MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1969

Differentialgeometrie im Großen

13.7. bis 19.7.1969

Die Tagung über Differentialgeometrie im Großen stand unter der Leitung von M.BARNER (Freiburg), S.S.CHERN (Berkeley) und W.KLINGENBERG (Bonn). Der größte Teil der Vorträge beschäftigte sich mit Beziehungen zwischen Krümmung und Topologie (Voraussetzungen über mittlere Krümmung, Schnittkrümmung, "pinching" etc.): CHARLAP, CHEEGER, GROMOLL, HEINTZE, KULKARNI, MATSUSHIMA, PRAKASH, SIMON, TSAGAS, TSUKAMOTO.

Über die Geometrie von Immersionen wurde von CHERN, DO CARMO, GARDNER, MÜNZNER, POHL, WILLMORE vorgetragen. Dabei wurde insbesondere auf Variationsprobleme (Minimalflächen, PLATEAUsches Problem, geschlossene Geodätische) eingegangen: CHERN, ELIASSON, HILDEBRANDT, KLINGENBERG und die Theorie der partiellen Differentialgleichungen unter geometrischem Aspekt von EBIN, ELIASSON, GARDNER, KAMBER und TONDEUR behandelt.

Viele dieser Probleme führen zwangsläufig zur Betrachtung von unendlich-dimensionalen Mannigfaltigkeiten: CRAEMER, EBIN, ELIASSON, KLINGENBERG, WEINSTEIN.

Andererseits wurden auch so klassischen Gegenständen wie Raumkurven (LITTLE, POHL, WILLMORE) und Zylindern (MÜNZNER) neue Seiten abgewonnen. Über Holonomiegruppen wurde von CHARLAP und GRAY, über die Struktur von Singularitäten von CALABI, SINGER und WEINSTEIN, über komplexe Distanzfunktionen von KOBAYASHI und über geodätischen Fluß von GREEN vorgetragen.

Vormittags gab es 1/2-stündige Vorträge mit speziellen Resultaten, nachmittags 1-stündige "surveys" über folgende Gebiete: Minimale Immersionen; Konvexität in der globalen RIEMANNschen Geometrie; Elliptische Operatoren; Geodätischer Fluß; Geschlossene Geodätische; Mannigfaltigkeiten nichtnegativer Krümmung.

Unter allgemeiner Zustimmung dankte WILLMORE beim Abschluß der Tagung den Tagungsleitern und dem Institut für die hervorragende Organisation und den harmonischen Verlauf dieser Tagung.

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Vortragsauszüge

S.Kobayashi: Invariant distanses on complex manifolds

Let M be a complex manifold and D the open unit disk in $\mathbb{C}$ withPoincaré distance(noneuclidean distance)ρ. The Caratheodory pseudo-distance c_M is defined by $c_M(p,q) = \sup \rho(f(p), f(q))$, where the supremum is taken with respect to all holomorphic mappings $f: M \to D$. We introduce another intrinsic pseudo-distance d_M as follows. Choose points $p=p_0, p_1 \ldots p_{k-1}$, $p_k = q$ in M, points a_i, b_i in D, (i=1...k), and holomorphic mappings $f_i: D \to M$ such that $f_i(a_i) = p_{i-1}$, $f_i(b_i) = p_i$. We set

$$d_M(p,q) = \inf \{ \rho(a_1, b_1) + \cdots + \rho(a_k, b_k) \},$$

where the infimum is taken with respect to all possible choices of $p_1, \ldots p_{k-1}, a_i, b_i, f_i$. Basic properties of c_M and d_M:

1. $c_D = d_D = \rho$
2. If M and N are complex spaces and $f: M \to N$ is holomorphic, then $d_N(f(p), f(q)) \leq d_M(p,q)$,for p,q$\in$M.

While c_M is the smallest pseudo-distance with properties 1. 2., d_M is the largest.

Examples

1. If M is a homogeneous space of a complex Lie group, then $d_M = 0$. We call M hyperbolic if d_M is a (true) distance(inducing the original topology of M).

