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Frobenius' Representations of Partitions and Related Problems.
by George E. Andrews

For the ordinary partition $5+5+5+3+2+1+1$, Frobenius writes

$$\begin{pmatrix} 4 & 3 & 2 \\ 6 & 3 & 1 \end{pmatrix}.$$

The Frobenius representation is derived from the correspondence:

1. Start with the Ferrers graph.
2. Delete main diagonal
3. Use portion of graph to right of diagonal for upper row of Frobenius rep$^{\underline{n}}$.
4. Use portion below diagonal (read columns) for lower row.

This representation of partitions makes Jacobi's Triple Product Identity nearly obvious:

$$\text{Let } \varphi(z) = \prod_{n=0}^{\infty}(1+zq^{n+1})(1+z^{-1}q^{n}), \quad z\neq 0, |q|<1.$$

Then $\varphi(z)$ is analytic in a deleted neighborhood of 0 and satisfies $\varphi(z) = zq\,\varphi(zq)$. Hence the Laurent series for $\varphi(z)$ must be (up to constant term) $A_0 \sum z^n q^{n(n+1)/2}$. However the constant term is clearly the generating function for all partitions written in Frobenius notation. Hence $A_0 = \prod (1-q^n)^{-1}$.

Indeed if we consider more generally $\begin{pmatrix} a_1 \cdots a_r \\ b_1 \cdots b_r \end{pmatrix}$ where the a_i are restricted to partitions of certain specifications A and the b_i to B, then the generating function for these two rowed objects is just $f_A(z,q)\, f_B(z^{-1},q)$, where $f_A(z,q)$ is the generating function for partitions of n into m parts subject to conditions A.

For example ~~let~~ $F_k(n)$ denotes the number of $\begin{pmatrix} a_1 \cdots a_r \\ b_1 \cdots b_r \end{pmatrix}$ such that $n = r + \Sigma a + \Sigma b$, each entry repeats at most k times in each row, and all entries are nonnegative. Then, e.g.

(1) $$\sum_{n \geq 0} F_2(n) q^n = \prod_{n=1}^{\infty} \frac{1}{(1-q^n)(1-q^{12n-2})(1-q^{12n-3})(1-q^{12n-9})(1-q^{12n-10})},$$

(2) $$\sum_{n \geq 0} F_3(n) q^n = \prod_{n=0}^{\infty} \frac{(1-q^{12n+6})^2}{(1-q^{6n+1})(1-q^{6n+2})^2(1-q^{6n+3})^3(1-q^{6n+4})^2(1-q^{6n+5})(1-q^{6n+6})}$$

$$\sum_{n \geq 0} F_1(n) q^n = \prod_{n=0}^{\infty} \frac{1}{1-q^{n+1}}.$$

For $k \geq 3$, the gen functions are modular on some subgroup of the modular group but are not simple infinite products.

The $F_k(n)$ have numerous nice results comparable to $p(n) = F_1(n)$.

(3) $$\sum_{n=0}^{\infty} F_2(n) q^n = \frac{\sum_{n=-\infty}^{\infty} \left(q^{(3n)^2} - q^{(3n+1)^2}\right)}{\prod_{n=1}^{\infty} (1-q^n)^2}.$$

(4) $5 \mid F_2(5n+3)$.

(5) $$\sum_{n=0}^{\infty} F_{k-1}(n) q^n = \frac{\sum_{j=-\infty}^{\infty} \sum_{r \geq |j|} (-1)^r q^{k\binom{r+1}{2} - (k-1)\binom{j+1}{2}}}{\prod_{n=1}^{\infty} (1-q^n)^2 (1-q^{kn})}.$$

I offer USA \$100.00 each for bijection proofs of (1) and (2), USA \$500 for combinatorial proof of (3), and USA \$20.00 each for purely combinatorial proofs of (4) and (5).

Schur functions and the invariant polynomials characterizing U(n) tensor operators

by
S. C. Milne

We give a direct formulation of the invariant polynomials ${}_{\mu}G_q^{(n)}(\,,\Delta_i\,,\,;\,,x_{i,i+1}\,,)$ characterizing U(n) tensor operators $\langle p, q, \ldots, q, 0, \ldots, 0\rangle$ in terms of the symmetric functions S_λ known as Schur functions. To this end we show after the change of variables $\Delta_i = \gamma_i - \delta_i$ and $x_{i,i+1} = \delta_i - \delta_{i+1}$ that

${}_{\mu}G_q^{(n)}(\,,\Delta_i\,,\,;\,,x_{i,i+1}\,,)$ becomes an integral linear combination of products of Schur functions $S_\alpha(\,,\gamma_i\,,)\cdot S_\beta(\,,\delta_i\,,)$ in the variables $\{\gamma_1, \ldots, \gamma_n\}$ and $\{\delta_1, \ldots, \delta_n\}$, respectively. That is, we give a direct proof that ${}_{\mu}G_q^{(n)}(\,,\Delta_i\,,\,;\,,x_{i,i+1}\,,)$ is a bisymmetric polynomial with integer coefficients in the variables $\{\gamma_1, \ldots, \gamma_n\}$ and $\{\delta_1, \ldots, \delta_n\}$. By making further use of basic properties of Schur functions such as the Littlewood-Richardson rule we prove several remarkable new symmetries for the yet more general bisymmetric polynomials ${}^{m}_{\mu}G_q^{(n)}(\gamma_1, \ldots, \gamma_n; \delta_1, \ldots, \delta_m)$. For example,

Let
$$ {}^{m}_{\mu}G_q^{(n)}(\gamma;\delta) = (-1)^{\binom{\mu+1}{2}q} \sum_{(\alpha,\beta)\in\Omega(\mu,q,m,n)} b_{(\alpha,\beta)} S_\alpha(\gamma_1, \ldots, \gamma_n)\, S_\beta(\delta_1, \ldots, \delta_m), \qquad (1.1a)$$

and
$$ {}^{m+1}_{\mu}G_q^{(n+1)}(\gamma;\delta) = (-1)^{\binom{\mu+1}{2}q} \sum_{(\alpha,\beta)\in\Omega(\mu,q,m+1,n+1)} c_{(\alpha,\beta)} S_\alpha(\gamma_1, \ldots, \gamma_n)\, S_\beta(\delta_1, \ldots, \delta_m), \qquad (1.1b)$$

where $\Omega_{(\mu,q,n,m)}$ is a set of ordered pairs (α,β) of partitions that depends on $\mu, q, m,$ and n; $b_{(\alpha,\beta)}$ and $c_{(\alpha,\beta)}$ are integers uniquely determined by (α,β); and S_α and S_β are Schur functions.

We then have

(i) $\Omega(\mu, q, m, n) \subseteq \Omega(\mu, q, m+1, n+1)$, (1.2a)

(ii) $(\alpha, \beta) \in \Omega(\mu, q, m, n)$ implies that $b_{(\alpha,\beta)} = c_{(\alpha,\beta)}$ (1.2b)

(iii) $\Omega(\mu, q, m, n) = \Omega(\mu, q, m+1, n+1)$ if and only if

$$n \geq (\mu+1)q \text{ and } m \geq (\mu+1+m-n)q, \qquad (1.2c)$$

(iv) the bound in (iii) is best possible, and (1.2d)

(v) the $n=(\mu+1)q$ and $m=(\mu+1+m-n)q$ case of (1.1a) gives the correct formula for ${}^{m+l}_{\mu}G^{(n+l)}_{q}(\gamma;\delta)$, for all integers l, when $(n+l)$ and $(m+l)$ variables are used in the Schur functions $S_\alpha(\gamma)$ and $S_\beta(\delta)$, respectively. (1.2e)

These new symmetries enable us to give an explicit formula for both ${}^{m}_{\mu}G^{(n)}_{1}(\gamma;\delta)$ and ${}_{1}G^{(n)}_{2}(\gamma;\delta)$. Out of the above work, several new combinatorial identities arise. For example:

Let $z_1, \ldots, z_n$ be arbitrary distinct real numbers and $(\lambda) \equiv (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_l \geq 0)$ a partition with $l \geq 1$ an integer. We then have:

$$\sum_{1 \leq j_1 < j_2 < \cdots < j_l \leq n} \left(S_{(\lambda_1,\lambda_2,\ldots,\lambda_l)}(z_{j_1}, z_{j_2}, \ldots, z_{j_l})\right) \cdot \prod_{\substack{1 \leq i \neq j \leq n \\ i \in S',\, j \in S^c}} (z_i - z_j)^{-1}$$

$$= S_{(\lambda_1 - n+l, \ldots, \lambda_l - n+l)}(z_1, z_2, \ldots, z_n), \qquad (1.3)$$

where $S' = \{j_1 < j_2 < \cdots < j_l\}$ and $S^c = \{j_{l+1}, \ldots, j_n\}$.

The left-hand side of (1.3) is 0 if and only if $0 \leq \lambda_l \leq n-l-1$.

Corollary 1.4. Set $\ell = 1$, then $S_{\lambda_1}(z_1, \dots, z_n)$ is just the homogeneous symmetric function $\beta_{\lambda}(z_1, \dots, z_n)$ and (1.3) becomes the important summation theorem of Biedenharn and Louck in J. Math Physics, (1970)

Corollary 1.5. Set $\lambda_i = n - \ell$, and $\ell = n-1$ in (1.3). Then the Schur function becomes an elementary symmetric function and we obtain (after trivial algebra),

$$1 = \sum_{m=1}^{n} \prod_{\substack{i=1 \\ i \neq m}}^{n} \left(1 - \frac{z_m}{z_i}\right)^{-1},$$ which is just the key identity given in Good's proof of Dyson's conjecture, (Also J. Math. Physics (1970)!!)

Finally, (1.3) is merely a special case of $\div$. (proof similar)

Let W' be a standard Weyl subgroup generated by simple reflections $\Delta' \subseteq \Delta$.

$$\sum_{W/W'} w\left(\left[\sum_{w'} w'\left(e^{\Lambda}\left(\prod_{\substack{\alpha > 0 \\ \alpha \in \Sigma'}} \frac{(e^{(\alpha/2)} - t e^{(-\alpha/2)})}{(e^{(\alpha/2)} - e^{(-\alpha/2)})}\right]\right) \cdot \prod_{\substack{\alpha > 0 \\ \alpha \in \Sigma - \Sigma'}} \frac{(e^{(\alpha/2)} - t e^{(-\alpha/2)})}{(e^{\alpha/2} - e^{-\alpha/2})}\right)$$

$$= \sum_{w \in W} w\left(e^{\Lambda} \prod_{\alpha > 0} \frac{(e^{\alpha/2} - t e^{-\alpha/2})}{(e^{\alpha/2} - e^{-\alpha/2})}\right)$$

Set $e^{\Lambda} \equiv 1$ and get an analog of Good's identity

Interplay Between Combinatorics and Topology. E. R. Canfield

First we cite two nice examples of "interplay" from the literature.

Ex 1 (Tucker, 1945) Let $\underbrace{\underset{\sim}{n} \times \cdots \times \underset{\sim}{n}}_{d \text{ factors}} \xrightarrow{\phi} \{\pm 1, \pm 2, \dots, \pm d\}$

be a ~~an arbitrary~~ labeling of $\underset{\sim}{n}^d$, where $\underset{\sim}{n} = \{0, 1, \dots, n\}$, with the property that $\phi(x) = -\phi(\vec{n} - x)$ for each x in the boundary of $\underset{\sim}{n}^d$. (Here, $\vec{n} := (n, \dots, n)$ and the boundary of $\underset{\sim}{n}^d$ consist of those x with at least one co-ordinate equal to zero or n.) Then there must exist two points x and x', whose co-ordinates differ by at most one, such that $\phi(x) = -\phi(x')$.

As a consequence of this combinatorial Lemma, one has topological ~~consequences~~ results: (i) a continuous $f: S^n \to S^n$ commuting with the antipodal cannot be extended to the inside of S^n; (ii) whenever $S^n = U_1 \cup \dots \cup U_{n+1}$, ($U_j$ open), some U_j contains an antipodal pair. [The latter is the Borsuk-Ulam Theorem (early 1930's).]

Ex 2 (Lovász, 1978) The Kneser Conjecture (1955) asserted that when the k-elements subsets of a set S having $2k+n$ elements are distributed into classes:

$$P_k(S) = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \cdots$$

such that no disjoint pair is placed in the same class, $n+2$ classes are required. [That $n+2$ classes suffice is seen by assigning the subsets to the first available class, using lexicographic order.]

Lovász proved this by defining a simplicial structure $(V, \mathcal{S})$ on vertices V which are the k-element subsets of S. The geometric realization, $\|\mathcal{S}\|$, of a simplicial structure $\mathcal{S}$ is the set of functions $x: V \to [0, 1]$ s.t. (a) $\sum_{A \in V} x(A) = 1$

$\|\mathcal{S}\|$ has the obvious topology as a subspace of $\mathbb{R}^{|V|}$ (b) $\mathrm{Sup}(x) \overset{\text{def}^n}{\equiv} \{A : x(A) > 0\} \in \mathcal{S}$

Lovász's structure permitted a continuous $f: S^n \to \|\mathcal{S}\|$ such that for any $p \in S^n$, any $A \in \mathrm{Sup}(f(p))$, any $B \in \mathrm{Sup}(f(-p))$ one has $A \cap B = \phi$. [Incidentally, a family $\langle A_0, \dots, A_m \rangle$ from $\mathcal{P}_k(S)$ is an m-simplex in Lovász's $\mathcal{S}$ provided $|\bigcup A_i| \le n+k$]
For $\mathcal{P}_k(S) = \mathcal{C}_1 \cup \cdots$, let $U_j \subseteq \|\mathcal{S}\| = \{x : \mathrm{Sup}(x) \cap \mathcal{C}_j \neq \phi\}$.
Then, $S^n = f^{-1}U_1 \cup \cdots$ and no $f^{-1}U_j$ can contain an antipodal pair. Thus, $\geq n+2$ classes are needed, by the Borsuk-Ulam Theorem.

We make the following <u>generalized Kneser conjecture</u>

GKC(t): If $\mathcal{P}_k(S) = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \cdots$, no $\mathcal{C}_j$ containing t pairwise disjoint subsets, with $|S| = tk + N$, then $\left[\frac{N}{t-1}\right] + 2$ classes are needed.

The "greedy algorithm" shows again that, if true, this is best possible.

Using a method similar to Bárány's simplification of Lovász's proof, we have verified GKC(t)

when $N = m(t-1) + (t-2)$.

When $N = m(t-1) + h$, $0 \le h < t-2$, the GKC(t) asserts

that $m+2$ classes are needed, but our method yields only $m+1$. Thus, for these values the correct answer is known to within 1.

J. Cigler, Some methods for q-identities.

A useful transformation for obtaining q-identities has been found by Paule, a student in Vienna:

$$\sum_{k\in\mathbb{Z}} a_k \begin{bmatrix}a+b\\a+k\end{bmatrix}\begin{bmatrix}a+b\\b+k\end{bmatrix} = \sum_j \frac{[a+b]!}{[a-j]!\,[b-j]!}\frac{q^{j^2}}{[2j]!} S_{2j}$$

with $$S_{2j} = \sum_{k\in\mathbb{Z}} \begin{bmatrix}2j\\j+k\end{bmatrix} q^{-k^2} a_k .$$

E.g. $a_k = (-1)^k q^{\frac{1}{2}k(5k+1)}$ gives

$$\sum_k (-1)^k q^{\frac{k}{2}(5k+1)} \begin{bmatrix}a+b\\a+k\end{bmatrix}\begin{bmatrix}a+b\\b+k\end{bmatrix} = \sum_j \frac{[a+b]!}{[a-j]!\,[b-j]!}\frac{q^{j^2}}{[j]!}(1-q)^j,$$

which is a finite form of the Rogers–Ramanujan identities. These are obtained by setting $a=b=n$, dividing by $\begin{bmatrix}2n\\n\end{bmatrix}$ and letting $n\to\infty$.

Generalizing a theorem by G.P. Egorychev another student, Krattenthaler, has given the following general form for q-inverse relations:

Define a Lagrange pair to be a pair $((f_n(z)),(F_n(z)))$ of sequences of formal power series satisfying $\frac{1}{2\pi i}\oint \frac{f_n(z)}{F_k(z)}\frac{dz}{z^{n-k+1}} = \delta_{nk}$.

Then $$a_n = \sum c_{nk} b_k \quad \text{with} \quad c_{nk} = \frac{\beta_k}{\alpha_n}\frac{1}{2\pi i}\oint f(z)\frac{g_n(z)}{H_k(z)}\frac{dz}{z^{n-k+1}}$$

$$\Longleftrightarrow \quad b_n = \sum d_{nk} a_k \quad \text{with} \quad d_{nk} = \frac{\alpha_k}{\beta_n}\frac{1}{2\pi i}\oint \frac{1}{f(z)}\frac{h_n(z)}{G_k(z)}\frac{dz}{z^{n-k+1}}$$

E.g. $g_n(z) = P_{2n+p}(1,-q^{n+1}z)$, $G_k(z) = \dfrac{P_{2k+2+p}(1,-q^k z)}{(1-q^{k+p}z^2)}$ gives

$$a_n = \sum_{k=0}^{n} q^{-k(n-k)} \begin{bmatrix}n+p+k\\n-k\end{bmatrix} b_k \quad \Longleftrightarrow$$

$$\Longleftrightarrow \quad b_n = \sum_{k=0}^{n} (-1)^{n-k} q^{\binom{n-k}{2}-(k-1)(n-k)} \begin{bmatrix}2n+p\\n-k\end{bmatrix} \frac{[2k+1+p]}{[n+k+p+1]} a_k$$

Lagrange Inversion and Sheffer systems on Free Monoids

by

Peter Kirschenhofer

We give a generalization of the Lagrange-Good-Formula to the case of special systems of formal power series in noncommuting variables:

Th: Let $A=\{a_1,\dots,a_\alpha\}$ be an at most countable alphabet, $f\in\mathbb{R}\langle\langle A\rangle\rangle$ and $(q_w)_{w\in A^*}$ a system of series $q_w\in\mathbb{R}\langle\langle A\rangle\rangle$ with $q_w = w * \varphi^{\underline{w}}$, where $*$ denotes the Shuffle product and $\varphi^{\underline{w}} = \prod^{*}_{1\le i\le\alpha} \varphi_i^{(w;a_i)}$ with φ_i $*$-invertible in $\mathbb{R}\langle\langle A\rangle\rangle$ and $(w;a_i)$ the number of occurrences of the letter a_i in w.
Then $f=\sum_{w\in A^*} c_w\cdot q_w$ where c_w equals the coefficient of w in

$$f*\varphi^{-\underline{w}} * \det(\delta_{i,j}+\varphi_i^{-1} * N_j\varphi_i),$$

N_j being the continuous linear Operator with $N_j w=(w;a_j)w$.

The proof uses a formula for special binomial systems in non-commuting variables (compare the talk of G. Baron):
If $(p_w)_{w\in A^*}$ is the (uniquely determined) binomial system with Δ-system (= derivation system) $(Q_w = D_w\cdot P^{\underline{w}})$, $P_i=\varphi_i(D)$ invertible then

$$p_w = \det(\delta_{i,j} + P_i^{-1}(N_j\varphi_i)(D))P^{-\underline{w}}w .$$

A generalization of Rodrigues' formula is given, too:
Th: The operator $T_{a_i,Q}$ mapping $T_{a_i,Q}\,p_w = p_{a_iw}$ is given by

$$T_{a_i,Q} = a_i\circ\varphi_i(Q) + \sum_{j=1}^{\alpha} N_j((\varphi_j^{-1} * \partial_{a_i}\varphi_j)(Q))$$

where $\partial_{a_i}(w^{(1)}w^{(2)}\dots w^{(n)}) = \delta_{a_i,w^{(1)}}\cdot w^{(2)}\dots w^{(n)}$ $\quad (w^{(j)}\in A)$.

Defining a Sheffer system related to the binomial system (p_w) by $s_w(X+Y) = \sum_{u,v}[w;u,v]s_u(X)p_v(Y)$

where X, Y are disjoint copies of A with $x_i y_j = y_j x_i$ for $x_i, y_j \in X, Y$ resp., $[w; u, v]$ denotes the number of possibilities to dissect the word w into the complementary subwords u and v, a number of well known theorems on ordinary Sheffer systems can be generalised. Of special interest in defining "natural" analogs of classical Sheffer sequences may be the following analog of a theorem due to J. Cigler in the commuting case:

<u>Th:</u> A system (s_w) with $\deg s_w = w$, $s_\varepsilon = \varepsilon$ is Sheffer related to (p_w) with shift operator $T_{a_i, Q}$ (see above) if and only if there exists a system of operators $(b_i(Q))_{1 \le i \le \alpha}$, $b_i(Q) \in \mathbb{R}\langle\langle D \rangle\rangle$ such that

$$s_{a_i w} = (T_{a_i, Q} - b_i(Q)) s_w .$$

Taking $s = \prod^{*}_{1 \le i \le \alpha} e_*(a_i \circ b_i)$, where $e_*(f) = \sum_{n \ge 0} \frac{f^{*n}}{n!}$, the "generating function" of s_w fulfills

$$\sum_w s_w(X)\, w(T) = \Big(\sum_w p_w(X)\, w(T)\Big) * s^{-1}(T) .$$

(X, T disjoint copies of A.)

On an adjacency property of graphs

by E. Triesch

A graph G has property $A(m,n,k)$ if its order is at least $m+n$ and if for any sequence of $m+n$ distinct points of G there exist at least k other points of G which are adjacent to the first m but not adjacent to the last n points. The minimum order of graphs with property $A(m,n,k)$ is denoted by $a(m,n,k)$. As we are only concerned with the case $k=1$, let

$$A(m,n) := A(m,n,1)$$

$$a(m,n) := A(m,n)$$

The adjacency property is defined and investigated in the following papers:

[1] A. Blass and F. Harary; First order properties of almost all graphs and simplicial complexes
J. Graph Theory 3 (1979) 225-240

[2] G. Exoo - F. Harary
The smallest graphs with certain adjacency properties
Discrete Math. 29 (1980) 25-32

[3] G. Exoo. On an adjacency property of graphs, J. Graph Th. 5 (1981) 371-378

In [3] the inequalities

$$(1) \qquad a(m,n) \le C_m n^{m+1} \log n$$

and
$$(2) \qquad \tfrac{3}{4} \cdot n^2 \le a(1,n) \le 4n^2 - 2n$$

are derived.

I proved the following inequalities:

Thm 1 If $G \in A(1,n)$ and (minimal degree of G) $= n+k$ $(k \geq 1)$ then
$$|V(G)| \geq n^2 + n(3-k) + k^2 - k + 2$$

and Thm 2: $a(m,n) \geq 1 + a(m-1,n) + a(m,n-1)$

As a corollary of Thm 1 you regain the lower bound in (2) (actually, $\frac{3}{4}n^2 + \frac{5}{2}n$ + positive constant).

From Thm 2, the inequality
$$a(m,n) \geq \binom{m+n+2}{m+1}$$
immediately follows which yields also a lower bound of the form
$$c_m' \, n^{m+1} \text{ for fixed } m.$$

Pictures and standard tableaux
by
Michael Clausen (jointly with F. Stötzer)

The notion of a picture appeared first in papers by James/Peel (J. Alg. 56) and Zelevinsky (J. Alg. 69 & Springer LN, 869) in connection with the representation theory of symmetric groups.
In the definition of a picture two orderings of $\mathbb{N}\times\mathbb{N}$ are involved:

$(a,b) \leq_P (c,d) :\Leftrightarrow (a \leq c \text{ and } b \leq d)$.

$(a,b) \leq_J (c,d) :\Leftrightarrow (\text{either } a<c \text{ or } a=c \text{ and } b \geq d)$.

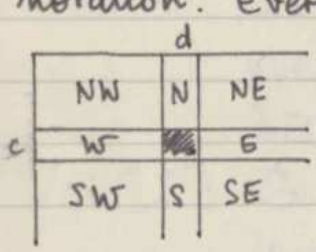
A function $f: X \to Y$ $(X, Y \subseteq \mathbb{N}\times\mathbb{N})$ is said to be PJ-standard iff f is an order morphism $f: (X, \leq_P) \longrightarrow (Y, \leq_J)$. Now let $A, B \subseteq \mathbb{N}\times\mathbb{N}$

$\left\{\begin{array}{l} T: A \to B \text{ is a picture} \\ \text{of shape } A \text{ and content } B \end{array}\right\} :\Leftrightarrow \left\{\begin{array}{l} T: A \to B \text{ is a bijection and} \\ T \text{ and } T^{-1} \text{ are PJ-standard.} \end{array}\right.$

$P(A,B)$ denotes the set of all pictures of shape A and content B.

Problem (from representation theory): Construct (!) $P(A,B)$.

Before we can give some characterizations of pictures we need some notation. Every $(c,d) \in \mathbb{N}\times\mathbb{N}$ decomposes $\mathbb{N}\times\mathbb{N}$ into disjoint parts:



We write $(a,b)\,(X, Y, \ldots, Z)\,(c,d)$ iff $(a,b) \neq (c,d)$ and (a,b) is in one of the parts $X, Y, \ldots, Z$ w.r.t. (c,d).
Example. $(a,b) \leq_P (c,d) \Leftrightarrow (a,b)\,(W, NW, N)\,(c,d)$.

$A \subseteq \mathbb{N}\times\mathbb{N}$ is P-convex $:\Leftrightarrow (x \leq_P y \leq_P z \wedge x, z \in A \Rightarrow y \in A)$.

$X\ (\subseteq A)$ is A-regular $:\Leftrightarrow \exists D: D$ P-convex, $(1,1) \in D$ and $X = A \setminus D$.

Let π_1, π_2 denote the natural projections $\mathbb{N}\times\mathbb{N} \twoheadrightarrow \mathbb{N}$ with $\pi_i: (a_1, a_2) \mapsto a_i$.

THEOREM. For a bijection $T: A \to B$ $(A, B \subseteq \mathbb{N}\times\mathbb{N})$ the following conditions are equivalent:

(1) T is a picture.

(2) $\forall\, x=(a,b),\ y=(c,d) \in A:$ $\left\{\begin{array}{l} x\,(E)\,y \Rightarrow T(x)\,(W, SW)\,T(y) \\ x\,(S, SE)\,y \Rightarrow T(x)\,(SW, S, SE)\,T(y) \\ x\,(SW)\,y \Rightarrow T(x)\,(SW, S, SE, E, NE)\,T(y) \end{array}\right.$

(3) $\begin{cases} \forall x \in A: & T[\{y \in A \mid x \lneqq_{\mathfrak{F}} y\}] \text{ is } B\text{-regular and} \\ \forall z \in B: & T^{-1}[\{y \in B \mid z \lneqq_{\mathfrak{F}} y\}] \text{ is } A\text{-regular.} \end{cases}$

(4) [further assumption: A, B P-convex]

$\pi_1 \circ T$ and $\pi_1 \circ T^{-1}$ are column strict (skew) reverse plane partitions and $\pi_2 \circ T$ and $\pi_2 \circ T^{-1}$ are row strict (skew) plane partitions. □

Cor. $\mathcal{P}(A,B) \neq \emptyset \Rightarrow$ A and B are row-finite, i.e. the intersection of A (resp. B) with every row of $\mathbb{N}\times\mathbb{N}$ has to be a finite set. □

For row-finite subsets of $\mathbb{N}\times\mathbb{N}$ we introduce the following equivalence relation: $A \approx A' :\Leftrightarrow \exists_{f: A' \rightarrowtail\!\!\!\twoheadrightarrow A} \forall_B \; \mathcal{P}(A,B)\circ f := \{T\circ f \mid T \in \mathcal{P}(A,B)\} = \mathcal{P}(A',B)$.

THEOREM. A, A' row-finite.

$$A \approx A' \iff \exists_{f: A' \rightarrowtail\!\!\!\twoheadrightarrow A} \forall x,y \in A': \begin{cases} x\,(E)\,y \Leftrightarrow f(x)\,(E)\,f(y) \\ x\,(S,SE)\,y \Leftrightarrow f(x)\,(S,SE)\,f(y) \\ x\,(SW)\,y \Leftrightarrow f(x)\,(SW)\,f(y). \end{cases}$$ □

THEOREM.

$$\mathcal{J} := \left\{ A \subset \mathbb{N}\times\mathbb{N} \;\middle|\; \begin{array}{l} A \text{ row-finite;} \\ \forall (i,j) \in A: \; (j' < j \Rightarrow \exists i': (i',j') \in A) \wedge (i' < i \Rightarrow \exists j': (i',j') \in A); \\ \forall (i,j) \in A: \; j \geq 2 \Rightarrow A \cap (\{(k,j) \mid k < i\} \cup \{(h,j-1) \mid h > i\}) \neq \emptyset \end{array} \right\}$$

is a $\approx$-transversal. □

In order to get estimates for $\mathcal{P}(A,B)$ we define for $A, C \in \mathcal{J}$:

$$A \precsim C \;:\Leftrightarrow\; \exists_{f: C \rightarrowtail\!\!\!\twoheadrightarrow A} \forall_{B \in \mathcal{J}} \; \mathcal{P}(A,B)\circ f \subseteq \mathcal{P}(C,B).$$

THEOREM. (i) $(\mathcal{J}, \precsim)$ is a poset.

(ii) $\{ \{(1,n),(2,n-1),(3,n-2),\dots,(n,1)\} \mid n \in \mathbb{N}\}$ is the set of all maximal elements in $(\mathcal{J}, \precsim)$.

(iii) $A \in \mathcal{J}$ is minimal in $(\mathcal{J}, \precsim)$ iff $A \in \mathcal{J}$ is totally ordered by $\lneqq_{\mathfrak{F}}$. □

Finally an algorithm was shown which constructs (besides other pictures) by suitable hook deformations the set $\mathcal{P}(A,B)$ for finite P-convex A and $B \in \mathcal{J}$.

COMBINATORIAL ASPECTS OF CONTINUED FRACTIONS AND ORTHOGONAL POLYNOMIALS

by

Philippe FLAJOLET

We show that the standard continued fraction expansions for power series which have the form

$$\sum a_n z^n = \cfrac{1}{1-\kappa_0 z - \cfrac{\lambda_1 z^2}{1-\kappa_1 z - \cfrac{\lambda_2 z^2}{\cdots}}} \qquad (1)$$

are a natural representation of ~~combin~~ generating functions associated to combinatorial structures called "path diagrams". Path diagrams are essentially weighted path in the upper half planes consisting of ascents $a = |^{1}_{1}$, descents $d = |^{1}_{-1}$ and level steps $q = |^{1}_{0}$. If $D_{k,l,n}$ is the number of path diagrams of length n, initial altitude k and final altitude l then:

$$\sum D_{0,0,n} z^n = \cfrac{1}{1-\kappa_0 z - \cfrac{\alpha_0 \delta_1 z^2}{1-\kappa_1 z - \cfrac{\alpha_1 \delta_2 z^2}{\cdots}}} \qquad (2)$$

where $\alpha_j, \delta_j, \kappa_j$ are weights associated to a, d and q steps at altitude j. With $D^{[h]}_{k,l,n}$ counting similarly path of height at most h, one has

$$\sum D^{[h]}_{0,0,n} z^n = \frac{P_h(z)}{Q_h(z)}, \qquad (3)$$

where P_h/Q_h is the h-th convergent of fraction (2). The P and Q polynomials appear in a number of enumerative results relative to path diagrams and for instance:

$$\sum D^{[h]}_{0,h,n} z^n = \frac{\alpha_0 \alpha_1 \cdots \alpha_{h-1} z^h}{Q_h(z)}. \qquad (4)$$

One can use (2), (3), (4) in two different ways. Either as a way of proving combinatorially continued fraction identities or as a means of getting specific results for particular choices of $\kappa_j, \alpha_j, \delta_j$.

For instance using bijective correspondances between certains systems of paths diagrams and set partitions or permutations, one obtains combinatorial proof of

$$\sum B_n z^n = \cfrac{1}{1 - 1z - \cfrac{1z^2}{1 - 2z - 2z^2 \cdots}} \qquad \sum n!\, z^n = \cfrac{1}{1 - 1z - \cfrac{1^2 z^2}{1 - 3z - 2^2 z^2 \cdots}} \quad \text{[Euler]}$$

(where B_n = coeff of $[\frac{x^n}{n!}]$ in $\exp(e^x - 1)$), and a variety of continued fractions expansions follow for ~~Stirling~~ generating functions of Stirling numbers, Eulerian number, Euler numbers ...

By (4) Taylor coefficients of inverses of classical orthogonal polynomials can also be interpreted and, for instance:

coeff $[z^n]$ in $\frac{k!\, z^k}{H_k(z)}$ with H_k (essentially) a Hermite polynomial, counts the number of way of placing n straps between $2n$ endpoints in a "tunnel" with capacity k.

Using again bijective correspondance and classical continued fractions, one finds:

The coefficient of $(-1)^n \frac{z^{2n}}{n!} \alpha^{2k}$ in the Jacobian elliptic function $cn(u, \alpha)$ counts the number of alternating permutations of $2n$ having k ~~peaks~~ valleys of even value.

Other applications include congruence properties of classical combinatorial quantities, expressions for Haenkel determinants, and q analogs. In particular one can obtain fractions related to Heine's expansion:

$$\sum [1]_q [3]_q \cdots [2n-1]_q z^n = \cfrac{1}{1 - \cfrac{q^0 [1]_q z^2}{1 - q^1 [2]_q z^2 \cdots}} \qquad \text{where } [a]_q = \frac{1-q^a}{1-q}$$

Thus embedding various results of Touchard, Carlitz ... in a unified framework.

Ref: Discrete Mathematics, 1980.

A Constructive Proof of the q-analog of Pfaff-Saalschutz

– D. M. Bressoud

Given two partitions of an integer: π $(0 \le a_1 \le a_2 \le \ldots \le a_n)$ and ψ $(0 \le b_1 \le \ldots \le b_m)$, define the crossing number of ψ with respect to π starting at s to be the largest integer, r, greater than or equal to 0 such that $b_r < a_{s-r+1}$.

It is proved by constructive methods that the generating function for pairs of partitions, π and ψ, satisfying

(1) π has n distinct parts, all ≥ 1 and $\le m+n+k$

(2) ψ has m parts (zeroes permitted), all parts $\le s+k$

(3) the crossing number of ψ with respect to π starting at s is r

is

$$q^{\binom{n+1}{2}} \begin{bmatrix} s \\ r \end{bmatrix} \begin{bmatrix} m+n-s \\ n-s+r \end{bmatrix} \begin{bmatrix} m+n+k-r \\ m+n \end{bmatrix} q^{(m-r)(s-r)}$$

where $\begin{bmatrix} A+B \\ A \end{bmatrix} = \prod_{i=1}^{A} \frac{(1-q^{B+i})}{(1-q^i)}$.

Summing over all r, and recognizing that the generating function for pairs of partitions satisfying (1) and (2) is

$$q^{\binom{n+1}{2}} \begin{bmatrix} m+n+k \\ m+n \end{bmatrix} \begin{bmatrix} m+s+k \\ m \end{bmatrix}$$

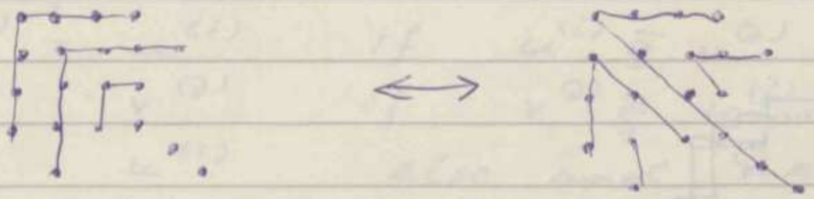
yields the q-analog of the Pfaff-Saalschutz summation.

What Is (or Should be) a Simple Combinatorial Proof of the Rogers-Ramanujan Identities

by George E. Andrews

A history of the interplay between bijective and algebraic-analytic proofs of classical partition identities was given. Three bijective proofs of Euler's theorem (The ptns of n into odd part are equinumerous with the ptns of n into distinct parts) were given, and their relationship to analytic generalizations were provided. E.G. Sylvester's "fish hook" bijection

7+7+3+1+1+1 9+5+4+2



corresponds to N.J. Fine's analytic result

$$\sum_{j\ge 0} \frac{t^{j+1} q^{2j+1}}{(1-tq)(1-tq^3)\cdots(1-tq^{2j+1})} = \sum_{n\ge 1} (1+q)\cdots(1+q^{n-1}) t^n q^n$$

and Fine's "refinement" of Euler (the ptns of n into distinct parts with largest $= k$ are equinumerous with the ptns of n into odd parts wherein the no. of parts $+ \frac{1}{2}$(largest part $-1) = k$). also

This interaction is observed, in work on Euler's Pentagonal Number Theorem (Euler, Legendre, Schur, Franklin), the Rogers-Ramanujan-Schur identities, and Schur's mod 6 theorem.

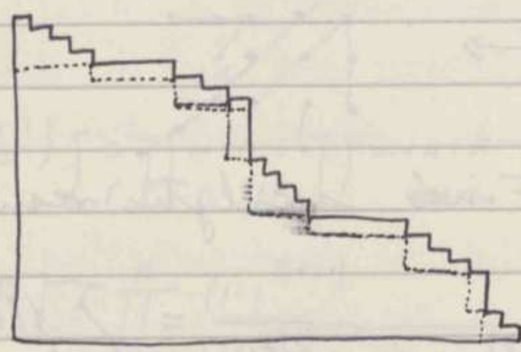
It is suggested that the mutual advance of combinatorial and algebraic methods is quite fruitful. Hopefully the recent bijective breakthrough by Garsia and Milne proving the R-R idents will foster this interaction.

Some Properties of the Majorization Order

Curtis Greene

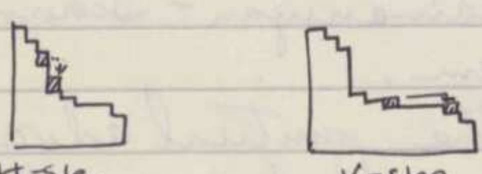
The majorization order $\leq$ on the set P_n of all partitions of n is defined as follows: if $\theta = \{\theta_1 \geq \theta_2 \geq \cdots\}$ and $\lambda = \{\lambda_1 \geq \lambda_2 \geq \cdots\}$, then $\theta \leq \lambda$ iff $\lambda_1 + \lambda_2 + \cdots + \lambda_i \geq \theta_1 + \theta_2 + \cdots + \theta_i$ for $i = 1, 2, \cdots$. We obtain simple combinatorial characterizations of two important functions on the lattice P_n: (i) the Möbius function, and (ii) the height function.

The first is based on a combinatorial decomposition of partitions (called the "staircase" decomposition), over which, in a certain sense, the Möbius function is multiplicative.



The arguments refine and "explain" earlier results of Brylawski and Bogart, which show that μ_{P_n} assumes only the values $\pm 1, 0$, with certain periodicities mod 3.

The height function of P_n is characterized by special kinds of maximal chains (called "HV-chains"), in which all of the covers of one type (called "H-steps") precede all covers of another type (called "V-steps").



Our main results (obtained jointly with D. J. Kleitman) are that (i) any pair $\theta \leq \lambda$ can be linked by an HV-chain; (ii) all of these chains have the same length; and (iii) this length is maximum.

Binomial Systems on Free Monoids

Gerd Baron

Starting from an at most countable alphabet A considered as an poset $\langle A, \varrho \rangle$ with corresponding strict partial order $\bar{\varrho}$ there are defined modules of formal power series and polynomials over a Ring R. ~~The~~ Products of words are defined by merging two words with regard to the partial order. That means the m-th item of the merged list w is computed by comparing the i-th item of the first list u and the j-th item of the second list v and a random generator

$$w^{(m)} = \begin{cases} u^{(i)} & \text{if } u^{(i)} \bar{\varrho} v^{(j)} \\ v^{(j)} & \text{if } v^{(j)} \bar{\varrho} u^{(i)} \\ u^{(i)} & \text{else and RANDOM} = 1 \\ v^{(j)} & \text{RANDOM} = 2 \end{cases}$$

Let $[w; u, v]_\varrho$ be the absolute frequency of the list w in all merged lists of u and v then $u *_\varrho v = \sum [w; u, v]_\varrho w$ is the ϱ-shuffle product. If ~~always~~ RANDOM = 1 the merged list is the ϱ-concatenation $u \circ_\varrho v$. So we get ring structures for the formal power series and polynomials. If $\varrho = \varrho_0$ the antichain the products are the well known shuffle product and the concatenation. So the coefficients $[w; u, v]_\varrho$ are generalized binomial coefficients. Further on we will us restrict to $\varrho = \varrho_0$. Using disjoint copies of A say $\bar{X}$ and $\bar{Y}$ we get the binomial theorem $w(\bar{X} + \bar{Y}) = \sum [w; u, v] u(\bar{X}) v(\bar{Y})$ and can therefore generalize binomial systems in the sense of ROTA. Defining corresponding observation systems many parts of the ROTA theory can be generalized, e.g. for a certain type of binomial systems the RODRIGUES formula. To this type belongs a generalisation of the NEWTON polynomials which give us the possibility to generalize the STIRLING numbers of the second kind. They are combinatorically interpreted as the number of partitions of a word w of initial type v, where $v^{(i)}$ the i-th letter of v is the first letter of the i-th part of the partition.

e.g. $w = a\,b\,c\,a\,b\,a\,c\,a\,b$ $\qquad v = a\,b\,c\,a$

$p_1 = aaba \quad p_2 = bc \quad p_3 = cb \quad p_4 = a$

$v = a \quad b \quad c \quad a$

Let $S(w,v)$ be the new STIRLING numbers then with $S(n,k)$ the old ones we have

$$S(|w|,k) = \sum_{|v|=k} S(w,v)$$

Umbral methods for multi-variate Hermite polynomials

by

Marilena Barnabei (jointly with A. Brini and G. Nicoletti)

Given an $n \times n$ symmetric matrix with real entries $A = (a_{ij})$, such that $\det(A) \neq 0$ and $\prod_i a_{ii} \neq 0$, the (n-variables) Hermite polynomials of variance A are defined by means of the generating function:

$$\sum_{\underline{d}} \frac{H_{\underline{d}}(x_1, \ldots, x_n)}{d_1! \ldots d_n!} t_1^{d_1} \ldots t_n^{d_n} = e^{\frac{1}{2}\varphi(x_1,\ldots,x_n) - \frac{1}{2}\varphi(x_1 - t_1, \ldots, x_n - t_n)}$$

where φ is the quadratic form associated to A.
The sequence $(H_{\underline{d}}(x_1, \ldots, x_n))$ turns out to be a sequence of Sheffer type, related to the sequence of binomial type

$$p_{\underline{d}}(x_1 \ldots x_n) = z_1^{d_1} \cdot z_2^{d_2} \ldots z_n^{d_n}$$

where $\quad z_i = \varphi_i(x_1, \ldots, x_n) = \sum_j a_{ij} x_j$.

The set of delta operators associated (in the sense of the Umbral Calculus) to such sequence is $(Z_1, \ldots, Z_n)$, where Z_i is the formal partial derivative with respect to the variable z_i. Hence we get

$$Z_i \, H_{\underline{d}}(x_1, \ldots, x_n) = d_i \, H_{\underline{d} - \underline{\delta}_i}(x_1, \ldots, x_n).$$

Moreover, if we define the Weierstrass operator as follows:

$$W := \exp\left(-\frac{1}{2}\varphi(Z_1, \ldots, Z_n)\right)$$

by general results of the Umbral Calculus we get

$$W z_1^{d_1} \ldots z_n^{d_n} = H_{\underline{d}}.$$

The following identity holds:

$$(*) \qquad H_{\underline{\underline{d}}+\underline{\underline{\delta}}_i} = (z_i - \varphi_i(Z_1, .., Z_n))H_{\underline{\underline{d}}} .$$

This yields the recursion:

$$H_{\underline{\underline{d}}+\underline{\underline{\delta}}_i} = z_i H_{\underline{\underline{d}}} - \sum_j a_{ij} d_j H_{\underline{\underline{d}}-\underline{\underline{\delta}}_j} .$$

Identity (*) allows us to prove the following generalized versions of the Rodrigues formula:

$$(1) \quad H_{\underline{\underline{d}}} = \prod_{i=1}^{n} (-1)^{d_i} e^{z_i^2/2a_{ii}} \left(\varphi_i(Z_1, .., Z_n)\right)^{d_i} e^{-z_i^2/2a_{ii}} .$$

$$(2) \quad H_{\underline{\underline{d}}} = (-1)^{\Sigma d_i} e^{\frac{1}{2}\varphi(x_1, .., x_n)} Z_1^{d_1} \cdots Z_n^{d_n} e^{-\frac{1}{2}\varphi(x_1, .., x_n)} .$$

Proof concepts for almost-all results by Walter Oberschelp — May 13, 1982

We introduce results of Fagin (J. Symb. Log. 41, 50-58) and Blass-Harary (J. Graph Th. 3, 225-40) concerning 0-1 laws for relative frequencies of first-order-defined n-element models in relations.
Then we interprete a generalisation of Lynch (Ann. Math. Log. 18, 91-135), where 0-1 laws are proved, if a superimposed structure (e.g. the successor mod n) can be used. The idea of „richness" (technically: k-extendibility with respect to the Ehrenfeucht game) is explicated, and a negative result for finite linear orders is compared with the successor case.

As a second generalisation we consider relative frequencies with respect to special relation theories defined by a condition $\mathfrak{D}$. $\mathfrak{D}$ is called Blass-Fagin (BF), if the limit exists and is 0 for every first order condition $\mathfrak{L}$. We exhibit the proof idea of Blass in the case, that $\mathfrak{D}$ is graph theory, and sketch, how things work, in the framework of parametric relations (W. Oberschelp, Lecture Notes Math. (Springer) 579 (ed. Foata), 297-307). The concept of richness appears again in the BF-proof for parametric conditions (W. Oberschelp, Oberwolfach 1980 and DMV-Meeting 1980 Dortmund) Beyond that we comment on results of K.J. Compton (Dissertation 1981). So far the exponential generating power series had convergence radius $R=0$. But Compton's results apply to $R>0$. Here exactly the case $R=\infty$ yields BF-conditions. We interprete this case „at the other end of the convergence scale" as „poorness" in structure, contrasted to richness in the former cases. Finally we interprete Compton's most interesting positive BF-example, viz. equivalence relations, and correspondingly partitions of n in the analogous unlabeled case.

W. Oberschelp

On ajoute des multiplicités et des couleurs au graphe des permutations (ordre d'Ehresmann, dit ordre de Bruhat dans le cas des groupes de Coxeter). Les arêtes simples peuvent être considérées comme des opérateurs, dépendant de 3 paramètres homogènes, sur l'algèbre des polynômes $\mathbb{Z}[a, b, .. c, d]$ =

$$D_{ab}(f) = \left[p \frac{f + qaf}{a - b} \right]^{1+\sigma} + r f^{\sigma}$$

où σ dénote la transposition $a \leftrightarrow b$, et similairement pour les paires de lettres consécutives, on définit D_{bc}, D_{cd}, ...

D'après le lemme de Tits, deux chemins ayant les mêmes points terminaux (et considérés comme des produits d'opérateurs) donnent le même opérateur.

On utilise ces opérateurs, à la suite de Demazure, Bernstein, Gelfand et Gelfand, pour étudier $\mathbb{Z}[a, b, c, d]$, et en particulier son quotient par l'idéal engendré par les polynômes symétriques en $a, b, \ldots c, d$. [ce quotient se trouve être l'anneau de cohomologie de la variété de drapeaux], ou l'autre quotient qui est l'anneau de Grothendieck de cette variété.

Une $\mathbb{Z}$-base de ces deux anneaux quotients consiste en les cycles de Schubert X_w, $w \in S_n$. On peut donner l'expression des X_w (en tant que polynômes), leur postulation, leur degré projectif, leurs intersections (formule de Pieri), exactement comme pour les grassmanniennes.

La différence la plus importante réside dans le fait que X_w n'est un déterminant que pour certaines permutations w.

On peut donner une expression générale qui englobe différentes formules classiques, comme par exemple la formule des caractères de Weyl, ou la classe des syzygies des variétés déterminantales, et le théorème de Bott.

