MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 16 / 1969

Gruppentheoretische Eigenschaften

1. 6. - 7. 6. 1 9 6 9

Die Tagungsleitung hatten R.Baer (Frankfurt), W. Gaschütz (Kiel) und K. Gruenberg (London). Das Tagungsthema "Gruppentheoretische Eigenschaften" umfaßt einen großen Teil der gegenwärtigen Gruppentheorie. Dabei versteht man unter einer gruppentheoretischen Eigenschaft eine isomorphismenabgeschlossene Klasse von Gruppen. Beispiele für solche Eigenschaften sind etwa (lokal) nilpotent, (lokal) auflösbar, (lokal) endlich, FC-Gruppe etc., aber auch jede Formation und jede Varietät. Diese Begriffe bilden auch schon den Rahmen für die Mehrzahl der Vorträge. Ferner wurde gesprochen in drei Vorträgen über Subnormalteilertheorie (dabei sei etwa, um den Zusammenhang mit dem Thema herzustellen, auf Roseblades koaleszente Klassen hingewiesen), über Eigenschaften allgemein in drei weiteren Vorträgen, über Dimensionsuntergruppen und, wie bei einer solchen Tagung nicht anders zu erwarten, über einige Gegenstände (BURNSIDE-Algebra, Stabilitätsgruppen etc.), die sich nicht unter das Thema subsummieren lassen.

Als besonders günstig erwies es sich, daß die Tagungsleiter schon frühzeitig einige Mathematiker dazu aufforderten, einen Überblick über ihr Forschungsgebiet zu geben. Die so zustande gekommenen Übersichtsvorträge von Barnes, Fischer, Hartley, Heineken, Kegel, P.M.Neumann, Robinson, Roseblade bildeten einen wichtigen Bestandteil der Tagung. Die Teilnehmer kamen aus Australien, England, Frankreich, USA und Deutschland.

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Vortragsauszüge

AMBERG, B.: Erweiterungen gruppentheoretischer Eigenschaften

Sei Θ eine Klasse von geordneten Paaren $(\mathfrak{X},\mathfrak{Y})$ faktorenvererblicher Eigenschaften $\mathfrak{X}$ und $\mathfrak{Y}$. Eine Gruppe G heiβt hyper-Θ-Gruppe, wenn jedes epimorphe Bild $H \neq 1$ von G einen Normalteiler $N \neq 1$ besitzt derart, daβ N eine $\mathfrak{X}$-Gruppe und $H/\mathcal{C}_H N$ eine $\mathfrak{Y}$-Gruppe ist für ein Paar $(\mathfrak{X},\mathfrak{Y})$ in Θ.

Satz 1 : Sind $\mathfrak{F}$ und hyper-Θ Eigenschaften Artinscher auflösbarer Gruppen, ist $\mathfrak{F}$ faktorenvererblich, so sind äquivalent:

(I) Jede Erweiterung einer $\mathfrak{f}$ -Gruppe durch eine hyper-Θ-Gruppe ist eine hyper-Θ-Gruppe.

(II) Jede endliche zerfallende Erweiterung eines eindeutig bestimmten minimalen Normalteilers mit der Eigenschaft $\mathfrak{f}$ durch eine hyper-Θ-Gruppe ist eine hyper-Θ-Gruppe.

Satz 2 : Ist hyper-Θ eine Eigenschaft auflösbarer Gruppen, so ist die Artinsche Gruppe G genau dann eine hyper-Θ-Gruppe, wenn gilt:

(*) Ist E eine endlich erzeugbare Untergruppe von G, so ist die Frattini-Faktorgruppe $E/\phi E$ eine hyper-Θ-Gruppe.

BARNES, D.W.: Formations of Lie algebras

A brief outline of the theory of formations, Schunck classes and projectors for Lie algebras, and comparison with the corresponding theory for finite groups. The principal differences are

(1) For fields of characteristic p, the conjugacy theorem holds only under the assumption that the intersection of the terms of the descending central series satisfies the $(p-1)^{st}$ Engel condition.