2. M is hyperbolic in the following cases:

1 M=bounded domain in $\mathbb{C}^n$

2 M= $P_1(\mathbb{C})$-3 points.

3 M= $P_2(\mathbb{C})$ - complete quadrilateral.

4 M= Kähler manifold with holomorphic sectional curvature $\leq k \leq 0$

5 Product of hyperbolic spaces.

6 M having a bounded domain as a covering manifold.

7 M= the space of holo. maps from a compact complex space into a hyperbolic space.

8 M= the space of moduli of a compact Riemann surface.

Applications The automorphisms group of a hyperbolic space is a Lie group.

The automorphism group of a compact hyperbolic space is finite.

A generalised big Picard theorem.(to higher dimensions)

Every compact hyperbolic manifold is a minimal manifold.

Reference Journal of Math. Soc. Japan.1967.

T.J. WILLMORE: <u>Integral theorems concerning Mean curvature of immersed manifolds</u>.

Let M and M' denote complete riemannian manifolds of dimensions n and m respectively, and suppose that M' is compact and without boundary. Let $f: M \to M'$ be a riemannian immersion, and let ζ be the mean curvature vector field of the immersion. We consider problems associated with minimising the integral

$\frac{1}{c}\int_M <\zeta,\zeta> *1$ for variable f, and suitable constant c

where the scalar product is defined over the space of vector fields along f.

Y. TSUKAMOTO: <u>Certain riemannian manifolds of pos. curvature</u>

<u>Theorem 1</u>: Let M be an n-dimensional complete simply connected riemannian manifold with sectional curvature K, $\frac{1}{4} < k \leq K \leq 1$, where k is a constant. If there exists a closed geodesic of length $2\pi/\sqrt{k}$ on M, then M is isometric to a sphere with constant curvature k.

<u>Theorem 2</u>: Let M be an even dimensional complete simply connected riemannian manifold with sectional curvature K, $\frac{1}{4} \leq K \leq 1$.

If there exists a closed geodesic of length 2π on M, then M is isometric to one of the compact symmetric spaces of rank 1 with canonical metric.

E.CALABI: <u>Über singuläre symplektische Strukturen</u>

Man betrachtet das Problem der Existenz einer symplektischen Struktur auf einer 2n-dimensionalen kompakten Mannigfaltigkeit X, wo vorausgesetzt ist, daß die zwei bekannten notwendigen Bedingungen schon erfüllt sind, nämlich daß

1) es eine fast komplexe Struktur in X gibt,

2) es eine 2-dimensionale Cohomologieklasse α in X gibt mit $\alpha^n \neq 0$.

Es sei ω eine geschlossene 2-Form, die die Klasse α darstellt, also $\int_X \omega^n > 0$ (in Bezug auf eine geeignete Orientierung). Dann ist der Ort der Nullstellen (=S) der 2n-Form ω^n ein Träger der Hindernisse für die Existenz einer symplektischen Struktur. Wenn man zunächst Transversalitätsbedingungen für S voraussetzt, dann gibt es das Problem, die Keime dieser Singularitäten von ω zu charakterisieren. Durch die von MARTINET und dem Verfasser gewonnenen Resultate ist die Klassifizierung dieser Singularitätenkeime im Fall n=2 vollständig gelöst.

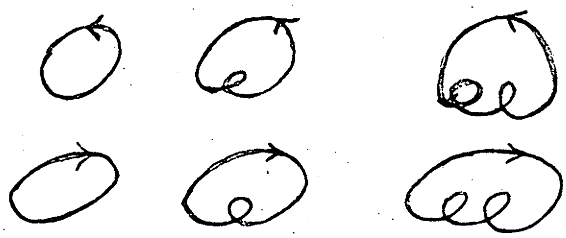
S.S.CHERN: Minimal submanifolds

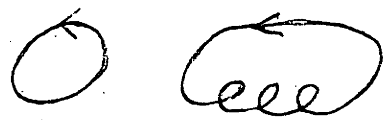
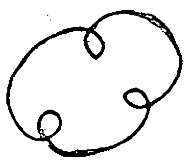
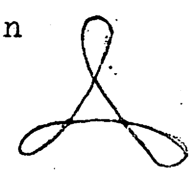
A survey of recent progress in the study of minimal submanifolds in a Riemannian submanifold, including : 1) Uniqueness theorems such as Bernstein and anti-Bernstein.2) A priori estimates of E. Heinz, Bombieri, DE Giorgi, Mirada, etc.

