MATHEMATISCHES FORTSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 5/1972

Arbeitsgemeinschaft über Fastringe und Fastkörper

30.1. bis 3.2.1972

Die zweite Arbeitsgemeinschaft über Fastringe und Fastkörper stand wieder unter der Leitung von G. Betsch (Tübingen). Die Vorträge und Diskussionen behandelten Themen aus der Theorie der Fastringe und Fastkörper; ein Vortrag hatte Levitzki-Radikale von Halbringen zum Gegenstand. Folgende Themen seien genannt:
Fastbereiche und scharf 2-fach transitive Permutationsgruppen, Darstellungstheorie und Radikaltheorie von Fastringen, Fastringe von Gruppenabbildungen, Geometrische Anwendungen planarer Fastringe.

<u>Teilnehmer</u>

G. Betsch, Tübingen
J.R. Clay, Tucson (Arizona)
E. Dobber, Amsterdam
C. Ferrero Cotti, Parma
G. Ferrero, Parma
W. Heise, Hannover
R.D. Hofer, Plattsburgh (N.Y.)
M. Holcombe, Belfast (Nordirland)
H. Karzel, Hannover
W. Kerby, Hamburg
D.A. Lawver, Tucson (Arizona)
C.G. Lyons, Harrisonburg (Virginia)
B.C. McQuarrie, Worcester (Massachusetts)
J.D.P. Meldrum, Edinburgh

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M. Thomsen, Hamburg
J. Timm, Bremen
H. Wähling, Hamburg
H. Wefelscheid, Hamburg
H.J. Weinert, Clausthal-Zellerfeld
H. Wielandt, Tübingen
J.L. Zemmer, Columbia (Missouri)

Vortragsauszüge

G. BETSCH: Sheaf representation of near-rings

J. Dauns and K.H. Hofmann developed the representation of rings by sections in a "field of spaces" or a sheaf (cf. Memoir AMS # 83 (1968)). We discuss various conditions on near-rings, which allow an application of the representation methods of Dauns-Hofmann to near-rings. In particular, we obtain a generalization of the main theorem in Dauns-Hofmann, Math. Zeitschr. 91, 103 - 123 (1966).

J.R. CLAY: Generating Balanced Incomplete Block Designs from Planar Near Rings

For each finite field F with order $p^n > 3$, p a prime, and for each divisor t of $p^n - 1$, $t \notin \{1, p^n - 1\}$, there exists a planar near ring $(F, +, *)$ whose blocks $F^* a + b$ form a balanced incomplete block design. The parameters of this design are dependent upon whether $t = p^m - 1$ or not.
From three finite abelian groups we construct a non abelian group $(N, \oplus)$. Under suitable conditions on the three abelian groups, $(N, \oplus)$ is the additive group of a planar near ring, showing that

the additive group of a planar near ring need not be abelian. With further restrictions on the abelian groups the blocks of the resulting planar near ring again yield a balanced incomplete block design.

C. FERRERO COTTI: Sugli stems in cui il prodotto è distributivo rispetto a se stesso

We study distributive near rings (i.e. near rings S with $xyz = xyxz = xzyz$ for all $x,y,z \in S$). Two-sided distributive near rings and semi-simple distributive near-rings are characterised. We get many results on distributive near-rings with a special ascending chain.

G. FERRERO: Applicazioni geometriche degli stems planari

Sia G uno stem planare. Le corrispondenze $\varphi_y : x \to yx$ non nulle formano un gruppo Φ. Diciamo che $a \in G$ è di prima categoria rispetto a Φ se $Ga = G(-a) + a$. L'elemento $a \in G$ è di prima categoria se e solo se $(Ga, +)$ è un gruppo (abeliano elementare). I blocchi della forma $Ga+b$ ($a \neq 0$; $a,b \in G$) formano un BIB-disegno se e solo se si verifica uno dei seguenti casi:

a) tutti gli elementi di G sono di prima categoria rispetto a Φ: allora il disegno è uno spazio affine,

b) Non ci sono elementi non nulli di prima categoria rispetto a Φ: allora si ha un disegno con $k = \lambda = |\Phi| + 1$.

Lo studio del secondo caso da luogo ad applicazioni ai gruppi di Frobenius.