(2) If the field is not algebraically closed, then not all saturated formations are locally definable.

(3) If the field is algebraically closed of characteristic 0, then the only Schunck classes are the zero class, the class of nilpotent algebras and the class of all soluble algebras.

BLESSENOHL, D. : Zur Theorie der Schunckklassen und gesättigten Formationen

Mit $\mathfrak{S}$ sei die Klasse der endlichen auflösbaren Gruppen bezeichnet. Sei $\mathfrak{F} \subseteq \mathfrak{S}$ eine Schunckklasse (= gesättigtes Homomorph) und $\mathfrak{G} \subseteq \mathfrak{S}$ irgendeine Klasse. Sei $\mathfrak{G}:\mathfrak{F} \subseteq \mathfrak{S}$ definiert durch: " $X \in \mathfrak{G}:\mathfrak{F} \iff$ Die $\mathfrak{F}$-Projektoren von X sind aus $\mathfrak{G}$". Dann gilt :

(1) Ist $\mathfrak{Y}$ eine gesättigte Formation und $\mathfrak{X} = \mathfrak{S}_\pi$ die Klasse der endlichen auflösbaren π-Gruppen, so ist $\mathfrak{Y}:\mathfrak{X}$ eine gesättigte Formation.

(2) Ist $\mathfrak{Y}$ eine Schunckklasse, so ist $(\mathfrak{Y}:\mathfrak{X}) \cap N(\mathfrak{X})$ eine Schunckklasse, wobei $N(\mathfrak{X})$ die Klasse der Gruppen mit normalem $\mathfrak{X}$-Projektor ist. Für $\mathfrak{X} = \mathfrak{S}_\pi$ lassen sich die $(\mathfrak{Y}:\mathfrak{X}) \cap N(\mathfrak{X})$-Projektoren leicht angeben: Sei G ein $\mathfrak{Y}$-Projektor einer π-Hallgruppe H von X und K eine π'-Hallgruppe von $N_X(G)$, dann ist GK ein $(\mathfrak{Y}:\mathfrak{X}) \cap N(\mathfrak{X})$-Projektor von X.

DADE, E.C. : Carter Subgroups and Fitting Heights of Finite Solvable Groups

By a simple example, one can see that it is impossible to approach this problem in the manner of Thompson. In fact, the example is a solvable group S of arbitrary nilpotent length and a group A of prime order acting as automorphisms of S, such that $C_S(A)$ always has nilpotent length 2 .

DOERK, K. : Zur Theorie der Formationen endlicher auflösbarer Gruppen

Ist F eine Formation, so hat $T(F) = \{H \mid H$ gesättigte Formation, $H \leq F\}$ maximale Elemente bezüglich $\leq$. Im allgemeinen gibt es mehr als ein maximales Element. Es werden nun zwei Klassen von ungesättigten Formationen angegeben, für die es genau ein maximales Element gibt:

1) NF ist das einzige maximale Element in $T(Y_F)$, wobei Y_F die Formation der F-SC-Gruppen ist.

2) Ist P_H die Formation derjenigen Gruppen, deren Praefrattinigruppen in der gesättigten Formation H liegen, so ist die Formation, die lokal durch $\{P_{H(p)}\}$ erklärt wird, das einzige maximale Element von $T(P_H)$.

FISCHER, B. : Classes of conjugate subgroups of finite soluble groups

Since Carter's discovery of self-normalizing nilpotent subgroups of finite soluble groups a number of canonical conjugacy classes of subgroups have been found. A survey was given on some results and open problems.

