MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 15/1971

Finite Geometries

4.4. bis 10.4.1971

Die diesjährige Tagung "Finite Geometries" stand unter der Leitung von Prof. D.R. Hughes (London) und Prof. H. Lüneburg (Kaiserslautern). In vielen Vorträgen wurden neue Ergebnisse aus der Theorie der endlichen Inzidenzstrukturen bekannt. Kennzeichnungen endlicher Ebenen wurden in einem anderen Teil der Ausführungen und Diskussionen behandelt. Drei Beiträge gaben eine Anwendung der Code- Theorie.

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Vortragsauszüge

J. VAN BUGGENHAUT: t-(k,v,λ)-designs with $\lambda > 1$

Three important problems for given t, k and λ:

- the multiplicities of the blocks of a given design
- the number of non-isomorphic designs of v points
- the existence of a design of v points where all blocks have multiplicity one.

We study these problems for 3-(4,v,3)-designs.
The necessary and sufficient condition is v even (Hanani 1963).
Several general constructions are known:

$$\left.\begin{array}{lcl} 3\text{-}(4,v,1) & \Longrightarrow & 3\text{-}(4,v,3) \\ 3\text{-}(6,v,1) & \Longrightarrow & 3\text{-}(4,v,3) \end{array}\right\} \text{ trivial}$$

$$\left.\begin{array}{lcl} 3\text{-}(4,t+1,1) & \Longrightarrow & 3\text{-}(4,2t,3) \\ 3\text{-}(4,t,3) & \Longrightarrow & 3\text{-}(4,2t,3) \\ 3\text{-}(\{4,6\},v,1) & \Longrightarrow & 3\text{-}(4,v,3) \end{array}\right\} \text{ Hanani 1963}$$

(↳ generalized Steiner system: blocks have 4 or 6 points)

$v = p^2+1$, p prime $> 3 \Longrightarrow 3\text{-}(4,v,3)$

$$\left.\begin{array}{l} 3\text{-}(4,v_1,3) \\ 3\text{-}(4,v_2,1) \end{array}\right\} \Longrightarrow 3\text{-}(4,v_1 \cdot v_2,3)$$

For each construction it is possible to compute the multiplicities of the blocks. A lower bound for the number of non-isomorphic designs of v points is deduced from the number of classes (following multiplicities of the blocks).
Excepted the designs deduced trivially from the 3-(6,v,1)-designs, the only known designs where all blocks of multiplicity one are obtained by the 6. construction ($v = p^2+1$).

R. P. BURN: The uniqueness of the harmonic construction

We obtain necessary and sufficient conditions on the coordinatizing quasi-field for the harmonic construction to be unique w. r. t. ((O), (∞)), in translation planes. We construct collineations in an arbitrary projective plane from limited assumptions about the harmonic construction. We characterise translation planes which are also dual translation planes of characteristic $\neq 2$ by means of the harmonic construction.

D. FOULSER: p-elements in translation planes

Let π be a translation plane of order p^r for $p \geq 3$, whose points are the elements of a vectorspace $V = V(2r,p)$ of dimension $2r$ over $GF(p)$. Let σ be a collineation of π such that $|\sigma| = p$, and the set of fixed points of σ, E_σ, is either an affine line (so σ is an elation of π) or a square-root subplane of π. Then σ is a "generalized elation" of V. The group G generated by two such collineations σ and τ of π, can be studied by use of an extension of a theorem of C. Hering (unpublished) and T.G. Ostrom (J. Alg., 14(1970),405-416).

Theorem Let σ and τ be 'generalized elations' of V with disjoint fixed-point subspaces E_σ and E_τ, respectively, each of dimension r. Let $p \geq 3$. Then $\sigma = \begin{pmatrix} I & I \\ 0 & I \end{pmatrix}$, $\tau = \begin{pmatrix} I & 0 \\ A & I \end{pmatrix}$ with respect to a suitable bases. Let $G = \langle \sigma, \tau \rangle$, and let $\mathcal{S}$ be the set of images of E_σ, E_τ under G. Then the following statements are equivalent:

1. $G \cong SL(2,u)$, for $u = p^s$, some s/r.
2. The matrix subring generated by A is a field, $GF(u)$.

