MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 19/1971

Gruppen und Geometrien

2.5. - 8.5.1971

Die Tagung "Gruppen und Geometrien" vom 2.5. bis 8.5.1971 stand unter der Leitung von Prof. Dr. D.G. Higman (Ann Arbor) und Prof. Dr. H. Salzmann (Tübingen). Ein großer Teil der gruppentheoretischen Vorträge war den Permutationsgruppen gewidmet. Überraschend war hierbei die Vielfalt der Methoden, die zur Anwendung kamen: Subnormalteiler (Knapp), Blockpläne (Cameron), S-Ringe und Charaktertheorie (Jones), Studium von Involutionen (Rowlinson, Buekenhout), Graphentheorie (Seidel), um nur einige zu nennen. Auch eines der ältesten und schwierigsten Probleme aus diesem Gebiet, die Untersuchung der Permutationsgruppen von Primzahlgrad, wurde erfolgreich angepackt (Neumann). Vorträge aus dem Gebiet der endlichen einfachen Gruppen ergaben unter anderem Charakterisierungen bekannter endlicher Gruppen (Timmesfeld, Stellmacher).

Die Geometrie-Vorträge handelten zum großen Teil von projektiven Ebenen, affinen Ebenen, Translationsebenen und Möbiusebenen, teils endlich, teils topologisch. Die Kollineationsgruppe der betreffenden Ebene - Permutationsgruppe oder Transformationsgruppe - spielte bei den meisten Untersuchungen eine zentrale Rolle. Da umgekehrt bei vielen der Gruppentheoretischen Vorträge geometrische Überlegungen mit herangezogen wurden, erwies sich die Themenzusammenstellung der Tagung als besonders glücklich.

Außer den Vorträgen bestand der Wert dieser Tagung aber auch in der Gelegenheit, anderen Mathematikern zu begegnen und mit ihnen zu diskutieren. So mag es durchaus sein - und ist zu hoffen -, daß manches Gespräch, welches auf dieser Tagung geführt wurde, seine Früchte noch bringen wird, wenn die Tagung schon längst vorüber ist.

Teilnehmer

Asche, D.S.	Melbourne/Australien
Baer, R.	Zürich
Baumann, B.	Bielefeld
Betten, D.	Tübingen
Blasig, V.	Bielefeld
Buekenhout, F.	Brüssel/B
Busekros, A.	Tübingen
Cameron, P.J.	Oxford/GB
Fleischer, K.-J.	Bielefeld
Foulser, D.	London/GB
Held, D.	Mainz
Higman, D.	Oxford/GB
Jones, G.A.	Oxford/GB
Knapp, W.	Tübingen
Komossa, S.	Bonn
Kurzweil, H.	Tübingen
Lingenberg, R.	Darmstadt
Livingstone, D.	Birmingham/GB
McDermott, J.P.J.	Newcastle/GB
Melchior, U.	Bochum
Morris, A.O.	Aberystwyth/GB
Neumann, P.	Oxford/GB
Prohaska, O.	Tübingen
Rowlinson, P.	Stirling/GB
Salzmann, H.	Tübingen
Seib, M.	Tübingen
Seidel, J.J.	Eindhoven/NL
Selinka, M.	Tübingen
Stellmacher, B.	Bielefeld
Timmesfeld. F.G.	Bielefeld
Wielandt, H.	Tübingen
Yaqub, J.	Tübingen

Vortragsauszüge

D. BETTEN: Eine Klasse 4-dimensionaler Translationsebenen mit 7-dimensionaler Kollineationsgruppe.

Für eine nicht desarguessche 4-dimensionale Translationsebene $\mathbb{P}$ mit der vollen Kollineationsgruppe Γ sei die Standgruppe Γ_o auf dem eigentlichen Punkt $o \in R^4$ 3-dimensional. Ferner halte die Zusammenhangskomponente $\Delta = (\Gamma_o)^1$ zwei Geraden W und S des Büschels $\mathcal{L}$ der Geraden durch o fest, wirke transitiv auf dem Raum der eindimensionalen Teilräume von S und fixiere genau zwei eindimensionale Teilräume von W. Dann wird die Ebene bis auf Isomorphie von folgendem Büschel erzeugt:

$$\mathcal{L}_{w,f} = \{S,W\} \cup \left\{ \begin{pmatrix} R\cos\varphi & R\sin\varphi \\ R^w(-\sin\varphi + f\cos\varphi) & R^w(\cos\varphi + f\sin\varphi) \end{pmatrix} ; \; R > 0, \; 0 \leq \varphi < 2\pi \right\}.$$