GÖBEL, R. : Klassen perfekter Gruppen

Ist $\mathfrak{n}$ eine gruppentheoretische Eigenschaft, so heißt eine Gruppe G eine $\mathfrak{n}$-perfekte Gruppe, wenn G der einzige Normalteiler N von G mit $\mathfrak{n}$-Faktorgruppe G/N ist. Für die Klasse $\mathfrak{n} = \mathfrak{A}$ der abelschen Gruppen ist $\mathfrak{n}$-Perfektheit die gewöhnliche Perfektheit. Wir beweisen den folgenden

Satz : Ist $\mathfrak{n}$ eine faktorenvererbliche gruppentheoretische Eigenschaft, so sind äquivalent:

(1) (a) Es gibt von 1 verschiedene $\mathfrak{n}$-perfekte Gruppen.
(b) $\mathfrak{n}$-Perfektheit vererbt sich auf subkartesische Produkte.

(2) Es ist $\mathfrak{n} = \mathfrak{I}$ die nur aus 1 bestehende Gruppenklasse.

JOHNSEN, K. : Die Burnside-Algebra einer endlichen Gruppe

Die G-Mengen einer endlichen Gruppe G bilden einen Ring B(G), wenn als Summe von G-Mengen die disjunkte Vereinigung und als Produkt das cartesische Produkt mit naheliegender G-Operation definiert wird. Es wurden Sätze über die arithmetische Struktur von B(G) und der Algebra $B(G) \otimes_{Z} K$ (K Körper) angegeben.

HARTLEY, B. : The Stability Group of a Series of Subgroups

A series of a group G is a set $(\Lambda_\sigma, V_\sigma \,;\, \sigma \in \Omega)$ of pairs of subgroups of G indexed by a totally ordered set Ω and such that (i) $V_\sigma \lhd \Lambda_\sigma \,(\forall\, \sigma \in \Omega)$, (ii) $\sigma < \tau \Longrightarrow \Lambda_\sigma \leq V_\tau$, (iii) $G-1 = \bigcup_{\sigma \in \Omega} (\Lambda_\sigma - V_\sigma)$; its stability group is the set of all automorphisms α of G such that for each $\sigma \in \Omega$, $x^{-1} x^\alpha \in V_\sigma$ for all $x \in \Lambda_\sigma$. We describe the way the structure of the stability group is influenced by the nature of the series, that is, by the order-type of Ω and the way the subgroups comprising the series are embedded in G .

HEINEKEN, H. : Engelprobleme

Engelprobleme sind alle Probleme, die mit der Gleichung $x^{(n)} o y = 1$ zusammenhängen. (1) Welche Gruppen gehören zu der von Gruenberg definierten Klasse E ? Man weiß, daß die auflösbaren Gruppen, die Gruppen mit Maximalbedingung und die mit Minimalbedingung dazugehören. (2) Welche Vererbungseigenschaften hat E ? Sie ist nicht faktorenvererblich aber normalteilervererblich. (3) Was weiß man für kleine n in der Gleichung ? Ab n = 4 herrscht große Ungewißheit.

KAPPE, W. : Properties which are inherited by products of relatively prime index.

Call H a t*-product of M and N if (i) M and N are normal (ii) there is a system T of generators for H such that t_1 mod M and t_2 mor N have finite and relatively prime order for all $t_i \in T$. If T = H satisfies (ii) we call H a t-product of M and N.

Theorem I. Suppose $E = \{f(p)\}$ is a locally defined formation.

(1) If each f(p) is inherited by t-products [t*-products] so is E.

(2) If each f(p) consists of primary groups only then each formation F $\leq$ E is inherited by t-and t*-products.

(3) If E is full and integrated and inherited by t-products [t*-products] then each f(p) is inherited by t-products [t*-products].

Theorem II. Suppose E is defined by a commutator identity, and LN is the class of locally nilpotent groups. Then E $\cap$ LN is inherited by t- and t*-products.

KEGEL, O.H. : Locally finite groups

Motivation for and comments on the following problems:

1. What can be said about the structure of a locally finite group G if every section of G has a satisfactory Sylow theory (for primes in a set π) ?

