MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH



Tagungsbericht 23/1968

Ergodentheorie

4.8. bis 10.8.1968

herein beschlossen, nur wenigen Gästen Vorträge einzuräumen, diese aber zu längeren Serien auszugestalten.
So hielt S.Kakutani (Yale University) mehrere Referate über veröffentlichte und unveröffentlichte Untersuchungen aus seinem Arbeitskreis. Man folgte damit posiviten Erfahrungen, die auf der Kombinatorik-Tagung 1967 -damals hatte es nur 4 Vortragende mit jeweils 4 Stunden Vortragszeit gegeben- gemacht worden

Leiter der Tagung war Prof.K.JACOBS (Erlangen). Es war von vorn-

Dies war die 2.0berwolfacher Ergodentagung und es wurde angeregt, eine weitere im Abstand von etwa 3 Jahren folgen zu lassen. Nachdem es auch diesmal nicht gelungen war, den Besuch einer sowjetischen Delegation zu erwirken, kam der Gedanke auf, die sowjetischen Kollegen zu bitten, die nächste Tagung in ihrem eigenen Land auszurichten. Dies hat sich bislang als erfolglos erwiesen. Im Augenblick ist eine von der Université de Rennes auszurichtende Ergodentagung unmittelbar vor dem Internationalen Mathematiker-Kongress in Nizza 1970 vorgesehen.

Die bei dieser Oberwolfacher Tagung anwesenden Gäste bildeten einen Teil der sich gegenwärtig stark vergrößernden Ergodiker-Familie. Eine Gesamttagung wäre bereits ein Mammutunternehmen. Neben einem solchen werden kleine Tagungen in Oberwolfach sicher ihren eigenen Rang behaupten.

Teilnehmer

waren.

À

R.Adler, Yorktown Heights/USA A.Brunel, Rennes/Frankr.

M.A.Akcoglu, Toronto/Canada D.L.Burkholder, Urbana/USA

T.Ando, Tübingen u.Sapporo/Jap.N.Dinculeanu, Bukarest/Rum.

G.Bray, Rennes/Frankr. H.Dinges, Frankfurt/Main





encented in the company of the company of

u babbalkos su byrd

Adelli, i sa s

The contract of the contract o

• PART CONTROL OF THE PROPERTY OF THE ARREST OF THE ARR

destrout de la company de particul de la company de la com

(a) The included of the second second of the second of the included of the included of the second of the second

• Responsibility of the particle of the par

F.Eicker, Freiburg/Brsg.

Ch.Grillenberger, Erlangen

D. Hanson, Columbia/USA

G. Helmberg, Eindhoven/Holl.

E. Hopf, Bloomington/USA

K. Jacobs, Erlangen

A. Jonescu-Tulcea, Evanston/USA

S.Kakutani, New Haven/USA

M. Keane, Erlangen

U.Krengel, Erlangen

W.Krieger, München

G.Maruyama, Tokyo/Japan

J. Neveu, Paris/Frankr.

A.Nijst, Eindhoven/Holl.

W. Parry, Coventry/Engl.

K.Post, Eindhoven/Holl.

H.Scheller, Erlangen

F.Simons, Eindhoven/Holl.

D.Stone, Rochester/USA

L. Sucheston, Columbus/USA

R. Theodorescu, Bukarest/Rum.

H. Totoki, Kyoto/Japan

R.Adler: <u>Isomorphisms of Markov shifts</u> (in collaboration with B.Weiss)

Let A = {1,...,N} be an alphabet of symbols and T = (t_{ij}) an N × N matrix of zeros and ones. Let E(T) denote the space of two-sided infinite sequences $\xi = (\dots \xi_{-1}, \xi_0, \xi_1, \dots)$ where $\xi_i \in A$ and $t_{\xi_n}, t_{n+1} = 1$ for all n; and let σ denote the shift transformation on E(T). The family ξ of measurable subsets of E(T) is given by $\xi = B(\bigcup_{i=0}^{\infty} \sigma^n \alpha)$ where α is the partition of E(T) into sets $\{\xi \mid \xi_0 = i\}$, $i = 1, \dots, N$. Assume further that T is irreducible; i.e. for every i,j ϵ A there exists n such that $t_{ij}^{(n)} > 0$. Let λ_T be largest positive characteristic value of T with x,y positive column and row characteristic vectors associated with λ_T , normalized so that $\xi x_i y_i = 1$. The vector $\pi = (x_1 y_1, \dots, x_N y_N)$ and the matrix $P = (P_{ij})$ where $P_{ij} = t_{ij} x_j / \lambda x_i$ define a Markov measure μ on ξ for which $h_{\mu}(\sigma) = \log \lambda_T$. This is the largest value of entropy the shift on E(T) can have for any σ -invariant measure and μ is the only

 $h_{\mu}(\sigma) = \log \lambda_{T}$. This is the largest value of entropy the shift on E(T) can have for any σ -invariant measure and μ is the only measure giving this value. The following conjecture was examined: $\lambda_{T} = \lambda_{T}$ \Rightarrow σ is metrically conjugate to σ . Partial results were obtained for $\lambda_{T} = 2$. The following example was

worked:
$$T = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$
 and $T' = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

The associated shifts were found to be conjugate.

. .

Power Stands of Span

A. Mijet Bindsome of Foll

A. Mijet Bindsome of Foll

A. Moser Bindsom of Columbia

D. Sided Bindsom of Volumbia

D. Sided Bindsom of Volumbia

A. Mosech Bindsom of Volumbia

A. Mosech Bindsom of Volumbia

A. Mosech Bindsom of Columbia

A. Mosech Bindsom of

R.Adlar. Isomorphisms of Mark (W. collaboration with B.W.das)

We as $(v_{i,j})$ of the oblique of coupling and $(v_{i,j})$ as $A \times A$ to remove in cooper has bronger (), and conductor to winds. The Allega Dunda Collins Collins of Allega Collins and Allega Statement occupante transfer ditida endi especiale e di Elban (n. Ef. e e di s with win ai (\mathbb{T}) 3 00 ofeache a franchin 1: \mathbb{F} via \mathbb{F} and $\mathbb{F}(\mathbb{T})$ 5 mo will right of a tea of other form of the matrix will be the constraint of the \mathbb{R}^n . The $z\in\mathbb{R}^n$ is a,..., which was therefore the forestable filty into some volume i, j \pm A therefore in the above the content of $\frac{(n)}{n!}$ > 0. Let λ_{ij} be largely ons course eviding generally Total Lands of a contract and relations used os popilskana ly dalm bata vasa vasaa valat in storeds wor (v_1^{-1}) = if xindsecond [bho $(v_1^{-1}v_1,\dots,v_{n-1}^{-1}v_1^{-1})$ = $v_1^{-1}v_1$. For $v_1^{-1}v_1^{-1}v_2$ white of D mong bourses which is being a factor of the property i=(s) which is the the largest value of ortropy the shift - vinc ent ar e bos ombowe dari-evol-, yne mat evel hat (Tyd),; asw south incompaison ofth confer all thereing cannot egently, at the property of the entire of the conjugate of the Partial our in association and invited only as a specific beauty of a section.

cotumpted to the control of the control of the control of the

M.A.Akcogulu: <u>Identification of the ratio ergodic limits for</u> the non conservative transformations

Let (X,F,μ) be a σ -finite measure space and T a positive linear contraction on $L_1(X,F,\mu)$. It is known that if T is a conservative transformation then it defines a sub σ -field I of F, consisting of the invariant subsets of X, and the identification of the ratio ergodic limits can be done in terms of I. If T is not necessarily conservative then it is shown that the σ -field I can be replaced by a field Σ . The members of Σ are "asymptotically invariant", in a natural sense, and the bounded ratio ergodic limits can be approximated by Σ -simple functions. Using this representation one obtains the identification of the limit in a way similar to the conservative case.

The details of this work, which was done jointly with R.W.Sharpe, will appear in the Transactions of the American Mathematical Society in 1968.

T.Ando: Invariant measures of a positive contraction in C(X)

We consider a compact <u>stonian</u> space X, i.e. the Banach lattice C(X) is conditionally complete, and a <u>positive</u> contraction T (in C(X)) with T1 = 1.

Theorem 1: The number of different ergodic measures with the same support as a given minimal-ergodic measure is either 1 or ∞ .

A measure ϕ (resp. the operator T) is called σ -additive, if $f_n \neq 0$ (in order sense) implies inf $|\phi|(f_n) = 0$ (resp. $Tf_n \neq 0$).

Theorem 2: Let T be σ -additive. The following assertions are equivalent:

- (a) Every invariant measure is σ -additive.
- (b) $n^{-1} \stackrel{n}{\Sigma} T^{j}f$ converges in norm for every f, and the subspace

{g : Tg = g} is finite dimensional.

Theorem 3: Let X admit a strictly positive, σ -additive measure, and let T be σ -additive. There exists no (non-trivial) σ -additive invariant measure, if and only if for each 0 < f there exists $0 < g \le f$ with order limit $n^{-1} \sum_{j=1}^{n} T^{j}g = 0$.