JOHN.A.LITTLE: Non-degenerate homotopies of curves on the unit 2-sphere.

THEOREM: There are 6 non-degenerate homotopy classes of curves on the unit 2-sphere.

Here we consider the homotopies: $h: S^1 \times I \to S^2$,such that the geodesic curvature of each curve h_t is not zero. Representatives are as follows



A key idea is to show that the curves  are non-degenerately homotopic.This is done ,essentialy, by putting the curve  over the top of the sphere to obtain  .

UDO.SIMON: Zum H-Satz in der mehr-dimensionalen Differentialgeometrie

$X: M_n \longrightarrow E_{n+m}$ sei eine C^r-Immersion einer n-dimensionalen, differenzierbaren,orientierbaren Mannigfaltigkeit der Klasse $C^r (r \geq 3)$ in den euklidischen Raum E_{n+m}.

Es werden kompakte Mannigfaltigkeiten $X(M_n)$ mit konstanter Krümmung H betrachtet(zur Definition von H vgl. etwa Eisenhart, Riemannian Geometry, S.169) und Bedingungen angegeben,unter denen $X(M_n)$ auf einer Hypersphäre liegt . Das Ergebnis enthält als Spezialfall den klassischen H-Satz für sternförmige Flächen.

N.PRAKASH: A note on immersed manifolds

Study of immersed manifolds and isometric imbeddings has been of immense interest to differential geometers during the past decade. It may be classified mainly under two headings, one which amounts to finding the least dimension of the space in which a given manifold could be immersed or isometrically imbedded and the other which deals with invariants such as total curvature and minimal total curvature associated with immersed manifolds.
In the present note we concern ourselves with second type of problem, for example by choosing a suitable structure on an immersed manifold we gather little more information regarding total curvature and minimal immersion than that provided in existing literature.

Y.MATSUSHIMA: On Hodge manifolds with zero first Chern class

Following two theorems will be proved:
1. Let Mbe a connected Hodge manifold whose first Chern class $c_1(M)$ is zero. Then the identity component G of the group of all holomorphic transformations of M is a complex torus of complex dimension $\frac{1}{2}b_1(M)$.
2. Let M be a connected Hodge manifold such that $c_1(M)=0$ and $b_1(M)>0$ Then there exists a finite covering manifold M′ of M such that $M'=A\times F$ where A is an abelian variety and F is a Hodge manifold such that $c_1(F)=b_1(F)=0$

F.W.KAMBER and Ph.TONDEUR: On the existence of certain types of invariant differential operaters.

Let M be a smooth or holomorphic manifold with structuresheaf $\underline{O}$. Consider a Lie algebra sheaf $\underline{L}$ of vectorfields on M, i.e. a sheaf of derivations of $\underline{O}$. By integeration, $\underline{L}$ defines a pseudogroup of local automorphisms of M, which acts on any naturally defined sheaf on M, e.g. vectorfields, differential forms, jets etc. This leads to the notion of a module over $\underline{L}$ and invariant maps between such modules. Consider vectorbundles E, F with section-sheaves $\underline{E}$, $\underline{F}$ over M and a k-order differential operator $D:\underline{E}\longrightarrow\underline{F}$. Assume D to be invariant i.e. compatible with respect to given actions of $\underline{L}$ on $\underline{E}$, $\underline{F}$. Then the symbol map $\sigma(D):\underline{S}^k\underline{T}^*\otimes_{\underline{O}}\underline{E}\longrightarrow\underline{F}$ is invariant, where $\underline{S}^k\underline{T}^*$ denotes the k^{th}-symmetric product of the covector-sheaf $\underline{T}^*$. Conversely, given such an invariant map $\sigma:\underline{S}^k\underline{T}^*\otimes_{\underline{O}}\underline{E}\longrightarrow\underline{F}$, we develop an obstruction theory for the existence of an invariant k^{th}-order differential operator $D:\underline{E}\longrightarrow\underline{F}$ with symbol map $\sigma(D)=\sigma$. For k=1 and $\sigma=id$, this is the problem of the existence of an invariant connection on on the vectorbundle E. This obstruction theory generalises Atiya's obstruction for the existence of complex analytic connections on a holomorphic vectorbundle [C.R.Acad.Sc.Paris, t.260(1965), 45-48]. The obstruction is analyzed in detail for a transitive Lie algebra sheaf and computed in some examples.