2. Does every simple locally finite group have a local system consisting of finite simple groups ?

3. Is there a non-linear locally finite simple group G with one of the following properties :

   a) For some prime p, there is a maximal elementary p-subgroup of G which is finite ?

   b) For some prime p, there is a Sylow p-subgroup of finite exponent in G ?

   c) For some prime p, there is a soluble Sylow p-subgroup in G ?

(The existence of such a group would imply that infinitely many "new" finite simple groups are involved in it.)

3'. Is there a non-linear locally finite simple group with some Sylow subgroup hyperabelian ?

4. Is every non-linear locally finite simple group a limit of finite symmetric groups ?

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LEDLIE, J.F. : On Free Metabelian D -Groups

For π a set of primes, a D_π-group is a group G with the property that, for every element g in G and for every prime p in π, g has a unique p-th root in G. The collection of all metabelian D_π-groups forms a variety of algebras, so the notion of free metabelian D_π-group is defined.

Two faithful representations of free metabelian D_π-groups are established : The first is inside a suitable power series algebra and resembles W.Magnus' representation of free groups inside a 'free' power series algebra; this representation implies that free metabelian D_π-groups are residually torsion-free nilpotent. The second representation is in terms of two-by-two matrices and is analogous to W.Magnus' representation of free metabelian groups using two-by-two matrices. These two representations are utilized to derive several properties of free metabelian D_π-groups.

LEVIN, F. : On the Laws of Free Nilpotent Groups

We demonstrate

i) a short proof that the variety of all nilpotent groups of class $\leq c$ $(c \geq 3)$ is generated by its free group of rank c-1 but not by its free group of rank c-2 ,

ii) a basis for the laws of a free nilpotent group of class c and rank c-2 .

MACDONALD, Sheila : The Laws of $PSL(2,p^n)$

It is relatively easy to see the type of laws required for a basis of a $PSL(2,p^n)$, the difficulty lies in actually calculating them in a specific case. We show how a property of the traces of the coresponding elements of $SL(2,p^n)$ assists in this calculation, and also give two types of law which hold for all p.

MAIER, R. : <u>Groups in which all quasinormal subgroups are normal</u>

Let G be a finite group. Q is <u>quasinormal</u> in G, ($Q \underset{q}{\le} G$), when QU=UQ for all subgroups U of G (Ore). Every quasinormal subgroup of G is subnormal in G. Let $S \trianglelefteq\trianglelefteq G$. S is <u>seminormal</u> in G ($S \underset{s}{\le} G$), when SX=XS for all $X \trianglelefteq\trianglelefteq G$. S is <u>strongly seminormal</u> in G ($S \underset{sts}{\le} G$), when $S \underset{s}{\le} W$ for all subgroups W containing S. G is a <u>q-n-group</u>, when $Q \underset{q}{\le} G$ implies $Q \trianglelefteq G$. Let $\mathfrak{Q}$ be the class of all groups G, such that all subgroups of G are q-n-groups. $\mathfrak{Q}$ is closed under taking subgroups and homomorphic images. K is called a <u>critical</u> group, when K is no q-n-group, but all proper subgroups and all proper factor-groups are q-n-groups. We characterize now the class $\mathfrak{Q}$:

Theorem I. The following statements are equivalent:

a) $G \in \mathfrak{Q}$.

b) $S \underset{s}{\le} U \le G \Longrightarrow F(U) \le N(S)$, ($F(U)$ = Fittingsubgroup of U).

c) $T \underset{sts}{\le} U \le G,\ T \le H(U) \Longrightarrow T \trianglelefteq U$, ($H(U)$ = hypercenter of U).

d) No Sylow subgroup P of G has a factor U/M ($M \trianglelefteq U \le P$) which is critical.