M.A ARCOGOLO. Marie of the trade of the conditor of the Arcogological Arc CHO! there land . D. Per Last do don 305

THE BEST OF TRANSPORT OF THE STREET OF THE PROPERTY OF THE STREET OF THE sewificativo (15) projector (数)。 「OBSERTED TO THE SERVICE (15) AND SERVI the state of the second state of the second ing a subject of the property of the second The of Fig. . Comment of the property and the following of the to a colling with real emode oil of mond coldware, the command of And our Josephyse Form of the standards add in highly one consider of And a part of the belong of a last common lemman of the common server. rajo e spekal samenia osta linggeri yd podemiy roppuse neb adimil reportantation one obtains the identification we the sight of ed this error constructive cast. The Changes of this work, which was made jointly wice thy Sharps will appears to a college of the American Mathematical That gradeous

(A) on the manual control of the con

We was fire a compact throughout A. L.e. the Baneau Lartece C(X) is bunditionally that the sposifive contraction f of into the comments

is an disk manageom of prate hootelije je krimin in ti smaoed. en forget en la brussen libogram Indinktioner i eigelenden to mas

Congress of the constitution and the set of governous entropy with expressing A (1 179) gabernes (1) [of local asking those above all)

Theorem 21 Inch To Additive Towns Inch as as only :deslaviup:

(a) Every is a comme is graddicive.

is declar and the La America and match at the series $f : \frac{1}{4} \int_{-1}^{1} f(d)$

· Isotalemonto e coercio de la mesca de la Commune evalibbe-- portation care en than a selection of the contract of the care maidibbase (injuing pass) or adjoin them a distribute to the Fidure of signas saudo la compacto de la la versa en la persona desinación.

G.Bray: About a theorem of mean ergodic convergence

Let σ be a locally compact connected abelian semi-group satisfying certain conditions such that we can imbed σ in a group $G = H \times \mathbb{R}^p$ where H is a compact abelian connected group. Let (S, Σ, μ) be a probability space, such that L^2 is separable, $(T_X)_{X \in \sigma}$ a semigroup of linear continuous operators which operates on each L^p, $1 \le p < \infty$ with $T^1: ||T_X||_p \le 1 \ \forall p \ 1 \le p < \infty$; $T^2 : \forall f,g \in L^2 \times \longrightarrow \langle T_x f,g \rangle$ is measurable; $T^3;T_xT_y = T_yT_x = T_{xy}$ and $T_x T_y^* = T_y^* T_x \forall x \in \sigma \text{ if we consider the } T_x \text{ as operations on } L^2$. Let U be the W-algebra generated by the $(T_X)_{X \in \sigma}$, K the hyperstonian spectrum of U $\phi_{X} \in C(K)$ the function corresponding to $T_{\mathbf{x}}$ in the Gelfand isomorphism, P the spectral measure of U . Denote by E' the clopen set which differs from E = $\{M \mid M \in K, \phi_x(M) = 1 \text{ for almost every all } x \in \sigma\}$ by a set of P-measure zero. The main result is: We can construct a directed increasing family of subsets of σ : $(B_j)_{j \in J}$ such that $\lim_{j \in J} \frac{1}{\lambda(B_j)} \int_{B_j} T_x f d\lambda(x) = P(E') (f) \quad \forall f \in L^p \text{ (convergence in } f \in L^p)$ the L^p norm sense) where P is a projection operator in L^p .

A.Brunel et M.Keane: <u>Théorèmes ergodiques pour une suite de</u> puissances d'une transformation

Soit (Ω, F, μ) un espace probabilisé, T une transformation: $\Omega \rightarrow \Omega$ conservant la mesure. On désigne aussi l'extension de T par la même lettre: Tf = f o T. Nous avons étudié les moyennes de Cesaro $\frac{1}{n} \sum_{i=1}^{n} T^{k_i}$ f pour $f \in L^1$ (Ω, F, μ) et une suite

croissante d'entiers $k_1 < k_2 < \dots$ et avons obtenu les résultats suivants:

- 1) Pour certaines suites que nous avons appelées suites uniformes: Théorème 1. Les conditions suivantes sont équivalentes:
 - 1) T est faiblement mélangeante.
 - 2) $\frac{1}{n} \sum_{i=1}^{n} T^{k_i} f \xrightarrow[n \to \infty]{} \int f d\mu$ (p.p.) pour tout $f \in L^1$

et toute suite uniforme (ki)i=1.2....





Approva o pickali dibai in kappear in Accidentia.

lot a ba a handi - report v er sted abelism semi-group f allreing certific cohestions rack that cas imbed a in a group \mathbb{C} and \mathbb{R}^{D} od o stolen, sull) religios<mark>a proma prom</mark>ensione de la servicio de promeso de la composición del composición de la composición de la composición de la composición de la composición del composición de la composición del composición del composición del composición de la composición d president for the supplication of the supplied (Tyle) and the supplied of the g. Por la Albania de Historia de La La Descriptor de la compresentación de la compresentación de la compresentación TARRELL STALLSTEELS TO SEE $p^2 + \exp_{i} e^{i \pi}$. The access the second $e^2 \sqrt{k} e^{i \pi}$ of Canob training on the same additional so the second some second some second some second some second some some ार विद्वार कर है है। इस है कि एक निर्माण के कि एक निर्माण के कि एक कि एक स्थानिक के लिए हैं है कि उस कि है है o paintenangers asidose, etc. (A) etc. et te autooga lecinode the statement in the second of the control of the state of the second of . I have the siepen see which differs from dos a vel ja sa lis greve asa i. (= (%)), (= (*)) Tiperi. Tourseso you switch study amage and the The $au_{\mathrm{const}}(z)$ is the decays for z .

The American model agreement to the consists of the con-

Sair course and appear parabolised, Then area stormarions with conservation of the conservation of the probability metric that the conservation of the probability metric that the conservation of the conser

ong strangs disposition of the state of spoods obtained lon offsetting and strangs of the offsetting and spoods of the strangs appealants of the strangs appealants of the strangs appealants of the strangs appealants of the strangs of the strangs

The second of the property of the second

Thomas, remarks the same



Ce résultat est un corollaire d'un théorème plus général qui affirme que pour une transformation T conservant la mesure:

Théorème 2.
$$\frac{1}{n} \sum_{i=1}^{n} T^{k_i} f \xrightarrow{n \to \infty} \tilde{f} (p.p.)$$
pout tout $f \in L^1$ et toute suite uniforme $(k_i)_{i=1,2,...}$

2) Pour des suites quelconques d'entiers nous avons établi la caractérisation suivante des transformations fortement mélangeantes:

Théorème 3. Les conditions suivantes sont équivalentes:

- 1) T est fortement mélangeante.
- 2) Pour tout f ∈ L¹ et toute suite croissante d'entiers k₁ < k₂<... il existe une suite reelle (c_i)_{i=1,2,...} qui possède les propriétés

$$\begin{cases} a, & 0 < c_n \end{cases}$$

$$b, & \sum_{n=1}^{\infty} c_n = +\infty, & \text{et l'on a:} \end{cases}$$

$$\frac{\sum_{i=1}^{n} c_i T^i f}{\sum_{i=1}^{n} c_i} \xrightarrow[n \to \infty]{} f d\mu \quad (p.p.)$$

D.L.Burkholder: Strong L₁ inequalities for quasi-linear operators on martingales

Consider martingales $f=(f_1,f_2,...)$ on some probability space (Ω,A,P) . Define the maximal function f^* by $f^*(\omega)=\sup_n |f_n(\omega)|$ and the square-root function S(f) by S(f)=

$$(\sum_{k=1}^{\infty} d_k^2)^{\frac{1}{2}}$$
 with $d_1 = f_1$ and $d_k = f_k - f_{k-1}$, $k \ge 2$. It is known that

(1)
$$c_p || S(f) ||_p \le || f^* ||_p \le c_p || S(f) ||_p$$

for 1 \infty, with c_p and C_p positive real numbers depending only on p [Burkholder, Martingale transforms, Ann.Math.Statist. 37 (1966) 1494-1504]. The present work, joint with Richard F.Gundy, indicates that (1) is also true for p=1 provided the martingale f has difference sequence d = (d_1, d_2, \ldots) of the





....

Co résolant est un desditain d'un amédeden plus gérépal qui affirar ond pour une velonfonteaue T coassivart un mosure:

of the Company of the second o

illand entre satur tretantis travant, aip pesite e e tri (S disperante pe for mersanasi neb e orinte maiduali larsmos sa

e esta propertione de la constitución de la constit

as the contract of the contra

Theoly and sweet the top of the second of th

Canadana and an analysis and a

and the contract of $(x_1, x_2, x_3) = 0$ and the property of the property of

 $\frac{2}{3} \left(\frac{2}{3} \right)^{\frac{3}{2}} w \sin d_{1} + \epsilon_{1} \cos \delta_{1} = \Gamma_{2} + \Gamma_{c+1} + \Gamma_{c+1} + \Gamma_{c} \cos \delta_{1} = \Gamma_{c+1} + \Gamma_{c+1} +$

 $\frac{1}{4} \frac{11}{12} (3) \circ 11 = \frac{1}{6} (3) \circ 11 = \frac$

for fire and interpolation and Compositive rund mashers top-modure outs of a composition of the composition

form $d_k = w_k e_k$ where w_k is A_{k-1} measurable, $E(e_k^2 | A_{k-1}) \ge \varepsilon > 0, ||e_k|| \le \varepsilon^{-1} < \infty, \ k \ge 1, \ \text{for some increasing sequence of sub-} \sigma \text{-fields of } A \ \text{relative to which } f \ \text{is a martingale.}$ In this case, c_1 and c_1 depend on ε . The operator s is quasilinear. A general theory of quasi-linear operators satisfying (1) for s is developed.

N.Dinculeanu: Algebraic models for measure preserving transformations

<u>Definition 1.</u> An object (Γ, U, ϕ) consisting of an abelian group Γ , an injective homomorphism $U: \Gamma \to \Gamma$ and a function of positive type $\phi: \Gamma \to \mathbb{C}$ such that: $\phi(\gamma) = 1$ iff $\gamma = 1, \phi$ o $U = \phi$, is called an algebraic ergodic system (a.e.s.).