Translation: Let G be a planar near-ring. The nonzero mappings $\varphi_y : x \to yx$ (of G into itself) form a group Φ. We say that $a \in G$ is of first category with respect to Φ if $Ga = G(-a) + a$. The element $a \in G$ is of first category if and only if $(Ga, +)$ is an (elementary abelian) group. The blocks of the form $Ga + b$ ($a \neq 0$; $a, b \in G$) form a BIB-design if and only if one of the following cases is verified:

a) all elements of G are of first category with respect to Φ: in this case the design is an affine space;

b) no nonzero element (of G) is of first category with respect to Φ: in this case we have a design with $k = \lambda = |\Phi| + 1$.

The investigation of the second case gives rise to applications of Frobenius groups.

R.D. HOFER: Simplicity of near-rings of continuous functions on topological groups

Let (G, +) be a topological group and let $\mathcal{N}(G)$ denote the near-ring of all continuous selfmaps of G. If G is disconnected and $\mathcal{N}(G)$ is simple then G is totally disconnected. If G is a totally disconnected group which contains a nonzero proper open subgroup then $\mathcal{N}(G)$ is simple. Let $\mathcal{N}_0(G)$ denote the sub-near-ring of $\mathcal{N}(G)$ consisting of all $f \in \mathcal{N}(G)$ such that $f(0) = 0$. If G is an S^*-group (that is, a group having the topology of an S^*-space; see Magill, Another S-admissible class of spaces, Proc. Amer. Math. Soc. 18 (1967), 295-298) or is disconnected, then $\mathcal{N}_0(G)$ is simple if and only if G is discrete. (Berman and Silverman proved that $\mathcal{N}_0(G)$ is simple when G is discrete.) Certain generalizations of the above results have been obtained.

W.M.L. HOLCOMBE: Endomorphism near-rings in General Categories

Let X be a group object in a category $\mathcal{C}$ (see Bucur and Deleanu: "Categories and Functors"). If $\mathcal{C}$ has finite products and a final object then $Mor_{\mathcal{C}}(X, X)$ is a near-ring. Dually for X a co-group object. Such near-rings were studied in various categories, including the category of "sets and maps", "pointed sets and pointed maps", "topological spaces and continuous maps", "homotopy category of topological spaces with base point", "groups and homs", "variety subcategories of groups".

Various possible applications were discussed, including ways of regarding homotopy and cohomology groups as near-ring modules over "standard" near-rings.

W. KERBY: Near-Domains and Sharply 2-Transitive Permutation Groups

It can be shown that every sharply 2-transitive permutation group is isomorphic to the group of linear transformations $x \to a+bx$ on a uniquely determined near-domain $(F,+,\cdot)$. To my knowledge, the question as to the existence of near-domains which are not near-fields is still open. Some of the more recent results relevant to this question are:

1) If char F=3, then F is a near-field;

2) If the multiplicative group $(F^*,\cdot)$ has finite conjugate classes, then F is a near-field;

3) Let char F > 2. Then F contains a maximal sub near-field E, namely $E = \{x \in F : 1 + x = x+1\}$, and $[F^* : E^*] = 1$ or $[F^* : E^*] = \infty$.

D.A. LAWVER: Existence of Near-rings in Special Cases (Near-rings on $Z(p^\infty)$).

A near-ring is a triple $(N,+,*)$ where $(N,+)$ is a group, $(N,*)$ a semi-group, and $a*(b+c) = (a*b) + (a*c)$, for all $a,b,c \in N$. It is well known that defining a near-ring on a group $(N,+)$ is equivalent to fixing a function $f:N \to End(N)$ where $a*b = (f(a))(b)$ and $f(a*b) = f(a) \circ f(b)$.
In section I we deal with the construction of near-rings over the quasi-cyclic groups $Z(p^\infty)$, p a prime. Here we know that $Z(p^\infty) = \bigcup_{n=0}^{\infty} H_n$ where $H_n = \left\langle \frac{1}{p^n} \right\rangle$. In this case $f:Z(p^\infty) \to End(Z(p^\infty))$ must satisfy: there is an n such that (i) $f(H_i) = \{\theta\}$, for $i < n$, (ii) $f(H_n) = f(Z(p^\infty))$, and (iii) the non-zero endomorphisms in $f(H_n)$ are automorphisms and form a group under composition.
In section II we indicate the impossibility of constructing near-rings with identity over certain classes of finite groups. Although these results are not new, the techniques, due to John Krimmel, are new and follow essentially from
THEOREM. Suppose that $(N,+,*)$ is a near-ring with identity such that every element has finite order. If $a,b \in N$ with $\ker f(a) \geq \ker f(b)$, then the order of a divides the order of b.