The critical groups are determined by the following

Theorem II. K is critical $\Longleftrightarrow K = N \rtimes Q,\ N = \langle n \rangle,\ Q = \langle s \rangle$,

$$n^{p^{\gamma-1}} = s^p = 1,\ n^s = n^{p^{\gamma-2}+1} \text{ with } \gamma \ge 3 \text{ and } p^\gamma \ne 8 .$$

MEYN, H. : <u>Zur Theorie der FC-Gruppen</u>

Unter Benutzung der folgenden gruppentheoretischen Eigenschaften und Operatoren

$\mathfrak{A}$ = abelsch, $\mathfrak{G}$ = endlich-erzeugbar, $\mathfrak{F}$ = endlich, $\mathfrak{K}$ = FC,

$\mathfrak{P}$ = periodisch, $\mathfrak{T}$ = torsionsfrei, $\mathfrak{S}$ = schicht-endlich und

$L\mathfrak{X}$ = lokal-$\mathfrak{X}$-Gruppen, $L^*\mathfrak{X}$ = lokal-normal-$\mathfrak{X}$-Gruppen,

$Z\mathfrak{X}$ = Gruppen G mit $G/Z \in \mathfrak{X}$, $K\mathfrak{X}$ = Gruppen G mit $G' \in \mathfrak{X}$

$\mathfrak{X}\,\mathfrak{X}'$ = Erweiterungen von $\mathfrak{X}$- mit $\mathfrak{X}'$-Gruppen und

$\mathfrak{X} ⓩ \mathfrak{X}'$ = zentrale Erweiterungen von $\mathfrak{X}$- mit $\mathfrak{X}'$-Gruppen

wird folgende Kette von Inklusionen bewiesen :

$$\mathfrak{B} \wedge \mathfrak{K} \subset Z\mathfrak{F} \begin{array}{c} \subset Z\mathfrak{S} \supset \\ \supset K\mathfrak{F} \subset \end{array} \mathfrak{K} \subset ZL^{*}\mathfrak{F} \begin{array}{c} \subset ZL\mathfrak{F} \subset LZ\mathfrak{F} = L\mathfrak{K} = LK\mathfrak{F} \supset \\ \supset L^{*}Z\mathfrak{F} \subset L^{*}K\mathfrak{F} \subset KL^{*}\mathfrak{F} \subset \end{array} KL\mathfrak{F} \subset K\mathfrak{O} \subset \mathfrak{O}\mathfrak{T}$$

Es ergibt sich u.a. ein einfacher Beweis eines Satzes von Černikow, der die FC-Gruppen wie folgt charakterisiert:

$$\mathfrak{K} = (\mathfrak{T} \wedge \mathfrak{A}) ⓩ L^{*}\mathfrak{F} .$$

MORAN, S.: <u>Dimension subgroups modulo n</u>

Let $D_{i,p^h}(G)$ denote i-th dimension subgroup of G modulo p^h and F_{i,p^h} denote the evaluation of $D_{i,p^h}(F)$ given by Lazard for a free group F. Then $D_{i,p^h}(F/N) = F_{i,p^h}N/N$ for an arbitrary p-group F/N and all $i \leq p$. However, there exists a finite p-group $F/N = \{x,y;\ x^{p^{h+1}} = y^{p^2} = 1,\ x^{p^{h-1}} = [x,y]\}$ with the property that $D_{p+1,p^h}(F/N) \neq F_{p+1,p^h}N/N$ for every $h \geq 2$ (except when $h = p = 2$).

NEUMANN, P.M. : <u>Pseudo-abelian varieties of groups</u>

There is a problem of great importance for the general theory of varieties - namely, does there exist a non-abelian variety which contains no non-abelian finite (equivalently no non-abelian soluble) groups ? (Problem 5, p.42, Hanna Neumann's book). This problem arises in extraordinarily many investigations in the theory of varieties. The point of my talk is to propound the

Vermutung: In such a variety (a pseudo-abelian variety), every group is a t-group (i.e. subnormal subgroups are normal).

