MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20/1970

Gruppen und Geometrien

14.6. bis 20.6.1970

Die Tagung "Gruppen und Geometrien" stand in diesem Jahr unter der Leitung von P. Dembowski (Tübingen), D. Higman (Ann Arbor) und H. Salzmann (Tübingen).

Der Themenkreis war wieder weitgefächert. So behandelten die Vorträge ebenso Probleme der topologischen Geometrie, der geometrischen Algebra wie auch der Gruppentheorie. Dabei bestand zwischen den erwähnten Gebieten keine scharfe Trennung, im Gegenteil wurde erneut die Verflechtung der angeschnittenen Fragen, unter anderem im Begriff der Permutationsgruppe, sichtbar. Eine große Anzahl der Vorträge beschäftigte sich mit Charakterisierungen der klassischen Gruppen sowie Untersuchung und Klassifizierung von einfachen Gruppen.

Die Möglichkeiten zur Kommunikation wurden von den in- und ausländischen Teilnehmern, auch abgesehen von den 22 Vorträgen, in reichem Maße ausgenutzt.

Teilnehmer

R. Baer, Zürich/Schweiz
H. Bender, Mainz
F. Buekenhout, Brüssel/Belg.
P. Dembowski, Tübingen
B. Fischer, Bielefeld
K.-J. Fleischer, Bielefeld
T. Gardiner, Bielefeld
D. Held, Wiesbaden
D. Higman, Ann Arbor/USA
D. Hunt, Coventry/England
A. Hoffer, Missoula /USA
Z. Janko, Mainz
W. Kantor, Chicago/USA
H. Kurzweil, Tübingen
D. Livingstone, Birmingham/Engl.
H. Mäurer, Darmstadt

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U. Melchior, Bochum
H. Neumann, Canberra/Australien
B.H. Neumann, Canberra/Austr.
P. Plaumann, Tübingen
O. Prohaska, Tübingen
P. Quattrocchi, Modena/Italien
H. Salzmann, Tübingen
R.-H. Schulz, Tübingen
M. Seib, Tübingen
Ch.S. Sims, New Brunswick
B. Stellmacher, Bielefeld
K. Strambach, Tübingen
F.G. Timmesfeld, Bielefeld
F.D. Veldkamp, Utrecht/Holland
H. Wielandt, Tübingen
G. Zappa, Florenz/Italien

Vortragsauszüge

H. Bender: Endliche Gruppen mit abelschen Sylow 2-Untergruppen

Es wurde über einen weiteren Beweis für das folgende, in einem Satz von John Walter enthaltene, Ergebnis berichtet:

SATZ: Sei G eine endliche einfache Gruppe mit abelschen Sylow 2-Untergruppen vom Range ≥ 3. Jede echte Untergruppe U von G besitze Normalteiler

$$1 \subseteq N \subseteq M \subseteq U$$

derart, daß N und U/M ungerade Ordnung haben und M/N direktes Produkt einer 2-Gruppe und einfachen Gruppen E_i ist, wobei $E_i \cong PSL_2(q)$ oder $C_{E_i}(t) = \langle t \rangle \oplus L$ mit $PSL_2(q) \leq L \leq P\Gamma L_2(q)$ gilt, und zwar für alle Involutionen t von E_i.

Dann gilt (i) oder (ii):

(i) G hat eine Untergruppe H gerader Ordnung, so daß $H \cap H^g$ gerade Ordnung hat für alle $g \in G - H$.

(ii) Für jede Involution t in G gilt $C_G(t) = \langle t \rangle \oplus L$, wobei L ist wie oben.