Dabei sind w und f reelle Zahlen mit $w > 1$, $f \geqq 0$ und $f^2 \leq 4w(w-1)^{-2}$. Umgekehrt existiert zu je zwei solchen Zahlen w und f eine Ebene $\mathbb{P}_{w,f}$, und zwei Ebenen $\mathbb{P}_{w,f}$ und $\mathbb{P}_{w',f'}$ mit $w,w' > 1$, $f,f' \geqq 0$ sind genau dann isomorph, wenn $w = w'$ und $f = f'$. Die Gruppen Δ, Γ_o, Γ_o/Δ und die effektive Wirkung von Γ_o auf dem Zylinder $\mathcal{L} - \{W,S\}$ wurden bestimmt.

F. BUEKENHOUT: 2-transitive groups in which involutions fix 4 points or less.

The following result was obtained. Let G be a 2-transitive group on Ω and let each involution of G fix 0,2 or 4 points of Ω. Let $8 \mid |\Omega|$. Then $|\Omega| = 8$ or 16 and G is a known group.

The corresponding result for $8 + |\Omega|$ was obtained by NODA. The transitive extensions of the preceding groups have also been classified.

P.J. CAMERON: Primitive permutation groups with multiply-transitive suborbits.

The following theorem is proved first: Let $\mathfrak{D}$ be a 3-(v, k, 1) design in which the design $\mathfrak{D}_p$ is a symmetric 2-design, for some point p . Then one of the following holds:

(i) $v = 4(l+1)$, $k = 2(l+1)$;

(ii) $v = (l+1)(l^2+5l+5)$; $k = (l+1)(l+2)$;

(iii) $v = 112$, $k = 12$, $l = 1$;

(iv) $v = 496$, $k = 40$, $l = 3$. This theorem is used in the proof of the following result:

Let G be a primitive, not 2-transitive permutation group on Ω , and suppose G_α is t-fold transitive on its orbit $\Gamma(\alpha)$, with $|\Gamma(\alpha)| = v > 2$ and $t \geq 2$. Then $(\Gamma^* \circ \Gamma)(\alpha)$ is a single suborbit of length $\frac{v(v-1)}{k}$, where

(a) $k \leq \frac{1}{2}(v-1)$; equality implies $k = 1$ or 2 .

(b) If $t \geq 3$ then either $k = 1$ or 2 or G has rank 3, with degree $(l+1)^2(l+4)^2$, $v = (l+1)(l^2+5l+5)$ and $k = (l+1)(l+2)m$ for some integer l .

(c) If $t \geq 4$ then $k = 1$ or 2.

(d) If $t \geq v-2$ (i.e. if $G_\alpha^{\Gamma(\alpha)}$ contains the alternating group) then either $k = 1$ or G has rank $\frac{v+1}{2}$ and an elementary abelian regular normal subgroup of order 2^{v-1}.

D. FOULSER: p-elements in translation planes.

Let Π be a translation plane of order p^r, for $p > 3$, (p a prime).

Definition: a Baer p-element of Π is a collineation of Π of order p which fixes a square-root subplane pointwise.

Theorem: $(p > 3)$. Π does not admit both affine elations and Baer p elements.

Definition: Let $V = V(2r,p)$ be a vectorspace of dimension $2r$ over $GF(p)$. A linear transformation σ of V is a generalized elation if 1) the fixed-point space E_σ of σ has dimension r, and 2) $(\sigma - 1)^2 = 0$; alternatinely, if σ has a matrix of the form $\begin{pmatrix} I & I \\ 0 & I \end{pmatrix}$.

Lemma: Let σ be a collineation of Π of order p which fixes a point $\mathcal{O}$ of Π. Then σ is a generalized elation of Π regarded as V, if and only if σ is an affine elation or a Baer p-element.

The proof of the Theorem uses this lemma, and a Theorem of T.G.Ostrom (J.Algebra 14 (1970), 405-416) and Christoph Hering (On elations of translation planes, to appear) concerning the group generated by two generalized elations (actually, the part of the Theorem which is used is due to L.E.Dickson).