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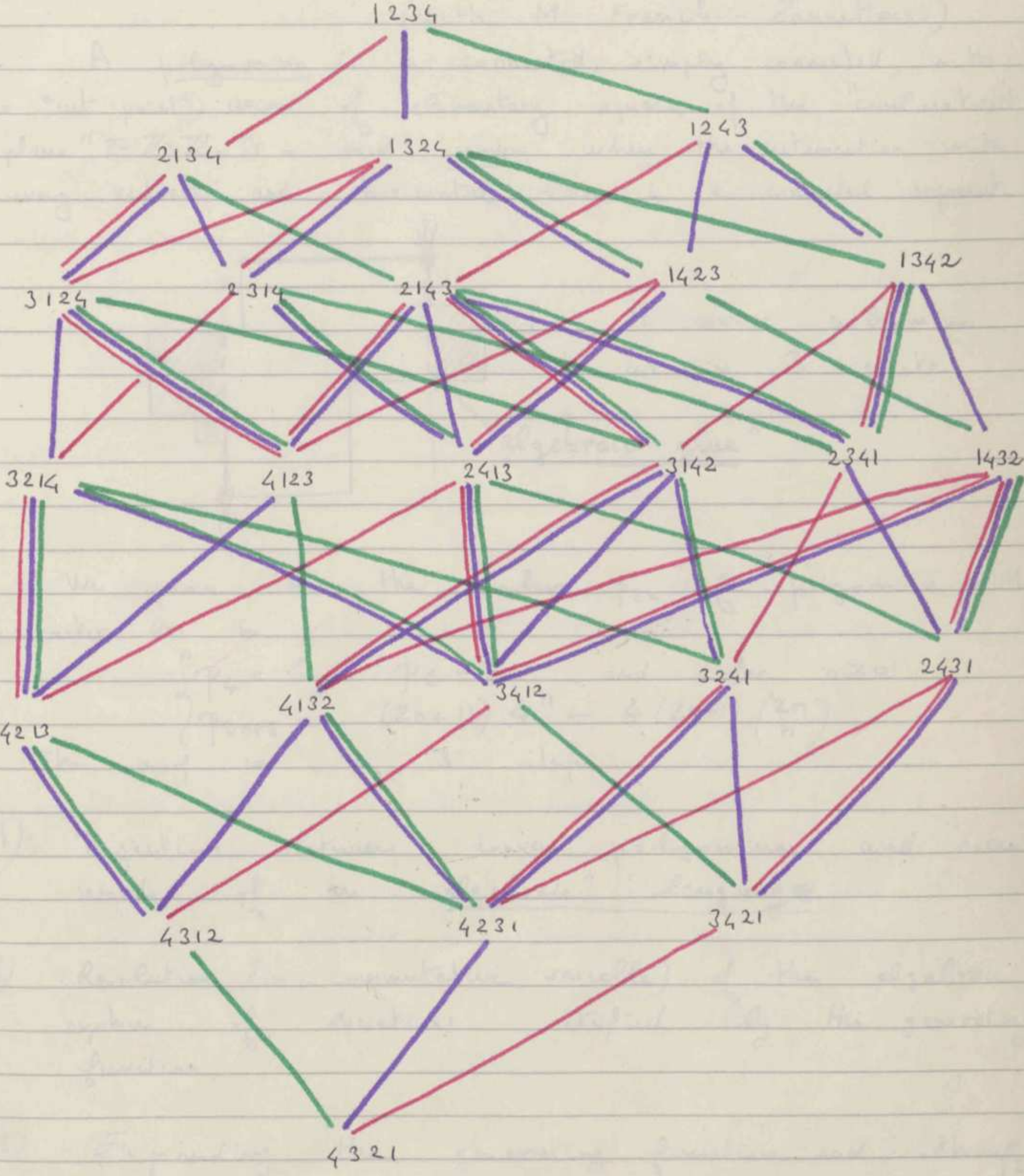
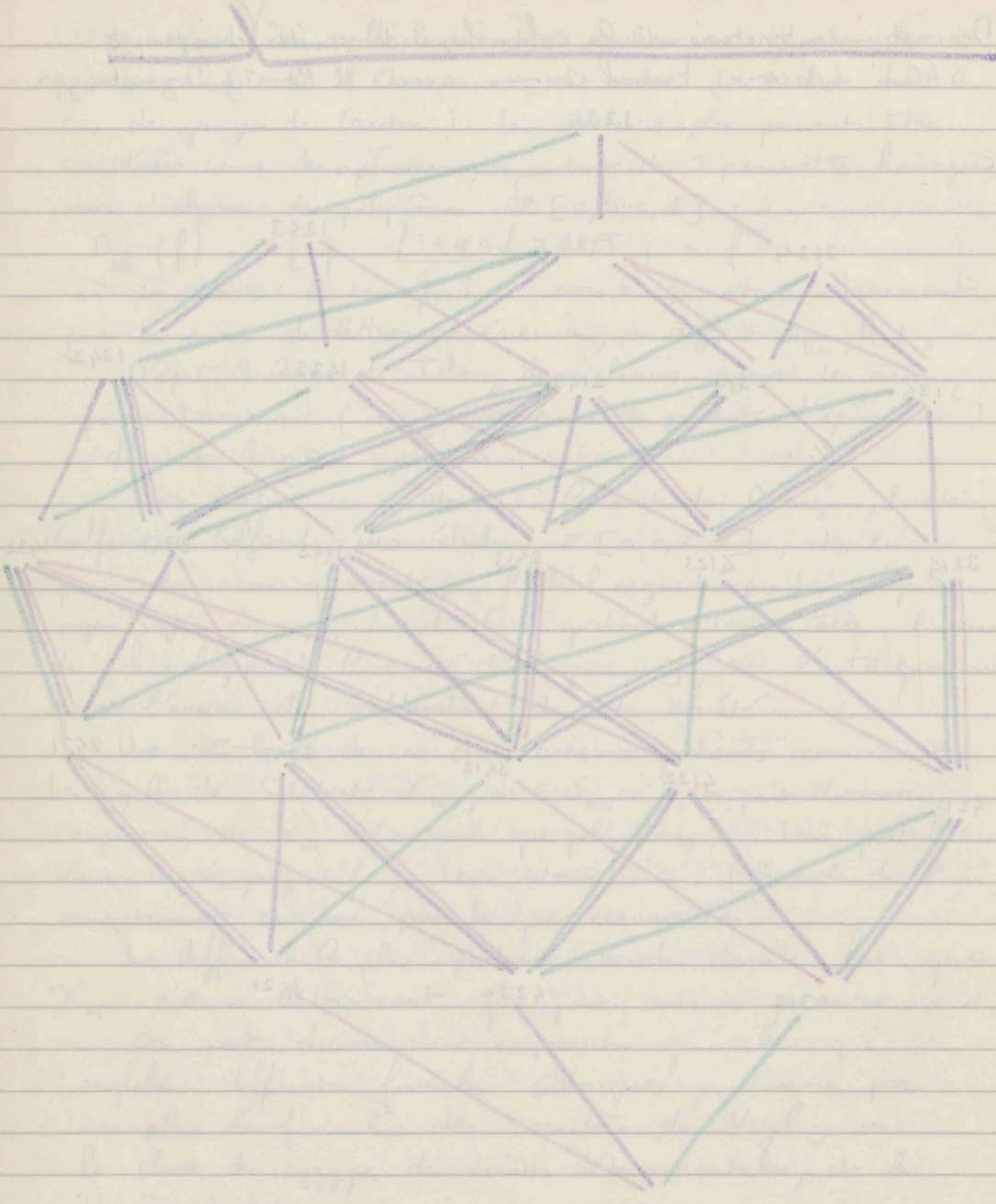
Description combinatoire de la cohomologie des variétés de drapeaux

Alain Lascoux, travail commun avec M.P. Schützenberger -

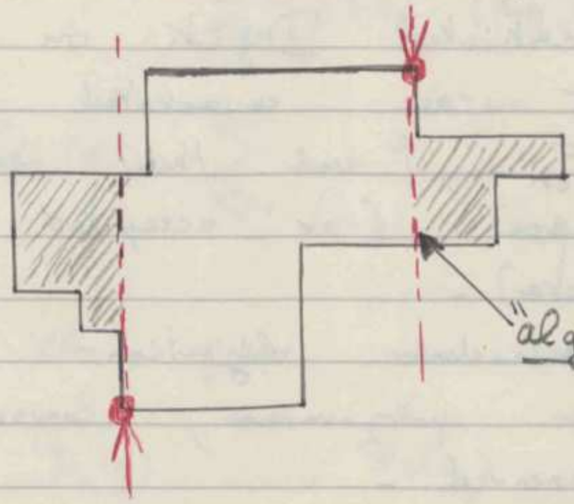




The number of convex polyominoes

G. Viennot (Bordeaux)
(with M. Franchi-Zannettacci)

A polyomino is a connected, simply connected, with no "cut point", union of elementary squares of the "combinatorist plane" $\Pi = \mathbb{Z}\times\mathbb{Z}$. It is said convex when the intersection with every vertical and horizontal lines is a connected segment.



A convex polyomino cut in 3 parts.

We prove that the number p_{2n} of polyominoes with perimeter $2n$ is:

$$\begin{cases} p_4 = 1, \quad p_6 = 2 \quad \text{and for } n \geq 0 \\ p_{2n+8} = (2n+11)\,4^n - 4(2n+1)\binom{2n}{n}. \end{cases}$$

The proof is in 3 steps.

1) Bijection between convex polyominoes and some words of an "algebraic" language.

2) Resolution (in commutative variables) of the algebraic system of equations satisfied by the generating function.

3) Expanding the generating function and obtaining p_{2n}

Step 1 is obtained by cutting the polyomino in 3 polyominos as shown in the picture. Three cases have to be considered (and thus 3 algebraic languages and 3 generating functions). The middle part is a pair of non-intersecting paths counted by the Catalan numbers $\frac{1}{n+1}\binom{2n}{n}$ and encoded by the classical algebraic language called "restricted Dyck on 2 letters". The two other part are enumerated by the Fibonacci numbers F_{2n}, and thus encoded by a rational language (or accepted by a finite state automata).

Step 1 introduce bijections such that by reconstructing the polyomino, convexity and algebricity is preserved.

Step 2 provide exercices for automata, rational ~~and~~ algebraic language theory course. (substitution, operator, construction of finite automata, ...)

§

A unique bijection (without splitting up in 3 types of polyominos) can be given for convex (and for row-convex polyominos, ~~according~~ counted according to the area and perimeter). Results of Klarner are deduced.

Some "simple" bijections related to the Rogers - Ramanujan - Schur identities

G. Viennot (Bordeaux)

These identities are:

(1) $$D_1(q) = \sum_{n\geq 0} \frac{q^{n^2}}{(1-q)\cdots(1-q^n)} = \prod_{n\geq 0} \frac{1}{(1-q^{5n+1})(1-q^{5n+4})}$$

(2) $$D_2(q) = \sum_{n\geq 0} \frac{q^{n^2+n}}{(1-q)\cdots(1-q^n)} = \prod_{n\geq 0} \frac{1}{(1-q^{5n+2})(1-q^{5n+3})}$$

We give an interpretation of the inverse $\frac{1}{D_1(q)}$ and $\frac{1}{D_2(q)}$ in term of "heaps of dominos", or equivalently weighted paths. In fact it appear that these interpretations (also given by Andrews) are a particular case of the Mac Mahon Master theorem.

The proof is bijective. The same bijection leads to interpret $D_2(q)/D_1(q)$ which appears to be a particular case of the classical formula to invert a matrix:

$$((1-A)^{-1})_{ij} = \frac{(-1)^{i+j} \operatorname{cof}_{ji}(1-A)}{\det(1-A)}$$

(with A infinite matrix)

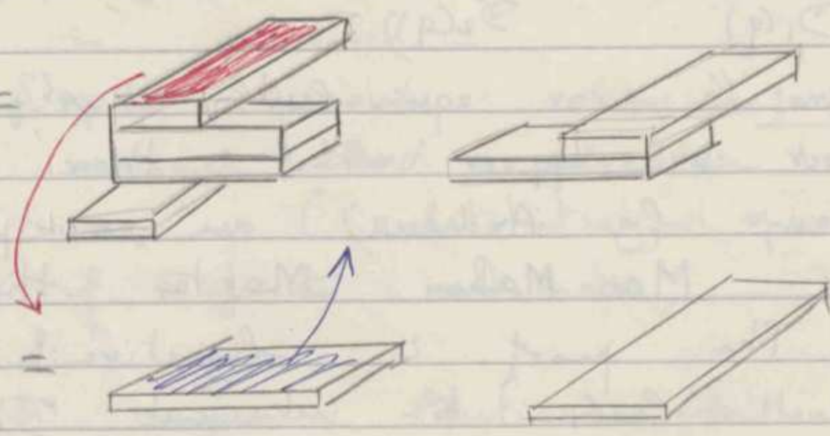
With the interpretation of weighted paths in terms of continued fractions (see P. Flajolet abstract) one obtains the famous continued fraction: (Ramanujan):

$$\frac{1}{1+\frac{q}{1+\frac{q^2}{1+\frac{q^3}{\dots}}}} = \frac{D_2(q)}{D_1(q)} = \frac{\prod_{n\geq 0}(1-q^{5n+1})(1-q^{5n+4})}{\prod_{n\geq 0}(1-q^{5n+2})(1-q^{5n+3})}$$

All these bijections are particular cases of a general bijection, including MacMahon Master theorem, inversion of matrices, and some bijections for general orthogonal polynomials. The "heaps of dominos", or more generally "pieces of puzzle" play the role of the Cartier-Foata flow and rearrangement monoid.

$$\frac{1}{\det(1-A)} =$$

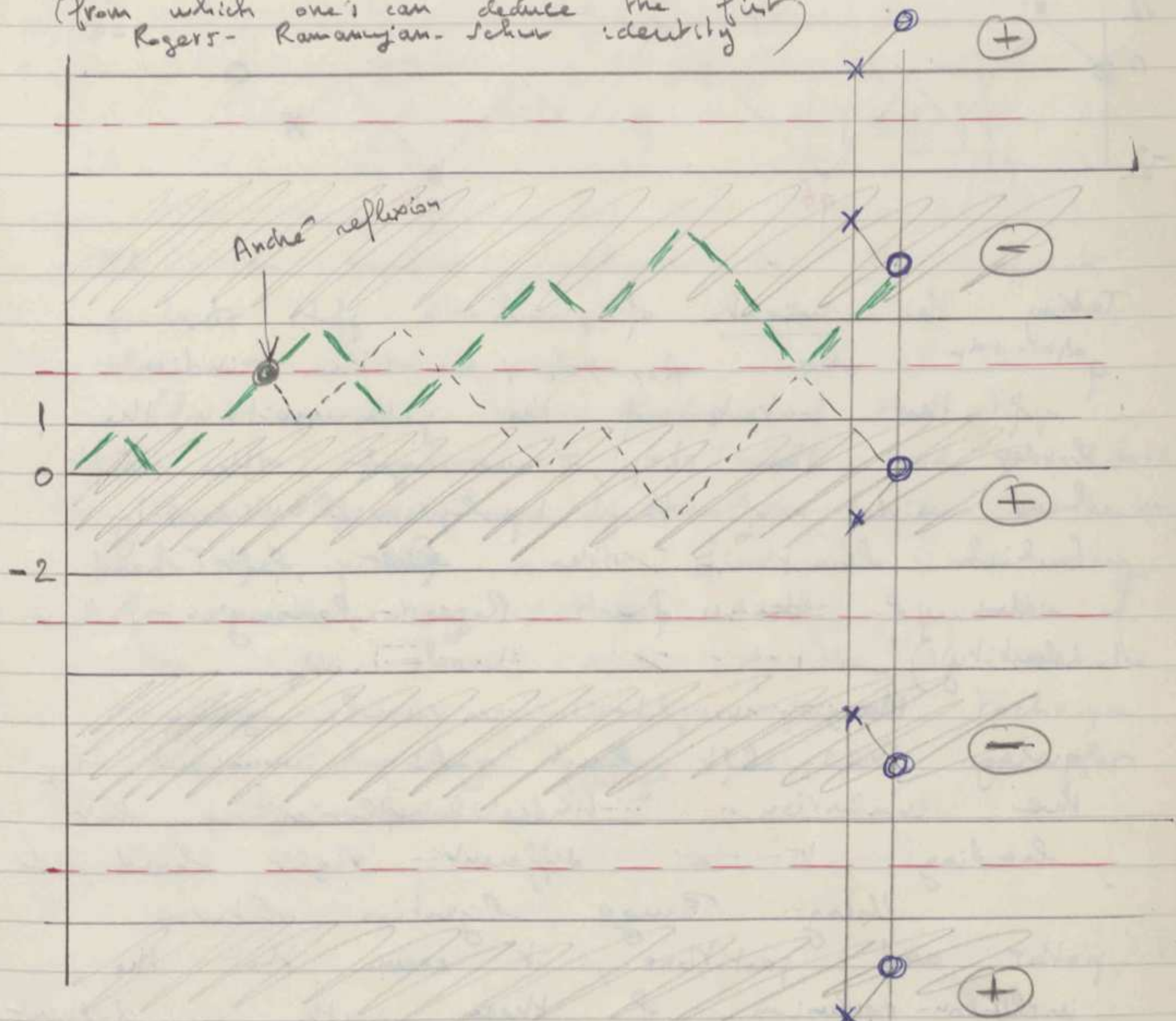
$$\det(1-A) =$$



Bijective proof for MacMahon Master Thm (and also inversion of matrices, orthogonality of polynomials, Moebius function for flow monoid, etc.) in the case of the Domino Puzzle.

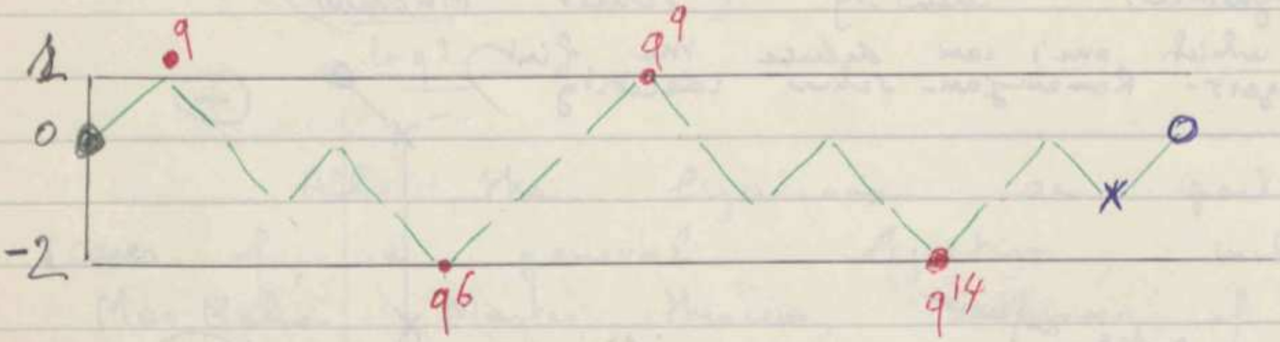
(see a forthcoming paper with S. Dulucq)

In the second part, we propose a picture with paths, proving for q=1 the following q-polynomial identity (Schur, Andrews): (from which one's can deduce the first Rogers-Ramanujan-Schur identity)



$$P_k(q) = \sum_{j\geq 0} q^{j^2} \begin{bmatrix} k-j \\ j \end{bmatrix}_q = \sum_{\lambda=-\infty}^{+\infty} (-1)^\lambda q^{\frac{\lambda(5\lambda+1)}{2}} \begin{bmatrix} k \\ \left[\frac{k-5\lambda}{2}\right] \end{bmatrix}_q$$

André reflexion ~~...~~ of paths prove this identity for q=1. (the left hand side is the Fibonacci numbers F_n, interpretated as paths bounded in the strip $-2 \le y \le 1$ with ending point O for even length and X for odd length:



Taking the weight of such a path as $q^{\alpha_1+\dots+\alpha_r}$ where $\alpha_1,\dots,\alpha_r$ are the indices of the contacts of the path with the border of the strip, one get the left hand side of the polynomial identity. (which limit is the ~~first~~ left hand side of the first Rogers-Ramanujan-Schur identity).

Many weights can be given giving the left hand side, invariant by the involution "André reflexion", but leading to a different right hand side.

Using Burge bijection between paths and partitions, it seems that the inclusion-exclusion of these paths is different from sieve methods ~~to pass~~ of Andrews and Bressoud.

~~By~~ ~~...~~ In fact, the left hand side of the 14 identities related to the hard-hexagonal model in statistical physics (obtained by Baxter and proved by Andrews) are also in the picture (with appropriate weights for the paths) as for example:

$$\sum_{i,j\geq 0} q^{\frac{i(3i+1)}{2}} \begin{bmatrix} k-2i-2j \\ i \end{bmatrix}_q \begin{bmatrix} i+j \\ j \end{bmatrix}_{q^2} q^j$$

$$= \sum_{\lambda=-\infty}^{+\infty} (-1)^\lambda q^{5\lambda^2-\lambda} \begin{bmatrix} k \\ [\frac{k-5\lambda}{2}] \end{bmatrix}_q$$

All the numbers involved in the infinite products of the right hand side are also in the picture.

(for example: the border of the strips are the numbers $\equiv 1,4 \pmod 5$ and $\equiv 2,3 \pmod 5$.
Also the duality $q \to q^{-1}$ used by Andrews can be seen on the weights.

The problem is to find a nice way for interpreting these infinite products of the right hand side of the 14 identities.

There must exist a general technique for handling ~~these~~ general weights, and for every moduli.

Extensions of Permutation Cycle Structure Results by Don Rawlings

The classical permutation cycle structure results may be generalized in a three step process:

(1) replace the notion of cycle with that of a basic component

(2) enumerate sequences by descents and basic components

(3) Finally, use the method developed in my paper which appears in the European Journal of Combinatorics (volume 2 pages 67-78, 1981) to convert the generating function for sequences into a generating function for permutations by descents, idescents, imajor index, and basic components.

As an example, the generating function for

$$A(n;t,s,q,z) = \sum_{\sigma \in S_n} t^{\text{ides}\,\sigma} s^{\text{des}\,\sigma} q^{\text{imaj}\,\sigma} z^{\text{basic comp. of}\,\sigma}$$

is

$$\sum_{n \geq 0} \frac{A(n;s,t,q,z)\,u^n}{(t;q)_{n+1}} = \sum_{r \geq 0} t^r \prod_{m=0}^{r} \left[1 - \frac{(1-s)uq^m - z}{1 - s\,\pi(m,r)}\right]^{-1}$$

where $\Pi(m,r) = \prod_{i=0}^{r-m-1} (1-(1-s)uq^{m+i+1})^{-1}$.

This generating function generalizes the q-Stirling numbers of the first kind and the q-Eulerian numbers simultaneously. To see the q-Stirling numbers, note that it is possible to derive the recurrence

$$A(n+1; 1,s,q,z) = zA(n;1,s,q,z) + sq\sum_{k=1}^{n} \begin{bmatrix} n \\ k-1 \end{bmatrix} q^{n-k}(1-s)^{n-k} A(k;1,s,q,z)$$

from the generating function. By setting $s=1$, this recurrence gives

$$A(n+1;1,1,q,z) = (z+q[n])\,A(n;1,1,q,z)$$
$$=(z+q[n])(z+q[n-1])\cdots(z+q[1])z.$$

Three observations concerning Hermite polynomials

Volker Strehl (Erlangen)

The usual combinatorial model for Hermite polynomials is extended in order to obtain simple combinatorial proofs of several identities of classical orthogonal polynomials by one method. The following examples are given: 1) the Szegö-identities relating HERMITE and LAGUERRE-polynomials, 2) the FRICOMI-identity relating polynomials defined by $(\frac{d}{dx})^n (1-x^2)^{-\lambda} = Q_n^\lambda(x) \cdot (1-x^2)^{-\lambda-n}$ ~~the~~ to GEGENBAUER-polynomials by a quadratic transformation, 3) two ways of getting generating functions for GEGENBAUER-polynomials in terms of JACOBI-polynomials (using the combinatorial model of FOATA-LEROUX).

Combinatorics of the Laguerre polynomials

Dominique Foata (Strasbourg)

The Laguerre polynomials $L_n^{(\alpha)}(x)$ $(n \geq 0)$ defined by

$$\sum u^n L_n^{(\alpha)}(x) = (1-u)^{-\alpha-1} \exp(-xu/(1-u))$$

have a combinatorial interpretation that can be used to prove most of the classical identities such as the Hille-Hardy formula, the Erdélyi formula. Furthermore, a multilinear version of the latter identity can be proved that is the analog for the Laguerre polynomials of the Kibble-Slepian formula for the Hermite polynomials. See D. Foata & V. Strehl, Une extension multilinéaire de la formule d'Erdélyi pour les produits de fonctions hypergéométriques confluentes, C.R. Acad. Sci. Paris, 293 (1981), 517-520 and Combinatorics of the Laguerre polynomials, Proc. Waterloo Conference, 1982, to appear.

On q-analogs of the Lagrange inversion formula and the Catalan numbers

Josef Hofbauer (Wien)

Let us consider the following q-analog of the notion of n-th power of a formal power series $\varphi(z)$:

$$\varphi_n'(z) = [n]\,\varphi_n(z)\,\overline{\varphi(z)} \qquad \text{and} \qquad \psi_n'(z) = q^{-n}[n]\,\psi_n(qz)\,\overline{\psi(z)}$$

The most general known example is

$$\varphi_n(z) = \frac{e_{q^s}((a[n]+b)z^s)}{e_{q^s}(bz^s)} = \frac{((\alpha-1)z^s;q^s)_\infty}{((\alpha-q^n)z^s;q^s)_\infty}$$

, which includes q-analog of e^{az}, $(1+z^s)^a$, ..

ψ_n's are obtained from φ_n's by replacing $q \to \frac{1}{q}$.

Then the coefficients a_n in the expansion

$$f(z) = \sum a_n \frac{z^n}{\varphi_n(z)\,\psi_n(z)} \qquad \text{are given by}$$

$$a_n = \frac{1}{[n]} f'(z)\,\varphi_n(z)\,\psi_n(qz)\Big|_{z^{n-1}}.$$

In this general form, this q-analog of the Lagrange formula, is due to Christian Krattenthaler.

The most important examples are

a) $\varphi_n(z) = (1-z)(1-qz)\cdots(1-q^{n-1}z) = (z)_n$, $\psi_n \equiv 1$ (Carlitz 1973)

b) $\varphi_n(z) = e_q(a[n]z)$, $\psi_n \equiv 1$ (Gipler 1980, related with q-Abel-identities of Jackson 1910)

c) $\varphi_n(z) = (az)_n$, $\psi_n(z) = (q^{-n}z)_n$

This example is closely related with the q-analogs of the classical orthogonal polynomials (e.g. the little q-Jacobi polynomials of Andrews and Askey), it can be applied to obtain nice q-analogs of the inverse relations of Čebyshev and Legendre type contained in Riordan's book.

In order to demonstrate the differences of this q-analog of the Lagrange formula with that of Garsia, where $\varphi_n(z) = \varphi(z)\varphi(qz)\cdots\varphi(q^{n-1}z)$, two versions of q-Catalan numbers have been compared:

1. version (Carlitz): $z=\sum^{\infty} C_{n-1} z^n (1-z)(1-qz)\cdots(1-q^{n-1}z)$

$$\Rightarrow \quad C_{n+1} = \sum_{k=0}^{n} q^{(k+1)(n-k)} C_k C_{n-k}$$

$$z = \sum \frac{C_n z^n q^{\binom{n-1}{2}}}{(1+z)(1+qz)\cdots(1+q^{2n-1}z)}$$

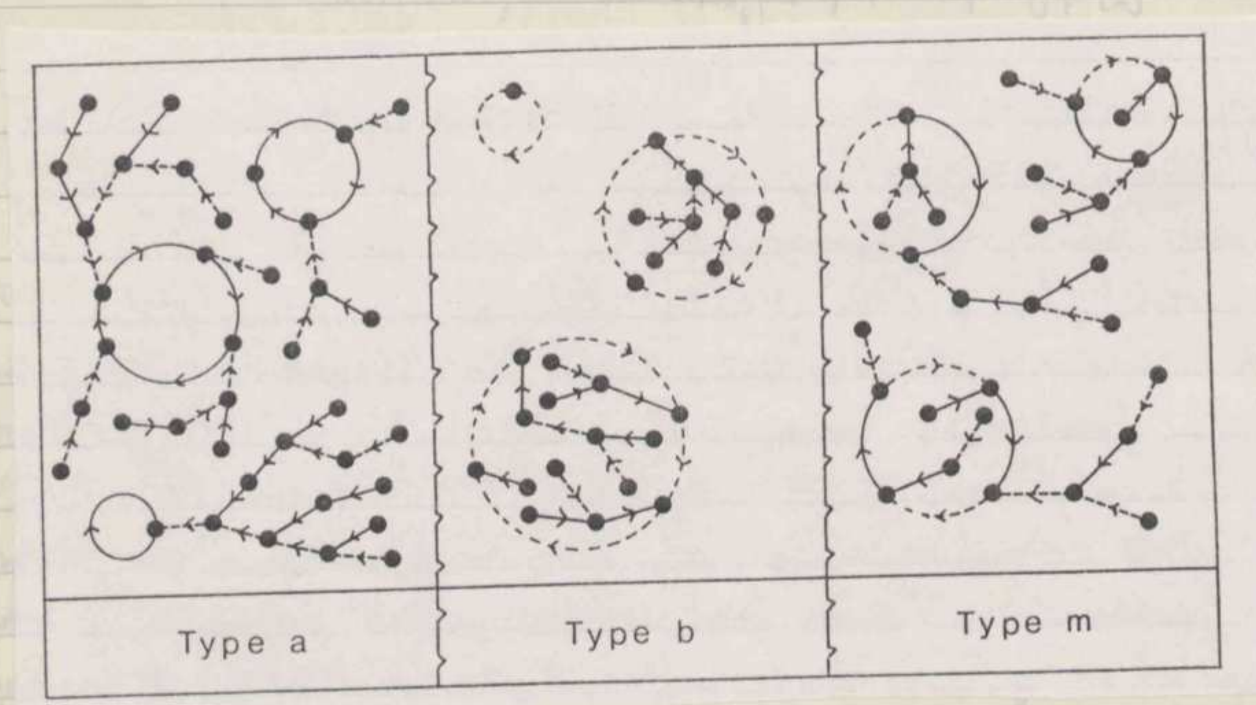
$$C_n = \sum q^{\mathrm{inv}\, w} \qquad w \in \mathcal{C}_n = \{0^{(n)}1^{(n)} : \text{every initial segment contains at least as many 0's as 1's}\}$$

2nd version: $z = \sum \frac{C_n(\lambda)}{q^{\binom{n}{2}}} \frac{z^n}{(1+\frac{z}{q^n})\cdots(1+\frac{z}{q})(1+q^{\lambda}z)\cdots(1+q^{\lambda+n-1}z)}$

$$\Rightarrow \quad C_n(\lambda) = \sum_{k=0}^{n} \frac{1}{[n]} \begin{bmatrix} n \\ k \end{bmatrix} \begin{bmatrix} n \\ k+1 \end{bmatrix} q^{\lambda k + k^2}$$

$$C_n(1) = \frac{1}{[n+1]} \begin{bmatrix} 2n \\ n \end{bmatrix} \qquad C_n(0) = \frac{1}{[n+1]} \begin{bmatrix} 2n \\ n \end{bmatrix} \frac{1+q}{1+q^n}$$

$$C_n(\lambda) = \sum_{w \in \mathcal{C}_n} q^{\mathrm{maj}\, w + (\lambda-1) d(w)} \qquad d(w) = \#\text{ of descents in } w$$



Jacobi endofunction.

Jacobi polynomials : combinatorial interpretation and generating function.

by Pierre Leroux

with Dominique Foata

to appear in Proc. Amer. Math. Soc.

The classical generating function for Jacobi polynomials is derived by purely combinatorial methods. The combinatorial interpretation comes from the explicit expression

$$n!\, P_n^{(\alpha,\beta)}(x) = n! \sum_{j=0}^{n} \binom{n+\alpha}{n-j}\binom{n+\beta}{j}\left(\frac{x+1}{2}\right)^{n-j}\left(\frac{x-1}{2}\right)^{j}$$

$$= \sum_{i+j=n} \binom{n}{i} (\alpha+1+j)_i\, (\beta+1+i)_j \left(\frac{x+1}{2}\right)^{i}\left(\frac{x-1}{2}\right)^{j}$$

Setting $X = \frac{x-1}{2}$ and $Y = \frac{x+1}{2}$, we define

$$\mathcal{P}_n^{(\alpha,\beta)}(X,Y) = \sum_{i+j=n} \binom{n}{i} (\alpha+1+j)_i\, (\beta+1+i)_j\, X^i Y^j$$

and show that

$$\sum_{n\geq 0} \mathcal{P}_n^{(\alpha,\beta)}(X,Y)\frac{u^n}{n!} = 2^{\alpha+\beta} R^{-1} (1-(X-Y)u+R)^{-\alpha} \cdot (1-(Y-X)u+R)^{-\beta}.$$

To do this we interpret $\mathcal{P}_n^{(\alpha,\beta)}(X,Y)$ as

the generating function of "Jacobi endofunctions" of $[n] = \{1,2,\ldots,n\}$, that is

- ordered set partition (A,B) of $[n]$
- injective functions $f: A \longrightarrow [n]$, $g: B \longrightarrow [n]$

with weights $(\alpha+1)^{c(f)}(\beta+1)^{c(g)} X^{|A|} Y^{|B|}$ where

$c(f)$ = number of cycles of f. See opposite page for a figure in which continuous arcs $\to$ represent f

⟵ and dotted arcs $\dashrightarrow$ represent g.

Enumeration of Partially Ordered Sets with Hooklengths - Bruce Sagan

The technique of Hillman & Grassl for providing a natural combinatorial proof of the hook generating function

$$\prod_{(i,j)\in\lambda} \frac{1}{1-x^{h_{ij}}}$$

for reverse plane partitions of shape λ is extended to cover shifted reverse plane partitions of shape $\lambda^* = (\lambda_1^* > \lambda_2^* — > \lambda_k^*)$ and posets whose Hasse diagram is a rooted tree. This accomplishes for the Hillman - Grassl algorithm what was done for the Schensted correspondence (see the abstracts for the 1979 Oberwolfach conference on Combinatorics) and can also be done for the Greene Nijenhuis & Wilf probabilistic algorithm for proving the hook formula $f_\lambda = n! / \prod_{(i,j)\in\lambda} h_{ij}$.

In addition the same methods are used to derive other partition generating functions, in particular

$\prod_{i=1}^{\infty} \frac{1}{(1-x^i)^i}$ for plane partitions of arbitrary shape

$\prod_{i=1}^{\infty} \frac{1}{(1-x^i y)^i}$ where y keeps track of the diagonal sum

$\prod_{i=1}^{\infty} \frac{1}{(1-x^i)^{\min(i,s)}}$ & $\prod_{i=1}^{\infty} \frac{1}{(1-x^i y)^{\min(i,s)}}$ for plane partitions with $\le s$ rows

$\prod_{i=1}^{s} \frac{1-x^{i+t}}{1-x^i}$ for partitions with $\le s$ parts each of size $\le t$

It was asked whether this technique could be applied to derive the generating functions for plane partitions "sitting inside an $r \times s \times t$ box" with or without diagonal or cyclic symmetry. This paper will appear shortly in the European J. of Combinatorics

Alternating Sign Matrices and Descending Plane Partitions

David P. Robbins with W.H. Mills and Howard Rumsey Jr.

(To appear in Journal of Combinatorial Theory)

An alternating sign matrix is a square matrix such that (i) all entries are 1, -1 or 0; (ii) every row and column has sum 1 and (iii) ~~the rows~~ in every row and column the non-zero entries alternate in sign.

A descending plane partition is a shifted plane partition with weakly decreasing rows, strictly decreasing columns, and which satisfy the property that the first element of each row is greater than the number of entries in its own row and ≤ the number entries in the preceding row.*

There is extremely strong evidence for the existence of a close connection between the set of all n by n alternating sign matrices and the descending plane partitions with all parts ≤ n. Andrews has shown in his paper on the Macdonald Conjecture that this set of descending plane partitions has cardinality $\prod_{k=0}^{n-1} \frac{(3k+1)!}{(n+k)!}$. There is in particular strong numerical evidence that this is the number of n by alternating sign matrices.

We described some refinements of this main conjecture and in addition a series of conjectures about ~~the~~ generating functions connected with alternating sign matrices.

In addition we described several theorems about both classes of objects.

* Here is an example of a descending plane partition:

```
888755
 5553
  43
   2
```

Proof of the Macdonald Conjecture

David P. Robbins with W.H. Mills and Howard Rumsey Jr.

(Has appeared in Inventiones Math., 1982)

A cyclically symmetric plane partition is a plane partition whose Ferrer's graph ~~is~~ (in 3-dimensions) is invariant under cyclic permutation of the coordinate axes ($x \to y$, $y \to z$, $z \to x$).

Macdonald has conjectured that the generating function for the set of cyclically symmetric plane partitions ~~cont~~ whose Ferrers graphs are contained in a box of size $m \times m \times m$ is a certain product of cyclotomic polynomials.

Andrews[1] has settled the case $q=1$ and in the course of his proof he introduced a new class of plane partitions which he called descending plane partitions and made a similar conjecture about the generating function for these partitions. It is the object of this paper to prove both of these conjectures.

[1] G.E. Andrews, Plane Partitions (III): The weak Macdonald Conjecture, Inventiones Math., 53 (1979) 193-225

HOW TO GET CUTE BIJECTIVE PROOFS OUT OF DULL INDUCTIVE PROOFS, DORON ZEILBERGER

Given two families of finite sets $\{A(n)\}$, $\{B(n)\}$, (where n is a discrete (possibly multi) index), it is required to prove $|A(n)| = |B(n)|$ $\forall n$. First find $a(n) = |A(n)|$ or rather a natural recurrence equation for $a(n)$, based on the natural recursive structre of $A(n)$. Do the same for $b(n)$. Find an inductive proof of $a(n) = b(n)$ assuming $a(i) = b(i)$, $i \le n-1$. Follow this algebraic proof step by step defining an algorithm $\pi(n) : A(n) \to B(n)$, based on $\pi(i)$, $i \le n-1$. Get a recursive algorithm written in "machine language" so to speak. Decompile it, taking advantage of the ready made subroutines in the human brain. Impress your friends with a cute algorithm.

Example: $F_n = F_{n-2} + F_{n-3} + \dots + 1$ (i.e. # dead rabbits $+1$ = # live rabbits).
$\mathcal{F}_n = \{(a_1 \dots a_k);\ a_i = 1, 2,\ a_1 + \dots + a_k = n\}$, $F_n = |\mathcal{F}_n|$, $F_n = F_{n-1} + F_{n-2}$ since $\mathcal{F}_n \leftrightarrow \mathcal{F}_{n-1} \cup \mathcal{F}_{n-2}$ by chopping the tail a_k.

Dull inductive proof: $F_n \underset{def}{=} F_{n-2} + F_{n-1} \underset{ind}{=} F_{n-2} + F_{n-3} + F_{n-4} + \dots + 1$ □

Bijective translation: <u>Alg $\pi(n)$</u>

Input: $(a_1 \dots a_k) \in \mathcal{F}_n$. 1) Look at a_k, and chop it. 2) If $a_k = 2$, then $\pi(n)[(a_1, \dots a_k)] \Leftarrow (a_1, \dots, a_{k-1})$; stop. 3) If $a_k = 1$, apply $\pi(n-1)$, i.e. $\pi(n)[(a_1, \dots a_k)] \Leftarrow \pi(n-1)[(a_1, \dots a_{k-1})]$.

After looking at this for a while it all boils down to: Look at first "2" from the right and chop it and all the 1's to its right. This is a cute algorithm if there ever was one.

Universal Algebra

May 16–22, 1982

Definability, generation, and decidability problems for varieties of modular lattices — C. Herrmann

Classes considered : $\mathcal{M}$: modular lattices , Arg : Arguesian lattices , $\mathrm{Con}\mathcal{M}$: lattice variety generated by all lattices of congruences of algebras in congruence modular varieties , $\mathcal{L}(R)$: lattice variety generated by lattices of R-submodules. $L(\mathbb{Z}_m) \subseteq L(\mathbb{Z}_n) \subseteq \mathrm{Con}\mathcal{M} \subseteq \mathrm{Arg} \subseteq \mathcal{M}$, $m|n$.

properties	$\mathcal{L}(\mathbb{Z}_p)$	$\mathcal{L}(\mathbb{Q})$	$\mathcal{L}(\mathbb{Z}_n)$ (else, $n=\infty$)	$\mathrm{Con}\mathcal{M}$	Arg	$\mathcal{M}$
finite equational basis	p prime ?	no	no	no		
generated by finite members	yes	no	yes		no (Freese)	no (Freese)
generated by finite dimensional members	yes	yes	yes		no (Freese)	no (Freese)
solvable word problem in $\le n$ generators	(Gel'fand) $n \le 4$	(Gel'fand) $n \le 4$		(Dedekind) $n \le 3$	(Dedekind) $n \le 3$	(Dedekind) $n \le 3$
unsolvable word problem in $\ge n$ generators	$n \geq 5$ (Hutchinson)	$n \geq 5$ (Hutchinson)	$n \geq 5$ (Hutchinson)	$n \geq 5$ (Hutchinson)	$n \geq 5$ (Hutchinson)	$n \geq 4$
solvable word problem for free lattices in $\le n$ generators	$n \le \infty$	$n \le \infty$	$n \le \infty$			
unsolvable word problem for free lattices in $\ge n$ generators						$n \geq 4$

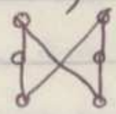
In particular, improving a bit on R. Freese's result on $F\mathcal{M}(5)$ we have

<u>Thm</u> The free modular lattice $F\mathcal{M}(4)$ in four generators has an unsolvable word problem.

Proof, let G be a two generator group with unsolvable word problem. From the submodule lattice $L({}_Q Q^2 \times R)$ – $Q = \mathbb{Z}_5 G \supseteq \mathbb{Z}_3 G = R$ – derive a modular lattice with four generators (changing the gluing maps between the sublattices $L({}_R R^4)$) in which G can be recovered via the von Neumann ring construction. Then, show that this construction can be modelled in $F\mathcal{M}(4)$. Freese's method of forcing group relations in a lattice is crucial.

<u>Thm</u> No finitely based variety of modular lattices which contains $\mathcal{L}(\mathbb{Q})$ is generated by its finite dimensional members.

Some Infinitary Free Lattices - I & II - G. Grätzer & D. Kelly

In Part I, the free $\mathfrak{m}$-lattice generated by the poset $H =$  is described. The letter $\mathfrak{m}$ denotes an infinite regular cardinal. An $\mathfrak{m}$-lattice (or $\mathfrak{m}$-complete lattice) is a poset L in which $\bigwedge X$ and $\bigvee X$ (meet and join respectively) exist for all $X \subseteq L$ with $0 < |X| < \mathfrak{m}$. This lattice, $CF_{\mathfrak{m}}(H)$ is put together from three building blocks: $CF_{\mathfrak{m}}(\underset{\sim}{2}+\underset{\sim}{2})$, A and B, where $A = \{\langle r,s\rangle \mid r<s;\ r,s$ dyadic rationals; $0 \le r,s \le 1;\ s-r = 2^{-n}$ with $n \geq \mathrm{ord}(s)\}$; B is defined ~~dually with~~ similarly with $r>s$. The ordering on A and B is componentwise.

Theorem: Let P be a countable poset. The following three conditions are equivalent:

(1) $CF_{\mathfrak{m}}(P)$ does not contain $F_{\mathfrak{m}}(3)$.

(2) P does not contain $\underset{\sim}{1}+\underset{\sim}{5}$, $\underset{\sim}{2}+\underset{\sim}{3}$ or $\underset{\sim}{1}+\underset{\sim}{1}+\underset{\sim}{1}$.

(3) $CF_{\mathfrak{m}}(P)$ can be embedded (as an $\mathfrak{m}$-sublattice) of $CF_{\mathfrak{m}}(H)$.

In the case that P is finite and $\mathfrak{m} = \aleph_0$, this theorem is due to I. Rival & R. Wille (J. f. reine u. angew. Math. 310(1979), 56-80).

Markov constructions of algebras - Kirby Baker

For a finite set S and subset T of $S \times S$, let $M_{S,T}$ denote with Markov chain $\{ \xi \in S^{\mathbb{Z}} : (s_i, s_{i+1}) \in T\ \forall i \}$, with left shift σ. It is useful to consider the case where S is an algebra (or partial algebra) and T is a subalgebra (or relative partial subalgebra) of $S \times S$. Then $M_{S,T}$ becomes an algebra (or partial algebra). This construction provides a unifying framework for varied examples: (1) arbitrarily long →

nonshortenable projectivities in varieties generated by finite nondistributive lattices; (2) McKenzie's proof that these same varieties lack definable principal congruences; (3) Park's construction of a non-finitely based finite idempotent commutative algebra; (4) Shallon's graph algebras; (5) a finite 2-unary algebra whose class of subdirect powers is not finitely axiomatizable (Gimpel, following another example of McKenzie).

INHERENTLY NONFINITELY BASED FINITE ALGEBRAS

George F. McNulty and Caroline R. Shallon

Peter Perkins calls a variety $\mathcal{V}$ <u>inherently</u> <u>nonfinitely</u> <u>based</u> iff $\mathcal{V}$ is locally finite and if $\mathcal{V} \subseteq \mathcal{W}$ with $\mathcal{W}$ a locally finite variety, then $\mathcal{W}$ is not finitely based. We present the following theorems.

THEOREM 0 If $\mathcal{V}$ is the variety generated by a finite groupoid which is nonassociative, nonabsorptive and possess both a zero and a unit, then $\mathcal{V}$ is inherently nonfinitely based.

[Here nonabsorptive means: if $\mathcal{V} \models x \approx \tau$, then τ is the variable x.]

THEOREM 1 Let $\mathcal{V}$ be a locally finite variety of groupoids. If either

all finite loopless graph algebras belong to $\mathcal{V}$

or

all finite looped graph algebras belong to $\mathcal{V}$

then $\mathcal{V}$ is inherently nonfinitely based.

Here a looped graph algebra is a groupoid with universe $A \cup \{0\}$ with $0 \notin A$ such that $0a = a0 = 00 = 0$ for all $a \in A$ and $ab = 0$ or a for $a, b \in A$ with $a \neq b$ and $aa = a$. A loopless graph algebra is like a graph algebra except $aa = 0$ for all $a \in A$.

Using these results and the techniques used to prove them it turns out that many small inherently nonfinitely based groupoids exist. This includes the known examples by Murskii and Višin. But we show that Lyndon's example fails to be inherently nonfinitely based.

Theorem 1 uses ideas of Perkins while Theorem 2 uses ideas of Murskii.

Semigroups of quotients of (semi-)lattices

F. Schmid

If S, T are commutative semigroups, $S \subseteq T$, T is called a semigroup of quotients of S (write $S \leq T$) iff for all $t_1 \neq t_2$, $t \in T$ there exists $s \in S$ such that $st_1 \neq st_2$ and $st \in S$. There exists a maximal semigroup $Q(S)$, such that $S \leq Q(S)$; whenever $S \leq T$, T embeds into $Q(S)$ over S. Fact: If S is a (meet-) semilattice, $S \leq T$ implies that T is a semilattice also. More remarkably, there is an "internal" description of $Q(S)$ in this case: $Q(S)$ may be identified with a certain collection of lower sets of S, ordered by set inclusion (this contrasts the situation for a general semigroup S). Further, $S \leq T$ implies that T is a join-extension of S. We investigate the structure of $Q(S)$ for various classes of semilattices. Sample of results:

(i) S a distributive lattice. $Q(S)$ is then also a distributive lattice, and the canonical embedding $S \hookrightarrow Q(S)$ preserves finite joins. If S is Boolean, $Q(S)$ is just the McNeille completion of S (Lambek).

Problem: For which distributive lattices S is $Q(S)$ complete?

(ii) S a finite semilattice. Then $Q(S)$ is a finite lattice. Let $P(S)$ be the lower set generated in S by the join-irreducibles, then $Q(S)$ is isomorphic with the lattice of all ideals of $P(S)$. Particularly, for a finite lattice S, $S \cong Q(S)$ iff every $x \in S$ has unique irredundant representation $x = j_1 \vee \cdots \vee j_n$, $j_k \in P(S)$ (where irredundant means that $j_r \vee j_s \notin P(S)$ whenever $1 \le r < s \le n$).

(iii) If S is any lattice, $Q(S)$ need not be a lattice. However, there exists a largest subsemilattice $E(S)$ of $Q(S)$ for which the canonical embedding $S \hookrightarrow E(S)$ preserves finite joins.

Problem: For which lattices S is $E(S)$ a lattice?

The finite congruence lattice problem or: What can Group Theory do for us?

Peter Köhler (Giessen)

There are three good reasons for the claim that Group Theory will play a crucial rôle in any attempt to solve the finite congruence lattice problem.

The first one is the celebrated Pálfy-Pudlak result stating that every finite lattice is isomorphic to the congruence lattice of a finite algebra if and only if every finite lattice can be embedded as an interval into the subgroup lattice of a finite group.

The second one is the recent example of a finite algebra having M_7 as congruence lattice - constructed by Walter Feit by exhibiting an appropriate group.

The third reason is the particular M_n-problem:

here we can show that some restrictions on the structure of a possible candidate having congruence lattice M_n, $n-1$ not a prime power, can be obtained using group-theoretical methods.

<u>Three-element groupoids with minimal clones</u>

Béla Csákány (Szeged)

The clones on a finite set form an atomic lattice whose atoms are called <u>minimal clones</u>. In this ~~paper~~ talk we ~~give~~ gave a complete list of those essentially distinct three-element algebras with one essentially binary operation whose clones of term functions are minimal.

For sets consisting of more than two elements the problem of listing the minimal clones is open. Our result may be considered as a first step towards the solution of this problem. Indeed, ~~the comp~~ the description of the maximal clones on a three-element set suggests how the maximal clones on a finite set can behave in general, and the same may be expected for minimal clones. On the other hand, it is known that any minimal clone on a three-element set is generated by an essentially at most ternary operation. The unary case is trivial, and we settle the binary case as follows:

Fix the base set $\{0,1,2\} = \underline{3}$, and denote the operation with Cayley table

	0	1	2
0	0	n_5	n_4
1	n_3	1	n_2
2	n_1	n_0	2

by the integer $\sum_{i=0}^{5} 3^i n_i$.

Then $\langle \underline{3}; f \rangle$ with f = 0, 8, 10, 11, 16, 17, 26, 33, 35, 68, 178, 624 have minimal clones of term functions, and if a three-element groupoid with essentially binary operation has minimal clone of term functions then it is either isomorphic or antiisomorphic to (exactly) one of the groupoids listed above.

On the representation of distributive semilattices

A. P. Huhn (Szeged)

E. T. Schmidt proved that every distributive lattice with 0 is isomorphic with the lattice of all compact congruences of a lattice. P. Pudlák gave another proof of the theorem. His proof is based on the fact that every distributive lattice is a direct limit of its finite sublattices D_γ. Representing this finite lattices simultaneously we get a directed system of lattices L_γ such that $D_\gamma \cong \mathrm{Con}(L_\gamma)$ and, for $\gamma \leq \delta$, the following diagram is commutative

$$\begin{array}{ccc} D_\gamma & \xrightarrow{\varepsilon_{\gamma\delta}} & D_\delta \\ \downarrow{\scriptstyle \varphi_\gamma} & & \downarrow{\scriptstyle \varphi_\delta} \\ \mathrm{Con}(L_\gamma) & \xrightarrow{\mathrm{Con}(\lambda_{\gamma\delta})} & \mathrm{Con}(L_\delta) \end{array},$$

where $\varepsilon_{\gamma\delta}$ denotes the embedding of D_γ to D_δ in the directed system of the D_γ's while $\lambda_{\gamma\delta}$ denotes the embedding of L_γ to L_δ in the directed system of the L_γ's. $\mathrm{Con}(\lambda_{\gamma\delta})$ is the induced embedding on the congruence lattices. The φ_γ are the isomorphisms $D_\gamma \to \mathrm{Con}(L_\gamma)$. Part of this theorem remains valid if the $\varepsilon_{\gamma\delta}$ are only semilattice embeddings. Namely, we can prove the simultaneous representation of two distributive semilattices. As a consequence we have, besides Schmidt's theorem, the following

Corollary. Every countable distributive semilattice with 0 is the semilattice of compact congruences of a lattice.

The proof combines a proof of E. T. Schmidt with the theory of free products of distributive lattices.

Completeness theorems for algebras with semiregular automorphism groups

Ágnes Szendrei (Szeged)

A finite algebra $\mathfrak{A} = \langle A; F \rangle$ is called demi-primal iff $\mathfrak{A}$ has no proper subalgebra and every function $A^n \to A$ $(n \geq 1)$ admitting the automorphisms of $\mathfrak{A}$ is a polynomial of $\mathfrak{A}$. It is clear that the automorphism group of a demi-primal algebra is semiregular, i.e. every nonidentity automorphism is fixed point free.

In order to get a demi-primality criterion for algebras with a given automorphism group Π (Π a semiregular permutation group on A), we have to determine the maximal subclones of the clone $\mathrm{Pol}(\Pi)$ consisting of all finitary functions on A which commute with the permutations in Π. It is not hard to show that every maximal subclone of $\mathrm{Pol}(\Pi)$ is of the form $\mathrm{Pol}(\Pi \cup \{\varrho\})$ where ϱ is either a permutation generating together with Π a semiregular permutation group, or one of Rosenberg's relations distinct from the permutations. To determine which of these relations in fact determine maximal subclones of $\mathrm{Pol}(\Pi)$ is much more difficult, and is known only in the following two special cases:

1) Π is of prime order (and hence $\mathrm{Pol}(\Pi)$ is a maximal clone)

   The description is a joint result with I. G. Rosenberg.