<u>Definition 2.</u> Two algebraic ergodic systems (Γ, U, ϕ) and (Γ', U', ϕ') are isomorphic if there exists a group isomorphism $\phi : \Gamma \to \Gamma'$ such that $\phi' \circ \phi = \phi$ and $\phi U = U' \phi$.

Example: Let (X, Σ, μ) be a probability measure space and $T: X \to X$ a measure preserving transformation.

- a) Let $\Gamma(\mu)$ be the set of (equivalence classes of) functions $f \in L^2(\mu)$ such that $|f| \equiv 1$; then $\Gamma(\mu)$ is a multiplicative group generating $L^2(\mu)$ and containing the circle group C.
- b) Let U_T be the operator on $L^2(\mu)$ induced by T. Then U_T is an injective homomorphism on $\Gamma(\mu)$ such that $U_Tc=c$ for $c\in C$.
- c) Put $\phi_{\mu}(f)=\int f\ d\mu$ for $f\in\Gamma(\mu)$. Then ϕ_{μ} is a function of positive type on $\Gamma(\mu)$ satisfying:
- $\phi_{u}(f) = 1 \text{ iff } f = 1 \text{ and } \phi_{u}(U_{T}f) = \phi_{u}(f)$

If $\Gamma \subset \Gamma(\mu)$ is a group invariant under U_T , then $(\Gamma, U_T, \phi_{\mu})$ is an a.e.s.. In particular, $(\Gamma(\mu), U_T, \phi_{\mu})$ and (C, U_T, ϕ_{μ}) are a.e.s..

<u>Definition 3.</u> An a.e.s. (Γ, U, ϕ) is an algebraic model for a measure preserving transformation T if there exists an injective homomorphism $J:\Gamma \to \Gamma(\mu)$ such that: $J\Gamma$ generates $L^2(\mu)$, $\phi = \phi_{\mu}$ o J and $JU = U_{m}J$.

<u>Definition 4.</u> An a.e.s. (Γ, U, ϕ) is discrete if $C \subset \Gamma$ and $\phi(\gamma) = \gamma$ for $\gamma \in C$ and $\phi(\gamma) = 0$ for $\gamma \notin C$.

Theorem 1. Two measure preserving transformations are conjugate iff they possess isomorphic algebraic models.





the property of the second of

<u>nnag formagnet en hande en de vollegelet en her beske hande en her et de v</u>ert beveren. De verken hande

A second position of the large content of the large section of the la

i de la composition La composition de la La composition de la

X e la company de la company d

and the second of the content of the

The District Court of the Court of the second of the second of the second of the Court of the Co

The problem of the second of t

internal of the contraction of the second of the second and the second of the contraction of the contraction

as the following of the control of (x,y,y), which is the (x,y) and (x,y)

where the first section of the contract of th

error tem som er stiller junger i Tvast synte busse om i p<u>erroell</u> Leiste i bret stat i Skrive som er beske i de skrive.



Theorem 2. Every a.e.s. is a model for a m.p.t..

Theorem 3. A m.p.t. T has a discrete model iff there exists a group $\Gamma' \subset \Gamma(\mu)$ which is an orthonormal base of $L^2(\mu)$ and $U_m\Gamma' \subset C\Gamma'$.

Theorem 4. An invertible m.p.t. is with discrete model iff it is conjugate to the superposition of a rotation and a continous automorphism on an abelian compact group equipped with Haar measure.

Ergodicity and transformations with discrete spectrum are also characterized by means of algebraic models.

H.Dinges: A pointwise ergodic theorem

Let (Ω,m) be a measure space and T a positive contraction of $L^1(\Omega,m)$, and let $x^1,\ldots,x^n\in L^1$, then

$$\int f(x^1,...,x^n) dm \ge \int f(Tx^1,...,Tx^n) dm$$

holds for every sublinear nonnegative f.

In particular if $x^{i} = T^{i}x$, $p^{i} = T^{i}p$ with $p \ge 0$, then

(*)
$$\int f(x^0, \dots, x^{n-1}; p^0, \dots, p^{n-1}) dm \ge \int f(x^1, \dots, x^n; p^1, \dots, p^n) dm \ge \dots$$

for every sublinear nonnegative f of 2n variables (n arbitrary). It was shown, that the maximal ergodic theorem for instance can be derived from inequalities of the type (*).

Several theorems were formulated, which lead to the following ergodic theorem:

If $x^i \in L^1(\Omega,m)$, $0 \le p^i \in L^1(\Omega,m)$ such that (*) holds, then $\frac{x^0 + \ldots + x^n}{p^0 + \ldots + p^n}$ converges almost surely as $n \to \infty$ on $\{\sum p^i > 0\}$.

If the limit is A, then

$$\frac{1}{n} \int (x^{0} + \ldots + x^{n-1} - A(p^{0} + \ldots + p^{n-1}))^{+} dm \xrightarrow[n \to \infty]{} 0.$$

Little information could be given about the hard part of the proof, in which there has to be shown, that there exist decompositions $x^{i} = x^{i}\psi + x^{i}(1-\psi)$; $p^{i} = p^{i}\psi + p^{i}(1-\psi)$

such that $(x^i\psi;p^i\psi)$ and $(x^i(1-\psi);p^i(1-\psi))$ fulfill (*) and have certain other properties.





Legicological de la proposición de la companiente del companiente del companiente de la companiente de

Andrew Communication of the Co

and the state of the control of the The control of Chr.Grillenberger: On the entropy and the spectrum of an almost periodic dynamical system

Prof.Jacobs has shown in his paper on "Almost periodic sources and channels" that the invariant average \bar{m} of a weakly almost periodic, uniformly mixing probability measure m on a compact metric space Ω with a topological automorphism T is ergodic, its spectrum is the group generated by all eigenvalues of the

sequences $(\int f \cdot T^t dm)_t$ integer for $f \in C(\Omega)$, and all flightvectors are strong flightvectors.

For the case of a two-sided Bernoulli space with finite alphabet A, T being the shift transformation and m the product of an almost periodic sequence (p^t) of probability vectors over A, the space of flightvectors is $N = L^2_{\overline{m}}(B_{\infty})^{\perp}$, where B_{∞} is the tail field, and in N T has Lebesgue spectrum with multiplicity N_o , except the trivial case in which m is a periodic point measure. (The result is valid also for markovian almost periodic m.)

In the same special case we obtain a formula for the Kolmogoroff-Sinai invariant in terms of the marginal distributions:

$$\hat{H}(\overline{m},T) = \overline{H(p^{t})} := \lim_{t \to \infty} \frac{1}{t} \sum_{s=0}^{t-1} H(p^{s})$$

D.L. Hanson: A mean ergodic theorem with general coefficients

Let $A_{N,K} \geq 0$ for N,K = 0,1,2,...; let (Ω,Σ,P) be a probability space. Let T be a measurable and measure preserving point transformation of Ω into Ω ; let d be the invariant subsets of Ω under T; and let L_2 be the collection of measurable square-integrable functions on (Ω,Σ,P) . The following theorem seems to be the "appropriate" one. It is a considerable improvement over the one presented in the author's talk. The improvements being suggested and proved by various people.

Theorem: If $\sum_{K} A_{N,K} = 1$ for all N, then $\sum_{K} A_{N,K} T^{K} f \rightarrow E\{f|d\}$ in L_2 -mean for all $f \in L_2$ and all (Ω, Σ, P, T) if and only if

(1)
$$\sum_{K=0}^{\infty} A_{N,K\alpha+j} + \frac{1}{\alpha} \text{ for } \alpha = 2,3,... \text{ and } j = 0,...,\alpha-1$$



<u>. Allega</u> de la lagrada del la lagrada el lagrada de la companyon de la compa CANAL THE RELIEF WHEN BY BY BY DO

Editor to the least a research of the same The Control of the second of t and the second of the second o AND THE ROYAL TO SEE THE SECOND SECON

PACH TO STORY OF THE STORY

ing. Kalanggan pangganggan panggan The section of the se The state of the section of description of the section of the se Test of the period of the The time of the fight sound account to the A TOTAL PROPERTY OF WELVE

of becommending the person of the . The set of the process of the particle of the set of the set of the set of the set of the second of the second

The second section of the second seco ± 2 is the specific of the second of the Constitution of the

<u>aferiut/100 (125) (2011) production (1</u> 1) the beautiful to a gradient

vultidagener e er en en en en en er er er er er er er en er en er en er en er en removed that the parameters of the control of the c 我们是我的人们,我们们就要把了"我"这一样的的是我的人的人们,但一个人的人们的人们的人们,这个人的人就是我的不是你的人 the property is the second and the second of namen de la cambie de la companya del la companya de la companya d

PROPERTY AND A CONTRACT OF THE PROPERTY OF THE he tought builted or here is a r Takke to believe engage Continue of the American Company of the

100 (\$13)20 · 100 · 100 The Kind that if the state of the state of the state of

imposes a destruction of con-The first of the property of (1.1 Property of the following



G.Helmberg: On mean recurrence time under a measure preserving flow.

M.Kac hat 1947 einen später verallgemeinerten Satz über mittlere Rückkehrzeit in eine Menge B eines Wahrscheinlichkeitsraumes unter wiederholter Ausübung einer maßtreuen Transformation T bewiesen. Eine analoge Zusage über mittlere Rückkehrzeit läßt sich für eine maßtreue Halbströmung $\{T_t\}_{t\geq 0}$ in einem Wahrscheinlichkeitsraum ableiten, doch müssen Mittelungsvorgang und Definition der Rückkehrzeit in geeigneter Weise modifiziert werden. Im ersten Teil des Referates wird die rein maßtheoretische Situation betrachtet, im zweiten Teil wird der Fall einer stetigen Halbströmung in einem kompakten metrischen Raum behandelt.