2! The subspaces E_σ, E_τ, $\sigma(E_\tau)$, $\tau(E_\sigma)$ can be embedded as lines in the Desarguesian affine plane of order p^r.

3. $\mathcal{S}$ is a partial spread (i.e., any two subspaces in are disjoint).

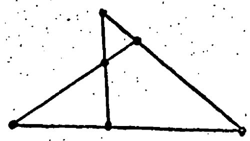
For translation planes, the cases in which conditions 1.-3. are not satisfied would be of particular interest.

M. J. GANLEY: Unitals in non-desarguesian planes

Theorem 1 If D is a finite commutative division ring with an involutary automorphism, then $\Pi(D)$ admits a unitary polarity.

We then consider the unitary polarity ρ so obtained when D is the Dickson division ring of order q^2, q odd. Let U be the corresponding unital with full automorphism group Aut U.

Lemma Either Aut U is 2-transitive on the points of U, or Aut U fixes a point of U.

Let C be the followilg configuration: 

Theorem 2 If $C \subset U$, then Aut U fixes a point of U.

Corollary Desarguesian unitals, in planes of odd order, do not contain such a configuration C (result due to O'Nan).

Finally, if q= 9 or 25, computation shows that U contains such a configuration, and so the unitals in these planes are not isomorphic to any other known unitals.

M. GIRARDI: Theorem D_9

D_9 is a special case of Desargues' theorem occurring when two vertices of one of the two homological triangles belong to a side of the other triangles.

Def. D_9 holds with respect to a fixed line 1 (or to fixed points A,B), if D_9 holds for every pair of homological triangles one of which has a side on the line 1 (or A,B as vertices).

Theorem 1 D_9 holds with respect to the fixed points A,B if and only if,starting from A,B , the harmonic construction is unique.

Theorem 2 In a grafic plane π in which D_9 holds with respect to a fixed line 1 and a fixed point $A \in 1$, in every frame $\omega VA\xi$ $(\xi \in 1)$, the coordinates form an abelian group. In this group either every element has order p (p prime) or all the elements are the infinite order.

A. HERZER: Projektivitäten in endlichen Fastkörperebenen

Sei p eine ungerade Primzahl und $q=p^n$; sei $F = F_{q,q^2}$ der Dicksonsche Fastkörper der Dimension 2 über seinem Zentrum $K = GF(q)$, sei Π die projektive Ebene über F und X eine Gerade von Π. Schließlich sei $\mathfrak{P}_X$ die Gruppe der Projektivitäten von X in sich. Dann ist $\mathfrak{P}_X$, betrachtet als Permutationsgruppe auf den Punkten von X, die symmetrische Gruppe. Der Beweis stützt sich auf das Lemma: $\mathfrak{P}_X$ enthält eine Untergruppe, welche q+1 Punkte festläßt und auf den übrigen primitiv operiert, aus welchem mit Hilfssätzen aus der Theorie der Permutationsgruppen folgt, daß $\mathfrak{P}_X$ die alternierende Gruppe enthält. Durch Joussen, 'Projectivities in finite nearfields'

(Vortrag auf derselben Tagung) ist schließlich die Existenz ungerader Permutationen gesichert.

X. HUBAUT: Designs with repeated blocks

A t-(k,v,λ) design is a triple (P,B,I), $I \subseteq P \times B$ such that $|P| = v$ and each t-set of P is incident to λ blocks of B, and each block of B is incident to k points of P. It may be seen as a set of v points with distinguished blocks of k points, each of them counted with multiplicity λ_i, such that every t-set is incident to λ blocks.
It is proved that it is possible to construct a t-(k,v,λ) design for arbitrary t and k when v and λ are large enough. The automorphism group of such designs is at least PΓL(t+1,q).

