MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/1971

Wahrscheinlichkeitstheorie

21.3. bis 27.3.1971

Auf der regelmäßigen Tagung über Wahrscheinlichkeitstheorie, diesmal geleitet von Herrn H.Dinges, Frankfurt/Main, wurden Ergebnisse aus folgenden Gebieten der Wahrscheinlichkeitstheorie vorgetragen: Konvergenz von Maßen, Grenzwertsätze für spezielle Prozesse, Martingale, Markov-Prozesse, Statistik, Ergodentheorie, Fluktuationstheorie, Grundlagen der Wahrscheinlichkeitstheorie, Konstruktion stochastischer Prozesse.
Einige Vorträge waren als Überblicke über die Forschungen des betreffenden Gebietes in den letzten Jahren angelegt.
Auch außerhalb der umfangreichen Vortragsreihe fand ein intensiver Gedankenaustausch statt.

<u>Teilnehmer</u>

J.Azéma, Orléans
L.Boneva, Sofia
R.Borges, Gießen
W.Bühler, Heidelberg
H.Carnal, Bern
K.Daniel, Frankfurt
H.Dinges, Frankfurt
L.Dubins, New York
R.M.Dudley, Cambridge
H.Engmann, Frankfurt
P.Gänßler, Köln
H.Grillenberger, Erlangen
W.Hansen, Erlangen
G.Helmberg, Eindhoven
K.Hinderer, Hamburg
K.Jacobs, Erlangen
D.A.Kappos, Athen
M.Keane, Rennes
D.G.Kendall, Cambridge
U.Krengel, Columbus
P.Mandl. Prag
V.Mammitzsch, München
P.A.Meyer, Freiburg
D.W.Müller, Göttingen
A.G.P.M.Nijst, Eindhoven
U.Oppel, München
R.Pyke, Seattle
D.Plachky, Münster
M.Rao, Aarhus
J.P.Raoult, Rouen
B.Rosén, Stockholm
K.Sarkadi, Budapest
C.L.Scheffer, Utrecht
C.P.Schnorr, Saarbrücken
F.H.Simons, Eindhoven
R.Theodorescu, Quebec
F.Topsoe, Kopenhagen
W.v.Waldenfels, Heidelberg
H.Walk, Stuttgart
E.A.Weiss, Bonn
P.Weiß, Linz
J.Wendel, London
H.Witting, Münster
H.Ziezold, Heidelberg

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Vortragsauszüge

J.Azéma: Réduites de surmartingales

On définit différentes notions de réduites pour les surmartingales-potentiel. Le point de vue envirагé a des applications en ce qui concerne les processus de Hunt. On peut ainsi démontrer un théorème de balayage pour les réduites extérieures et donner une interprétation probabiliste au balayage des fonctionelles additives.

L.Boneva/D.G.Kendall: Spline transformations in statistics

Presentation(through examples, with lantern slides) of three new techniques for the diagnostic investigation of the probability densities in one or two dimensions generating data whether grouped (as histograms) or not.

W.Bühler: Relationship structure in branching processes

For supercritical branching processes - under mild restrictions on the offspring distribution (with mean $m>1$) and the distribution of life lengths - the generation of a random individual alive at time t tends to be $N(mt,mt)$. The generation of the last common ancestor of two individuals has a limit distribution. Thus their degree of relationship - i.e. the length of the path joining them in the family pedigree - is asymptotically $N(2mt,2mt)$. If we trace an individual's ancestry back by time s we go L_s generations back and obtain D_t relatives whose last ancestor common with the first individual was alive at time t-s. We have $L_t \sim N(mt,mt)$, $P(D_t e^{-(m-1)t} \leq x) \longrightarrow \sum \frac{k \cdot p_k}{m} F^{(k-1)}(x)$ where $F^{(i)}$ is the i-th convolution of the d.f. of $W=\lim Z(t)e^{-(m-1)t}$. Similarly going back by n generations, i.e. by time T_n, we have T_n is asymptotically normal and

$$\lim_{n\to\infty} P(e^{-bn}D_n \leq x) = \begin{cases} 0 & \text{if } b < (m-1)/m \\ 1/2 & \text{if } b = (m-1)/m \\ 1 & \text{if } b > (m-1)/m \end{cases} .$$