D. HELD: A correct computation of $|J_1|$.

In his computation of the order of J_1, JANKO has to rule out the case that the normalizer N of a S_5-subgroup is contained in the centralizer C of an involution. The character theory for N contains a mistake because the basis of the vector space of all class functions of N which vanish outside special classes is given incorrectly. Nevertheless, the case $N \subseteq C$ can be ruled out using the SUZUKI order formula for a generalized character of norm three of N. All orders lead to contradictions.

D.G. HIGMAN: On generalized quadrangles.

If a generalized quadrangle has $K > 2$ points on each line and $R > 2$ lines through each point, then $R - 1 \leq (K - 1)^2$.

G.A. JONES: Primitive Groups of Prime-power Degree.

The following results are obtained, concerning primitive but not doubly transitive groups of degree p^3, where p is a prime:

1) if G is such a group, then all minimal transitive subgroups of G are regular, and the Sylow p-subgroups of G have order at most p^4 and (for $p \neq 3$) exponent p;
2) if the Sylow p-subgroups of such a group G have order p^4, then they have class three and contain elementary abelian regular subgroups, provided $p \neq 3$;
3) a nonabelian group R of order p^3 and exponent p^e is a B-group (i.e. cannot be a regular subgroup of such a group G) if and only if $p = 2$, or $p > 3$ and $e = 2$.

The techniques used include S-rings, 2-closure, and character theory.

W. KNAPP: <u>On uniprimitive permutation groups.</u>

Let (G,Ω) be a uniprimitive permutation group and $G_\alpha^{\Delta(\alpha)}$ a subconstituent of (G,Ω) fore some $\alpha \in \Omega$. In certain cases the structure of G_α can be determined by the knowledge of $G_\alpha^{\Delta(\alpha)}$ by using subnormal subgroups or the ZJ - Theorem of G.GLAUBERMAN. A structure theorem for the case $G_\alpha^{\Delta(\alpha)} \geq A^{\Delta(\alpha)}$ and $|\Delta(\alpha)| \geq 6$ can be obtained by subnormal theory. The ZJ-Theorem can be used especially for the case $|\Delta(\alpha)|$ a prime number.

J.P.J. McDERMOTT: <u>On 3/2 transitive groups.</u>

A (k,m)-group is a $\frac{3}{2}$-transitive group of rank $k + 1$ and degree $1 + km$. Call such a group a Z(k,m)-group if a point-stabilizer is a FROBENIUS group which acts naturally on each of its non-trivial orbits.

Theorem: Let G be a non-soluble Z(k,m)-group on Ω, $(k > 1)$ and suppose $|G_{\alpha\beta}| = 2$ for $\alpha, \beta \in \Omega$. Then $m = 1 + 2^\mu$ for some $\mu > 2$ and G is permutation-isomorphic to $SL(2,2^\mu)$ represented on the cozets of a dihedral group of order 2m. Attempts to extend this result to the case "$|G_{\alpha\beta}|$ even" lead to the following general problem:

Let G be any(non-regular) primitive group on Ω. Is it true that given $\alpha, \beta \in \Omega \, \exists \gamma \in \Omega$ and $j \in \{1, \ldots k\}$ such that $\{\alpha, \beta\} \subseteq \Gamma_j(\gamma)$? Here $\Gamma_1(\gamma), \ldots, \Gamma_k(\gamma)$ are the non-trivial G_γ - orbits. The terms of the usual composition, the question asks if $\bigcup_{i=1}^{k} \Gamma_i^* \circ \Gamma_i = \bigcup_{i=1}^{k} \Gamma_i$.

U. MELCHIOR: On the isomorphism $S_6 \cong PS_p4(2)$.

Apurely geometrical proof was given of the fact, that the stabilizer of an hyperoval $\mathcal{J}$ in PG(2,4), permuting the points of $\mathcal{J}$ symmetrically acts on the other points like $PS_p4(2)$ on PG(3,2).

A.O. MORRIS: Schur Multiplier of Generalized symmetric Group.

For integers $m, n \geq 0$, the generalized symmetric group, denoted by $S(n, m)$ is defined to be the wreath product of the cyclic group Z_m and the symmetric group $S(n)$. The following was proved:

Theorem: Let $H^2(S(n, m), \mathbb{C}^*)$ be the Schur Multiplier of $S(n, m)$.