In section III we give constructions of near-rings in three broad classes of groups.

C.G. LYONS: Endomorphism Near Rings

For an arbitrary near ring R with nontrivial idempotent e it is known that $R = A + M$ where $A = \{r - er \mid r \in R\}$ and $M = \{er \mid r \in R\}$. By examining the summands A and M, necessary and sufficient conditions that A is an ideal are determined. It is shown that an additive generating set for R determines additive generating sets for A and M. And, if R is a C-ring both A and M are near rings and the procedure for determining additive generating sets is shown to be iterative. Thus, the problem of constructing a C-ring from an idempotent and an additive generating set reduces to constructing its summands.
This technique of multiple decomposition is used to construct the endomorphism near ring of G where G is a dihedral group of order 2n, n even. It is further shown that $I(D_{2n}) \subsetneq A(D_{2n}) \subsetneq E(D_{2n})$ contrary to the case for n odd.
As a final result it is shown that if H is a fully invariant abelian summand of the group G then E(H) embeds in E(G) as a group direct summand.

B.C. MCQUARRIE: Near Rings that are N-systems

In proving that the addition in a near field is abelian, B.H. Neumann used a set of six axioms which are different from those usually used to define a near field. Systems that satisfy the six axioms are called N-systems. In essence an N-system is a (left) near ring with halvable identity in which the right cancellation law holds. Ligh has shown that all finite N-systems are near fields. It had been an open question whether or not there exist proper N-systems (those which are neither rings nor near fields). In this paper we show the existence of proper N-systems and exhibit some of their properties.

J.D.P. MELDRUM: Representation theory of d.g. near-rings

The results as presented in the talk given at the Oberwolfach conference were incorrect. The following summary gives the corrected results.

Let R be a left near-ring and let S be a semigroup of distributive elements generating R. A group G is an (R,S) group if there exists a homomorphism θ from (R,S) into the near-ring generated by the endomorphisms of G such that $S\theta$ is a semigroup of endomorphisms of G. Let $\underline{V}$ be a variety of groups. We say $R\varepsilon\underline{V}$ if $(R,+)\varepsilon\underline{V}$. If (R,S) has a faithful representation on $G\varepsilon\underline{V}$, then $R\varepsilon\underline{V}$.

Theorem: In every non-abelian variety $\underline{V}$, there exists a d.g. near-ring (R,S) which does not have a faithful representation.

Upper and lower faithful d.g. near-rings for (R,S) are defined and shown to exist.

The general question of adjoining an identity to a d.g. near-ring (R,S) remains open, if we do not insist that S has to remain distributive in the larger near-ring.

J. MISFELD: Zur Konstruktion topologischer Fastkörper

Alle bisher bekannten unendlichen Fastkörper (definiert durch Schiefkörperaxiome ohne eines der beiden Distributivgesetze) lassen sich nach einem von Dickson [1905] für endliche angegebenen und von Karzel [1965] auf unendliche Fastkörper verallgemeinerten Verfahren aus Schiefkörpern konstruieren ("Dicksonsches Konstruktionsverfahren"). Es wurden (gemeinsam mit J. Timm erarbeitete) notwendige und hinreichende Bedingungen dafür angegeben, wann man mit Hilfe dieses Verfahrens aus topologischen Körpern topologische Fastkörper konstruieren kann.

R. MLITZ: Verallgemeinerte Jacobson-Radikale in Polynom-Kompositionsfastringen

Man betrachtet den Polynomring in einer Unbestimmten über einem kommutativen Ring mit 1, und zwar als Fastring mit den Operationen der Addition und der Komposition. Um von diesem Fastring verallgemeinerte Jacobson-Radikale berechnen zu können, werden zunächst die von BETSCH (1963) angegebenen 3 Definitionen verallgemeinerter Jacobson-Radikale auf Fastringe, in denen die 0 nur einseitig invariant ist, übertragen. Man erhält so formal 9 Radikale, von denen allerdings mindestens 2 zusammenfallen. Alle diese Radikale sind subhereditär, in dem Sinn, daß für gewisse idealähnliche Unterfastringe M eines Fastrings N gilt: $J(M) \supseteq J(N) \cap M$. Sodann werden einige dieser Radikale für Polynomfastringe berechnet bzw. abgegrenzt. Die hier erhaltenen Resultate stellen eine Erweiterung jener von CLAY und DOI (1971) dar, die solche Polynomfastringe über Körpern der Ordnung $p>2$ betrachteten.