A.Ionescu-Tulcea: <u>Lifting for abstract valued functions and</u> separable stochastic processes

Let (Ω, F, μ) be a complete probability space. Let $M_R^{\infty}(=M_R^{\infty}(\Omega, F, \mu))$ be the algebra of all measurable bounded functions $f:\Omega \to R$. For f,g ϵ M_R we write f ϵ g if f and g coincide μ -almost everywhere. Let now E be a completely regular space and let $C_p(E)$ be the algebra of all continuous functions h: E + R. A function f: $\Omega \rightarrow E$ will be called weakly measurable if h o f is measurable for each h \in $C_R(E)$. We denote by M_E^{∞} the set of all f: $\Omega \to E$ such that: 1) f is weakly measurable and 2) $\overline{f(\Omega)}$ is compact. For f,g in M_E^{∞} we write f Ξ g if h o f Ξ h o g (in M_R^{∞}) for each $h \in C_R(E)$. The notion of lifting is extended from the "real space" $exttt{M}_{\mathsf{R}}^{\infty}$ to the "abstract space" M_{E}^{∞} . Let $\rho: M_{R}^{\infty} \to M_{R}^{\infty}$ be a lifting of M_{R}^{∞} . A mapping $\rho': M_E^{\infty} \to M_E^{\infty}$ is called a lifting of M_E^{∞} associated with ρ if i) $\rho'(f) \equiv f$; ii) $f \equiv g$ implies $\rho'(f) = \rho'(g)$; iii) $\rho(h \circ f) = h \circ \rho'(f)$ for all $f \in M_E^{\infty}$, $h \in C_R(E)$ (as a matter of fact, condition iii) is the defining equation of ρ '). It can be shown that there exists a unique lifting of M_E^{∞} associated with $\rho_{\textrm{F}}$ this lifting will be denoted by $\rho_{\textrm{F}}.$ The lifting $\rho_{\textrm{F}}$ is then applied to obtain a separable modification of a stochastic process: If $(X_t)_{t \in T}$ is a stochastic process defined on (Ω, F, μ) with values in \bar{E} , then the process $(Y_t)_{t \in T}$, where $Y_t = \rho_E(X_t)$ for each $t \in T$ is a separable modification of $(X_t)_{t \in T}$.



Maivessend engast trong on the color of the Dressen to the Color of th

envalture, - in modernome archega ches escile residente escile escile escile escile escile escile escile escil Compared to the standard of the contract of the first of

2/0.1

。 1977年(1977年)(1978年)在1990年後8日(1982年)(1977年)(1977年)(1977年)(1977年)(1977年)(1977年)(1977年)(1977年)(1977年)(1977年)(197 TO THE FOREIGN AND THE STATE OF THE STATE OF

na - Cerani vagaul seel, e gestiis visas e contribues e contribié de contrib

The Claiman at the Charles of the Company of the Co naideachaidean air i san an deile chaireach de bha an aireachad r<mark>ždedo moviže</mark> jeu nosto proje ližnosto praklado ež jodo no iznosto projektorije. . Alabandad music ascual requires requires approximately of a section of Foreign

in the confidence of Service Community of the Community of th and algebraic to the constituent of the contract of the contra

ter somme a la plus di collegation de partir de la partir de la partir de la collegation della collegation de la collegation de la collegation de la collega istoriosmosi (n. 1903). Paristorios de la composición del composición de la composición de la composición del composición de la composició A CONTRACT TO THE REAL PROPERTY OF THE STATE OF THE STATE

tin kilometri (1904), kilometri kilometri kilometri kilometri kilometri kilometri kilometri kilometri kilometr Kilometri 有一种的一种工作的,不可以**是一种工作,**是一种工作,但是一种工作,但是一种工作。 · (A) while of the control of the state

And the second of the second o

tion that we was for the male form of which will be a first for the contract of the first of the contract of t

1750 - 1997年 (1750年) - 1957年 (1750年) - 1957年 (1750年) - 1957年 (1750年) - 1957年 (1750年)

 (H_{i}) , $\zeta = 0$, which is an indicated and (H_{i}) and (H_{i}) and (H_{i}) and (H_{i}) and (H_{i}) iedogoja koj jednosti pre**istiko iz** postava po**raziona proprio preis** postava preistava. The set of the second of the sign of the enterty will be prefer to a second of the sec OF THE SEARCH COME AND SERVICE OF THE SEARCH COME. a bine a comparation of the property of the common testing of the

El Carrier e est reputation de la company de

K.Jacobs: On sequences of Toeplitz type

For the construction of almost periodic functions on the line Toeplitz used (Ann. 1928) a combinatorial device similar to the following

leading to the 0-1-sequence

01000101010001000100010101 ...

A general device of this kind is based on a sequence

$$\eta^{(1)}, \eta^{(2)}, \eta^{(3)}, \dots$$

of sequences of symbols 0,1,∞(="hole") which

1) are periodic, 2) begin with 0 and 1 and 3) contain ∞ (of course, infinitely often, then).

Construct a sequence $\eta_{(1)}, \eta_{(2)}, \ldots$ such that

$$\eta_{(1)} = \eta^{(1)}$$

 $\eta_{(n)} = \eta_{(n-1)}$ with $\eta^{(n)}$ filled into the "holes".

Clearly the $\eta_{(n)}$ are successive "completions" of each other, and finally all "holes" are stuffed such that in the limit an almost periodic sequence η containing only 0 and 1 is obtained. Let ρ_{∞} (ξ) the mean frequency of ∞ 's in the 0-1- ∞ -sequence ξ . Then

$$\rho_{\infty}(\eta_{(n)}) = \rho_{\infty}(\eta^{(1)}) \cdot \ldots \cdot \rho_{\infty}(\eta^{(n)})$$

We have the following

Theorem: If $\rho_{\infty}(\eta_{(n)}) \to 0$, then η is strictly ergodic, and the attached unique invariant measure m_{η} has pure point spectrum. In case $\rho_{\infty}(\eta_{(n)}) \not \to 0$, then one obtains easily examples for almost periodic, but not strictly ergodic sequences, e.g. one given by Oxtoby.

K.Jacobs: Riemannian dynamical systems

Let $\underline{\alpha}$ be compact metric and $\emptyset \ddagger \underline{\alpha} \in \underline{\alpha}$, m a finite measure in α , and T a m-preserving, m-almost everywhere continuous mapping $\alpha + \alpha$. Then the dynamical system (α, T, m) is called Riemannian.





A REMARK AND AND A STATE OF THE STATE OF THE

in value est no ha s**ignosti** editerio qui e a non totala tale integral de la compositorio della compositori

en de la composition La composition de la

i desperatores in a francis se en la companio de persona de la companio del companio del companio de la companio del companio del companio de la companio del c

the state of the s

A constant of the control of the contr

en de la composition La composition de la

The state of the s

order a degleration of the control of the control

in a substance of the second of the property of the second of the second

In such a system m-almost every point ω is "of permanent T-continuity", i.e. T is continuous on ω , ω T,.... Such a point is called

- 1) an almost periodic visitor, if
 - a) orb(w) carries m
 - b) for every neighbourhood U of ω there is some L > O such that $\{\omega^T^t,\ldots,\omega^T^{t+L-1}\}\cap U \neq \emptyset$ (t=0,1,...).
- 2) a regular visitor, if for every m-almost clopen set $F \subseteq \Omega$ and every $\epsilon > 0$ there is a $t_0 > 0$ such that $t \ge t_0$

implies
$$\left|\frac{1}{t}\sum_{u=0}^{t-1} 1_{F}(\omega T^{S+u}) - \frac{m(F)}{m(\Omega)}\right| < \epsilon$$
 (s = 0,1,...)

It is easily seen:

- A) The induced system on a m-almost clopen set Ω ' is again Riemannian and every $\omega \in \Omega$ ' visiting only the interior of Ω ', is still an almost periodic (resp. regular) visitor, if ω was so for the original system.
- B) Map Ω into $\underline{\hat{\Omega}} = \underline{\Omega} \times \underline{\Omega} \times \ldots$ by $\phi : \omega \rightarrow \hat{\omega} = (\omega, \omega T, \omega T^2, \ldots),$

let $\hat{m} = m\phi$, $\hat{T} = \text{shift.}$ Then ϕ carries an almost periodic visitor ω into an almost periodic $\hat{\omega}$, and an a.p. regular visitor ω into a strictly ergodic point $\hat{\omega}$.

Exerting first a suitable version of A) to a suitable circle rotation, a strictly ergodic system with pure point spectrum looses all its non-trivial eigenvalues, but there is still plenty of almost periodic visitors; thus in performing B) we obtain a weakly mixing strictly ergodic dynamical system.

Probably one can even put the system into finite-state shift space in this special case.

S.Kakutani: Examples of weakly but not strongly mixing transformations

Let (Y,B,μ) be the Lebesgue measure space on the unit interval Y = [0,1]. Let $Y = Y_1 \cup Y_2$ (disj.) be a partition of Y. Let $X = Y_1 \cup Y_2 \cup Y_2'$ be a two-story "skyscraper" built over Y with respect to the partition $Y = Y_1 \cup Y_2$ (disj.).

Let (X,B,μ) be the corresponding measure space on X (defined in an obvious way). Let ψ be a m.p.t. defined on (Y,B,μ) .