(All formulae given are for the Markovian case, the results

H.Dinges: Laws of large numbers

Let $(\Omega,\mathfrak{A},P)$ be a σ-finite measure space and $y_1, y_2, \ldots; p_1, p_2, \ldots$ integrable random variables, $p_i \geqslant 0$ such that

$$\int h(y_1, y_2, \ldots; p_1, p_2, \ldots)dP \geqslant \int h(y_2, y_3, \ldots; p_2, p_3, \ldots)dP$$

for every positive sublinear h on $R^\infty \times R^\infty$; then

a) $\dfrac{y_1+y_2+\cdots+y_n}{p_1+p_2+\cdots+p_n}$ converges (to a function ψ say) a.e.,

b) $\dfrac{1}{n}\int_{\{\Sigma p_i=\infty\}} | y_1+\cdots+y_n - \psi\cdot(p_1+\cdots+p_n)| \, dP \longrightarrow 0$.

The analogy to the individual ergodic limit theorem and the equivalence of a) to the Chacon-Ornstein-theorem were explained in the talk.

L.Dubins: Inequalities for stochastic processes

For suitable classes of stochastic processes, it is often possible to obtain bounds, sometimes sharp, to the probabilities of various events. The vehicle for obtaining these inequalities are the elementary properties of semimartingales, thet is, the elementary properties of subfair gambling strategies.

R.M.Dudley: Metric convergence of probability measures

Let ϱ be the Prokhorov metric for probability measures and β the "BL norm" metric defined by

$$\beta(\mu,\nu) := \sup\left\{ \left|\int f \, d(\mu-\nu)\right| : \forall x, y \neq z, \ |f(x)| + \frac{|f(y)-f(z)|}{d(y,z)} \leq 1 \right\}$$

We consider the speed of convergence in known limit theorems. First let $X_1, X_2, \ldots$ be independent identically distributed random variables with values in R^k, $E|X_1| < \infty$, $EX_1 = 0$, $\mu_n = L(X_1+\cdots+X_n/n)$, so that $\mu_n \longrightarrow \delta_0$ weakly. If $E|X_1|^t < \infty$, then $\varrho(\mu_n,\mu) = O(n^{\frac{1-t}{1+t}})$ and for every $\varepsilon > 0$, $\varrho(\mu_n,\mu) = O(n^{\frac{1-t}{t}+\varepsilon})$. Here the powers $(1-t)/(1\ t)$ and $(1-t)/t$ are best possible, according to results of L.Baum and M.Katz (1963), D.L.Hanson and others (1966, 1969).

Secondly let $X(t)$ be a standard Poisson process, $0 \leqslant t \leqslant 1$, and let $X_n(t) = \sum_{j=1}^{[nt]} X_{nj}$ where X_{nj} are independent, $Pr(X_{nj}=1) = \frac{1}{n} = 1 - Pr(X_{nj}=0)$. Let $\mathcal{D}$ be the space of functions on $[0,1]$ with at worst jump discontinuities with the metric as defined e.g. in

Billingsley's book. Then $\varrho(L(X_n),L(X)) = O(\frac{1}{n}) \neq o(\frac{1}{n})$ and equally $\beta(L(X_n),L(X)) = O(\frac{1}{n}) \neq o(\frac{1}{n})$.

