MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1968

Ergodentheorie

4.8. bis 10.8.1968

Leiter der Tagung war Prof.K.JACOBS (Erlangen). Es war von vornherein beschlossen, nur wenigen Gästen Vorträge einzuräumen, diese aber zu längeren Serien auszugestalten.
So hielt S.Kakutani (Yale University) mehrere Referate über veröffentlichte und unveröffentlichte Untersuchungen aus seinem Arbeitskreis. Man folgte damit posiviten Erfahrungen, die auf der Kombinatorik-Tagung 1967 -damals hatte es nur 4 Vortragende mit jeweils 4 Stunden Vortragszeit gegeben- gemacht worden waren.

Dies war die 2.Oberwolfacher Ergodentagung und es wurde angeregt, eine weitere im Abstand von etwa 3 Jahren folgen zu lassen. Nachdem es auch diesmal nicht gelungen war, den Besuch einer sowjetischen Delegation zu erwirken, kam der Gedanke auf, die sowjetischen Kollegen zu bitten, die nächste Tagung in ihrem eigenen Land auszurichten. Dies hat sich bislang als erfolglos erwiesen. Im Augenblick ist eine von der Université de Rennes auszurichtende Ergodentagung unmittelbar vor dem Internationalen Mathematiker-Kongress in Nizza 1970 vorgesehen.

Die bei dieser Oberwolfacher Tagung anwesenden Gäste bildeten einen Teil der sich gegenwärtig stark vergrößernden Ergodiker-Familie. Eine Gesamttagung wäre bereits ein Mammutunternehmen. Neben einem solchen werden kleine Tagungen in Oberwolfach sicher ihren eigenen Rang behaupten.

Teilnehmer

R.Adler, Yorktown Heights/USA
M.A.Akcoglu, Toronto/Canada
T.Ando, Tübingen u.Sapporo/Jap.
G.Bray, Rennes/Frankr.
A.Brunel, Rennes/Frankr.
D.L.Burkholder, Urbana/USA
N.Dinculeanu, Bukarest/Rum.
H.Dinges, Frankfurt/Main

Gefördert durch DFG Deutsche Forschungsgemeinschaft

F.Eicker, Freiburg/Brsg.
Ch.Grillenberger, Erlangen
D.Hanson, Columbia/USA
G.Helmberg, Eindhoven/Holl.
E.Hopf, Bloomington/USA
K.Jacobs, Erlangen
A.Jonescu-Tulcea, Evanston/USA
S.Kakutani, New Haven/USA
M.Keane, Erlangen
U.Krengel, Erlangen
W.Krieger, München
G.Maruyama, Tokyo/Japan
J.Neveu, Paris/Frankr.
A.Nijst, Eindhoven/Holl.
W.Parry, Coventry/Engl.
K.Post, Eindhoven/Holl.
H.Scheller, Erlangen
F.Simons, Eindhoven/Holl.
D.Stone, Rochester/USA
L.Sucheston, Columbus/USA
R.Theodorescu, Bukarest/Rum.
H.Totoki, Kyoto/Japan

R.Adler: Isomorphisms of Markov shifts (in collaboration with B.Weiss)

Let $A = \{1,..,N\}$ be an alphabet of symbols and $T = (t_{ij})$ an $N \times N$ matrix of zeros and ones. Let $E(T)$ denote the space of two-sided infinite sequences $\xi = (\dots\xi_{-1},\xi_0, \xi_1\dots)$ where $\xi_i \in A$ and $t_{\xi_n,\xi_{n+1}} = 1$ for all n; and let σ denote the shift transformation on $E(T)$. The family $\mathcal{E}$ of measurable subsets of $E(T)$ is given by $\mathcal{E} = B(\bigcup_{-\infty}^{\infty} \sigma^n\alpha)$ where α is the partition of $E(T)$ into sets $\{\xi | \xi_0 = i\}$, $i = 1,\dots,N$. Assume further that T is irreducible; i.e. for every $i,j \in A$ there exists n such that $t_{ij}^{(n)} > 0$. Let λ_T be largest positive characteristic value of T with x,y positive column and row characteristic vectors associated with λ_T, normalized so that $\Sigma x_i y_i = 1$. The vector $\pi = (x_1y_1,\dots,x_Ny_N)$ and the matrix $P = (P_{ij})$ where $P_{ij} = t_{ij}x_j/\lambda x_i$ define a Markov measure μ on $\mathcal{E}$ for which $h_\mu(\sigma) = \log \lambda_T$. This is the largest value of entropy the shift on $E(T)$ can have for any σ-invariant measure and μ is the only measure giving this value. The following conjecture was examined: $\lambda_T=\lambda_{T'} \Rightarrow \sigma$ is metrically conjugate to σ'. Partial results were obtained for $\lambda_T = 2$. The following example was worked:

$$T = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \quad \text{and} \quad T' = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

The associated shifts were found to be conjugate.

M.A.Akcoglu: Identification of the ratio ergodic limits for the non conservative transformations

Let (X,F,μ) be a σ-finite measure space and T a positive linear contraction on $L_1(X,F,\mu)$. It is known that if T is a conservative transformation then it defines a sub σ-field I of F, consisting of the invariant subsets of X, and the identification of the ratio ergodic limits can be done in terms of I. If T is not necessarily conservative then it is shown that the σ-field I can be replaced by a field Σ. The members of Σ are "asymptotically invariant", in a natural sense, and the bounded ratio ergodic limits can be approximated by Σ-simple functions. Using this representation one obtains the identification of the limit in a way similar to the conservative case.
The details of this work, which was done jointly with R.W.Sharpe, will appear in the Transactions of the American Mathematical Society in 1968.

T.Ando: Invariant measures of a positive contraction in C(X)

We consider a compact stonian space X, i.e. the Banach lattice C(X) is conditionally complete, and a positive contraction T (in C(X)) with T1 = 1.
Theorem 1: The number of different ergodic measures with the same support as a given minimal-ergodic measure is either 1 or ∞.

A measure ϕ (resp. the operator T) is called σ-additive, if $f_n \downarrow 0$ (in order sense) implies $\inf |\phi|(f_n) = 0$ (resp. $Tf_n \downarrow 0$).

Theorem 2: Let T be σ-additive. The following assertions are equivalent:
(a) Every invariant measure is σ-additive.
(b) $n^{-1} \sum_1^n T^j f$ converges in norm for every f, and the subspace

$\{g : Tg = g\}$ is finite dimensional.

Theorem 3: Let X admit a strictly positive, σ-additive measure, and let T be σ-additive. There exists no (non-trivial) σ-additive invariant measure, if and only if for each $0 < f$ there exists $0 < g \leq f$ with order limit $n^{-1} \sum_1^n T^j g = 0$.