Fisee-® and thing as the second of the seco

THE CONTRACTOR OF THE SHOP IN A SECOND STATE OF

e de la composition La composition de la La composition de la

In a section of the control of the control of the control of the aggregation of the control of t

rando en la composición de la composición del composición de la co

rectivate of the control of the cont

Herbon First Level Carlot (# 19 decide 19 decide) Level Carlot (19 decide) Section (19

Herger in reference to the first of the second of the seco

-roteasnt pri je vin gret noa a televi nen in energi e i ee dasked de likin

Environment (included not expert to the property of the control of

and it is the contraction of the





Let ϕ be the m.p.t. defined on (X,B,μ) by using the method described in the preceding talk. It is possible to prove that ϕ is weakly but not strongly mixing in the following two cases: Case I: ψ is the transformation defined on Y by

$$\psi(y) = y + \frac{1}{2} \quad \text{if } 0 < y < \frac{1}{2} ;$$

$$\psi(y) = y - (1 - 2^{-n} - 2^{-(n+1)}) \quad \text{if } 1 - 2^{-n} < y < 1 - 2^{-(n+1)}, \ n = 1, 2, \dots;$$

$$Y_1 = \bigcup_{n=0}^{\infty} (1 - 2^{-2n}, 1 - 2^{-(2n+1)}), \ Y_2 = \bigcup_{n=0}^{\infty} (1 - 2^{-(2n+1)}, 1 - 2^{-(2n+2)}).$$

Case II: $\psi(y) = y + \alpha$, where α is a transcendental number, $\alpha = \sum_{n=1}^{\infty} 10^{-(2^n-1)}$; $Y_1 = (0,\beta)$, $Y_2 = (\beta,1)$, where β is a real number whose decimal expansion $\beta = \sum_{m=1}^{\infty} b_m 10^{-m}$ satisfies $b_m = 5$ if $m = 2^n-1$ for some n.

S.Kakutani: <u>Induced measure preserving transformations and</u> related topics

Let (X,B,μ) be an atomless measure space with $0<\mu(X)\le\infty$, and let ϕ be an ergodic measure preserving transformation (m.p.t.) defined on it. Let Y be a measurable subset of X with $\mu(Y)>0$, $\mu(X-Y)>0$. For almost all $y\in Y$, there exists a positive integer n=n(y) such that $\phi^i(y)\notin Y$, $i=1,\ldots,n-1$, and $\phi^n(y)\in Y$. Put $\psi(y)=\phi^{n(y)}(y)$ for a.e. $y\in Y$. Then ψ is an ergodic m.p.t. defined on $(Y,B,\mu).\psi$ is called the m.p.t. induced on Y by ϕ . If we put $Y_n=\{y|y\in Y,n(y)=n\},\ n=1,2,\ldots,\ then Y=\bigcup_{n=1}^\infty Y_n\ (\text{disjoint})\ \text{and}\ X=\bigcup_{n=1}^\infty \int_{i=0}^{n-1}\phi^i(Y_n)\ (\text{disj.}).$ (It is n=1 i=0 possible that $\mu(Y_n)=0$ for some n, and also that $\mu(Y_n)=0$ for all $n\geq n_0$ for some n_0).

Conversely. let (Y,B,μ) be a measure space with $0 < \mu(Y) \le \infty$, and let ψ be an ergodic m.p.t. defined on it. Let $Y = \bigcup_{n=1}^{\infty} Y_n \text{ (disj.) be a (finite or countably infinite) partition of Y. Construct a "skyscraper" over Y in such a way that there exist exactly n-1 floors <math>Y_n^{(1)}$, $Y_n^{(2)}$,... $Y_n^{(n-1)}$ over $Y_n^{(0)} = Y$, $n=1,2,\ldots$





Sonder Bas gried va (2.6,7) or Perlaboration and Free and the Part of the state BORRO CINCOLD ROBLES OF THE SECRET SCHOOL SECTION SCHOOL SECTION SECTIONS en de la companya de la co

on the finance of the second o Therefore Business and the second of the sec

and the first of the second of

DAS LEGGLETE GLEGGER, EN CHANGE EL EN LEGGLE DE LEGGLE DE GRAN DE LEGGLE DE LEGGLES DE LEGGLES DE LEGGLES DE L 至。"他是是是这个

1 is 1 = 2 (A) = 3 (A) The first teachers of the second of the seco O Pastae Office District

The second of th A the Court of the e de la marria de la composición de la

Edited on a moved was a construction of a decided on a section of the second The state of the s

Programme and the second secon

THE PROPERTY OF THE PARTY OF TH TOT UBILITY OF THE RESERVE OF THE SET WILLIAM SET OF THE SET OF TH

is a figure was the company of the c

Let x_n be a "vertical" mapping which maps $Y_n^{(i-1)}$ onto $Y_n^{(i)}$, i=1,...,n-1. Put $X = \bigcup_{n=1}^{\infty} \bigcup_{i=0}^{n-1} Y_n^{(i)}$ (disj.) and consider the measure space (X,B,μ) on X (B and μ are defined in an obvious way so that χ_n becomes a m.p.t. of $Y_n^{(i-1)}$ onto $Y_n^{(i)}$, i = 1, ..., n-1). Put $\phi(x) = \chi_n(x)$ if $x \in Y_n^{(i)}$ for some n and i $(0 \le i \le n-2)$ and $\phi(x) = \psi(\chi_n^{-(n-1)}(x))$ if $x \in Y_n^{(n-1)}$ for some n. Then ϕ is an ergodic m.p.t. defined on (X,B,μ) and it is easy to see that the ergodic m.p.t. defined on the skyscraper X over Y with respect to the partition $Y = \bigcup_{n=1}^{\infty} Y_n$ (disj.) and the base transformation ψ . This relation between ϕ and ψ is denoted by $\phi > \psi$. If we denote by [ol the class of all m.p.t. which are spatially isomorphic with ϕ , then we may define the relation $[\phi] > [\psi]$ to mean that $\exists \phi_0 \in [\phi], \exists \psi_0 \in [\psi]$ such that $\phi_0 > \psi_0$. This is obviously a transitive relation (i.e. $[\phi_1] > [\phi_2]$, $[\phi_2] > [\phi_3] \Rightarrow [\phi_1] > [\phi_3]$). Theorem 1: There exists ϕ_3 with $[\phi_3] > [\phi_1]$, $[\phi_3] > [\phi_2]$ if and only if there exists ϕ_4 with $[\phi_1] > [\phi_4]$ and $[\phi_2] > [\phi_4]$. We write $[\phi_1] \sim [\phi_2]$ (equivalent) if one and hence both of the conditions in theorem 1 are satisfied.

Theorem 2:

 $[\phi] \sim [\psi]$ if and only if the classes of flows built under a function over $\phi \in [\phi]$ and $\psi \in [\psi]$ are identical (by spatial isomorphism).

S.Kakutani: Ergodic measure preserving transformations defined on an infinite measure space

Let (X,B,μ) be the Lebesgue measure space on the real line X=R. The existence of an ergodic m.p.t. defined on (X,B,μ) is shown by using the method of skyscraper. We note that in this way we get all ergodic m.p.t. on (X,B,μ) by taking all $Y\in B$ with $0<\mu(Y)<\infty$ and all partitions $Y=\bigcup_{n=1}^\infty Y_n$. Let ϕ be an ergodic m.p.t. defined on (X,B,μ) with $\mu(X)=\infty$ and let $A\in B$, $0<\mu(A)<\infty$. Put $A_n=A_n(\phi)=\bigcup_{i=0}^{n-1}\phi^i(A)$, $n=1,2,\ldots$. Then $\mu(A_n)+\infty$, $\mu(A_n)/n+0$.





The second secon

Company to mediate the second of the second

Senilob anoldem to proportion with the following the Arthurst of the Arthurst

Theorem 1. For any ϕ , there exists a sequence $\{n_k\}$ of positive integers, $n_k < n_{k+1}$, $k = 1, 2, \ldots$, such that

lim inf $\mu(A_{\kappa}(\phi))/k > 0$ for any $A \in B$ with $\mu(A) > 0$.

On the other hand, from the skyscraper construction, it follows that there exists an ergodic m.p.t. ψ and a set A with $0 < \mu(A) < \infty$ such that $\lim_{k \to \infty} \mu(A_{n_k}(\psi)) = 0$. This shows:

Theorem 2. There exist infinitely many ergodic m.p.t. defined on the Lebesgue measure space (with X = R) no two of which are spatially isomorphic.

Various examples of ergodic m.p.t. with interesting numbertheoretical properties were discussed in this talk.

S.Kakutani: Spectral analysis of the Morse dynamical system

Let Ω = Π {+1,-1} be the set Ω of all two-sided infinite $n \in \mathbb{Z}$ sequences ω = { $\omega_n \mid n \in \mathbb{Z}$ } with ω_n = +1 or -1, $n \in \mathbb{Z}$. Ω is a totally disconnected compact metrizable space with respect to the usual product topology. Define the <u>shift</u> transformation σ on Ω by $(\sigma(\omega))_n = \omega_{n+1}$, $n \in \mathbb{Z}$ and the <u>involution</u> τ by $(\tau(\omega))_n = -\omega_n$, $n \in \mathbb{Z}$. Put

 $\xi_0 = +1$, $\xi_1 = -1$ and $\xi_{2n} = \xi_n$, $\xi_{2n+1} = -\xi_n$, n=1,2,...

Then we obtain the Morse sequence in which the usual 0 and 1 are replaced by +1 and -1.

Put $\xi_{-n} = \xi_{n-1}$, n=1,2,... and $\omega_0 = \{\xi_n | n \in Z\} \in \Omega$.

Consider the orbit closure $\Omega_0 = \overline{\text{orb}(\omega_0)} = \{\sigma^n(\omega_0) | n \in Z\}$. Then Ω_0 is invariant under σ and τ , and (Ω_0, σ) is a strictly ergodic dynamical system. This is called the Morse dynamical system.