NEUMANN, P.M. : Varieties of groups as categories

A survey of those parts of the general area of varieties of groups which are particularly motivated by questions from universal algebra and category theory. In particular recent work on the problems:

1. When are left-cancellable morphisms in a variety surjective ?
2. Which varieties admit a "generalized free product with amalgamated subgroup" ?
3. Is there a cohomology theory for a variety, which classifies extensions in dimension 2 ?
4. Given that there are such, how many are there ?
5. In particular, are the cohomologies defined by Burr and Beck, by André, by Rhinehart, etc. all the same ?

NEWELL, M. : Soluble min - by - max Groups

A group G is called a min-by-max group if it is an extension of a group satisfying the minimal condition on subgroups by a group which satisfies the maximal condition on subgroups. We call a group critical if it is an extension of a periodic abelian group by a torsionfree abelian group and prove :

"A radical group G is min-by-max if and only if its nilpotent critical subgroups are min-by-max."

POMMER, H. : Fortsetzung von Funktoren bei topologischen Gruppen

Es sei S die Abbildung, die jeder endlichen Gruppe die Menge aller ihrer p-Sylowgruppen zuordnet. S läßt sich auf die Klasse aller pro-endlichen Gruppen fortsetzen.

In diesem Vortrag werden zwei Wege gezeigt, wie man diese konkrete Situation verallgemeinern kann. Dabei erweist sich die kategorien-theoretische Sprechweise am angemessensten. In der Anwendung der gewonnenen Ergebnisse wird auch von subnormalen Untergruppen und Sylowsystemen (bzw. Verallgemeinerungen davon) die Rede sein.

POWELL, M.B. : "Disjunctions" of Groups

If $\{w_i\}_{i=1}^{n}$ is a set of words of a free group F, we shall say that the group G satisfies the disjunction $w_1 \vee w_2 \vee \dots \vee w_n$ if for every homomorphism $\phi : F \longrightarrow G$, $w_i \phi = 1$ for at least one value of i. From a disjunction we may construct words which are laws of every group satisfying the disjunction, and, what is more important, which allow us to recover the disjunction in certain groups - e.g. non-abelian simple groups - having these words as laws. This procedure may be applied to derive laws of the two-dimensional linear groups (over a finite field) which are sufficient to ensure that the variety they define is locally finite.

[The above is essentially due to R.M. Bryant.]

RAE, A. : Soluble groups possessing a normal local system of finite subgroups

A local system Σ of finite subgroups is normal if $U,V \in \Sigma$, $U \leq V \Longrightarrow U \trianglelefteq \trianglelefteq V$. Let $\mathcal{L}$ denote the class of groups possessing such a system.

Problem : Are maximal p-subgroups of $\mathcal{L}$-groups isomorphic ?

Theorem : If $G \in \mathfrak{A}^n \cap \mathcal{L}$ (soluble and $\mathcal{L}$) then G modulo its Hirsh-Plotkin radical is f.c. The locally nilpotent residual of G is generated by finite normal subgroups of G.

Cor. : Maximal p-subgroups of soluble $\mathcal{L}$-groups are isomorphic modulo the Hirsh-Plotkin radical of G.

Examples : There exists an ($\mathfrak{A}^3 \cap \mathcal{L}$)-group whose maximal 2-subgroups are not locally conjugate; and an ($\mathfrak{A}^2 \cap \mathcal{L}$)-group of exponent 6 whose maximal 2- and maximal 3-subgroups are abelian and self normalizing.

ROBINSON, D.J.S. : <u>Groups which are minimal with respect to normality being intransitive</u>

A <u>$\mathfrak{T}$-group</u> is a group in which normality is a transitive relation; other groups are called <u>non-$\mathfrak{T}$-groups.</u> A <u>minimal non-$\mathfrak{T}$-group</u> is a non-$\mathfrak{T}$-group all of whose proper subgroups are $\mathfrak{T}$-groups. While the existence of infinite minimal non-$\mathfrak{T}$-groups is problematic, the finite minimal non-$\mathfrak{T}$-groups can be completely determined and this classification accounts for all minimal non-$\mathfrak{T}$-groups which are locally finite or 2-groups or SJ-groups in the sense of Kuroš.