2) Π is a regular permutation group

   In this case it follows (without making use of Rosenberg's primality criterion) that every maximal subclone of $\mathrm{Pol}(\Pi)$ is of the form $\mathrm{Pol}(\Pi \cup \{\sigma\})$ for some subset σ of A, or for some equivalence σ on A, or the affine relation σ determined by Π provided Π is elementary abelian.

On congruence lattices of complemented modular lattices

E.T. Schmidt (Budapest)

I consider the following question: is every distributive algebraic lattice isomorphic to the congruence lattice of a complemented modular lattice? For finite distributive lattices we have:

THEOREM. For every finite distributive lattice D there exists a complemented modular lattice K such that the congruence lattice of K is isomorphic to D and K is a sublattice of the lattice of all subspases of a countably infinite dimensional vector space over a finite field.

For infinite D the problem is unsolved, but some ideas were presented. We follow Pavel Pudlák's approach which reduce the problem to investigations of the representations of finite distributive lattices. We need to use continuous geometries instead of the subspace lattices of vector spaces.

Probability, linear extensions and distributive lattices

Ivan Rival (Calgary)

Ordered sets and even distributive lattices occur often in scheduling and sorting problems. A set of inequalities (e.g. a<b, c<d, etc.) in an ordered set P can be regarded as an "event" and can then be identified with the set of all linear extensions of P in which these inequalities are satisfied. If all linear extensions of P are taken as equally likely we have a probability measure. Recently, L.A. Shepp (Annals of Probability (1982)) proved this conjecture of I. Rival and B. Sands:

$$Pr(a<b \mid a<c) \geq Pr(a<b),$$

where $Pr(a<b)$ equals the number of linear extensions of P in which $a<b$ and $Pr(a<b \mid a<c)$ is the corresponding usual conditional probability. The proof is a clever use of distributive lattices.

Congruence relations of concept lattices

Rudolf Wille

Lattices can be interpreted as hierarchies of concepts. This fundamental interpretation may be formalized as follows: A context is understood as a triple (G,M,I) where G and M are sets, and I is a binary relation between G and M; the elements of G and M are called objects and attributes, respectively. If gIm for $g \in G$ and $m \in M$ we say: the object g has the attribute m. Following traditional philosophy we define a concept of (G,M,I) as a pair (A,B) with $A \subseteq G$, $B \subseteq M$, and $A = \{g \in G \mid gIm \text{ for all } m \in B\}$, and $B = \{m \in M \mid gIm \text{ for all } g \in A\}$; A and B are called the extent and the intent of the concept (A,B), respectively. The hierarchy of concepts is captured by the definition: $(A_1,B_1) \leq (A_2,B_2) :\Leftrightarrow A_1 \subseteq A_2 \, (\Leftrightarrow B_1 \supseteq B_2)$. All concepts of (G,M,I) together with the order $\leq$ form a complete lattice, the concept lattice $\underline{\mathfrak{B}}(G,M,I)$. A basic problem is to determine the concept lattice for a given context. With respect to this problem the study of congruence relations and subdirect products of concept lattices leads to a reduction of the determination procedure for $\underline{\mathfrak{B}}(G,M,I)$ if $\underline{\mathfrak{B}}(G,M,I)$ can be subdirectly decomposed. The main result is that congruence relations and subdirect decompositions of a concept lattice can be directly obtained from its context without knowing the concept lattice. R. Wille

Congruence relations of relational systems.

Dietmar Schweigert

An equivalence $\Pi \subseteq A^2$ is called a congruence of a system $(A;\varrho)$, ϱ n-ary relation, $n > 1$ if for all $\varrho(a_1,\ldots,a_n)$, $a_1 \Pi b_1, \ldots, a_{n-1} \Pi b_{n-1}$ there exists $b_n \in A$ such that $\varrho(b_1,\ldots,b_n)$ and $a_n \Pi b_n$. For $(A;\varrho)$, $(B;\bar{\varrho})$ $f: A \to B$ is a relational homomorphism if 1) from $\varrho(a_1,\ldots,a_n)$ it follows $\bar{\varrho}(f(a_1),\ldots,f(a_n))$ 2) for $\bar{\varrho}(f(d_1),\ldots,f(d_n))$ there is $c \in A$ such that $\varrho(d_1,\ldots,d_{n-1},c)$ and $f(d_n) = f(c)$. We can show homomorphism theorems and that the lattice $C(A;\varrho)$ of the congruences of $(A;\varrho)$ is complete. Call a relation ϱ flexible if $\varrho(a_1,\ldots,a_{n-1},x)$ is solvable for all $a_1,\ldots,a_{n-1} \in A$. In this case the lattice $C(A;\varrho)$

is algebraic and we have a decomposition of $(A;\varrho)$ as a subdirect product of subdirect irreducible systems. We study classes of flexible systems which are closed under flexible subsystems, relational homomorphisms and direct products. To describe these classes we consider formulas for predicate symbols R_i of the following form: 1) $R_i(x_1,\dots,x_n)$
2) $R_{j_1}(x_1,\dots,x_{n_{j_1}}) \wedge \dots \wedge R_{j_s}(x_1,\dots,x_{n_{j_s}}) \longrightarrow R_t(x_1,\dots,x_{n_t})$
ordered in such a way that $x_{n_{j_r}}$ does not appear in any $R_{j_k}(x_1,\dots,x_{n_k})$ for $1 \leq k < r$.

GEOMETRIC UNIVERSAL ALGEBRAS

R. Padmanabhan, WINNIPEG.

Let us consider the following phenomenon which occurs in several "different" areas of mathematics: If $\langle G;\Omega\rangle$ is a mathematical structure admitting a natural binary operation $\mu: G\times G \to G$ with two-sided identity e i.e. $\mu(x,e) = \mu(e,x) = x \quad \forall x \in G$, then μ induces a natural abelian group structure associated with $\langle G;\Omega\rangle$. Thus (1) If G is a completely irreducible algebraic curve over an algebraically closed field k and μ a morphism then $\langle G;\mu,e\rangle$ is an algebraic abelian group; (2) If G is a topological space and μ continuous in both arguments (so called an H-space or a Jonsson-Tarski topological algebra) then the fundamental group $\langle\pi(G);e\rangle$ is abelian; (3) If μ is an affine operation $\alpha x+\beta y+k$ then it is obvious that $\langle G; \mu, e\rangle$ is an abelian group when $\mu(x,y) = x+y-e$. Thus it is natural to ask for a common universal algebraic formulation of the implication

$$\{\mu(x,e) = \mu(e,x) = x\} \models_{G} \{\mu \text{ induces an abelian group operation}\}.$$

With this in mind, we give a few formal rules of derivation for an equational theory such that (i) these rules of derivations are formally valid for all the above mentioned categories and (ii) under these rules of derivations, one can derive the abelian group laws for μ from the one variable law $\{\mu(x,e) = \mu(e,x) = x\}$. A geometric universal algebra, is, by definition, an algebra $\mathfrak{A} = \langle A; F\rangle$ whose first order theory satisfies such implications:

(1) $\exists b_i \, \forall x_j \; f(x_1, x_2, .., x_n, b_1, .., b_m) = b_{m+1} \Rightarrow \forall x_j \, \forall y_i \, \forall z_j \; f(\vec{x}, \vec{y}) = f(\vec{z}, \vec{y})$

(2) $f(\phi_{i1} x_1, \phi_{i2} x_2, .., \phi_{in} x_n) = g(\phi_{i1} x_1, \phi_{i2} x_2, .., \phi_{in} x_n)$

when $\phi_{ij}(x) : A \to A$ are algebraic functions such that $\phi_{ii} = id_A$

then $\mathfrak{A} \models$ the identity $f(x_1, x_2, .., x_n) = g(x_1, x_2, .., x_n)$

(3) If $f(x,e) = f(y,e) \Rightarrow x = y$ then $f(x,u) = f(y,u) \Rightarrow x = y \quad \forall u$.

The implication (1) has been studied in the literature quite extensively: thus it is the Rigidity Lemma of D. Mumford for complete algebraic curves; J. Mycielski has proved a similar version for connected topological algebras; W. Taylor has recently investigated a similar condition (Term condition) in the context of universal algebras. Peter Gumm and others in the Darmstadt school have studied the term function condition in the context of commutators in universal algebras.

Compatible orderings of lattice-ordered algebras

L. Szabó (Szeged)

By a compatible ordering of an algebra $\langle A; F\rangle$ we mean a partial order ρ on A preserved by every operation in F

THEOREM. Let $\mathfrak{A} = \langle A; F\rangle$ be an algebra having two binary local algebraic functions $\wedge$ and $\vee$ such that $\langle A; \wedge, \vee\rangle$ is a lattice and every operation in F preserves the natural ordering $\leqq$ of $\langle A; \wedge, \vee\rangle$. If $\prec$ is a compatible ordering of $\mathfrak{A}$, then $\Theta_1 = (\prec \cap \leqq) \circ (\prec \cap \leqq)^{-1}$ and $\Theta_2 = (\prec \cap \geqq) \circ (\prec \cap \geqq)^{-1}$ are congruence relations of $\mathfrak{A}$ with $\Theta_1 \cap \Theta_2 = \omega$. Thus $\mathfrak{A}$ is a subdirect product of $\mathfrak{A}/\Theta_1$ and $\mathfrak{A}/\Theta_2$. Moreover, $a \prec b$ iff $[a]\Theta_i \prec_i [b]\Theta_i$, $i = 1,2$, where $\prec_1 = ((\Theta_1 \vee \Theta_2)/\Theta_1) \cap \geqq_1$ and $\prec_2 = ((\Theta_1 \vee \Theta_2)/\Theta_2) \cap \leqq_2$. ($\leqq_i$ is the natural ordering of $\langle A/\Theta_i; \wedge, \vee\rangle$, $i = 1,2$.) If $\prec$ is a lattice ordering (i.e. $\langle A; \prec\rangle$ is a lattice) then $\Theta_1 \circ \Theta_2 = A \times A$, and thus $\mathfrak{A} \cong \mathfrak{A}/\Theta_1 \times \mathfrak{A}/\Theta_2$ and $\prec_1 = \geqq_1$, $\prec_2 = \leqq_2$.

Finitely Boolean representable varieties

Emil W. Kiss (Budapest)

A subalgebra $\mathfrak{L}$ of an algebra $\mathfrak{A}$ is called <u>very skew</u> if $\mathfrak{L}$ is skew in each direct decomposition of $\mathfrak{A}$. It is proved that a finite neutral simple algebra $\mathfrak{A}$ in a modular variety is quasi primal iff there is a bound on the cardinalities of the very skew subalgebras of the finite direct powers of $\mathfrak{A}$. With this characterization a short, elementary proof of a result of S. Burris and R. McKenzie stating that each variety Boolean representable by a finite set of finite algebras is the join of an abelian and a discriminator variety is obtained.

Affine Equational Classes and Affine Equational Logic

Robert W. Quackenbush (Winnipeg)

Let K be an equational class. An algebra $\mathfrak{A} = \langle A; F\rangle$ is quasi-affine if for some abelian group $\langle A; +\rangle$, each $f \in F$ is an affine transformation with respect to $+$; $\mathfrak{A}$ is affine if in addition $x-y+z$ is a term function. K is (quasi-) affine if each $\mathfrak{A} \in K$ is (quasi-) affine.

Theorem (Ch. Herrmann): K is affine iff K is modular and for each $\mathfrak{A} \in K$, $\Delta(\mathfrak{A}) = \{(a,a) \mid a \in A\}$ is a congruence class of $\mathfrak{A}^2$.

Variation: K is affine iff K is regular and hamiltonian (each subalgebra is a congruence class). The variation comes immediately to mind upon recalling the characterization of K being equivalent to R-Mod (for some R): K is pointed, point-regular and hamiltonian.

Generalization is the following rule of inference in equational logic: for terms t, α_i, α_i' $(1 \le i \le n)$, β_j, β_j' $(1 \le j \le m)$, from $t(\underline{\alpha}, \underline{\beta}) = t(\underline{\alpha}, \underline{\beta}')$

infer $t(\underline{\alpha}', \underline{\beta}) = t(\underline{\alpha}', \underline{\beta}')$.

$ET(K)$ is the equational theory of K and $G(K)$ is the smallest equational theory containing K and closed under generalization.

Theorem (R. McKenzie): If K is permutable and $ET(K) = G(K)$, then K is affine.

Theorem (W. Taylor): If K is n-permutable and $ET(K) = G(K)$, then K is permutable.

Theorem: If K is n-modular and $ET(K) = G(K)$, then K is n-permutable (same n).

Theorem: If $Mod(G(K))$ is the class of all quasi-affine algebras in K, and $\mathfrak{A} \in K$, then $\mathfrak{A}$ is quasi-affine iff $\Delta(\mathfrak{A})$ is a congruence class of $\mathfrak{A}^2$.

On Coordinatizing Arguesian Lattices

by Alan Day and Doug Pickering (Thunder Bay)

A spanning n-diamond in a modular lattice L is a sequence $(x_1, \ldots, x_n, x_{n+1})$ satisfying $\sum_{j \neq i} x_j = 1$ (all i) and $x_i \cdot \sum(x_k : k \neq i, j) = 0$. The canonical examples are

A) $(n-1)+2$ pts in general position in an $(n-1)$-dimensional projective geometry

B) $(Re_1, \ldots, Re_n, R(\Sigma e_i))$ in $\mathcal{L}({}_R R^n)$ for any ring R

c) $(\mu_1[M], \ldots, \mu_n[M], \delta[M])$ in $\mathcal{L}({}_R M^n)$ for any module M where μ_i $i=1,\ldots,n$ is the canonical injection $\mu_i : M \longrightarrow M^n$ and $\delta : M \longrightarrow M^n$ is the diagonal.

An Arguesian lattice is a (modular) lattice satisfying $(a_0+b_0)(a_1+b_1)(a_2+b_2) \leq a_0(a_1+c_2(c_0+c_1)) + b_0(b_1+c_2(c_0+c_1))$ where $c_i = (a_j+a_k)(b_j+b_k)$.

THEOREM 1: The auxiliary ring of a spanning n-diamond, $(D; \oplus, \otimes, 0, t)$ is indeed a ring, if L is Arguesian and $n \geq 3$.

THEOREM 2: There is a meet-preserving coordinatization map from any "hyperplane" $\Sigma(x_k : k \neq i, j)$ into $\mathcal{L}({}_D D^{n-1})$ which is join-preserving if the diamond satisfys Artmann's upper frame complementability condition

Varieties of modular ortholattices.

Günter Bruns (Hamilton, Ont.)

Let MOn $(n \geq 1)$ be the modular ortholattice consisting of $2n$ incomparable elements and the bounds and let $MO0$ be the one-element ortholattice.

__Theorem.__ If $\mathfrak{K}$ is a variety of modular ortholattices which is not contained in the variety $[MO2]$ generated by $MO2$ then $MO3 \in \mathfrak{K}$

__Conjecture.__ Every variety of modular ortholattices which is different from all $[MOn]$ $(0 \leq n \leq \omega)$ contains a projective plane (with orthocomplementation)

Some recent developments in the theory of partial algebras

Peter Burmeister (Darmstadt)

During the last years interest in partial algebras has increased in Computer Science because of some applications in this field (context sensitive programming languages, specification of data types). This has given new impact to the development of the theory of partial algebras which needs model theoretic concepts on a very early stage. Hoehnke and others on one side, Obtułowicz develop a theory in a Lawvere-style, others like Kupka, Reichel and Kaphengst now do it more in a set theoretical framework. Andréka, Németi, Sain, Pasztor and John have used some basic category theoretical concepts to do partial algebra theory. — A good basis for a model theory for partial algebras seems to be the concept of "existence-equations" (E-equations) $t \overset{e}{=} t'$ (t, t' terms in the usual sense of some type Δ): We say that a partial algebra $\underline{A}$ satisfies $t \overset{e}{=} t'$ with respect to a valuation $\mu: X \to A$ of the set of variables in $\underline{A}$ (briefly $\underline{A} \models t \overset{e}{=} t' [\mu]$) iff the values $t^{\underline{A}}(\mu)$ and $t'^{\underline{A}}(\mu)$ of the corresponding partial term functions do exist and are equal: $t^{\underline{A}}(\mu) = t'^{\underline{A}}(\mu)$.

All other formulas built on E-equations as atomic formulas in the usual sense are then treated as usual with a two-valued semantics (cf. H. Thiele 1966). Besides E-equations elementary implications of the form $\bigwedge_{i=1}^{n} t_i \overset{e}{=} t_i \Rightarrow t_0 \overset{e}{=} t_0'$ (existentially conditioned E-equations, briefly: ECE-equations) ~~or~~ $\bigwedge_{i=1}^{n} t_i \overset{e}{=} t_i' \Rightarrow t_0 \overset{e}{=} t_0'$ (QE-equations) are of special interest, especially ECE-equations take the rôle of equations in partial algebra theory for basic axioms. For these concepts Birkhoff-type theorems of the kind $Mod\, F(\underset{\sim}{K}) = \mathbb{H}\,\mathbb{S}\,\mathbb{P}^{+}\underset{\sim}{K}$ exist, where $\mathbb{S}$ stands for closed subalgebras, $\mathbb{P}^{+}$ for reduced products (or only products in the case F = E-equations) and $\mathbb{H}$ stands for weak homomorphic images ($F=$ E-equations), closed homomorphic images ($F=$ ECE-equations) or isomorphisms ($F=$ QE-equations), respectively. Aso for the syntactical part Birkhoff-type theorems exist in these cases. – Moreover, within this language together with a model theoretic interpretation of the category theoretical concept of a factorization system one gets good descriptions for the most important attributes of homomorphisms between partial algebras. (For more details see Preprint № 582 of the Fachbereich Mathematik, Technische Hochschule Darmstadt).

The notion of codimension for Heyting algebras

Jürgen Schulte Mönting, Univ. Tübingen

The only Heyting algebra which can be embedded into every algebraically closed Heyting algebra is the Boolean algebra $\underline{2}$, whereas every non-Boolean algebraically closed Heyting algebra is an adequate matrix for the intuitionistic propositional logic, i.e. generates the variety $\mathcal{H}$ of all Heyting algebras.

A more detailed analysis of the structure of algebraically closed Heyting algebras is given by the following characterization theorem:

<u>Theorem</u> A Heyting algebra can be embedded into every algebraically closed Heyting algebra $\hat{H}$ of codimension $\langle \hat{d}, \hat{c} \rangle$ if and only if it is cocompact, countable and locally finite and has a codimension not greater than $\langle \hat{d}, \hat{c} \rangle$.

The cocompactness has only to be required in the case of infinite codimension. This condition prevents a Heyting algebra from behaving like a Boolean algebra freely generated by countably many elements.

The codimension of a Heyting algebra H is a pair $\langle d, c \rangle$, $c, d \in \omega \cup \{\infty\}$ where c is the number of minimal prime filters on H, and d is the number of those minimal prime filters which do not contain the filter D of dense elements.

This second-order notion has a first-order counterpart which is the one to compute with: In a Heyting algebra of codimension $\langle d, c \rangle$ there exists an orthogonal antichain $\langle a_\mu \mid \mu < c \rangle$ satisfying $a_\mu \vee a_\nu = 1$ $(\mu \neq \nu)$ $a_\mu' = 0$ for $\mu < d$, a_μ strictly regular (i.e. $[a_\mu, 1] \cap D = \{1\}$) for $d \leq \mu < c$.

Codimensions are partially ordered by the product order. The Heyting algebras of fixed codimension form a universal subclass of $\mathcal{H}$.

This concept seems to be a useful tool for a structure theory of Heyting algebras.

A combinatorial property of free algebras.

Wilfrid Hodges (Bedford College, London), joint with J. Baldwin, J. Berman, A. Glass.
(To appear in Algebra Universalis)

An endomorphism base of the algebra A is a set $X \subseteq A$ such that every ~~permutation~~ map $f: X \to X$ extends to an endomorphism $f^*: A \to A$, so that $f^* g^* = (fg)^*$ and $1_X^* = 1_A$. X is an endomorphism base over $Y \subseteq A$ iff moreover each f^* fixes Y pointwise.

THEOREM. If A is a free algebra in a variety with countable language, $X, Y \subseteq A$ and $|Y| \leq \omega < |X|$, then X contains an uncountable endomorphism base of A over Y.

In the theorem, (i) 'free algebra' can be replaced by 'free power of a countable algebra', and (ii) 'countable' ($\leq \omega$) can be replaced by 'of cardinality κ' where κ is any uncountable regular cardinal.

Typical corollaries: (1) A free boolean algebra contains no uncountable chain (Horn, 1968). (2) If a free power of a countable group, in some variety of groups, contains an uncountable set X of elements and an element g such that for all $x \neq y$ in X either $[x,y] = g$ or $[y,x] = g$, then the elements of X pairwise commute. (3) If $\varphi(v_1, \ldots, v_n)$ is a positive formula with parameters in the free algebra A, and X is an uncountable subset of A such that every n-element subset of X can be listed as $(x_1, \ldots, x_n)$ which satisfies φ, then X has an uncountable subset in which every n-tuple (possibly repeating) satisfies φ. (4), (5), ...

Approximation in universal algebra

Hans Kaiser (Technische Universität Wien)

When one analyses interpolation of functions on $\mathbb{R}$ with values in $\mathbb{R}$ from the topological point of view, one is led to the following concept: Let $\langle A, \Omega, \mathfrak{T}\rangle$ be a topological universal algebra, $\langle F_k(A), \Omega\rangle$ the full k-ary function algebra over A endowed with the product topology of $(A, \mathfrak{T})$. $\langle A, \Omega, \mathfrak{T}\rangle$ is said to have the approximation property iff the algebra $P_k(A)$ of k-ary polynomial functions with the induced topology is dense in $F_k(A)$ for all $k \in \mathbb{N}$.

In order to exclude trivialities we assume that all topological algebras considered satisfy T_2. (This approach is due to G. Kowol). The main purpose of the lecture is to give a description of all topological algebras satisfying T_2 in congruence permutable varieties as a corollary of the following theorem:
A topological universal algebra $\langle A, \Omega, \mathfrak{I} \rangle$ satisfying T_2 has the approximation property iff. there is a Malcev-function $m: A^3 \to A$ which has the approximation property, there is an $a \in A$ and a non-constant $q: A^2 \to A$ such that $q(x, a) = q(a, x) = a$ for all $x \in A$ which has the approximation property and for every non-trivial congruence relation Θ on A ~~there~~ we have $[a]\Theta = A$.
In addition to that Jacobson's density theorem ~~is derived~~ for rings of linear transformations of vectorspaces over skewfields is derived in this setting.

PROPERTIES OF HOMOMORPHISMS AND QUOMORPHISMS BETWEN PARTIAL ALGEBRAS

B. Wojdyło (Toruń, Poland)

Details - see Preprint Nr. 657, TH Darmstadt, 1982
(authors: P. Burmeister and B. Wojdyło)

Let $\underline{A} = (A, (f^{\underline{A}})_{f \in \Omega})$ and $\underline{B} = (B, (f^{\underline{B}})_{f \in \Omega})$ be partial algebras of type $\Delta = (n_f)_{f \in \Omega}$.

A <u>quomorphism</u> of $\underline{A}$ into $\underline{B}$ is any partial mapping $h: A \dashrightarrow B$ s.t.
$(\forall f \in \Omega)(\forall \underline{a} \in A^{n_f})[\underline{a} \in dom\, f^{\underline{A}} \wedge \underline{a} \in (dom\, h)^{n_f} \wedge f^{\underline{A}}(\underline{a}) \in dom\, h \to$
$\to h \circ \underline{a} \in dom\, f^{B} \wedge h(f^{\underline{A}}(\underline{a})) = f^{\underline{B}}(h \circ \underline{a})]$

An universally defined quomorphism is called a <u>homomorphism</u>.
Consider the following list of properties of quomorphisms between partial algebras:
i (injective), a (almost onto: $[h[A]]_{\underline{B}} = \underline{B}$), o (onto),

The lattice $\underline{L}(H)$:

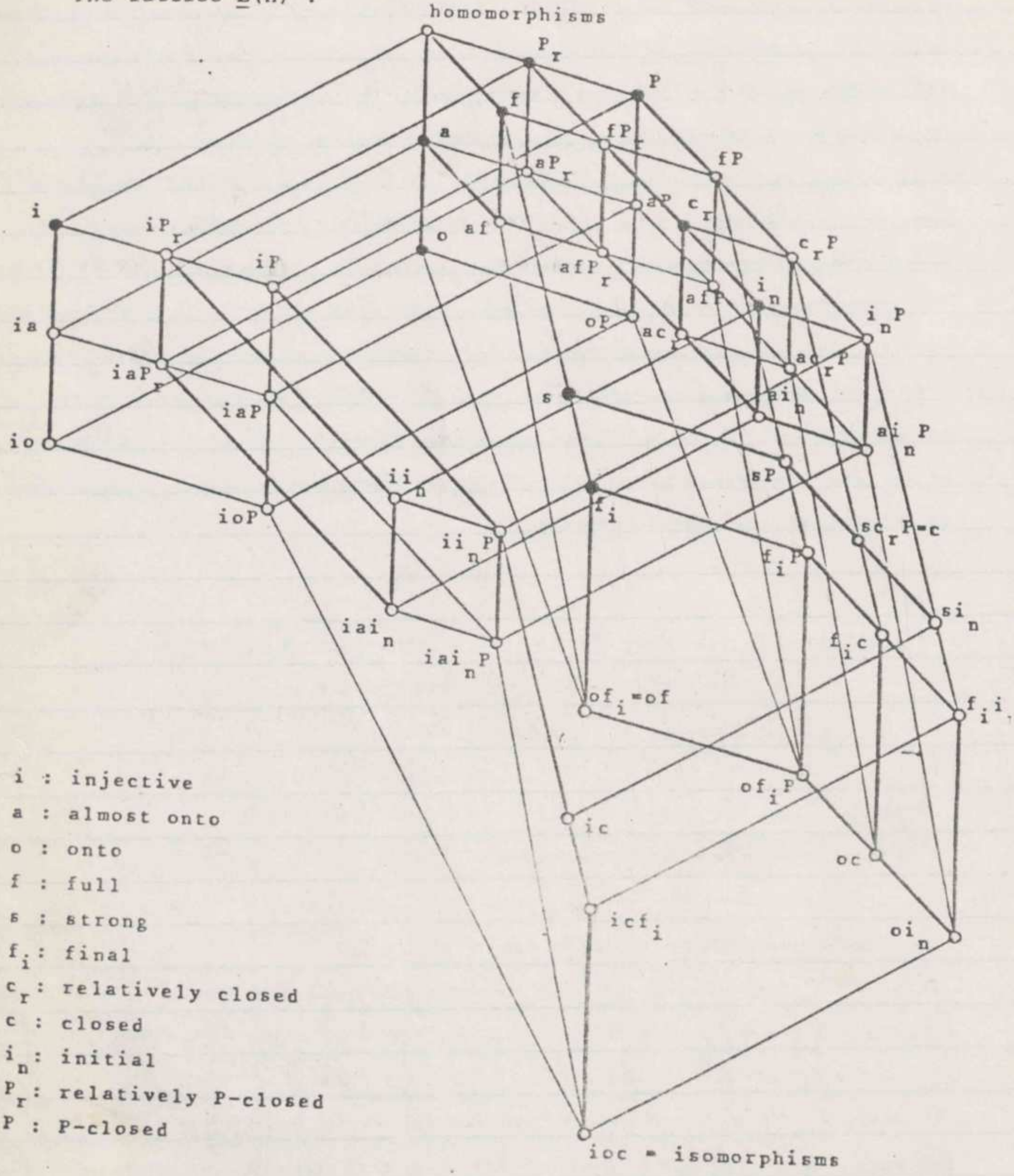


Figure 1

f (full: $f^B(h\circ\underline{a}) = h(a) \to (\exists \underline{a}' \in A^{n_f})[\underline{a}' \in (\mathrm{dom}\, h)^{n_f} \wedge \mathrm{dom} f^A \wedge f^A(\underline{a}') \in \mathrm{dom}\, h \wedge h\circ\underline{a} = h\circ\underline{a}']$
s (strong: $f^B(h\circ\underline{a}) = b \Rightarrow$ ——— " ———)
fi (final: $f^B(\underline{b}) = b \Rightarrow (\exists \underline{a} \in A^{n_f})(\exists a \in A)[f^A(\underline{a}) = a \wedge h\circ\underline{a} = \underline{b} \wedge h(a) = b]$)
Cr (relatively closed: $f^B(h\circ\underline{a}) = h(a) \Rightarrow f^A(\underline{a}) = a'$ for some $a' \in \mathrm{dom}\, h$)
c (closed: $f^B(h\circ\underline{a}) = b \Rightarrow$ ——— " ———)
in (initial: $f^B(h\circ\underline{a}) = h(a) \Leftrightarrow f(\underline{a}) = a \wedge \underline{a} \in (\mathrm{dom}\, h)^{n_f} \wedge a \in \mathrm{dom}\, h$)
Pr (relatively P-closed, where $P \subseteq T$ is a subset of a set T of terms with variables X: $(\forall p \in \downarrow P)(\forall \underline{a} \in A^X)[p^B(h\circ\underline{a}) = h(a)$ for some $a \in A \to$
$\to \underline{a} \in \mathrm{dom}\, p^A \wedge \{q^A(\underline{a}) \mid q \in \downarrow\{p\}\} \subseteq \mathrm{dom}\, h]$
P (P-closed: $(\forall p \in \downarrow P)(\forall \underline{a} \in A^X)[p^B(h\circ\underline{a}) = b$ for some $b \in B \Rightarrow \underline{a} \in \mathrm{dom}\, p^A \wedge$
$\wedge \{q^A(\underline{a}) \mid q \in \downarrow\{p\}\} \subseteq \mathrm{dom}\, h]$
u (universally defined: $\mathrm{dom}\, h = A$).

Starting from this properties of quomorphisms we can define new ones by combining the given ones, thus getting names for special quomorphisms. We present an „algorithm" for building such names, namely:
There are given six sets of basic words („< >" designates the empty word):

(1) {< >, i}　　　　(4) {< >, Pr, Cr, ~~c~~ in}
(2) {< >, a, o}　　　(5) {< >, P}
(3) {< >, f, s, fi}　　(6) {< >, u}

A name of some new property is a word being a sequence of elements (exactly one element from each set) from above sets occuring in the following ordering: (1) (2) (3) (4) (5) (6) quom.
The classes of homomorphisms belonging to the ~~notions~~ properties of hom. definable by above „procedure" build the lattice $\underline{L}(H)$ - see Fig. 1.
It is a concept lattice (in the sense of R. Wille)
for homomorphism.
A concept lattice for quomorphism $\underline{L}(Q) = \underline{L}(H) \times \underline{2}$.

Varieties of relation algebras.

Bjarni Jónsson, Vanderbilt University.

The class <u>RA</u> of all relation algebras (in the sense of Alfred Tarski) is a congruence distributive and congruence permutable variety in which every subdirectly irreducible ~~alg~~ algebra is simple (In fact, <u>RA</u> is a discriminator variety.) Every finite, simple member of <u>RA</u> is splitting; in particular, this is true of $\mathfrak{R}(n)$, the full relation algebra on n elements.

<u>Thm</u> 1. Every embedding of $\mathfrak{R}(n)$ into a simple relation algebra is an isomorphism.

<u>Thm</u> 2. A simple relation algebra A is isomorphic to $\mathfrak{R}(n)$ iff there exists an element a in A such that

$$a;a \leq a, \quad a + a^{\smile} \geq 0', \quad a^{n} = 0, \quad a^{n-1} \neq 0.$$

<u>Thm</u> 3. The conjugate variety of $\mathfrak{R}(n)$, $\underline{RA}^*(n)$, is a dual atom in the lattice of all subvarieties of <u>RA</u>.

<u>Thm</u> 4. The identity

$$a^{n-1} \leq 1;((a;a)a^{-} + a^{-}a^{\smile -}0' + a^{n});1$$

is an equational basis for $\underline{RA}^*(n)$ mod <u>RA</u>.

<u>Open problems</u>. 1. Are the algebras $\mathfrak{R}(n)$ the only finite, simple relation algebras that cannot be properly embedded in larger simple relation algebras?

2. Are the varieties $\underline{RA}^*(n)$ the only lower covers for <u>RA</u>?

3. If $m \neq n$, then $\underline{RA}^*(m) \cap \underline{RA}^*(n)$ is obviously a lower cover for $\underline{RA}^*(n)$. Does $\underline{RA}^*(n)$ have other lower covers?

4. The lattice of subvarieties of <u>RA</u> has three atoms. How many varieties are there on the next level?

Ideals in universal algebras
Aldo Ursini, Siena University

K Fix an equational class of algebras with a constant 0.
An ideal term $p(\vec{x},\vec{y})$ is a term such that
$$p(\vec{x},\vec{0})=0 \text{, in } K.$$
For $\mathfrak{A}\in K$, a subset I of A such that $0\in I$ is an ideal of $\mathfrak{A}$ if for all ideal terms $p(\vec{x},\vec{y})$, for all $\vec{a}\in A$, $\vec{i}\in I$, $p(\vec{a},\vec{i})\in I$.
K has ideal determined congruences (K is ideal determined, for short) if for all $\mathfrak{A}\in K$, I ideal of $\mathfrak{A}$ there is exactly one congruence θ of $\mathfrak{A}$ such that $I=[0]\theta$.
If K is ideal determined, congruences are modular.
Th. 1. Being ideal determined is a Mal'cev condition.
Th. Define a term $t(\vec{x},\vec{y},\vec{z})$ to be a commutator term if $t(\vec{x},\vec{y},\vec{0})=0=t(\vec{x},0,\vec{z})$ hold in K. Define the commutator of two ideals I,J of $\mathfrak{A}\in K$ as follows
$$[I,J]=\{t(\vec{a},\vec{i},\vec{j}) \mid t(\vec{x},\vec{y},\vec{z}) \text{ a commutator term, } \vec{a}\in A, \vec{i}\in I, \vec{j}\in J\}$$
Th. 2. The congruence corresponding to $[I,J]$ is equal to the commutator of the corresponding congruences, whenever I,J are ideals of K, and K is ideal determined.

The size of Congruence lattices of models of a first-order theory.

Sauro Tulipani (Univ. of Camerino)

Given a first-order theory T in a countable language without relation symbols, define for every infinite cardinal λ: $C_T(\lambda) = \sup\{|Con(A)| : A \in Mod(T),\ |A| = \lambda\}$

$L_T(\lambda) = \sup\{\text{length } Con(A) : A \in Mod(T),\ |A| = \lambda\}$

THM 1 For every theory T one of the following cases holds:

Case (i) $L_T(\lambda) = ded(\lambda) \leq C_T(\lambda) \leq 2^\lambda$ for every infinite cardinal λ

Case (ii) $(\exists n < \aleph_0)$ $L_T(\lambda) = n$, $C_T(\lambda) = \lambda$ " " "

Case (iii) $(\exists m, n < \aleph_0)$ $L_T(\lambda) = n$, $C_T(\lambda) = m$ " " "

Further properties of $L_T(\lambda)$ and $C_T(\lambda)$ can be proved for theories with Definability of Compact Congruences (DCC) for the special case of $\aleph_0$-categorical theories.

The inequality in Case (i) cannot be improved. In fact, there are examples of $\aleph_0$-categorical theories for which $C_T(\lambda) = ded(\lambda)$ for every infinite cardinal λ. However, if T is a stable theory which has DCC, then $C_T(\mu) > \mu$ for some μ implies $C_T(\lambda) = 2^\lambda$ for every λ. This stems on the following

THM 2 Let T be a theory such that for every positive integer k there exist a model A of T and a congruence of A with a minimal set of generators of cardinality k. Then, for every infinite cardinal λ there exists a model B of T such that $|B| = \lambda$ and the semilattice $(P(\lambda), \cup)$ of power-set of a set of cardinality λ can be embedded in Con (B).

OPEN: If $C_T(\mu) > ded(\mu)$ for some infinite cardinal μ implies alway $C_T(\lambda) = 2^\lambda$ for every infinite cardinal λ.

Planes in Dilworth truncations

Jiří Tůma, Prague

Let us consider a finite geometric (i.e. point and semimodular) lattice L. Denote by L_k the lattice obtained from L by indentifying all elements with rank $\leq k-1$. In general, L_k will not be geometrical lattice. Dilworth found a canonical construction which extends L_k to a new geometrical lattice $D(L_k)$ having the same points, and which preserves as many properties of L_k as it can: covering relation, meets, and all joins which do not damage semimodularity. If B is the boolean lattice of all subsets of a finite set, then $D(B_2)$ is isomorphic to a partition lattice. We give a partition-like representation of elements in $D(B_k)$ for all k.

A geometric lattice is a minor of a geometric lattice L if it is a join-subsemilattice of L preserving covering relation. Tutte's deep characterization of graphic matroids gives a finite list of all minimal forbidden minors of partition lattices (i.e. of all minimal geometric lattices which are not minors of any partition lattice). We show that for all $k \geq 3$ there are infinitely many minimal geometric lattices of rank 3, which are forbidden in all $D(B_k)$. Further properties of minors of $D(B_k)$ are given

Jiří Tůma

VARIETIES WITH EQUATIONALLY DEFINABLE PRINCIPAL CONGRUENCES – A STUDY OF THE DEDUCTION THEOREM IN ALGEBRAIC LOGIC

W.J. BLOK, P. KÖHLER, D. PIGOZZI*

MOST OF THE FAMILIAR VARIETIES THAT ARISE IN LOGIC TURN OUT TO HAVE EQUATIONALLY DEFINABLE PRINCIPAL CONGRUENCES (EDPC) IN FACT IT CAN BE SHOWN THAT EVERY VARIETY THAT COMES FROM THE ALGEBRAIZATION OF A DEDUCTIVE SYSTEM SATISFYING SOME REASONABLE VERSION OF THE DEDUCTION THEOREM MUST HAVE EDPC. THIS SUGGESTS THE FOLLOWING CLOSELY CONNECTED PROBLEMS: (I) UNDER WHAT ADDITIONAL CONDITIONS CAN A VARIETY WITH EDPC BE GIVEN THE FORM OF ONE ARISING FROM LOGIC; (II) CAN THE VARIETIES OF LOGIC THEMSELVES BE BE CHARACTERIZED IN NATURAL ALGEBRAIC TERMS. THESE PROBLEMS REQUIRE AN ANALYSIS OF THE ALGEBRAIZATION PROCESS ITSELF.

A DEDUCTIVE SYSTEM $\mathcal{L}$ IS <u>ALGEBRAIZABLE</u> IFF THERE EXISTS A SYSTEM Δ_i, $i<m$, (IN TWO PROPOSITIONAL VARIABLES) OF FORMULAS WITH THE FOLLOWING PROPERTY: FOR EACH SET Γ OF FORMULAS THE SYSTEM OF DEDUCTIVE RELATIONS $\Gamma \vdash_{\mathcal{L}} \varphi \Delta_i \psi$, $i<m$, DEFINES A CONGRUENCE RELATION ON THE FORMULA ALGEBRA WHICH, IN TURN, UNIQUELY DETERMINES Γ. THE <u>GÖDEL RULE</u> HOLDS IN $\mathcal{L}$ IFF, FOR ANY PAIR OF FORMULAS φ, ψ, ONE CAN DEDUCE THEIR EQUIVALENCE, I.E, $\varphi, \psi \vdash_{\mathcal{L}} \varphi \Delta_i \psi$, $i<m$; WITH A FEW NOTABLE EXCEPTIONS ALL THE SPECIFIC DEDUCTIVE SYSTEMS CONSIDERED IN THE LITERATURE ADMIT THE GÖDEL RULE. THE <u>GENERALIZED DEDUCTION THEOREM</u> HOLDS IN $\mathcal{L}$ IFF THERE EXISTS A SYSTEM OF FORMULAS $\rightarrow_i$, $i<m$, SUCH THAT $\Gamma, \varphi \vdash_{\mathcal{L}} \psi$ IFF $\Gamma \vdash_{\mathcal{L}} \varphi \rightarrow_i \psi$, $i<m$.

THEOREM. LET $\mathcal{L}$ BE AN ALGEBRAIZABLE DEDUCTIVE SYSTEM AND V ITS ASSOCIATED ALGEBRA (i) $\mathcal{L}$ ADMITS THE GÖDEL RULE IFF V IS 1-REGULAR FOR SOME ~~REGULAR~~ CONSTANT 1. (ii) $\mathcal{L}$ ADMITS THE GENERALIZED DEDUCTION THEOREM IFF V HAS EDPC

LET V BE A 1-REGULAR VARIETY. V IS A VARIETY OF WEAK BROUWERIAN SEMILATTICES WITH FILTER PRESERVING OPERATIONS (WBSO) IF IT HAS BINARY TERMS $\to$ AND $\cdot$ (CALLED WEAK RELATIVE PSEUDO COMPLEMENTATION AND ~~MEET~~ WEAK MEET RESPECTIVELY) SATISFYING THE FOLLOWING CONDITIONS (i) $a \to b$ AND $b \to a$ DEFINES A CONGRUENCE $\approx$ ON THE REDUCT $\langle A, \to, \cdot, 1 \rangle$ SUCH THAT $\langle A, \to, \cdot, 1 \rangle / \approx$ IS A BROUWERIAN SEMILATTICE (ii) THE 1-IDEALS OF $\mathfrak{A}$ ARE EXACTLY THE SUBSETS OF A OF THE FORM $\bigcup F$ WHERE F IS A FILTER OF $\langle A, \to, \cdot, 1 \rangle / \approx$.

THEOREM LET V BE A CONGRUENCE-PERMUTABLE AND 1-REGULAR VARIETY. IF V HAS EDPC THEN V IS A WBSO VARIETY.

CONGRUENCE PERMUTABILITY IS NECESSARY IN ORDER TO DEFINE WEAK MEET. NON-TRIVIAL EXAMPLES OF WBSO VARIETIES ARE DISCRIMINATOR VARIETIES AND MODAL ALGEBRAS SATISFYING THE IDENTITY $x^{0^{m+1}} = x^{0^m}$ FOR SOME NATURAL NUMBER m.

EMBEDDING IN GLOBALS OF GROUPS AND SEMILATTICES

Matthew Gould, Vanderbilt University

Borrowing the ~~~~ terminology of Trnková, we use the term "global of S" to denote the semigroup consisting of all non-void subsets of a semigroup (S,·), under the "complex product", $A \cdot B = \{ab \mid a \in A, b \in B\}$.

Trnková proved in 1975 that every commutative semigroup S is embeddable in the global of $(\mathbb{N}^{|S| \cdot \aleph_0}, +)$. Since $\mathbb{N} \subseteq \mathbb{Z}$, we have S embeddable in the global of a group. In 1979, A. Liau asked for a finite analogue of this result and proved that <u>if every z-semigroup is embeddable in the global of a finite abelian group, then every finite commutative semigroup is so embeddable</u>, where by "z-semigroup" is meant a finite, commutative semigroup S with 0 satisfying any of the following equivalent conditions: (i) 0 is the only idempotent in S;
(ii) $S^n = 0$ for some $n \in \mathbb{N}$;
(iii) For each x there is some $n(x) \in \mathbb{N}$ such that $x^{n(x)} = 0$.

Observing that the embeddability property is preserved under the formation of subdirect products, and that every homomorphic image of a z-semigroup is a z-semigroup, we immediately improve Liau's result by reducing the problem to <u>subdirectly irreducible</u> z-semigroups. Refining a result of Schein, we note that a z-semigroup S is subdirectly irreducible if and only if distinct non-zero elements have distinct annulets, that is, for $a \in S \setminus \{0\}$, the map $a \longrightarrow \{x \in S \mid ax = 0\}$ is one-to-one. Reaching a dead end, we then take another approach, namely to construct the free z-semigroup of height K on n generators (the <u>height</u> of a z-semigroup S is the smallest n satisfying (ii) above). Jointly with J. Iskra we have proved that these free z-semigroups are indeed embeddable as required. It remains to deal with factor semigroups of these free ones; as yet we have only partial results.

A third approach is to ask which commutative finite semigroups S can be embedded in the global of a <u>semilattice</u>. It is easy to see that such semigroup must be <u>combinatorial</u>, that is, no subsemigroup having more than one element can be a group.

Denoting Lau's question by Q.1, we pose

Q.2: Is every finite, commutative, combinatorial semigroup embeddable in the global of a finite semilattice?

~~Along with~~ Jointly with J. Iskra, we have proved the analogues of the above results (First, that Q.2 holds in general if it holds for all subdirectly irreducible z-semigroups; second, that Q.2 holds for the free z-semigroups), and the following

Theorem. If Q.2 has an affirmative solution, then so has Q.1.

Although it is not directly relevant, we also have the

Theorem. If S and T are finite semilattices having isomorphic globals, then $S \cong T$.

(The corresponding result for ~~[illegible]~~ groups — finite or infinite — is rather trivial and was first noted by Tamura and Shafer ~~[illegible]~~ in 1967.)

TWO SIDES OF CONGRUENCE MODULAR VARIETIES

H. Peter Gumm (TH Darmstadt)

It has been observed for a number of years that algebras in modular varieties split into two cases, once you put some severe restrictions on them. It seems that the algebras in a modular variety form a continuous spectrum whose one end consists of the subvariety of affine algebras while the other end consists of those algebras whose subdirect powers have distributive congruence lattices. The commutator operation on congruence lattices has provided the proper tool to enable one distinguishing these two cases. We give several examples of this fact and present a Mal'cev type condition which is indeed just Jonsson's condition for distributivity and Mal'cev's characterization of permutability glued together. At the distributive end of the spectrum we present the following theorem which generalizes the famous Jónsson Lemma and its subsequent improvements due to Hagemann and Herrmann and the one of Hrushovskii. To this end we define $\xi(\Theta) = \bigvee\{\alpha \mid \exists_{\beta \gneq \Theta}\ [\alpha,\beta] \leq \Theta\}$, and $\xi(\mathfrak{A}) := \xi(\sigma)$. We obtain the Theorem: If $\mathfrak{A}$ is finitely subdirectly irreducible in $\mathcal{V}(K)$, then $\mathfrak{A}/_{\xi(\mathfrak{A})} \in HSP_u(K)$. The prime congruences turn out to be precisely those Θ with $\xi(\Theta) = \Theta$, so if $\sqrt{\mathfrak{A}}$ is the prime radical of $\mathfrak{A}$ we obtain:

Corollary: Let $\mathfrak{A} \in HSP(K)$, then $\mathfrak{A}/_{\sqrt{\mathfrak{A}}} \in P_S HSP_u(K)$.

At the other end of modular varieties we look at congruences $\alpha \geq \beta$ with $[\alpha,\beta] = 0$. On every β-class $[a]\beta$ an affine algebra $\mathfrak{A}^\nabla[\beta]_a$ can be defined such that $(a,b) \in \alpha$ implies $\mathfrak{A}^\nabla[\beta]_a \cong \mathfrak{A}^\nabla[\beta]_b$. In particular, for $\alpha = 1$ we find: $\mathfrak{A}^\nabla[\beta] \in HSP_f(\mathfrak{A})$ and $\beta \cong \mathfrak{A} \times \mathfrak{A}^\nabla[\beta]$.

Lattice-ordered loops

Trevor Evans (Emory Univ. Atlanta)

The problem studied is in the general area of the effect of order conditions on the algebraic structure of algebras in a variety of lattice-ordered algebras. We determine the subvariety, in terms of identities, generated by all fully-ordered loops in the variety of lattice-ordered loops. This subvariety is characterized by the further identity

$$\{(x-y) + (y-x)\Theta\}^+ = 0 \quad \text{for all inner mappings } \Theta$$

The known results for groups and commutative groups are special cases of this

Idempotent Entropic Algebras

J.D.H. Smith, jointly with A.B. Romanowska

Idempotent entropic algebras are algebras in which each element forms a singleton subalgebra (idempotence) and in which each operation is a homomorphism (entropicity). Typical models are semilattices, and convex subsets of a finite-dimensional Euclidean space under the operations of weighted mean. Semilattice words in an alphabet correspond to subsets of the alphabet, while weighted means of an alphabet may be considered as probability distributions on it. Thus in general idempotent entropic algebras may be regarded as a universal algebraic approach to "choice and chance".

Current work of the authors on these algebras has investigated the algebraic structure of subalgebra systems of idempotent entropic algebras, including freeness results, an algebraic description of the approximation of a subalgebra by its finitely generated subalgebras, and a variety of structure theorems for the system of subalgebras.

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Infinitesimal Deformation Of Two-Dimensional Cusp-Singularities

Kurt Behnke, Universität Hamburg

Let K be a real quadratic number field, let $M \subset K$ be a complete lattice, say $M = \mathbb{Z} + \mathbb{Z}\omega$, $0 < \omega' < 1 < \omega$, and let U be an infinite cyclic group of algebraic units of K, preserving M. The group of 2×2 matrices of the form $\begin{pmatrix} \varepsilon & \mu \\ 0 & 1 \end{pmatrix}$, $\varepsilon \in U$, $\mu \in M$, acts on the product $H \times H$ of upper half planes by $(z_1, z_2) \mapsto (\varepsilon z_1 + \mu, \varepsilon' z_2 + \mu')$ freely and discontinuously, and the quotient manifold X' can be completed by adding a singular point ∞ to give a normal complex surface X. The singularity (X, x) at infinity is called a <u>cusp singularity</u>.

The resolution of X has as exceptional set either a rational curve with a node, or a cycle of nonsingular rational curves of self-intersection numbers $b_0, \dots, b_{r-1}$, where $\omega = [[\overline{b_0, \dots, b_{r-1}}]]$ is the purely periodic continued fraction development of ω.

Let $M^* = \mathbb{Z} \oplus \mathbb{Z}\omega^*$, $\omega^* = \frac{2-\omega'}{1-\omega'}$. M^* is strictly equivalent to the complementary lattice of M, and the local ring at the cusp consists of the convergent Fourier series $\sum_{\gamma \in (M^*)^+} c_\gamma \exp(2\pi i(\gamma z_1 + \gamma' z_2))$, which are invariant under U; that means $c_\gamma = c_{\gamma\varepsilon}$, for all $\varepsilon \in U$, and bounded at ∞.

Let $\omega^* = [[\overline{a_0, a_1, \dots, a_{s-1}}]]$

Let $A_{-1} = \omega^*$, $A_0 = 1$, $A_{k+1} - A_{k-1} = a_k A_k$. Then the lattice points $\{A_k\}_{k \in \mathbb{Z}}$ are minimal set of generators for the semigroup $(M^*)^+$, A_s generates U and $A_s \cdot A_k = A_{s+k}$. Hence:

<u>Theorem</u> (Nakamura, Ueno): Let $F_\ell(z) = \sum_{k=-\infty}^{\infty} \exp\{2\pi i (A_{ks+\ell} z_1 + A'_{ks+\ell} z_2)\}$.

Then for $s \geq 3$, $F_0, \dots, F_{s-1}$ are a minimal set of generators of the maximal ideal of the local ring at ∞.