Let μ be the unique normalized invariant measure on (Ω_0,σ) . μ is also τ -invariant. Let $L^2(\Omega_0) = L^2(\Omega_0,\mu)$ be the complex L^2 -space on Ω_0 with respect to μ . Let V_σ , V_τ be the unitary operators defined on $L^2(\Omega_0)$ by $V_\sigma f(\omega) = f(\sigma^{-1}(\omega))$, $V_\tau f(\omega) = f(\tau^{-1}(\omega))$.

A function $f \in L^2(\Omega_0)$ is called an <u>even</u> function if $V_\tau f = f$, an <u>odd</u> function if $V_\tau f = -f$. Denote by \mathcal{M}_e and \mathcal{M}_o the sets of all even and odd functions from $L^2(\Omega_0)$. Then, \mathcal{M}_e and \mathcal{M}_o are closed linear subspaces of $L^2(\Omega_0)$, mutually orthogonal, and together span





ordinalista de la julio pode la participación de seculo de la propertional de la propertional de la propertion De la contraction de

and the second of the second o

under 1915 de l'implication de la communication de la communication de la communication de la communication de Richard de la communication de Enterent de la communication de

and the structure of th

Horosame (1907) To the some and distribute the company of the control of the distribute of the control of the c

y disentant beathrophy in the second order. The least of the first of the contract of the second order o

Vifui de la companya del companya del companya de la companya del companya de la companya de la companya del companya de la companya del companya de la companya de la companya de la companya del companya de la compan

The specification of the speci

the space $L^2(\Omega_0)$. From the fact that σ and τ commute, if follows that both \mathcal{M}_e and \mathcal{M}_o are invariant under V_σ .

Theorem. V_σ has a pure point spectrum on \mathcal{M}_e (its eigenvalues are given by $\lambda = j 2^{-n}$, j, n=0,1,2,...).

V has a continuous singular spectrum on \mathcal{M}_o .

M.Keane: Generalized Morse sequences

Let b = $b_1 cdots b_m$ and c = $c_1 cdots c_n$ be sequences of zeros and ones (i.e. blocks). We define b × c = $b^c 1 b^c 2 cdots b^c n$, where b^o = b and b^1 = $1 - b_1, cdots, 1 - b_m$. Then the Morse sequence x = 0110100110010110... may be written as an infinite product of blocks: x = 01 cdots 01 cdots 01 cdots. Sequences of the form x = $b^1 cdots b^2$..., where each b^k is a block beginning with zero, are called recursive sequences. Recursive sequences are almost periodic and define in a natural way an "orbit" in the two-sided shift space on zeros and ones. We give necessary and sufficient conditions for the strict ergodicity of such recursive sequences (which are called generalized Morse sequences).

U.Krengel: On mixing in infinite measure spaces

Let (Ω, Ω, μ) be a σ -finite measure space. A sequence (A_n) of measurable sets is called remotely trivial, if the σ -algebra $R(A_n) = \bigcap_{k=1}^{\infty} B_k(A_n)$ is trivial, where $B_k(A_n)$ is generated by $A_k, A_{k+1}, \ldots (A_n)$ is called semi remotely trivial (s.r.t.) if every subsequence contains a remotely trivial subsequence. A measure preserving transformation T is called mixing if $(T^{-n}A)$ is s.r.t. for all A with $\mu(A) < \infty$, it is called completely mixing if $(T^{-n}A)$ is s.r.t for all $A \in \Omega$. Mixing in infinite measure spaces is equivalent with being of zero type. T is completely mixing iff

Examples: Markov shifts on a unilateral product space for null-recurrent, aperiodic ergodic Markov chains. For invertible transformations, however, complete mixing implies the existence of a





andled the common education of the formula and the company of the contraction of the cont

Discrete the course of the cou

SON THE COMPANY OF THE PARTY OF

The Company of the second control of the sec

Live To? Googe Coucum, F. W. Live, W. C. Do Howell, J. W. C. Line, XH.
 Hamed Note in the Section of the Coucum Section Secti



finite invariant measure. This negatively answers the problem posed by Mrs.Dowker at Oberwolfach in 1965. For mixing transformations an analogue of the theorem of Blum and Hanson on mean convergence for expressions

$$\frac{1}{n} \sum_{1}^{n} T^{k_{i}} f (k_{1} < k_{2} < ...)$$

can be proved. For completely mixing transformations we have a theorem which generalizes and strengthens a theorem of Orey on the convergence of $\sum\limits_{k}|p_{i_1,k}^{(n)}-p_{i_2,k}^{(n)}|$ to zero. The work was done jointly with Sucheston.

W.Krieger: On non-singular transformations of a measure space

We consider a Lebesgue measure space (M,B,m). By an automorphism of (M,B,m) we mean a B-measurable transformation of (M,B,m) that together with its inverse is non-singular with respect to m. We study an equivalence relation between these automorphisms which we call the weak equivalence. Two automorphisms S and T are weakly equivalent if there is an automorphism U such that for almost all $x \in M$, U maps the S-orbit of x onto the T-orbit of Ux. Ergodicity, the existence of a finite invariant measure resp. of a σ -finite invariant measure as well as the non-existence of such measures are invariants of weak equivalence. We solve the problem of weak equivalence for a class of automorphisms that comprises all ergodic automorphisms that admit a σ -finite invariant measure and also certain ergodic automorphisms that do not admit such a measure.

D.Maharam: On iterates of positive operators

An abstract measure space consists of a space S, a Borel field \mathcal{L}_S of subsets of S, and a σ -ideal $\eta_S \in \mathcal{L}_S$ such that \mathcal{L}_S / η_S satisfies the countable chain condition. We denote by \mathcal{F}_S^+ the family of all extended real nonnegative "measurable" functions on S [i.e. functions f such that $(f \geq \alpha) \in \mathcal{L}_S$ for all real α .], by \mathcal{F}_S , $\{f \mid (f \dagger 0) \in \eta_S\}$, and by \mathcal{F}_S^+ , \mathcal{F}_S / η_S . Let $(\mathcal{T}, \mathcal{M}, \mu)$ be a measure space (with σ -finite Lebesgue measure μ): We define the "standard product" $(R, \mathcal{L}_R, \eta_R) = (S, \mathcal{L}_S, \eta_S) \times (\mathcal{T}, \mathcal{M}, \mu)$ or





Her Johnson Ber Brothstein Steiner der Steine der Steine der Steine Steine der Steine der Steine Steine

and the companies of the companies of the second of the companies of the c

metalgromedia, no vii. (in, 5.6) concert various above and a respect to the discussion of the discussion of the concert of the concert of the discussion of the converted and the converted and the discussion of the discussion of the converted and the converted and the discussion of the converted and the converted and

and the control of the control of the control of

From Euler specific of the top of the control of t





[R \times T, for short] as follows: R = S \times T, \mathcal{L}_R = Borel field generated by the sets H × K , H $\in \mathcal{L}_S$, K $\in \mathcal{H}$. If f $\in \mathcal{F}_R^+$, then for each fixed $s \in S$, f(s,t) is measurable in t, and the function mf defined on S by mf(s) = $\int_{\mathbb{T}}$ f(s,t)dt is $\mathcal{L}_{\mathbb{S}}$ -measurable. Now define $n_R = \{A \mid A \in \mathcal{L}_R, m_{\chi_A} \in \mathcal{J}_S\}$, where χ_A is the characteristic function of A. (R, \mathcal{L}_{R} , n_{R}) is an abstract measure space, and m induces (by reducing modulo null functions) a linear map M called the standard integral from F_R^+ onto F_S^+ . In particular, let $(S^*, \mathcal{L}, \eta^*)$ be an abstract measure space and ϕ^* a linear map of $F_{S^*}^+$ onto itself such that (i) (S*, \mathcal{L}^* , η^*) is the standard product $(S_1, \mathcal{L}_{1^{n_1}}) \times (T_1, \mathcal{M}_{1^{n_1}})$, where T_1 is the measure-theoretic product of an arbitrary number of copies of the unit interval; (ii) \exists "measurable" 1-1 point map $\xi: S^* \to S_1$ [i.e., ξ maps \mathcal{L}^* onto \mathcal{L}_1 and η^* onto η_1] such that for $f^* \in F_{S^*}^+$, $\phi^* f^* = \xi^{-1} M_1 f^*$, where M_1 is the standard integral of F_{S^*} onto F_{S_1} . By an easy induction we now define (S_n, \mathcal{X}_n , η_n) and (T_n, \mathcal{M}_n , μ_n) (n=1,2,...) so that (i) \mathcal{L}_{n+1} , η_{n+1} , \mathcal{M}_{n+1} are the images under ξ of \mathcal{L}_n , η_n , \mathcal{M}_n , and ξ is measure-preserving between \mathcal{M}_n and \mathcal{M}_{n+1} ; (ii) S_n is the standard product $S_{n+1} \times T_{n+1}$, and so S^* is the standard product $S_n \times T_{n_1}^*$ where T_n^* is the measure theoretic product $T_1 \times ... \times T_n$. It follows that for $f^* \in F^{*+}$, and for each n > 0, $\phi^{*n}f^{*} = \xi^{-n}M_{n}f^{*}$ where M_{n} is the standard integral from $F_{S}^{+}*$ to $F_{S_n}^*$, i.e. for $f \in \mathcal{F}^{*+}$, $s^* \in S^*$, we have modulo null sets, $\Phi^{*n}f^{*}(S^{*}) = \int_{T_{1}} \dots \int_{T_{n}} f^{*}(t'_{1}, \dots, t'_{n}, \xi^{-n}(S^{*}))(dt_{1}) \dots (dt_{n}).$

Now let (S, \mathcal{K}, n) be an arbitrary abstract measure space and Φ a map of $F_S^+ \to F_S^+$ such that (i) $f_n \in F_S^+$, $\alpha_n \ge 0$ $(n=1,2,\ldots) \Rightarrow \Phi(\Sigma \alpha_n f_n) = \Sigma \alpha_n \Phi f_n$; (ii) $\exists f_n \in F_S^+$ such that $\Sigma f_n = 1$ a.e. and $\Phi f_n < \infty$ a.e. Then $\exists (S^+, \mathcal{K}^+, n^+)$ and $\Phi f_n < \infty$ a.e. Then $\exists (S^+, \mathcal{K}^+, n^+)$ and $\Phi f_n < \infty$ a.e. Then $\exists (S^+, \mathcal{K}^+, n^+)$ and $\Phi f_n < \infty$ as in the preceding paragraph, and a function $K^+ \in F_S^+$ such that S can be "imbedded" in S^+ in such a way that, for each $f \in F_S^+$ considered as an element of F_S^+ , $\Phi^n f = \Phi^{*n}(K^* f)$.