A.WEINSTEIN: <u>Singularities of functions and exponential mappings</u>

A 1-1 correspondence is constructed between equivalence classes of germs of families of functions parametrized by a manifold M_n and germs of maximal isotropic submanifolds of the symplectic manifold T^*M. The projection of such a manifold onto M is called a quasi-exponential mapping. The exponential mapping of a riemannian manifold and the Gauss mapping of a submanifold of Euclidean space may be considered as quasi-exponential mappings. The quasi-exponential point of view is useful for establishing generic properties of the geometrically defined exponential mappings.

M.DO CARMO: <u>Minimalsubmanifolds of the sphere.</u>

The continuation of the survey on minimal submanifolds was concerned with the special situation of isometric minimal immersion $x: S^n_k \to S^m_1 \subset \mathbb{R}^{m+1}$ of n-spheres S^n_k of constant sectional curvature k into S^m_1. A qualitative description of such immersions was sketched which is roughly as follows: The coordinate functions of x are spherical harmonics of degree s on S^n_1, for n=2, and arbitrary s, for n arbitrary and $s \leq 3$, the immersions are unique modulo a rigid motion. On the other hand for each $n \geqslant 3$ and $s \geqslant 4$, there exists a $N(n,s)$-parameter family of distinct such immersions filling a convex, compact set and $\inf_{n,s} N(n,s) \geqslant 18$.

D.B.EBIN: <u>Groups of diffeomorphisms and the motion of incompressible-Fluids.</u>

The space of C^k maps from a compact manifold M^n to some other manifold N is an infinite dimensional manifold modelled on the Banach space of C^k sections of some pull-back of T(N) over M. Similarly there are manifolds of mappings of class $C^{k+\alpha}$ and L^p_k $(k > n/p)$.
If $s > n/2+1$, $\mathcal{D}^s = \{\eta \in L^2_s(M,M) / \eta \text{ is a } C^1 \text{ diffeomorphism}\}$ is open in $L^2_s(M,M)$, and it is a group with a natural weak riemannian structure induced by such a structure on M. Also if μ is the riemannian volume of M, and $\mathcal{D}^s_\mu = \{\eta \in \mathcal{D}^s / \eta^*\mu = \mu\}$, then $\mathcal{D}^s_\mu$, as a riemannian manifold of $\mathcal{D}^s$ has a right invariant metric and smooth affine connection.

R.S.KULKARNI: <u>Curvature Structures</u>
A report on equivalence problem and the problems arising therefrom. Applications to conformal transformations, F.SCHUR's theorem etc.

E.HEINTZE: <u>Die Krümmung des Raumes SU(5)/Sp(2)xT</u>

Die einzigen bekannten einfach zusammenhängenden, homogenen Räume positiver Krümmung sind die symmetrischen Räume vom Rang 1 und die beiden BERGER-Räume $V_1=Sp(2)/SU(2)$ und $V_2=SU(5)/Sp(2)xT$. Bekanntlich beträgt das pinching der symmetrischen Räume 1/4 und das von V_1 1/37. In diesem Vortrag wurde gezeigt: $k_{V_2}=16/29*37$.

L.W.GREEN: <u>Geodesic flows</u>

This was a survey on the dynamical aspects of geodesic flows, emphasizing the existence of dense (in the tangent bundle) geodesics and the possibility of ergodicity. The techniques rewiewed were
1. symbolic description of the geodesics,
2. rotation numbers and the theorem of ARNOLD,
3.the C-systems of ANOSOV,
4. the abstraction of E.HOPF's method of asymptotic geodesics to what may be called "asymptotic coerciveness", as applied to homogenous spaces and riemannian manifolds of strictly negative curvature.

W.KLINGENBERG: <u>Closed geodesics.</u>
M compact riemannian manifold.1. Method for constructing closed geodesics,find them in the space ΛM of parametrized curves on M. Here homology methods ,homotopy methods and category methods do apply. For example: $\pi_k(M) \neq 0 \Rightarrow \pi_{k-1}(\Lambda M, \Lambda^0 M) \neq 0$
($\Lambda^0 M$ = Space of trivial curves). Hence a non-trivial element $\pi_{k-1}(\Lambda, \Lambda^0)$ determines a non-trivial geodesic.Under more restrictive assumptions one can show the existence of ∞ closed geodesics.
2. Method uses a geodesic flow $\varphi_t: T_1M \to T_1M$ .Examples:
K<o on M implies the closed geodesics in T_1M are dense.