To compute the vectorspace T^1 of infinitesimal deformations of a cusp-singularity, use the exact sequence of sheaves $0 \to \Theta_X \to i^*\Theta_{\mathbb{C}^s} \to N \to 0$, which gives T^1 as the kernel of $H^1(X', \Theta_X) \xrightarrow{\chi} H^1(X', i^*\Theta_{\mathbb{C}^s})$. Here the map χ is given by $\chi(\vartheta) = (\vartheta F_0)\frac{\partial}{\partial x_1} + \dots + (\vartheta F_{s-1})\frac{\partial}{\partial x_s}$ by the chain rule.
Denote by A the vectorspace of Fourier-series $\sum_{\gamma \in M^*} c_\gamma \exp(2\pi i(\gamma z_1 + \gamma' z_2))$, by D the space of derivations $A \cdot \frac{\partial}{\partial z_1} \oplus A \cdot \frac{\partial}{\partial z_2}$.
U acts on D by conjugation, and we prove the following:

<u>Theorem</u>. $H^1(X', \Theta_X)$ is the cokernel of the map $D \xrightarrow{\varepsilon-1} D$,
$H^1(X', i^*\Theta_{\mathbb{C}^s})$ is the cokernel of the map $A^s \xrightarrow{\varepsilon-1} A^s$
and the commutative diagram

$$\begin{array}{ccc} & D & \xrightarrow{\chi} & \bigoplus_{\delta=1}^{s} A \frac{\partial}{\partial x_\delta} \\ & {\scriptstyle \varepsilon-1}\downarrow & & \downarrow{\scriptstyle \varepsilon-1} \\ & D & \xrightarrow{\chi} & \bigoplus_{\delta=1}^{s} A \frac{\partial}{\partial x_\delta} \\ & {\scriptstyle \varepsilon-1}\downarrow & & \downarrow{\scriptstyle \varepsilon-1} \\ T^1_X \hookrightarrow & H^1(X', \Theta_X) & \xrightarrow{\chi} & H^1(X', i^*\Theta_{\mathbb{C}^s}) \end{array}$$

gives an explicit description of T^1_X.

Embedding of curves and cuspidal rational curves

David Eisenbud, Brandeis University.

This is a report of recent work of mine with <u>Joe Harris</u>.

<u>Theorem 1</u> Let C be a general curve of genus g over $\mathbb{C}$, $g \neq 0, 1, 3$. C can be embedded as a ~~sm~~ curve of degree d in projective space if and only if $d \geq \frac{3}{4}g + 3$.

This and other results on general linear series on general curves can be deduced from corresponding theorems on ~~to~~ general (geometrically) rational curves with g ordinary cusps. In particular, a simpler proof of the Brill-Noether theorem than that due to Griffiths-Harris, which used nodal rational curves can be given, and the ramification in general embeddings can be determined.

K-theory for complexes of modules.

Hans-Bjørn Foxby, Københavns Universitet

For any category $\underline{X}$ of complexes of modules over a ring A the abelian group $Æ(\underline{X})$ is presented by generators $[P]$, only depending on the isomorphism class of $P \in \underline{X}$, subject to the relation $[P]=0$ if P is exact, and to the relation $[P]=[\bar{P}]+[\tilde{P}]$ whenever there is an exact sequence $0 \to \bar{P} \to P \to \tilde{P} \to 0$ in $\underline{X}$.

Let $S_1, \ldots, S_d \subseteq \text{Center} A$ be multiplicatively closed, let $\underline{P}_S$ denote the category of bounded complexes P of f.g. projective (left) A-modules, such that $S_v^{-1}P$ is exact for all $v=1,\ldots,d$, and let $\underline{P}_S^d$ be the subcategory consisting of complexes of the form $0 \to P_d \to \cdots \to P_1 \to P_0 \to 0$.

<u>Theorem 1</u>. The canonical homomorphism: $Æ(\underline{P}_S^d) \to Æ(\underline{P})$ is an isomorphism.

For $d=1$ the inverse can be given explicitly (and this gives rise to an exact sequence of groups $K_1(A) \to K_1(S_1^{-1}A) \to Æ(\underline{P}_S) \to K_0(A) \to K_0(S_1^{-1}A)$.)

Now assume that A is local, and that M and N are f.g. A-modules, such that $\operatorname{pd} M < \infty$ and $\dim(M \otimes N) = 0$. The <u>intersection multiplicity</u> is $\chi(M,N) := \Sigma_i (-1)^i \operatorname{length} \operatorname{Tor}_i(M,N)$.

<u>Theorem 2</u>. If $\operatorname{grade} M \leq 1$ <u>or</u> $\dim N \leq 1$, then (0) $m+n \leq d$, (1) $\chi(M,N)=0$ if $m+n<d$, and (2) $\chi(M,N)>0$ if $m+n=d$, where $d=\dim A$, $m=\dim M$, and $n=\dim N$.

<u>Corollary 1</u>. (0), (1), and (2) hold always, if either $\dim A \leq 2$ or A is regular and $\dim A \leq 4$. (Here the last part has also been proved by Hochster.)

<u>Corollary 2</u>. (1) holds always, if A is regular and $\dim A \leq 5$. (This has also been proved by Dutta.) My proof uses:

<u>Lemma</u>. If A is regular and (M,N) is a counter example to (1) with $m+n$ minimal, then $d+m+n$ is even.

The proof of the Lemma, as well as the proof of Theorem 2', use the groups $\underline{A}(\underline{X})$ for various categories $\underline{X}$.

The Hodge-Index-Theorem in Arakelov's Intersection-Theory (Faltings)

Suppose X/R is a semistable curve of genus g over the integers R of a number-field K. Arakelov has defined an intersection product for divisors on a compactification of X, obtained by adding fibres over the infinite places of K (see Izv. Akad. Nauk. SSSR, $\underline{38}$ (1974)).
We prove a Riemann-Roch and a Hodge-index-theorem for this product.
The Riemann-Roch deals with the volume of a fundamental domain in $\Gamma(X, \mathcal{O}(D)) \otimes_{\mathbb{Z}} \mathbb{R}$, with respect to the lattice $\Gamma(X, \mathcal{O}(D))$, for a divisor D. For its formulation we construct a canonical volume-form on $\Gamma(X, \mathcal{O}(D)) \otimes_{\mathbb{Z}} \mathbb{R}$.
The Hodge-index-theorem is proved by relating the self-intersection of a divisor to its Néron-Tate-height in the Mordell-Weil group. For this we need the Riemann-Roch.

A lifting result for finiteness of local cohomology.
M. Brodmann

Let M be a finitely generated module over a power-series ring $k[[X_1, \dots, X_s]] = R$. Let $k[[Z,T]] \to R$ be a homomorphism, $\mathfrak{m}$ the maximal ideal of R. Assume that f is regular with respect to $M/\Gamma_{\mathfrak{m}}(M)$ for all $f \in \mathfrak{m}$

$\in [Z,T]k[[Z,T]]-(0)$, and that $H_m^{i-1}(M/fM)$ is finitely generated. Then $H_m^i(M)$ is finitely generated.

An easy proof of Grothendiecks finiteness theorem follows

The depth of the module of differentials of a generic determinantal singularity.

Udo Vetter, Universität Osnabrück - Abt. Vechta

Let K be a field, (X_j^i) an (m,n)-matrix of indeterminates over K, r an integer such that $1 \leq r < \min\{m,n\}$ and $R := K[X_j^i]_{(X_j^i)} / I_{r+1}$, where $I_g = I_g(X_j^i)$ denotes the ideal generated by all g-minors of (X_j^i). By $D_K(R)$ we will denote the module of Kähler-differentials of R over K. Then one can prove

$$\operatorname{depth} D_K(R) = \dim K[X_j^i]/I_r(X_j^i) + 2.$$

It follows that $D_K(R)$ is a second syzygy. Since one easily gets that $\operatorname{depth} D_K(R)_{\mathfrak{p}} = 2$ for the (prime) ideal $\mathfrak{p}$ of the singular locus of R, $D_K(R)$ is not a third syzygy. The formula also implies a negative answer to questions of Buchsbaum and Robbiano, resp., concerning the behaviour of $\operatorname{depth} I_{r+1}^s / I_{r+1}^{s+1}$ for $s \geq 2$.

The formula has been proved by using the results on "algebras with straightening laws" due to De Concini, Eisenbud and Procesi. These results also can be used in order to obtain results on $\operatorname{depth} \operatorname{Hom}_R(D_K(R), R)$ and the vanishing of $\operatorname{Ext}_R^i(D_K(R), R)$.

Residuenkomplex und reguläre Differentialformen

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Sei k ein bewerteter Körper der Charakteristik 0, A eine lokale analytische k-Algebra. Als den Residuenkomplex $D_\Omega(A)$ definieren wir den Komplex $\operatorname{Hom}_{\Omega_R}(\Omega_A, \Omega_R^{\cdot+m} \otimes_R C^{\cdot+m}(R))$, wobei $R := k\langle\langle X_1, \dots, X_n \rangle\rangle$ eine reguläre Potenzreihenalgebra und $R \xrightarrow{\pi} A$ ein endlicher Homomorphismus der Codimension m ist. $C^\cdot(R)$ bezeichne den Cousinkomplex von R. $D_\pi(A)$ (Residuenkomplex) ist bis auf kanonische Isomorphie unabhängig von der Darstellung π und ist ein Komplex graduierter Ω_A-Moduln (Ω_A : De Rham-Algebra) mit äußerer Differentiation d. Die 0-te Kohomologie wird mit ω_A bezeichnet und ist ein Ω_A-Modul mit äußerer Differentiation d. Bezüglich der Restklassendarstellung $R = k\langle\langle X_1..X_n\rangle\rangle \twoheadrightarrow A$ der Codimension m kann ω_A als Modul, dessen Elemente gewisse "Residuensymbole" $\left[\begin{smallmatrix} x^* \\ F_1,..,F_m \end{smallmatrix}\right]$, $x \in \Omega_R^{\cdot+m}$, $F_1..F_m \in \mathrm{Ker}(R\to A)$ maximale R-Sequenz, sind. Es gibt einen Homomorphismus $c_A : \Omega_A \to \omega_A$, der bei vollständigem Durchschnitt $A = R/(F_1..F_m)$ durch $\xi \mapsto \left[\begin{smallmatrix} \xi \wedge dF_1 .. dF_m \\ F_1 .. F_m \end{smallmatrix}\right]$.

Mit Hilfe von Residuensymbolen können einige Aussagen ~~[illegible]~~ bei vollständigem Durchschnitt mit isolierter Singularität gemacht werden, nämlich (1) $\mathrm{Der}_k(A)$ wird von der Eulerderivation und den trivialen determinantiellen Derivationen erzeugt, (2) Die De Rham-Kohomologie $H_{DR}(\omega) = k \oplus (\omega_A^{g-1})_0 \oplus (\omega_A^{g})_0$.

Growth of Bass numbers and of Betti numbers of local rings

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The Betti numbers $b_i = \dim_k \mathrm{Tor}_i^R(k,k)$ and the Bass numbers $\mu_i = \dim_k \mathrm{Ext}_R^i(k,R)$ of a local ring (R,m,k) can be used to characterize respectively regular rings and complete intersections (c.i.), and Gorenstein rings. Denoting by $\ll$ coefficientwise inequalities of formal power series, it is known that

$$(1+t)^{\mathrm{edim} R}/(1-t^2)^{\mathrm{edim} R - \dim R} \ll P_R(t) := \sum_i b_i t^i \ll (1+t)^{\mathrm{edim} R} \Big/ 1 - \sum_{i=1}^{\mathrm{edim} R} c_i t^{i+1}$$

In particular, the sequence $\{b_i\}_{i\geq 0}$ is non decreasing for $i > \mathrm{edim}\, R$, and its growth is at least polynomial and at most exponential. We prove now other rate of growth can occur:

<u>Theorem.</u> Either $\{b_i\}$ grow polynomially, which happens if and only if R is a c.i., or they grow exponentially, i.e. there exist an $N>0$ and real integers $C, D > 1$ such that for $i > N$: $C^i < b_i \leq D^i$.

This had been conjectured by the author, and partial results had been reported at an Oberwolfach Tagung in 1979. Since then Y. Félix and J.-C. Thomas had proved the claim for localizations at the irrelevant ideal of graded rings over a field of char. 0. A corollary is the proof of the following conjecture of Golod and Gulliksen: denoting by r_R the radius of convergence of $P_R(t)$, the following possibilities occur: (1) $r_R = \infty \iff R$ is regular; (2) $r_R = 1$ if and only if R is c.i. (3) $0 < r_R < 1$ in all remaining cases.

The proof of the theorem among other things uses ideas from the work of Félix-Halperin-Thomas in rational homotopy theory. To make them applicable we construct a theory of "minimal models" for DG algebras with divided powers, which in particular makes it possible to compute with the Lie algebra of elements dual to the indecomposable elements of $\mathrm{Tor}^{-}(k,k)$. It turns out that a reasonable theory of "homotopy Lie algebras" for DG Γ algebras can be constructed. Parallels with considerations in algebraic topology are valuable for proving algebraic results. Conversely, algebraic techniques can be used to prove results in topology, like the following (which depends on a local result of G. Levin and the speaker):

Theorem Let M be an n-dimensional compact, simply connected formal manifold (e.g. a Kähler manifold, by a result of Deligne-Griffith-Morgan-Sullivan). Let $D^n \subset M$ be a small open disc, and $\bar{M} = M - D^n$. Then one has the isomorphism of graded Lie algebras

$$\pi_*(\Omega \bar{M})/(\bar{\omega}) \xrightarrow{\sim} \pi_*(\Omega M) \otimes \mathbb{Q}$$

where $\bar{\omega}$ is an (explicitely given) element of degree $n-2$.

Using this the rational homotopy groups of several manifolds can be explicitely computed (including the example of complete intersections of dim ≥ 2, in $\mathbb{P}^m$, which has been established directly by Niessendorfer).

Two remarks on flatness and tangential flatness.

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This is a report of recent work of mine with U. Orbanz.

The 1. remark is concerned with (necessary and sufficient) conditions for flatness of a morphism $f: X \to Y$ having Cohen-Macaulay-fibres. Most of these conditions involve (generalized) Hilbert-functions. A stronger property than the flatness of f is the flatness of the induced morphism in the tangent cones. This property may be called tangential flatness. The geometric implications or interpretations of this stronger property seem to be less clear than for ordinary flatness. The 2. remark gives a (necessary and sufficient) condition for a flat morphism $f: X \to Y$ with regular base Y to be tangentially flat. The link between the motivations of these 2 remarks is a recent paper of Shepherd-Barron, in which he gives a numerical criterion for flatness, assuming regular fibres. But the assumption of regular fibres makes a flat morphism tangentially flat. So the results of Shepherd-Barron are special cases of our first remark (and in part they are special cases of a recent preprint of B. Herzog):

Theorem 1: Let $f: (A,M) \to (B,N)$ a local homom. of local rings. Let $I = MB$, $\bar{B} = B/I$, $d = \dim \bar{B}$, for any prime $P \subset B$ with $P \supset I$ we put $\bar{P} = P/I$. Assume that $\bar{B}$ is Cohen-Macaulay. Then the following conditions are equivalent: (i) f flat

(ii) for any s.o.p. $\underline{x}$ for I we have: $H^{(d)}[\underline{x}, I, B] = e(\underline{x}, \bar{B}) H^{(0)}[A]$

(iii) " " : $H^{(0)}[\underline{x}R + I, B] = e(\underline{x}, \bar{B}) H^{(d)}[A]$

(iv) for all $P \in \mathrm{Min}(I)$ we have: $H^{(0)}[IB_P, B_P] = e(\bar{B}_{\bar{P}}) H^{(0)}[A]$

(v) $A \to B_P$ is flat for all $P \in \mathrm{Min}(I)$.

Theorem 2: Same notations. Let $\overline{gr_N B} = gr_N B / gr(f)(gr_M^+(A)) \xrightarrow{\;\gamma\;} gr_{\bar{N}} \bar{B}$ the canonical epimorph. (where $gr(f): gr_M(A) \to gr_N B$). Then if A is regular, the following conditions are equivalent:

(i) $gr(f)$ is flat

(ii) f is flat and γ is an isomorphism.

A more general frame of Thm 2 was indicated

Un théorème de Riemann Roch local (L. Szpiro)

Il s'agit de l'énoncé suivant : (qui n'est démontré à cet instant que pour les anneaux locaux des variétés algébriques par W. Fulton, et pour les anneaux gradué par C. Peskine et votre serviteur)

Soit $L_\cdot$ un complexe parfait sur un anneau local noethérien A, $X = \operatorname{Spec} A$ $Y = \operatorname{Support}(H(L_\cdot))$, $K_\cdot(-)$ le foncteur groupe de Grothendieck des modules de type fini sur — et $A_\cdot(-)$ le groupe de Chow de — tensorisé par $\mathbb{Q}$ alors on a un diagramme commutatif

$$\begin{array}{ccc} K_\cdot(X) & \xrightarrow{\tau_X(\cdot)} & A_\cdot(X) \\ \chi(\cdot)\downarrow & & \downarrow \operatorname{Op}(L_\cdot)\cap(\cdot) \\ K_\cdot(Y) & \xrightarrow{\tau_Y(\cdot)} & A_\cdot(Y) \end{array}$$

R.R.

où τ est l'opérateur de Todd et $\operatorname{Op}(L_\cdot)$ est un opérateur gradué. τ et $\operatorname{Op}(L_\cdot)$ possèdent les propriétés fonctorielles qu'on devine.

On peut après avoir compris (même démontré !) ce théorème se pencher sur mes deux conjectures favorites : Soit $L_\cdot$ un complexe parfait à homologie de longueur finie alors

C.1 $\chi(L_\cdot^\vee) = (-1)^{\dim A}\chi(L_\cdot)$ où $L_\cdot^\vee = \operatorname{Hom}(L_\cdot, A)$

C.2 $\chi(L_\cdot^{(p)}) = p^{\dim A}\chi(L_\cdot)$ quand la caractéristique de A vaut p. et $L_\cdot^{(p)}$ le "Frobéniusé" de $L_\cdot$

En particulier R.R. implique que

$$\chi(L_\cdot^{(p^n)})/p^{nd} \xrightarrow[n\to\infty]{} \text{limite} = \bar{\chi}(L)$$

et C1 et C2 se ramènent à montrer que

$$(*) \quad \bar{\chi}(L) = \chi(L)$$

On peut remarquer que si on connaissait (*) on aurait – grâce à R.R – une démonstration de : $\chi(L.\otimes N) = 0$ dès que $\dim N < \dim A$.

Réf W. Fulton "Intersection theory" to appear Springer-Verlag 2 volumes

C. Peskine L. Szpiro Syzygies et multiplicités CRAS mai 1974

L. Szpiro "Sur la theorie des complexes parfait" à paraitre Proceedings Symposium Commutative Alg. Durham 1981. R.Y. Sharp Editor

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How to make a complex exact: The existence of generic free resolutions and related objects.

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In order to focus all the equational conditions, which are satisfied by the entries of the matrices a^k in a finite free resolution (FFR) $F: 0 \longrightarrow R^{b_n} \xrightarrow{a^n} R^{b_{n-1}} \longrightarrow \dots \longrightarrow R^{b_1} \xrightarrow{a^1} R^{b_0}$

over a commutative ring R, into a single object, Hochster coined the notion of a <u>generic</u> FFR: A pair (R, F) as above is called generic of type $(b_n, \dots, b_0)$ if every such FFR G over a commutative ring A can be obtained from F by an extension $\varphi: R \to A$. Hochster (and Huneke) completely solved the problem for $n \leq 2$ and showed that for these cases there even exist universal FFRs: the extension $\varphi: R \to A$ is always unique. Moreover he conjectured that generic FFRs exist for every possible type $(b_n, \dots, b_0)$ and that the underlying ring can always be taken as a finitely generated $\mathbb{Z}$-algebra.

We prove Hochster's conjecture with "finitely" replaced by "countably". The proof is based on a very simple exactification technique which can also be used to produce generic models for many other types of objects like complexes with certain exactness conditions, periodic free resolutions, perfect resolutions etc. The existence of one object of a given "type" always ensures the existence of a generic model. For certain acyclic complexes the underlying ring of a generic model can not be chosen as a noetherian ring, we believe however that Hochster's conjecture holds for FFRs.

PM-Rings.

Maria Contessa - Università di Roma - ITALY.

A ring A is a <u>pm-ring</u> if every prime ideal is contained in a unique maximal ideal.

Theorem. | A direct product of any family of pm-rings is still a pm-ring.

Two proofs, one topological and one algebraic, are given.

The algebraic one is based on a new characterization of these kind of rings.

A structure theorem for noetherian reduced pm-rings is also done.

Mixed Hodge Structures of an isolated singularity and the purity theorem.

Let $y \in Y$ be an isolated singular point on an analytic germ of variety. Let $p : Y' \to Y$ be a desingularisation of Y and S' the normal crossing divisor (N.C.D) over y. Then.

Proposition: We have an exact sequence of mixed Hodge structure (M.H.S)

$$H^i_y(Y) \xrightarrow{p^* + i^* \circ i_*} H^i_{S'}(Y') \oplus H^i(y) \xrightarrow{i'^* \circ i'_* - p^*} H^i(S') \xrightarrow{\partial} H^{i+1}_y(Y)$$

where $i : y \to Y$ and $i' : S' \to Y'$ are embeddings.

Let $S' = \cup_i S'_i$ be union of smooth irreducible and proper components. Consider the complex ${}_W E_1^{\cdot, q}$: ($S'^{(p)}$ denote $\amalg_{i_0 < \dots < i_p} S'_{i_0}$

$$\to H^{2p+q-2}(S'^{(-p)}) \dashrightarrow H^{q-4}(S'^{(1)}) \longrightarrow H^{q-2}(S'^{(0)}) \xrightarrow{\varphi} H^q(S'^{(0)}) \longrightarrow H^q(S'^{(p-1)}) \dashrightarrow$$

$$E_1^{p,q} (p<0) \qquad E_1^{-1,q} \qquad E_1^{0,q} \qquad E_1^{1,q} \qquad E_1^{p,q} (p>0)$$

(the terms on the left: like for $H^*_{S'}(Y')$; the terms on the right: like for $H^*(S')$)

where to the left the differentials are Gysin morphisms like for the logarithmic complex in [H II] by Deligne Publ. Math. IHES N°40 (1972), and to the right the differentials are restrictions morphisms, dual to those at left. The morphism φ is the dual of the intersection matrix

$$I : \oplus_q H_q(S'_i) \longrightarrow \oplus H_{q-2}(S'_i) \quad \text{defined by } I(a)_i = a \cap_{Y'} S'_i .$$

Proposition: The cohomology of the above complex $E_1^{*,q}$ gives the terms of weight q of $H^{*+q}_y(Y, \mathbb{C})$.

Gabber has proved a purity theorem on Mc Pherson cohomology in car $p>0$.

we refer to notes by Deligne at the IHES. We deduce the following interpretation after discussions with Deligne:

Proposition: Semi-purity: The weights of $H^i_y(Y,\mathbb{C})$ are $\geq i$ for $i > n$, and dually the weights of $H^i_y(Y,\mathbb{C})$ are $< i$ for $i \leq n$.

Corollary: The complex

$$H^{q-2}(S'^{(0)}) \xrightarrow{\varphi} H^q(S'^{(0)}) \longrightarrow \cdots \to H^q(S'^{(p-1)}) \cdots \to H^q(S'^{(n-1)}) \to 0$$

is exact for $q > n$.

A dual statement is true for $q \leq n$.

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Les points doubles rationnels des surfaces

Il s'agit de points singuliers isolés de surface obtenus en passant au quotient dans $\mathbb{C}^2$ par l'action d'un sous-groupe fini G de $SL(2,\mathbb{C})$. La surface S obtenue peut être désingularisée en $\tilde{S} \xrightarrow{q} S$. Le graphe dual du diviseur exceptionnel $\Gamma(G)$ est du type A_n, D_n, E_6, E_7, E_8. Soit $c: G \hookrightarrow SL(2,\mathbb{C})$ la représentation canonique de G. En examinant l'action de la multiplication par c sur l'ensemble des représentations irréductibles non trivial de G, Mac Kay construit un diagramme qui n'est autre que $\Gamma(G)$. Par inspection de ces diagrammes on peut donc associer à toute représentation irréductible ρ de G une composante irréductible du diviseur exceptionnel d_ρ. De plus le cycle $Z = \sum_i rg(\rho)\, d_\rho$ n'est autre que le cycle fondamental de la singularité. Dans un travail commun avec G. Gonzalez nous donnons une description géométrique de cette correspondance.

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— Differential calculus and characteristic classes —

~~We~~ give explicit formulas for the fundamental classes of a Cohen Macaulay subvariety of a given ~~smooth~~ variety, generalizing the formula $\left[\frac{df_1 \ldots df_r}{f_1 \ldots f_r}\right]$ of the complete intersection case. For this we use the following basic construction: if $L_1 \xrightarrow{\varphi} L_0 \to M$ is a presentation of a R-module M, choose bases of L_1, L_0 s.t. the matrix of φ is $A = (a_{ij})$. Then the morphism $d\varphi \in \mathrm{Hom}(L_1, L_0 \otimes \Omega^1_R)$ has an image γ^1_M in $\mathrm{Ext}^1(M, M \otimes \Omega^1_R)$ which is independent of the different choices: it is the Atiyah class of M (which corresponds to the extension of the principal ~~o~~parts of M). Then one shows that $\frac{(\gamma^1_A)^r}{r!}$ is the fundamental class of A in $\mathrm{Ext}^r(A, \Omega^r_R) \simeq \mathrm{Ext}^r_r(A, \Omega^r_R \otimes A)$. One can also relate the ~~local~~ intersection theory of modules with the the evaluation of ~~the~~ classes.

This construction has a global analogue which can be related to the Chern classes of sheaves and which gives explicit formulas for the Chern classes in Čech cohomology. Comparing the local and global theories, one obtains easily the Grothendieck formula relating ~~the~~ chern class of the ring of a subvariety and the fundamental of this subvariety.

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Microgéométrie

. Soient X un espace analytique et $Y \subset X$ un sous espace fermé. Pour tout faisceau F sur X on définit $Sp(F)$: un complexe de faisceaux sur le cône normal de Y dans X, dont la cohomologie est localement constante sur les génératrices épointées du cône. Ce complexe spécialisé redonne lorsque Y est un diviseur de X défini par une équation, le complexe des cycles évanescents de F.

. Soient Y un espace analytique et $E \to Y$ un fibré vectoriel complexe. A tout complexe de faisceaux $F^{\cdot}$ sur E, à cohomologie localement constante sur les génératrices épointées, on peut associer le transformé de Fourier géométrique de $F^{\cdot}$ qui est un complexe de même nature sur le fibré dual.

. Soient X une ~~espace~~ variété analytique ~~etc~~, Y une sous-variété, F un complexe de faisceaux sur X. Le microlocalisé de F est le complexe $\mathcal{F}(SpF)$. C'est un complexe de faisceaux sur le fibré conormal de Y dans X. Lorsqu'on prend pour F le faisceau $\mathcal{O}_X$, on obtient ainsi le faisceau $C^{\mathbb{R}}_{Y|X}$ introduit par Sato, Kawai, Kashiwara.

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Ein Verschwindungssatz für gewisse Kohomologiegruppen

Sei X ein komplexer Raum, $Y \subset X$ ein kompakter komplexer Unterraum, der durch das kohärente Ideal $\mathcal{J} \subset \mathcal{O}_X$ definiert sei. Unter welchen Bedingungen verschwinden die Kohomologiegruppen $H^i(W, \mathcal{J}^k \mathcal{F})$ für große k und kohärentes $\mathcal{F}$, wobei W eine gewisse offene Umgebung von Y in X ist. Ist nun $Y \subset X$ ein lokal vollständiger Durchschnitt und $\mathcal{F}$ ein kohärenter $\mathcal{O}_X$-Modul, derart daß $\mathcal{J}$ lokal von einer regulären $\mathcal{F}$-Sequenz erzeugt ist und ist ferner das Normalenbündel von Y in X ein Bündel vom Typ $\{p,q\}$, so kann man für gewisse $i \in \mathbb{N}$ einen solchen Verschwindungssatz beweisen. Der Beweis benutzt ein Deformationsargument. Es reicht dann aus einen analogen Satz in einer sehr speziellen Situation zu beweisen.

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Konstruktion verseller Deformationen in der analytischen Geometrie
(Bericht über gemeinsame Arbeit mit S. Kosarew)

Es wurden die Grundzüge einer Theorie skizziert, mit der im Prinzip "jedes" analytische Deformationsproblem behandelt werden könnte. Für die meisten bisher bekannten Fälle (Deformationen von kompakten komplexen Räumen, Deformationen von kohärenten Moduln mit kompaktem Träger, Deformationen von isolierten Singularitäten ....) läßt sich das Verfahren bereits jetzt mit Erfolg anwenden.

J. Bingener (Regensburg)

Deformations of cones over flag varieties

Let G be a semi-simple group over $\mathbb{C}$, $\rho : G \to Gl(V)$ an irred. representation in the finite-dim. vectorspace V. If $B \subseteq G$ is a Borel-subgroup of G, there is a line $\ell \subseteq V$, such that $\rho(B)\ell = \ell$. The variety $\overline{\rho(G).\ell} \subseteq V$ is called the minimal cone of ρ. $C(\rho) = \overline{\rho(G).\ell}$ can also be thought of as the cone over the image of $G \xrightarrow{|\ell_\rho|} \mathbb{P}(V)$.

The question is: For which ρ, $C(\rho)$ is rigid?

Using Bott's thm. and results of M. Demazure on the rigidity of the flag-varieties G/B, we give a complete list of the non-rigid cones:

Thm. Corresponding to the root system the only ~~irred~~ non-rigid cones correspond to the characters in $X(T)$: (T a max. torus of G)

A_n $(n \geq 3)$	$\varpi_1 + \varpi_n$, the adjoint represent.
A_3	ϖ_2, $2\varpi_2$, the Plücker-embedding of Grass(2,4) and its second Veronese
B_n $(n \geq 4)$	ϖ_1, $2\varpi_1$, the regular repr. and its second Veronese emb.
C_n, D_n $(n \geq 3)$	"
E_6, E_7, E_8, F_4, G_2	all representations are rigid.

In the non-rigid cases all deformations can be described very easily in terms of the group.

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Ein Satz über endliche Erweiterungen von normalen analytischen Algebren und einige Anwendungen.

Sei k ein bewerteter Körper der Charakteristik Null; ist $A \to B$ eine endliche Erweiterung von normalen analytischen k-Algebren, so läßt sich zeigen, daß der Modul der Zariski-Differentiale auf A ein direkter A-Summand des Moduls der Zariski-Differentiale auf B ist. Als erste Anwendung beweisen wir eine Aussage über analytisch-verzweigte Überlagerungen isolierter Hyperflächensingularitäten. Als zweite Anwendung des obigen Satzes beweisen wir einige Aussagen über solche normale Singularitäten A, die von einem konvergenten Potenzreihenring $B = k\langle x_1, \ldots, x_s \rangle$ überlagert werden, derart, daß der Strukturhomomorphismus $A \to B$ nicht-ausgeartet ist. Es wird gezeigt, daß der Modul der k-Derivationen auf A sowie der Modul der Zariski-Differentiale auf A Macaulay-Moduln sind. Hieraus folgt dann, daß die Kotangentenmoduln $\mathrm{Ext}^i_A(D_k(A), A)$ sowie $\mathrm{Ext}^i_A(D_k(A), \omega_A)$, $\omega_A :=$ kanonischer Modul von A, für $i = 1, \ldots, d-2$ verschwinden, falls der singuläre Ort von A von der Kodimension $d \geq 3$ ist. Insbesondere sind solche Singularitäten A starr, die in der Kodimension 2 regulär sind. Ferner folgt: Ist A nicht regulärer vollständiger Durchschnitt, so ist der singuläre Ort von A rein-2-kodimensional. Eine ähnliche Aussage gilt für fast vollständige Durchschnitte A.

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Equations and Syzygies of projective curves (after Rob Lazarsfeld)

Let $C \subset \mathbb{P}^r_k$ be a reduced and irreducible projective curve, $S = k[x_0, \ldots, x_r]$ the homogeneous coordinate ring of $\mathbb{P}^r_k$, and I_C the homogeneous ideal of the curve. Following Castelnuovo, Mumford, and others, we say that I_C is p-regular if $H^1(\mathbb{P}^r, I_C(p-1)) = 0$ and $H^2(\mathbb{P}^r, I_C(p-2)) = H^1(\mathbb{P}^r, \mathcal{O}_C(p-2)) = 0$. It is not difficult to show that I_C is p-regular if and only if I_C is generated by forms of degree $\leq p$ and, for each l, the l^{th} sysygy of I_C as an S-module is generated by forms of degree $\leq p+k$, or again, if and only if $I_C \cap (x_0, \ldots, x_r)^p$ has a linear free resolution (this circle of ideas is ~~exampl~~ exposed, for example, in a forthcoming paper by S. Goto and the author.)

Theorem (Lazarsfeld) If C as above is contained in no hyperplane, then I_C is $[(\text{degree } C) - r + 2]$-regular.

This result was proved by Castelnuovo for smooth curves in $\mathbb{P}^3$ and by Peskine-Gruson for arbitrary curves in $\mathbb{P}^3$ by different methods. Lazarsfeld's proof is essentially to approximate the resolution of I_C by the Eagon-Northcott complex associated to the presentation matrix of the graded module corresponding to a general line bundle of degree $\deg C - r + g + 1$ on the normalization of C.

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Epimorphism Problems

We discuss partial results about the following question raised by Abhyankar (at least in special cases)

Question (Epimorphism problem). Let $x_1, \cdots, x_n \in k^{[m]}$ = the polynomial ring in m variables over a field k. Let $k[x_1, \cdots, x_n] = k^{[m]} = k[u_1, \cdots, u_m]$. Does there exist an automorphism $\sigma : k[X_1, \cdots, X_n] \to k[X_1, \cdots, X_n]$ (= the polynomial ring in n variables over k) such that

$$\left(\sigma(X_1), \cdots, \sigma(X_n)\right)_{X_i \to x_i} = (u_1, \cdots, u_m, 0, \cdots 0) \, ?$$

There are easy counterexamples if $\text{cha}\cdot k > 0$, namely $x_1 = u_1^{p^2}$, $x_2 = u_1 + u_1^p$ in $n = 2$, $m = 1$ and this can be easily generalized to arbitrary n, m.

So.

Assume $\text{cha}\, k = 0$ and without loss of generality k-algebraically closed.

The following cases are known.

1. $n = 2$, $m = 1$ – Abhyankar-Moh epimorphism theorem.
2. $n = 3$, $m = 2$. Let us rewrite the notation.

$$\varphi : k[X, Y, Z] \to k[u, v] \to 0$$

$$(f(X, Y, Z)) = \ker \varphi. \quad \text{Consider } f \text{ in } k[X, Y][Z]$$

2.1 If f is linear in Z over $k[X, Y]$ then yes.

2.2 If $f = a_0 Z^d + a_1 Z^{d-1} + \cdots + a_d$ and $a_0, \cdots, a_{d-1}$ have a common factor in $k[X, Y]$ then yes. (With Russell)

2.3 If $f = aZ^2 + bZ + c$ and a, b do not meet as curves i.e. $(a, b) = (1)$ then yes. (Student of Russell)

3. $n = 3$, $m = 1$. Notation. $\varphi : k[X, Y, Z] \to k[t] \to 0$ $P = \ker \varphi$.

$$\Delta = \{ r \mid ~~\deg \varphi(h)~~ \deg \varphi(h) = r, \ h \text{ variable in } k[X, Y, Z] \}$$

Object is to show $\Delta = \{0, 1, 2, \cdots\}$.

3·1 It is possible to arrange that $\varphi(\alpha Z+\beta)=t$ where $\alpha,\beta\in k[X,Y]$, $\alpha\in k[X]$. Then
$\deg \varphi(\sqrt{\alpha})+s \in \Delta$ for all $s\geq 0$ where
$\sqrt{\alpha}$ denotes the reduced expression of α.
In particular Δ misses only finitely many numbers if any.

3·2 Moreover one can arrange $\varphi(\alpha Z+f(Y))=t$ for suitable $f(Y)\in k[Y]$ (after automorphism). If $\deg_Y f(Y)\leq 2$ then <u>Yes</u>.

Surface results (2) are discussed or related in Finding and Cancelling Variables, (with P. Russell), J. Alg. 57, No 1, 1979, 151–166.

Other results will be published elsewhere.

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Cotangent functors of curve singularities

Let k be a perfect field. We consider a one-dimensional analytic k-algebra $R\simeq k[[X_1,\dots,X_m]]/I$ which is the residue class ring of an analytic k-algebra S modulo a regular sequence $z_1,\dots,z_n$. Let $A=k[[X]]\subseteq R$ be a noetherian normalization of R such that $Q(R)/Q(A)$ is separabel.

<u>Theorem 1</u>: Suppose S_φ is regular for all primes of height 1, and let $l\geq 0$ be an integer, then

$$(-1)^l\sum_{i=0}^{l}(-1)^i\, l(T_i(R/A,R))\geq(-1)^l\, l(\Sigma(R/A)/R),$$

and $(-1)^l\sum_{i=1}^{l}(-1)^i\, l(T^i(R/A,R))\geq(-1)^{l-1}\, l(\Sigma(R/A)/R)$

where the T_i, T^i denote the cotangent functors, $\mathcal{C}(R/A)$ the complementary module and $l(--)$ the length of a module.

Corollary: If the defining ideal of R is in the linkage class of a complete intersection, then

$$l(\tau\Omega_{R/k}) = l(\operatorname{Coker} c_R) + l(\tau I/I^2),$$

here $c_R : \Omega_{R/k} \to \omega_R$ is the canonical map into the module of regular differentials

Theorem 2: Suppose S_q is a Gorenstein ring for all primes of height 1, and let $l \geq 1$ be an integer, then

$$(-1)^l \sum_{i=1}^{l} (-1)^i l(T^i(R/A, R)) \geq (-1)^{l+l} \sum_{i=0}^{l+l} (-1)^i l(T_i(R/A, R))$$

J. Herzog (joint work with R. Waldi)

Equisingular deformation of Hamburger-Noether-expansion

Let k be an algebraically closed field of arbitrary characteristic. To every pair x, y in $tk[[t]]$, $(x,y) \neq (0,0)$, one can associate an infinite matrix

$$HN(x,y) = \begin{pmatrix} p_i \\ r_i \\ \alpha_i \end{pmatrix}_{1 \leq i < \infty}$$

where $p_i, \tau_i \in \mathbb{N} \cup \{\infty\}$, $\alpha_i \in k$ (cf. Russel, Hamburger-Noether expansion. manu. math (1980). Now let $\mathcal{A}$ be the category of complete local k-algebras with residue field k. Let $f_0 \in k[[X,Y]]$ be irreducible, $B_0 = k[[X,Y]]/(f_0) \subset k[[t]]$. A deformation f of f_0 over A is a powerseries $f \in A[[X,Y]]$ such that $\bar{f} = f_0$ where $\bar{\ }$ means reduction modulo the maximal ideal of A. A pair $x, y \in A[[t]]$ such that $f(x,y) = 0$ is a parametrization of A. Starting from $HN(\bar{x},\bar{y})$ one can define the characteristic sequence $\tau(HN(\bar{x},\bar{y}))$ (cf Russel) which in turn gives a minimal set of generators of the value semigroup Γ_0 of B_0.

An $f \in A[[X,Y]]$, $\bar{f} = f_0$, is called an equisingular deformation of f_0 if f possesses a parametrization x, y for which ~~the~~ the algorithm leading to $HN(\bar{x},\bar{y})$ is working uniquely. Then one has

Prop 1: $\tau(HN(x,y)) = \tau(HN(\bar{x},\bar{y}))$.

In case of $ch(k) = 0$ this gives the same definition as Wahl (Puiseux expansions and deformations).

One can prove

Theorem 1. Let f be an equisingular deformation of f_0 over A, x, y a parametrization of f which admits a HN-expansion. Then

$\Gamma := \{ m \mid m \in \mathbb{N},$ there exists $g \in A[[x,y]]$

such that $m = o(g) = \Phi(\bar{g})\} = \Gamma_0$.

Corollary. The value-semigroup Γ is independent of the parametrisation.

Now let $Y = \operatorname{Spec} A$, X a scheme over Y and $s : Y \to X$ a section for the structure morphism π. The quadruple $\mathfrak{F} = (X, Y, \pi, s)$ is called a plane branch if there exist $f \in A[[x,y]]$ such that $\bar{f}$ is irreducible and there is ι such that

$$\begin{array}{ccc} X & \xrightarrow[\iota]{\sim} & \operatorname{Spec}(A[[x,y]]/(f)) \\ & \searrow \quad \swarrow & \\ & Y & \end{array}$$

commutes. $\mathfrak{F}$ is called Γ-equisingular if Γ is the value semigroup of $k[[x,y]]/(\bar{f})$ and f is an equisingular deformation of $\bar{f}$.

Theorem 2. Let $\mathfrak{F}, \mathfrak{G}$ be two Γ-equisingular families. There exists $\nu_0 = \nu_0(\Gamma)$ such that an isomorphism

$$\mathfrak{F}_\nu \to \mathfrak{G}_\nu$$

induces an isomorphism

$$\mathfrak{F} \to \mathfrak{G}$$

(Here $\mathfrak{F}_\nu, \mathfrak{G}_\nu$ are the families defined by $I^{\nu+1}$ where I is the ideal defining $s(Y)$)

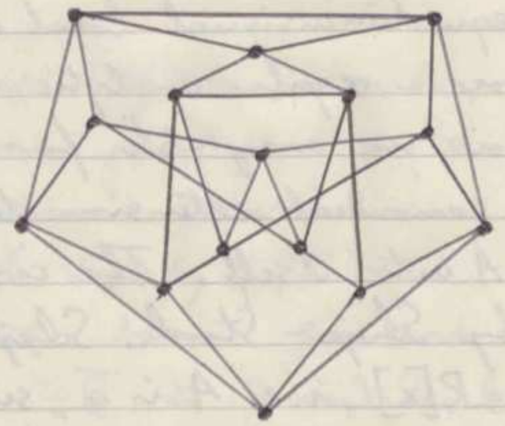
K. Kiyek

A generalization of the Zariski discriminant criterion and the extension of derivations on analytic algebras

If A is a reduced equidimensional local analytic algebra over $\mathbb{C}$, which is finite over a regular analytic algebra R over $\mathbb{C}$ of the same dimension, one may ask for a given k-derivation $\delta \in Der_k(R)$ if its canonical extension to the total quotient ring of A maps A into itself. The case of a normal A has been settled by Scheja - Storch. Scheja - Regel have investigated the case $A = R[x]$, i.e. A is a „simple extension" of R. On the other hand the classical Zariski discriminant criterion in the nonnormal case $A = R[x]$ suggests that one should look for conditions on δ to map the relative Lipschitz - saturation $\tilde{A}^R$ of A with respect to R into itself (instead of A as above). Such conditions are derived in two cases which are considered separately: Firstly one looks for the most general conditions on A for the Zariski criterion to hold in essentially the original form (δ- stability of the discriminant locus of A over R). Secondly the general case of an arbitrary A is considered and it turns out that not only the discriminant locus of A over R has to be δ- stable but also (eventually) some embedded prime ideals.

E. Böger

GRAPHENTHEORIE



30.5.82 — 5.6.82

Clique-reguläre Graphen mit verbotenen Kreisen

Def: Seien $3 \leq k, l \in \mathbb{N} \ni \lambda$ und $G = (\underset{\cdot}{G}, \underline{G})$ sei ein endlicher, schlichter Graph mit $x \in \underset{\cdot}{G}$. Es gelte:

1.) Jede Clique von G ist ein K_k.

2.) Jede Kante von G ist in genau einer Clique von G enthalten.

3.) Der Clique-Graph von G enthält keinen C_i für $3 \leq i \leq l$.

Dann bedeute:

$\mathfrak{G}_k^{(l)} := \{ G \mid G \text{ erfüllt } 1.), 2.) \text{ und } 3.) \}$

$n_G := |\underset{\cdot}{G}|$

$k_G := |\{ Y \mid Y \text{ ist Clique in } G \}|$

$\gamma(x) := |\{ Y \mid Y \text{ ist Clique in } G \wedge x \in Y \}|$

$k_\lambda^{(l)} := \min \{ k_G \mid G \in \mathfrak{G}_k^{(l)} \wedge \bigwedge_{x \in G} \gamma(x) = \lambda \}$

Satz 1 : $\bigwedge_{3 \leq k \in \mathbb{N}} \bigwedge_{\varepsilon > 0} \bigvee_{G \in \mathfrak{G}_k^{(5)}} \frac{n_G}{k_G} < \varepsilon$.

Satz 2 : $k_\lambda^{(4)} \leq \lambda \cdot k^{k(\lambda - 1)}$.

Vermutung : $3_3^{(4)} = 55$.

Egmont Köhler, Hamburg

Monochromatic subgraphs in edge-coloured complete graphs.

Given an arbitrary edge-colouring of a complete graph, one can ask the question: how large a monochromatic complete subgraph can always be found? Attempts at answering such questions form the subject matter of the partition calculus. The lecture will deal with some results of this general nature.

Richard Rado, 1-6-1982.

Eine Variation des Satzes von Menger für Wege der Länge ≥ 3.

Sei G ein schlichter Graph und $S \subseteq V(G)$, dann sei $\langle u, S, v \rangle_G^n$ gleichbedeutend mit: Jeder u-v-Weg in G der Länge $\geq n$ trifft.

L. Montejano und V. Neumann-Lara zeigten in ihrer noch nicht veröffentlichten Arbeit "A variation of Mengers theorem for long paths" folgenden Satz:

" Sei $n \geq 2$. Falls aus $\langle u, S, v \rangle_G^n$ $|S| \geq h$ folgt, so existieren $\left\{\frac{h}{3n-5}\right\}$ offendisjunkte u-v-Wege der Länge $\geq n$ in G."

Es werden Graphen mit t Wegen der Länge $\geq n$ angegeben, für die $|S| \geq (n-1)t$ gilt, und für den Fall $n=3$ wird gezeigt, daß sogar $\left\{\frac{h}{3}\right\}$ offendisjunkte u-v-Wege der Länge ≥ 3 existieren.

Michael Hager, Berlin

Two classical problems involved by the construction of triangular imbeddings

(A. Bouchet, Le Mans)

Soit un graphe G et un entier $m > 1$. Notons $G_{(m)}$ le graphe obtenu en remplaçant chaque sommet x de G par m sommets indépendants $x_1, x_2, \ldots, x_m$ et en définissant une arête $[x_i, y_j]$ si, et seulement si, $[x, y]$ est une arête de G.

Supposons connue une immersion triangulaire de G dans une surface S. Notons K le complexe simplicial défini par cette immersion. Nous désirons construire, si cela est possible, un complexe simplicial $\tilde{\mathbb{K}}$ attaché à une immersion triangulaire de $G_{(m)}$ dans une surface $\tilde{S}$ qui a la même caractéristique d'orientabilité que S'. Nous obtiendrons ainsi une formule de genre minimum pour $G_{(m)}$.

Nous nous intéressons plus particulièrement à la construction de $\tilde{\mathbb{K}}$ comme revêtement de $\mathbb{K}$ ramifié en chaque sommet de $\mathbb{K}$. Cela est possible lorsque 2, 3 et 5 ne divisent pas m (à paraître dans J. Combinat. Theory, Series B). Si m est divisible par 5, la construction sera possible si la conjecture de Tutte sur les 5-flots est démontrée.

Supposons maintenant que l'espace de $\mathbb{K}$ est la sphère, $m = 3$ et $\mathbb{K}^1$ est distinct du graphe complet à 4 sommets. La possibilité de la construction équivaut à l'existence d'un coloriage des sommets de $\mathbb{K}^1$ avec 4 couleurs (résultat obtenu le mois dernier avec D. Bénard et J.L. Fouquet). Ainsi il suffit d'appliquer le théorème des 4 couleurs.

<u>Conjecture</u>. — La construction est toujours possible pour $m = 3$, à part un nombre fini de cas.

A. Bouchet

<u>Eine nützliche Verwendung von Graphen in der Algebra</u>

Dies ist ein Bericht über neue Arbeit von DOV TAMARI.

Es sei B eine endliche Menge T zusammen mit einer Abbildung aus $T \times T$ in T. Mann nenne B <u>associativ</u> falls eine Halbgruppe eksistiert in welche B eingebettet ist.

Mit Hilfe der Halbgruppe $S(B) = F_{|B|}/\mathcal{E}_B$ sieht man leicht dass B nicht associativ $\Longleftrightarrow \exists\, p, q \in T,\ p \neq q$: p kann mit Hilfe der Abbildungen von B in q überführt werden.

Jeder solcher Überführung von p in q entspricht eine genau definierte schrittweise Konstruktion eines endlichen 3-regulären Graphen in der Ebene. Auch gibt es dann immer eine Überführung von p in q dessen Graph 3-zusammenhängend ist.

Somit kann nicht-Associativität (falls vorhanden) durch systematische Untersuchung der endlichen 3-regulären 3-zusammenhängenden Graphen in der Ebene immer durch ein endliches Verfahren nachgewiesen werden.

Es ist aber nicht möglich für jedes endliche B eine Zahl n_B zu finden mit folgender Eigenschaft: wenn keins der 3-regulären 3-zusammenhängenden ebenen Graphen mit $\leq n_B$ Punkten anzeigt dass B nicht associativ ist, dann ist B associativ. Das Problem, ein rekursives Verfahren zu finden das für jedes B entscheidet ob B associativ ist oder nicht, ist nämlich äquivalent mit dem Wort Problem für Halbgruppen.

G. A. Dirac

Eine Verwandtschaft zwischen gewissen Multigraphen und den Tschebyscheffschen Polynomen

Zwei Familien von Multigraphen werden folgenderweise definiert. Die Ecken sind die Paare von ganzen Zahlen (j,k), wo $j \geq 0$ und entweder $0 \leq k \leq j$ (einseitiger Fall) oder $-j \leq k \leq j$ (zweiseitiger Fall). In beiden Fällen gehen im allgemeinen bis an die Ecke (j,k) $a+b+1$ Bögen, zwar einer aus der Ecke $(j-1,k-1)$, a aus der Ecke $(j-1,k)$ und b aus der Ecke $(j-1,k+1)$. Man nimmt sich vor, die Anzahl $B(j,k)$ der Wege von $(0,0)$ bis zu (j,k) zu berechnen. Es kann dann folgenderweise verfahren werden. Man startet vom Tschebyscheffschen Polynom erster Art $T_i(x)$ oder zweiter Art $U_i(x)$, je nachdem man sich im zwei- oder einseitigem Falle befindet. Man ersetzt x durch $(x-a)/2\sqrt{b}$, multipliziert durch den Faktor $b^{i/2}$ und bezeichnet den Koeffizient von x^j mit $A(i,j)$. Dann gilt, mit $i<k$, folgender Satz: $\sum_{j=i}^{k} A(i,j)B(j,k) = 0$. Verschiedene Sonderfälle, besonders im einseitigem Fall, ergeben wohlbekannte Zahlreihen als $B(j,0)$: z. B. Catalan, Motzkin, Schröder, usw.