G.Bray: About a theorem of mean ergodic convergence

Let σ be a locally compact connected abelian semi-group satisfying certain conditions such that we can imbed σ in a group $G = H \times \mathbb{R}^p$ where H is a compact abelian connected group. Let (S,Σ,μ) be a probability space, such that L^2 is separable, $(T_x)_{x \in \sigma}$ a semi-group of linear continuous operators which operates on each L^p, $1 \leq p < \infty$ with T^1: $\|T_x\|_p \leq 1$ $\forall p$ $1 \leq p < \infty$;
T^2 : $\forall f,g \in L^2$ $x \longrightarrow \langle T_x f,g\rangle$ is measurable; T^3;$T_x T_y = T_y T_x = T_{xy}$ and $T_x T_y^* = T_y^* T_x$ $\forall x \in \sigma$ if we consider the T_x as operations on L^2.
Let U be the W^*-algebra generated by the $(T_x)_{x \in \sigma}$, K the hyper-stonian spectrum of U $\phi_x \in C(K)$ the function corresponding to T_x in the Gelfand isomorphism, P the spectral measure of U .
Denote by E' the clopen set which differs from
$E = \{M | M \in K, \phi_x(M) = 1$ for almost every all $x \in \sigma\}$ by a set of P-measure zero. The main result is: We can construct a directed increasing family of subsets of σ: $(B_j)_{j \in J}$ such that

$$\lim_{j \in J} \frac{1}{\lambda(B_j)} \int_{B_j} T_x f d\lambda(x) = P(E')(f) \quad \forall f \in L^p$$ (convergence in the L^p norm sense) where P is a projection operator in L^p.

A.Brunel et M.Keane: Théorêmes ergodiques pour une suite de puissances d'une transformation

Soit (Ω,F,μ) un espace probalilisé, T une transformation: $\Omega \to \Omega$ conservant la mesure. On désigne aussi l'extension de T par la même lettre: $Tf = f \circ T$. Nous avons étudié les moyennes de Cesaro $\frac{1}{n} \sum_{i=1}^{n} T^{k_i} f$ pour $f \in L^1(\Omega,F,\mu)$ et une suite croissante d'entiers $k_1 < k_2 < \ldots$ et avons obtenu les résultats suivants:

1) Pour certaines suites que nous avons appelées suites uniformes:

Théorème 1. Les conditions suivantes sont équivalentes:

1) T est faiblement mélangeante.

2) $\frac{1}{n} \sum_{i=1}^{n} T^{k_i} f \xrightarrow[n\to\infty]{} \int f d\mu$ (p.p.) pour tout $f \in L^1$ et toute suite uniforme $(k_i)_{i=1,2,\ldots}$.

[illegible] semi-groups [illegible]

Let G be a [illegible] ordered abelian semi-group [illegible] satisfying certain conditions such [illegible] can imbed [illegible] in a group [illegible] R^p [illegible] compact [illegible] ordered group. [illegible] separability [illegible] is separable [illegible] group of linear [illegible] $1 \leq p <$ [illegible]

[illegible] measurable [illegible] consider [illegible]

Let G be the [illegible] [illegible]

spectral measure of [illegible] corresponding [illegible] the spectral measure of [illegible] which differs from [illegible] over all [illegible] by a set [illegible]

[illegible] operator [illegible]

[illegible] ergodiques pour une suite [illegible] transformation

Soit [illegible] un espace probabilisé, T une transformation [illegible] On désigne [illegible] La [illegible] Ceci [illegible] pour [illegible] nous avons obtenu les résultats [illegible] nous avons appelées suites [illegible] équivalentes [illegible]

Ce résultat est un corollaire d'un théorème plus général qui affirme que pour une transformation T conservant la mesure:

<u>Théorème 2.</u> $\frac{1}{n} \sum_{i=1}^{n} T^{k_i} f \xrightarrow[n\to\infty]{} \bar{f}$ (p.p.)

pout tout $f \in L^1$ et toute suite uniforme $(k_i)_{i=1,2,\ldots}$

2) Pour des suites quelconques d'entiers nous avons établi la caractérisation suivante des transformations fortement mélangeantes:

<u>Théorème 3.</u> Les conditions suivantes sont équivalentes:

1) T est fortement mélangeante.
2) Pour tout $f \in L^1$ et toute suite croissante d'entiers $k_1 < k_2 < \ldots$ il existe une suite reelle $(c_i)_{i=1,2,\ldots}$ qui possède les propriétés

$$\begin{cases} a, & 0 < c_n \searrow \\ b, & \sum_{n=1}^{\infty} c_n = +\infty, \end{cases} \quad \text{et l'on a:}$$

$$\frac{\sum_{i=1}^{n} c_i T^i f}{\sum_{i=1}^{n} c_i} \xrightarrow[n\to\infty]{} \int f \, d\mu \quad \text{(p.p.)}$$

D.L.Burkholder: <u>Strong L_1 inequalities for quasi-linear operators on martingales</u>

Consider martingales $f = (f_1, f_2, \ldots)$ on some probability space (Ω, A, P). Define the maximal function f^* by $f^*(\omega) = \sup_n |f_n(\omega)|$ and the square-root function $S(f)$ by $S(f) = (\sum_{k=1}^{\infty} d_k^2)^{\frac{1}{2}}$ with $d_1 = f_1$ and $d_k = f_k - f_{k-1}$, $k \geq 2$. It is known that

(1) $\quad c_p \| S(f) \|_p \leq \| f^* \|_p \leq C_p \| S(f) \|_p$

for $1 < p < \infty$, with c_p and C_p positive real numbers depending only on p [Burkholder, Martingale transforms, <u>Ann.Math.Statist.</u> 37 (1966) 1494-1504]. The present work, joint with Richard F.Gundy, indicates that (1) is also true for p=1 provided the martingale f has difference sequence $d = (d_1, d_2, \ldots)$ of the

form $d_k = w_k e_k$ where w_k is A_{k-1} measurable, $E(e_k^2 | A_{k-1}) \geq \varepsilon > 0$, $||e_k|| \leq \varepsilon^{-1} < \infty$, $k \geq 1$, for some increasing sequence of sub-σ-fields of A relative to which f is a martingale. In this case, c_1 and C_1 depend on ε. The operator S is quasi-linear. A general theory of quasi-linear operators satisfying (1) for p = 1 is developed.

N.Dinculeanu: Algebraic models for measure preserving transformations

Definition 1. An object (Γ, U, ϕ) consisting of an abelian group Γ, an injective homomorphism $U: \Gamma \to \Gamma$ and a function of positive type $\phi: \Gamma \to \mathbb{C}$ such that: $\phi(\gamma) = 1$ iff $\gamma = 1$, $\phi \circ U = \phi$, is called an algebraic ergodic system (a.e.s.).

Definition 2. Two algebraic ergodic systems (Γ, U, ϕ) and (Γ', U', ϕ') are isomorphic if there exists a group isomorphism $\Phi : \Gamma \to \Gamma'$ such that $\phi' \circ \Phi = \phi$ and $\Phi U = U' \Phi$.

Example: Let (X, Σ, μ) be a probability measure space and $T: X \to X$ a measure preserving transformation.

a) Let $\Gamma(\mu)$ be the set of (equivalence classes of) functions $f \in L^2(\mu)$ such that $|f| \equiv 1$; then $\Gamma(\mu)$ is a multiplicative group generating $L^2(\mu)$ and containing the circle group C.

b) Let U_T be the operator on $L^2(\mu)$ induced by T. Then U_T is an injective homomorphism on $\Gamma(\mu)$ such that $U_T c = c$ for $c \in C$.

c) Put $\phi_\mu(f) = \int f \, d\mu$ for $f \in \Gamma(\mu)$. Then ϕ_μ is a function of positive type on $\Gamma(\mu)$ satisfying:
$\phi_\mu(f) = 1$ iff $f = 1$ and $\phi_\mu(U_T f) = \phi_\mu(f)$
If $\Gamma \subset \Gamma(\mu)$ is a group invariant under U_T, then (Γ, U_T, ϕ_μ) is an a.e.s.. In particular, $(\Gamma(\mu), U_T, \phi_\mu)$ and (C, U_T, ϕ_μ) are a.e.s..