F. Buekenhout: Transitive groups in which involutions fix one or three points

Let G be a finite transitive permutation group acting on a set S of v elements. Let each involution in G fix one or three points. Then the only possibilities are

(i) G $\triangleright$ T, T is elementary abelian and regular of order 9 or 27; there are three 2-transitive G's and two other groups; the latter are primitive of degree 27, order $3^4 \cdot 2^3$ and $3^4 \cdot 2^2$.
(ii) G is primitive, normalizes a simple group N which is one of the following: A_7, $v = 7$; A_7, $v = 15$; PSL_2 (5), $v = 15$; PSL_2 (7), $v = 7$; PSL_2 (9); $v = 15$; PSL_2 (11), $v = 11$; PSL_2 (13), $v = 91$; M_{11}.
(iii) G is imprimitive, isomorphic to PSL_2 (11) acting on 55 points.
(iv) G has an imprimitivity class of 3 elements; an involution fixing one of the points in such a class fixes them all.
(v) G has an imprimitivity class of length $\frac{v}{3}$, O(G) fixes this class, is transitive on it; a Sylow S_2 of G has just one involution.
(vi) O(G) is transitive on S (G imprimitive), a S_2 has just one involution.

P. Dembowski: Products of involutions

Let G be any group and denote by I(G) the set of all involutions in G. Define

$$\mathfrak{P}(G) := \{ o(xy) \mid x,y \in I(G),\ x \neq y \}.$$

Theorem: If G is finite and $\mathfrak{P}(G) = \{2, n\}$ for some integer $n > 2$, then $n = 4$.

A proof of this was given. In the course of this proof, some other results were obtained concerning "dihedrally odd" groups, i.e. such G for which $\mathfrak{P}(G)$ contains no even integer other than 2.

K.-J. Fleischer: Kennzeichnung von Sp (2n, 4); $O^{\pm}$ (2n, 4)

Sei G = $\langle D \rangle$ eine endliche Gruppe, wobei D ein normaler Komplex von Involutionen ist. Außerdem gelte für zwei Elemente $d,e \in D$ stets $d \cdot e = e \cdot d$ oder $o(d \cdot e) \in \{3,5\}$. Dann heißt D eine Menge von $\{3,5\}$ - Transpositionen von G.

SATZ: Sei G = $\langle D \rangle$, wobei D eine Konjugiertenklasse von $\{3,5\}^+$ - Transpositionen von G ist. ($^+$ bedeutet, daß 3 und 5 als Ordnungen wirklich vorkommen).

Außerdem gelte:
(Z) Sind d, x, e $\in$ D mit o (d x) = 5, so ist C_D (e) $\cap \langle d,x \rangle \neq \emptyset$.
Dann ist
(a) G $\cong$ Sp (2n, 4) für ein n $\geq$ 1 $\Longleftrightarrow$ $\exists$ c, d $\in$ D mit cd $\in$ D .
(b) G $\cong$ $O^{\pm}$ (2n, 4) für ein n $\geq$ 2 $\Longleftrightarrow$ cd $\notin$ D für alle c, d $\in$ D.

D. Held: M_{24} - like Centralizers

In this lecture we have considered non-abelian finite simple groups which possess a 2-central involution t such that the centralizer C (t) of t is an extension of an extra-special group of order 2^7 by L_2 (7). The only groups with such a property are GL (5,2), M_{24} and R_3 (M_{24}).

D. Hunt: A Characterization of M (22)

The following theorem was proved:
Let G be a finite simple group with an involution t such that C_G (t) $\cong$ $C_{M(22)}$ (π) where π is a 3-transposition in M (22), the simple group of B. Fischer of order $2^{17} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$. Than G $\cong$ M (22).

π is not central in the Sylow 2-subgroup but
$|C_G(\pi)| = 2^{16} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$ and $C_G(\pi)/\langle\pi\rangle \cong$ PSU (6,2).

Z. Janko: On Groups of Characteristic 2

It was discussed how the general classification problem of determining all finite simple groups leads in a natural way to a concept of finite simple groups of characteristic 2. Also a result of D. Gorenstein together with a lecturer's result about simple groups of characteristic 2 was discussed.