· · · · · ·

- Miles and Element in the Marin Community of the Marin Community House the series was a series of the second of the series the second state of the second u du situativa a della seggio di seggio d La constanti di administrati di seggio d og karar momenta og til skrift med som en gjen skrijen i skrive som skrive som en karar. grains were also the control of the respectful to the control of the con the area to will be a finite of the first factors and the contributions and the first field of the contributions and the contributions are also because of the contributions and the contributions are also because of the contributions and the contributions are also because of the contributions are a ារីជាអ្នកស្និត្តស្នាក់ក្នុងស្នាស់ សម្រេច ស្រែក ស្ ស្រីជាអនុសាធិនត្រូវបានស្នាស់ ស្រែក ស្រ ស្រីក្រុម s and the group of the second Constitution (The Conference of the particle of the Conference of the grant of the second control of the second of the secon n de de la composition della c g p<mark>ods sin 75 ob fo</mark>r elleg for glass to the form of the second side energy of the cooper of the energy of the en go a la Maria Mala de la companya d La companya de la companya del companya de la companya del companya de la c gegen in a section of

A REPORT OF THE PROPERTY OF TH

n terméternit non anno l'internation de l'éposition de l'éposition de l'éposition de la communité de la commun Communité de la communité de l Communité de la communité de l ["S is imbedded in S*" means \exists isomorphism θ of $\mathcal{L}|$ η into the measurable/null sets of some measurable set $A \in \mathcal{L}^*$ such that for all $f \in F_S^+$, $\theta \Phi f = \Phi^* \theta f$]. Thus, the iterates of any operator satisfying (i) and (ii) above can be given a concrete representation using the integral formula of the last paragraph.

G.Maruyama: The canonical version of a flow

Let (X,B_{μ},μ) be a topological probability space, where X is a Hausdorff space, B_{μ} the μ -completion of the Borel field on X, and μ a Radon measure on B_{μ} . T_t , $t \in T = (-\infty,\infty)$ is a canonical flow on X, iff it is a group of 1-1 measure preserving mappings of X onto itself, and the map $(t,x) \in T \times X \to T_t \times X$ is continuous. Under mild conditions, a measurable flow on an arbitrary measure space is isomorphic to a canonical flow restricted to a set of outer measure 1. In general, any continuous flow is algebraisomorphic to a canonical flow.

A.Nijst: Some remarks on conditional entropy

We discuss an integral representation of conditional entropy which generalizes a well-known result of this sort and we show that this representation theorem implies that additivity of conditional entropy and conditional independence of σ -fiels are equivalent.

Also we obtain by this representation theorem with the aid of a lemma of Sinai a simple proof of the decomposition theorem of the Kolmogorov-Sinai invariant.

W. Parry: Compact abelian group extensions of dynamical systems

Let X be compact metric, S a homeomorphism and let G be a compact abelian group acting continuously on X such that G commutes with S. If G acts freely we say (X,S) is a G-extension of (Y,T) where Y = X/G and T is the homeomorphism induced by S on Y. Conditions are given for (X,S) to be minimal or uniquely ergodic when (Y,T) enjoys the same property. In particular we show that a G extension is "likely" to lift the property considered (when X is connected) in the sense that $\{g\colon gS \text{ is uniquely ergodic } \{minimal\}\}$ contains a dense G_{δ} given that T is uniquely ergodic



envolving per te volument of the second of t

The second second of the second secon

Reprise Charles in the analysis of the property

Vydredna Eseraktricano a reclusión i como a recentrión e como de co

and a juga to take a second compared or extrementation of the compared of the

Designation of the Company of the Co

Josephson Josephson of the second energy and the second energy and





["S is imbedded in S*" means \exists isomorphism 0 of $\mathcal{L}|$ n into the measurable/null sets of some measurable set $A \in \mathcal{L}^*$ such that for all $f \in F_S^+$, $\theta \circ f = \Phi^* \circ f$. Thus, the iterates of any operator satisfying (i) and (ii) above can be given a concrete representation using the integral formula of the last paragraph.

G.Maruyama: The canonical version of a flow

Let (X,B_{μ},μ) be a topological probability space, where X is a Hausdorff space, B_{μ} the μ -completion of the Borel field on X, and μ a Radon measure on B_{μ} . T_t , $t\in T=(-\infty,\infty)$ is a canonical flow on X, iff it is a group of 1-1 measure preserving mappings of X onto itself, and the map $(t,x)\in T_{\times}X\to T_tx\in X$ is continuous. Under mild conditions, a measurable flow on an arbitrary measure space is isomorphic to a canonical flow restricted to a set of outer measure 1. In general, any continuous flow is algebraisomorphic to a canonical flow.

A.Nijst: Some remarks on conditional entropy

We discuss an integral representation of conditional entropy which generalizes a well-known result of this sort and we show that this representation theorem implies that additivity of conditional entropy and conditional independence of σ -fiels are equivalent.

Also we obtain by this representation theorem with the aid of a lemma of Sinai a simple proof of the decomposition theorem of the Kolmogorov-Sinai invariant.

W. Parry: Compact abelian group extensions of dynamical systems

Let X be compact metric, S a homeomorphism and let G be a compact abelian group acting continuously on X such that G commutes with S. If G acts freely we say (X,S) is a G-extension of (Y,T) where Y = X/G and T is the homeomorphism induced by S on Y. Conditions are given for (X,S) to be minimal or uniquely ergodic when (Y,T) enjoys the same property. In particular we show that a G extension is "likely" to lift the property considered (when X is connected) in the sense that $\{g\colon gS \text{ is uniquely ergodic } \{minimal\}\}$ contains a dense G_{δ} given that T is uniquely ergodic





Her over the thin the composition in the set to select the selection of th

our formation and the first terms of the contract of the contr

And the state of the communication of the control o

Yaginda St. Especification of the CHIEF Control of the Chief

Vigorator Laborita in the statement of the interest of the problem of the statement of t

in the Block and middle some code as a consideration of the code and t

ranger and the first for an electromagnetic particles of the second second of the seco

The segment of the Control of the Co





[minimal] . This suggests the definition of a stable G-extension: when gS is homeomorphic to S for all g in an open set. Hence stable extensions always lift the required property. This is applied to affine transformations of certain three dimensional manifolds called nilmanifolds.

K.Post: Some elementary investigations on measurable transformations

a) The transformation $Tx \equiv 2x \mod 1$ on the unit interval with measure μ defined for all Borel sets E by

$$\mu(E) = \int_{E} \frac{1}{x} d\lambda(x),$$

 λ being Lebesgue-measure, has the property

$$0 < \mu(E) < \infty \implies \mu(E) < \mu(T^{-1}E) < 2\mu(E)$$
.

This example provides an answer on a question by G.Helmberg (Tagung über Ergodentheorie, 1965).

b) If T is a measurable transformation on an arbitrary measure space (X,\mathcal{R},μ) satisfying $\mu(T^{-1}A)\leq\mu(A)$ for all $A\in\mathcal{R}$, then any $E\in\mathcal{R}$, for which $E\subset\bigcup_{n=1}^\infty T^{-n}E$ (μ) must have the property $\mu(T^{-1}E)=\mu(E)$. This result, due to F.Simons, provides a short proof of the implication

T conservative
$$\mu(T^{-1}A) \leq \mu(A) \text{ for all } A \in \mathcal{R} \ \} \implies \mu(T^{-1}A) = \mu(A) \text{ for all } A \in \mathcal{R} \ .$$

cf. G.Helmberg, über konservative Transformationen Math.Ann. 165, 44-61 (1966)

L.Sucheston: On convergence of information in spaces with infinite invariant measure, abstract of talk

We extend the ergodic theorems of information theory ($\underline{Shannon-MacMillian-Breiman}$ theorems) to spaces with an infinite invariant measure. An L_1 difference theorem and pointwise ratio theorem are proved, for the information of spreading partitions. For the validity of the theorems it is assumed that the supremum







 f^* of the conditional information given the increasing "past" is integrable. Simple necessary and sufficient conditions for the integrability of f^* are obtained in special cases: If the initial partition is composed of one state of a null-recurrent Markov chain, then f^* is integrable if and only if the partition of this state according to the first return times has finite entropy. (Paper written in collaboration with <u>E.M.Klimko</u>, to appear in Z.Wahrscheinlichkeitstheorie vol.9 (1968).)

H.Scheller: A short proof of Abramov's theorem on the presentation of entropy for induced transformations

Given a normed dynamical system (Ω,B,m,T) -T not necessarily invertible - and $E \in B$, let be $r_E(a)$ the first recurrence time (equal to 0 on E^c); let $\mathcal{R}(E)$ resp. $\sigma(E)$ be the σ -fields generated by r_E resp. E and let T_E be the induced transformation defined by T^E .