G. Kreweras, Paris VI

Die Jordan'sche Normalform der Adjazenzmatrix spezieller gerichteter Graphen

Sind G_1, G_2 gerichtete Graphen gleicher Knotenzahl n und $A(G_1)$, $A(G_2)$ ihre Adjazenzmatrizen, so sollen G_1 und G_2 äquivalent heißen genau dann, wenn $A(G_1)$ und $A(G_2)$ die gleiche Jordan'sche Normalform $\mathfrak{F}$ besitzen.
Die Ähnlichkeitsklasse $\mathfrak{A}$, in der $A(G_1)$ und $A(G_2)$ liegen, wird durch $\mathfrak{F}$ (d.h. durch die Serie der entsprechenden Elementarteiler) eindeutig beschrieben.
Für die Klasse der gerichteten Bäume kann nun analog ein System graphentheoretischer Invarianten angegeben werden, die die Klasse $\mathfrak{K}$ mit $G_1, G_2 \in \mathfrak{K}$ eindeutig beschreiben; insbesondere gibt es in $\mathfrak{K}$ ein Graphen (es ist der mit minimaler Kantenanzahl), der die Klasse $\mathfrak{K}$ repräsentiert.
Dieser Graph besitzt als Adjazenzmatrix gerade $\mathfrak{F}$.

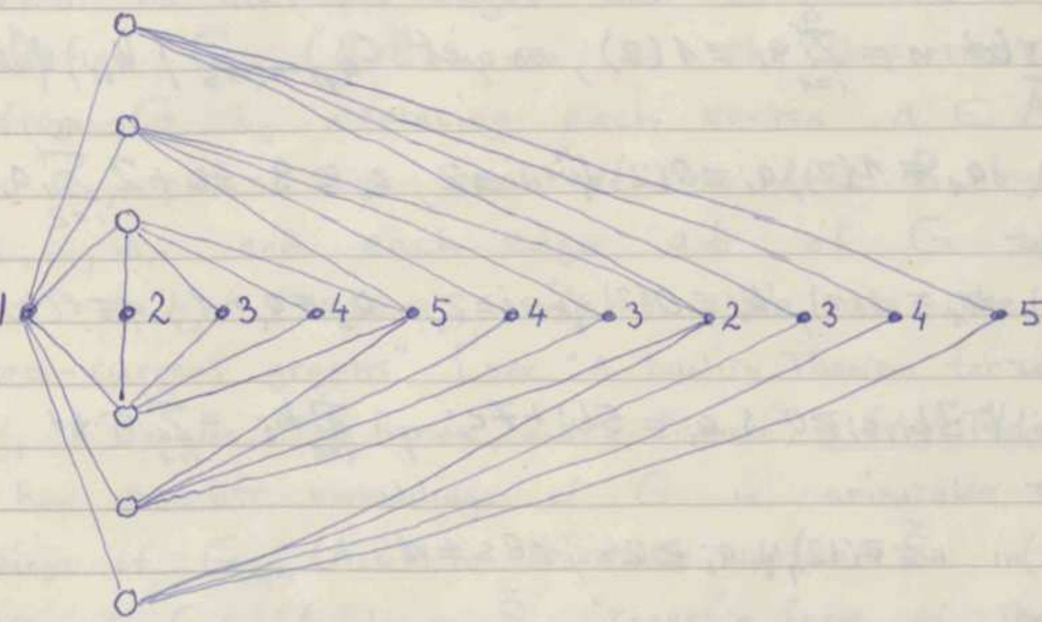
Peter Hennig

Die Spaltungszahl des vollständigen paaren Graphen

Wenn man aus einem Graphen H einen neuen Graphen G konstruiert, indem man zwei nicht benachbarte Ecken identifiziert, so nennen wir diese Operation eine Ecken-Identifizierung. Die Umkehrung heißt Ecken-Spaltung. Es sei S eine geschlossene Fläche. Die Spaltungszahl (splitting number) $sp_S(G)$ eines Graphen G bezüglich S ist die kleinste Zahl von Ecken-Spaltungen, die nötig sind, um G in einen in S einbettbaren Graphen zu transformieren. Es gilt

$$sp_S(K_{m,n}) = \max\left(\left\lceil \frac{(m-2)(n-2)}{2} \right\rceil - 2 + E(S),\ 0 \right).$$

Hierin ist $K_{m,n}$ der vollständige paare Graph mit m und n beliebig und $E(S)$ die Euler'sche Charakteristik der Fläche S. Die Zeichnung illustriert das Beispiel $K_{6,5}$ für S = Ebene. Ecken mit gleicher Nummer



sind zu identifizieren. Wenn S die Ebene und G der vollständige Graph K_m ist, so ist die Spaltungszahl auch bekannt für $m \leq 32$ und für mindestens eine Restklasse $m \pmod 3$. Diese Ergebnisse für $K_{m,n}$ und K_m sind in Zusammenarbeit mit B. JACKSON erzielt worden.

1. Juni 1982

Gerhard Ringel
Santa Cruz, Kalifornien

Bemerkungen zum Oberwolfacher Problem

Ein Graph F "teile" einen Graphen G, wenn es eine Zerlegung von G in zu F isomorphe Faktoren gibt. Die Frage nach den 2-regulären Teilern des vollständigen Graphen ist als "Oberwolfacher Problem" (Ringel, 1967) bekannt.

Bezeichnen wir mit $C_{a_1,a_2,\dots,a_s}$ den aus Kreisen C_{a_i} der Länge $a_i \geq 3$

bestehenden Graphen, so stützen wir die Vermutung, daß die „trivialen" notwendigen Bedingungen, nämlich $n=\sum_{i=1}^{s} a_i \equiv 1(2)$, für $C_{a_1,\dots,a_s} \mid K_n$ mit Ausnahme zweier Fälle $C_{4,5} \nmid K_9$, $C_{3,3,5} \nmid K_1$ auch hinreichend sind.

<u>Satz I</u> Ist $n = \sum_{i=1}^{s} a_i \equiv 1(2)$, so gilt $C_{a_1,\dots,a_s} \mid K_n$, falls

(i) $a_1 \equiv 1(2),\ a_i \equiv 0(2)$ für $i \geq 2$, $a_1 \geq 9-4s+2\sum_{i=2}^{s} a_i$, $n \equiv 1(4)$

oder

(ii) $a_1 \equiv 1(2),\ a_i \equiv 0(2)$ für $i \geq 2$, $a_2 = a_1+1$, $n \equiv 1(4)$

oder

(iii) $\forall i: \exists b_i, c_i \geq 0 : a_i = 5b_i + 7c_i$; $\sum_{i=1}^{s} b_i = \sum_{i=1}^{s} c_i + 1$

oder

(iv) $n \equiv 7(12), \forall i: a_i \geq 24$, $48 s \leq n$

oder

(v) $\exists\, t \geq 4$, $n \equiv 1(t)$, $\frac{n-1}{t} \equiv 1(2), \forall i: a_i \geq 2t^2$ ist.

Der Beweis ergibt sich aufgrund der Transitivität von „$\mid$" nach Bestimmung der 2-regulären Teiler gewisser 2r-regulärer Hilfsgraphen.

Als Nebenergebnis haben wir

<u>Satz II</u> (i) $C_{a_1,\dots,a_s} \mid K_{n,n} \Rightarrow 2n = \sum_{i=1}^{s} a_i$, $n \equiv 0(2)$, alle $a_i \equiv 0(2) :\Leftrightarrow (T)$.

(ii) (T) ist hinreichend für $C_{a_1,\dots,a_s}$ falls

α) $n \equiv 0(4)$ oder

β) $n \equiv 2(4)$ und $\# a_i \equiv 2(4)$ kleiner gleich $2\lfloor \frac{n}{8} \rfloor$ ist.

(iii) $C_{6,6} \nmid K_{6,6}$.

Wolf Piotrowski

Excess-current graphs and embeddings of bipartite graphs in orientable surfaces

Let G be a finite connected bipartite graph with no multiple edges and with the bipartition A, B. Define $G_{m,n}(A,B)$ to be the (bipartite) graph obtained from G by replacing each vertex $a \in A$ by m copies $a_1, \dots, a_m$ and each vertex $b \in B$ by n copies $b_1, \dots, b_n$ and each edge ab of G by the mn edges $a_i b_j$ $(1 \leq i \leq m,\ 1 \leq j \leq n)$. We use the method of "excess-current graphs" [see "A Duality Theorem for Graph Embeddings", B. Jackson, T.D. Parsons, and T. Pisanski, J. Graph Theory 5 (1981)] to show how to lift embeddings of G in orientable surfaces S to embeddings of $G_{m,n}(A,B)$ in orientable surfaces $\tilde{S}$ in such that every face of $G_{m,n}(A,B)$ in $\tilde{S}$ "covers" a face of the same size (of G in S) —— such liftings are henceforth called facelifts.

An $\mathcal{F}$-factor of a graph is a spanning subgraph each component of which belongs to the family $\mathcal{F}$.

Theorem 1: Every embedding of G in an orientable surface S facelifts to an embedding of $G_{m,n}(A,B)$ in some orientable surface $\tilde{S}$ in each of the following cases:

(1) G has a $\{P_4\}$-factor

(2) m and n are both odd and G has a $\{P_k \mid k \geq 4\}$-factor

(3) $G = [H(A',B')]_{2,2}$ for some bipartite H with a 1-factor

Theorem 2: $G_{m,n}[A,B]$ has a quadrilateral embedding in $\tilde{S}$ if

(1) m and n are both even

(2) G has a $\{P_4\}$-factor and G has an (orientable) quadrilat embedding

(3) m and n are both odd, G has an (orientable) quadrilateral embedding and G has a $\{P_k \mid k \geq 4\}$-factor.

This work is done jointly with Mohammed Abu-Sbeih.

T. D. Parsons
USA

The edge-reconstruction problem for infinite graphs

C. Thomassen was the first who gave examples showing that the infinite version of the edge-reconstruction conjecture is false. We improved this result by giving simpler examples showing the same. Furthermore, we proved the following. Let G and H be two infinite graphs that have (up to isomorphism) the same family of edge-deleted subgraphs. Then

(1) G and H have the same number of components,
(2) G and H have the same degree-sequence,
(3) $G \cong H$ in case that G is a locally finite tree containing no subdivision of the regular tree of degree three,
(4) $G \cong H$ in case that G is locally finite and almost r-regular, i.e., there are at most a finite number of vertices of degree $\neq r$ (for $r \in \mathbb{N} \cup \{0\}$),
(5) $G \cong H$ if G is a locally finite graph with a finite number of components which possesses at least two non-stable components. (A nonempty connected graph A is called stable if for each $e \in E(G)$ there exists a component A' of A − e with $A' \cong A$.)

All these results have finite analogs. Note that (5) generalizes the well-know theorem that each finite graph with more than 3 edges is edge-reconstructible in case that it contains at least two non-trivial components.

Thomas Andreae

Unendliche lokalendliche hypohamiltonsche Graphen

Ein unendlicher Graph G heißt hamiltonsch, wenn er einen zweiseitig unendlichen Weg besitzt, der alle Ecken von G enthält. G ist hypohamiltonsch, wenn G nicht hamiltonsch, aber G-v für jede Ecke v ∈ G hamiltonsch ist. G ist lokalendlich, wenn jede Ecke endlichen Grad hat. Thomassen untersuchte unendliche hypohamiltonsche Graphen und stellte dabei die Frage nach der Existenz lokalendlicher solcher Graphen. Ich beantworte diese Frage positiv.

Monika M. Schmidt

Existence of graphs with given set of r-neighborhoods

The problem of whether there exists a graph satisfying a particular set of local constraints can often be reduced to a finite set of questions of the following sort: given a finite set Φ of finite rooted graphs, is there a graph G such that the set of r-neighborhoods of vertices of G is precisely Φ?

We show that this kind of question, though in general recursively unsolvable, becomes solvable when a bound is imposed on the lengths of cycles in G. The result continues to hold when G is allowed (or required) to be infinite, connected, or both; in fact in the infinite cases the cycle restriction can often be dropped.

The theorems hold for graphs "decorated" in various ways, and a colored-edge version is used to demonstrate the solvability of a problem in the theory of hypergraphs involving degree-sets of k-trees.

Peter Winkler
Emory U.
Atlanta, GA U.S.A.

Satz von Kuratowski und eine ähnliche Fragestellung

$M(\Gamma, >)$ bezeichne die Menge aller bezüglich $>$ minimalen Graphen von Γ, $\overline{\Gamma}_0$ die Menge aller nicht planaren Graphen, $>_{\ddot{u}}$ die Unterteilungsrelation. Dann gibt es zwei Formulierungen des Satzes von Kuratowski:
I. $G \in \overline{\Gamma}_0 \Leftrightarrow (\exists H \in M(\overline{\Gamma}_0, >_{\ddot{u}})$ mit $G >_{\ddot{u}} H)$, II. $G \in \overline{\Gamma}_0 \Leftrightarrow (\exists H \in M(\overline{\Gamma}_0, \supseteq)$ mit $G \supseteq H)$.
Von Ringel stammt die interessante Frage, ob man die Menge $\overline{\Gamma}_1$ aller nicht 1-planaren Graphen in ähnlicher Weise (wie beim Satz von Kuratowski) durch bestimmte „minimale" Graphen charakterisieren kann, wobei ein Graph genau dann 1-planar heißt, wenn er sich in der Ebene so zeichnen läßt, daß jede Kante höchstens eine andere Kante trifft. Hauptergebnis ist: (1) für $\overline{\Gamma}_1$ eine „Aufspaltung" des Satzes von Kuratowski in dem Sinne, daß (I) nicht gilt und (nur) (II) gilt und (2) daß $M(\overline{\Gamma}_1, \supseteq)$ und auch $M(\overline{\Gamma}_1, >_{\ddot{u}})$ unendlich sind. Zu (1) gilt der allgemeine Satz: $G \in \overline{\Gamma} \Leftrightarrow (\exists H \in M(\overline{\Gamma}, >)$ mit $G > H)$ gilt genau dann, wenn die Graphenmenge Γ und die Relation $>$ die beiden Bedingungen erfüllen: 1. $(G \in \Gamma \wedge G > G') \Rightarrow G' \in \Gamma$ (beachte: $\overline{\Gamma}$ bedeutet die Menge aller nicht in Γ enthaltenen Graphen) und 2. jede unendliche „Kette" von Graphen $G_1 > G_2 > \cdots > G_n > \cdots$ bricht ab, d.h. von einem N ab sind alle G_n, $n \geq N$ miteinander isomorph. Daß Γ_1 nicht die 1. Bedingung hier erfüllt folgt leicht aus der Tatsache, daß aus jedem Graphen stets ein 1-planarer Graph entsteht, wenn man seine Kanten nur hinreichend oft unterteilt. Das Beispiel der 1-planaren Graphen zeigt also (im Gegensatz zu den planaren Graphen), daß die Charakterisierung einer Graphenmenge im Sinne von (I) schon von vornherein unmöglich und im Sinne von (II) möglich, aber nur mittels unendlich vieler Graphen möglich sein kann. Klaus Wagner, Wodanstr. 57, 5000 Köln 91

A characterization of conference graphs

A finite graph G is said to have property $P^{\geq t}_{m,n}$ (resp. $P^{t}_{m,n}$) if G has at least $m+n+t$ vertices and if, for any sequence of $m+n$ distinct vertices of G, there are at least t (resp exactly t) other vertices adjacent to the first m and non adjacent to the last n vertices of the sequence

It is known (Exoo, 1981) that, given m, n and t, almost all graphs have property $P_{m,n}^{\geq t}$.

Obviously G has property $P_{m,n}^{t}$ iff $\overline{G}$ has property $P_{n,m}^{t}$, so that we may assume $m \geq n$. G has property $P_{1,0}^{t}$ iff G is regular of degree t. G has property $P_{2,0}^{1}$ iff G is a friendship graph (Erdös, Renyi and Sós, 1967). G has property $P_{2,0}^{t}$ $(t \geq 2)$ iff G is a strongly regular graph with parameters $\lambda = \mu = t$ (Bose and Shrikhande, 1970). G has property $P_{m,0}^{t}$ $(m \geq 3)$ iff $G = K_{m+t}$ (Carstens and Kruse, 1977) Theorem: G has property $P_{m,n}^{t}$ $(m, n \geq 1)$ iff $m = n = 1$ and G is a conference graph (i.e. a strongly regular graph with parameters $v = 4t+1$, $k = 2t$, $\lambda = t-1$, $\mu = t$)

J. Doyen (Brussels)

Retracts of hypercubes

We consider loopless undirected graphs without multiple edges. A hypercube is a weak cartesian power of the complete graph K_2, that is, the covering graph of the lattice of all finite subsets of some set. An induced subgraph H of a graph G is a retract of G if there is an edge-preserving map f from G onto H such that $f|_H$ is the identity map on H. R. Nowakowski & I. Rival and D. Duffus & I. Rival have shown that amongst the retracts of hypercubes are all trees and the covering graphs of finite distributive lattices. Now, more generally, a median graph G is a connected graph such that for any three vertices $u, v,$ and w, there exists a unique vertex x which lies simultaneously on some shortest (u,v)-, (v,w)-, and (w,u)-paths.

THEOREM: A graph is median if and only if it is the retract of some hypercube.

This result suggests the following classification scheme for graphs: a variety is any class of graphs closed under the formation of retracts and (weak cartesian) products. Then by the Theorem the class of median graphs is just the variety generated by K_2.

Hans-J. Bandelt
Universität Oldenburg

Imbeddings of Cayley graphs and the genus of a group

Let A be a finite group and let $\gamma(A)$ (respectively $\sigma(A)$) be the smallest genus of any surface containing an imbedded Cayley graph for A (respectively, on which A acts faithfully). The parameter γ was introduced by A. White in 1972 and the parameter σ by the author in 1981 (JCT to appear), although Burnside's genus of a group is closely related (it demands the action preserve orientation). We consider the relationship between γ and σ. The following are known:

(1) $\gamma(A) \leq \sigma(A)$ for all A and for abelian A, $\gamma(A) << \sigma(A)$

(2) $\gamma(A)=0 \Leftrightarrow \sigma(A)=0$ (Maschke 1896)

(3) $\sigma(A)=1 \Leftrightarrow$ A is a quotient of one of the 17 Euc. space groups

(4) $\gamma(A)=1 \Leftrightarrow$ " " or one of three groups of orders 24, 48, 48 (these three groups have been shown in last month not to be quotients of space groups) (Proulx 1978)

(5) $|A| \leq 168(\sigma(A)-1)$ (Hurwitz 1892)

(6) $|A| \leq 168(\gamma(A)-1)$ (Tucker 1980)

(7) $\gamma(A)=2 \Leftrightarrow A=\langle x,y,z : x^2=y^2=z^2=(xy)^2=(yz)^3=(xz)^8=[y,(xz)^4]=1\rangle$

The Hurwitz-type result (6) suggests that the number of Cayley graphs of given genus $\gamma>1$ is finite (this would be a special case of a conjecture of Babai). An example of Nick Wormald shows there are infinitely many Cayley graphs of genus 2, however our work still seems likely to be able to show that for $\gamma>2$ there are only finitely many Cayley graphs of genus γ.

Tom Tucker
Colgate University (USA)

MATROID INTERSECTIONS IN GRAPH THEORY /by András FRANK/

The matroid intersection problem (i.e. finding a maximum cardinality or, more generally, maximum weight common independent set of two matroids) is well solved from both theoretical and algorithmical point of view.

We show that the following graph-theoretical problems can be formulated with the help of matroid intersections. Thus we have good characterizations and good algorithms for those problems:

1. Find a minimum weight subset of arrows of a digraph to cover all the directed cuts. /Related to Lucchesi-Younger thm/.
2. Find a minimum weight k-strongly connected orientation of an undirected graph /related to Nash-Williams orientation thm/.
3. Extend a digraph by a minimum number of arrows so as to have k arrow disjoint paths from a source to each other nodes.
4. THM. A digraph G can be covered by k branchings /: directed forest with in-degrees ≤ 1) ⟺ G can be covered by k forests and each indegree is ≤ k.

András Frank
Budapest
Research Inst. for Telecommunication

ON WEAKLY ACYCLIC DIGRAPHS

Let $D=(V,A)$ be a digraph, and let $P_{AC}(D) \subseteq \mathbb{R}^A$ denote the convex hull of all incidence vectors of acyclic arc sets $B \subseteq A$. By definition, acyclic arc sets do not contain directed cycles. This implies that for every directed cycle $C \subseteq A$ the inequality $x(C) := \sum_{a \in C} x_a \leq |C|-1$ is valid with respect to $P_{AC}(D)$. Setting

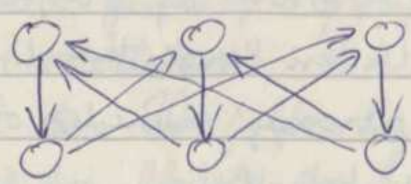
$$P_C(D) := \{x \in \mathbb{R}^A \mid 0 \leq x_a \leq 1 \ \forall a \in A, \ x(C) \leq |C|-1 \ \forall \text{ directed cycles } C \subseteq A\}$$

we conclude that $P_{AC}(D) \subseteq P_C(D)$ for every digraph D.

A digraph $D=(V,A)$ is called weakly acyclic if $P_{AC}(D) = P_C(D)$ holds; otherwise D is called strongly cyclic. Clearly, acyclic digraphs are weakly acyclic; and it follows from the Lucchesi–Younger–Theorem that planar digraphs are weakly acyclic. We present several further classes of such digraphs.

Using the ellipsoid method and a shortest path algorithm (as separation subroutine) we can show that the weighted feedback arc set problem (resp. the acyclic subgraph problem) can be solved in polynomial time. This generalizes a result of Lucchesi for planar digraphs.

A digraph $D=(V,A)$ is called minimally strongly cyclic if D is strongly cyclic and $D-a$ is weakly acyclic for all $a \in A$. We can show that for all $n \geq 6$ there are minimally strongly cyclic digraphs of order n. The smallest such digraph is the following orientation of $K_{3,3}$:



Martin Grötschel
Universität Bonn

A heuristic algorithm for finding a Hamilton cycle in a graph.

In this talk the concept of
HYBRID ALGORITHM

was introduced. A simple renumbering algorithm may be combined with an algorithm like that of Posá. Both algorithms have the property that they may stop without finding a Hamilton cycle in a Hamiltonian graph. By examples it can be shown that the alternate use of both algorithms may overcome impasses. If application of one of both algorithms alone leads to a halt, the other algorithm may take over.

The general problem that was posed is:

P: What are the properties of hybrid algorithms?

It should be remarked that we can, of course, investigate any combination of heuristics. Some of them, like Posá's algorithm, have a non-deterministic flavour in the sense that, in the construction of longer and longer paths, at each step an arbitrary choice is made from the points where one can go to. If ever a polynomial algorithm is to be found (⇒ P = NP), I think it will be a hybrid algorithm with at least one heuristic of this nature.

Cornelis Hoede.
(University of Twente)

A Classification of Reflexive Graphs

A graph variety is a class V of graphs which contains all direct products of members of V and which contains all retracts of members of V. In this way we hope to achieve a classification scheme: order all graph varieties by inclusion. This approach has been used recently by E. M. Jawhari, R. J. Nowakowski, M. Pouzet and I. Rival, especially for those graphs which have a loop at each vertex — reflexive graphs.

Ivan Rival
(Grenoble, France)

On subdivisions of n-connected graphs

L. Lovász and myself independently proved that if G is an n-connected graph, $k_1 \le k_2 \le \ldots \le k_n$ are positive integers with $\sum_{i=1}^{n} k_i = v(G)$ (the number of the vertices of G) then for every sequence of vertices $v_1, \ldots, v_n$ there exists a partition of $V(G)$ into classes $V_1, \ldots, V_n$ such that $|V_i| = k_i$, $v_i \in V_i$ and the induced subgraphs $G(V_i)$ are connected. On the other side, if G has such a partition for any sequence of vertices $v_1, \ldots, v_n$ (the numbers $k_1, \ldots, k_n$ are fixed), then sharp lower bounds are proved for the connectivity number $k(G)$. Sometimes it implies the n-connectivity of G, e.g. if $k_1 \le k_2 \le \ldots \le k_{n-1} \le 2$. The investigations are analogue to ones of Dirac, Mesner and Watkins, Green.

Ervin Győri
(Budapest, Hungary)

Längste Kreise in 3-zusammenhängenden Graphen

In 2-zusammenhängenden Graphen G hat man $n(C_{max}) \geq 2d_{min}$ oder $G - C_{max} = \emptyset$; dabei ist C_{max} ein Kreis maximaler Länge $n(C_{max})$ in G und d_{min} der Minimalgrad in G. Dies ist ein wohlbekanntes Resultat von G.A. Dirac. Für längste Kreise in 3-zusammenhängenden Graphen G kann man zeigen: $n(C_{max}) \geq 3d_{min} - 3$ oder $G - C_{max}$ ist kantenlos. Es werden weiter „lokale" Versionen und Möglichkeiten der Verschärfung bei stärkeren Zusammenhangsforderungen angesprochen.

Heinz Adolf Jung

Strip-These

Ein zusammenhängender unendlicher Graph heißt Strip (Streifen), wenn er einen zusammenhängenden Untergraph Δ (mit Rand $\partial\Delta$) enthält und einen Automorphismus φ hat, sodaß $0 < |\partial\Delta| < \infty$, $\varphi[\Delta \cup \partial\Delta] \subseteq \Delta$, und $\Delta \setminus \varphi[\Delta]$ endlich ist. Einige Eigenschaften von Strips werden vorgestellt. Auch werden hinreichende Bedingungen dafür gegeben, daß ein zusammenhängender unendlicher Graph ein Strip ist. So ein Graph ist genau dann Strip, wenn er lokalfinit ist und einen Automorphismus besitzt mit endlich vielen Orbits. (Gemeinsame Arbeit mit H. A. Jung.)

Mark Watkins
(Syracuse NY, USA)

Handliche Graphe.

Bedeute c eine übliche k-Färbung des Graphen $G(V,E)$, also $c: V \to \{1,2,\ldots k\}$ wobei $c(x) \neq c(y)$ wenn $xy \in E$. Betrachten wir eine weitere Zuordnung $b: V \to \{1,2,\ldots,k\}$, die k-Beschränkung heissen will. Ein Graph G wird handlich k-färbbar genannt, wenn jede k-Beschrenkung von G eine k-Färbung von G zulässt nämlich eine Färbung c, mit $c(v) \neq b(v)$ für alle $v \in V$. Ein Graph G heisst k-handlich wenn er k-chromatisch ist und wenn jede nichtkonstante k-Beschränkung von G eine k-Färbung zulässt.

Satz 1. Für $k=3$ sind die k-handlichen Graphe genau die ungeraden Kreise.

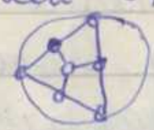
Satz 2 Wenn G_1 und G_2 k_1 bzw. k_2-handlich sind dann ist der Graph $G_1 = G_2$ (alle Ecken von G_1 mit allen Ecken von G_2 verbunden) (k_1+k_2)-handlich, (Dirac konstruktion)

Satz 3 Wenn ein paarer Graph handlich k-färbbar ist, dann ist er ein Teil des Graphen $K_{2,n}$, wobei die zwei Punkte der kleinen Zelle noch mit einem geraden Weg verbunden sind und auf jeder Ecke ein beliebiger Baum noch sitzt. Folgerung: K_k ist handlich für $k \geq 2$.

Satz 4. Wenn G folgende Eigenschaften hat: (i) k-kantenkritisch, (ii) $G-e$ handlich k-färbbar, (iii) $e=xy$ ist so daß eine jede k-Beschränkung und für jede wahl von $a \neq b(x)$ eine k-Färbung zuläßt mit $c(x)=a$; und für jede nichtkonstante Beschränkung kann man a so wählen daß $c_1(x)=c_2(x)=a$, aber $c_1(y) \neq c_2(y)$ — dann kann man zwei Kopien von G benützen und die Hajos konstruktion mit e durchführen und einen k-handlichen graphen bekommen.

Beispiel x  y hat diese Eigenschaften und man kann successive Θ_3 =  bekommen,

Satz 5. K_k besitzt alle verlangte Eigenschaften.

Beispiele von 4-handlichen kritischen Graphen:
K_4, W_5 = , W_{2n+1}, FW_3 = , FW_{2n+1}
Θ_n, Grötzsch Graph.

Anwendungen Bestimmung von k-Färbbarkeit, und (TOFT) k-Kritischsein von Hypergraphen.

J. Schönheim. (D. KELLY u. R. WODROW wirkten mit)

Extremal Problems Involving a Graph and Hypergraphs.

Let G be a finite loopless graph with k vertices. To each vertex i of G associate a set $\mathcal{F}_i$ of subsets of the set $X=\{1,2,\cdots,n\}$. We say the association has property I if:-

$i\neq j$, $F_i\in\mathcal{F}_i$, $F_j\in\mathcal{F}_j$, (i,j) is an edge of G $\Longrightarrow$ $F_i\cap F_j\neq\emptyset$.

Similarly we say the association has property U, ≠, ⊄ if the condition $F_i\cap F_j\neq\emptyset$ above is replaced respectively by $F_i\cup F_j\neq X$, $F_i\neq F_j$, F_i is not a proper subset of F_j. There are 4 properties and hence 16 cases. For each case we bound both $\sum_{1\leq i\leq k}|\mathcal{F}_i|$ and $\min_{1\leq i\leq k}\{|\mathcal{F}_i|\}$. Examples give lower bounds

and theory gives upper bounds. We have G directed or undirected, 2 cases. Sometimes we assume G has only two vertices and one edge. Many, but not all, of the 256 problems are solved.

David E Daykin (with Peter Frankl.)

An application of graph theory in geographical data handling.

A related name is computer aided cartography or geocoding. About 10 years back the problem arose of mapping a "real-world" network into a digital model readable by a computer. Land surveyors read into the computer a SEGMENT FILE and an AREA FILE. We shall show below, that by introducing a suitable algorithm the AREA FILE need not be read into the computer and the cost of encoding the area file is avoided.

The discussion of redundancy in data structures may be formulated as a graph problem:

Given: A graph G which is finite, planar, 2-fold connected, without loops, without multiple edges. The (x,y)-coordinates of each vertex of G is given in a coordinate system of $\mathbb{R}^2$.

Wanted: A list of faces with corresponding facebounding circuits.

Solution: A theorem and an algorithm.

The Theorem: Let G be as above and give to each edge an (arbitrary) orientation. Then: $\forall \vec{e} \in E(G)$ $\exists$ C_1, C_2 two distinct circuits containing e. Traversed in the orientation of $\vec{e}$ one of the circuits will be clock-wise oriented and the other circuit will be counterclockwise oriented.

The algorithm is quite simple.
The above gives rise to a conjecture

Def A system of circuits $C_1, C_2, \dots, C_p$ in a graph G is called a 2-fold circuit partition of G if
(1) $C_i \neq C_j$ for $i \neq j$
(2) $\forall e \in E(G) \; \exists i \neq j : e \in C_i \wedge e \in C_j$.

Conjecture
G is finite and each connected component of G is 2-fold connected, $\neq$ circuit, $\neq$ o–o
$\Downarrow$
G has a 2-fold circuit partition.

Examples: planar graphs, $K_{3,3}$, K_5

Question: Find details about such a partion (e.g. # of ways to do it, # of circuits)

P. D. VESTERGAARD
AALBORG, DENMARK

with Johs. Vibe-Pedersen and Erik Stubkjær

Cops and Robber.

Several people have recently investigated the game of cops and robber. Result 1: The class $\mathcal{C}$ of cop-winning graphs is a variety. Problem: Find the irreducibles. Result 2:

$G \in \mathcal{C}$ iff G can be reduced to K_1 by successively removing the pitfalls (p is a pitfall iff $p \cup N(p) \subseteq N(d)$ for some other vertex d). Let $g(G)$ be the smallest number of cops needed to catch the robber. Result 3: If G has minimal degree $\geq n$ and girth ≥ 5 then $g(G) \geq n$. Examples: Petersen graph, Dodecahedron. Result 4: If G is planar then $g(G) \leq 3$. (joint work with M. Fromme)

M. Aigner
Berlin

Kantenrekonstruktion unendlicher, Weg-endlicher Graphen

Satz: Jeder Weg-endliche Graph mit unendlicher Kantenmenge ist stark Kanten-rekonstruierbar.

Beweisidee: Wesentlich sind die Begriffe Ordnung und Kern eines Weg-endlichen Graphen G. ($o(G)$, Ker G)
$o(G)$ ist eine eindeutig bestimmte Ordinalzahl, die jedem Weg-endlichen Graphen G zugeordnet werden kann, beginnend mit $o(G) := 0 \iff G$ endlich, und dann induktiv fortfahrend. Damit in Zusammenhang steht der Begriff des Kerns von G, der ein eindeutig bestimmter endlicher Teilgraph ist. Ker G ist in fast allen Kanten-verminderten Teilgraphen wiederzuerkennen.
Wir zeigen, daß aus der Annahme, es gebe $G \not\cong H$, aber G schwach Hypomorph zu H, eine weitere Aussage folgt. Von dieser weisen wir durch transfinite Induktion nach der Ordnung nach, daß sie nie erfüllt ist.

R. Schmidt
Hannover

On the number of eulerian orientations of undirected graphs.

If G is a loopless $2k$-regular undirected graph on n points, the number $\varepsilon(G)$ of eulerian orientations of G satisfies:

$$\left(2^{-k}\binom{2k}{k}\right)^n \le \varepsilon(G) \le \left(\sqrt{\binom{2k}{k}}\right)^n, \qquad (*)$$

and these ground numbers are best possible (as functions of k). The number $\varepsilon(G)$ is related to the permanent of a matrix. The upper bound in $(*)$ can be straightforwardly derived from Brègman's upper bound for permanents of $(0,1)$-matrices. The lower bound in $(*)$, however, is better than the one which follows from known lower bounds on permanents.

The methods are similar to those used in counting 1-factors and 1-factorizations in bipartite graphs, where it is conjectured that, if G is a k-regular bipartite graph on $2n$ points, then:

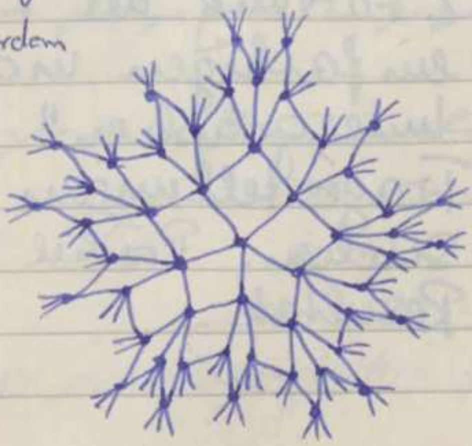
(1) the number of 1-factors in G is at least $\left(\frac{(k-1)^{k-1}}{k^{k-2}}\right)^n$,

(2) the number of 1-factorizations in G is at least $\left(\frac{k!^2}{k^k}\right)^n$

Conjecture (1) is true for $k=1,2,3$ (Voorhoeve), while conjecture (2) is true if $k=2^a.3^b$. Both conjectured lower bounds are best possible.

A. Schrijver
Amsterdam

Mosaikgraphen



Mosaikgraphen sin Teil-graphen von im allgemeinen unendlichen Graphen, die planar sind, die q-regulär

sind und deren Flächen nur p-Ecke sind. Nur für $(p,q) = (3,3), (3,4), (3,5), (4,3), (5,3)$ ergeben sich endliche Graphen, nämlich die der Platonischen Körper. Über kombinatorische Eigenschaften der Mosaikgraphen ist sehr wenig bekannt.
Es wird hier die maximale Anzahl $M(n)$ von Kanten für n Knoten untersucht. Für $(3,6), (4,4), (6,3)$ ist $M(n) = n + \{\frac{1}{2}(n + \sqrt{6n})\}, = 2n + \{2\sqrt{n}\}, = 3n + \{\sqrt{12n-3}\}$, also immer $= \frac{p}{2}n + O(\sqrt{n})$. Im allgemeinen ist eine geschlossene Formel noch nicht gelungen, jedoch gilt $M(n) \approx \frac{p}{2}n + (p-2)n$, so daß die Anzahl von "Randkanten" nur bei den regulären Parkettierungen $O(\sqrt{n})$ und sonst $O(n)$ ist.

Heiko Harborth
Braunschweig

Recent results in Ramsey theory

Die endliche Version des Satzes von Ramsey, nämlich $\forall k\, m\, \exists r\;\; r \longrightarrow (m)^k_2$ hat Verallgemeinerungen auf Vektorraumverbände, Abelsche Gruppenverbände Algebrenverbände etc. Die Aussage dass "zu jedem endlichen Graphen M ein Graph R existiert, welcher folgender Bedingung genügt: Zu jeder 2 Färbung der Kanten von R gibt es einen einfarbigen induzierten zu M isomorphen Untergraphen" gibt Anlass entsprechende Fragestellungen für og. Verbände zu untersuchen. Prömel (1982) bewies entsprechende Resultate.

Als Analog zum Canonization Lemma von Erdős Rado ~~in~~ bewiesen Erdős Graham ein Canonization Lemma für arithmetische Progressionen.

SATZ (Deuber, Graham, Prömel, Voigt) Zu natürlichen Zahlen t, l und zu jeder Färbung der Gitterpunkte in $\mathbb{N}^t$ gibt es einen linearen Unterraum $U \subseteq \mathbb{Q}^t$ und einen ~~Quadrat~~ Würfel W_d

$$W_d = \left\{ (a_1 \cdots a_t) + \sum_{i=1}^{t} \lambda_i (0 \cdots d \cdots 0) \;/\; \lambda_i \in \{0, \cdots l-1\} \right\}$$

so dass in W_d zwei Punkte genau dann gleich gefärbt sind, wenn sie in derselben Restklasse mod U liegen

W. Deuber Bielefeld

Minimale Ramsey-Graphen für Dreiecke

Ein Graph G heißt minimaler K_3-Graph, wenn bei jeder 2-Färbung der Kanten von G ein einfarbiger Teilgraph K_3 auftritt, jedoch jeder echte Teilgraph von G eine 2-Färbung der Kanten ohne einfarbigen K_3 besitzt. Es werden einige unendliche Klassen minimaler K_3-Graphen angegeben.

Ingrid Mengersen, Braunschweig

$3K_2$-decomposition of a graph

A Graph $G = G(V, E)$ is said to have an H-decomposition if it is the union of edge-disjoint isomorphic copies of the graph H.

The following two conditions for $G = G(V,E)$ to have a $3K_2$-decomposition are obviously necessary.

(1) $|E| = 3k$, $k \geq 1$

(2) $\deg v \leq k$, $\forall v \in V(G)$

It is proved that the necessary conditions are also sufficient excluding a list of 26 graphs.

Yehuda Roditty (with A. Bialostocki)
Tel Aviv, ISRAEL.

Shortness exponents for polytopes which are k-gonal modulo n.

Exceptionally, this talk has no connection with its title.
Usually people avoid intersecting edges when embedding graphs into manifolds. And if they simply cannot avoid them, then they do not like them too much. The next theorem makes these people happy.

Let G be embedded into a surface S with intrinsic metric, such that the edges are shortest paths and edge-cuts are allowed. Let $V(G)$ be the vertex set and $I(G)$ the set of edge-intersections (cutpoints) of G. Of course $\bigcup_G V(G) = S$.

Theorem. There are convex surfaces S of class C^1, such that

$$\bigcup_{\substack{\text{all } G \\ \text{embedded} \\ \text{in } S}} I(G)$$

is of first Baire category in S.

T. Zamfirescu

Zum Thema Kanten-Färbung bei regulären Graphen

I. G sei ein schlingenfreier, für ein g: $2 \leq g \in \mathbb{N}$ g-regulärer Graph, der Kanten-Färbungen mit g Farben besitzt (: benachbarte Kanten haben verschiedene Farben). Bei einer gegebenen Färbung F bedeutet dann TT (: Tausch-Teilgraph) einen Teilgraphen, dessen Kanten man neben den durch F gegebenen 'Erstfarben' solche 'Zweitfarben' zuordnen kann, dass bei jeder Ecke des TT die Zweitfarben eine discordante Permutation der Erstfarben sind (discordant hier: 2.-Farbe ≠ 1.-Farbe). Es gilt der <u>Satz</u>: Aus einer beliebigen Färbung F erhält man <u>alle anderen</u> Färbungen von G, indem man bei allen TT die Erstfarben durch die Zweitfarben ersetzt.
Die einfachsten TT sind die 2-farbigen Kreise, – und bei ∞-en Graphen die (evtl. vorhandenen) 2-farbigen beiderseitig ∞-en Wege, mit denen man ja <u>seit Kempe</u> operiert. Es wird eine Klasse von (bipartiten) Graphen angegeben, bei denen man <u>aus keiner Färbung</u> <u>durch</u> blosse Benützung von <u>2-farbigen</u> TT alle anderen Färbungen bekommt.
<u>II</u>. Die zuletzt erwähnten Graphen besitzen Färbungen, bei denen <u>für je 2 Farben</u> die Kanten mit diesen beiden Farben einen <u>Hamilton-Kreis</u> bilden: solche Färbungen nennen wir <u>H-Färbungen</u>; für <u>g = 3</u> nennen wir einen Graphen, der eine H-Färbung besitzt, einen <u>3H-Graphen</u>, – und wenn <u>jede</u> Färbung eine H-Färbung ist, einen

reinen 3H-Graphen; u.a. gilt der

Satz: (1) G ist genau dann ein 3H-Graph, wenn G 3 Hamilton-Kreise enthält, die so verlaufen, daß jede Kante zu genau 2-en von ihnen gehört;

(2) für e = 2, 4, 6, ... gibt es 3H-Graphen – darunter auch planare – mit e Ecken;

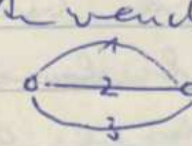
(3) für e = 2, 6, 10, ... gibt es auch bipartite 3H-Graphen, – ausgenommen  sind sie nicht-planar;

(4) für e = 4, 8, 12, ... gibt es keine bipartiten 3H-Graphen;

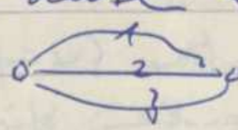
(5) für e ≤ 8 sind alle 3H-Graphen rein, für e ≥ 10 gibt es reine und nicht-reine 3H-Graphen.

Beweis der Existenz-Aussagen durch Beispiele.

Speziell: Das Dodekaeder ist ein reiner 3H-Graph.

III. Es werden 2 sehr einfache Prozesse gezeigt, durch deren wiederholte Anwendung man aus dem 3-gefärbten Graphen 

jeden 3H-Graphen und jede seiner Färbungen gewinnen kann.

IV. Es wird ein Prozess (~ "farbentreues trennendes Spalten einer Kante") und 3 Spezialfälle gezeigt, durch deren wiederholte Anwendung man aus dem 

jeden 3-färbbaren 3-regulären Graphen und alle seine Färbungen gewinnen kann.

Th. Kaluza, Hannover

Linear algebra and hypergraphs

Using linear algebraic methods I could prove some old and some new theorems like

① Suppose $\mathcal{F} = \{F_1, \dots, F_m\}$ is a family of subsets of (the n-set) X. If $m > \sum_{i=0}^{k} \binom{n}{i}$, then $\exists\, Y \subset X$, $|Y| = k+1$, such that

$$\left|\{F \cap Y : F \in \mathcal{F}\}\right| = 2^{k+1}$$ (this was originally proved by N. Sauer)

② Let t be an integer and suppose $|F_i \cap F_j| \neq t$ for $\mathcal{F} = \{F_1, F_2, \dots, F_m\}$. Then for $n > n_0(t)$ we have

a) $n+t+1$ is even

$$m \le \sum_{i=0}^{t} \binom{n}{i} + \sum_{i=\frac{n+t+1}{2}}^{n} \binom{n}{i}$$

b) $n+t+1$ is odd

$$m \le \sum_{i=0}^{t} \binom{n}{i} + \sum_{i=\frac{n+t+2}{2}}^{n} \binom{n}{i} + \binom{n-1}{\frac{n+t}{2}},$$

this is answering a question of ~~Erd~~ Erdős.

The results were obtained partly in collaboration with Z. Füredi, J. Pach, and N. M. Singhi.

Peter Frankl
CNRS, Paris

Some problems concerning the chromatic number

P. Erdős and A. Hajnal asked the following question: Does there exist a constant $\varepsilon > 0$ with the following property? If every subgraph H of a graph G can be made bipartite by omission of at most $\varepsilon |H|$ edges (here by $|H|$ the number of vertices of H is denoted), then $\chi(H) \le 3$

The aim of a lecture is to give a negative answer to this question and deal with the similar problem for hypergraphs.

The first was done also by L. Lovász who used a different example.

U. Rödl (Prague)

Differential - Differenzengleichungen, Anwendungen und numerische Probleme

6.6. – 12.6. 1982

Improved absolute stability of predictor - corrector methods for retarded differential equations

The absolute stability of predictor - corrector type methods is investigated for retarded differential equations. The stability test equation is of the form $dy(t)/dt = \omega_1 y(t) + \omega_2 y(t-\omega)$ where ω_1, ω_2 and ω are constants ($\omega > 0$). By generalizing the conventional predictor-corrector methods it is possible to improve the stability region in the $(\omega_1 \Delta t, \omega_2 \Delta t)$-plane considerably. In particular, methods based on extrapolation - predictors and backward differentiation - correctors are studied. Stability plots and numerical results are presented.

Peter J. van der Houwen
MC, Amsterdam

Zellenfunktionen, Kohärenz und erweiterte Symmetrien bei Differenzenmethoden

Die Verwendung von Zellenfunktionen ist "dual" zu derjenigen von Knotenfunktionen. Einer Zelle wird eine Zahl zugeordnet, welche dort den Mittelwert der kontinuierlichen Lösungsfunktion annähert. Durch Extremalprinzipien definierte Gebietsfunktionale werden im umgekehrten Sinne abgeschätzt als mit Knotenfunktionen.

Eine Differenzenmethode heisst "Kohärent", wenn die Gleichungen zu verschiedenen Maschenweiten einander nicht widersprechen. Kohärente Methoden geben bei elementarsten Problemen genaue Lösungen, i. A. aber besonders gute Näherungen.

Nicht nur in symmetrischen Gebieten, auch bei "erweiterter Symmetrie" kann man die Anzahl der Unbekannten reduzieren: nicht das Gebiet selbst, sondern die Klasse der darin zugelassenen Funktionen weist eine gewisse Symmetrie auf.

Joseph Hersch
E.T.H., Zürich.

Estimating the global error of Runge-Kutta approximations for ordinary differential equations

The user of a code for solving the initial value problem for ordinary differential systems is normally left with the difficult task of assessing the accuracy of the numerical result returned by the code. Even when the code reports an estimate of the global error, the question may remain whether this estimate is correct, i.e. whether the user can rely on the estimate. We will discuss a simple idea of measuring the reliability of the global error estimate with the aim of assisting the user in the validation of the numerical result. The idea is put into practice with the existing code SERK (ACM Algorithm 504) developed by Shampine and Watts.

Jan G. Verwer
Mathematisch Centrum
Amsterdam

Zeitlich verzögerte automatische Kontrolle des freien Randes bei Zweiphasen-Stefan-Problemen.

Untersucht wurde ein Zweiphasen-Stefan-Problem, bei dem der Wärmefluß über die beiden festen Ränder durch Heizungsanlagen automatisch kontrollierbar ist. Die Heizer werden elektrisch durch Photozellen, die die Entwicklung des freien Randes beobachten, ein- bzw. ausgeschaltet. Dabei auftretende zeitliche Verzögerungen führen zu Delays. Das zugehörige mathematische Modell lautet:

$$\begin{aligned}
&\alpha_1 u_{xx} - u_t = 0 \quad, \text{ in } \Omega_T^-(s), && \alpha_2 v_{xx} - v_t = 0 \quad, \text{ in } \Omega_T^+(s),\\
&u(x,0) = \varphi(x) \quad, a \le x \le b, && v(x,0) = \varphi(x) \quad, b \le x \le c,\\
&\left.\begin{array}{l} u_x(a,t) = -f_1(t) \\ u(s(t),t) = 0 \end{array}\right\} , 0 \le t \le T, && \left.\begin{array}{l} v_x(c,t) = -f_2(t) \\ v(s(t),t) = 0 \end{array}\right\} , 0 \le t \le T,
\end{aligned}$$

mit $s(0) = b$, der Energiebedingung $\dot{s}(t) = -\delta_1 u_x(s(t),t) + \delta_2 v_x(s(t),t)$, $0 \le t \le T$, auf dem freien Rand, und den Steuergleichungen

$$\left.\begin{array}{l}
\beta_1 \dot{f}_1(t) + f_1(t) = \frac{1}{2}\,[1 - \mathrm{sgn}\,(s(t-\tau_1) - s_1(t-\tau_1))] \\
\beta_2 \dot{f}_2(t) + f_2(t) = \frac{1}{2}\,[1 + \mathrm{sgn}\,(s(t-\tau_2) - s_2(t-\tau_2))]
\end{array}\right\} , \quad 0 \le t \le T,$$

wobei $\alpha_i, \beta_i, f_i, \delta_i, s_i, \tau_i$, $i = 1,2$, und φ gegeben sind. Ferner ist $s(t)$ für $t \in [-\max(\tau_1, \tau_2), 0]$ bekannt.

Durch Transformation in eine mengenwertige Fixpunktgleichung wird die Existenz einer "Lösung" für hinreichend kleines $T > 0$ gezeigt. Numerische Beispiele zeigen, daß das Modell realistisch ist. Die Resultate wurden gemeinsam mit K.-H. Hoffmann, Augsburg, erzielt.

Jürgen Sprekels,
Universität Augsburg.

Numerical solutions of Functional Differential Equations: Asymptotic behaviour and characteristic roots.

In general, solutions of FDE's are not smooth, but will have jump discontinuities in their derivatives up from the first. However, in many cases these jumps can be calculated in position and value. Thus they may be substracted from the solution, and so an FDE with a smooth solution can be calculated with a high order by a linear multistep method.

Two questions will be discussed:

a) If the solution is not smooth enough, what happens to the order of the numerical approximation?

b) If the solution is more than smooth enough, is it possible to derive an asymptotic expansion of the error in terms of the stepsize (Gragg - Stetter theory)?

Maarten de Gee
Mathematisch Instituut
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Utrecht, The Netherlands.

Monotone dynamische Systeme

Ein dynamisches System $\dot{x} = F(x)$, $x \in W \subset \mathbb{R}^n$, $W = \mathring{W}$ heißt monoton, wenn gilt "$x \leq y \Rightarrow x(t) \leq y(t)\ \forall t \geq 0$" soweit die Lösungen definiert sind; das System ist streng monoton wenn gilt: "$x \lneqq y \Rightarrow x(t) < y(t)\ \forall t > 0$" soweit $x(t), y(t)$ definiert sind.

Nach einem Satz von M.W. Hirsch gilt für fast alle

Punkte x, deren Vorwärtsorbit $O_+(x)$ kompakten Abschluß in W hat, daß alle Häufungspunkte von $O_+(x)$ stationäre Punkte sind.
Für alle praktischen Zwecke sind also fast alle Lösungen $x(t)$, die vom Rand von W wegbleiben, von stationären Punkten nicht zu unterscheiden.
Für den Satz von Hirsch geben wir einen neuen Beweis, der im Gegensatz zu Hirsch' ursprünglichem Beweis mit elementaren Hilfsmitteln auskommt.