R.M.Dudley: Binomial tests for goodness of fit

Let $(S,\mathcal{T})$ be a measurable space decomposed as $S = \bigcup_{j=1}^{m} A_j$, A_j disjoint in $\mathcal{T}$. Let P be a known probability measure on $(S,\mathcal{T})$ and let Q be an unknown probability for which we observe a sample $X_1,\ldots,X_n$; the X_i are independent with $L(X_i) = Q$. To test the hypotheses $P = Q$, we consider the deviation of k_j, the number of X_i in A_j, from the P-expected number $nP(A_j) = np_j$. For any integer $t \geq 0$, $Pr(k_j \geq t) = E(t,n,p_j) := \sum_{r=t}^{n} \binom{n}{r} p^r q^{n-r}$ where $q=1-p$. Let $E = \min_j E(t,n,p_j)$ and define an integer t_j by $E(t_j-1,n,p_j) > E \geq E(t_j,n,p_j)$. Let U_j be the event $t_j \leq k_j$ and let $U = \bigcup_{j=1}^{m} U_j$. Then we define the E-test thus: we reject the hypotheses $P=Q$ at the level s (e.g. $s = .05$ or $.01$ or $.001$) if $Pr(U) \leq s$ (on the hypotheses P). It is possible to estimate $Pr(U)$ within $s^2/2$ by $\sum_{j=1}^{m} E(t_j,n,p_j)$ which in case $p_j \equiv \frac{1}{m}$ simplifies to $mE(K,n,\frac{s}{m})$, $K = \max_j k_j$. There is a symmetrical test (B-test) for the lower deviation $k_j < np_j$ using lower binomial tails. The two tests can be combined into the EB-test where we consider the least probable deviation of k_j from np_j. Binomial probabilities can presently be calculated much more accurately than the true probabilities needed for the χ^2-test.

H.Engmann: Existence of L^1-contractions making given random variables admissible

If $y_1, y_2,\ldots; p_1, p_2,\ldots$ are nonnegative integrable r.v. on a σ-finite measure space $(\Omega,\mathcal{A},P)$ the following two statements are equivalent:

(i) $\int_\Omega H(y_1, y_2,\ldots; p_1, p_2,\ldots)dP \geq \int_\Omega H(y_2, y_3,\ldots; p_2, p_3,\ldots)dP$ if $H(s_1, s_2,\ldots;t_1, t_2,\ldots) = \max_{m\in M}(\sum_{n\in N} \max_{i\in I} \sum_{j=1}^{i} (a_{mnj}s_j - b_{mnj}t_j)) \geq 0$ with s_n, t_n real, a_{mnj}, $b_{mnj} \geq 0$, (M,N finite $\subset \mathbb{N}$, $i \in \mathbb{N}$).

(ii) There is a sequence $T_1 \geq T_2 \geq \ldots$ of positive contractions of $L^1(\Omega,\mathcal{A},P)$ making $y_1, y_2,\ldots; p_1, p_2,\ldots$ admissible (i.e. $T_n y_{n-1} \geq y_n$, $T_n p_{n-1} \leq p_n$).

The result allows to replace the conditions in the ratio limit theorem of Cuculescu and Foias which make use of contractions

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by conditions on the joint distribution of the y_n and p_n. In the case of the Chacon ratio limit theorem too an equivalence between the existence of the contraction used there and a property of the joint distribution of the random variables occuring in the assumptions of the theorem can be established.

P. Gänßler: <u>Zur Charakterisierung gleichgradig σ-stetiger Familien regulärer Maße</u>

Ist $X = (X,\mathfrak{T})$ ein topologischer Hausdorffraum und $rca(X,\mathfrak{F})$ die Familie aller regulären, abzählbar additiven, reellwertigen auf der Borelalgebra $\mathfrak{F}$ definierten Mengenfunktionen (regulär: $\forall G \in \mathfrak{T} \wedge \forall \varepsilon > 0 \; \exists K$ kompakt, $K \subset G$, so daß $|\mu(A)| < \varepsilon \;\; \forall A \subset G \setminus K$), so wird folgendes gezeigt (G bzw. K bezeichnen i.f. stets offene bzw. kompakte Teilmengen von X) :

Satz 1: $\mathfrak{M} \subset rca(X,\cdot)$ ist genau dann gleichgradig σ-stetig, wenn eine der folgenden untereinander äquivalenten Bedingungen erfüllt ist:

(i) $\forall G \;\wedge\; \forall \varepsilon > 0 \;\; \exists K \subset G$, so daß $\sup\limits_{\mu \in \mathfrak{M}} |\mu|(G \setminus K) < \varepsilon$.

(ii) $\forall A \in \mathfrak{F} \;\wedge\; \forall \varepsilon > 0 \;\; \exists K \subset A$, so daß $\sup\limits_{\mu \in \mathfrak{M}} |\mu|(A \setminus K) < \varepsilon$.