I. PIEPER: On a class of near-modules

A normal local near-module is a local algebra F in which one distributive law is not valid (cf. H. Karzel and I. Pieper, Bericht über geschlitzte Inzidenzgruppen, J. Ber. der Deutsch. Math. Verein.). We suppose different conditions for the sub-nearmodule K+N, generated by the underlying sfield and the set N of non-units.

In the finite case (F,.) is a commutative local algebra iff the units of K+N lie in the centre $\mathcal{C}(U)$ of the group of units U of F. There is a weaker statement for the infinite case.

If we assume the units of K+N to lie in the nucleus $\mathcal{N}(U)$ of U in F then there is an ideal A_N of F in N such that the difference structure F/A_N is a local algebra. F/A_N is commutative when F is finite and N commutative.

With these results we can show that there is no normal local nearmodule F with commutative N and $(K+N) \cap U \subseteq \mathcal{N}(U)$, $U \not\subseteq \mathcal{N}(U)$, which is constructed from a commutative local algebra with the same set of units by the Dickson-Karzel-Timm derivation method.

G. PILZ: On the Construction of Near-Rings from a Z- and a C- Near-Ring

Let N be a near-ring and N_Z (N_C) be its maximal Z- (C-) subnear-ring. It is well known that each element of N can be uniquely represented as a sum of an element from N_Z and an element from N_C.

In this paper the converse problem is considered: let N_1 (N_2) be a Z- (C-) near-ring, respectively, and form $(N,+) := (N_1,+) \oplus (N_2,+)$. A multiplication "." such that $(N,+,.)$ is a near-ring with $N_Z = N_1$ and $N_C = N_2$ is called admissable. How can these admissable multiplications be characterized? - It is shown that there exists an admissable multiplication in any case and that the admissable multiplications are exactly the uniquely determined extensions of all near-module multiplications $N \times N_2 \to N$ fulfilling 3 conditions. This characterisation becomes very simple if N_2 is a distributively generated near-ring whose distributive elements distribute over whole N. Applications to identities in N and their relations to identities in N_2, to affine near-rings and to ordered near-rings are mentioned.

S.D. SCOTT: Non-nilpotent ideals of near-rings with minimal condition

Let N be a near-ring with minimal condition on right N-subgroups and Q(N) the sum of the nilpotent right ideals of N. Let T be a minimal ideal of N. If T is non-nilpotent then $Q(N) \cap T = \{0\}$. This means that T is a finite direct sum of minimal non-nilpotent right ideals of N and that there exists a right ideal R of N such that $N = T \oplus R$.

The proof is accomplished as follows: If it is shown that $Q(N) \cap T$ is a direct summand of T the rest follows. To show $Q(N) \cap T$ is a direct summand of T we take a minimal non-nilpotent right ideal R_1 of N contained in T and by considering a minimal non-nilpotent right N-subgroup M of N contained in R_1 we show that $R_1 = M$ and R_1 is a minimal right ideal of N. It follows that $T = R_1 \oplus R_2$ where R_2 is a right ideal of N such that $Q(N) \cap T \leq R_2$. In this manner it is shown that $Q(N) \cap T$ is a direct summand of T.

J. TIMM: Free Near-Algebras

For the concept of near-algebras see H.D. Brown, Yamamuro, Timm (4) in B^+. Some theorems connecting near-algebras and left-uniquely solvable near-rings were stated. Applying the theory of universal algebras the existence of free near-algebras in some classes of near-algebras was shown. The polynomial ring over a field turned out to be a retract of several free near-algebras and using this fact the concept of degree and the Euclidean algorithm were generalized to free near-algebras of some classes of algebras.

The ideals of this free near-algebras may be characterized by means of C-primal ideals.

Applications of this theory to the case of archimedean ordered near-algebras and the construction-problem for non-Dickson nearfields were discussed.

(B^+ = Bibliography of Betsch/Malona/Clay Jan. 1972).

