(N_t)$). They are interested in the time $\tau = \tau(N_o, N_1, N_2)$ during which the process N_t remains in the interval $N_1 < N_t < N_2$ if N_o is the initial number of particles. It can be shown that $(E\tau)^{1/N}$ tends to a constant $\gamma > 1$ when $N \to \infty$, $N_o = a_o N$, $N_1 = a_1 N$, $N_2 = a_2 N$. where a_o, a_1 and a_2 are fixed numbers and $p(N_t)$ satisfies some simple regularity conditions. This result apparently is a special case of some unknown general limit theorem for population-size-dependent branching processes.

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