Differentialgleichungen, die in natürlicher Weise auf streng monotone dynamische Systeme führen, erhält man z.B. bei der Beschreibung von Krankheitsverläufen in mehreren Populationen.

Ludwig J. Cromme

Universität Göttingen

Über die Existenz positiver periodischer Lösungen bei einem linearen Diffusionsmodell.

Es wird ein lineares Diffusionsmodell betrachtet, das beschrieben wird durch ein System linearer Differentialgleichungen

$$\dot{x}(t) = A(t)\,x(t) + b(t), \quad t \in \mathbb{R}, \qquad (D)$$

mit nicht-negativen konstanten Koeffizienten a_{ij} für $i \neq j$, $1 \leq i, j \leq n$, $a_{ii}(t) = -\sum_{j \neq i} a_{ij} - d_i(t)$, $1 \leq i \leq n$, p-periodischen, nicht-negativen Funktionen $d_i(t)$ bzw. $b_i(t)$, $1 \leq i \leq n$, welche stückweise stetig bzw. stetig sind. Solche Modelle treten z.B. bei der mathematischen Beschreibung des Vorganges der Hämodialyse auf. Unter einer Irreduzibilitäts=bedingung, die verhindert, daß das System (D) im Falle $a_{ij} > 0 \Leftrightarrow a_{ji} > 0$ in unabhängige Systeme zerfällt, wird gezeigt, daß (D) genau eine absolut stetige, p-periodische, in allen Komponenten positive

Lösung besitzt, wenn gilt: $\sum_{i=1}^{n} a_i \neq 0$ und $\sum_{i=1}^{n} b_i \neq 0$

W. Kratz, TH Darmstadt

Eine Eigenwertaufgabe mit einer Funktional-Differentialgleichung

Für die Schar $(0 \leq \alpha \leq 1)$ von Eigenwertaufgaben mit einer Funktional-Differentialgleichung

$$\left.\begin{array}{l} -u''(x) = \lambda \{ \alpha u(1-x) + [1-\alpha] u(x) \} \text{ in } 0 < x < 1 \\ u(0) = 0, \quad u'(1) = 0 \end{array}\right\} \quad (1)$$

kann die Lösung angegeben werden (Fallunterscheidung: $0 \leq \alpha < \frac{1}{2}$; $\alpha = \frac{1}{2}$; $\frac{1}{2} < \alpha \leq 1$). Für die Eigenwertaufgabe

$$\left.\begin{array}{l} -\Delta u(x,y) = \lambda u(1-x, 1-y) \text{ in } 0 < x, y < 1 \\ u(0,y) = u(x,0) = 0; \quad u_n(1,y) = u_n(x,1) = 0 \end{array}\right\} \quad (2)$$

werden obere und untere Eigenwertschranken nach Ritz und Temple-Collatz bzw. Lehmann-Maehly berechnet; letztere unter Hinzunahme eines von Goerisch angegebenen Stufenverfahrens, das auf einer Einbettung von (2) in eine zu (1) analoge Schar von Eigenwertaufgaben beruht.

Julius Albrecht, TU Clausthal

Projektionsmethoden zur Approximation von Funktionaldifferentialgleichungen

Es wird ein ein Überblick, unter besonderer Berücksichtigung der eigenen Ergebnisse, über jüngste Resultate zur dynamischen Approximation von Funktionaldifferentialgleichungen (FDG) und assoziierte Optimierungs- und Identifikationsprobleme gegeben. Dabei wird die vorliegende FDG als abstraktes Cauchyproblem in einem geeignet gewählten Hilbertraum formuliert, so daß Methoden der (Operator-) Halbgruppentheorie anwendbar werden. Insbesondere kann der Satz von Trotter-Kato dazu benützt werden, zu zeigen, daß unter der der Voraussetzung der Konsistenz eines

Approximationsschemas dessen Stabilität äquivalent zur Konsistenz ist. Dieser Zugang gestattet es, die Approximation von FDG durch Treppenfunktionen, wie sie durch lange Zeit in der Ingenieurliteratur vorgeschlagen wurden, und Splineapproximationen in einer einfachen Form zu behandeln. Approximationsverfahren für neutrale FDG können auf ähnliche Weise entwickelt werden. Eine besondere Bedeutung kommt auch dem linear-quadratischen Optimierungsproblem zu; hier ist es notwendig, eine simultane Approximation der FDG und der assoziierten Riccati-Integralgleichung zu erzielen.

Karl Kunisch, T.U. Graz.

Faktorisierung linearer Differenzengleichungen mit Anwendungen auf Matrizen.

Lineare D-Gln. mit variablen Koeffizienten über Z werden auf ihre Struktur hin untersucht. Auf Grund von Regularitätsvoraussetzungen gelingt eine vollständige Faktorisierung in D.Gln. erster Ordnung. Bei Zwei-Punkt-Randwertaufgaben, speziell bei periodischen Randbedingungen sind derartige Zerlegungen nach dem bei Differentialgleichungen bekannten Floquet'schen Prinzip relativ leicht zu bestimmen. – Die Faktorisierungen sind geeignet, Schranken für gewisse Interpolationsoperatoren mit Splines zu konstruieren und insbesondere Beschränktheitsaussagen zu gewinnen. Sie sind ferner für die numerische Lösung der zugehörigen Gleichungssysteme von erheblicher Effizienz.

Günter Meinardus, U Mannheim.

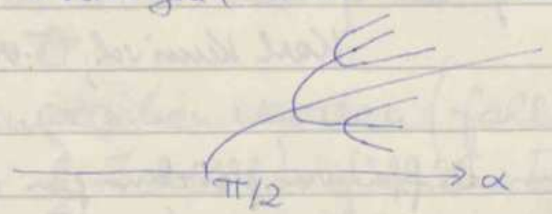
Verzweigung von periodischen Lösungen bei Funktionaldifferentialgleichungen

1977 bewies R D Nussbaum Nichteindeutigkeit periodischer Orbits für eine Klasse von Funktionaldifferentialgleichungen der Form

$$\dot{x}(t) = \alpha f(x(t-1)),$$

mit $\alpha > \pi/2$, $f'(0) = -1$, f nicht monoton. Die Untersuchung solcher Nichtlinearitäten ist durch – einfache – Modelle physiologischer Kontrollmechanismen nahegelegt.

Numerische Studien von KP Hadeler, von H Jürgens, HO Peitgen, D. Saupe haben in den folgenden Jahren die Vermutung gestützt, daß die periodischen Lösungen, die in einer Hopf-Verzweigung bei $\alpha = \pi/2$ entstehen, sich weiter verzweigen, wenn α wächst:



Wir beweisen nun, daß solche höheren Verzweigungen tatsächlich existieren – in einer Klasse von Nichtlinearitäten f, die z.B. $-\sin$, und die ungerade Fortsetzung von $0 \le x \mapsto -x(1-x)$ enthält.

Hans-Otto Walther, U München

Die komplexe Dynamik einer Differential-Differenzengleichung aus der Biologie

Die Differentialgleichung mit verzögertem Argument

$$\dot{x}(t) = f(x(t-1)) - \alpha x(t)$$

ist unabhängig von mehreren Forschern (Lasota & Washewska, Mackey & Glass, Coleman & Renninger) aufgestellt worden, um Prozesse der Blutbildung, des Populationswachstums, der Atmung, neuronaler Aktivität und der Stoffwechselregulation zu modellieren. Wir zeigen, daß die Lösungen dieser Gleichung einen bemerkenswerten Reichtum von Strukturen entfalten, in Abhängigkeit von der Nichtlinearität $f: \mathbb{R}_+ \to \mathbb{R}_+$, dem Zerfallsparameter $\alpha > 0$ und den Anfangsbedingungen. Es können eine oder mehrere stationäre Lösungen in Verbindung mit Hysteresisphänomenen vorkommen, sowie stabile und instabile Grenzzyklen. Diese Zyklen können sehr kompliziert sein, indem sie eine beliebige Anzahl von Extrema innerhalb der kleinsten (positiven) Periode haben können. Es gibt sogar Situationen, in denen zu einer gegebenen Funktion f und einem gegebenen Wert α unendlich viele aperiodische, quasi-zufällige Lösungen existieren. Ein solcher chaotischer Fluß kann ergodisch und mischend sein. Die Existenz aller dieser Eigenschaften wird für eine Klasse von Funktionen f und Parameter α bewiesen.

Literatur: U. an der Heiden, H.-O. Walther: Existence of chaos in control systems with delayed feedback, J. Diff. Equs. (to appear); U. an der Heiden, M.C. Mackey: The dynamics of production and destruction – Analytic insight into complex behavior (to appear).

Uwe an der Heiden

Universität Bremen

Berechnung vieler Lösungen für diskrete Modelle aus der Biologie und der Chemie.

Es wurde ein diskretes Modell der Form $Ax = \lambda F(x, \lambda)$ mit einem verwickelten Verzweigungsbild vorgestellt. Für jedes λ sollten möglichst viele Lösungen berechnet werden. Hierzu wurde ein Verfahren angegeben, welches auf der Spaltung des Gleichungssystems in zwei Teile beruht. Jeder Teil gibt Anlaß zu einem Lösungsgebilde. Beide werden zu einem Lösungsgebilde des gesamten Systems zusammengefügt.

Emil Roth, Konstanz.

"Maximum principles for linear difference-differential operators in periodic function spaces"

I deal with maximum and minimum principles for first and second order difference-differential equations with constant coefficients in periodic function spaces. I give necessary and sufficient conditions for the mono=tonicity of the inverse operator $L_n^{-1}(a, b, \tau, T)$, where

$$L_n(a,b,\tau,T)u(t) =: u^{(n)}(t) + a\,u(t) + b\,u(t-\tau) \qquad n = 1,2$$

with $a, b \in \mathbb{R}$ and $\tau \in [0,T]$, which maps the space C_T^n of the T-periodic functions of class C^n into C_T^0.
Since for $b=0$ the problem reduces to an ordinary one, I consider bounds for b depending on a, τ and T, in order the maximum (or the minimum) principle to hold. I show that there exists a function $\beta(\cdot,\cdot)$ such that $L_n^{-1}(a,b,\tau,T)$ is monotone if and

only if $-aT^n < bT^n \le \beta(aT^n, \tau/T)$. Besides I give a numerical ~~procedure~~ method to compute the function β or, at least, to bound it.
The results can be used for seeking periodic solutions of equations such as

$$u^{(n)}(t) = f(t, u(t), u(t-\tau)) \qquad n = 1, 2 .$$

Mario Zennaro
Università di Trieste

Kardinale Splines, die linearen Differenzengleichungen genügen

Für kardinale exponentielle und logarithmische Splines s_m wird mit Hilfe der inversen Laplace- bzw. Mellin-Transformation eine komplexe Kurvenintegraldarstellung mit nicht kompaktem Integrationsweg etabliert. Eine Anwendung des Residuenkalküls liefert das asymptotische Grenzverhalten der Folge $(s_m)_{m \geq 1}$ für $m \to \infty$. Vgl. Complex Contour Integral Representation of Cardinal Spline Functions. Contemporary Math., Vol. 7. Providence, R.I.: AMS 1982.

Walter Schempp (Siegen)

Faktorisierung total positiver Bandmatrizen

(gem. Arbeit mit A.S. Cavaretta, C.A. Micchell., P.W. Smith)

Wir betrachten eine Klasse von linearen Differenzengleichungen, die typischerweise von Splineinterpolation bezüglich biinfiniter periodischer Knotenfolgen herrührt. In Matrizenform führt dies zur Untersuchung biinfiniter, total positiver m-Band Block-Toeplitz-Matrizen, $A = (A_{i-j})_{i,j \in \mathbb{Z}}$, $A_i = (a_{ek}^{(i)})_{e,k=1}^{N}$, etwa, wobei $A_j = 0$, für $j<0$, $j>q$ für ein $q \in \mathbb{N}$. Definiert man das ($(N \times N)$-matrizenwertige) Symbol $A(z) = \sum_{j=0}^{q} A_j z^j$, so läßt sich zeigen, daß $\det A(z)$ nur reelle Nullstellen des Vorzeichens $(-1)^N$ hat, so daß $Ax = y$ auf l_∞ invertierbar ist, genau dann wenn $\det A((-1)^N) \neq 0$ gilt. Tatsächlich ist die Bestimmung der Nullstellen von $\det A(z)$ eine Folge des allgemeineren Resultats, daß sich jede ~~strikte~~ total ~~m-Band~~ positive biinfinite strikte m-Band-Matrix A in ein Produkt von m total positiven 1-Band-Matrizen faktorisieren läßt. Ferner wird die Eindeutigkeit solcher Faktorisierungen diskutiert.

Wolfgang Dahmen
(Bielefeld)

Fehlerabschätzungen beim Caratheodory-Fejer-Verfahren

Zunächst werden zwei Abschätzungen des Fehlers angegeben, der entsteht, wenn man aus der Lösung des Caratheodory-Fejérschen Minimumproblems durch Abschneiden der negativen Potenzen von z ein Approximationspolynom an ein Polynom höheren Grades bildet, und zwar einerseits für exponentiell fallende und andererseits für monoton fallende Koeffizienten des gegebenen Polynoms.

Ausgehend davon wird gezeigt, daß bei diesem Vorgehen für wachsende Polynomgrade und wachsende Anzahl von Koeffizienten im Caratheodory-Fejer-Verfahren eine asymptotisch beste Polynomapproximation entsteht.

Manfred Hollenhorst
(Gießen)

Der Einfluß der Interpolation auf den globalen Fehler bei retardierten Differentialgleichungen

Betrachtet wird das retardierte Anfangswertproblem

$$y'(x) = f(x, y(x), y(x-\tau)) \quad \text{für } x \geq x_0,$$
$$y(x) = \varphi(x) \quad \text{für } x \leq x_0,$$

wobei die Verzögerung $\tau = \tau(x, y(x)) \geq 0$ zustandsabhängig sein darf. Bei der numerischen Behandlung dieses Problems wird der Funktionswert $y(x-\tau)$ am retardierten Argument i.a. durch einen mittels Interpolation gewonnenen Funktionswert $u(x-\tau)$ ersetzt, wodurch man eine "benachbarte Differentialgleichung" erhält. Unter Einbeziehung dieser Tatsache gelingt es, eine Abschätzung für den globalen Fehler anzugeben, die nur numerisch kontrollierbare Größen enthält, insbesondere die Integrations- und Interpolationsfehler der verwendeten Verfahren.

Herbert Arndt (Bonn)

Anwendung von Differential-differenzgleichungen bei der Lösung von Differentialgleichungen durch Reihentwicklungen.

We are interested in solving a linear ordinary differential equation $L_D y(x) = 0$ by a series $\sum_0^\infty \alpha_r u_r(x)$ [where $\underset{\sim}{u}(x) = \{u_r(x)\}$ is a chosen sequence of functions]. This method succeeds only when we obtain a difference equation for the sequence $\underset{\sim}{\alpha} = \{\alpha_r\}$ which can be solved either explicitly or numerically. The function sequence $u(x)$ must satisfy a difference - differential equation of the form $L_D u = M_\Delta u$ where M_Δ is a difference operator. Then the equation for α is $M_\Delta^* \alpha = 0$ where M_Δ^* is the adjoint of M_Δ in a suitable sequence space.

The important question is: how can we choose the expansion functions $u_r(x)$ so that $\underset{\sim}{u}(x)$ satisfies an equation $L_D u = M_\Delta u$ so that M_Δ (and hence M_Δ^*) is as simple as possible? (e.g. of lowest possible order).

F. M. Arscott
(Winnipeg, Canada)

Solution of tridiagonal linear systems with a parallel computer.

The solution of tridiagonal linear systems of equations with the aid of a MIMD (multiple - instruction multiple - data stream) parallel computer with two processors is considered. Of the methods discussed it appears that reduction to bidiagonal form is the most efficient direct and the Jacobi - iteration the most efficient iterative method. Numerical results substantiate these conclusions.

Gerhard Joubert
(Eindhoven, Niederlande)

Spezielle lineare Gleichungssysteme, die bei der Lösung von Differenzialgleichungen mit finiten Elementen entstehen.

Es handelt sich um die numerische Lösung linearer Gleichungssysteme, deren Matrizen symmetrisch und positiv definit sind und die Form $M = \begin{pmatrix} A & C \\ C & B \end{pmatrix}$ haben, wo A, B und C schwach besetzte quadratische Toeplitz Matrizen mit einer speziellen Bandstruktur sind, deren Elemente von einem positiven Parameter abhängen.

Solche Gleichungssysteme entstehen z. B. bei der Lösung einer Stokes' Gleichung für viskose, fast incompressible Flüssigkeit mit Dirichletschen Randbedingungen auf einem rechteckigen Gebiete, wenn die Methode der finiten Elemente benutzt wird, wobei in die Aufgabe ein Penalisationsparameter ε für den Druck der Flüssigkeit eingeführt wird.

Als ein Beispiel wurde ein solches System mit einer Matrix der Ordnung 30 gelöst, wobei für ε die Werte $10^{-1}, 10^{-2}, \ldots 10^{-5}$ genommen wurden, und ein anderes System der Ordnung 198.

Olga Přerné, Prag

Ein Epidemie-Modell mit diskreter Parasitenzahl

In den klassischen Epidemie-Modellen wird die Population in Klassen (gesund, krank, immun) eingeteilt, deren Dichten Differentialgleichungen genügen, die nach dem Massenwirkungsgesetz gebildet werden. Hier wird ein Modell entwickelt, bei dem die Wirtspopulation nach dem Alter und der diskreten Zahl der Parasiten pro Wirt klassifiziert wird. Ein solches Modell ist sinnvoll, wenn jeweils nur wenige Parasiten auftreten und diese die Mortalität des Wirtes beeinflussen. Über eine erzeugende Funktion führt dieses Modell auf eine partielle Differentialgleichung 1. Ordnung mit Integraltermen. Bisheriges Ergebnis: Lokale Existenz, Bifurkation stationärer Zustände, Stabilität des trivialen stationären Zustandes bis zur Bifurkation hin

(gemeinsame Arbeit mit K. Dietz) K.P. Hadeler, Tübingen

Zur Methode der periodischen Zahlen.

We explain the method by an example: Find the number u_n of diophantine triangles of perimeter n. Our polyhedron method gives fast the form of u_n:

$$48u_n = n^2 + [a, b]n + [a', b', 0] + [a'', b'', c'', 0]$$

where the periodic number $u_n = [u_1, u_2, \dots, u_k]$ equals the u_i whose $i \equiv n$, mod k. We determine these numbers by some initial values of u_n and v_n, associated by our reciprocity law. With the notation of the nearest integer $u_n = \left\| \frac{n^2 + [6, 0]n}{48} \right\|$. It satisfies the linear recurrence

$$\{(1-u^2)(1-u^3)(1-u^4)\} = 0, \qquad u^r \sim u_{n-r}.$$

E. Ehrhart

Strasbourg (France)

Improvement of a Mesh Selection Algorithm for Collocation Methods by Smoothing

In this contribution a problem encountered in the mesh selection algorithm of the code of Ascher-Christiansen-Russell when applied to a stiff system is discussed. Based on an analysis of the collocation method for a stiff model problem an extension of the mesh selection algorithm is proposed. This extended algorithm smoothes the obtained collocation approximation by suitable interpolation. The capability of this method is demonstrated by a numerical example.

M. van Veldhuizen
Vrije Universiteit
Amsterdam.

Numerische Behandlung der Plattenaufgabe mit kritischen Randpunkten.

In dieser Arbeit handelt es sich um eine Formel der finiten differenzen Methode herzustellen, welche den negativen Effekt von Singularitäten am Rande des Definitionsgebiet, an der Lösung der Plattengleichung beseitigt. Man benutzt dazu das Prinzyp, welches man an der Lösung der Laplace-Gleichung angewendet hat.

Die Verbesserungen an den Werten der Lösungen ist von grossem Wert da man den Rechenaufwand verringert und deshalb Zeit und Speicher an der Rechenmaschine spart.

Hermann ALDER
Universidad de Concepción
Chile.

Anwachsende Schwingungen bei einigen Differenzengleichungen mit Verzögerungsglied

Es wird eine Differenzengleichung mit Verzögerungsglied betrachtet:

$$y(n) = y(n-1) - y([\alpha n]) \qquad (n = 1, 2, 3, \cdots),$$

wobei α eine gegebene Konstante mit $0 < \alpha < 1$ ist ($[z]$ = grösste ganze Zahl $\leq z$). $n = 0$ ist singuläre Stelle. Man hat zu unterscheiden $0 < \alpha < \frac{1}{2}$ und $\frac{1}{2} \leq \alpha < 1$. Für $\alpha \geq \frac{1}{2}$ gibt es als für alle n gültige Lösung nur $y(n) \equiv 0$, aber für $n \geq n_0$ bei passendem n_0 unter einer zusätzlichen Annahme über Anfangswerte schwingungsartige Lösungen mit unbeschränkt wachsenden „Amplituden" und „Schwingungsdauern". Derselbe Schwingungscharakter tritt auch für $0 < \alpha < \frac{1}{2}$ für alle $n > 0$ und bei der zugehörigen kontinuierlichen Differentialgleichung $y'(x) = -c\, y(\alpha x)$ mit $c > 0$ auf.

Lothar Collatz, Hamburg.

Zur Einschliessung der Lösungen bei erzwungenen retardierten Schwingungen

Für Gleichungen der Form

$$u''(t) + a\, u(t) + b\, u(t - \tau(t)) = f(t)$$

[$f(t)$ gegebene Funktion der Periode T, $\tau(t)$ gegebene Verzögerungsfunktion, a, b Konstanten, $u(t)$ gesucht] wurden unter gewissen Voraussetzungen über die Koeffizienten von BELLEN und ZENNARO Maximum-, bzw. Minimum-Prinzipien hergeleitet, welche die Grundlage für Monotonie-Aussagen und damit für die numerische Berechnung periodischer Lösungen $u(t)$ mit Hilfe von Approximations- und Optimierungs-Methoden ergeben. Es wird über numerische Erfahrung berichtet: dabei liefert die Methode mit geringem Rechenaufwand enge Einschliessungsintervalle für $u(t)$.

Lothar Collatz, Hamburg.

Maschenunabhängigkeit beim Newton-Verfahren und Schrittweitensteuerung

Große Systeme von Differenzengleichungen entstehen bei der Diskretisierung von Differentialgleichungen. Zur Lösung dieser nichtlinearen Systeme verwendet man häufig Newton-Verfahren. Die Zahl der Iterationen für das Differential- und Differenzenproblem stimmen bei „gleicher" Startfunktion und vorgeschriebener Genauigkeit für genügend kleines h überein. Diese Tatsache wird ausgenutzt, um in Zwischenschritten von h_0 über $h_1, \ldots,$ bis h die Näherung z^h effektiv zu berechnen.

Klaus Böhmer, Marburg

Eine Anwendung einer Differenzengleichung in der Medizin.

Die reifenden roten Blutkörperchen werden wegen ihrer netzartigen Struktur Retikulozyten genannt und auf grund ihrer Erscheinung in vier Klassen eingeteilt. In einer Reihe medizinischer Untersuchungen wurde die Anzahl der Retikulozyten dieser Klassen bei Neugeborenen als Funktion des Kindesalters ermittelt. Die Klassen werden von einem jeden Teilchen sukzessiv durchlaufen, die Verweilzeiten in den einzelnen Gruppen sind jedoch nicht direkt bestimmbar.

Es wird nun unter geeigneten Annahmen ein deterministisches Modell für die Entwicklung der Retikulozyten aufgestellt, das die Verweilzeiten als Parameter enthält. Das Modell liefert eine Integrodifferenzengleichung für die Dichte der Retikulozyten als Funktion des Kindesalters und Retikulozytenreifezustandes. Einführen der Produktionsfunktion der Leber gestattet es, die Parameteridentifikation auf die Minimierung der Norm des Fehlers in einer Konsistenzrelation zurückzuführen.

Als Ergebnis erhält man eine Schätzung der Leberfunktion mit der medizinischen Konsequenz, daß kurz vor der Geburt die Leberproduktion von Retikulozyten stark angeregt und kurz nach der Geburt wieder abgeschaltet wird.

Helmut Werner, Bonn

Darstellungstheorie und Lokale Kohomologie

13. – 19. Juni 1982

[illegible]

… the trace of a Lefschetz trace formula, expressing the number of fixed points of … [illegible] … of F on the $H^i(X)$. Such a theory was developed by Grothendieck, Artin and others … [illegible]

I. G. Macdonald

Introduction to Deligne-Lusztig theory

Let G be a connected reductive group defined over the field with q elements, with Frobenius map $F: G \to G$. Let $G^F = \{g \in G : Fg = g\}$. The Deligne-Lusztig theory studies the characters of the finite group G^F. For each F-stable maximal torus T of G and each character θ of T^F, the Deligne-Lusztig generalized character $R_T^G(\theta)$ … [illegible] … The character …

Darstellungstheorie und l-adische Kohomologie

13. – 19. Juni 1982

Introduction to l-adic cohomology

The conjectures of A. Weil in diophantine geometry (~~Numbers of~~ Solutions of equations in finite fields, Bull. A.M.S. 1949) indicated strongly that there should exist a cohomology theory for algebraic varieties defined over a field of arbitrary characteristic; the cohomology groups $H^i(X)$ should be vector spaces of finite dimension over some field of characteristic 0, and should have the usual formal properties which would in particular imply the truth of a Lefschetz trace formula expressing the number of fixed points of a morphism $X \xrightarrow{F} X$ as the alternating sum of the traces of F on the $H^i(X)$. Such a theory was developed by Grothendieck, Artin and others in the early 1960's, by generalizing the classical notions of sheaf cohomology to define étale cohomology for an arbitrary scheme X. The lectures were a brief introduction to this subject, designed for the non-expert, and covered the statements of some of the main theorems, including that of the trace formula in its general form, for an arbitrary l-adic sheaf of coefficients

I. G. Macdonald

Introduction to Deligne – Lusztig theory

Let G be a connected reductive group defined over ~~the~~ field with q elements with Frobenius map $F: G \to G$. Let $G_F = \{g \in G;\ g^F = g\}$. The Deligne – Lusztig theory studies the characters of the finite group G_F. For each F-stable maximal torus T of G and each character θ of T_F the Deligne-Lusztig generalized character $R_{T,\theta}$ of G_F was defined. The character formula for $R_{T,\theta}$ in terms of Green functions on certain subgroups of G was stated, and also the scalar product formula for $(R_{T,\theta}, R_{T',\theta'})$. The degrees of the $R_{T,\theta}$ and their character values on semisimple elements of G_F were also described. The way in which the $R_{T,\theta}$ give a partition of the set of all irreducible characters of G_F into geometric conjugacy classes was described, each class containing just one character of degree prime to p. (if p is not a bad prime for G). The degrees of these latter characters of G_F were described in terms of the semisimple classes in the dual group $G^*_{F^*}$. Finally a brief discussion of Lusztig's work on the unipotent characters of G_F was given.

R. W. Carter.

Trigonometric sums and representations of Weyl groups

According to Springer's hypothesis (proved by Kazhdan, cf. the subsequent talk) the character values $R_{T,\theta}(u) = R_{T,1}(u)$ on unipotent elements can be computed as trigonometric sums on the G_F-orbit of a strongly regular element $A' \in (\mathrm{Lie}\, T)_F$. Using ℓ-adic cohomology and some geometric reductions one can express $R_{T,1}(u)$ as the alternating sum of the traces of a twisted Frobenius $F^* r_B^i(w)^{-1}$ on the cohomology groups $H^i(\mathcal{B}_u, \mathbb{Q}_\ell)$, where $\mathcal{B}_u$ is the set of Borel subgroups of G containing u, and where $r_B^i : W \to \mathrm{Aut}(H^i(\mathcal{B}_u, \mathbb{Q}_\ell))$ is a representation of the Weyl group $W = N(T)/T$. This representation of W commutes with the natural action of the disconnected centralizer $C(u) = Z_G(u)/Z_G(u)^\circ$. Let $\{u_1, \ldots, u_m\}$ be a set of representatives for the unipotent conjugacy classes of G. Then for each irreducible representation χ of W there is exactly one u_i and an irreducible representation φ of $C(u_i)$ such that χ occurs in the φ-isotypic component of the top cohomology $H^{top}(\mathcal{B}_u, \mathbb{Q}_\ell)$.

P. Slodowy

Green functions and Deligne–Lusztig characters (after Kazhdan).

This talk contained a review of a paper by D. A. Kazhdan (Israel J. of Math. 28 (1977), 272–286), which contains another approach to the Deligne–Lusztig characters $R_{T,\theta}$. This approach leads to character formulas on unipotent elements. However, restrictions on p and q are needed.

The main problem is to show that the class function on G_F which is the candidate for $R_{T,\theta}$, is a virtual character of G_F. This is done by a suitable application of Brauer's theorem. The most difficult part of the proof is to show that the restriction of this function to the group U^F of rational points of a maximal connected unipotent $\mathbb{F}_q$-subgroup of G is a virtual character of U^F. To do this, l-adic cohomology is involved. It is used to prove the following result. Let X be an algebraic variety defined over $\mathbb{F}_q$. Assume there is a closed filtration $X = X_0 \supset X_1 \supset \dots$ such that for all i there exists a morphism $f_i : X_i - X_{i+1} \longrightarrow Y_i$ whose non-empty fibers are all isomorphic to a fixed affine space $\mathbb{A}^d$ (the filtration is not assumed to be defined over $\mathbb{F}_q$!). Then the number $|X^F|$ of rational points of X is divisible by q^d.

T. A. Springer

Introduction to middle intersection cohomology.

M. Goresky and R. MacPherson have associated new topological invariants, in the form of homology or cohomology groups, to certain singular spaces X (e.g. admitting a Whitney stratification with axiom B, in particular algebraic varieties or complex analytic spaces). The space X is endowed with a filtration

$$X = X_n \supset X_{n-2} \supset X_{n-3} \supset \dots \supset X_0 \supset X_{-1} = \emptyset$$

by closed subspaces X_i such that $S_j = X_j - X_{j+1}$ is either empty or a j-manifold (the j-th stratum). In particular S_n is a n-manifold ($X_{n-1} = X_{n-2}$ by convention). There is moreover a local triviality condition: around $x \in S_j$, the stratification is a product of S_j by a stone over a stratified space (the link). To this and a suitable sequence of integers (the perversity) are associated cohomology groups. This talk was devoted to one such, the middle intersection cohomology, so far the most important for applications to algebraic varieties. Accordingly it was assumed that $S_j = \emptyset$ for odd j's.

First, the simplicial definition was recalled. Then I went to the sheaf theoretic point of view and gave various characterisations (up to quasi-isomorphism) of the intersection cohomology sheaf IC^*, and some of the main properties of the intersection cohomology groups $IH^*(X;R)$, where R is the underlying ground ring. In particular, when R is a field, IC^* is (Verdier)-self dual, hence there is a perfect pairing

$$IH^i(X;R) \times IH_c^{n-i}(X;R) \longrightarrow R \quad (i \in \mathbb{N}),$$

where c refers to cohomology with compact supports. Finally, the extension to local coefficients was described: given a locally constant sheaf E on S_n, whose stalks are finitely generated R-modules, there is similarly an intersection cohomology sheaf $IC^*(E)$ and a perfect pairing (when R is a field)

$$IH^i(X;E) \times IH_c^{n-i}(X;E')$$

where E' is the locally constant sheaf contragredient to E.

Ref: M. Goresky - R. MacPherson, "Intersection Homology Theory" Topology 18 (1980), 135-162; Intersection Homology II (preprint); Cheeger, Goresky, MacPherson: L^2 cohomology and Intersection homology (recent Annals of Math Studies, ed. by S.T. Yau); various preprints.

A. Borel

Determination of Green functions.

Let G be a connected reductive group over $\mathbb{F}_q$ with Frobenius endomorphism F. If char $(\mathbb{F}_q)$ and q are large enough, Springer and Kazhdan have shown that the Green functions of G can be expressed in terms of F and some representations of the Weyl group W on the l-adic cohomology of the varieties $\mathcal{B}_u = \{B \mid B$ Borel subgroup of G, $B \ni u\}$, $u \in G^F$ unipotent. In the top cohomology groups, the actions of F and W can be computed. It has been noticed by Shoji that these informations and the results of Borho and MacPherson on Springer's representations can be used to transform the orthogonality relations into a system of equations for the Green functions. These can be used to compute the Green functions of exceptional groups. For classical groups Shoji has a geometric argument which gives more equations, and in theory the system of equations can also be solved. As applications, we get the results (already known for classical groups) that the Green functions are polynomials and that $\mathcal{B}_u$ has no odd cohomology.

N. Spaltenstein

Hecke algebras and their application to representations of finite Chevalley groups

The properties of the Hecke algebra of a permutation representation of a finite group, on the cosets of a subgroup, were summarized, along with formulas for degrees and character values for irreducible constituents of the permutation character, in terms of the values of irreducible characters of the Hecke algebra. With these ideas as background, the Hecke algebra $\mathcal{H}(G,B)$ of a finite Chevalley group G, and a Borel subgroup B, was described, along with the presentation of $H(G,B)$ due to Iwahori. Changing the point of view, the generic Hecke algebra $\mathcal{H}$ associated with a finite Coxeter group (W,R) was defined, so that if (W,R) is the Weyl group of G, then $\mathcal{H}(G,B)$ and $\mathbb{Q}W$ are both obtained as specializations of $\mathcal{H}$. We then have the deformation theorem, that $\mathcal{H}(G,B) \cong \mathbb{C}W$, and the parametrization of characters of $\mathcal{H}^K$ and the components of 1_B^G in terms of the irreducible characters of W. Generic degrees associated with characters of W were defined; they turn out (in general) to be polynomials which specialize to give the degrees of constituents of 1_B^G. A generic multiplicity formula was also given, with an application to determine the effect of the duality operation on components of 1_B^G. The possibility of extending these results to components of $\tilde{\lambda}^G$, for a cuspidal irreducible character of a Levi subgroup L_J of P_J, was indicated, using Howlett & Lehrer's result that $\mathrm{End}_{\mathbb{C}G}\,\tilde{\lambda}^G$ can be obtained as a specialization of a generic algebra associated with a subgroup of the Weyl group, depending on λ. An introduction was given to Kazhdan and Lusztig's results on representations of a generic Hecke algebra, using the idea of a W-graph for a finite Coxeter group W. The lecture concluded with a statement of Lusztig's theorem, that there exists an explicit isomorphism $\mathcal{H}^{\mathbb{Q}(\sqrt{u})} \cong \mathbb{Q}(\sqrt{u})W$. (For further discussion and references for part of this material, the reader may consult a survey article by the author (Bull. Amer. Math. Soc. vol 1 (N.S.)).

C.W. Curtis

Weyl group representations and Weyl group Fourier transformations

The Green functions studied by Springer in his article "Trigonometric sums, Green functions, and representations of Weyl groups" are basically the Fourier transform of the characteristic function of a regular semi-simple orbit in a semi-simple Lie algebra of over a finite field, evaluated at a nilpotent element. Also, the result of Borho - MacPherson, saying that for $\tilde{N} \xrightarrow{\pi} N$ the Springer resolution of the nilpotent variety $R\pi_*(\mathbb{Q}_\ell)[\dim N]$ decomposes as $\bigoplus_{(\mathcal{O},\varphi)} \underline{IC}^{\cdot}(\bar{\mathcal{O}}, L_\varphi) \otimes V_{(\mathcal{O},\varphi)}$, where $\mathcal{O}$ runs through nilpotent orbits in N, φ through equivariant representations of the finite group $\pi_0(\mathcal{O})$, L_φ denoting the associated local system on $\mathcal{O}$, and $V_{(\mathcal{O},\varphi)}$ a space where the Weyl group W acts irreducibly, has been recently derived by Kashiwara, using the formal Fourier transform of the $\mathcal{D}_{\mathfrak{g}}$ - holonomic module with regular singularities describing the equations satisfied by an invariant eigendistribution on $\mathfrak{g}$.

One may prove a variant of this result of Kashiwara, working with Deligne's Fourier transform for (perverse) sheaves in characteristic p: if $E \xrightarrow{p_1} X$ is a vector bundle over X smooth, of relative dimension m, and $G \in D^b_c(E, \bar{\mathbb{Q}}_\ell)$ one defines $\mathcal{F}(G) = R p_{2,*}(p_1^* G \otimes \mathcal{L}_{\psi,F})[m]$ where

$$\begin{array}{ccc} & E \times_X E^* \xrightarrow{\ F\ } \mathbb{A}^1 & \\ {}^{p_1}\swarrow & & \searrow^{p_2} \\ E & & E^* \end{array} \qquad F(x,\xi) = \langle x, \xi \rangle$$

$\mathcal{F}$ defines an equivalence from the category of perverse sheaves on E to the category of perverse sheaves on E^*. It is essentially compatible with Verdier duality. For $p: \tilde{\mathfrak{g}} = G \times^B \mathfrak{b} \longrightarrow \mathfrak{g}$, $R p_* \mathbb{Q}_{\ell,\tilde{\mathfrak{g}}}$ easily decomposes according to $\hat{W}$. By Fourier transform, so does $R\pi_* \mathbb{Q}_{\ell,\tilde{N}}$.

This Fourier transform for perverse sheaves promises to be a useful tool for the study of trigonometric sums (see a letter of Laumon to Katz).

J-L Brylinski   June 1982

Decomposition of the R_T^θ's.

Let G be a connected reductive algebraic group defined over F_q. Deligne and the author have constructed for each maximal torus $T \subset G$ defined over F_q and for each character $\theta: T(F_q) \to \bar{\mathbb{Q}}_\ell^*$ a virtual representation R_T^θ of the finite group $G(F_q)$. The lecture was concerned with the problem of decomposing these virtual representations in the case where G has connected centre. The main tool used is the intersection cohomology of Deligne - Goresky - Macpherson. Let X_w be the locally closed subvariety of G/B defined by $\{gB \mid g^{-1}F(g) \in BwB\}$, and let $\bar{X}_w$ be its Zariski closure. The following description of its intersection cohomology was given: $\sum (-1)^i \mathbb{H}^i(\bar{X}_w)\, u^{i/2} = \sum_{E \in \hat{W}} \mathrm{Tr}\,(\sum_{y \leq w} P_{y,w} T_y, \tilde{E})\, R_E$ as elements in the representation ring of $G(F_q)$ tensored by $\mathbb{Q}[u^{1/2}]$. Here $\tilde{E}$ denotes the representation of the Hecke algebra corresponding to an irreducible representation E of W, $P_{y,w}$ are the polynomials defined by Kazhdan and the author for any two elements in a Coxeter group and $R_E = |W|^{-1} \sum_{w \in W} \mathrm{Tr}(w, E) \sum (-1)^i H^i_c(X_w)$. (An analogous result in cohomology with compact support of X_w was proved by Asai and Digne - Michel. However, use of $\mathbb{H}^i$ gives more precise results and generalizes well to non-unipotent representations) The following theorem was stated and a proof was indicated. If E, E' are irreducible representations of W, then $R_E, R_{E'}$ are disjoint if and only if E, E' are in distinct two sided cells of W. This gives rise to a partition of the set of unipotent representations into families, one for each two sided cell of W. The representations in a given family can be parametrized by the elements of a set $\mathcal{M}(\Gamma)$ where Γ is a finite group associated to the cell and $\mathcal{M}(\Gamma) = \{(x, \sigma) \mid x \in \Gamma \text{ up to conjugacy}, \sigma \in Z(x)^\wedge\}$. The multiplicities of these representations in the R_E can be expressed in terms of a Fourier transform on $\mathcal{M}(\Gamma)$. This implies explicit character formulas for all unipotent representations of $G(F_q)$ on semisimple elements.

This generalizes to non-unipotent representations of $G(F_q)$ under the assumption that G has connected centre. Here one uses intersection cohomology of $\bar{X}_w$ with coefficients in local systems ~~…~~ of rank 1.

G. Lusztig , June 1982.

Modular representations of finite Chevalley groups (in equal characteristic)

Let G be a connected semi-simple algebraic group defined over $\mathbb{F}_q$ with Frobenius endomorphism F. For the sake of simplicity assume G to be simply connected and split. This talk gave a survey over the representation theory of the finite group G^F over $\overline{\mathbb{F}_q}$ and described its relations with the representation theories of G and of its Frobenius kernels $_FG$. It contained a description of Lusztig's conjecture how to express the formal characters of the simple G-modules in terms of the formal characters of the Weyl modules and how to obtain from this information the composition factors of the Deligne-Lusztig characters (for G^F) reduced mod p and of the universal highest weight representations for $_FG$. Furthermore it showed how the characters of the principal indecomposable modules for G^F as well as for $_FG$ might be computed from a knowledge of these character formulas. Finally some results on the decomposition of the reduction mod p of the unipotent characters of G^F were mentioned.

Jens C. Jantzen

Blocks in classical groups (Unequal characteristic)

This talk is an exposition of some results obtained (jointly with P. Fong) on the r-blocks of general linear, unitary, symplectic and orthogonal groups over $\mathbb{F}_q$, where r is an odd prime not dividing q. First, let $G = GL(n,q)$, and let e be the order of q mod r. The unipotent characters of G are parametrized by partitions of n. If λ is a partition of n, let χ^λ be the corresponding unipotent character. The first theorem is that χ^λ, χ^μ are in the same r-block if and only if λ, μ have the same e-core. Then, the r-blocks are classified.

There is a "Jordan decomposition theorem" for blocks similar to the Jordan decomposition of characters of $GL(n,q)$. Finally the characters in a block can be classified. These theorems were stated and the main ideas in the proofs were indicated. Analogous theorems hold for the unitary groups. Finally some work in progress for symplectic and orthogonal groups was described; for example the r-blocks can be classified in these groups also.

Bhama Srinivasan

A Duality Operation for Representations of finite Chevalley Groups.

A duality operation in the character ring $\mathrm{ch}(\mathbb{C}H)$ of a finite group H is a $\mathbb{Z}$-automorphism of period 2 preserving the inner product of characters, and thus permuting, up to sign, the irreducible characters. For a finite Coxeter system (W,R), such an operation is given by $\mu \to \mu\varepsilon$, where ε is the sign representation, which can also be expressed as $\mu\varepsilon = \sum_{J \subseteq R} (-1)^{|J|} \mu|_{W_J}^{W}$, using Solomon's formula for ε. Now let G be a finite Chevalley group. The adjoint operations of truncation $T_J : \mathrm{ch}\,\mathbb{C}G \to \mathrm{ch}\,\mathbb{C}L_J$ and induction $I_J : \mathrm{ch}\,\mathbb{C}L_J \to \mathrm{ch}\,\mathbb{C}G$, for a Levi subgroup L_J of a standard parabolic subgroup P_J, $J \subseteq R$, with unipotent radical V_J, are defined by: $T_J\zeta(x) = |V_J|^{-1} \sum_{v \in V_J} \zeta(xv)$, and $I_J\mu = \mathrm{Ind}_{P_J}^{G} \tilde{\mu}$, where $\tilde{\mu}$ is the lift of $\mu \in \mathrm{ch}\,\mathbb{C}L_J$ to P_J with V_J in the kernel. Now define, for $\zeta \in \mathrm{ch}\,\mathbb{C}G$, $\zeta^* = \sum_{J \subseteq R} (-1)^{|J|} I_J T_J \zeta$. This operation commutes with truncation: $T_J(\zeta^{*G}) = (T_J\zeta)^{*L_J}$, and using this fact it can be proved that $\zeta \mapsto \zeta^*$ is a duality operation. Some applications to character theory were indicated, including Alvis's interpretation of Springer's formula that $\rho^* = |G|_{p'} \chi_U$, where χ_U is the characteristic function on the unipotent set U, and ρ is the regular character, and Alvis' proof, for $q \gg 0$, of MacDonald's conjecture that for $\zeta \in \mathrm{Irr}\,G$, $\zeta(1)^{-1} \sum_{u \in U} \zeta(u) = \pm q^m$, if $G = \underline{G}(\mathbb{F}_q)$, for some $m \in \mathbb{Z}$. (See D. Alvis J. Alg. 74 (1982), 211-222.)

C.W. Curtis

A duality operation ...., second part.

In this second part a brief review was given of some additional results on the duality, viz.

(a) a homological interpretation of the duality, in terms of homology of a system of coefficients on the Tits building of G (after Deligne - Lusztig, J. Alg. 74 (1982), 284-291);

(b) a result of N. Kawanaka for the (similarly defined) duality operation for class functions on the finite Lie algebra $\mathfrak{g}$ associated with G. This result connects the Fourier transform on $\mathfrak{g}$, restricted to the nilpotent set, with the duality operation (N. Kawanaka, Fourier transforms of nilpotently supported invariant functions on a simple Lie algebra over a finite field, preprint).

T.A. Springer.

Algebraische Gruppen

21. - 27. Juni 1982

Green Polynomials of classical groups.

Let G be a connected reductive group defined over $\mathbb{F}_q$, $A \in \mathfrak{g}$, a nilpotent element of Lie algebra of G. For a closed subvariety $\mathcal{B}_A$ of the variety of Borel subgroup $\mathcal{B}$, Springer representation of the Weyl group W on l-adic cohomology $H^i(\mathcal{B}_A, \bar{\mathbb{Q}}_l)$ can be defined. For $A \in \mathfrak{g}^F$ (F is Frobenius endomorphism) Green functions $Q_{T_w}(A)$ is described using Springer rep'ns as follows. $Q_{T_w}(A) = \sum_{i \geq 0} (-1)^i \operatorname{Tr}(F^* r_i(w)^{-1}, H^i(\mathcal{B}_A))$.
In this talk, we give a systematic way of computing Green functions for Chevalley groups. This depends on the following three properties.
① Springer correspondence. ② Th. of Borho - Macpherson.
③ For $\phi \in \widehat{C(A)}$ (where $C(A) = Z_G(A)/Z_G^0(A)$), if ϕ-isotypic subspace of $H^{2d_A}(\mathcal{B}_A)$ ($d_A = \dim \mathcal{B}_A$) is zero, then the same is true for every $H^i(\mathcal{B}_A)$.
Actually, for each case, Springer correspondence is given explicitly (Shoji, Alvis, Lusztig, Spaltenstein), and the property ③ is serious in the case of classical groups. It is proved using the local triviality of the map $\mathcal{B}_A \to P_A$ for suitable subvariety of G/P and the classification of $\widehat{C(A)}_0 = \{\phi \in \widehat{C(A)} \mid H^{2d_A}(\mathcal{B}_A)_\phi \neq 0\}$.
As a corollary, Green functions turn out polynomials in q whose coefficient are independent of p. Also, we get a basis of the space of uniform functions of G^F whose support are in the set of unipotent element. In particular, characteristic functions on the set $O(u)^F$ (u = unip. $O(u)$ = G-orbit in $\bar{\mathbb{F}}_q$) are uniform. These methods for computation are also applied to the non-split group of type D_n. (See also the talk of Spaltenstein at last week (p.158))

T. Shoji

Invariant eigendistributions and holonomic systems

In this talk, first Kashiwara's main theorems about the Harish-Chandra systems on semisimple Lie algebras were introduced. Let $\mathfrak{g}$ be a complex semisimple Lie algebra and $p_1, \dots, p_r$ be generators of invariant polynomials on the dual of $\mathfrak{g}^*$. For $\lambda \in \mathfrak{h}^*$ = the dual of a Cartan subalgebra, we consider the system of LDE:
$(p_i(\partial_x) - p_i(\lambda))u = 0 \quad (i = 1, \dots, r)$, $L_A u = 0 \quad (A \in \mathfrak{g})$
where L_A is the vector field whose value at $x \in \mathfrak{g}$ equals $[A, X]$. Let $\mathcal{M}_\lambda$ be the $\mathcal{D}_{\mathfrak{g}}$-module given by this system (call it the Harish-Chandra system). Then it is easily seen that $\mathcal{M}_\lambda$ is a holonomic system in the sense of M. Sato. Let $\mathfrak{g}_{rs}$ be the set of regular semisimple elements in $\mathfrak{g}$. Then the local system $\mathcal{E}_\lambda := \mathcal{H}om_{\mathcal{D}_{\mathfrak{g}_{rs}}}(\mathcal{M}_\lambda|\mathfrak{g}_{rs}, \mathcal{O}_{\mathfrak{g}_{rs}})$ = the local solutions on $\mathfrak{g}_{rs}$, has a W-module structure (W = the Weyl group). The first theorem: $\mathbb{R}\mathcal{H}om_{\mathcal{D}_{\mathfrak{g}}}(\mathcal{M}_\lambda, \mathcal{O}_{\mathfrak{g}}) \simeq \underline{IC}^\cdot(\mathcal{E}_\lambda)[-\dim\mathfrak{g}]$ = the Deligne-Goresky-MacPherson middle intersection complex for $\mathcal{E}_\lambda$. Thus we have an analytic construction of the Springer representation. The second theorem concerns the "Fourier transform" $\mathcal{M}_0^F$ of the Harish-Chandra system $\mathcal{M}_0$ for $\lambda = 0$. We have a quasi-iso. $\mathbb{R}\mathcal{H}om_{\mathcal{D}_{\mathfrak{g}}}(\mathcal{M}_0^F, \mathcal{O}_{\mathfrak{g}}) \simeq \underline{IC}^\cdot(\mathcal{E}_\lambda)[-\dim\mathfrak{g} - \operatorname{rank}\mathfrak{g}]\big|_{\mathcal{N}}$ where $\mathcal{N}$ is the nilpotent variety of $\mathfrak{g}$. The RHS was decomposed by Borho-MacPherson according to the W-action, but here, we have the decomposition: $\mathcal{M}_0^F \simeq \bigoplus_{\chi \in \hat{W}} V_\chi \otimes \mathcal{M}(\chi)^F$ which corresponds to the decomposition $\mathcal{M}_0 \simeq \bigoplus_{\chi \in \hat{W}} V_\chi \otimes \mathcal{M}(\chi)$. Here the $\mathcal{D}_{\mathfrak{g}}$-module $\mathcal{M}(\chi)^F$ is a simple holonomic system supported by the closure of a single nilpotent orbit $\mathcal{O}(\chi) \subset \mathcal{N}$ and corresponds to the $\underline{IC}^\cdot$ of some local system on $\mathcal{O}(\chi)$. This recovers the original Springer correspondence between $\hat{W}$ and the local systems on nilpotent orbits.