Definition 3. An a.e.s. (Γ, U, ϕ) is an algebraic model for a measure preserving transformation T if there exists an injective homomorphism $J: \Gamma \to \Gamma(\mu)$ such that: $J\Gamma$ generates $L^2(\mu)$, $\phi = \phi_\mu \circ J$ and $JU = U_T J$.

Definition 4. An a.e.s. (Γ, U, ϕ) is discrete if $C \subset \Gamma$ and $\phi(\gamma) = \gamma$ for $\gamma \in C$ and $\phi(\gamma) = 0$ for $\gamma \notin C$.

Theorem 1. Two measure preserving transformations are conjugate iff they possess isomorphic algebraic models.

Theorem 2. Every a.e.s. is a model for a m.p.t..
Theorem 3. A m.p.t. T has a discrete model iff there exists a group $\Gamma' \subset \Gamma(\mu)$ which is an orthonormal base of $L^2(\mu)$ and $U_T\Gamma' \subset C\Gamma'$.
Theorem 4. An invertible m.p.t. is with discrete model iff it is conjugate to the superposition of a rotation and a continous automorphism on an abelian compact group equipped with Haar measure.
Ergodicity and transformations with discrete spectrum are also characterized by means of algebraic models.

H.Dinges: A pointwise ergodic theorem

Let (Ω,m) be a measure space and T a positive contraction of $L^1(\Omega,m)$, and let $x^1,\dots,x^n \in L^1$, then

$$\int f(x^1,\dots,x^n)\,dm \geq \int f(Tx^1,\dots,Tx^n)\,dm$$

holds for every sublinear nonnegative f.
In particular if $x^i = T^i x$, $p^i = T^i p$ with $p \geq 0$, then

$$(*) \quad \int f(x^o,\dots,x^{n-1};p^o,\dots,p^{n-1})dm \geq \int f(x^1,\dots,x^n;p^1,\dots,p^n)dm \geq \dots$$

for every sublinear nonnegative f of 2n variables (n arbitrary).
It was shown, that the maximal ergodic theorem for instance can be derived from inequalities of the type (*).
Several theorems were formulated, which lead to the following ergodic theorem:
If $x^i \in L^1(\Omega,m)$, $0 \leq p^i \in L^1(\Omega,m)$ such that (*) holds, then

$$\frac{x^o+\dots+x^n}{p^o+\dots+p^n} \text{ converges almost surely as } n\to\infty \text{ on } \{\sum p^i > 0\} .$$

If the limit is A, then

$$\frac{1}{n}\int (x^o+\dots+x^{n-1}-A(p^o+\dots+p^{n-1}))^+dm \xrightarrow[n\to\infty]{} 0 .$$

Little information could be given about the hard part of the proof, in which there has to be shown, that there exist decompositions $x^i = x^i\psi + x^i(1-\psi)$; $p^i = p^i\psi + p^i(1-\psi)$
such that $(x^i\psi;p^i\psi)$ and $(x^i(1-\psi);\ p^i(1-\psi))$ fulfill (*) and have certain other properties.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Chr.Grillenberger: On the entropy and the spectrum of an almost periodic dynamical system

Prof.Jacobs has shown in his paper on "Almost periodic sources and channels" that the invariant average $\bar{m}$ of a weakly almost periodic, uniformly mixing probability measure m on a compact metric space Ω with a topological automorphism T is ergodic, its spectrum is the group generated by all eigenvalues of the sequences $(\int f\cdot T^t\, dm)_{t \text{ integer}}$ for $f \in C(\Omega)$, and all flightvectors are strong flightvectors.
For the case of a two-sided Bernoulli space with finite alphabet A, T being the shift transformation and m the product of an almost periodic sequence (p^t) of probability vectors over A, the space of flightvectors is $N = L^2_{\bar{m}}(B_\infty)^{\perp}$, where B_∞ is the tail field, and in N T has Lebesgue spectrum with multiplicity $\aleph_0$, except the trivial case in which m is a periodic point measure. (The result is valid also for markovian almost periodic m.)
In the same special case we obtain a formula for the Kolmogoroff-Sinai invariant in terms of the marginal distributions:

$$\hat{H}(\bar{m},T) = \overline{H(p^t)} : = \lim_{t\to\infty} \frac{1}{t} \sum_{s=0}^{t-1} H(p^s)$$

D.L.Hanson: A mean ergodic theorem with general coefficients

Let $A_{N,K} \geq 0$ for $N,K = 0,1,2,\ldots$; let (Ω,Σ,P) be a probability space. Let T be a measurable and measure preserving point transformation of Ω into Ω; let d be the invariant subsets of Ω under T; and let L_2 be the collection of measurable square-integrable functions on (Ω,Σ,P). The following theorem seems to be the "appropriate" one. It is a considerable improvement over the one presented in the author's talk. The improvements being suggested and proved by various people.
Theorem: If $\sum_K A_{N,K} = 1$ for all N, then $\sum_K A_{N,K}\, T^K f \to E\{f|d\}$ in L_2-mean for all $f \in L_2$ and all (Ω,Σ,P,T) if and only if

(1) $$\sum_{K=0}^{\infty} A_{N,K\alpha+j} \to \frac{1}{\alpha} \text{ for } \alpha = 2,3,\ldots \text{ and } j = 0,\ldots,\alpha-1$$

and

(2) $$\sum_{\{k|k\gamma \bmod [0,1) \in [a,b)\}} A_{N,K} = b-a$$ for all $a,b,\gamma \in [0,1)$ such that $a < b$ and γ is irrational.

G.Helmberg: On mean recurrence time under a measure preserving flow.

M.Kac hat 1947 einen später verallgemeinerten Satz über mittlere Rückkehrzeit in eine Menge B eines Wahrscheinlichkeitsraumes unter wiederholter Ausübung einer maßtreuen Transformation T bewiesen. Eine analoge Zusage über mittlere Rückkehrzeit läßt sich für eine maßtreue Halbströmung $\{T_t\}_{t \geq 0}$ in einem Wahrscheinlichkeitsraum ableiten, doch müssen Mittelungsvorgang und Definition der Rückkehrzeit in geeigneter Weise modifiziert werden. Im ersten Teil des Referates wird die rein maßtheoretische Situation betrachtet, im zweiten Teil wird der Fall einer stetigen Halbströmung in einem kompakten metrischen Raum behandelt.

A.Ionescu-Tulcea: Lifting for abstract valued functions and separable stochastic processes

Let (Ω,F,μ) be a complete probability space. Let $M_R^\infty(=M_R^\infty(\Omega,F,\mu))$ be the algebra of all measurable bounded functions $f:\Omega \to R$. For $f,g \in M_R^\infty$ we write $f \equiv g$ if f and g coincide μ-almost everywhere. Let now E be a completely regular space and let $C_R(E)$ be the algebra of all continuous functions $h: E \to R$. A function $f: \Omega \to E$ will be called weakly measurable if $h \circ f$ is measurable for each $h \in C_R(E)$. We denote by M_E^∞ the set of all $f: \Omega \to E$ such that: 1) f is weakly measurable and
2) $\overline{f(\Omega)}$ is compact. For f,g in M_E^∞ we write $f \equiv g$ if $h \circ f \equiv h \circ g$ (in M_R^∞) for each $h \in C_R(E)$.
The notion of lifting is extended from the "real space" M_R^∞ to the "abstract space" M_E^∞. Let $\rho: M_R^\infty \to M_R^\infty$ be a lifting of M_R^∞. A mapping $\rho': M_E^\infty \to M_E^\infty$ is called a lifting of M_E^∞ associated with ρ if i) $\rho'(f) \equiv f$; ii) $f \equiv g$ implies $\rho'(f) = \rho'(g)$; iii) $\rho(h \circ f) = h \circ \rho'(f)$ for all $f \in M_E^\infty$, $h \in C_R(E)$ (as a matter of fact, condition iii) is the defining equation of ρ'). It can be shown that there exists a unique lifting of M_E^∞ associated with ρ; this lifting will be denoted by ρ_E. The lifting ρ_E is then applied to obtain a separable modification of a stochastic process: If $(X_t)_{t \in T}$ is a stochastic process defined on (Ω,F,μ) with values in E, then the process $(Y_t)_{t \in T}$, where $Y_t = \rho_E(X_t)$ for each $t \in T$ is a separable modification of $(X_t)_{t \in T}$.