V. JHA: A characterization of finite semifield plane

Theorem : Let Π be a finite translation plane of order $\neq 16$. Π_o a subtranslation plane of Π (containing the special line of Π). If Π has a group of collineations which leaves Π_o pointwise fixed the following must hold:
a) Π_o is a desarguesian Baer subplane of Π.
b) Π is derivable with respect to $\Pi_o \cap l_\infty$. The derived plane can be coordinatized by a semifield of dimension ≤ 2 over its middle nucleus.
In fact every plane coordinatized by a semifield of dimension ≤ 2 over its middle nucleus can be obtained by deriving a plane Π satisfying the hypotheses of the theorem.

W. JONSSON: The collineation group of the 2-(56,11,2) design of Hall, Lane and Wales

Using the rank three representation of PSL(3,4), Hall, Lane and Wales constructed a 2-(56,11,2) design. This design D is associated with the Steiner system 3-(22,6,1) whose group is the automorphism group of the Mathieu group M_{22}. This representation allows one to see that D has a collineation group isomorphic to a subgroup of index three in the full group of collineations and correlations of the projective plane of order four. A group theoretical argument shows that this is the full group of D.

J. JOUSSEN: On projectivities in finite nearfield planes

Let be $F = F_{q,q^2}$ the finite Dickson nearfield belonging to the Dickson pair (q,2), q a prime power, q odd. Let be $\Pi = \Pi(F)$ the projective plane over F, and X a line of Π. Let $\mathcal{P}_X$ denote the group of all projectivities of X onto itself. Then:

i) $\mathcal{P}_X$ is quadruply transitive on X.

ii) $\mathcal{P}_X$ contains odd permutations.

iii) If q^2-2 is a prime number, then $\mathcal{P}_X = \mathcal{S}_X$, the symmetric group on the points of X.

V. C. MAVRON: On the construction of certain affine designs

Denote by AD(k,m) an affine design with parameters $2-(km,k,\frac{k-1}{m-1})$, $k \neq 1$. A G-class of an AD(k,m), Π, is a parallel class such that for any block c in the class, Π_c is an $AD(km^{-1},m)$. Given Γ, Σ which are AD(k,3)'s and a bijection Θ between their point sets, then an AD(3k,3), Π, is constructed. If Θ is an isomorphism it is proved that if the number of G-classes in Σ is n, then Π has 3n+1 G-classes precisely. By suitable choices of Θ non-isomorphic affine designs with the same parameters may be constructed.

It can be shown that on AD(3k,3), Π, can be constructed in this way if and only if Π admits two axial involutions α, β such that i) α, β have parallel and distinct axes and ii) α, β fix precisely the same parallel classes.

U. MELCHIOR: Partial t-designs and tetragonal configurations

The relation between a certain generalisation of inversive planes and the tetragonal configurations introduced by J.Tits was discussed. A characterization of the tetragonal configuration related to the orthogonal polarity in finite projective space PG(4,q), q odd, was given.