D.CRAEMER: Homology of the space of closed curves on the real projective space.

Let M be a compact riemannien manifold. On the Hilbert manifold $\Lambda(M)$ of closed H^1-curves on M one has the energy integral E. With the help of degenerate MORSE-theory, which is equivariant with respect to some appropriate group-actions, one can calculate the homology of $\Lambda(M)$. In the case $M=P^n$=real proj. space we get:

$$H_m(\Lambda(P^n))=H_m(P^n)\bigoplus_{r=1}^{\infty}H_{m-(2r-1)(n-1)}(V_2(R^{n+1}))\bigoplus_{r=0}^{\infty}H_{m-(2r+1)(n-1)}(V_2(R^{n+1}))$$

where $V_2(R^{n+1})$ denotes the Stiefel-manifold of orthonormal 2-frames in R^{n+1}. For $\mathbb{Z}_2$-coefficients we have $H_m(V_2(R^{n+1}))=H_m(S^n x S^{n-1})$, therefore:

$$H_m(\Lambda(P^n))=\begin{cases}\mathbb{Z}_2 & o\leq m\leq n, m\neq n-1\\ \mathbb{Z}_2 x \mathbb{Z}_2 x \mathbb{Z}_2 & m=n-1\\ \mathbb{Z}_2 x \mathbb{Z}_2 & m\equiv o \mod(n-1), m\neq n-1\\ \mathbb{Z}_2 x \mathbb{Z}_2 & m\equiv 1 \mod(n-1), m\neq n\\ o & \text{otherwise}\end{cases}\qquad (n\geq 3)$$

M.SINGER: Mod 2 Index Theory

Let F_1 denote the set of skew adjoint Fredholm operators on a real Hilbert space. Let $ind_1: F_1\rightarrow \mathbb{Z}_2$ be given by $ind_1: A\mapsto \dim \ker A \mod 2$.

THEOREM: ind_1 is continous and induces a bijection on components of F_1.

COROLLARY: If A is an elliptic skew adjoint differential operator acting on smooth sections of a real-vector bundle over a compact manifold X, then $ind_1 A$ is invariant under perturbation. In particular, $ind_1 A$ depends only on $\sigma(A)$ as an element of $KR^{-1}(TX)$

The index Theorem for ind_1 was stated and some examples were given, espacially the example $A=\Sigma \oplus D_j$ with $D_j= * d+(-1)^j d *$ on $C^{\infty}(\Lambda^{2j})$ is skew adjoint for dim $X=4k+1$. In this case, $ind_1 A=k_X(R)$ the Kervaire semicharacteristic over the reals. The operator A was interpreted in terms of the bundle of Clifford algebras over X. The following theorem of M.F.Atiyah was discussed: Let T_1, T_2 be a pair of vector fields on X(of dim 4k+1). Suppose the singularity set $S=\{x\in X; \{T_1(x), T_2(x)\}$ spans a subspace of dim$<2\}$ is finite. Then $\sum_{s\in S} t(s)=k_X(R)$, where $t(s)\in\pi_{4k}(V_{4k+1,2})=Z_2$ measures the singularity at s. The proof of the following special case was given: S empty $\Rightarrow k_X(R)=o$.

H.F.MÜNZNER: Eine globale Kennzeichnung der Zylinder.