(iii) $\begin{cases} \text{(a)} \; \forall K \wedge \forall \varepsilon > 0 \;\; \exists G \supset K, \text{ so daß } \sup\limits_{\mu \in \mathfrak{M}} |\mu|(G \setminus K) < \varepsilon. \\ \text{(b)} \; \forall \varepsilon > 0 \; \exists K, \text{ so daß } \sup\limits_{\mu \in \mathfrak{M}} |\mu|(X - K) < \varepsilon. \end{cases}$

(iv) Für jede Folge paarweise disjunkter K_j, $j \in \mathbb{N}$, ist $\lim\limits_{j \to \infty} (K_j) = 0$ gleichmäßig bzgl. $\mu \in \mathfrak{M}$

Bemerkung: (iv) impliziert die folgende Bedingung (v):

(v) Für jede Folge G_j, $j \in \mathbb{N}$, regulär offener Mengen mit paarweise disjunkten abgeschlossenen Hüllen ist $\lim\limits_{j \to \infty} (G_j) = 0$ gleichmäßig bzgl. $\mu \in \mathfrak{M}$.

Satz 2: Ist $X = (X,\mathfrak{T})$ ein regulärer Hausdorffraum, so impliziert umgekehrt (v) jede der Bedingungen von Satz 1, und für jede Folge $(\mu_n)_{n \in \mathbb{N}}$ regulärer Maße, für welche die Zahlenfolge $(\mu_n(G))_{n \in \mathbb{N}}$ für jede regulär offene Menge G konvergiert, konvergiert $(\mu_n(A))_{n \in \mathbb{N}}$ für jede Borelsche Menge A (und durch $\mu_o(A) := \lim\limits_{n \to \infty} \mu_n(A)$, $A \in \mathfrak{F}$, wird ein reguläres Maß μ_o auf $\mathfrak{F}$ definiert).

G.Helmberg: Über die Umkehrung des Ergodensatzes von E.Hopf

Es seien T ein submarkovscher Operator in $L^1(E,\mathfrak{F},\pi)$ bzw. dual in $L^\infty(E,\mathfrak{F},\pi)$ und P, C, D resp. der positive, konservative und dissipative Teil von E bezüglich T und π.

Satz 1: Die folgenden Aussagen sind äquivalent:

a) $\forall h \in L^\infty \ \exists \lim_{n\to\infty} \frac{1}{n+1}\sum_{k=0}^{n} T^k h$ f.ü.

b) P=C und $T^k 1_D \downarrow 0$ f.ü.

Satz 2: Die folgenden Aussagen sind äquivalent:

a) $\forall g \in L^1 \ \exists \lim_{n\to\infty} \frac{1}{n+1}\sum_{k=0}^{n} gT^k$ f.ü.

b) $\forall g \in L^1$ supp $\limsup_{n\to\infty} \frac{1}{n+1}\sum_{k=0}^{n} gT^k \subset P$.

In beiden Situationen lassen sich die Grenzwerte ähnlich wie bei Chacon-Ornstein identifizieren.

M.Keane: A classification of stationary measures on sequence spaces

By introducing the notion of a g-measure, a characterization of shift-invariant measures on sequence spaces is given and a construction principle, starting from a "nice" function g on the sequence space, for an invariant measure μ_g which is ergodic and even strongly mixing under the shift. Applications and suggestions for applications were given in the area of harmonic analysis, probability theory, ergodic theory and statistics.

K.Jacobs: Recent results in ergodic theory (a survey)

0) Preliminaries: flows and mappings globally investigated by measure-theoretical, topological, differential methods. Historical sketch.

1) Measure-theoretical isomorphy: Ornstein's result: Bernoulli and even weak Bernoulli (including ergodic finite state Markovian) measures with equal entropy are isomorphic. Related topics.

2) Strict ergodicity: Jewett's result: every weakly mixing system can live in a strictly ergodic set. Krieger's extension to ergodic systems with finite entropy. Jacobs' extension to the continuous-time case. The combinatorial symbol sequence constructions of Thue, Morse, Kakutani, Keane, Jacobs, Grillenberger.