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W. Kantor: Unitary polarities

Let $\mathfrak{P}$ be a projective plane of order q^2 and ϑ a unitary polarity of $\mathfrak{P}$. Let Γ be a collineation group preserving ϑ. Then $\mathfrak{P}$ is desarguesian if either

(1) There are at least 3 non-collinear absolute points such that $|\Gamma(x, x^{\vartheta})| = q$ for each of these points; or

(2) Γ is transitive on the flags (x, L) with x absolute and L non-absolute.

H. Kurzweil: p-Automorphismen von auflösbaren p'-Gruppen

SATZ: Eine Gruppe P von Primzahlordnung p operiere auf einer endlichen auflösbaren p'-Gruppe G. Ist f die Fittinglänge von G und h die von $C_G(P)$, so gilt

$$f \leq h + c, \quad \text{mit } c \leq 4$$

Unter zusätzlichen Voraussetzungen kann die Abschätzung von c verbessert werden. Der "wahre" Wert von c liegt zwischen 2 und 4.

Setzt man anstelle von auflösbar π-separabel; anstelle der Fittinglänge die π-Länge in obigem Satz, so gilt

$$f \leq h + 2$$

Es ist $f \leq h + 1$, falls $|C_G(P)| \equiv 1\ (2)$ oder folgende Voraussetzung gilt: Ist (2 p-1) eine Potenz einer Primzahl t, so ist die t-Sylowgruppe von G abelsch.

D. Livingstone: On some of the exceptional finite simple groups

The character tables of the simple groups known to be involved in the large Conway group have all been determined.

Some progress has been made towards the determination up to conjugacy of all maximal subgroups of the exceptional groups involved. In particular the classification has been completed for the Higman-Sims group, the Mc Laughlin group and the third Conway group.

U. Melchior: On Finite Laguerre planes

For a Laguerre plane $\mathcal{L}$ of finite order the tangent-relation on the circles is transitive if and only if $\mathcal{L}$ is of even order. Let Δ be the group of those automorphisms of $\mathcal{L}$ mapping all circles to tangent ones.

Theorem: Let $\mathcal{L}$ be a Laguerre plane of even order. Then the stabilizer Δ_c acts for all circles C 2-fold transitive on the points incident with C if and only if $\mathcal{L}$ is classical, i.e. $\mathcal{L}$ is the geometry of non-isotropic plane sections of a quadratic cone in some $\mathbb{P}(3,2^e)$.

B.H. Neumann: The isomorphism problem for algebraically closed groups

A system of equations $f(x_1, \ldots, a_1, \ldots) = 1$ and inequations $g(x_1, \ldots, a_1, \ldots) \neq 1$ in variables $x_1, \ldots$ and constants $a_1, \ldots$ in a group A is consistent over A if it can be satisfied in some group that contains A . We call A algebraically closed (W.R. Scott) if every finite consistent such system over A can be solved in A itself. There are $2^{\aleph_0}$ isomorphism classes of countable algebraically closed groups; but not a single one has ever been explicitly constructed. No algorithmic method is known that will distinguish two countable algebraically closed groups from one another; and it is conjectured that non exists. The main contents of the paper is some reasoning in support of this conjecture. A full account is to be published in Proc. Conf. Decision Problemsin Group Theory, Irvine, Calif., September 1969, North Holland Publ. Co., Amsterdam 1971.

H. Neumann: The current state of knowledge on laws in groups

I reported on the question what classes of groups have the property that the laws of every group in the class have a finite basis. Past work shows this to be true for all nilpotent groups, for finite groups (and some others, close to finite in the sense of this problem), metabelian groups and groups obtainable from any finite number of these by taking cartesian products, subgroups and factor groups. Recent work has shown, however, that not every group has this property; I reported on these results, due to A.Yu. Ol'shankii, M.-R. Vaughan-Lee and A.I. Adjan who provided independent and different examples. The upshot is that it is now known that groups with no finite basis for their laws exist in the class of all soluble groups of length four, but that the problem remains open for groups that are soluble of length three. I mentioned some further relevant results and problems; I also added some remarks on the laws of finite simple groups.