W. MIELANTS: On the number of topologies on a finite set

Starting with results obtained by D. Klarner in his paper: 'The number of graded partially ordered sets' J. Combin. theory 1969(12-19)), and using a certain characteristic number of a finite topological space,namely his small inductive dimension, I found the following lower bound of the number of topologies on a finite set with cardinality n: $N(n) \geq \sum_{h=0}^{n-1} \sum_{m=1}^{n} S(n,m)[A(h,m)+G(h,m)]$ where $S(n,m)$ denotes the Stirlingnumbers of 2^o kind; $G(h,n) = \sum_{k=1}^{n-h-1} A(h,n-k)\,\varphi(h)^k$ with $\varphi_n(h) = \sum_{a=0}^{h-3} 2^a(2^{h-a-2}-1)$; $A(h,n) = B(h+1,n)-B(h,n)$ with $B(h,n) = \sum_{k=0}\binom{n}{k} c(h,k)\,d(h-1,n-k)$ with $d(h,n) = \sum_{(n_1 \ldots n_k)} (-1)^k \binom{n}{n_1 \ldots n_k} \prod_{i=1}^{k} c(h,n_i)$ where the sum extends on all compositions $(n_1, \ldots, n_k)$ of n in a unrestricted number of positive parts and with $c(h,n) = \sum_{[n_1 \ldots n_h]} \binom{n}{n_1 \ldots n_h} 2^{n_1 n_2 + n_2 n_3 + \ldots + n_{h-1} n_h}$ where the sum extends on all compositions $[n_1, \ldots, n_h]$ of n in exactly h non-negative parts.

O. PROHASKA: Finite derivable nets

A net $\mathbb{N}$ of degree m+1 and order m^2 is called derivable, if there is a net $\mathbb{N}^*$ defined on the same points as $\mathbb{N}$, such that the following conditions hold:

(1) Two points are joined in $\mathbb{N}$ if and only if they are joined in $\mathbb{N}^*$.

(2) A line of $\mathbb{N}$ meets a line of $\mathbb{N}^*$ in 0 or m points.

Then for any line N^* of $\mathbb{N}^*$ the substructure $\mathbb{N}(N^*)$ of $\mathbb{N}$, consisting of the points of N^* and the lines of $\mathbb{N}$ meeting N^* in m points, forms an affine plane of order m, with the property

(3) Any point not on N^* is on a unique line of $\mathbb{N}(N^*)$.

The substructures of $\mathbb{N}$ of the form $\mathbb{N}(N^*)$ for a line N^* of $\mathbb{N}^*$ are called the Baer subplanes of $\mathbb{N}$.

The following theorems hold:

Theorem 1: The Baer subplanes of a derivable net are desarguesian.

Theorem 2: If an affine plane $\mathbb{A}$ of order m^2 contains a derivable net $\mathbb{N}$ of the same order and admits a collineation group leaving $\mathbb{N}$ invariant and acting as a rank-3-permutation group on the points of $\mathbb{A}$ then $\mathbb{A}$ is either desarguesian or a Hall plane.

R.-H. SCHULZ: Can $PSU(3,q^2)$ act as a collineation group of an affine plane of order q^3 ?

The following theorem has been proved :

Theorem: Let $\mathfrak{A}$ be a finite affine plane of order q^3 with $q=2^s$ and $s \equiv 1 \bmod 2$. Then there does not exist a collineation group Δ of $\mathfrak{A}$ fixing an affine point and inducing the group $PSU(3,q^2)$ on

the line at infinity.
When $s \equiv 0 \mod 2$ it is nor known whether there exists an affine plane admitting such a $\triangle$. If there is, $\triangle$ must possess exactly 4 line- and 4 point-orbits in the projective closure of $\mathfrak{A}$; the possible lengths of these orbits are known.

N. J. A. SLOANE: Sphere packing and error-correcting codes

Simple constructions are given for packing equal spheres in n-dimensional Euclidean space E^n using error-correcting codes. Many new packings are obtained, including nonlattice packings in E^{10}, E^{11}, E^{13} which seems to be the first known examples of packings having a greater density than the best known lattice packings. Packings in $E^n, n=2^m$, are described having density $\triangle$ satisfying $\log \triangle \sim -\frac{n}{2} \log \log n$.
For large n these seem to be the densest packings which have been explicitly constructed.

N. J. A. SLOANE: On the existence of a projective plane of order 10

If a projective plane of order 10 exists, let $\mathfrak{A}$ denote the (111,56) binary error-correcting code generated by the rows of the incidence matrix. It is known that the weight distribution of the code $\mathfrak{A}$ is uniquely determined by the number of codewords of weights 12,15,16. It is the object of this talk to report that the number of codewords of weight 15 is zero, thus reducing the number of unknown weights to two. Part of this calculation was carried out by computer.