3) A^*-homeomorphisms: Bowen's result: the number N_n of points of period n yields $\lim \frac{1}{n}\log N_n$ for the topological entropy and, in case of C-density, of exactly one invariant probability which gives a K-system.

4) Sinai's dispersive billard on the torus minus a finite number of well disjoint strictly convex domains is a K-flow, according to Sinai's paper in the Uspekhi 1970.

U.Krengel: Erzeugende in der Ergodentheorie

This lecture was a survey on results of Krieger and myself on the existence of finite generators for measure preserving and nonsingular transformations and flows. Let T be a nonsingular invertible and ergodic transformation of a probability space $(X, \underline{F}, \mu)$. A partition $\xi = \{A_1, \dots, A_n\}$ of X into disjoint measurable sets A_i is called a generator if $\underline{F}$ is generated up to nullsets by $\{T^k \xi, k=0, \pm 1, \pm 2, \dots\}$ and a strong generator if $\underline{F}$ is generated up to nullsets by $\{T^k \xi, k=0,1,2,\dots\}$. The existence of a generator of size n is equivalent to the property that T is isomorphic to a shift for a process with n states. The process is deterministic if and only if ξ is a strong generator.
Theorem 1 (Krieger): If T is measure preserving T has a finite generator if and only if the entropy h(T) of T is finite. The smallest size n of a generator is between exp(h(T)) and exp(h(T)) + 1.
Theorem 2: If T doesn't preserve any finite invariant measure equivalent to μ, then there exists a strong generator of size 2. Both theorems can be refined in that ξ can be required to belong to an arbitrary exhaustive sub-σ-algebra $\underline{G}$ of $\underline{F}$, i.e. a $\underline{G}$ with $T^{-1}\underline{G} \subset \underline{G}$ such that $\{T^k \underline{G}, k=0,1,\dots\}$ generates $\underline{F}$.
Theorem 3: For flows $\{T_t, -\infty < t < \infty\}$ there always exists a strong generator of size 2 in an arbitrary exhaustive sub-σ-algebra.

Theorem 3 has probabilistic consequences which seem difficult to obtain by other means. Astationary process (X_k) with finitely many states is called forward deterministic if X_1 is a measurable function of $(X_0, X_{-1}, X_{-2}, \dots)$, i.e. if the σ-algebra generated by $(X_0, X_{-1}, \dots)$ is the full σ-algebra. This is the case if and only if the past tail σ-algebra $\underline{F}(-\infty)$ equals $\underline{F}$. The process (X_k) is called completely nondeterministic if $\underline{F}(-\infty)$ is trivial. (X_k) is called backward deterministic (completely nondet.) if the time-reversed process (X_{-k}) is forward deterministic (completely nondet.). It is known that (X_k) is forward deterministic if and only if it is backward deterministic. Theorem 3 implies that for continuous time the situation is different: There exist stationary processes (X_t) with 2 states, the pathes of which have only finitely many jumps in any finite time-interval, and such that (X_t) is forward deterministic, but backward completely nondeterministic. Such a process (X_t) has the interesting property that (X_t) is forward deterministic, but every discrete subprocess $\{X_{n\delta}, n=0,\pm 1,\pm 2,\dots)$ $(\delta > 0)$ is forward completely nondeterministic. References can be found in my paper on generators in the Proc. of the Midwestern Conf. on Ergodic Theory, Springer Lect. Notes Vol. 160.

P.Mandl: Adaptive Steuerung Markovscher Ketten

Auf einer Markovkette, deren Übergangswahrscheinlichkeiten von einem unbekannten Parameter α abhängen, wird eine additive Ertragsfunktion definiert. Als Kriterium für die Güte der Steuerung wird der mittlere Ertrag pro Zeiteinheit verwendet. Ist die Kette unzerlegbar, so läßt sich jedem α eine optimale stationäre Steuerung $\hat{z}(i;\alpha)$ zuordnen (i bezeichnet den Zustand der Kette). Man nimmt eine Schätzung α_n^*, n=1,2,. (insbesondere die Maximal-

Likelihood-Schätzung) des tatsächlichen Wertes α_o des unbekannten Parameters und bildet die Steuerung $\hat{z}(i;\alpha_n^*)$. Einige Resultate zeigen, daß diese Regel im allgemeinen zu guten Steuerungen führt.