K.Jacobs: On sequences of Toeplitz type

For the construction of almost periodic functions on the line Toeplitz used (Ann. 1928) a combinatorial device similar to the following

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ...
 1   1   1   1   1   1   1   1   1   1
   0       0       0       0       0
       1               1
```

leading to the 0-1-sequence

010001010100010001000100010101 ... .

A general device of this kind is based on a sequence

$$\eta^{(1)} , \eta^{(2)} , \eta^{(3)} ,...$$

of sequences of symbols $0,1,\infty$(="hole") which 1) are periodic, 2) begin with 0 and 1 and 3) contain ∞ (of course, infinitely often, then).

Construct a sequence $\eta_{(1)}, \eta_{(2)},...$ such that

$\eta_{(1)} = \eta^{(1)}$

$\eta_{(n)} = \eta_{(n-1)}$ with $\eta^{(n)}$ filled into the "holes".

Clearly the $\eta_{(n)}$ are successive "completions" of each other, and finally all "holes" are stuffed such that in the limit an almost periodic sequence η containing only 0 and 1 is obtained. Let $\rho_\infty(\xi)$ the mean frequency of ∞'s in the 0-1-∞-sequence ξ. Then

$$\rho_\infty(\eta_{(n)}) = \rho_\infty(\eta^{(1)})\cdot\ldots\cdot\rho_\infty(\eta^{(n)})$$

We have the following

Theorem: If $\rho_\infty(\eta_{(n)}) \to 0$, then η is strictly ergodic, and the attached unique invariant measure m_η has pure point spectrum. In case $\rho_\infty(\eta_{(n)}) \not\to 0$, then one obtains easily examples for almost periodic, but not strictly ergodic sequences, e.g. one given by Oxtoby.

K.Jacobs: Riemannian dynamical systems

Let $\underline{\Omega}$ be compact metric and $\emptyset \neq \Omega \subseteq \underline{\Omega}$, m a finite measure in Ω, and T a m-preserving, m-almost everywhere continuous mapping $\Omega \to \Omega$. Then the dynamical system (Ω,T,m) is called Riemannian.

In such a system m-almost every point ω is "of permanent T-continuity", i.e. T is continuous on ω, $\omega T, \ldots$. Such a point is called

1) an almost periodic visitor, if
   a) $\overline{\mathrm{orb}(\omega)}$ carries m
   b) for every neighbourhood U of ω there is some $L > 0$ such that $\{\omega T^t, \ldots, \omega T^{t+L-1}\} \cap U \neq \emptyset$ $(t=0,1,\ldots)$.
2) a regular visitor, if for every m-almost clopen set $F \subseteq \Omega$ and every $\varepsilon > 0$ there is a $t_0 > 0$ such that $t \geq t_0$ implies $\left| \frac{1}{t} \sum_{u=0}^{t-1} 1_F(\omega T^{s+u}) - \frac{m(F)}{m(\Omega)} \right| < \varepsilon$ $(s = 0,1,\ldots)$

It is easily seen:

A) The induced system on a m-almost clopen set Ω' is again Riemannian and every $\omega \in \Omega'$ visiting only the interior of Ω', is still an almost periodic (resp. regular) visitor, if ω was so for the original system.

B) Map Ω into $\hat{\underline{\Omega}} = \underline{\Omega} \times \underline{\Omega} \times \ldots$ by

$$\phi : \omega \to \hat{\omega} = (\omega, \omega T, \omega T^2, \ldots),$$

let $\hat{m} = m\phi$, $\hat{T}$ = shift. Then ϕ carries an almost periodic visitor ω into an almost periodic $\hat{\omega}$, and an a.p. regular visitor ω into a strictly ergodic point $\hat{\omega}$.

Exerting first a suitable version of A) to a suitable circle rotation, a strictly ergodic system with pure point spectrum looses all its non-trivial eigenvalues, but there is still plenty of almost periodic visitors; thus in performing B) we obtain a weakly mixing strictly ergodic dynamical system.

Probably one can even put the system into finite-state shift space in this special case.

S.Kakutani: <u>Examples of weakly but not strongly mixing transformations</u>

Let (Y,B,μ) be the Lebesgue measure space on the unit interval $Y = [0,1]$. Let $Y = Y_1 \cup Y_2$ (disj.) be a partition of Y. Let $X = Y_1 \cup Y_2 \cup Y_2'$ be a two-story "skyscraper" built over Y with respect to the partition $Y = Y_1 \cup Y_2$ (disj.).

Let (X,B,μ) be the corresponding measure space on X (defined in an obvious way). Let ψ be a m.p.t. defined on (Y,B,μ).

Let ϕ be the m.p.t. defined on (X,B,μ) by using the method described in the preceding talk. It is possible to prove that ϕ is weakly but not strongly mixing in the following two cases:
Case I: ψ is the transformation defined on Y by

$$\psi(y) = y + \frac{1}{2} \quad \text{if } 0 < y < \frac{1}{2} \;;$$

$$\psi(y) = y-(1-2^{-n}-2^{-(n+1)}) \quad \text{if } 1-2^{-n} < y < 1-2^{-(n+1)},\ n=1,2,\dots;$$

$$Y_1 = \bigcup_{n=0}^{\infty} (1-2^{-2n}, 1-2^{-(2n+1)}),\quad Y_2 = \bigcup_{n=0}^{\infty} (1-2^{-(2n+1)}, 1-2^{-(2n+2)}).$$

Case II: $\psi(y) = y+\alpha$, where α is a transcendental number, $\alpha = \sum_{n=1}^{\infty} 10^{-(2^n-1)}$; $Y_1 = (0,\beta)$, $Y_2 = (\beta,1)$, where β is a real number whose decimal expansion $\beta = \sum_{m=1}^{\infty} b_m 10^{-m}$ satisfies $b_m = 5$ if $m = 2^n-1$ for some n.

S.Kakutani: Induced measure preserving transformations and related topics

Let (X,B,μ) be an atomless measure space with $0 < \mu(X) \leq \infty$, and let ϕ be an ergodic measure preserving transformation (m.p.t.) defined on it. Let Y be a measurable subset of X with $\mu(Y) > 0$, $\mu(X-Y) > 0$. For almost all $y \in Y$, there exists a positive integer $n = n(y)$ such that $\phi^i(y) \notin Y$, $i=1,\dots,n-1$, and $\phi^n(y) \in Y$. Put $\psi(y) = \phi^{n(y)}(y)$ for a.e. $y \in Y$. Then ψ is an ergodic m.p.t. defined on (Y,B,μ). ψ is called the m.p.t. induced on Y by ϕ. If we put $Y_n = \{y \mid y \in Y, n(y)=n\}$, $n=1,2,\dots$, then $Y = \bigcup_{n=1}^{\infty} Y_n$ (disjoint) and $X = \bigcup_{n=1}^{\infty} \bigcup_{i=0}^{n-1} \phi^i(Y_n)$ (disj.). (It is possible that $\mu(Y_n) = 0$ for some n, and also that $\mu(Y_n) = 0$ for all $n \geq n_0$ for some n_0).

Conversely. let (Y,B,μ) be a measure space with $0 < \mu(Y) \leq \infty$, and let ψ be an ergodic m.p.t. defined on it. Let $Y = \bigcup_{n=1}^{\infty} Y_n$ (disj.) be a (finite or countably infinite) partition of Y. Construct a "skyscraper" over Y in such a way that there exist exactly n-1 floors $Y_n^{(1)}, Y_n^{(2)}, \dots Y_n^{(n-1)}$ over $Y_n^{(0)} = Y$, $n=1,2,\dots$.