Secondly as applications, we can determine the Fourier transform of nilpotent orbital measures $\mu_{\mathcal{O}}$ for any nilp. orbits $\mathcal{O}$ (This was discovered first by Barbasch-Vogan for "special" orbits). This leads to some consequences which generalize ~~the~~ some results by King and Joseph. concerning the relation among the $\hat{W}$, nilpotent orbits and irreducible characters of infinite dim. representations of G. (For analogue over finite fields, see Brylinski's lecture given in the last week, p. 160.) Ryoshi Hotta

On representations of Hecke algebras of affine Weyl groups

The study of unramified principal series representations of p-adic reductive groups are generalized by Matsumoto in terms of Hecke algebras. Let G be a connected semisimple grp (for simplicity, of adjoint type) over $\mathbb{C}$ and T be a maximal torus. Then the Weyl grp W of (G. T) naturally acts on X(T), the character grp of T and one can construct $\tilde{W}$, the semidirect product of W by X(T). This grp $\tilde{W}$ is an affine Weyl grp (a Coxeter grp). So one can define the Hecke algebra $\mathcal{H} = \mathcal{H}(\tilde{W}. q)$ with a parameter $q \in \mathbb{C}^{\times}$. Matsumoto constructed a nice family of $\mathcal{H}$-modules (principal series representations) M_s parametrized by every element s of T. Concerning to this module M_s, one can give a criterion for irreducibility in terms of the parameter s. This criterion resembles the irreducibility criterion for spherical principal series representations of real groups given by Kostant. Also this result suggests the connection between the set of equivalence classes of irreducible $\mathcal{H}$-modules and the set $F(q) = \{(s, N) \mid s \in G,$ semisimple; $N \in \mathrm{Lie}\, G$, nilpotent ; $\mathrm{Ad}(s)N = q^{-1}N\}/\overset{G}{\sim}$, what is called Deligne-Langlands conjecture. The general parametrization of irreducible $\mathcal{H}$-modules seems to be difficult. But in case $q=1$ (hence $\mathcal{H} = \mathbb{C}[\tilde{W}]$), one can construct $\tilde{W}$-representations on cohomology groups $H^i(\mathcal{B}_g)$ of the fixed point subvariety $\mathcal{B}_g$ ($g \in G$) of the flag variety by using Lusztig's method. In exactly the same way of Springer representations, one can obtain all irreducible representations of $\tilde{W}$ in the top cohomologies. In this case, the connection of irreducible $\mathcal{H}$-modules and $F(1) = \{$the set of conjugacy classes$\}$ are obvious.

Shin-ichi Kato

COHEN-MACAULAYNESS FOR MULTI-CONES OVER SCHUBERT VARIETIES

Let G be a semi-simple simply-connected Chevalley group over a field k; let T be a maximal torus in G and B a Borel subgroup $\supset T$. Let W be the Weyl group of G relative to T. Let Q be a parabolic subgroup of G containing B and W_Q the Weyl group of Q. For $w \in W/W_Q$, let $X(w)$ (= Zariski closure of $BwQ \pmod{Q}$ with the canonical reduced structure) be the Schubert variety in G/Q. Let $Q = P_1 \cap \cdots \cap P_r$, where P_i, $1 \le i \le r$ are maximal parabolic subgroups of G. Let L_i be the ample generator of $\mathrm{Pic}(G/P_i)$, $1 \le i \le r$ and let $L = \bigotimes_{i=1}^{r} L_i^{a_i}$, $a_i \in \mathbb{Z}^+$, be a positive line bundle on G/Q. Let $R(w) = \bigoplus_{L \ge 0} H^0(X(w), L)$

Then we have Theorem 1: If G is of type A_n, then $R(w)$ is C.M.
[Let G be a classical group. For $w \in W$, call $X(w)$, a Kempf variety, if ① $\pi|_{X(w)} : X(w) \longrightarrow \mathrm{Im}\, X(w)$

is equi-dimensional (where $\pi : G/B \longrightarrow G/P$ is the canonical projection map from G/B onto G/P, P being the maximal parabolic subgroup corresponding (S_{α_1}) and ② Fibers of $\pi|_{X(w)}$ are Kempf varieties in lower rank. Then we obtain a characterization of Kempf varieties by means of standard monomials and as a consequence we obtain Theorem 2. For G being of type A, B, C or D and $X(w)$ being a Kempf variety, the ring $R(w)$ is Cohen-Macaulay.

V. Lakshmibai

A finer decomposition of Bruhat cells.

Let G be a semisimple algebraic group over an alg. closed field k. Let $B \supseteq T$ be respectively a Borel subgroup and a maximal torus. Let $W = N(T)/T$ be the Weyl group. One the has the Bruhat decomposition of ~~G (contradicts of~~ G/B) into cells which are parametrized by elements of W; $G/B = \bigcup_{y \in W} By \cdot B$. Each Bruhat cell $By \cdot B$ is isomorphic to an affine space $k^{l(y)}$ where l is the length function in W. This talk gave a further (and finer in some sense) decomposition of $By \cdot B$ into sets $\{A_{\underline{\sigma}}\}_{\underline{\sigma} \in \mathfrak{S}}$ ($\mathfrak{S}$ is some indexing set) each of which is isomorphic to a product of an affine space $k^{m(\underline{\sigma})}$ and a torus $(k^*)^{n(\underline{\sigma})}$. Further, ~~One~~ there exists a map $\pi : \mathfrak{S} \to W$ such that $A_{\underline{\sigma}} \subseteq w_0 B w_0 \pi(\underline{\sigma}) \cdot B$ (where w_0 is the maximal element of W). This in particular gives a description of $By \cdot B \cap w_0 B w_0 x \cdot B$ for $x \leq y$; this intersection is of interest in several different contexts eg. in Kazhdan Lusztig polynomials. The set $\mathfrak{S}$ is the set of certain special subexpressions of a fixed reduced expression of y. One further considers the closures (in $By \cdot B$) of $A_{\underline{\sigma}}$'s. One then gets a partial order $\leq$

in $\mathcal{S}$ such that $\bar{A}_{\underline{\sigma}} = \bigcup_{\underline{\sigma} \le \underline{\tau}} A_{\underline{\tau}}$. One has an explicit description of this order $\le$. The ~~discussion applied~~ so far is applicable to the case of an affine Weyl group with minor changes.

One now considers the situation in an arbitrary Coxeter group (W, S). The set $\mathcal{S}$, ~~and~~ the map $\pi: \mathcal{S} \to W$ and the order $\le$ still makes sense and one ~~looks at~~ the applications in this case. The first application is to give an explicit description of the polynomials '$R_{x,y}$' which occur in the context of Kazhdan-Lusztig polynomials. viz. $R_{x,y}(q) = \sum_{\underline{\sigma} \in \mathcal{S}, \pi(\underline{\sigma}) = x} q^{m(\underline{\sigma})} \cdot (q-1)^{n(\underline{\sigma})}$ (where $m(\underline{\sigma})$, $n(\underline{\sigma})$ are as mentioned before)

Another application ~~to~~ to the L-shellability of the Bruhat ordering. One further proves that there is a subset $\mathcal{S}_0$ of $\mathcal{S}$ such that $\pi: \mathcal{S}_0 \to W(y)$ ($W(y) = \{x \in W \mid x \le y\}$) is an isomorphism of posets. Thus the order $\le$ on $\mathcal{S}$ 'covers' the Bruhat ordering on W.

Vinay V. Deodhar

Let G be a semisimple algebraic group over a field K, algebraically closed of characteristic $\neq 2$. Let $\Theta: G \to G$ be an involutive automorphism H the fixpoint subgroup of Θ. Then one can find a projective variety X with the following properties:

1) $G/N(H) \hookrightarrow X$ as a dense open set and the action of G on $G/N(H)$ extends to an action of G on X.
2) Every orbit closure in X is smooth (in particular X is smooth)
3) $X - G/N(H)$ is a union of l hypersurfaces which are orbit closures smooth and meet transversally, l the rank of G/H.
4) The G-orbits of X correspond to the subsets of the set of restricted simple roots

Further one can describe each orbit closure as a locally trivial fibration on a suitable G/P (P parabolic) with fibers a compactification of the same type for the adjoint group associated to the Levi component L of P and an induced involution θ_L in L.

In the case $G = SL(4)$, $\theta(x) = {}^t x^{-1}$ one can use the above compactification to establish rigorously Schubert's computation of the number 666,841,088 of quadrics in $\mathbb{P}^3$ tangent to 9 quadrics in general position.

Claudio Procesi

Representations with a free algebra of invariants.

Let G be a complex connected semi-simple linear algebraic group and $\pi: G \to GL(V)$ a rational representation. Assume that π does not contain the trivial representation.

In the talk the proof, due to V.L. Popov (Izv. Akad. Nauk SSSR, 46 (1982), 347–371), of the following result was discussed: If the algebra of invariants $\mathbb{C}[V]^G$ is free, i.e. is a polynomial algebra with homogeneous generators, then there are, for a fixed G, only finitely many possibilities for the isomorphism class of V.

Popov's proof gives explicit bounds. It uses properties of Poincaré series.

T. A. Springer

INVARIANTS OF UNIPOTENT GROUPS

A regular unipotent subgroup U of Gl_n is given by a subset Ψ of the root system $\Phi = \{(i,j) \mid 1 \le i,j \le n,\ i \ne j\}$ such that Ψ is a strict ordering of the set $\Omega = \{1,\dots,n\}$. Conjecture: If Gl_n acts on $k[X] = k[X_{ij} \mid 1 \le i \le n,\ 1 \le j \le N]$ in the natural way, then $k[X]^U$ is generated by the invariant minors of the matrix X. A necessary and sufficient condition is given for the stronger property that $k[X]^U$ is spanned by the invariant standard bitableaux; this proves the conjecture in many cases. By Grosshans' criterion, in these cases, the U-invariants are finitely generated whenever Gl_n (or Sl_n) acts rationally on a finitely generated k-algebra. This gives a positive (characteristic free) answer to Hilbert's 14^{th} problem in many cases.

Klaus Pommerening

Existence and non-existence of finite presentations for some classes of arithmetic groups over global function fields.

Let G be an almost simple alg. gp., defined over a global function field k with ring O_S of S-integers and Γ a S-arithmetic subgroup of G. Denote by s the number of primes in S ($0 < s < \infty$), by r the rank of G and by $\hat{r}_i$ the rank of $\hat{G}_i = G \otimes_k k_{v_i}$ for $v_i \in S$, if $s = 1$ we write only $\hat{r}$.

Then the following list of results is known:

I, Γ is not finitely generated $\Leftrightarrow$ $s = 1$, $r = \hat{r} = 1$.

II, 1, $r = 0$: Γ is always finitely presented (f.p.)

2, $r = 1$, $s \geq 2$: Γ is f.p $\Leftrightarrow \sum_{i=0}^{s} \hat{r}_i \geq 3$
(for $G = SL_2$ this is due to U. Stuhler)

$r = 1$, $s = 1$, $\hat{r} = 2$: there exist examples of not f.p. groups Γ.

3, $r = 2$, $s = 1$, $\hat{r} = 2$: Γ is not f.p (for classical G)

4, $r = 2$, G split, $G \neq$ type G_2, $O_S = \mathbb{F}_q[t, t^{-1}]$ ($s = 2$)
Γ is f.p (Hurrelbrink)

5, $r \geq 3$, G split, $O_S = \mathbb{F}_q[t]$ ($s = 1$): Γ is f.p
(Rehmann - Soulé)

The ideas of the proofs for case 2? and 3? were given.

Helmut Behr (Frankfurt a. M.)

Free subgroups of semi-simple groups.

In connection with various developments arising out of the Hausdorff paradox (1914), T. J. Dekker asked (1956-58) whether there exists a free subgroup (non-commutative is always understood) $F \subset SO(n+1)$ acting on S^n freely if n is odd, with commutative isotropy groups if n is even. He observed this was certainly so if $n \equiv -1$ (4) in the first case, $n \neq 4$ in the second one. In answer to Dekker's first question, P. Deligne and D. Sullivan proved recently that $SU(n)$ contains a free subgroup acting freely on S^{2n-1}. They use the existence of division algebras with involutions of the second kind, of arbitrary degree, over number fields. As a generalization of this result, which also settles affirmatively Dekker's second question, I showed that a compact connected semi-simple group G contains a free subgroup F which acts freely on G/U if $rk\, U < rk\, G$ and one F' which has commutative isotropy groups on G/U if $rk\, G = rk\, U$. For F' one needs only to take a free subgroup of a principal three-dimensional subgroup. The main point for the existence of F is the following theorem, in which G is now a connected semi simple group over any field k.

<u>Theorem</u>: Let $m \geqq 2$ and $w(X_1, \dots, X_m)$ a non-trivial element in the free group over $X_1, \dots, X_m$. Let $f_w : G^m \to G$ be the map defined by $g = (g_i) \longmapsto w(g_1, \dots, g_m)$. Then f_w is dominant.

This is first proved for SL_n by induction on $n \geqq 2$, using the existence of a division algebra of $d^\circ\, n^2$ over some infinite field of the same char. as k, and then for general G by induction on $\dim G$.

From this and the unirationality of G over k (Grothendieck), one derives notably that if k has infinite transcendence degree over its prime field, then $G(k)$ contains a free subgroup F such that any $x \in F - \{1\}$ generates a Zariski-dense subgroup in a maximal torus of G. It follows that F acts freely on $G(k)/U(k)$ whenever U is a closed k-subgp. of rank $<$ rk G.

A. Borel

Birational properties of varieties of semi-simple groups

Let f be a nongenerate quadratic form on n-dimensional vector space V over a field K, char$(K) \neq 2$. Let Spin(f) is the spinor group over K of the form f. At the Congress in Helsinki P. Deligne was formulated following question: Is the variety Spin(f) K-rational, in particular for $K = \mathbb{R}$? For a long time it was conjectured that the varieties of simply connected groups are always rational. However, the author showed that the varieties $SL_{\mathcal{D}}(n)$ determined by the group $SL(n, \mathcal{D})$, where $\mathcal{D}$ is a division ring of finite K-rank, can be not K-rational. With the connection Deligne's question is proved following theorems.

Theorem 1. Let $f(z) = z_1^2 + x_1 z_2^2 + \cdots + x_{n-2} z_{n-1}^2 + (x_1 x_2 \cdots x_{n-2}) z_n^2$ be a quadratic form on n variables over $K = \mathbb{Q}(x_1, x_2, \ldots, x_{n-2})$. Then the variety Spin$(f)$ is not rational over K for $n \equiv 2 \pmod 4$, $n \geq 6$.

Theorem 2. If $f(z) = \sum_{i=1}^{n} z_i^2$ then Spin(f) is rational over arbitrary field K

theorem 3. The variety Spin(f) is K-rational for arbitrary locally compact (nondiscrete) field K.

V. P. Platonov (Minsk USSR)

Tensor Products and Filtrations for Rational Representations of Algebraic Groups.

Let G be a connected affine algebraic group over k, an algebraically closed field. Let B be a Borel subgroup containing T, a maximal torus and X the character group of T. For $\lambda \in X$ we denote ~~by~~ also by λ the one dimensional B module on which T acts with weight λ. For $\lambda \in X$, $Y(\lambda) = \mathrm{Ind}_B^G(\lambda)$ – the induced G module. A G-module V has a good filtration (g.f.) if there is a filtration $0 = V_0, V_1, V_2 \dots$ of V s.t., for each i, V_i/V_{i-1} is either zero or $Y(\lambda_i)$ for some $\lambda_i \in X$.

Conjecture If V has a g.f. then $V|P$ has a g.f. for any parabolic subgroup P of G, moreover if V' is also a G module with a g.f. then $V \otimes V'$ has a g.f.

We prove the conjecture for $p > 41$ (for arbitrary p if G is classical; for $p > 2$, F_4, E_6; for $p > 19$ for E_7 and $p > 41$ for E_8). That $V \otimes V'$ has a g.f. for G of type A_ℓ or p large compared with the Coxeter number was proved by Wang Jian-pan.

S Donkin (Cambridge, England)

Partial resolutions of nilpotent varieties (Joint work with W. Borho)

Let $\pi : \tilde{\mathcal{N}} \to \mathcal{N}$ be the Springer resolution of the nilpotent variety $\mathcal{N}$ of a reductive algebraic group G with Weyl group W. In previous work, we showed that the endomorphism ring of $R\pi_* \underline{\mathbb{Q}}_{\tilde{\mathcal{N}}}$ is naturally isomorphic to the group ring of W. Now we consider the partial resolution $\xi: \tilde{\mathcal{N}}^P \to \mathcal{N}$ obtained by replacing the Borel subgroups in the construction of $\tilde{\mathcal{N}}$ by parabolic subgroups conjugate to P. If $\eta: \tilde{\mathcal{N}} \to \tilde{\mathcal{N}}^P$ is the projection, we have End $R\eta_* \underline{\mathbb{Q}}_{\tilde{\mathcal{N}}}$ is naturally isomorphic to the group ring of the Weyl group of the Levi part of P. As a corollary, we compute that the cohomology of the Steinberg fiber $\xi^{-1}(x)$ is the W invariant part of the cohomology of the Springer fiber $\pi^{-1}(x)$.

R. MacPherson

Adjoint Quotients for Kac-Moody Groups and Deformations of Singularities.

Let G be the group associated to a Kac-Moody algebra with simply connected root datum of rank r (cf. the talk of Tits). Using the traces of the fundamental representations of G we define an adjoint quotient $\chi: \mathcal{G} \to \mathbb{C}^r$ on the set $\mathcal{G}$ of trace class elements of G. We analyse the fibers of the restriction of χ to the subset $\mathcal{G}^0$ of elements conjugate into a Borel subgroup B, and we obtain a complete classification of the conjugacy classes in $\mathfrak{g}^\circ$. These results allow a partial embedding of the semiuniversal deformation of simply elliptic or cusp singularities of degree ≤ 5 into the map χ. The base of these deformations was described by Looijenga as a partial compactification of an orbit space of the Weyl group W. An analysis of the restriction of χ to the normalizer N of T relates the boundary components of this compactification to the cosets of T in N.

Peter Slodowy

Groups associated with Kac-Moody algebras

Let $\mathfrak{R}$ be the data consisting of a free abelian group Λ, a finite system $(\alpha_i)_{i\in I}$ of elements of Λ and a system $(h_i)_{i\in I}$ (in 1-1 correspondence with the previous one) of elements of the $\mathbb{Z}$-dual $\Lambda^\vee$ of Λ, such that the matrix $A=(A_{ij})=(\langle\alpha_j,h_i\rangle)$ is a generalized Cartan matrix, i.e. $A_{ii}=2$, $A_{ij}\in\mathbb{Z}$, $A_{ij}\leq 0$ if $i\neq j$ and $A_{ij}=0 \Rightarrow A_{ji}=0$.

If A is "definite" (i.e. product of a positive definite symmetric matrix by an invertible diagonal matrix), the theory of Chevalley associates to such a data $\mathfrak{R}$ a groupe scheme over $\mathbb{Z}$ (the Chevalley scheme), hence a functor $\mathfrak{G}_{\mathfrak{R}}$ from the category of rings to the category of groups. Can one extend that to an arbitrary system $\mathfrak{R}$?

The case where A is "semi-definite" suggests that one must rather try to define two functors $\mathfrak{G}_{\mathfrak{R}}$ and $\widehat{\mathfrak{G}}_{\mathfrak{R}}$ from the category of rings to the category of topological groups, where $\widehat{\mathfrak{G}}_{\mathfrak{R}}(R)$ is the completion of $\mathfrak{G}_{\mathfrak{R}}(R)$ (whenever the latter is defined: it is not clear that $\mathfrak{G}_{\mathfrak{R}}(R)$ will have a natural meaning for an arbitrary ring R). For example, suppose that $I=\{-,+\}$, $\Lambda=\mathbb{Z}$ identified with its dual in the obvious way, $h_\pm=\pm 1$, $\alpha_\pm=\pm 2$, hence $A = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$. The corresponding Kac-Moody algebra over $\mathbb{C}$ is known to be $\mathfrak{sl}_2(\mathbb{C}) \otimes \mathbb{C}[T, T^{-1}]$, hence the natural guess $\mathcal{G}_{\mathfrak{R}}(\mathbb{C}) = SL_2(\mathbb{C}[T,T^{-1}])$. On the other hand, the Iwahori-Matsumoto-Bruhat-T. theory associates A to the group $SL_2(\mathbb{C}((T)))$, which will be $\widehat{\mathcal{G}}_{\mathfrak{R}}(\mathbb{C})$ in this case.

The above program has been carried out to the following extent: to every $\mathfrak{R}$ are naturally associated two functors $\mathcal{G}_{\mathfrak{R}}$, $\widehat{\mathcal{G}}_{\mathfrak{R}}$ from the category of principal ideal domain to the category of topological groups. The case of fields had been treated earlier under various characteristic restrictions and for special choices of A by Moody, Teo and Marcuson, and for matrices A of "affine type" by Garland.

In the lecture, the definition of $\mathcal{G}_{\mathfrak{R}}$ and $\widehat{\mathcal{G}}_{\mathfrak{R}}$ was sketched and various questions concerning them were discussed, such as: BN-pairs in $\mathcal{G}_{\mathfrak{R}}(K)$ and $\widehat{\mathcal{G}}_{\mathfrak{R}}(K)$ (K a field) and the corresponding Bruhat decompositions (the decompositions BNB and B^-NB, which are "equivalent" in the classical case, become here essentially different); elementary description of $\widehat{\mathcal{G}}_{\mathfrak{R}}(R)$ in the "semi-definite case"; algebro-geometric structure of $\widehat{\mathcal{G}}_{\mathfrak{R}}(\mathbb{C})$ (an ind-pro-variety); Schubert varieties (over $\mathbb{C}$) and Demazure desingularisation.

J. Tits

On Zariski-dense subgroups of simple algebraic groups.

The following strong approximation result was stated and its corollaries discussed

Theorem. Let k be an algebraic number field, G an absolutely almost simple simply connected algebraic group defined over k.

Let Γ be a Zariski-dense subgroup of $G(k)$. Then there exist a subfield k_1 and a finite set S of primes of k_1, containing all archimedean ones, such that

G is defined over k_1 and

the closure of Γ in $\prod^{res}_{v\notin S} G(k_v)$ contains an open subset of $\prod^{res}_{v\notin S} G(k_{1,v})$.

Parts of the proof of this result and its applications were obtained in collaboration with Ch. Matthews and L. Vaserstein. The proof uses classification of finite simple groups.

B. Weisfeiler

Cohomology of arithmetic groups and special values of L-functions

In the theory of modular symbols one obtains information concerning the special values of L-functions attached to modular forms by integrating the modular forms against certain cycles. The result can be interpreted as an intersection number and thus yields the algebraicity of the value $L(f,1)$ after dividing it by a transcendental period. This method goes back to Eichler, Shimura and Manin.

In my talk I presented this method ~~from~~ and some variations from a more abstract and unified point of view. One uses the theory of representations of the group of finite adeles $GL_2(\mathbb{A}_f)$ to interprete the intersection numbers in terms of an intersection intertwining operator, between two irreducible $GL_2(\mathbb{A}_f)$ modules. Then the L-values enter as normalizing factors between this intertwining operator and another one constructed from local data. This method allows us to prove results that are more general and more precise than those previously known. There is also some hope that we may generalise this to some higher dimensional groups.

G. Harder

Cohomology of arithmetic subgroups of SL_2 and automorphic forms

Let Γ be a torsionfree arithmetic subgroup of a semi-simple algebraic $\mathbb{Q}$-group $\underline{G}$ with $rk_{\mathbb{Q}}\underline{G} > 0$. The real Liegroup $G = \underline{G}(\mathbb{R})$ operates freely and discontinuously on the associated symmetric space $X = G/K$ ($K \subset G$ maximal compact subgroup) resp. on the space $\Omega^*(X)$ of $\mathbb{C}$-valued differential forms on X. The Eilenberg-Maclane cohomology groups $H^*(\Gamma, \mathbb{C})$ may identified with the cohomology $H^*(\Gamma\backslash X, \mathbb{C})$ of the subcomplex $\Omega^*(X)^\Gamma$ of Γ invariant elements in $\Omega^*(X)$. We discussed the attempt to relate these cohomology groups with the theory of automorphic forms, there in particular the theory of Eisensteinseries (as developed by Selberg and Langlands) For this purpose one studies the natural restrictions $r_P^*: H^*(\Gamma, \mathbb{C}) = H^*(\Gamma\backslash\bar{X}, \mathbb{C}) \longrightarrow H^*(e'(P), \mathbb{C})$ of the cohomology of the Borel-Serre compactification $\Gamma\backslash\bar{X}$ of $\Gamma\backslash X$ on the cohomology of a face $e'(P)$ in the boundary $\partial(\Gamma\backslash\bar{X})$ of $\Gamma\backslash\bar{X}$ (P a proper parabolic $\mathbb{Q}$-subgroup of G). We described the conditions under which one can associate to a given cuspidal class in $H^*(e'(P), \mathbb{C})$ a non-trivial class in $H^*(\Gamma\backslash X; \mathbb{C})$, which is represented by the value of a suitable Eisensteinseries at a special point Λ_0. Various methods were indicated to decide if the Eisensteinseries in question has a pole at this point λ_0 or not. In some cases this question is related to the problem of unitarizibility of Langlands' quotients in the theory of irreducible admissible representations of G. The general results one can obtain by these various methods give us a complete picture in the case SL_3:

<u>Thm</u>: Let $\Gamma = \Gamma(m) \subseteq SL_3(\mathbb{Z})$ a full congruence subgroup of level $m \geq 3$. Then one has a direct sum decomposition

$$H^*(\Gamma \backslash X, \mathbb{C}) = H^*_{cusp}(\Gamma \backslash X, \mathbb{C}) \oplus H^*_{Eis}(\Gamma \backslash X, \mathbb{C})$$

in the cusp cohomology and a space which is generated by Eisensteincohomologyclasses. These classes have a closed, harmonic representative which is either a value of a suitable Eisenstein-series or a residue of such at a point λ_0. H^*_{Ein} restricts isomorphically onto $\mathrm{Im}\, r^*: H^*(\Gamma \backslash \bar{X}, \mathbb{C}) \longrightarrow H^*(\partial(\Gamma \backslash \bar{X}), \mathbb{C})$.

The structure of $H^*_{Ein}(\Gamma(m) \backslash X, \mathbb{C})$ as a $SL_3(\mathbb{Z}/m\mathbb{Z})$-module is also obtained by these methods.

I take this opportunity to mention a result on $H^*_{cusp}(\Gamma(m) \backslash X, \mathbb{C})$ obtained in joint work with R. Lee: If $m \equiv 3 \mod 8$ and $m \equiv -1 \mod 3$ then $\dim_{\mathbb{C}} H_{cusp}(\Gamma(m) \backslash \bar{X})$ is greater than $m(m+1)$.

J. Schwermer

June 25, 82.

P-invariant distributions on $GL(n)$.

Let F be a local nonarchimedean field, $G = GL(n,F)$ be general linear group, P be the subgroup of matices which last raw is equal to $(0,0,\dots,0,1)$. Consider the adjoint action of G on the space $X = Mat(n,F)$, $ad\,g(x) = gxg^{-1}$.

Theorem. Let $\mathcal{E}$ be a distribution on X invariant under the action $ad\cdot$ of the group P. Then it is invariant under the action ad of the whole group G.

Remarks. 1. The same is true for the adjoint action of G on $X = G$.

2. The theorem is obviously false for finite F. I think it is true for $F = \mathbb{R}$ or $\mathbb{C}$.

Corrolary. Let (π, E) be a smooth irreducible representation of G in the vector space E, $(\hat{\pi}, \hat{E})$ be the contragradient representation. Then any P-invariant bilinear pairing $B : \widetilde{E} \times E \to \mathbb{C}$ is G-invariant and hence is proportional to the standard pairing.

Corrolary. Let (π, H) be a unitary topologically irreducible representation of G in the Hilbert space H. Then the restriction π to P is also topologically irreducible. If π is nondegenerate, the scalar product in H can be written as a standard integral in a Kirrillov's model of H.

Let T be the space of polynomials over F, $\sigma : X \to T$ be the characteristic map $\sigma(x)$ = characteristic polynomial of x. In the proof I use Gelfand-Kazhdan's principal: If the statement of the theorem is true for any fiber $X_t = \sigma^{-1}(t)$, then it is true for all X.

For instance, consider the case $n=2$. Let x be an element of X, O_x its G-orbit, C_x its centralizer, so that $O_x = G/C_x$. P-invariant distributions on O_x correspond to C_x-eigendistributions on G/P with eigencharacter $\nu(x) = |\det x|$. The space $G/P = A \setminus 0$, where $A = \{(a_1, a_2) | a_i \in F\}$. If x is anisotropic, C_x acts transitively on $A \setminus 0$, so there is only one ν-invariant distribution - the Haar measure μ, which is (G,ν)-invariant. If X is split, there are several orbits, but only one ν-invariant distribution μ. But if x is $\left(\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right)$, there is additional distribution. It correcponds to the P-invariant distribution on $O_x \subset X \setminus 0$ equal to $|a_{12}|^{-1} \delta_{a_{11}=a_{21}=a_{22}=0}$, where δ- is δ-function on a line. But surpraisingly it can not be extended to a P-invariant distribution on X. The easyest proof uses Fourier transform.

In general case, theorem implies the following statement.

<u>Statement</u>. Let A^* be the space of colomn-vectors of the length n, $Y^* = X \times A^*$ and δ be the product of adjoint action on X and standard action on A^*. Then any G-invariant distribution $\mathcal{E}$ on Y^* is concentrated on the subspace $X \times 0 \subset Y^*$.

Using this statement for $m<n$ il inductively prove main theorem.

Joseph Bernstein.

Some conjectures for root systems and finite Coxeter groups.

A brief account of some conjectures generalizing those of Dyson and Mehta, and the evidence in their favour.

(C1) Let R be a reduced (finite) root system. For each root $\alpha \in R$ let e^α be the corresponding formal exponential. Then the constant term in the Laurent polynomial $\prod_{\alpha \in R} (1-e^\alpha)^k$ (k a positive integer) should be equal to $\prod_{i=1}^{\ell} \binom{kd_i}{k}$, where $d_1, \ldots, d_\ell$ are the degrees of the fundamental invariants of the Weyl group. ("Dyson's conjecture" is the case where R is of type A_n.) (C1) is true for R of classical type (all k) by virtue of an integral formula of Selberg; also for all R and $k = 1, 2$.

More generally:

(C2) With R etc. as above, let q be an indeterminate. Then the constant term (i.e., not involving any e^α) in $\prod_{\alpha \in R^+} \prod_{i=1}^{k} (1-q^{i-1}e^{-\alpha})(1-q^i e^\alpha)$ should be $\prod_{i=1}^{\ell} \begin{bmatrix} kd_i \\ k \end{bmatrix}$, where $\begin{bmatrix} n \\ r \end{bmatrix}$ is the Gaussian polynomial (or q-binomial coefficient) which reduces to $\binom{n}{r}$ when $q = 1$.

(C3) Let W be a finite group of isometries of $\mathbb{R}^n$ generated by reflections. For each reflection $r \in W$ let $h_r(x) = 0$ be the equation of the reflecting hyperplane, and let $P(x) = \prod h_r(x)$. Then with $P(x)$ suitably normalized, the integral $\int_{\mathbb{R}^n} e^{-|x|^2/2} |P(x)|^{2k} dx$ should be equal to $(2\pi)^{n/2} \prod_{i=1}^{n} \frac{(kd_i)!}{k!}$, where d_i are the degrees of the fundamental invariants of W acting on $\mathbb{R}^n$. (Mehta's conjecture is the case where W is the symmetric group, acting by permutations of the coordinates.) (C3) is true for W of type A, B and D, also for W dihedral.

I G Macdonald

Arithmetic and cohomology of reductive group schemes over local regular rings of dimension ≥ 1.

Let X be an integral noetherian scheme, $k(X)$ the field of rational functions on X, G a reductive group scheme over X. Let E be a locally isotrivial principal homogeneous space for G over X.

Def. We say that E is rationally trivial if $E(k(X)) \neq \emptyset$.

Conjecture (Serre - Grothendieck, 1958, 1966). If X is a regular scheme then any rationally trivial principal homogeneous space for G over X is locally trivial in the Zariski topology on X.

Theorem. Assume that one of the following conditions holds:

(i) $\dim X = 1$ (G is arbitrary reduct. X-group);

(ii) $X = \operatorname{Spec} R$, where R is a complete local ring (G is arbitrary reductive group)

(iii) $\dim X = 2$, G is quasisplit over X

Then the conjecture is true.

Corollary 1. Let X be a smooth curve over an algebraically closed field k, G is a reductive k-group. Then $H^1(X_{zar}, G) \xrightarrow{\sim} H^1(X_{et}, G)$.

Corollary 2. Let R be a local ring, K its quotient field, G', G two semisimple group schemes over R such that there exists a K-isomorphism of their general fibres $\varphi_K : G' \otimes_R K \xrightarrow{\sim} G \otimes_R K$. Then there exist $\alpha \in \operatorname{Aut}_{K\text{-gr}}(G \otimes_R K)$ and R-isomorphism $\varphi : G' \to G$ which extend $\alpha\varphi_K$, i.e. $\varphi \otimes_R K = \alpha \circ \varphi_K$.

Yevsey Nisnevich (Cambridge, USA)

RIESZ SPACES AND OPERATOR THEORY.

28 JUNI – 3 JULI 1982

Positive projections in classical Banach lattices

If P is a positive projection on an arbitrary Banach lattice E then the range $H = P(E)$ is a closed linear subspace of E satisfying the following two conditions:

i) H is a lattice with respect to the ordering induced by E on H (not necessarily a sublattice of E!)

ii) $\forall_{e \in E} \exists_{h_0 \in H} \{h \in H : h \leq e\} \leq h_0$

Under what assumptions are these conditions also sufficient to ensure the existence of a positive projection $P: E \to E$ with range H? It turns out that there is a positive projection with $P(E) = H$ in each of the following cases (provided that (i),(ii) hold):

1. E is an L^p-space, $1 \leq p < \infty$.
2. H is finite dimensional, E is an arbitrary Banach lattice.
3. $E = \mathcal{C}_0(X)$ for some locally compact space X and for any two points $x, y \in \{z \in X : \exists_{h \in H} h(z) \neq 0\}$ $\varepsilon_x|_H$ is not a real multiple of $\varepsilon_y|_H$ where $\varepsilon_x, \varepsilon_y$ denote the respective Dirac measures.

K. Donner

ON THE BOUNDEDNESS OF THE HILBERT TRANS.

If $\mathbb{X}$ is a Banach space, $L_p(\mathbb{R}, \mathbb{X})$ is the Bochner space $f: \mathbb{R} \to \mathbb{X}$ s.t. $(\int \|f(r)\|^p dr)^{1/p} < \infty$.

$(H_\delta f)(t) = \int_{-\infty}^{t-\delta} \frac{f(r)}{t-r} dr + \int_{t+\delta}^{+\infty} \frac{f(r)}{t-r} dr$ and

$(Hf)(t) = \lim_{\delta \downarrow 0} (H_\delta f)(t)$ when it exists.

Burkholder: H is bounded if $\mathbb{X}$ satisfies the unconditional martingale difference sequence property, equivalent to: there is a biconvex ζ on $\mathbb{X} \times \mathbb{X}$ such that $\zeta(0,0) > 0$ and if ~~~~ $\|x\| \leq 1 \leq \|y\|$, then $\zeta(x,y) \leq \|x+y\|$.

Bourgain. If H is bounded, then $\mathbb{X}$ satisfies the above property.

We show that if H is well-defined, then $\mathbb{X}$ is superreflexive and H is bounded.

M. E. Lacey

Non-order bounded linear operators in Riesz spaces

We investigate the behaviour of different classes of linear operators $T: E \to F$ (E and F Riesz spaces) which have the property that they are order bounded when considered as mappings into F^u, the universal completion of F. This abstract formulation is adequate to generalize some classical and recent results to the setting where no measure space is involved. We report on two classes in this setting: Carleman operators and extended abstract kernel operators. This includes characterizations and properties for these classes.

Peter van Eldik

EQUILIBRIA IN MARKETS WITH A RIESZ SPACE OF COMMODITIES

Using the theory of Riesz spaces, we present a new proof of the existence of competitive equilibria for an economy having a Riesz space of commodities.

C. D. Aliprantis

POSITIVE COMPACT OPERATORS

Let E be a BANACH LATTICE, and let $T: E \to E$ be a positive, compact operator. IF $S: E \to E$ is an operator such that $0 \le S \le T$, then we ask what effect does the compactness of T have on S? The following is the main result answering this question.

THEOREM Let E be a Banach Lattice, and let $S, T : E \to E$ be two operators such that $0 \le S \le T$. IF T is compact then:

1. S^3 is a compact operator (although S^2 need not be compact).

2. S^2 is a Dunford-Pettis operator (although S need not be).

3. S is a weak Dunford-Pettis operator.

O. Burkinshaw

INTERSECTIONS OF IDEALS IN SIMPLEX SPACES

Let K be a compact Choquet simplex and let $A(K)$ denote the Banach space of all affine, continuous real-valued functions on K with the supremum norm. A subset E of ∂K (the set of extreme points of K) is facially closed if $E = \partial F$ for some closed face F of K. K has the property that $f^{-1}(0) \cap \partial K$ is facially closed for all $f \in A(K)$ if and only if it has property (P): the intersection of any family of ideals in $A(K)$ is an ideal. It is an open question, dating back to a paper of Effros in 1967, whether (P) implies that $A(K)$ is a Riesz space. Gleit (1972) gave an affirmative answer to this question when K is metrisable.

In this talk we give a simple proof of Gleit's result and also an extension:-

Theorem Suppose that whenever $q \in \overline{\partial K} \setminus \partial K$ there exists a compact set E with $E \setminus \{q\} \subseteq \partial K$ and q an accumulation point of E. Then (P) implies that $A(K)$ is a Riesz space.

A. J. Ellis

UNITAL EMBEDDING OF F-ALGEBRA'S

Let A be a uniformly complete semiprime f-algebra. Then the following are equivalent:

(i) A can be embedded as an order ideal in its f-algebra $\mathrm{Orth}(A)$ of orthomorphisms

(ii) A has the M.D. property (i.e., if $0 \le u \le vw$, $0 \le v, w \in A$, then there exist $p, q \in A$ such that $u = pq$, $0 \le p \le v$, $0 \le q \le w$)

(iii) A has property (*) (i.e., if $0 \le u \le v^2$, $0 \le v \in A$, then there exists $0 \le w \in A$ such that $u = w v$). The proof of this theorem is based on:

Theorem In a uniformly complete semiprime f-algebra A $\sqrt{uv}$ exists for all $0 \le u, v \in A$.

This theorem is a generalisation of a theorem due to M. Henriksen and D.G. Johnson, stating that $\sqrt{u}$ exists for all positive elements u of a uniformly complete unital f-algebra

C.B. Huijsmans.

Mappings of Certain Riesz Spaces

Theorem Suppose L is a Riesz space. The following two statements are equivallent:

(1) L is Riesz isomorphic to a function space L′ with the property that if f and g belong to L′ then there is a finite disjoint subset A of L′ such that each of f and g is a linear combination of the points of A, and

(2) If ρ is a positive linear functional defined on a directed subspace M of L, ρ can be extended to L (as a positive linear functional).

Theorem If X is a perfectly normal Baire space or X is ccc compact or X is a P-space then bounded pointwise convergence implies order convergence in $C(X)$

Theorem $C(X)$ is almost σ-complete and every convergence implies pointwise convergence if and only if X is a P-space

[signature]

Representations of groups by positive operators.

A bounded strongly continuous representation U of a locally compact abelian group G on a Banach lattice E is called a lattice action if each U_t is a lattice isomorphism. Such an action is called markovian if in addition there exists an U-invariant topological order unit. U is called irreducible if $\{0\}$ and E are the only closed U-invariant lattice ideals. Let G^* be the dual group and let $\sigma(U)$ be the spectrum of U (in the sense of Arveson).

Results: Let U be a lattice action. Then $\mathbb{1} \in \sigma(U)$ and $\chi \in \sigma(U)$ always implies $\overline{\{\chi^n : n \in \mathbb{Z}\}} \subset \sigma(U)$. Let in addition U be irreducible and Markovian. Then $\sigma(U) = [G/\ker U]^* =$ annihilator of $\ker U$ in G^*.

For a Markovian action U we find necessary and sufficient conditions for $\sigma(U)$ to be equal to G^*.

A point $\chi \in \sigma(U)$ is called a Fredholm point iff every approximate eigenvector (x_α) corresponding to χ possesses an accumulation point. For a bounded Radon measure μ set $U_\mu = \int U_t \, d\mu(t)$. Then the following is true: If $z \in \sigma(U_\mu)$ is a Riesz-point then $D = \{\chi \in \sigma(U) : \hat{\mu}(\chi) = z\}$ is nonvoid, finite, and consists of Fredholm points of U. As a corollary we obtain: Assume that there exists a $\mu > 0$ such that the spectral radius $r_\sigma(U_\mu)$ is a Fredholm point of U_μ. Then the Banach lattice E is the direct sum of finitely many orthogonal bands $E_1, \dots, E_n$ which all are invariant under U. Moreover $U_j := U|_{E_j}$ is irreducible and ~~ker~~ $G/\ker U_j$ is compact, hence for $f \in L^1(G)$ all U_f are compact. This generalizes results of Wolff, Greiner resp.

Manfred Wolff, Tübingen

Non order bounded disjointness preserving operators on uniformly complete Riesz spaces.

Let E be a uniformly complete Riesz space on which there exists a non order bounded linear operator with the property that $x \perp y \Rightarrow Tx \perp y$. E contains an atomless σ-inextensible principal projection band, and hence cannot support a locally convex locally solid Hausdorff topology. There is such an operator on an inextensible Riesz space if and only if it does not have the property that, for each weak order unit e and $x \geq 0$, x is a disjoint supremum of components of e. There are atomless inextensible Riesz spaces with this property. The corresponding problem for σ-inextensible Riesz spaces is open.

A.W. Wickstead.

Components of positive operators

Let L and M be Dedekind complete Riesz spaces and denote by $\mathcal{L}_b(L,M)$ the Riesz space of all order bounded linear operators from L into M. For any $0 \leq T \in \mathcal{L}_b(L,M)$ the Boolean algebra of components of T is denoted by $\mathcal{B}_T$. Components of T which are of the form $\bigvee_{i=1}^{n} Q_i T P_i$ (P_i and Q_i order projections in L and M respectively) will be called simple components, and the collection of all simple components of T is denoted by $\mathcal{A}_T$. Clearly, $\mathcal{A}_T$ is a subalgebra of $\mathcal{B}_T$.

We introduce some notation. Let $\mathcal{B}$ be a Boolean algebra and X a sublattice of $\mathcal{B}$. Put $X^{\uparrow} = \{ b \in \mathcal{B} : \exists\, x_\alpha \in X \text{ s.t. } x_\alpha \uparrow b \}$ and similarly define $X^{\downarrow}$. Furthermore, $X^{\uparrow\omega} = \{ b \in \mathcal{B} : \exists\, x_n \in X \ (n=1,2,\ldots) \text{ s.t. } x_n \uparrow b \}$ and in like manner define $X^{\downarrow\omega}$. The main result is as follows.

Theorem. Let L and M be Dedekind complete Riesz spaces with ${}^{\perp}(M_n^{\sim}) = \{0\}$. For any $0 \leq T \in \mathcal{L}_b(L,M)$ we then have $\mathcal{B}_T = \mathcal{A}_T^{\uparrow\omega\downarrow\uparrow}$;

Gefördert durch Deutsche Forschungsgemeinschaft

and if T is in addition order continuous then $\mathcal{B}_T = \mathcal{A}_T^{\uparrow\omega\downarrow\omega}$.

We mention some applications of the above result. Let L and M be Banach lattices. For any $0 \le T \in \mathcal{L}_b(L,M)$ let $(T)^-$ be the closure in the r-norm of the set of all operators of the form $\sum_{i=1}^n R_i T S_i$ (where $S_i \in \mathcal{L}_b(L)$, $R_i \in \mathcal{L}_b(M)$ $(i=1,\dots,n)$).

Theorem Let L and M be Dedekind complete Banach lattices with ${}^\perp(M_n^*) = \{0\}$. If $0 \le T \in \mathcal{L}_b(L,M)$ with order continuous norm (i.e., $T \ge S_\alpha \downarrow 0 \Rightarrow \|S_\alpha\| \downarrow 0$), then it follows from $0 \le S \le T$ in $\mathcal{L}_b(L,M)$ that $S \in (T)^-$.

As is known, if L^* and M have order continuous norms, then compact operators and Dunford-Pettis operators from L into M have order continuous norm.

Corollary Let L and M be Dedekind complete Banach lattices with L^* and M having order continuous norms, and suppose that $0 \le S \le T$ in $\mathcal{L}_b(L,M)$.
(i) If T is compact, then $S \in (T)^-$ (in particular this implies that S is compact, a result of P. G. Dodds and D. H. Fremlin).
(ii) If T is Dunford-Pettis, then $S \in (T)^-$ (in particular S is likewise Dunford-Pettis).

Ben de Pagter

~~Some order theoretical aspects of disintegration~~

~~In two different situations it is shown how an abstract integration procedure leads to disintegration:~~

Daniell-Stone Integration and Abstract Hardy Alg. Theory

In a recent paper [Arch. Math. 38 (1982), 258-265] LEINERT developed the Daniell-Stone integral extension procedure without the lattice condition. He started from a vector space E of realvalued functions on a set X, not supposed to be a lattice under the pointwise operations, and a positive linear functional $I: E \to \mathbb{R}$, subject to some continuity condition. The present author [Math. Ann. 258 (1982), 447-458] uses a fortified continuity condition on I and applies a different construction of $L^1(I|E) \supset E$ and the extension $\hat{I}: L^1(I|E) \to \mathbb{R}$. Our main result is the fact, familiar in the case of a vector lattice E, that the functions in $L^1(I|E)$ can be represented in terms of limits of isotonic sequences of functions in E^+.
The new set-up and the main result had been inspired by a typical example arising in the abstract Hardy algebra situation in the sense of Barbey-König [LNM Vol. 593]. Here $L^1(I|E)$ becomes the space of conjugable functions, to be defined in an appropriate sense.

Heinz König, Saarbrücken

Riesz spaces, vector measures and conical measures.

Let μ be a conical measure on the l.c.t.v.s. X such that every conical measure ν with $0 \le \nu \le \mu$ has resultant $r(\nu)$ in X. Let $K_\mu = \{r(\nu) : 0 \le \nu \le \mu\}$. If L is a Riesz space with order unit e, and if $A: L \to X$ is a linear map for which $A([0,e])$ has $\sigma(X,X')$ compact closure, then there is a conical measure μ on X such that the $\sigma(X,X')$ closure of $A([0,e])$ is precisely K_μ. Conversely, each K_μ is even the order continuous image of an order interval in a Dedekind complete Riesz space. The techniques used in the proof of the above are from the duality theory of Riesz spaces and the results place the Kluvanek characterization of the range of a vector measure within a purely order theoretic setting.

Peter Dodds, Bedford Park.

Some order theoretical aspects of disintegration.

In two different situations it is shown how an abstract integration procedure leads to disintegration.

1. From Hahn-Jordan-decomposition to disintegration.

The key argument leading easily to the desired disintegration in the context of spectral theory (in Hilbert space or Freudenthal or Alfsen-Shultz as well) is formalized as follows:

Let $(P_t)_{t\in\mathbb{R}}$ be an increasing, right continuous family in some Boolean σ-algebra $\mathcal{Z}$ and denote by $\pi: (S,\Sigma) \to \mathcal{Z}$ its Loomis-Sikorsky homomorphism. Then there is a measb. function $f: S \to \overline{\mathbb{R}}$ with $P_t = \pi(\{f \le t\})$, $\forall t\in\mathbb{R}$; moreover f is (essentially) given by an integral representation $f = \int t\,\mu(dt)$ for some Borel measure on $\mathbb{R}$ with values in the vector lattice of Σ-measb. functions on S.

2. A "non-commutative" Strassen disintegration theorem.

Let $(E,\le)$ be a monotone σ-complete ordered vector space with weak order unit u ($\sim \mathcal{P}(E)$ Boolean σ-algebra of split projections); let (S,Σ) be a measurable space and $\pi: \Sigma \to \mathcal{P}(E)$ a σ-homomorph. ($\sim \pi(\cdot)u: \Sigma \to E_+$: vector measure); let $\mu \in E_+^*$ σ-order-continuous positive linear functional ($\sim \sigma := \mu\circ\pi(\cdot)u : \Sigma \to \mathbb{R}_+$: measure). Define $x \le y$ μ-a.e. $:\Leftrightarrow \mu(\pi(A)x) \le \mu(\pi(A)y)$, $\forall A\in\Sigma$.

Let $p: F \to E$ (F any $\mathbb{R}$-vector space) be sublinear mod "$\le \mu$-a.e." and let $\Phi \in F^*$ be dominated by $\mu\circ p$.

Then (if μ is strictly positive and $\mathcal{L}^1(\sigma) \subset \mathcal{L}^1(\pi(\cdot)u)$ in case u is not a strong order unit) Φ can be disintegrated below p, i.e. $\Phi = \mu\circ\varphi$ for some linear $\varphi : F \to E$ (even with $\operatorname{im}\varphi \subset \operatorname{im} \mathcal{S}d\pi(\cdot)u$, the local center at u) such that $\varphi \leq p$ pointwise μ-a.e.

A non-trivial application arises in the case $F = E =$ selfadjoint part of some W^*-algebra A, $\mu \in A_{*+}$ any positive linear functional in the predual, π the Loomis-Sikorsky homomorph. for the Boolean algebra of central projections, and $p : x \mapsto |x| := \sqrt{x^*x}$. Although $x \mapsto |x|$ is not sublinear with respect to the natural ordering in E (unless A is commutative), it is with respect to "$\leq \mu$-a.e.".

W. Hackenbroch, Regensburg

A problem about irreducible operators

Let L be a Dedekind complete Riesz space, $L^\sim$ its order dual and $L_n^\sim$ the band of all order continuous members of $L^\sim$. Furthermore, let $\mathcal{L}_r(L)$ be the Dedekind complete Riesz space of all regular linear operators in L. If L is an ideal in the Riesz space of all real μ-measurable functions on some σ-finite measure space (X,μ), then the band $(L_n^\sim \otimes L)^{dd}$ in $\mathcal{L}_r(L)$ is exactly the space of all regular kernel operators T in L (i.e. there exists $T(x,y)$ on $X\times X$ such that $(Tf)(x) = \int_X T(x,y) f(y)\, d\mu(y)$ for all $f \in L$). Accordingly, in the general case, $(L_n^\sim \otimes L)^{dd}$ is called the band of "abstract" kernel operators in L.

The positive operator T in L is called irreducible (band-irreducible) if T leaves no band in L invariant except $\{0\}$ and L and T is called strongly irreducible if, for any $u > 0$ in L, the image Tu is a weak unit in L. Furthermore, for $0 \leq T \in (L_n^\sim \otimes L)^{dd}$, we call T super irreducible if T is a weak unit in $(L_n^{\sim} \otimes L)^{dd}$. For (non-abstract) kernel operators the following holds:

Superirr ⇒ strongly irr ⇒ irr,

and none of the conclusions in the converse direction holds. Furthermore, if the positive kernel operator T is strongly irreducible, then T^2 is superirreducible. For "abstract" kernel operators the same results hold, but the proof for the T^2-result requires heavy machinery (L is Riesz isomorphically represented as a space of measurable functions; T becomes then a non-abstract kernel operator). The problem referred to in the title is to find a direct proof. This would lead to a direct proof of the Ando-Krieger theorem about the spectral radius of a positive irreducible abstract kernel operator in a Dedekind complete Banach lattice.