Using a lemma reducing the set of σ -fields needed for the computation of $\hat{H}(T)$ (valid for sweep out sets) and using the relations $H(T_E) = H(T_{ET}^{-1})$, $H(\mathcal{R}(E)) \leq 2H(\sigma(E))$, we derive from the inclusions

 $\bigvee_{t=1}^{\infty} B_{o} T_{E}^{-t} \subseteq \bigvee_{t=1}^{\infty} B_{o} T^{-t} \subseteq \mathcal{R}(E) \vee \bigvee_{t=1}^{\infty} B_{o} T_{E}^{-t} \text{ (valid within E and all }$

σ-fields $B_o \supseteq \Re(E)$) the formula (1) $\hat{H}(T_E)$ - $2H(\sigma(E)) \le \hat{H}(T) \le H(T_E)$ + $H(\sigma(E))$. Applying (1) to arbitrarily small sweep out subsets of E - observe $(T_E)_F = T_F$ - we obtain a generalized version of Abramov's theorem: $\hat{H}(T_E) = \hat{H}(T)$ for all sweep out sets.

F.H.Simons: Sweep-out sets and strong generators

Let (X,\mathcal{R},μ) be a σ -finite measure space, and let T be a non-singular measurable transformation in (X,\mathcal{R},μ) . T is said to be periodic on a set $A\in\mathcal{R}$ if there exists a natural number n such that for all measurable $B\subseteq A$ we have $T^{-n}B\supseteq B[\mu]$; the least number n satisfying this condition is the period of T on A. T is said to be aperiodic, if the only sets on which T is periodic are μ -null sets.





Tipoli galasta a si tra ven jerilijango i topoli ili septimi ili s

. Ordern berede stieren sie die 1900 van die 1 Die 1900 van die 1

viliases como formito e la como de la como d

Side of the Address of the Control of the Contro

The size G matrix A and A and A and A and A are some size A and A and A are some size A and A are s

CARROLLES, COLONDO BEEN OF CARROLS



As a generalization of theorems of Parry we can prove the following:

Theorem 1: The following statements are equivalent:

- a) T is aperiodic
- b) There exists a countable infinite sweep-out set partition of X.

Theorem 2: If moreover μ is non-atomic, then the following statements are equivalent.

- a) There exists a strong generator for ${\mathcal R}$
- b) \Re is countably generated; there exists a partition ζ_0 such that $T^{-1}\Re \sim \zeta_0 = \Re \{\mu\}$; T is aperiodic.

H.Totoki: A special flow which is a K-flow

Let T be a Bernoulli shift on (Ω, \mathcal{L}, P) where $(\Omega, \mathcal{L}, P) = \prod_{-\infty}^{\infty} (E, \mathcal{L}_E, P_E)$ with non-trivial probability space (E, \mathcal{L}_E, P_E) . Let θ be an integrable function on Ω such that $\inf_{\omega} \theta(\omega) > 0$. Construct the special flow $\{S_t\} = (T, \theta)$. If $\theta(\omega)$ is a bounded function of the O-th coordinate $X_O(\omega)$ of ω , $\theta(\omega) = \hat{\theta}(X_O(\omega))$, then there are the following two possibilities:

- (1) $\{S_t\}$ has non-constant eigenfunctions (in the case when the distribution of $\Theta(\omega)$ is of lattice type), or
- (2) $\{S_t\}$ is a K-flow (in the case when the distribution of $\Theta(\omega)$ is of non-lattice type).

Problems:

1. K.A.Post

Given any sequence of objects, (some of which may coincide) does there exist a uniformly distributed sequence of numbers on the unit interval with the same coincidence - pattern? Necessary conditions are obviously

- (i) the asymptotic density of all objects is zero.
- (ii) for any ϵ > 0 there are only finitely many objects, the frequency quotient of which attains values > ϵ .





en de la verge de la company de la la company de la co En 1888 de la company de l

and the sine of the sounder on the second of the first of the first of the second of t

non magnetiagos estas Darje e am timpo esta un liberal de la comencia de la comencia de la comencia de la come La comencia de la co

one signatura de la companya della c

100 modelni i kantin i kantin i kaliki in katifi modelni i kantin i kan

which had a polymorphic and a standard

(a) The mineral content of the conte

neoficial property of the company of the company of the first section of the company of the comp

(4) Fig. 1. Application of the control of the co





2. L.Sucheston

Let X_0, X_1, \ldots be a null-recurrent Markov chain and set

$$f_{kk}^{(n)} = P(X_1 + k, X_2 + k, ..., X_{n-1} + k, X_n = k | X_0 = k).$$

Is the relation

$$-\sum_{n} f_{kk}^{(n)} \log f_{kk}^{(n)} < \infty$$

a class property of the chain?

3. L.Sucheston

There exists an infinite measure space analogue of the Shannon-Mc-Millan-Breiman theorem (cf. E.M.Klimko and L.Sucheston, Z.Wahr-scheinlichkeitstheorie, vol.9, 1968). Can this be applied to obtain an analogue of the Kolmogorov-Sinai theorem, for an appropriately defined concept of entropy?

4. L. Sucheston

Call the following statement an "approximate" ergodic theorem: Let $0 \le f_i + f$, $0 \le g_i + g > 0$, let T be an operator as e.g. in the Chacon-Ornstein theorem, and assume

(*) sup
$$f_i \in L_1$$
, sup $g_i \in L_1$.

Then $\sum_{0}^{n-1} T^{i} f_{i} / \sum_{0}^{n-1} T^{i} g_{i}$ converges a.e. Given f_{i} such that sup $f_{i} \notin L_{1}$, produce g_{i} and T such that $\sum_{0}^{n-1} T^{i} f_{i} / \sum_{0}^{n-1} T^{i} g_{i}$ diverge

a.e. To avoid trivial counterexamples, assume that the measure space is sufficiently rich to support a sequence of independent functions (cf. also Blackwell and Dubins, Illinois J.Math.1, 508-514, 1963).

5. L. Sucheston

Extend the Blum-Hanson mean ergodic theorem for subsequences to "mixing" operators on uniformly convex Banach spaces. Recall that for such spaces X Kakutani (Tohoku Math.J.) proved that $x_n \in X$, $\frac{1}{n} \sum_{i=0}^{n-1} x_i + \text{weakly, implies } \frac{1}{n} \sum_{i=0}^{n-1} x_k + \text{converges strongly for a subsequence } x_k$





antenation and the second second second of the second of t - Trainey is grown in the company of

The model made and the electronic and the late the late the control of the eight of

The second displaying the second of the seco

The first property of the second of the seco

The measure that the measure is a superior that the measure

nadacqəbiti tə ilin iştiqəri in türk bir dəlir yüylişilili in in arasını

THE STATE OF THE S

A STANDER

6. S. Kakutani

Can one always build a skyscraper over a given transformation so that the resulting transformation is weakly but not strongly mixing?

7. S. Kakutani, v. Neumann

Let
$$b(n) = \exp \left(2\pi i \left\{\frac{\varepsilon_1}{p_1} + \frac{\varepsilon_2}{p_2} + \dots + \frac{\varepsilon_k}{p_k}\right\}\right),$$

where
$$0 < p_1 < p_2 < \dots > \infty$$
 is fixed, and $n = \epsilon_1 + \epsilon_2 p_1 + \epsilon_3 p_1 p_2 + \dots + \epsilon_k p_1 \dots p_{k-1}, 0 \le \epsilon_i < p_i$.

Is orb(b) strictly ergodic? Does it have continuous spectrum?

8. W.Parry

Determine the relation between the spectrum of a given transformation and that of its group extensions.

9. R.Adler

If T and T' are (0,1)-matrices with common largest eigenvalue, are the corresponding shifts σ and σ' (with their respective measures of maximal entropy) conjugate?

10. K. Jacobs

Does there exist an invariant measure on Lip (1,1) which gives a K-flow?

11.D.Stone

Determine an invariant for non-singular conjugacy (i.e.T and T' are non-singularly conjugate if \exists S non-singular but not necessarily measure preserving such that TS = ST').

12.Ch.Grillenberger

Calculate the entropy of an a.p. mean Markov measure.

13.M.Keane

Can a strictly ergodic system have infinite entropy?

14.U.Krengel

If T is a measure preserving transformation in a σ -finite measure space, does there always exist a set E such that the return partition of E for T has finite entropy. By return partition we mean the partition of E into the sets $R_k = \{\omega : \omega \text{ returns to E at time } k \text{ for the first time}\}$.





· 性軟體實行政學數學物質和包含學的學術。 (1985年) · 1985年(1985年) · 1985年(1985年) · 1985年(1985年) · 1985年)

general Engineer San Landing Control

· 医囊膜管肠炎 等的,所以为此,自然为其实,或所谓,不能而为,以为其。 e catalogue d'attention et et partir à la company d'encept de

CONNECTED TO SECTION OF CONTRACT OF THE SECTION OF

HARRICA COMPLETE A COLLEGE OF A FOLLOWING CONTROL OF A COLLEGE OF A CO

Lander Bright Control HIR there are the control of the feet of the control of the contro

ARREST SE CONTRACTO CONTRACTO 为建筑工作。在1962年中的1967年中,1967年_{,19}67年,1967年(1967年)。

 $(\Phi_{\mathcal{A}}(\mathbf{x},\mathbf{y})) = \frac{1}{2} (\mathbf{x} + \mathbf{y})$

en de la composition La composition de la and the sum arms on the second

The property of the property o