Es sei $P:\mathbb{R}^3\to\mathbb{R}^2$ die Projektion $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\longmapsto\begin{pmatrix}x_1\\x_2\end{pmatrix}$ .Betrachte Flächen

$x:M^2\longrightarrow\mathbb{R}^3$ mit den Eigenschaften: (1) $K\geq 0$. (2) $(k_1-c)(k_2-c)\leq 0$, wobei k_1, k_2 Hauptkrümmungen und $c\in\mathbb{R}$, $c\neq 0$. (3) $P\circ x: M^2\to R^2$ ist Immersion. Für die dritte Komponente $\hat{x}_3$ der Parallelfläche $\hat{x}=x+\frac{1}{c}n$ gilt dann ein Mini-Max Prinzip: Die Mengen $T_1=\{p\in M/\hat{x}_3(p)=\inf \hat{x}_3(M)\}$, $T_2=\{p\in M/\hat{x}_3(p)=\sup\hat{x}_3(M)\}$ besitzen keine kompakten Zusammenhangskomponenten (können aber durchaus $\neq\emptyset$ sein). Der Beweis benutzt die topolog. Struktur des Krümmungsliniennetzes zur Beherrschung der wesentlichen Singularitäten von $\hat{x}$ (Nabelpunkte von x). Bei Benutzung der Tatsache, dass sich nicht kompakte vollständige Flächen mit $K\geq 0$, $K\not\equiv 0$ nach Sacksteder im wesentlichen (d.h. bis auf einen eventuellen zylindrischen Teil, der sich ins Unendliche erstreckt) einwertig projizieren lassen und in Projektionsrichtung unendlich hoch sind, liefert das Mini-Max-Prinzip: Auf jeder nicht kompakten, vollständigen Fläche in $\mathbb{R}^3$ mit (1) und (2) gilt $K\equiv 0$ (d.h. sie ist zylindrisch). Dieser Satz verallgemeinert ein Resultat von Klotz und Ossermann, bei dem anstelle von (2) H=const. vorausgesetzt wird.

S. HILDEBRANDT: On the Plateau problem for surfaces with variable mean curvature.

We consider the following problem: Let H(x) be a function of class $C^1(K)$ where $K=\{x\,|\,|x|\leq 1\}\subset E^3$=3-dim Euclidean space. Suppose that Γ is a rectifiable Jordan curve contained in K. Determine a mapping: $x:\bar{B}\to\Gamma$, where $B=\{\omega=u+iv: |\omega|<1\}$, $x=x(u,v)=x(\omega)=(x^1,x^2,x^3)$, s.t. $x\in C^0(\bar{B})\cap C^2(B)$, $D(x)=\iint_B|\nabla x|^2\,du\,dv<\infty$, and $\Delta x=x_{uu}+x_{vv}=2H(x)(x_u\wedge x_v)$ as well as $|x_u|=|x_v|$, $x_u\cdot x_v=0$ in B, and $x:\partial B\to\Gamma$ is a topological mapping. As E.Heinz has remarked, there is no solution in general for $k=\sup_{x\in K}|H(x)|>1$. We prove: There is always a solution of the problem provided that $k\leq 1$.

G. TSAGAS: ON the cohomology ring of a pinched riemannian manifold of dimension 4.

Let M be a compact riemannian orientable manifold. We assume that the manifold admits a metric whose sectional curvature is k-pinched, then the cohomology ring of the manifold has some properties:

THEOREM. Let dimM=4. If k>2/11, then there exists no zero element of the cohomology group $H^2(M,\mathbb{R})$, whose square belongs to zero class.

COROLLARY. If M is homeomorphic to $S^2\times S^2$, then $k\leq 2/11$.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

ALFRED GRAY: Weak holonomy Groups

The notion of a weak holonomy group G of a Riemannian manifold M is defined. The case when G is a compact connected Lie group acting transitively and effectively on some sphere is considered. If $G \neq U(n), SU(n), G_2$ or $Spin(9)$, then it is proved that G is a subgroup of the holonomy group of M. The most interesting weak holonomy groups are G_2 and $U(n)$. If M has weak holonomy group $U(n)$, then M is a nearly Kähler manifold, i.e., $\nabla_X(J)(X)=0$ for vector fields X on M where ∇ and J denote the Riemannian connection and almost complex structre of M, respectively. A great deal can be said about the topology and geometry of such manifolds. For example, assume that M is a compact nearly Kähler manifold which is not Kählerian. If the sectional curvature of M satisfies a certain positivity condition, then the second betti- number of M vanishes.

JEFF CHEEGER: The differentiable Pinching Problem for symmetric Spaces of rank one.