Let χ_n be a "vertical" mapping which maps $Y_n^{(i-1)}$ onto $Y_n^{(i)}$, $i=1,\dots,n-1$. Put $X = \bigcup_{n=1}^{\infty} \bigcup_{i=o}^{n-1} Y_n^{(i)}$ (disj.) and consider the measure space (X,B,μ) on X (B and μ are defined in an obvious way so that χ_n becomes a m.p.t. of $Y_n^{(i-1)}$ onto $Y_n^{(i)}$, $i = 1,\dots,n-1$). Put $\phi(x) = \chi_n(x)$ if $x \in Y_n^{(i)}$ for some n and i ($0 \leq i \leq n-2$) and $\phi(x) = \psi(\chi_n^{-(n-1)}(x))$ if $x \in Y_n^{(n-1)}$ for some n. Then ϕ is an ergodic m.p.t. defined on (X,B,μ) and it is easy to see that the ergodic m.p.t. defined on the skyscraper X over Y with respect to the partition $Y = \bigcup_{n=1}^{\infty} Y_n$ (disj.) and the base transformation ψ. This relation between ϕ and ψ is denoted by $\phi > \psi$. If we denote by $[\phi]$ the class of all m.p.t. which are spatially isomorphic with ϕ, then we may define the relation $[\phi] > [\psi]$ to mean that $\exists\, \phi_o \in [\phi], \exists\, \psi_o \in [\psi]$ such that $\phi_o > \psi_o$. This is obviously a transitive relation (i.e. $[\phi_1] > [\phi_2]$, $[\phi_2] > [\phi_3]$ $\Rightarrow$ $[\phi_1] > [\phi_3]$).

Theorem 1: There exists ϕ_3 with $[\phi_3] > [\phi_1]$, $[\phi_3] > [\phi_2]$ if and only if there exists ϕ_4 with $[\phi_1] > [\phi_4]$ and $[\phi_2] > [\phi_4]$.

We write $[\phi_1] \sim [\phi_2]$ (equivalent) if one and hence both of the conditions in theorem 1 are satisfied.

Theorem 2:

$[\phi] \sim [\psi]$ if and only if the classes of flows built under a function over $\phi \in [\phi]$ and $\psi \in [\psi]$ are identical (by spatial isomorphism).

S. Kakutani: Ergodic measure preserving transformations defined on an infinite measure space

Let (X,B,μ) be the Lebesgue measure space on the real line $X = R$. The existence of an ergodic m.p.t. defined on (X,B,μ) is shown by using the method of skyscraper. We note that in this way we get all ergodic m.p.t. on (X,B,μ) by taking all $Y \in B$ with $0 < \mu(Y) < \infty$ and all partitions $Y = \bigcup_{n=1}^{\infty} Y_n$. Let ϕ be an ergodic m.p.t. defined on (X,B,μ) with $\mu(X) = \infty$ and let $A \in B$, $0 < \mu(A) < \infty$. Put $A_n = A_n(\phi) = \bigcup_{i=o}^{n-1} \phi^i(A)$, $n=1,2,\dots$.

Then $\mu(A_n) \uparrow \infty$, $\mu(A_n)/n \downarrow 0$.

Theorem 1. For any ϕ, there exists a sequence $\{n_k\}$ of positive integers, $n_k < n_{k+1}$, $k = 1,2,\ldots$, such that

$$\liminf \mu(A_{n_k}(\phi))/k > 0 \text{ for any } A \in B \text{ with } \mu(A) > 0.$$

On the other hand, from the skyscraper construction, it follows that there exists an ergodic m.p.t. ψ and a set A with $0 < \mu(A) < \infty$ such that $\lim \mu(A_{n_k}(\psi)) = 0$. This shows:

Theorem 2. There exist infinitely many ergodic m.p.t. defined on the Lebesgue measure space (with X = R) no two of which are spatially isomorphic.

Various examples of ergodic m.p.t. with interesting number-theoretical properties were discussed in this talk.

S.Kakutani: Spectral analysis of the Morse dynamical system

Let $\Omega = \prod_{n\in Z} \{+1,-1\}$ be the set Ω of all two-sided infinite sequences $\omega = \{\omega_n | n \in Z\}$ with $\omega_n = +1$ or -1, $n \in Z$. Ω is a totally disconnected compact metrizable space with respect to the usual product topology. Define the shift transformation σ on Ω by $(\sigma(\omega))_n = \omega_{n+1}$, $n \in Z$ and the involution τ by $(\tau(\omega))_n = -\omega_n$, $n \in Z$. Put

$$\xi_0 = +1, \quad \xi_1 = -1 \text{ and } \xi_{2n} = \xi_n, \quad \xi_{2n+1} = -\xi_n, \; n=1,2,\ldots .$$

Then we obtain the Morse sequence in which the usual 0 and 1 are replaced by +1 and -1.

Put $\xi_{-n} = \xi_{n-1}$, $n=1,2,\ldots$ and $\omega_0 = \{\xi_n | n \in Z\} \in \Omega$.

Consider the orbit closure $\Omega_0 = \overline{orb(\omega_0)} = \overline{\{\sigma^n(\omega_0) | n \in Z\}}$.

Then Ω_0 is invariant under σ and τ, and (Ω_0,σ) is a strictly ergodic dynamical system. This is called the Morse dynamical system.

Let μ be the unique normalized invariant measure on (Ω_0,σ). μ is also τ-invariant. Let $L^2(\Omega_0) = L^2(\Omega_0,\mu)$ be the complex L^2-space on Ω_0 with respect to μ. Let V_σ, V_τ be the unitary operators defined on $L^2(\Omega_0)$ by $V_\sigma f(\omega) = f(\sigma^{-1}(\omega))$, $V_\tau f(\omega) = f(\tau^{-1}(\omega))$.

A function $f \in L^2(\Omega_0)$ is called an even function if $V_\tau f = f$, an odd function if $V_\tau f = -f$. Denote by $\mathfrak{M}_e$ and $\mathfrak{M}_o$ the sets of all even and odd functions from $L^2(\Omega_0)$. Then, $\mathfrak{M}_e$ and $\mathfrak{M}_o$ are closed linear subspaces of $L^2(\Omega_0)$, mutually orthogonal, and together span

the space $L^2(\Omega_o)$. From the fact that σ and τ commute, if follows that both $\mathfrak{M}_e$ and $\mathfrak{M}_o$ are invariant under V_σ.

Theorem. V_σ has a pure point spectrum on $\mathfrak{M}_e$ (its eigenvalues are given by $\lambda = j\ 2^{-n}$, j, n=0,1,2,...).

V has a continuous singular spectrum on $\mathfrak{M}_o$.

M.Keane: Generalized Morse sequences

Let $b = b_1 \ldots b_m$ and $c = c_1 \ldots c_n$ be sequences of zeros and ones (i.e. blocks). We define $b \times c = b^{c_1} b^{c_2} \ldots b^{c_n}$, where $b^o = b$ and $b^1 = 1-b_1, \ldots, 1-b_m$. Then the Morse sequence x = 0110100110010110... may be written as an infinite product of blocks: $x = 01 \times 01 \times 01 \times \ldots$. Sequences of the form $x = b^1 \times b^2 \ldots$, where each b^k is a block beginning with zero, are called recursive sequences. Recursive sequences are almost periodic and define in a natural way an "orbit" in the two-sided shift space on zeros and ones. We give necessary and sufficient conditions for the strict ergodicity of such recursive sequences (which are called generalized Morse sequences).

U.Krengel: On mixing in infinite measure spaces

Let $(\Omega, \mathfrak{A}, \mu)$ be a σ-finite measure space. A sequence (A_n) of measurable sets is called remotely trivial, if the σ-algebra $R(A_n) = \bigcap_{k=1}^{\infty} B_k(A_n)$ is trivial, where $B_k(A_n)$ is generated by $A_k, A_{k+1}, \ldots$ (A_n) is called semi remotely trivial (s.r.t.) if every subsequence contains a remotely trivial subsequence. A measure preserving transformation T is called mixing if $(T^{-n}A)$ is s.r.t. for all A with $\mu(A) < \infty$, it is called completely mixing if $(T^{-n}A)$ is s.r.t for all $A \in \mathfrak{A}$. Mixing in infinite measure spaces is equivalent with being of zero type. T is completely mixing iff