Then, if m is odd

$$H^2(S(n, m), \mathbb{C}^*) \cong \begin{cases} Z_2 & \text{if } n \geq 4 \\ 0 & \text{if } n < 4 \end{cases}$$

and if m is even

$$H^2(S(n, m), \mathbb{C}^*) \cong \begin{cases} Z_2 \times Z_2 \times Z_2 & \text{if } n \geq 4 \\ Z_2 \times Z_2 & \text{if } n = 3 \\ Z_2 & \text{if } n = 2 \\ 0 & \text{if } n = 1 . \end{cases}$$

P.M. NEUMANN: Transitive permutation groups of prime degree.

This was intended as an exposition of the

Theorem: If G is an insoluble permutation group, transitive and of prime degree p, and if a

Sylow p-normalizer $\mathbb{N}(P)$ has even order then G is 3-fold transitive,

but it turned into an expository account of groups of prime degree, including a sketch proof that amongst all the "known" simple groups only the rather obvious ones do arise.

O. PROHASKA: Finite derivable nets.

A net $\mathbb{N}$ is called derivable if it has a replacement $\mathbb{N}^*$ such that the lines of $\mathbb{N}^*$ are subplanes of $\mathbb{N}$. For N^* a line of $\mathbb{N}^*$ denote the corresponding subplane of $\mathbb{N}$ by $\mathbb{N}(N^*)$. Then:

Theorem: $\mathbb{N}(N^*)$ is a desarguesian affine plane.

An affine plane $\mathbb{A}$ is derivable if it contains a derivable net $\mathbb{N}$.

Then:

Theorem: Assume the affine plane $\mathbb{A}$ is derivable with respect to a net $\mathbb{N}$ and admits a rank-3-collineation group leaving $\mathbb{N}$ invariant. Then $\mathbb{A}$ is desarguesian or a Hall plane.

P. ROWLINSON: Involutions fixing a small number of points.

Let G be a simple group permuting primitively the elements of Ω , $1<|\Omega|<\infty$. Suppose that G has just one conjugacy class of involutions. Let f be the number of points of Ω fixed by an involution b in G , and let T be a Sylow 2-subgroup of G . If $1 \leq f \leq 5$ then one of the following holds.

(1) a certain subgroup K of G is strongly embedded,

(2) T is dihedral,

(3) T is semi-dihedral,

4) $f \in \{4,5\}$, $T \cong Z_2 \times Z_2 \times Z_2$ and $C(b) \neq \langle b \rangle \times R$ with
$R \cong PSL(2,q)$ $(q \geq 5)$,

(5) $f \in \{4,5\}$, $T \cong Z_2 \times Z_2 \times Z_2 \times Z_2$,

(6) $f \in \{4,5\}$, $T \cong Z_4 \, {}^{2}Z_2$,

(7) $f \in \{4,5\}$, $T \cong T_o \in \mathcal{S}_2(M_{12})$,

(8) $f \in \{4,5\}$, $T \cong Z_o \in \mathcal{S}_2(PSL(3,4))$,

(9) $f = 5$, $C(b) = \langle b \rangle \times R$ where $R \cong A_5$.

H. SALZMANN: 4-dimensional Translation Planes.

Let $\mathcal{P} = (P, \mathcal{L})$ denote a compact projective plane, dim $P = 4$. Then

(1) The collineation group $\Gamma_{\mathcal{P}}$ is a Lie group of dimension ≤ 16 .

(2) If $\Delta \leq \Gamma_{\mathcal{P}}$, dim $\Delta \geq 13$, then dim $\Delta = 16$, Δ is transitive, and $\mathcal{P}$ is arguesian.

(3) Any group $\Delta \leq \Gamma_{\mathcal{P}}$ with $9 \leq$ dim $\Delta \leq 12$ has a fixed point or a fixed line; $\mathcal{P}$ is arguesian if dim $\Delta \geq 10$.

(4) If dim $\Delta = 9$, $\mathcal{P}$ is a translation plane or the dual of a translation plane [and hence $\mathcal{P}$ is arguesian by a result of D. BETTEN] .

M. SEIB: Doubly Transitivity in finite projective planes.

A proof of the following theorem was given:

Theorem: Let $\mathbb{P}$ be a projective plane of even order s^2, and let U be a subset of s^3+1 distinct points of $\mathbb{P}$. If $\mathbb{P}$ admits a collineation group G which maps U onto itself and acts doubly transitive on the points of U then $\mathbb{P}$ is desarguesian and G is isomorphic to a subgroup of $P\Gamma U(3,s^2)$ containing $PSU(3,s^2)$.

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J.J. SEIDEL: Lines, codes, graphs, and groups.