P.A.Meyer: Applications of Ray's resolvents

The theory of Ray's resolvents (Ray's paper, Ann. of Math. 1959) has become very useful in dealing with quite general Markov processes. Examples of applications to a right continuous strong Markov process X are:

a) C.T.Shih's theorem on the approximation of hitting times of sets by hitting times of finely open sets.

b) Accessibility of stopping times: if T is a stopping time such that either X_{T_-} does not exist or $X_{T_-} = X_T$, then T is accessible.

D.W.Müller: Fluktuationen konvergenter Prozesse

Sei $X_1, X_2, \ldots$ eine Folge identisch verteilter unabhängiger Zufallsvariabler mit $EX_1 = 0$, $EX_1^2 = 1$. Bezeichne N_ε die (zufällige) Anzahl der n, für welche $|n^{-1}(X_1 + \ldots + X_n)| > \varepsilon$ (>0). Für $\varepsilon \to 0$ hat $\varepsilon^2 N_\varepsilon$ eine Grenzverteilung; diese hat die Dichte

$$f(t) = f^+(t) - \sum_{n>c} (2n+1)!\,(2n+1)^2 \left[f^+(t(2n+1)^2) - 2\right] * \prod_{j=0}^{n} \left[f^+(t(2j+1)^2) - \delta\right];$$

hierbei bezeichnet $f^+(t) = (2/\pi t)^{1/2} e^{-1/2t} - \mathrm{erfc}(t)$ die Grenzdichte des entsprechenden einseitigen Problems, $\mathrm{erfc}(t) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \int_t^\infty e^{-\frac{u^2}{2}}\,du$, δ ist die Diracsche Deltafunktion. f(t) stellt die Dichte der Zufallsvariablen $\int_0^\infty I_{[|B(t)|>t]}\,dt$ dar, die auf einem Wiener-Prozess B mit $P(B(0)=0)=1$ definiert ist. Wesentlich für den Beweis ist die Identität

$$L(a,\lambda) + \lambda \int_0^\infty E^a\, I_{[|B(s)| > B(0)+s]} \left\{e^{-\lambda(T(s)-s)}\, L(T(s)+a, \lambda\right\} ds = 1,$$

welche von

$$L(a,\lambda) = E^a \exp\left(-\lambda \int_0^\infty I_{[|B(u)| > B(0)+u]}\, du\right) \qquad (a, \lambda > 0)$$

und

$$T(s) = \inf\{u \geq s : |B(u)| = B(0)+u\} \qquad (\leq +\infty)$$

erfüllt wird. Ein analoger Grenzwertsatz gilt für den Prozess der aufeinanderfolgenden Mediane einer Folge von (identisch) gleichmäßig verteilten unabhängigen Zufallsvariablen.

A.G.P.M.Nijst: Conditional mean information and conditional mean entropy

Let $(X,\mathcal{R},P)$ be a probability space and let $\mathcal{R}_0$ and $\mathcal{R}_1$ be sub-σ-fields of $\mathcal{R}$. Then the concepts of conditional information and conditional entropy of $\mathcal{R}_1$ with respect to $\mathcal{R}_0$ are well known. Furthermore if T is a measure preserving transformation on $(X,\mathcal{R},P)$ and ξ is a measurable partition of X with finite entropy because of MacMillan's theorem the mean information of ξ is defined as a a.e. and L^1-norm limit. Hence we can speak about the concepts of mean information and mean entropy of ξ. A definition of conditional mean information and conditional mean entropy of a partition ξ with finite entropy conditioning by an invariant σ-field is given which generalizes a definition of Neveu. Some properties of conditional mean information and conditional mean entropy are discussed which lead to a generalization of a theorem of Neveu.