Calabi and Gromoll have shown that a sufficiently pinched simply connected Riemannian manifold is diffeomorphic to the standard sphere. We extend this result by showing that a Riemannian manifold (kähler manifold) which is sufficiently pinched in the appropriate sense is diffeomorphic to a compact symmetric Space of rank one. The arguement treats the cases corresponding to the various model spaces essentially simultaneously.

LEONARD S. CHARLAP: The Group of Affinities of Compact Flat Riemannian Manifolds.

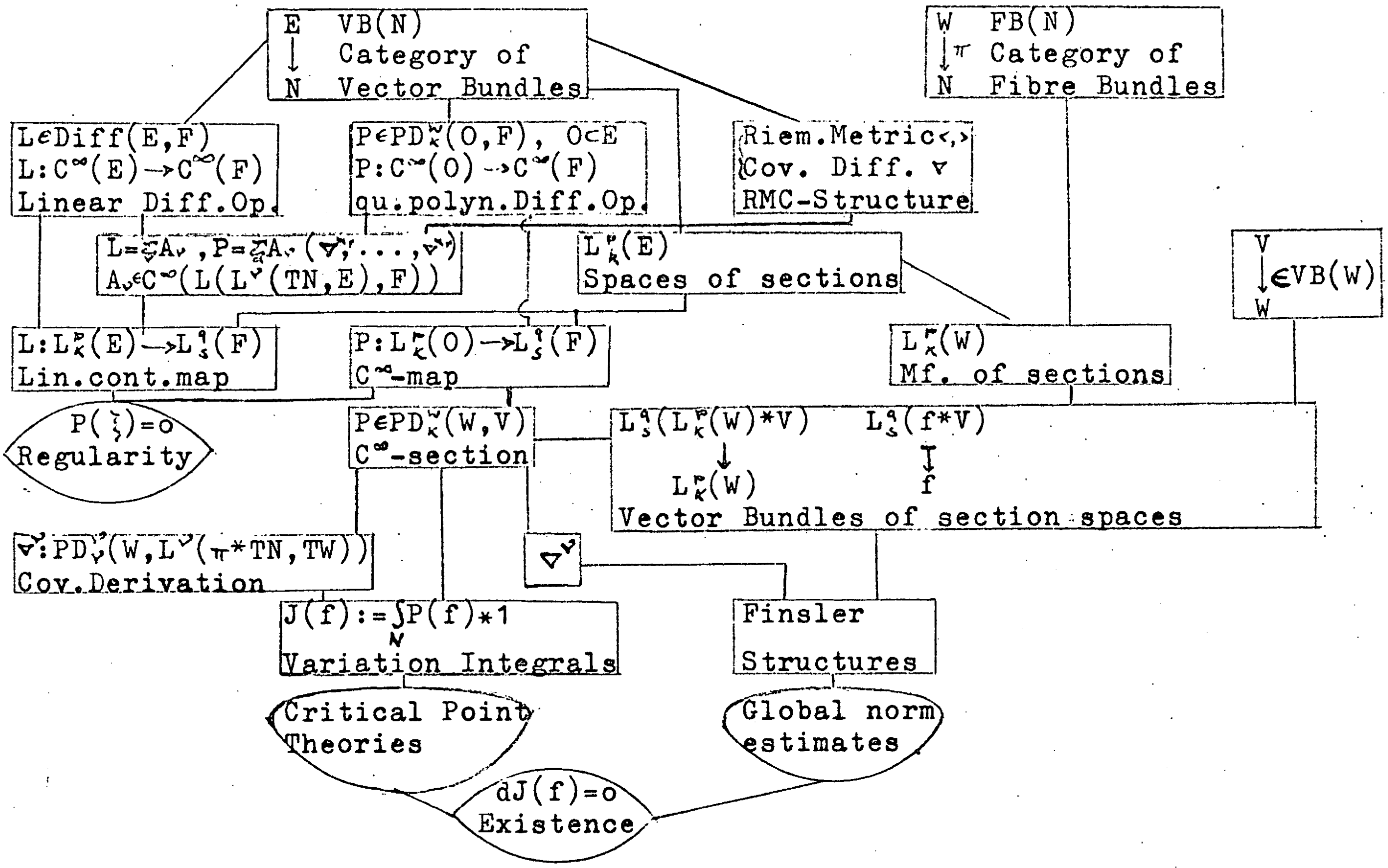
Let X ba a compact flat riemannian manifold with holonomy group $\Phi(X)$. Then it is known that X is covered by an n-dim. Torus T whose group of deck transformations is isomorphic to $\Phi(X)$. Let $Aff(X)$ be the Lie group of affinities (diffeo. that preserve the connection) of X and $Aff^0(X)$ its identity component. Then it is not hard to see that $Aff^0(X)$ is a torus whose dimension equals the first betti-number of X.