The title stands for the following subjects:

1) sets of equiangular lines in Euclidean spaces,

2) binary and q-ary error-correcting codes,

3) strongly regular and strong graphs,

4) automorphism groups of such systems.

The subjects are introduced, and interrelations between them are discussed.

B. STELLMACHER: Eine Kennzeichnung der Gruppen A_6 und U(4,2).

Sei G eine endliche Gruppe, die von einer Konjugiertenklasse D von Elementen der Ordnung 3 erzeugt wird, und es gelte:

(a) Für $a,b \in D$ ist $a^b = a$ oder $\langle a,b \rangle \simeq A_4$, A_5 oder $SL(2,3)$

(b) $\langle C_D(a) \rangle$ ist abelsch und $\langle C_D(a) \rangle \neq \langle a \rangle$,

Dann ist $G/Z(G) \simeq A_6$ oder $U(4,2)$.

Dieser Satz soll als Induktionsverankerung dazu verwendet werden, Gruppen (z.B. $Sp(2n, 2)$, $O^{\pm}(2n, 2)$, Sz, Co_1) durch eine Konjugiertenklasse von Elementen der Ordnung 3 zu kennzeichnen.

F.G. TIMMESFELD: $\{3,4\}$ - transpositions in finite groups.

Let G be a finite group generated by a class of conj. involutions D, which satisfy.

(i) For all $d,e \in D$ follows $o(de) \in \{1,2,3,4\}$

(ii) All numbers occur

(iii) If $o(de) = 4$, then $(de)^2 \in D$.

Let $O_2(G) = Z(G) = 1$, then G is isomorphic to a Chevalley or Steinberg - group or GF(2), but not to PSU(n,2).

H. WIELANDT: Unspreadable subgroups of primitive permutation groups.

Let G be a primitive permutation group on a finite set Ω, and U the set of ("unspreadable") subgroups which possess fixed points but are trivial on some orbit ($\neq \{\alpha\}$) of G_α whenever α is a fixed point of H. Using the fact that a maximal unspreadable subgroup is subnormal in each G_α in which it is contained one obtains the theorem: Let G be primitive on Ω, and $\alpha \in \Omega$. Then the subgroup $U^{(\alpha)}$ which is generated by all unspreadable subgroups of G which are contained in G_α has a normal Sylow subgroup with prime power index. Hence, for instance each subgroup whose order is divisible by 3 distinct prime factors is spreadable.

J. YAQUB: Strongly ovoidal Möbius planes.

Def^n. The Möbius plane M is "strongly ovoidal" if and only if

1) $M = M(\Omega)$ where Ω is an ovoid in $R_3(F)$, F commutative,

2) each central automorphism of M is induced by a linear collineation of $R_3(F)$.

Th^m. 1. If $M = M(\Omega)$ is strongly ovoidal and if, for some $(p,\overset{C}{c})$, M is (p,C) transitive and (p,q) transitive for all $q \neq p$ on C, then either 1) M is miquelian or 2) ch F = 2 and Ω can be represented as: $\{(0,0,1,0)\} \cup \{(x,y,z,1) \mid z = a\varphi(x) + G\,\varphi(y)\}$ where i) $\varphi : F \longrightarrow F$ is a (field) monomorphism, ii) $ab^{-1} \notin \operatorname{Im} \varphi$, iii) $\varphi(x) = x \Longleftrightarrow x = 0$ or 1, iv) if $k \neq 0$ then there exists $x \neq 0$ such that $\varphi(x) = k\,x$, v) $\varphi(x) \not\equiv x^2$.

In the latter case M is of Hering class III 2.

<u>Th^m. 2.</u> If $M = M(\Omega)$ is strongly ovoidal with ch $F \neq 2$ and if, for some (p,C), M is (p,C) transitive and (p,q) half-transitive for all $q \neq p$ on C, (but not (p,q) transitive), then F is ordered and Ω can be represented as: $\{(0,0,1,0)\} \cup \{(x,y,z,1) \mid z = k\,x^2 + l\,y^2$ if $x \geq 0$, $z = k'\,x^2 + l\,y^2$ if $x < 0\}$, where k, l, k' have the same sign and $k' \neq k$. (M is of Hering class IV 1).

<u>Remarks</u> 1) Theorem 1 generalizes a result of N. Krier on Möbius planes of finite even order. 2) Examples of Theorem 1 and 2 can be constructed over suitable infinite F.

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