R.Pyke: Constructions of a.s. convergent stochastic processes

A survey on the literature on constructions of partial-sum and empirical processes from Brownian motion. The implicit constructions of Skorokhod, Dudley and Wichura were reviewed, as well as the explicit constructions made possible by methods of several authors for stopping Brownian motion to give specific distributions. Assuming fourth moments, refined constructions are provided by Strassen(1967) and Kiefer (1969). Generalizations to multidimensional state and/or parameter spaces were considered indicating the extensions of Strassen's L.I.L. to the latter case.

J.P.Raoult: Lebesgue decomposition of sequences of measures

Let be given a sequence $((\Omega_n,\mathcal{A}_n))$ of measurable spaces and, for every n, two σ-finite signed measures, μ_n and ν_n. We say that the sequence (μ_n) is, with respect to the sequence (ν_n), contiguous if and only if

$$(\forall \varepsilon>0)(\exists \eta>0)(\exists N\in\mathbb{N})(\forall n>N)(\forall A_n\in\mathcal{A}_n)\left[|\nu_n|(A_n)<\eta \Rightarrow |\mu_n|(A_n)<\varepsilon\right]$$

asymptotically orthogonal if and only if

$$\lim_{n\to\infty}\left[\inf_{A_n\in\mathcal{A}_n}(|\mu_n|(\complement A_n)+|\nu_n|(A_n)\right]=0.$$

($|\ |$ means total variation).

We study existence and uniqueness, being given a sequence

$((\Omega_n, \mathcal{A}_n, \mu_n, \nu_n))$, of a couple $((\mu_n'), (\mu_n''))$ of sequences of measures such that for every n, $\mu_n = \mu_n' + \mu_n''$, (μ_n') is contiguous to (ν_n); (μ_n'') is asymptotically orthogonal to (ν_n).
A topological necessary and sufficient condition of existence of such a decomposition is given. We also give a sufficient condition, using probabilistic tools.

B.Rosén: <u>A central limit theorem for sampling with unequal probabilities</u>

To each item s, s = 1,2,...,N, in a collection is associated a variate value a_s and a draw probability proportionate p_s. Items are drawn one after the other without replacement, so that at each draw the probability of drawing item s is proportional to p_s if item s remains in the collection. Let Z_n be the sum of the variate values in a sample of size n. Approximation formulas were given for EZ_n and $\mathrm{cov}(Z_m, Z_n)$. Furthermore the joint distribution $L(Z_{n_1}, Z_{n_2}, \ldots, Z_{n_d})$ is, under general conditions, approximately gaussian.

C.L.Scheffer: <u>Limits of projective systems of compact Hausdorff spaces connected by submarkov stochastic kernels</u>

Let $(X_i, P_{ij})_I$ be a directed projective system of compact spaces, where the P_{ij} are continuous submarkov dispersions. Let D be the category of Hausdorff spaces with continuous submarkov dispersions as morphisms.
Theorem: The projective limit $\{P_i : X \to X_i\}_I$ of such a projective system always exists in the category D. The space X is either compact or locally compact; it is compact iff one has $\sup_i \inf_{j \geq i} \| P_{ij} 1_i \| > 0$; in particular X is compact if all the P_{ij} are markovian.

K.P.Schnorr: <u>Verteilungsunabhängige Invarianzeigenschaften von Zufallsfolgen</u>

Sei $K(\mu, A)$ die Menge der ollektive im Raum der unendlichen binären Folgen zum W-Maß μ und zu der abzählbaren Menge A von Auswahlregeln. Ville zeigte 1937, daß es in $K(\mu, A)$ stets Folgen gibt, welche das Gesetz vom iterierten Logarithmus nicht erfüllen. Dennoch läßt sich der von Mises'sche Ansatz harmonisch in die neuere Theorie der Zufälligkeit (Kolmogoroff, Martin-Löf, Schnorr) einbetten. Die von Mises'schen Kollektive in der Church'schen Präzisierung erfüllen gerade alle effektive Zufallsgesetze exponentieller Ordnung. Dies sind für den Statistiker die wichtigst n Zufallsgesetze. Dagegen ist das Gesetz vom iterierten Logarithmus nicht von exponentieller Ordnung. Indem man den Begriff der Auswahlregel erweitert, kann man Zufallsfolgen durch verteilungsunabhängige Invarianzeigenschaften charakterisieren. Der von Mises'sche Ansatz liefert somit eine von mehreren äquivalenten Definitionen von Zufallsfolgen.