H. WÄHLING: Automorphismen Dicksonscher Fastkörper

Man nennt das geordnete Paar (F, F^{φ}) zweier Fastkörper (FK) einen Dicksonschen FK (DFK) und den Fastkörper F^{φ} regulär, wenn F ein Körper ist und wenn $\varphi : a \to a_{\varphi}$ eine Abbildung (Dickson-Abbildung oder gekoppelte Abbildung) von F^* in die Automorphismengruppe A(F) von F ist, die der Bedingung $(ab^{a_{\varphi}})_{\varphi} = b_{\varphi}\, a_{\varphi}$ für alle $a, b \in F^*$ genügt. Welche Konsequenzen haben Voraussetzungen über die Mächtigkeit der sogenannten D-Gruppe $\Gamma_{\varphi} = \{a_{\varphi} \mid a \in F^*\}$ von (F, F^{φ}) für die Automorphismen von F^{φ} ?

(A) Es seien (F, F^{φ}) und (F', F'^{ψ}) zwei DFK, $F^{\varphi} \neq N(2, 3)$ (der echte FK mit 9 Elementen) und $|\Gamma_{\varphi}| < |F|$, $|\Gamma_{\psi}| < |F'|$. Dann ist jeder Isomorphismus von F^{φ} auf F'^{ψ} ein Isomorphismus von F auf F'.

(B) (F, F^{φ}) sei ein DFK mit $|\Gamma_{\varphi}| < |F|$ und $F^{\varphi} \neq N(2, 3)$.

Jeder galoissche Teil-Fastkörper H von F^{φ} ist galoissch in F. Wenn F kommutativ ist und algebraisch über H, stimmt die volle Galoisgruppe $\mathcal{G}$ von F^{φ} über H mit der von F über H überein.

Setzt man außerdem voraus, daß $|\Gamma_{\varphi}|^2 < |H|$ im Fall $|H| < \omega_0$, $|F| \geq \omega_0$, so ist die Menge $\mathcal{F}$ aller H enthaltenden Teil-FK von F^{φ} mit der Menge aller H enthaltenden Teilkörper von F identisch, und die Abbildung $E \to \mathcal{G}(E) = \{\gamma \in \mathcal{G} \mid x^{\gamma} = x$ für alle $x \in E\}$ von $\mathcal{F}$ in die Menge aller (in der endlichen Topologie) abgeschlossenen Untergruppen von $\mathcal{G}$ ist bijektiv.

H. WEFELSCHEID: Zur Konstruktion scharf 3-fach transitiver Permutationsgruppen mit Hilfe von Fastkörpern

Definiert man K-T-Felder $(F(+, \cdot, \sigma)$ durch

I) $F(+,\cdot)$ ist ein Fastbereich (neardomain),

II) σ ist ein involutorischer Automorphismus von $F^*(\cdot)$ mit der Eigenschaft:

$$1 - \sigma(1+a) = \sigma(1+\sigma(a)) \quad \forall a \in F \setminus \{0,-1\},$$

dann operieren die Abbildungen der Form

α: $x \to a + mx$ mit $a, m \in F$, $m \neq 0$ für $x \in F$ und $\alpha(\infty) = \infty$,

β: $x \to b + \sigma(a + mx)$ mit $b, a, m \in F$, $m \neq 0$ für $x \in F$ und $\beta(\infty) = b$

scharf 3-fach transitiv auf der Menge $F \cup \{\infty\}$.
Umgekehrt ist jede scharf 3-fach transitive Gruppe isomorph zur Gruppe der Transformationen der Form α, β eines eindeutig bestimmten K-T-Feldes. K-T-Felder, die keine Körper sind, lassen sich folgendermaßen konstruieren:
Es sei $F(+,\cdot)$ ein kommutativer Körper, $\tau \in \operatorname{Aut} F(+,\cdot)$ und G eine Untergruppe von $F^*(\cdot)$, so daß gilt:

i) τ ist eine Involution,

ii) $\tau(G) = G$,

iii) $[F^*:G] = 2$.

Dann ist $F(+, o, \sigma)$ ein K-T-Feld, wenn man definiert:

$$a \circ b = \begin{cases} a \cdot b & \text{für } a \in G \\ a \cdot \tau(b) & \text{für } a \notin G \end{cases}$$

und $\sigma(a) = a^{-1}$ (wobei a^{-1} das Inverse von a bezüglich (.) bezeichnet). Alle endlichen K-T-Felder sowie die Beispiele von Tits und Karzel sind in dieser Weise konstruiert. Man kann mit dieser Methode unendliche K-T-Felder beliebiger Charakteristik konstruieren und auch alle endlichen in unendliche einbetten.