$$\int 1_{T^{-n}A}\, f d\mu \to 0 \text{ for } f \in L_1^o = \{f \in L_1 : \int f = 0\}.$$

Examples: Markov shifts on a unilateral product space for null-recurrent, aperiodic ergodic Markov chains. For invertible transformations, however, complete mixing implies the existence of a

finite invariant measure. This negatively answers the problem posed by Mrs.Dowker at Oberwolfach in 1965. For mixing transformations an analogue of the theorem of Blum and Hanson on mean convergence for expressions

$$\frac{1}{n}\sum_{1}^{n} T^{k_i} f \quad (k_1 < k_2 < \dots)$$

can be proved. For completely mixing transformations we have a theorem which generalizes and strengthens a theorem of Orey on the convergence of $\sum_k |p^{(n)}_{i_1,k} - p^{(n)}_{i_2,k}|$ to zero. The work was done jointly with Sucheston.

W.Krieger: <u>On non-singular transformations of a measure space</u>

We consider a Lebesgue measure space (M,B,m). By an automorphism of (M,B,m) we mean a B-measurable transformation of (M,B,m) that together with its inverse is non-singular with respect to m. We study an equivalence relation between these automorphisms which we call the weak equivalence. Two automorphisms S and T are weakly equivalent if there is an automorphism U such that for almost all $x \in M$, U maps the S-orbit of x onto the T-orbit of Ux. Ergodicity, the existence of a finite invariant measure resp. of a σ-finite invariant measure as well as the non-existence of such measures are invariants of weak equivalence. We solve the problem of weak equivalence for a class of automorphisms that comprises all ergodic automorphisms that admit a σ-finite invariant measure and also certain ergodic automorphisms that do not admit such a measure.

D.Maharam: <u>On iterates of positive operators</u>

An abstract measure space consists of a space S, a Borel field $\mathcal{L}_S$ of subsets of S, and a σ-ideal $n_S \subseteq \mathcal{L}_S$ such that $\mathcal{L}_S / n_S$ satisfies the countable chain condition. We denote by $\mathcal{F}^+_S$ the family of all extended real nonnegative "measurable" functions on S [i.e. functions f such that $(f \geq \alpha) \in \mathcal{L}_S$ for all real α.], by $\mathfrak{Z}_S$, $\{f \mid (f \neq 0) \in n_S\}$, and by F^+_S, $\mathcal{F}_S / n_S$. Let $(T, \mathfrak{M}, \mu)$ be a measure space (with σ-finite Lebesgue measure μ): We define the "standard product" $(R, \mathcal{L}_R, n_R) = (S, \mathcal{L}_S, n_S) \times (T, \mathfrak{M}, \mu)$ or

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

[R × T, for short] as follows: $R = S \times T$, $\mathcal{L}_R$ = Borel field generated by the sets $H \times K$, $H \in \mathcal{L}_S$, $K \in \mathcal{M}$. If $f \in \mathcal{F}^+_R$, then for each fixed $s \in S$, $f(s,t)$ is measurable in t, and the function mf defined on S by $mf(s) = \int_T f(s,t)dt$ is $\mathcal{L}_S$-measurable. Now define $\eta_R \equiv \{A | A \in \mathcal{L}_R, m\chi_A \in \mathfrak{Z}_S\}$, where χ_A is the characteristic function of A. $(R, \mathcal{L}_R, \eta_R)$ is an abstract measure space, and m induces (by reducing modulo null functions) a linear map M called the <u>standard</u> <u>integral</u> from F^+_R onto F^+_S. In particular, let $(S^*, \mathcal{L}^*, \eta^*)$ be an abstract measure space and ϕ^* a linear map of $F^+_{S^*}$ onto itself such that (i) $(S^*, \mathcal{L}^*, \eta^*)$ is the standard product $(S_1, \mathcal{L}_1, \eta_1) \times (T_1, \mathcal{M}_1, \mu_1)$, where T_1 is the measure-theoretic product of an arbitrary number of copies of the unit interval; (ii) $\exists$ "measurable" 1-1 point map $\xi : S^* \to S_1$ [i.e., ξ maps $\mathcal{L}^*$ onto $\mathcal{L}_1$ and η^* onto η_1] such that for $f^* \in F^+_{S^*}$, $\phi^* f^* = \xi^{-1} M_1 f^*$, where M_1 is the standard integral of F_{S^*} onto F_{S_1}. By an easy induction we now define $(S_n, \mathcal{L}_n, \eta_n)$ and $(T_n, \mathcal{M}_n, \mu_n)$ $(n=1,2,...)$ so that (i) $\mathcal{L}_{n+1}$, η_{n+1}, $\mathcal{M}_{n+1}$ are the images under ξ of $\mathcal{L}_n$, η_n, $\mathcal{M}_n$, and ξ is measure-preserving between $\mathcal{M}_n$ and $\mathcal{M}_{n+1}$; (ii) S_n is the standard product $S_{n+1} \times T_{n+1}$, and so S^* is the standard product $S_n \times T^*_{n_1}$ where T^*_n is the measure theoretic product $T_1 \times \ldots \times T_n$. It follows that for $f^* \in F^{*+}$, and for each $n > 0$, $\phi^{*n} f^* = \xi^{-n} M_n f^*$ where M_n is the standard integral from $F^+_{S^*}$ to $F^+_{S_n}$, i.e. for $f \in \mathcal{F}^{*+}$, $s^* \in S^*$, we have modulo null sets,

$$\phi^{*n} f^*(s^*) = \int_{T_1} \cdots \int_{T_n} f^*(t'_1, \ldots, t'_n, \xi^{-n}(s^*))(dt_1)\ldots(dt_n).$$

Now let $(S, \mathcal{L}, \eta)$ be an arbitrary abstract measure space and Φ a map of $F^+_S \to F^+_S$ such that (i) $f_n \in F^+_S$, $\alpha_n \geq 0$ $(n=1,2,...)$ $\Longrightarrow$ $\Phi(\Sigma\alpha_n f_n) = \Sigma\alpha_n \Phi f_n$; (ii) $\exists$ $f_n \in F^+_S$ such that $\Sigma f_n = 1$ a.e. and $\Phi f_n < \infty$ a.e. Then $\exists$ $(S^*, \mathcal{L}^*, \eta^*)$ and ϕ^* as in the preceding paragraph, and a function $K^* \in F^+_{S^*}$ such that S can be "imbedded" in S^* in such a way that, for each $f \in F^+_S$ considered as an element of $F^+_{S^*}$, $\Phi^n f = \phi^{*n}(K^* f)$.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

["S is imbedded in S^*" means $\exists$ isomorphism Θ of $\mathcal{L}|\eta$ into the measurable/null sets of some measurable set $A \in \mathcal{L}^*$ such that for all $f \in F_S^+$, $\Theta\Phi f = \Phi^* \Theta f$]. Thus, the iterates of any operator satisfying (i) and (ii) above can be given a concrete representation using the integral formula of the last paragraph.

G.Maruyama: <u>The canonical version of a flow</u>

Let (X,B_μ,μ) be a topological probability space, where X is a Hausdorff space, B_μ the μ-completion of the Borel field on X, and μ a Radon measure on B_μ. T_t, $t \in T = (-\infty,\infty)$ is a canonical flow on X, iff it is a group of 1-1 measure preserving mappings of X onto itself, and the map $(t,x) \in T{\times}X \rightarrow T_t x \in X$ is continuous. Under mild conditions, a measurable flow on an arbitrary measure space is isomorphic to a canonical flow restricted to a set of outer measure 1. In general, any continuous flow is algebra-isomorphic to a canonical flow.