F.H.Simons: <u>Backward Markov processes induced by a measurable nonsingular transformation</u>

P written to the left of a function denotes a (sub-) Markov operator in $L^{\infty}(X,\mathcal{R},\mu)$, or its extension to the class $M^{+}(X,\mathcal{R},\mu)$ of nonnegative $\mathcal{R}$-measurable functions; P written to the right of a function denotes the corresponding (sub-) Markov operator in $L^{1}(X,\mathcal{R},\mu)$ or its extension to $M^{+}(X,\mathcal{R},\mu)$. If $\mu_0 \approx \mu$, μ_0 σ-finite, then there exists a Markov operator $P^{\leftarrow}_{\mu_0}$ acting from t=1 to t=0 if and only if $\mu_0 P$ is σ-finite, and in that case $fP^{\leftarrow}_{\mu_0} = \frac{d\mu_0}{d\mu}\left(P\left(\frac{f}{(d\mu_0/d\mu)}P\right)\right)$ for all $f \in M^{+}(X,\mathcal{R},\mu)$. Let T be a nonsingular measurable transformation on $(X,\mathcal{R},\mu)$ and suppose μ σ-finite on $(X,T^{-1}\mathcal{R})$. Let P be the forward process associated with T ($Pf = f \circ T$ for $f \in M^{+}$). The mapping $P^{\leftarrow}_{\mu_0} \longrightarrow h = \frac{(d\mu_0/d\mu)}{E_{T^{-1}\mathcal{R}}\frac{d\mu_0}{d\mu}}$ is a one-to-one mapping of the class of all backward processes /associated with P and the class of functions $h \in M^{+}$ for which $h > 0$ and $E_{T^{-1}\mathcal{R}}h = 1$. Moreover $fP^{\leftarrow}_{\mu_0} = \frac{d\mu T}{d\mu} h(f \circ T)$ for all $f \in M^{+}$. If $\mu(X) = 1$, T ergodic measure preserving, then $P^{\leftarrow}_{\mu_0}$ is dissipative, unless $P^{\leftarrow}_{\mu_0} = P^{\leftarrow}_{\mu}$.

F.Topsoe: <u>Measure spaces connected by correspondences</u>

$\varphi: X \rightarrow Y$ is a correspondence if $\varphi(x)$, for each x, is a nonempty subset of Y. When X and Y, assumed to be topological spaces, are provided with measures μ and η respectively a notion of measure preserving correspondence $\varphi: X \rightarrow Y$ was introduced ($\mu(\varphi^{S}B) \leq \eta B \leq \mu(\varphi^{W}B)$; $B \subseteq Y$). This notion was studied in detail for upper semi-continuous and compact-valued correspondences. A theorem - aiming at the construction of stochastic processes - on existence of projective limits for projective systems of measure spaces connected by correspondences was announced.

H.Walk: Wahrscheinlichkeitstheoretische Methoden bei der Approximation durch lineare positive Operatoren

Zunächst werden auf wahrscheinlichkeitstheoretischem Wege der Korowkinsche Satz für lineare positive Operatoren (mit einer geringen Zusatzvoraussetzung) und ein weiteres Resultat bewiesen. Dabei ergibt sich, daß eine vom Verfasser angegebene hinreichende Bedingung zur Erfassung unbeschränkter Funktionen f in Sätzen Korowkinschen Typs bei $f \geq 0$ auch notwendig ist.
Für eine unter verwendung von Zufallsvariablen definierte Klasse linearer positiver Operatoren werden asymptotische Güteaussagen (bei der Existenz links- und rechtsseitiger Ableitungen) mitgeteilt und als bestmöglich erkannt, ebenso gleichmäßige Güteaussagen mit Hilfe eines Stetigkeitsmoduls; hierbei wird eine asymptotisch beste Konstante ermittelt. Hilfsmittel sind zentrale Grenzwertsätze, insbesondere eine von Feller 1968 angegebene Verallgemeinerung des Satzes von Barry-Esseen.

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