P. Plaumann: Lokal monothetische Gruppen

Eine im kleinen kompakte Gruppe G heißt lokal monothetisch, wenn jede von zwei Elementen erzeugte Gruppe monothetisch ist.

SATZ: Eine im kleinen kompakte Gruppe G ist genau dann lokal monothetisch oder isomorph zur additiven Gruppe der reellen Zahlen, wenn alle Faktoren von G eine abelsche Gruppe von Automorphismen haben.

H. Salzmann: Collineation groups of 4-dimensional planes

Let $\mathfrak{P} = (P, \mathcal{L})$ be a compact, 4-dimensional topological projective plane. Then, in the compact-open topology, the full collineation group $\Gamma_{\mathfrak{P}}$ is a Lie group of dimension at most 16. Its connected component consists of continuous collineations only and acts regularly on the set of quadrangles. If $\mathfrak{P}$ admits a connected point-transitive collineation group or a group Δ of dimension ≥ 10, then $\mathfrak{P}$ is arguesian. Each point-transitive group contains the unitary group $PSU_3(\mathbb{C}, f)$ of a hermitian form f of index 0 and hence is even flag transitive. If $\dim \Delta \geq 13$, then Δ contains the projective group $PSL_3(\mathbb{C})$; a group Δ satisfying $9 \leq \dim \Delta \leq 12$ fixes a point or a line.

Ch. Sims: The generation of permutation groups

Let G be a permutation group on the finite set Ω. A sequence of points $X = \alpha_1, \ldots, \alpha_k$ is a base for G if no non-identity element of G fixes X. Define $G^{(i)}$ to be the stabilizer of $\alpha_1, \ldots, \alpha_{i-1}$ in G. A subset S of G is a strong generating set relative to a base

X if $G^{(i)} = \langle S \cap G^{(i)} \rangle$, $i = 1, \ldots, k$. Once a strong generating set for G has been found, the computation of such things as centralizers of elements by computer becomes relatively easy for $|\Omega| \leq 2000$. The problem of determining for a given set S of permutations and sequence X whether or not X is a base for the group $G = \langle S \rangle$ and if so whether S is a strong generating set for G relative to X has not yet been satisfactorily solved.

B. Stellmacher: Eine Kennzeichnung von PGU (n, 2) und PSp (2n, 3)

Es wurde folgender Satz gezeigt:
Sei G eine endliche Gruppe mit den Eigenschaften
(1) G wird von einer Konjugiertenklasse D von Elementen der Ordnung 3 erzeugt; zwei Elemente a, b $\in$ D genügen der Gleichung $a^b = a$ oder $b^a = a^{5'}$.
(2) $Z(G) = 1$, und G' ist einfach.
Dann ist $G \cong PGU(n, 2)$ für $n \geq 4$ oder $G \cong PSp(2n, 3)$ für $n \geq 2$; und D ist bis auf die inverse Konjugiertenklasse D^{-1} eindeutig bestimmt.

K. Strambach: Rechtsdistributive Quasigruppen auf Mannigfaltigkeiten

SATZ: Auf keiner kompakten Mannigfaltigkeit M kann ein stetiges Produkt "$\circ$" so erklärt werden, daß gilt
(1) Es ist $(a \circ b) \circ c = (a \circ c) \circ (b \circ c)$ für beliebige a, b, c $\in$ M.
(2) Die Gleichungen $a \circ x = b$ und $y \circ a = b$ sind für a, b $\in$ M eindeutig lösbar.

F.G. Timmesfeld: Finite groups generated by $\{3,4\}$ - transpositions

Let D be a set of involutions of the finite group G, satisfying the following properties.
1. $D = d^G$ for d $\in$ D.
2. $\langle D \rangle = G$.
3. for d, e $\in$ D : $o(d\,e) \in \{1, 2, 3, 4\}$ and all numbers occur.
4. if $o(d\,e) = 4$, than $(d\,e)^2 \in D$.