H.J. WEINERT: Levitzki-Radicals of Semirings

Let $R = (R, +, \cdot)$ be a semiring with commutative addition. For each right ideal A of R we denote by

$$\bar{A} = \{x \in R \mid x + a_1 = a_2 \quad \text{for some } a_1, a_2 \in A\}$$

the k-closure of A.
For each (two-sided) ideal A of R the quotient semiring R/A is defined by the congruence relation $x \equiv y\ (A) \Leftrightarrow x + a_1 = y + a_2$ for some $a_1, a_2 \in A$. Then $R/A = R/\bar{A}$ and $\bar{A} \supseteq A$ is an annihilating zero of R/A, even if R has no zero or has a zero 0 which does not annihilate. Suppose that R has kernel (intersection of all two-sided ideals) $K \neq \emptyset$. If R has a zero 0, we have $0 \in \bar{K}$, and $K = \bar{K} = \{0\}$, if 0 is annihilating. We define the concepts "nil", "locally nilpotent", "nilpotent", and "annihilator (right) ideal" with respect to $\bar{K}$ (and not with respect to K, c.f. A. Costa, Sur la théorie générale des demi-anneaux I, Sém. Dubreil-Pisot, Paris 1961, exposé no 24). The unions of all locally nilpotent left, right, and two-sided ideals coincide, and form the Levitzki radical L(R) of R. Then one obtains the important lemma $L(R) = \overline{L(R)}$, which yields for instance the following results: (1) $L(R/L(R)) = \{0\}$. (2) If R is a nil semiring satisfying the ACC on left annihilators, then R is locally nilpotent. (3) If R satisfies the ACC on left and right annihilators, then any nil sub-semiring of R is nilpotent. These results improve and generalize results of E. Barbut (Fund. Math. 68 (1970), 261-269).

H. WIELANDT: How to single out function near rings

In order to single out sub-near rings of the near ring N(G) of all self-mappings of a given group G, there are three familiar procedures.

(1) Pick a subgroup $H \leq G$, put $N_H(G) := \{f: G \to G \mid H^f \subseteq H\}$.

(2) Pick a normal subgroup $K \trianglelefteq G$, put
$N_{G/K}(G) := \{f: G \to G \mid (Kg)^f \subseteq Kg^f\}$.

(3) Pick an endomorphism ε of G, put $N_\varepsilon(G) := \{f: G \to G \mid f\varepsilon = \varepsilon f\}$.

There is a general method which contains (1) - (3) as special cases:

(1^k) Pick a cardinal number k and a subgroup of the direct product G^k of k copies of G. Make N(G) act on G^k component-wise. Pick a subgroup $H \leq G^k$ and define $N_H(G)$ by the same formula as in (1).

The most useful special cases seem to occur for k = 2 (e.g. (2) and (3)), and possibly k = 3. REMAK's investigation of subgroups of G^2 and G^3 might be useful (J. reine angew. Math. 166).
Also the methods developed for investigating permutation groups P by discussing the P-invariant k-relations (Lecture Notes, Dept. of Math., Ohio State University, Columbus 1969) will be applicable ((1^k) defines a near-ring by means of an invariant "linear" k-relation, namely $H \leq (G^k, +)$).

J.L. ZEMMER: Valuation near-rings

Let K be a near-field. A sub near-ring N of K is called a total near-ring provided N is a sub near-ring of K with the property for each x ε K, x ≠ 0, either x or x^{-1} ε N. Some of the elementary properties of total near-rings are investigated. It is seen that a total near-ring is a local near-ring in the sense of C.J. Maxon, On local near-rings, Math. Z., 106 (1968), 197-205. Necessary and sufficient conditions, involving the group of units of N, are given that N ≠ J(N), where J(N) is the radical of N as defined by J. Beidleman, On near-rings and near-ring modules, Ph.D. Thesis, Pennsylvania State University, 1964. It is shown that the concept of total near-ring is not strong enough to have geometric significance. By placing suitable restrictions on the group of units in N, one is led to the concept of valuation near-ring which does have geometric connections. From this one is led to define a left valuation on a near-field and a place of a near-field. The relations between these and their geometric applications are discussed. Examples of a total near-ring and a valuation near-ring are given.

G. Betsch (Tübingen)