THEOREM: $Aff(X)/Aff^0(X)$ satisfies an exact sequence

$$0 \longrightarrow H^1(\Phi(X);\pi_1(T)) \longrightarrow Aff(X)/Aff^0(X) \longrightarrow N_\alpha/\Phi(X) \longrightarrow 1$$

where N_α is a subgroup of the normaliser of $\Phi(X)$ in $Aut(\pi_1(T))$. N_α is described explicitely as in the action of $N_\alpha/\Phi(X)$ on $H^1(\Phi(X);\pi_1(T))$ while partial information is obtained as to the class in $H^2(N_\alpha/\Phi; H^1(\Phi(X),\pi_1(T)))$ which describes the extension.

H.ELIASSON: Variation Integrals in Fibre Bundles



W.F.POHL: The differential geometry of secants.

In order to find relations among the various measures for submanifolds of eucl.dian spaces one considers the secant lines and their limits, the tangent lines. One parametrizes these with various other abstract spaces, analogous to the tangent bundle of an abstract differentiable manifold. One considers these using the methods of topology, Morse theory and integral geometry. From the point of view of topology one obtains theorems on the numbers of singularities [see POHL, "On a theorem related to the four-vertex problem", Annals of Math. 84 (1966), 356-367: S.JONES, "The two-vertex theorem for space curves", thesis, University of Minnesota, 1969.). From the point of view of Morse-theory one also obtains singularity theorems (see Benjamin HALPERN, "Global theorems for closed plane curves," to appear.). From the point of view of integral geometry one obtains a variety of results including the isoperimetric inequalities/and differential topological formulas (see POHL, "The self linking number of a closed space curve", Journal of Maths, Mech, 17 (1968), 975-985, "Some integral formulas for space curves and their generalisations," Annals of Math. 90 (1968), 1327-1345, POHLand T.BACHOFF, "On a generalisation of the isoperimetric inequality," to appear: J.WHITE "Self linking and the Gauss integral in higher dimensions, Annals of Math., to appear, "Some differential invariants of submanifolds of euclidean space," to appear.

R.GARDNER: The technique of Integral formula in the geometry of Immersions

Let M be a compact orientable m-dimensional manifold without boundary. When M is riemannian with metric ds^2 the Hodge * mapping may be utilized to construct integral formulas of the type $0=\int_{M_m} d*\omega$, where w is a liniear differential form. In particular three methods are singled out :

1. w=df for f: $M_m \to R$
2. w=<z,dY> for z,Y: $M_m \to Y$, where z,Y are sections of a vector bundle with metric <, > and covariant derivative D.
3. For Z a fixed vector field and F a quadratic differential form on M, define w by duality on a vector field Y by w(Y)=F(Z,Y).

D.GROMOLL: Convexity in global differential geometry.

Let M be a complete riemannian manifold. A connected subset C of M is called convex if for any compact $A\subset M$ there is a $\delta>0$ such that for all $p,q\in C\cap A$ and any geodesic c: $[0,1]\to M$ from p to q of length$\leq\delta$ we have $c[0,1]\subset C$. Let $C\neq\Phi$ be compact convex and the sectional curvature K be non-negative on C, $\partial C\neq\Phi$. Then C can be contracted continuously over convex sets onto a compact totally geodesic submanifold S without boundary, the SOUL of C. If $C\subset M$ is convex with respect to arbitrarily long geodesics, then C is called totally convex. For any noncompact M with $K\geq 0$ everywhere there is a construction leading to continuous filtration of M by compact totally convex sets having a common SOUL S. In particular, M is diffeom. to the normal bundle of S. For further results see J.Cheeger and D.Gromoll " the structure of complete manifolds of nonnegative curvature", Bull. Amer.Math.Soc.74(1968),1147-1150.

Berichterstatter: Asha & Diether Craemer (Bonn)