ROBINSON, D.J.S. : <u>Finiteness conditions and solubility</u>

This is a survey of progress in the theory of soluble and generalized soluble groups subject to finiteness conditions. First of all soluble groups whose abelian factors have their components all of finite rank are discussed. Then some of the generalizations of the classical theorems of Mal'cev and Černikov (on soluble groups whose abelian subgroups satisfy the maximal or minimal condition) are described. Finally P. Hall's theory of finitely generated soluble groups is discussed and some applications given.

ROSE, J.S. : Splitting of extensions and the faithful irreducible representations of some monolithic groups

Proposition: Let G be a finite monolithic group whose unique minimal normal subgroup has order 2, $|G| > 2$. Let p be a prime $\equiv -1 \pmod 4$. Then any faithful irreducible representation of G over GF(p) has even degree.

ROSEBLADE, J.E. : Centralizers in finitely generated nilpotent groups

Let G be a finitely generated abelian by nilpotent group. I shall discuss and prove two theorems which I have proved with my student J.C.Lennox.

(A) There exists $N = N(G)$ so that any subgroup H of G has upper central height at most N.

(B) There exists $N = N(G)$ so that for all $i \neq 0$ and all x in G the inclusion $C_G(x^i) \leq C_G(x^N)$ holds.

(B) has as a consequence that

(C) G satisfies the maximum condition for centralizers.

Both (A) and (B) extend easily to the class $(\mathfrak{N}^2\mathfrak{F})$ Max - n.

ROSEBLADE, J.E. : The join problem for subnormal subgroups

Let $\mathfrak{X}$ be a class of groups. $\mathfrak{X}$ is coalescent $\Longleftrightarrow$ whenever H and K are subnormal $\mathfrak{X}$-subgroups of a group G then $\langle H,K \rangle$ is also a subnormal $\mathfrak{X}$-subgroup of G. A survey will be given of the present state of knowledge about coalescent classes.

SANDLING, R. : The Dimension Subgroup Problem

I will prove two results about this problem :

1) Every finite p-group is contained in one which satisfies the dimension subgroup conjecture.

2) Abelian-by-cyclic groups satisfy the conjecture.

I will also show that the subgroups attached to the Lie powers of the argumentation ideal give some evidence for the possible truth of the conjecture and that they suggest that even stronger theorems may hold.

SCHOENWAELDER, U.F.K. : Normal Subgroups Containing Their Centralizer

We showed how one can obtain the following two results (and similar ones) from a general principle formulated in terms of subgroup theoretical properties. Every finite p-group G has a characteristic subgroup N with $\mathfrak{C}_G(N) = \mathfrak{Z}(N)$ and $N/\mathfrak{Z}(N) \subseteq \Omega_1 \mathfrak{Z}(G/\mathfrak{Z}(N))$ [Feit - Thompson]. Every hyperabelian group G has a normal subgroup N with $\mathfrak{C}_G(N) = \mathfrak{Z}(N)$ and $N/\mathfrak{Z}(N)$ abelian [Baer].

The parameters involved are a sublattice ν of the lattice of normal subgroups of the group G, subgroup theoretical properties α and β on ν , a $[\nu,\alpha]$-immersed ν -subgroup R of G with $(R,G,G) \in \beta$ and a centralizer function $c: \nu \longrightarrow \nu$ replacing $\mathfrak{C}_G$. Suitable hypotheses on α,β and c guarantee the the existence of a ν -subgroup N with $(c_0N,N,G) \in \alpha$, $(c_0N,cN,G) \in \beta$ and cN/c_0N centralized by $Z = \mathfrak{Z}(G \bmod R)$; here $c_0N = N \cap cN$.