A.Nijst: <u>Some remarks on conditional entropy</u>

We discuss an integral representation of conditional entropy which generalizes a well-known result of this sort and we show that this representation theorem implies that additivity of conditional entropy and conditional independence of σ-fiels are equivalent.

Also we obtain by this representation theorem with the aid of a lemma of Sinai a simple proof of the decomposition theorem of the Kolmogorov-Sinai invariant.

W.Parry: <u>Compact abelian group extensions of dynamical systems</u>

Let X be compact metric, S a homeomorphism and let G be a compact abelian group acting continuously on X such that G commutes with S. If G acts freely we say (X,S) is a G-extension of (Y,T) where Y = X/G and T is the homeomorphism induced by S on Y. Conditions are given for (X,S) to be minimal or uniquely ergodic when (Y,T) enjoys the same property. In particular we show that a G extension is "likely" to lift the property considered (when X is connected) in the sense that {g: gS is uniquely ergodic [minimal]} contains a dense G_δ given that T is uniquely ergodic

["S is imbedded in S^*" means $\exists$ isomorphism θ of $\mathcal{L}|\eta$ into the measurable/null sets of some measurable set $A \in \mathcal{L}^*$ such that for all $f \in F_S^+$, $\theta\Phi f = \Phi^* \theta f$]. Thus, the iterates of any operator satisfying (i) and (ii) above can be given a concrete representation using the integral formula of the last paragraph.

G.Maruyama: The canonical version of a flow

Let (X,B_μ,μ) be a topological probability space, where X is a Hausdorff space, B_μ the μ-completion of the Borel field on X, and μ a Radon measure on B_μ. T_t, $t \in T = (-\infty,\infty)$ is a canonical flow on X, iff it is a group of 1-1 measure preserving mappings of X onto itself, and the map $(t,x) \in T\times X \rightarrow T_t x \in X$ is continuous. Under mild conditions, a measurable flow on an arbitrary measure space is isomorphic to a canonical flow restricted to a set of outer measure 1. In general, any continuous flow is algebra-isomorphic to a canonical flow.

A.Nijst: Some remarks on conditional entropy

We discuss an integral representation of conditional entropy which generalizes a well-known result of this sort and we show that this representation theorem implies that additivity of conditional entropy and conditional independence of σ-fiels are equivalent.

Also we obtain by this representation theorem with the aid of a lemma of Sinai a simple proof of the decomposition theorem of the Kolmogorov-Sinai invariant.

W.Parry: Compact abelian group extensions of dynamical systems

Let X be compact metric, S a homeomorphism and let G be a compact abelian group acting continuously on X such that G commutes with S. If G acts freely we say (X,S) is a G-extension of (Y,T) where Y = X/G and T is the homeomorphism induced by S on Y. Conditions are given for (X,S) to be minimal or uniquely ergodic when (Y,T) enjoys the same property. In particular we show that a G extension is "likely" to lift the property considered (when X is connected) in the sense that {g: gS is uniquely ergodic [minimal]} contains a dense G_δ given that T is uniquely ergodic

[minimal] . This suggests the definition of a stable G-extension: when gS is homeomorphic to S for all g in an open set. Hence stable extensions always lift the required property. This is applied to affine transformations of certain three dimensional manifolds called nilmanifolds.

K.Post: Some elementary investigations on measurable transformations

a) The transformation $Tx \equiv 2x \bmod 1$ on the unit interval with measure μ defined for all Borel sets E by

$$\mu(E) = \int_E \frac{1}{x}\, d\lambda(x),$$

λ being Lebesgue-measure, has the property

$$0 < \mu(E) < \infty \Rightarrow \mu(E) < \mu(T^{-1}E) < 2\mu(E).$$

This example provides an answer on a question by G.Helmberg (Tagung über Ergodentheorie, 1965).

b) If T is a measurable transformation on an arbitrary measure space $(X, \mathfrak{R}, \mu)$ satisfying $\mu(T^{-1}A) \leq \mu(A)$ for all $A \in \mathfrak{R}$, then any $E \in \mathfrak{R}$, for which $E \subset \bigcup_{n=1}^{\infty} T^{-n}E \quad (\mu)$ must have the property $\mu(T^{-1}E) = \mu(E)$. This result, due to F.Simons, provides a short proof of the implication

$$\left.\begin{array}{l} T \text{ conservative} \\ \mu(T^{-1}A) \leq \mu(A) \text{ for all } A \in \mathfrak{R} \end{array}\right\} \Rightarrow \mu(T^{-1}A) = \mu(A) \text{ for all } A \in \mathfrak{R} .$$

cf. G.Helmberg, Über konservative Transformationen Math.Ann.165, 44-61 (1966)

L.Sucheston: On convergence of information in spaces with infinite invariant measure, abstract of talk

We extend the ergodic theorems of information theory (Shannon-MacMillian-Breiman theorems) to spaces with an infinite invariant measure. An L_1 difference theorem and pointwise ratio theorem are proved, for the information of spreading partitions. For the validity of the theorems it is assumed that the supremum

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

f^* of the conditional information given the increasing "past" is integrable. Simple necessary and sufficient conditions for the integrability of f^* are obtained in special cases: If the initial partition is composed of one state of a null-recurrent Markov chain, then f^* is integrable if and only if the partition of this state according to the first return times has finite entropy. (Paper written in collaboration with E.M.Klimko, to appear in Z.Wahrscheinlichkeitstheorie vol.9 (1968).)

H.Scheller: A short proof of Abramov's theorem on the presentation of entropy for induced transformations

Given a normed dynamical system (Ω,B,m,T) -T not necessarily invertible - and $E \in B$, let be $r_E(a)$ the first recurrence time (equal to 0 on E^c); let $\mathcal{R}(E)$ resp. $\sigma(E)$ be the σ-fields generated by r_E resp. E and let T_E be the induced transformation defined by T^{r_E}.