Then D is called a conjugacy class of $\{3,4\}^+$ -transpositions of G. The following theorem was proved.

Let D be a conjugacy class of $\{3,4\}^+$ -transpositions of G. Let $O_2(G) = 1$.
Iff there exists a subset $T \subsetneq D$, $|T| \neq 1$ with $T \cap T^d = \left\{ \begin{matrix} T \\ \emptyset \end{matrix} \right.$, $d \in D$ than G/Z(G) is isomorphic to one of the following groups.

1. GL (n 2)
2. $G_2(2)'$
3. ${}^3D_4(2)$
4. Sp (6,2)
5. $SO^-(8,2)$

F.D. Veldkamp: Projective planes over rings

It is possible to define projective planes by means of free modules of dimension 3 over certain "decent" rings, e.g. over semiprimary rings. A system of axioms is given for such planes, in which only certain pairs of points are required to have a unique line in common, nothing being said about other pairs of points. The axioms also include the existence of transvections and dilatations. Any plane satisfying these axioms can be coordinatized by an associative ring. Thus results of Klingenberg and others are generalized.

H. Wielandt: More normal subgroups of finite transitive permutation groups

Let Ω be a finite set, $n := |\Omega| > 1$, and G a transitive permutation group on Ω. For $g \in G$ put $f(g, \Omega) = |\{\alpha \in \Omega \mid \alpha^g = \alpha\}|$.
Jordan showed that $\sum_{g \in G} f(g, \Omega) = |G|$. If we define
$L := gp\{g \in G \mid f(g, \Omega) = 0\}$ and $S := gp\{g \in G \mid f(g, \Omega) \geq 2\}$
(the groups generated by the "long" resp. "short" elements of G) than $L \geq S$, and L turns out to be the generalized Frobenius Kernel G^* of G,
$G^* := G \setminus \bigcup_{\alpha \in \Omega} (G_\alpha - N_\alpha)$ where $N_\alpha = gp\{G_{\alpha\beta} \mid \beta \in \Omega, \beta \neq \alpha\}$
If S is intransitive on Ω, let Ω' be the set of orbits of S. Then G/S acts faithfully on Ω' as a Frobenius group or a regular group.
This suggests studying the following normal series of G :
$G \trianglerighteq L \trianglerighteq S = G_2 \trianglerighteq G_3 \trianglerighteq \ldots \trianglerighteq G_n = 1$
where $G_k := gp\{g \in G \mid f(g, \Omega) \geq k\}$.

G. Zappa: Varieties generated by finite supersoluble groups

Let V be the variety generated by finite supersoluble groups whose exponent divides $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots p_s^{\alpha_s}$ ($p_1 > p_2 > \dots > p_s$ prime numbers).

Let be $q_i = n/p_i^{\alpha_i}$, $m_i = p_1^{\alpha_1} \cdots p_i^{\alpha_i}$. Then V is defined by the following laws:

$$x_1^n, \left(x_1^{\frac{n}{m_i}} x_2^{\frac{n}{m_i}}\right)^{m_i}, \left[[x_1, x_2]^{q_i}, x_3^{p_i^{\alpha_i}(p_i-1)}\right] \quad (i = 1, \dots r) \qquad (1)$$

Consequently, the laws of V have a finite base.
Let X be the subvariety of V s.t:

$G \in X \Longleftrightarrow G \in V$ and the p_i-Sylow subgroups of G are nilpotent of class $\leq c_i$ (i = 1, ..., r).

Then X is defined by the laws (1) and by the laws:

$$\left[x_1^{q_i}, \dots\dots, x_{c_i+1}^{q_i}\right]^{m_{i-1}} \quad (i = 1, \dots, r)$$

X is a Cross variety.

R.-H. Schulz, Tübingen