SEITZ, G.M. : Modular p-Groups and Formations

I will report on joint work with Professor C.R.B. Wright. If P is a modular p-group ($p > 2$) then we prove that either P is abelian or there are subgroups $P_1 < P_2$ of P such that P_1, P_2 are characteristic in P, $P_1 \geq \phi(P)$ (the Frattini-subgroup of P) and P_2/P_1 is centralized by the full automorphism group of P. Combining this theorem with work of H.N. Ward and work of B.Huppert we obtain restrictions on the $\mathfrak{F}$-residual of solvable groups.

STONEHEWER, S.E.: The Join of Finitely many Subnormal Subgroups

Let $\mathfrak{X}$ be a class of groups closed with respect to taking normal subgroups and epimorphic images. Suppose that $\mathfrak{X}$ contains the class of abelian groups. Let $G = < H_i;\ i = 1,2,\dots,m >$, $H_i \lhd^n G$, $1 \leq i \leq m$.

Theorem A. If $H_i \in \mathfrak{X}$, $1 \leq i \leq m$, then G has a finite series, of length $f = f(m,n)$, with residually-$\mathfrak{X}$-factors.

Taking $\mathfrak{X}$ to be the class of soluble groups of derived length $\leq d$, we have the following result, answering questions of Robinson, Roseblade and Stonehewer.

Corollary. The join of finitely many subnormal soluble subgroups is soluble, of derived length bounded by a function of the separate derived lengths and subnormality indices.

Theorem B. Let $H_i^* \lhd H_i$, $H_i/H_i^* \in \mathfrak{X}$, and suppose H_i^* has no non-trivial $\mathfrak{X}$-quotients, $1 \leq i \leq m$. Then $G^* = < H_i^*;\ 1 \leq i \leq m > = H_1^* H_2^* \dots H_m^* \lhd G$, G/G^* has a finite series, of length $f = f(m,n)$, with residually-$\mathfrak{X}$-factors, and G^* has no non-trivial $\mathfrak{X}$-quotients.

When $\mathfrak{X}$ is the class of soluble groups of derived length $\leq d$, this generalizes a result of Wielandt (1955), where it was assumed, in addition, that G has a composition series.

TOMKINSON, M.J. : Formations of Locally Soluble FC-Groups

A periodic locally soluble FC-group G possesses covering $\mathfrak{F}$-subgroups and any two such subgroups are locally conjugate in G. These subgroups are constructed from the covering $\mathfrak{F}$-subgroups of the finite factor groups of G using the fact that G/Z is residually finite. To prove that the subgroups constructed have the necessary covering property one first consider the properties of $\mathfrak{F}$-normalizers of periodic locally soluble FC-groups.

WIELANDT, H. : Subnormalität und Vertauschbarkeit

1. In einer endlichen Gruppe kann man die Existenz von Normalteilern mitunter aus dem Satz von Kegel erschließen: Ist $AB \neq G$; $A,B \neq 1$; A^x vert B^y, so ist G nicht einfach. Etwas schärfer kann man zeigen: Aus $G = AB^G = BA^G$ und A^x vert B^y folgt $G = AB$. Zusammen mit einer Verallgemeinerung der Normenreihe ("Für $A,B \leq G$ setze man $A_o = B_o = G$, weiter $A_n = A^{B_{n-1}}$, $B_n = B^{A_{n-1}}$") findet man: Ist G endlich, $A,B \leq G$ und für alle $x,y \in G$ stets A^x vert B^y, so ist $A^B \cap B^A$ subnormal in G. Offen bleibt die Frage nach allen Subnormalitätseigenschaften, die aus diesen Voraussetzungen folgen.

2. Zu einer brauchbaren Definition verallgemeinert subnormaler Untergruppen kommt man vielleicht, indem man in endlichen Gruppen die subnormalen Untergruppen durch Vertauschbarkeitseigenschaften kennzeichnet.

Dieter Blessenohl (Kiel)