Using a lemma reducing the set of σ-fields needed for the computation of $\hat{H}(T)$ (valid for sweep out sets) and using the relations $H(T_E) = H(T_{ET^{-1}})$, $H(\mathcal{R}(E)) \leq 2H(\sigma(E))$, we derive from the inclusions

$$\bigvee_{t=1}^{\infty} B_o T_E^{-t} \subseteq \bigvee_{t=1}^{\infty} B_o T^{-t} \subseteq \mathcal{R}(E) \vee \bigvee_{t=1}^{\infty} B_o T_E^{-t}$$ (valid within E and all σ-fields $B_o \supseteq \mathcal{R}(E)$) the formula (1) $\hat{H}(T_E) - 2H(\sigma(E)) \leq \hat{H}(T) \leq$ $\leq H(T_E) + H(\sigma(E))$. Applying (1) to arbitrarily small sweep out subsets of E - observe $(T_E)_F = T_F$ - we obtain a generalized version of Abramov's theorem: $\hat{H}(T_E) = \hat{H}(T)$ for all sweep out sets.

F.H.Simons: Sweep-out sets and strong generators

Let $(X,\mathcal{R},\mu)$ be a σ-finite measure space, and let T be a non-singular measurable transformation in $(X,\mathcal{R},\mu)$. T is said to be periodic on a set $A \in \mathcal{R}$ if there exists a natural number n such that for all measurable $B \subset A$ we have $T^{-n}B \supset B$ $[\mu]$; the least number n satisfying this condition is the period of T on A. T is said to be aperiodic, if the only sets on which T is periodic are μ-null sets.

As a generalization of theorems of Parry we can prove the following:

<u>Theorem 1:</u> The following statements are equivalent:

a) T is aperiodic

b) There exists a countable infinite sweep-out set partition of X.

<u>Theorem 2:</u> If moreover μ is non-atomic, then the following statements are equivalent.

a) There exists a strong generator for $\mathfrak{R}$

b) $\mathfrak{R}$ is countably generated; there exists a partition ζ_o such that $T^{-1}\mathfrak{R} \vee \zeta_o = \mathfrak{R}[\mu]$; T is aperiodic.

H.Totoki: <u>A special flow which is a K-flow</u>

Let T be a Bernoulli shift on $(\Omega, \mathfrak{B}, P)$ where $(\Omega, \mathfrak{B}, P) = \prod_{-\infty}^{\infty}(E, \mathfrak{B}_E, P_E)$ with non-trivial probability space $(E, \mathfrak{B}_E, P_E)$. Let θ be an integrable function on Ω such that $\inf_\omega \theta(\omega) > 0$. Construct the special flow $\{S_t\} = (T,\theta)$. If $\theta(\omega)$ is a bounded function of the 0-th coordinate $X_o(\omega)$ of ω, $\theta(\omega) = \hat{\theta}(X_o(\omega))$, then there are the following two possibilities:

(1) $\{S_t\}$ has non-constant eigenfunctions (in the case when the distribution $\theta(\omega)$ is of lattice type), or

(2) $\{S_t\}$ is a K-flow (in the case when the distribution of $\theta(\omega)$ is of non-lattice type).

<u>Problems:</u>

1. <u>K.A.Post</u>

Given any sequence of objects, (some of which may coincide) does there exist a uniformly distributed sequence of numbers on the unit interval with the same coincidence - pattern?

Necessary conditions are obviously

(i) the asymptotic density of all objects is zero.

(ii) for any $\varepsilon > 0$ there are only finitely many objects, the frequency quotient of which attains values $> \varepsilon$.

2. L.Sucheston

Let $X_0, X_1, \ldots$ be a null-recurrent Markov chain and set

$$f_{kk}^{(n)} = P(X_1 \neq k,\ X_2 \neq k, \ldots, X_{n-1} \neq k,\ X_n = k \mid X_0 = k).$$

Is the relation

$$-\sum_n f_{kk}^{(n)} \log f_{kk}^{(n)} < \infty$$

a class property of the chain?

3. L.Sucheston

There exists an infinite measure space analogue of the Shannon-Mc-Millan-Breiman theorem (cf. E.M.Klimko and L.Sucheston, Z.Wahrscheinlichkeitstheorie, vol.9, 1968). Can this be applied to obtain an analogue of the Kolmogorov-Sinai theorem, for an appropriately defined concept of entropy?

4. L.Sucheston

Call the following statement an "approximate" ergodic theorem: Let $0 \leq f_i \to f$, $0 \leq g_i \to g > 0$, let T be an operator as e.g. in the Chacon-Ornstein theorem, and assume

(*) $\sup f_i \in L_1$, $\sup g_i \in L_1$.

Then $\sum_0^{n-1} T^i f_i \big/ \sum_0^{n-1} T^i g_i$ converges a.e. Given f_i such that $\sup f_i \notin L_1$, produce g_i and T such that $\sum_0^{n-1} T^i f_i \big/ \sum_0^{n-1} T^i g_i$ diverge a.e. To avoid trivial counterexamples, assume that the measure space is sufficiently rich to support a sequence of independent functions (cf. also Blackwell and Dubins, Illinois J.Math.1, 508-514, 1963).

5. L.Sucheston

Extend the Blum-Hanson mean ergodic theorem for subsequences to "mixing" operators on uniformly convex Banach spaces. Recall that for such spaces X Kakutani (Tohoku Math.J.) proved that $x_n \in X$, $\frac{1}{n}\sum_0^{n-1} x_i \to$ weakly, implies $\frac{1}{n}\sum_0^{n-1} x_{k_i}$ converges strongly for a subsequence x_{k_i}.

6. S.Kakutani

Can one always build a skyscraper over a given transformation so that the resulting transformation is weakly but not strongly mixing?

7. S.Kakutani, v.Neumann

Let
$$b(n) = \exp\left(2\pi i\left\{\frac{\varepsilon_1}{p_1} + \frac{\varepsilon_2}{p_2} + \dots + \frac{\varepsilon_k}{p_k}\right\}\right),$$
where $0 < p_1 < p_2 < \dots \to \infty$ is fixed, and $n = \varepsilon_1 + \varepsilon_2 p_1 + \varepsilon_3 p_1 p_2 + \dots$
$$\dots + \varepsilon_k p_1 \dots p_{k-1},\ 0 \leq \varepsilon_i < p_i.$$
Is orb(b) strictly ergodic? Does it have continuous spectrum?

8. W.Parry

Determine the relation between the spectrum of a given transformation and that of its group extensions.

9. R.Adler

If T and T' are (0,1)-matrices with common largest eigenvalue, are the corresponding shifts σ and σ' (with their respective measures of maximal entropy) conjugate?

10. K.Jacobs

Does there exist an invariant measure on Lip (1,1) which gives a K-flow?

11. D.Stone

Determine an invariant for non-singular conjugacy (i.e.T and T' are non-singularly conjugate if $\exists$ S non-singular but not necessarily measure preserving such that TS = ST').

12. Ch.Grillenberger

Calculate the entropy of an a.p.mean Markov measure.

13. M.Keane

Can a strictly ergodic system have infinite entropy?

14. U.Krengel

If T is a measure preserving transformation in a σ-finite measure space, does there always exist a set E such that the return partition of E for T has finite entropy. By return partition we mean the partition of E into the sets $R_k = \{\omega : \omega$ returns to E at time k for the first time$\}$.

Ch.Grillenberger (Erlangen)