S. L. SNOVER: Generalized block intersection numbers for t-designs and related codes

Using generalized block intersection numbers for t-designs and results from coding theory, some sub-t-designs are found for the 3-cube, the inversive plane over GF(3), the small and large Steiner systems, S(12,6,5) and S(24,8,5), and the Nordstrom-Robinson code.

J. A. THAS: Ovaloidic translation planes

With every ovaloid O of the projective space $\mathcal{P}(3,q)$, $q=2^h$ and $h>1$, there corresponds a 1-spread $\mathcal{S}(O)$ which belongs to a linear complex of lines of $\mathcal{P}(3,q)$. The finite translation plane $\mathcal{T}$ which corresponds with $\mathcal{S}(O)$ is called an ovaloidic translation plane. Conversely, with every 1-spread $\mathcal{S}$ of the projective space $\mathcal{P}(3,q)$, $q=2^h$, $h > 1$, which belongs to a linear complex of lines of this space, there corresponds an ovaloid O of $\mathcal{P}(3,q)$ (such that $\mathcal{S} = \mathcal{S}(O)$).

If O is the ovaloid of Tits-Suzuki-Segre, then $\mathcal{T}$ is the translation plane which is uniquely determined by the following properties: (a) $\mathcal{T}$ is finite of order q^2, with $q=2^{2r+1}$ and $r \geq 1$; (b) the kernel of $\mathcal{T}$ is GF(q); (c) $\mathcal{T}$ admits a collineation group isomorphic to Sz(q). If O is an elliptic quadric of $\mathcal{P}(3,q)$, $q=2^h$, $h > 1$, then $\mathcal{T}$ is the affine plane $\mathcal{A}(2,q^2)$.

Remark: The projective space $\mathcal{P}(3,q)$, $q=p^h$, together with a linear complex of lines L of $\mathcal{P}(3,q)$, form a tactical configuration W(q), with parameters $v=(q+1)(q^2+1)=b$ and $k=q+1=r$. This tactical configuration W(q) is always self-dual when p=2, and is never self-dual when p is odd.

H. UNKELBACH: Eine Charakterisierung der Hughes-Ebenen

Es wurde bewiesen:

Ist π eine nicht-desarguessche projektive Ebene der Ordnung q^2, q ungerade Primzahlpotenz, $q \neq 19$, und ist $G \cong PSL(3,q)$ eine Kollineationsgruppe von π, so ist π eine Hughes-Ebene.

H. WERNER: Affine coordinatization of finite geometries

For any (partial) algebra A one has the 'affine geometry' of A with the weak parallelism Π^A ($\Pi^A(p|M)$ denotes the parallel through p to the subspace generated by M).

Theorem 1 : A finite geometry with weak parallelism Π is the affine geometry of some algebra A iff Π satisfies : $\Pi(p|\emptyset)=p$, $\Pi(p|\Pi(q|M)) \subseteq \Pi(p|M)$, $q \in \Pi(p|p,q)$ and $\Pi(p|M) \subseteq \Pi(p|M \cup N)$.

In such a geometry a mapping $\delta : A \longrightarrow A$ is called dilatation if each $\delta(M)$ is contained in some $\Pi(q|M)$.

Theorem 2 : The dilatations of $\Gamma(A)$ are the admissible functions of A.

Problem : For which algebra A are the dilatations of $\Gamma(A)$ even the algebraic functions of A ? Those algebras are called affine complete. Examples : vector-spaces, semisimple rings, post-algebras, Boolean algebras and functionally complete algebras.

Theorem 3 : A finite algebra A is functionally complete iff

$$p(x,y,z)=\begin{cases} x & y=z \\ z & y\neq z \end{cases}$$ is an algebraic function on A.

H. Unkelbach (Mainz)

Gefördert durch
DFG Deutsche Forschungsgemeinschaft