A. C. Zaanen, Leiden

Über Faltungsoperatoren und Multiplikatoren

Der Vortrag befaßt sich mit der Darstellung von Markov-Operatoren. Das zentrale Ergebnis lautet:

<u>Theorem</u>. Es sei S eine kompakte abelsche Halbgruppe mit der Eigenschaft, daß die stetigen Semicharaktere die Punkte trennen. Ferner existiere in S ein Punkt s_0 mit $\chi(s_0) \neq 0$ für alle stetigen Semicharaktere χ. Es sei T ein Markov-Operator auf $C(S)$ mit der Eigenschaft, daß jeder Semicharakter eine Eigenfunktion von T ist. Dann gibt es auf der Translationshülle S^* von S ein Wahrscheinlichkeitsmaß μ, so daß sich T wie folgt als Faltungsoperator darstellen läßt:

$$(Tf)(s) = \int_{S^*} f(s \cdot t)\, d\mu(t) \quad \text{für alle } f \in C(S) \text{ und } s \in S.$$

Als Anwendung wird gezeigt, daß gewisse Multiplikatoren auf kommutativen halbeinfachen Konvolutionsmaßalgebren als Faltungsoperatoren dargestellt werden können.

E. Scheffold, Darmstadt

Lipschitz Conditions for Operators.

For $1 \le p < \infty$ let $T \in \mathcal{B}(L_p)$, and for Δ any measurable subset of $[0,1]$ with measure $|\Delta|$ we consider the following conditions: (1) $\|P_\Delta T\|_p \le \varphi(|\Delta|)$, (2) $\|TP_\Delta\|_p \le \varphi(|\Delta|)$, and (3) $\|P_{\Delta'} T P_\Delta\|_p \le \varphi(|\Delta|)\psi(|\Delta|)$. Here P_Δ is the projection induced by Δ and $\psi, \varphi : [0,1] \to R^+$ are increasing, and continuous at 0 with $\varphi(0) = \psi(0) = 0$. ① If $\|TP_\Delta\|_p \le |\Delta|^{1/p}$ + $1 < p < \infty$ then T is representable: $(Tf)(x) = \int_0^1 g(x,y) f(y)\,dy$ where $\|T\| \sim \left(\int_0^1 \left(\int_0^1 g(x,y)^{p'} dy\right)^{p/p'} dx\right)^{1/p} < \infty$. ② If $\|TP_\Delta\|_p \le \Delta^{1/r}(1 - \ln|\Delta|)^{1/\alpha}$, then $T : L_{q,1;1/\alpha} \to L_p$ where $1/q = 1/p - 1/p$, and conversely. ③ If $\|TP_\Delta\|_2 \le |\Delta|^{1/2}(1 - \ln|\Delta|)^{1/\alpha}$ where $2 < \alpha < \infty$ then T is compact and moreover the singular numbers $\{s_n(T)\} \in l_{q,\infty}$ where $1/q = 1/2 - 1/\alpha$. Conversely if the singular numbers of an operator $T \in \mathcal{B}(L_2)$ are in $l_{q,\infty}$ then a unitary equivalent of T satisfies the condition above.

… Allen (College Station)

The Jordan Decomposition for Vector Measures.

Two methods are presented which yield Jordan decomposition theorems for vector measures with values in a Banach lattice. The first method is based on a common approach to vector measures and linear operators whereas the second one relies on factorization theorems which reduce the decomposition of vector measures to that of linear operators. Main result: Suppose μ is a vector measure on a ring of sets with values in a Banach lattice with property (P). Then μ is the difference of two positive orthogonal vector measures, if μ has bounded variation.

Klaus D. Schmidt (Mannheim)

A proof was given of a variational inequality that depends only on the Hahn-Banach theorem in a finite dimensional space. A new definition of directional derivative was discussed and a characterization was given of the directional derivatives of convex, hemicontinuous functions. A generalization of Fan's minimax inequality was given which can be used to improve the result given in a previous talk by Aliprantis on the existence of free disposal equilibrium prices in the model of a Walrasian economy based on a first space of commodities.

Stephen Simons (Santa Barbara)

Lattice-isometries in Riesz spaces.

Let $d(x,y) = |x-y|$ be the generalized distance in the Riesz space E. In order to bild the related geometry (metric g.) it is natural to investigate the mappings $T: E \to E$ that are distance-preserving (lattice-isometries). Since a lattice-isometries $S: E \to E$ can be expressed as $S = t \circ T$, where t is a translation, T a latt.-isometry and $T(0)=0$. We are thus led to the problem of studying those l-isometries that leave the 0 fix (homogeneous l-isometries, the set of which is $H(E)$). Since $T \in H(E)$ is linear, we are reduced to study modulus preserving linear operators in E ($|Tx| = |x|, \forall x \in E$). Homogeneous l-isometries are closely related to projection bands, as shown by the following

Theorem: Let E be a Riesz space (and denote by $\mathcal{P}(E)$ the set of all band projections). Then: (i) if $T \in H(E)$, then there exist a unique $P \in \mathcal{P}(E)$ s.t. $T = 2P - I$. Conversely, $P \in \mathcal{P}(E) \Rightarrow 2P - I \in H(E)$. (ii) With the canonical ordering of $\mathcal{L}(E)$, $H(E)$ is a Boolean algebra, that is isomorphic to the Boolean algebra $\mathcal{P}(E)$.

Part (ii) allows us to relate the lattice theoretic properties of E (completeness and projection properties) with the Boolean properties of $H(E)$. The consideration of $H(E)$ can be useful for the knowledge of the structure of the Riesz space, and specially f-algebras with multiplicative unity.

If $T \in H(E)$, then $T \in L^r(E)$ and $|T| = I$. Does the property $|T| = I$ characterize in $L^r(E)$ the h. l-isometry? !

Theorem: If E is an Arch. Riesz s., then:

i) The following are equivalent: a) $T \in H(E)$, b) $T \in L^r(E)$, $|T|$ exists, $|T| = I$
(of course, if E is DC, then: $T \in H(E) \Leftrightarrow |T| = I$)

ii) TFAE: a) $T \in H(E)$, b) $T \in \mathrm{Orth}\, E$, $|T| = I$, c) $T \in \mathrm{Orth}\, E$, $T^2 = I$

Next we have studied homogeneous l-isometries in special kinds of Riesz spaces. If $u > 0$, consider $\mathcal{B}_u(E) = \{x \mid x \wedge (u-x) = 0\}$. Then, if $b(x)$ stands for the band generated by x, and $N_u = \{x \in \mathcal{B}_u(E) \mid x \text{ is a proj. element}\}$

Theorem Let E be a vector lattice with a weak unit $u > 0$ ($b(u) = E$). Then:

i) With the ord. of E, N_u is a Boolean algebra, subalgebra of $\mathcal{B}_u(E)$

ii) $H(E) \simeq N_u$ (consq. u, u' weak units $\Rightarrow N_u \simeq N_{u'}$)

iii) $T \in H(E) \Rightarrow \exists! e \in N_u \mid T = P_e - P_{u-e}$ (P_e = band projection onto $b(e)$).
Moreover, $e = \frac{1}{2}(Tu + u)$

Joan Trias
ETSAV, Universitat Politècnica de Barcelona (Spanien)

Generalization of the Oettli/Prager-Theorem to Riesz Spaces.

It is shown that the classical Oettli/Prager-theorem concerning a system of linear equations can be generalized to operator equations in Riesz spaces. Theorem: Let X and Y be Riesz spaces, Y Dedekind complete, $\mathbb{A} := \{L \mid L: X \to Y \text{ linear, order bounded}\}$. For any $A \in \mathbb{A}$, $\alpha \in \mathbb{A}^+$, $y \in Y$, $\eta \in Y^+$, $\bar{x} \in X$, the following assertions are equivalent

(1) $\exists\ \bar{A} \in [A-\alpha, A+\alpha],\ \bar{y} \in [y-\eta, y+\eta] : \bar{A}\bar{x} = \bar{y}$

(2) $|A\bar{x} - y| \leq \alpha|\bar{x}| + \eta$

This theorem can be applied e.g. to linear integral equations; it allows to check whether or not some approximate solution $\bar{x}$ of $Ax = y$ is acceptable within prescribed tolerances α and η.

H. Fischer, München

On the asymptotic behavior of positive semigroups

We show what order structure and positivity can do for stability theory. In particular we ~~show~~ investigate strongly continuous, irreducible semigroups $\{T(t)\}_{t \geq 0}$ of bi-Markov operators on $L^1(X,\mu)$ and ask what conditions on the spectrum of the generator A (e.g., (a): 0 is isolated in $P\sigma(A) \cap i\mathbb{R}$, (b): 0 is isolated in $\sigma(A) \cap i\mathbb{R}$, (c): 0 is a pole of the resolvent) imply the existence of a partially periodic semigroup of positive operators $S(t)$ such that $\lim_{t\to\infty} (T(t) - S(t)) = 0$ for one of the standard operator topologies such as (A): weak operator topology, (B): weak operator topology, (C): operator norm.

Rainer Nagel (Tübingen)

Fredholm theory for Operators with a trace.

Let $(\mathfrak{A}, \underline{A})$ be a quasi-normed operator ideal and let τ be a continuous trace defined on $\mathfrak{A}$. Then

$$\delta^{(p)}(\mu) := \exp\left[\int_{\gamma(\mu)} w^{p-2}\tau[T^p R(\tfrac{1}{w};T)]dw\right],$$

with $R(\lambda;T)$ $(\lambda=\mu^{-1})$ the resolvent of the operator T and $\gamma(z)$ a rectifiable curve from zero to z, is a Fredholm divisor of T, with T an operator such that $T^p \in \mathfrak{A}$. This entire function has finite rank $\rho \leq 2p-1$. If τ is a r-spectral trace on $\mathfrak{A}(E,E)$ then the genus of $\delta^{(p)}$ is less than or equall to $pr-1$. The example of kernel operators completely of finite double norm on Banach function spaces is then considered, and the theory is applied to them to derive classical results of Zaanen, Coleman, Smithies. et

J. Grobler (Potchefstroom).

AN ORDER THEORETICAL CHARACTERIZATION OF THE FOURIER TRANSFORMATION

Let G, G_1, G_2 be locally compact groups. $C^b(G) \supset P(G)$ denotes the cone of all continuous positive definite functions on G, $B(G) := \operatorname{span} P(G) \subset C^b(G)$ the Fourier-Stieltjes Algebra and $A(G) \subset B(G)$ the Fourier Algebra, finally $L^1(G)$ is defined as usual via Haar measure.

$X(G)$ $(X = A, B$ or $L^1)$ is ordered by two cones: a) $X(G)_+$ the cone of all pointwise positive functions and b) $X(G)_p$ the cone of all positive definite functions in $X(G)$ $X = A, B$; resp. for $X = L^1$: $L^1(G)_p := \overline{co}\{f^{*}*f^* \mid f \in L^1(G)\}$.

Theorem. Let $T: X(G_1) \to X(G_2)$ be linear & bijective, $X = A, B$ or L^1. The following are equivalent:

(i) $TX(G_1)_+ = X(G_2)_+$, $TX(G_1)_p = X(G_2)_p$

(ii) There exists $c>0$ and a top. group isomorphism or anti-

isomorphism $\alpha : G_2 \to G_1$ such that
$Tf = c\, f \circ \alpha \quad (f \in X(G_1))$.

The Theorem implies that the "biordered space" $(X(G), X(G)_+, X(G)_p)$ is a complete isomorphism invariant ($X = L^1$, A or B). This is of special interest ~~sin~~ for $X = B$, since the space $(B(G), B(G)_+, P(G))$ is very easy to define. In particular, Haar measure is <u>not</u> needed for its definition.

The following order theoretical characterization of the Fourier Transformation is a consequence:

<u>Corollary</u>. Let $F : L^1(G_1) \to A(G_2)$ be linear and bijective. The following are equivalent:

(i) $F L^1(G_1)_+ = A(G_2)_p$, $F L^1(G_1)_p = A(G_2)_+$

(ii) G_1 & G_2 are abelian and there exists a top group isomorphism $\alpha : G_2 \to \hat{G}_1$ such that $Tf = c \cdot (\mathcal{F} f) \circ \alpha$ $(f \in L^1(G_1))$, where $\mathcal{F} : L^1(G_1) \to A(\hat{G}_1)$ denotes the Fourier Transformation.

Wolfgang Arendt, Tübingen.

Factorization of positive multilinear operators

We extend Nikišin's and Maurey's factorization theorems for positive linear operators to positive multilinear operators. Our main result can be summarized as follows:

<u>Theorem</u>. If $B : L_{p_1} \times \cdots \times L_{p_n} \to L_q$ $(q \geq 0)$ is a positive n-linear operator and $r \geq 1$ is such that $r^{-1} = \Sigma_1^n p_k^{-1}$ and $r \geq q$, then there exists $\varphi \in L_s$ with $\varphi > 0$ a.e., where $s^{-1} = q^{-1} - r^{-1}$ such that $\frac{1}{\varphi} \cdot B(L_{p_1} \times \cdots \times L_{p_n}) \subset L_r$

The proofs employ the positive projective tensor products of

Banach lattices, as developed by D. Fremlin.

Anton R. Schep (Columbia)

Automorphisms of regular completions of operator algebras

This is a report on joint work with K. SAITÔ.

For simplicity the results are stated for unital C^*-algebras although this restriction is not essential. Let A be a unital C^*-algebra and let $\hat{A}$ be its regular completion. Then each self-adjoint element b in $\hat{A}$ is the ~~supre~~ supremum of the set of self-adjoint elements in A which b dominates. Furthermore, $\hat{A}$ is monotone complete and is monotone generated by A. When A is commutative and so of the form $C(X)$, for some compact Hausdorff space X, then the self-adjoint part of $\hat{A}$ may be identified with the Dedekind completion of the Riesz space $C(X)$.

Each $*$-automorphism α of A has a unique extension to a $*$-automorphism $\hat{\alpha}$ of $\hat{A}$

Theorem Let A be a simple C^*-algebra. Then α is an outer $*$-automorphism of A if, and only if, $\hat{\alpha}$ is an outer $*$-automorphism of $\hat{A}$.

J. D. M. Wright (Reading)

An extension theorem for extended orthomorphisms

Recently Mathieu Meyer and myself have obtained the next result:

Theorem. If E is an Archimedean f-algebra and F a quasi-unital Riesz subspace, then every $T \in \mathrm{Orth}^{\infty}(F)$ can be extended to $\tilde{T} \in \mathrm{Orth}^{\infty}(E)$ in the sense that $T = \tilde{T}$ on some order dense ideal of F. If F is order dense in E, super quasi-unital and the ideal generated by F is super order dense in E, then every $T \in \mathrm{Orth}^{\sigma}(F)$ has an extension $\tilde{T} \in \mathrm{Orth}^{\sigma}(E)$ such that $T = \tilde{T}$ on some super order dense ideal of F.

(F is (super) quasi-unital if the ideal in F generated by $\{x \in F;\ xy = x$ for some $y \in F_+\}$ is (super) order dense in F; $T \in \mathrm{Orth}^{\sigma}(E)$ if $T \in \mathrm{Orth}^{\infty}(E)$ and can be defined on a super order dense ideal of E).

The next corollaries (in which E is any Archimedean Riesz space) show how powerful is this result.

Corollary 1. If $\mathrm{Orth}^{\sigma}(E)$ is order dense in $\mathrm{Orth}^{\infty}(E)$, then $\mathrm{Orth}^{\sigma}(\mathrm{Orth}^{\sigma}(E)) = \mathrm{Orth}(\mathrm{Orth}^{\sigma}(E)) = \mathrm{Orth}^{\sigma}(E)$.

(These equalities with "∞" instead of "σ" are always true).

Corollary 2. If $\mathrm{Orth}(E)$ is order dense in $\mathrm{Orth}^{\infty}(E)$, then $\mathrm{Orth}^{\infty}(\mathrm{Orth}(E)) \subseteq \mathrm{Orth}^{\infty}(E)$ and $\mathrm{Orth}^{\sigma}(\mathrm{Orth}(E)) \subseteq \mathrm{Orth}^{\sigma}(E)$.

Corollary 3. If E is uniformly complete and F is any Riesz subspace of E, then every $T \in \mathrm{Orth}^{\infty}(F)$ has an "extension" $\tilde{T} \in \mathrm{Orth}^{\infty}(E)$.

Duhoux M. (Louvain-la-Neuve)

Quasi-compact positive operators

Let T be a bounded linear operator on a Banach space E. We denote the fixed space $\{x \in E : Tx = x\}$ by $F(T)$ and $n^{-1}\sum_{i=0}^{n-1} T^i$ by T_n, $n \in \mathbb{N}$. If there exist a compact operator K and a natural number m with $\|T^m - K\| < 1$ then T is said to be quasi-compact.

Theorem. Let T be a positive linear operator on $C(X)$, where $C(X)$ is a Grothendieck space. If $\sup \|T_n\| < \infty$ and $\dim F(T') < \infty$, then T is quasi-compact and (T_n) converges uniformly to an operator of finite rank.

Heinrich P. Lotz
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PROBABILITY IN BANACH SPACES

4 JULY — 10 JULY 1982

Weighted Empirical and Brownian Processes

If $u(t) \geq 0$ is measurable on $(0,1]$ and B_t denotes a standard Brownian motion, let X_t denote the weighted Brownian motion, $X_t = u(t) B_t$, $0 < t \leq 1$. Necessary and sufficient conditions are found for the random field

$$\phi(f) \equiv \int_0^1 f(t) X_t \, dt$$

to determine a weak distribution with a continuous covariance functional with respect to the $L^q(0,1)$ metric, $1 < q \leq 2$. Similar conditions are obtained for the Brownian bridge process and for the empirical processes, $F_n(t)$, determined by i.i.d. uniform variables. As an application, explicit conditions on $u(t)$ are obtained which are necessary and sufficient for the following LIL-type result:

Fix p, $2 \leq p < \infty$. With probability 1, $\| u(t)(F_n(t) - t) \|_p = O\left(\sqrt{\frac{\log\log n}{n}}\right)$.

Victor Goodman
Bloomington, Indiana

Vecteurs aléatoires gaussiens à valeurs dans certains espaces de Banach.

Soit $(E, \|\cdot\|)$ un espace de Banach séparable ; on suppose que le carré de sa norme est deux fois directionnellement dérivable hors de l'origine et que la dérivée seconde $D^2(u)$, $u \neq 0$ reste bornée. On montre que dans ces conditions, les 3 propriétés suivantes sont équivalentes pour toute forme bilinéaire symétrique positive et continue sur $E' \times E'$: (1) Il existe une décomposition $\Sigma b_n \otimes b_n$ de Γ telle que $\sup_{u \neq 0} \Sigma \langle D^2(u), b_n \otimes b_n \rangle < \infty$, (2) Γ est la covariance d'un vecteur gaussien X à valeurs dans E. (3) Il existe une décomposition $\Sigma b_n \otimes b_n$

de Γ telle que $\Sigma \|b_n\|^2$ soit fini. On conjecture que ces propriétés sont équivalentes à : (5) Il existe une mesure de probabilité μ sur le bord S de la boule unité de E' telle que

$$\sup_{y \in S} \int \sqrt{\log \frac{1}{\mu\{z \in S : \Gamma(z,z) - 2\Gamma(y,z) + \Gamma(y,y) < u^2\}}}\, du < \infty .$$

X. Fernique
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An almost sure invariance principle for triangular arrays of B-valued random variables

We give a simple proof of the probability invariance principle for triangular arrays of i.i.d. random variables with values in a separable Banach space, recently proved by de Acosta, and improve this result to an almost sure invariance principle. (jt. work with A. Dabrowski & W. Philipp)

H. Dehling
Universität Göttingen

Empirical processes on large classes of sets or functions

Given $\mathcal{F} \subset \mathcal{L}^1(A, \mathcal{A}, P)$ with $\|\delta_x\|_{\mathcal{F}} := \sup_{f \in \mathcal{F}} |f(x)| < \infty$ P-a.e., $P_n := \frac{1}{n}(\delta_{x_1} + \dots + \delta_{x_n})$, x_j i.i.d. (P), limit theorems for P_n uniformly over $\mathcal{F}$ become limit theorems in a Banach space $(\ell^\infty(\mathcal{F}), \|\cdot\|_{\mathcal{F}})$. Conversely if $(S, \|\cdot\|)$ is a Banach space, $\mathcal{F} \subset S'$ (the dual) such that $\|x\| = \|\delta_x\|_{\mathcal{F}}$, $x \in S$, let X_j be random elements of S such that $f(X_j)$ are measurable and $E|f(X_j)| < \infty$, $f \in \mathcal{F}$. Then limit theorems for $X_1 + \dots + X_n$ in S can be written as limit theorems for P_n uniformly over $\mathcal{F}$.

Let P = Lebesgue measure on I^d, $I = [0,1]$, $d = 2$ or 3. If $d = 2$

let $\mathcal{C} := \{C : \langle x,y\rangle \in C,\ u \le x,\ v \le y \Rightarrow \langle u,v\rangle \in C\}$ (lower layers).
If $d=3$ let $\mathcal{C}$ = all convex sets. Theorem. In either case, for all $\delta > 0$ there is a $c > 0$ such that

$$\Pr\{\sup_{C\in\mathcal{C}} |(P_n - P)(C)| > c\sqrt{\log n}/(\log\log n)^{\frac{1}{2}+\delta}\} \to 1,\ n \to \infty.$$

R. M. Dudley
M. I. T., Cambridge, Mass.

Complex ~~convexity~~ martingales

Let $(E, \|\ \|)$ be a complex quasi-normed space whose quasi-norm is uniformly continuous on the unit ball of E. For $0<p<\infty$ we define a modulus of complex uniform convexity:

$$h_p^E(\varepsilon) = \inf\left\{1 - \|x\| : \frac{1}{2\pi}\int_0^{2\pi} \|x + e^{i\theta}y\|^p d\theta \le 1,\ \|y\| = \varepsilon\right\}.$$

For a large class of spaces, including normed spaces, all the moduli are equivalent, and h_p^E is $h_p^{L_p(E)}$.

These moduli are related to the behaviour of E-valued martingales which also reflect the complex structure of ~~E~~ a normed space E. Let (Ω, Σ, P) be a probability space with filtration $(\Sigma_n)_{n=0}^\infty$: let $(\eta_n)_{n=1}^\infty$ be an adapted sequence, each uniformly distributed on $|z|=1$, and with η_n independent of Σ_{n-1}, let $1 \le p < \infty$ and let $(v_n)_{n=0}^\infty$ be an adapted sequence of E-valued random variables, with $E(v_n|\Sigma_{n-1}) = E(v_n|\Sigma_{n-1}, \eta_n)$ for $n>0$, and $E\|v_n\|^p < \infty$. Then if $x_n = v_0 + \sum_{j=1}^n \eta_j v_j$, $(x_n)_{n=0}^\infty$ is a complex h_p-martingale. Using these martingales, renorming theorems analogous to those of Enflo and Pisier are established.

This is joint work with W J Davis and N Tomczak-Jaegermann

D J H Garling,
Cambridge, England.

CLT and WLLN in certain Banach spaces.

For B a type 2 Banach lattice, we obtain a relationship between CLT in B and WLLN in the Banach lattice of its squares. We obtain also two sided estimates of $E\|S_n\|^p$ which in l_p, $l_p(l_r)$ spaces $1 \le p < \infty$ give rise for the WLLN (hence also the CLT). As a consequence of these estimates we also solve the domain of attraction problem in l_p. Several examples and counterexamples are provided.
This is joint work with Joel Zinn.

Evarist Giné
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The Jordan Decomposition for Vector Measures.

Two methods are presented which yield Jordan decomposition theorems for vector measures with values in a Banach lattice. The first method is based on a common approach to vector measures and linear operators whereas the second one relies on factorization theorems which reduce the decomposition of vector measures to that of linear operators.
Main result: Suppose μ is a vector measure of bounded variation on a ring of sets with values in a Banach lattice with property (P). Then μ is the unique difference of two positive orthogonal vector measures of bounded variation.

Klaus D. Schmidt (Mannheim)

New results about type and cotype.

Let X be any Banach space and let C be its stable type p constant ($1<p<2$), possibly equal to ∞.
Then for any $\varepsilon>0$, X contains a subspace $(1+\varepsilon)$-isomorphic to ℓ_p^k for all $k < \eta_p(\varepsilon)\, C^{p'}$ with $\frac{1}{p}+\frac{1}{p'}=1$, where $\eta_p(\varepsilon)>0$ depends only on p and $\varepsilon>0$.
This result yields a quantitative version of a theorem of Maurey and the author which states that X is not of type p-stable iff X contains, for each $\varepsilon>0$ and each k, a subspace $(1+\varepsilon)$-isomorphic to ℓ_p^k.

Gilles Pisier

Some results on the Cluster set $C(\{\frac{S_n}{a_n}\})$ and the LIL

The cluster set $C(\{\frac{S_n}{a_n}\})$ is examined under conditions necessary for the bounded law of the iterated logarithm, and necessary and sufficient conditions for the LIL are obtained in spaces satisfying a certain comparison principle. In particular, it is shown that there is a complete blending of the compact LIL and the bounded LIL in Hilbert space. (this is joint work with A. de Acosta)

Jim Kuelbs

Invariance principles for sums of Banach space valued random elements and empirical processes

This is joint work with R. M. Dudley. We establish almost sure and probability invariance principles for sums of independent not necessarily measurable random elements with values in a not necessarily separable Banach space. We then show that empirical processes readily fit into this general framework. Thus we bypass the problems of measurability and topology characteristic for the previous theory of weak convergence of empirical processes.

The results can be extended in part to mixing sequences of random elements and to sequences of independent not necessarily identically formed random elements.

Walter Philipp, Urbana, IL

Limit theorems in the Banach space c_0.

The central-limit theorem and the law of the iterated logarithm are studied for a c_0-valued random variable X. The idea of the proofs is to carry the central-limit or iterated logarithm property for X to the same limit property for a suitable random variable T(X) which takes its values in C[0,1]. To this random variable T(X) we then apply the "mesures majorantes" method.

Bernard Heinkel
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On distribution of distributions for sample functions of Gaussian random processes

The main theorem:

Let $X(\omega, t)$ be a Gaussian zero-mean random process and the parametric set T itself is a measure space $T = (T, \Sigma, P)$, the process X being measurable. For every $\omega \in (\Omega, \mathfrak{A}, \gamma)$ consider the distribution $P_\omega \overset{def}{=}$ $\overset{def}{=} PX^{-1}(\omega, \cdot)$ of the sample function $X(\omega, \cdot)$ (the local times). Let $\varkappa(\mu, \nu)$ denotes the Kantorovich distance. The behaviour of the random variable $\varkappa = \varkappa(P_{\omega_1}, P_{\omega_2})$ is investigated, where $\omega_1, \omega_2 \in (\Omega, \mathfrak{A}, \gamma)$ are independly chosen elements. The process $X(\omega, t)$ is supposed to obey the following conditions: 1) the sample functions belong to the space $L^2(T, \Sigma, P)$ almost surely. 2) the correlation operator of the process X is majorised by the unit operator operator (condition of normalisation)

Theorem: There exist such absolute constants C_1, C_2 that the distribution of the random variable $\varkappa$ coinsides with the distribution relative to the standard Gaussian one-dimentional distribution of a function $\varphi: \mathbb{R} \to \mathbb{R}$ which satisfies the Lipschitz condition with the constant C_1 and the condition $|\varphi(0)| \le C_2$

V. Sudakov

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On the law of the iterated logarithm in certain smooth Banach spaces

The following generalization of the law of the iterated logarithm in Hilbert space of V. Goodman, J. Kuelbs and J. Zinn is presented: Let $(B, \|\cdot\|)$ a 2-uniformly smoothable real separable Banach space and X a B-valued random variable. Then X satisfies the bounded law of the iterated logarithm if and only if X is centered, $E\left\{\frac{\|X\|^2}{L_2\|X\|}\right\} < \infty$

and the expectations $E\{f^2(X)\}$, $\|f\|_{B'} \le 1$, are uniformly bounded; X satisfies the compact law if and only if X satisfies the bounded law and the random variables $f^2(X)$, $\|f\|_{B'} \le 1$, are uniformly integrable. Applications are given to growth rates for sums of independent and identically distributed random variables taking values in B.

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Remarks on various recent definitions of Feynman Integrals.

After a short review of the work on Feynman Integrals, we present an analytic ~~extensi~~ continuation of the Albevario-Høgh-Krogh integral $\mathcal{F}_{AH}^{\sigma}$, $\mathrm{Re}\,\sigma \geq 0$. We show that $\mathcal{F}_{AH}^{1/2}$ is equal to $\mathcal{F}_{AH}$, the AH-Feynman Integral. This result includes the work of A. Truman and recent work of G. Johnson and Kallianpur-Bromley on the relation between Cameron-Feynman Integral $\mathcal{F}_C$ and $\mathcal{F}_{AH}$. We show that given $\mathcal{F}_{AH}^{\lambda}$ for $\lambda > 0$ we can construct a Brownian Motion using the work of E. Nelson. The method used is that of Laplace transform.

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Pointwise Translation and the General Central Limit Problem

Let $X_{n1}, \ldots, X_{nk_n}$ be a u.a.n. array of independent random vectors on $\mathbb{R}^d$ and put $S_n = X_{n1} + \cdots + X_{nk_n}$. There exist vectors $v_n \in \mathbb{R}^d$ s.t. $\mathcal{L}(S_n - v_n) \to \gamma$ iff (I) a tail probability condition, (II) a truncated variance condition and (III) a centering condition hold. The condition (III) is superfluous in that (I) and (II) always imply (III) iff the limit law γ has the property that the only infinitely divisible laws which are pointwise translates of γ are actually vector translates. Not all infinitely divisible laws have this property. We characterize those which do.

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Bounded law of the iterated logarithm for the weighted Empirical distribution process for the non-i.i.d case

Theorem: Let $\psi(t)$, $t \geq 0$ be a non-negative, non-decreasing, left continuous function with right hand limits. Let $\{X_k\}$ be a sequence of non-negative, independent random variables and let $\{\varepsilon_k\}$ be a Rademacher sequence, independent of $\{X_k\}$. Let $Z_k(t) = \psi(t)[I(X_k \geq t) - P(X_k \geq t)]$, $S_m = \sum_{k=1}^{m} \varepsilon_k \psi(X_k)$ and $\mathcal{S}_m(t) = \sum_{k=1}^{m} Z_k(t)$. Let $\{b_m\}$ be an increasing sequence of positive numbers with $\lim_{m\to\infty} b_m = \infty$ and assume that

(1) $\overline{\lim_{m\to\infty}} \frac{|S_m|}{b_m} < \lambda$ a.s. for some $\lambda > 0$.

Furthermore, assume that

$$(2) \quad \overline{\lim_{m\to\infty}} \sup_{\psi(t) > 2\lambda b_m} b_m^{-1} \psi(t) \sum_{k=1}^{\infty} P(X_k \geq t) \leq 6\lambda .$$

Then

$$(3) \quad \overline{\lim_{m\to\infty}} \sup_{t>0} \frac{|S_m(t)|}{b_m} < 9920 \lambda \quad a.s.$$

(This is joint work with M. B. Marcus.)

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Necessary and Sufficient Conditions for the Continuity of Strongly Stationary p-stable processes

The necessary and sufficient condition of Dudley and Fernique for the a.s. continuity of stationary Gaussian processes is extended to strongly stationary p-stable processes, $1 < p < 2$. This is joint work with G. Pisier.

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ℓ^p-spaces in subspaces of L^1

This is a joint work of B. Maurey and S. Guerre.
If E is a subspace of L^1 and $p(E)$ is the supremum of real p's such that E is of Rademacher type p, it is well known that $\ell^{p(E)}$ is finitely representable in E that was proved by H.P. Rosenthal and in a more general case by B. Maurey and G. Pisier. We prove that in fact $\ell^{p(E)}$ is isomorphic to a subspace of E. This theorem uses the theory of stable Banach spaces which was developped by J.L. Krivine and B. Maurey.

Sylvie Guerre (Paris VI)

Asymptotic behavior of martingales in Banach spaces

This is a survey of results obtained since 1978. We report on results concerning a.s. behavior of $M_n/n^{1/p}$ and on the behavior of the corresponding maximal function. Results are compared with the available theorems in the case of independent increments.

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Bad rates of convergence for the C.L.T in Hilbert Space

We show that one can smoothly renorm the Hilbert space H such that the rate of convergence in the CLT becomes very bad. More precisely, let us fix a sequence $\xi_n \to 0$ and $\varepsilon > 0$. We can then construct a norm $N(\cdot)$ on the Hilbert space and a bounded random variable X on H with the following properties:

a) The norm $N(\cdot)$ is $(1+\varepsilon)$ equivalent to the usual norm. It is infinitely many times differentiable, and each differential is bounded on the unit sphere.

b) If (X_i) denotes indep copies of X, and if γ is the Gaussian measure with the same cov. as X, then the inequality

$$\sup_{t>0} \left| P\left(N\left(n^{-\frac{1}{2}} \sum_{i=1}^{n} X_i\right) \leq t\right) - \gamma\left(x : N(x) \leq t\right) \right| \geq \xi_n$$

occurs for infinitely many n.

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On One parameter proofs of multiparameter convergence theorems

Let $L(1) \supset L(2) \supset \dots \supset L(m)$ be Orlicz spaces over a probability space $(\Omega, \mathcal{F}, P)$, and let $T(k,n)$, $k = 1, 2, \dots, m$; $n \in \mathbb{N}$ be positive linear operators on $L(k)$ such that for $k = 2, \dots, m$, if $X \in L^+(k)$ then (a) $\lim_n T(k,n)X = T(k,\infty)X$ exists a.s. and b) $\sup_n T(k,n)X \in L(k-1)$. Then for each $X \in L(m)$, $\lim_{s_i \to \infty} T(1,s_1) \dots T(m,s_m)X = T(1,\infty) \dots T(m,\infty)X$ a.s. Let $I^m = \underbrace{\mathbb{N} \times \mathbb{N} \times \dots \mathbb{N}}_{m \text{ times}}$ and $(\mathcal{F}_s, s \in I^m)$ be an increasing filtration. For $s = (s_1, \dots, s_m)$, set $\mathcal{F}^k_{s_k} = \bigvee \mathcal{F}_s$ where $\bigvee$ is over all values of all indices but s_k. Let $T(k, s_k) = E[\cdot \mid \mathcal{F}^k_{s_k}]$; one obtains a generalization of Cairoli's theorem choosing $L(m) = L \log^{m-1} L$. The multiparameter Dunford-Schwartz follows by letting $T(k, s_k)$ be appropriate averages. Similarly one obtains a multiparameter Rota theorem and (letting all $L(k) = L_p$, $1 < p < \infty$) multiparameter Akcoglu and (letting $L(k) = L(2)$) multiparameter Stein theorems.

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A Central Limit Theorem for Chemical Reactions with Diffusion

Two mathematical models of chemical reactions with diffusion for a single reactant in a one-dimensional volume are compared, namely the deterministic and the stochastic model. The deterministic model is given by a partial differential equation, the stochastic one by a space-time jump Markov process. By the law of large numbers the consistency of the two models is proved. The deviation of the stochastic model from the deterministic model is estimated by a central limit theorem. This limit is a distribution-valued Gauss-Markov process and can be represented as the mild solution of a certain stochastic partial differential equation.

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Speed of convergence in functional limit theorems in Banach space

Let E be a separable Banach space, let $X_i; i \in \mathbb{Z}$ an i.i.d. sequence of E-valued random vectors, such that $EX_1 = 0$ and the fourth moment of X_1 exist. Let F denote a smooth functional defined on E. Then the convergence rate in the central limit theorem for the regions $\{x \in E : F(x) < z\}$ is $O(n^{-1/2})$ provided

that the gradient of F fulfills some additional conditions.
Applications are made to star-shaped regions in Hilbert space, empirical processes and L^{2p}-spaces $1 \leq p < \infty$.

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Asymptotic Formulas for Gaussian Spherical Integrals

Let H be a separable Hilbert space and μ a Gaussian measure on H. For the case of the (μ-)analytic function $p: H \to \mathbb{R}$, $p(x) = \|x\|$, results of the following type are presented in detail.

1) $dp(\mu) / d\lambda \in C^\infty(\mathbb{R})$.
2) The Gaussian surface measure exists on $S(r) = p^{-1}(r)$ in the sense of Minkowski's formula.

In particular the following asymptotic formula for the μ-area of $S(r)$ holds

$$\sim \sum_{k=1}^{\infty} \Big\{ \prod_{j \neq k} (\lambda_k - \lambda_j)^{-1} \Big\} e^{-r^2/2\lambda_k},$$

where λ_k are the eigenvalues of the covariance operator of μ (here with multiplicity 1, to simplify the formula).

Alexander Hertle (Mainz)

Gaussian measures and large deviations

We show the following result

Theorem: Let μ be a mean zero Gaussian measure on a separable Banach space E. Let A_μ be its covariance operator, $\mathcal{H}_\mu$ be its reproducing kernel Hilbert space and $\tilde{\mu}: E \to \mathbb{R}^+$ be the energy functional corresponding to μ (that is: $\tilde{\mu}(x) = \frac{1}{2}\|x\|^2_{\mathcal{H}_\mu}$ if $x \in \mathcal{H}_\mu$ and $\tilde{\mu}(x) = +\infty$ if not). Let $(\mu_n)_n$ be a sequence of mean zero Gaussian measures on E converging weakly to μ. Then:

(1) The covariance operator A_{μ_n} of μ_n converges to A_μ in the space of nuclear operators from E' into E (equipped with the nuclear norm);

(2) If, for each subset B of E, $\tilde{\mu}(B)$ denotes the inferior bound of $\tilde{\mu}$ on B,

(a) for each closed set F of E, $\overline{\lim}\, \frac{1}{n} \operatorname{Log} \mu_n(\sqrt{n}\, F) \leq -\tilde{\mu}(F)$;

(b) for each open set U of E, $\underline{\lim}\, \frac{1}{n} \operatorname{Log} \mu_n(\sqrt{n}\, U) \geq -\tilde{\mu}(U)$.

Simone Chevet
Université de Clermont

Polar sets of Gaussian Processes

Let $\{X(t),\ t \in S\}$, $S \subset \mathbb{R}^N$ be a Gaussian centered process defined on S with continuous paths. Let $X_1, \dots, X_d$ iid with $\mathcal{L}(X_i) = X$ and put $X^d = (X_1, \dots X_d)$ then consider K a compact non ~~void~~ empty of $\mathbb{R}^d$.
We know that K is polar iff $P\{\exists t \in S : X_t \in K\} = 0$.
We examin the conditions under which K is polar.
We obtain a sufficient condition expressed in terms of Hausdorff-measure of K. We also get a lower bound of $P\{\exists t \in S : \|X_t - K\| < \alpha\}$.

Weber Michel
Université de Strasbourg.

Un principe de symétrisation dans les espaces de Gauss.

Nous introduisons une symétrisation dans les espaces de Gauss analogue à la symétrisation de Steiner dans les espaces euclidiens. Cela nous permet de donner une démonstration simple et directe de l'inégalité de Borell :

$$\forall A \in \mathcal{B}(\mathbb{R}^n),\ \forall r>0 : \quad \Phi^{-1}\circ\gamma_n\big(A + r B_n(0,1)\big) \geq \Phi^{-1}\circ\gamma_n(A) + r .$$

Nous obtenons aussi une inégalité du type de celle de Brunn-Minkowski pour les ensembles convexes dans un espace de Gauss : si A et B sont deux parties convexes non vides de $\mathbb{R}^n$, alors

$$\forall \lambda\in[0,1],\quad \Phi^{-1}\circ\gamma_n\big(\lambda A + (1-\lambda)B\big) \geq \lambda\,\Phi^{-1}\circ\gamma_n(A) + (1-\lambda)\,\Phi^{-1}\circ\gamma_n(B).$$

Antoine Ehrhard
Université Louis-Pasteur
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Norm-dependent positive definite functions on Banach spaces

In 1938 I. J. Schoenberg proved the following result:
A continuous function $f:\mathbb{R}_+ \to \mathbb{R}$ such that $f(\|x\|)$ is positive definite on an infinite dimensional Hilbert space has the form

$$f(t) = \int_0^\infty e^{-\lambda t^2}\, d\mu(\lambda)$$

for some finite non-negative measure μ on $\mathbb{R}_+$. We show that this result is true for any infinite dimensional Banach space.
(Joint work with J. P. R. Christensen, København.)

Paul Ressel
Universität Eichstätt

A new method to prove Strassen's log log invariance principle and its application to stationary sequences of B-valued r v's

The main result of this talk is of the following type: For a "wide class" of stationary sequences $(X_n)_{n \in \mathbb{N}}$ of B-valued random variables (B a real separable Banach space) satisfying $E(f(X_1)) = 0$ and $E(f(X_1)^2) < \infty$ for all $f \in B^*$ (= dual of B) the following two statements are equivalent:

(a) There exists a mean zero Gaussian measure with covariance operator V and the sequence $((n \log\log n)^{-1/2} \sum_{h=1}^{n} X_n)_{n \geq 3}$ is with probability one conditionally $\|\cdot\|$-compact. (Here V depends on (X_n) in a prescribed way.)

(b) Without changing its distribution one can redefine the sequence (X_n) on a new probability space on which there exists a Brownian motion $(W(t))_{t \geq 0}$ such that

$$\|\sum_{n \leq t} X_n - W(t)\| = o((t \log\log t)^{1/2}) \quad \text{a.s.}$$

Erich Berger
Universität Göttingen

Stochastic integration and p-smoothable Banach spaces

Let $X = (X_t)_{t \in [0,1]}$ be a homogeneous Gaussian process with values in a Banach space E, and let F be a second Banach space. X is called an L^2-integrator if for every progressively measurable process $Y = (Y_t)_{t \in [0,1]}$ with values in $L(E, F)$ and that $E \int \|Y_t\|^2 dt < \infty$ the stochastic integral $\int Y dX$ exists as in the finite-dimensional case. Then one can prove that X is an L^2-integrator if and only if F has an equivalent 2-uniformly smooth norm. Similar characterisation of p-uniformly smoothable Banach spaces for $p < 2$ can be obtained by using suitable Lévy-processes instead of Gaussian processes as integrator processes.

G. Dettweiler
Universität Tübingen

Probabilistic limits theorems in general spaces

Let $(\Gamma, \mathcal{G}, \leq)$ be a set with a σ-algebra and a "partial" ordering. If there exist an increasing measurable cofinal from $(\mathbb{N}^{\mathbb{N}}, \mathcal{B}(\mathbb{N}^{\mathbb{N}}), \leq)$ into $(\Gamma, \mathcal{G}, \leq)$ we say that $(\Gamma, \mathcal{G}, \leq)$ is smoothly ordered. Let $(\Omega, \mathcal{F}, P)$ be a probability space and Λ a set of real functions on Λ, then Λ is said to be smoothly filtering upwards if there exist a smoothly ordered space and a map $\varphi: \Gamma \times \Omega \to \mathbb{R}$ so that (i) $\varphi(\alpha, \omega) \leq \varphi(\beta, \omega)$ $\forall \alpha \leq \beta$ $\forall \omega$, (ii) $\forall f \in \Lambda$ $\exists \alpha$: $\varphi(\alpha, \omega) \geq f(\omega)$ $\forall \omega$, (iii) $\varphi(\alpha, \cdot) \in \Lambda$ $\forall \alpha$, (iv) $\{(\alpha, \omega) \mid \varphi(\alpha, \omega) > a\} \in \mathcal{S}(\mathcal{G} \otimes \mathcal{F})$ $\forall a \in \mathbb{R}$.

Theorem If Λ is smoothly filtering upwards on $(\Omega, \mathcal{F}, P)$ then $F(\omega) = \sup\{f(\omega) \mid f \in \Lambda\}$ is P-measurable, and if $\int_* f dP > -\infty$ for some $f \in \Lambda$ then

$$\int f dP = \sup_{f \in \Lambda} \int_* f dP = \sup_{f \in \Lambda} \int^* f dP = \sup_{f \in \Lambda_0} \int f dP$$

where $\Lambda_0 = \{f \in \Lambda \mid f \text{ is } P\text{-measurable}\}$.

This result has a broad spectrum of application to limit theorems of stochastic processes in general spaces

J. Hoffmann-Jørgensen
Århus Universitet.

Variationsrechnung

12. – 16. Juli 1982

"On the Morse Number of Minimal Disc Surfaces Spanning a wire in $\mathbb{R}^3$"

Let $\alpha: S^1 \to \mathbb{R}^3$ be a wire on the boundary of a convex body with $\Gamma^\alpha = \alpha(S^1)$. Let $E: \mathfrak{N}(\alpha) \to \mathbb{R}$ be Dirichlet's functional where $\mathfrak{N}(\alpha)$ is the space of all harmonic surfaces spanning Γ^α. Then by jiggling α a bit we may assume that there are a finite number of "non-degenerate" immersions (Böhme – T), spanning Γ^α. Let $u_1 \dots u_N$ be those which are embedded and $u_{N+1} \dots u_m$ those which are immersed and not embedded & suppose that $\tau_1 \dots \tau_N$, $\tau_{N+1}, \dots \tau_m$ be the Morse indexes i.e. the dimension on which $D^2E(\alpha): T\mathfrak{N}(\alpha) \times T\mathfrak{N}(\alpha) \to \mathbb{R}$ is negative.
Define $\mu(E, \alpha) = \sum_{i=1}^{N} (-1)^{\tau_i}$, $\mu(I, \alpha) = \sum_{i=N+1}^{m} (-1)^{\tau_i}$

Then $\mu(E, \alpha) = 1$, $\mu(I, \alpha) = 0$.

Cor 1. If any suff. smooth wire on the boundary of a convex body admits two strict embedded minima, then there exists a third embedded disc surface which is not a strict minimum.

Cor 2. Using a result of Meeks (or ?) if Γ^α admits a non-immersed disc surface then Γ^α admit at least three embedded minima.

Cor 3. If Γ^α (extreme) admits a disc non embedded immersed surface which is a strict minimum it admits another immersed non-embedded which is not a strict minimum.

Tony Tromba 14.7.

H-surfaces in Lorentzian manifolds

We considered the existence of space-like slices of prescribed mean curvature H in a Lorentzian manifold M, which is topologically a product $M = I \times N$, where $I \subset \mathbb{R}$ and N is a compact, connected Riemannian manifold, with metric $ds^2 = -dt^2 + g_{ij}(x,t)\,dx^i dx^j$. Under the hypotheses that singularities at the endpoints of the time interval ("big bang" and "big crunch") will provide barriers, we proved the existence of globally defined hypersurfaces of prescribed mean curvature H for any bounded H. Furthermore, we constructed a "foliation type" family $(S_\tau)_{\tau \in \mathbb{R}}$ of surfaces of constant mean curvature τ.

Claus Gerhardt (Heidelberg)

Some recent developments in the theory of minimal surfaces

This talk focussed on recent developments in the regularity theory of minimal surfaces and on the related applications to existence theory for minimal surfaces in Riemannian manifolds. Specifically, the following two theorems were discussed:

Theorem A (Joint work with R. Schoen, Comm. P. & Appl. Math. 1981)

Given a stable embedded minimal hypersurface M contained in the truncated cylinder $B_\rho \times (-\varepsilon\rho, \varepsilon\rho)$ with $\partial M \subset \partial B_\rho \times (-\varepsilon\rho, \varepsilon\rho)$ (B_ρ = ball of radius ρ in $\mathbb{R}^n$), and suppose the n-dimensional volume $|M| \le \Gamma \rho^n$. Then for $\varepsilon = \varepsilon(n, \Gamma) > 0$ sufficiently small, we have $M \cap (B_{\rho/2} \times \mathbb{R})$ = a union of C^2 graphs, each satisfying the minimal surface equation.

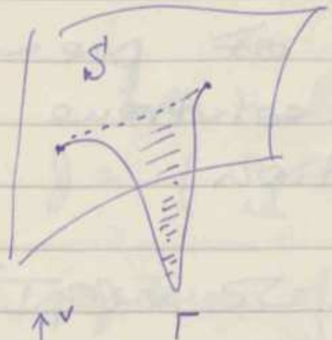
The theorem actually holds if M has some possible a-priori singularities, provided that at least $\mathcal{H}^{n-2}(\operatorname{sing} M \sim \partial M) = 0$.

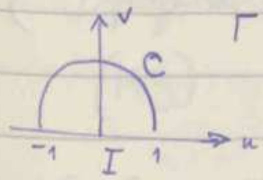
Theorem B (Joint work with F.J. Almgren, Pisa Journal, 1979)

Given a bounded domain $A \subset \mathbb{R}^3$ with $|\partial A| < \infty$, and let $\{M_k\}$ be a sequence of embedded discs with $\partial M_k \subset \partial A$, $M_k \sim \partial M_k \subset A$, and suppose that M_k is within ε_k of minimizing area with respect to its own boundary: $|M_k| \le |M| + \varepsilon_k$ $\forall$ embedded disc M such that $\partial M = \partial M_k$. If $\varepsilon_k \downarrow 0$ and if the M_k converge to a measure μ in the sense that $\int_{M_k} f \, d\mathcal{H}^2 \to \int_A f \, d\mu$ for each continuous function f having compact support in A, then for each $\xi \in \operatorname{support} \mu \cap A$ there is a $\sigma > 0$ and a positive integer n and a C^2 embedded minimal surface Σ such that $\int_A f \, d\mu = n \int_\Sigma f \, d\mathcal{H}^2$ whenever support $f \subset B_\sigma(\xi)$.

L. Simon 15-7-82

Some observations on minimal surfaces with free boundaries





Let $\mathfrak{x} : B \to R^3$ be a minimal surface which is bounded by a configuration $\langle \Gamma, S \rangle$ consisting of an arc Γ and a surface S without boundary. That is:

$$\Delta \mathfrak{x} = 0 \quad , \quad |\mathfrak{x}_u|^2 = |\mathfrak{x}_v|^2 \quad , \quad \mathfrak{x}_u \cdot \mathfrak{x}_v = 0 \quad \text{in } B$$

$$D_B(\mathfrak{x}) = \iint_B |\nabla \mathfrak{x}|^2 \, du \, dv < \infty \quad , \quad \mathfrak{x}|_C \text{ maps } C \text{ mono-}$$

tonically & continuously onto Γ, $\mathfrak{x}|_I$ maps I a.e. into S.

Suppose that S satisfies an R-sphere-condition (in particular, $S \in C^3$).

Then we have the following results (joint work with J.C.C. Nitsche):

Theorem 1. The length $L(\Sigma)$ of the free trace $\Sigma := \{\mathfrak{x}(u) : u \in I\}$ is bounded by $L(\Gamma) + (c/R) \, D_B(\mathfrak{x})$, where c is a number < 7, provided that $\mathfrak{x}$ is stationary for $\langle \Gamma, S \rangle$ and does not possess a branch point of odd order on I.

Cor. $\mathfrak{x}$ is continuous on $\overline{B}$ if $\mathfrak{x}$ is stationary and has no branch points of odd order on ~~B~~ I.

Theorem 2. If S bounds a star-shaped, H-convex set Ω and if $\Gamma \subset \Omega \cup S$ then $\mathfrak{x}(\overline{B}) \subset \Omega \cup S$ and has no branch points on I provided that $\mathfrak{x}$ is bounded by $\langle \Gamma, S \rangle$.

Theorem. If $\mathfrak{x}$ minimizes D_B among all surfaces bounded by $\langle \Gamma, S \rangle$ then $\mathfrak{x}$ has no branch points of odd order on I. If, in addition, S is real analytic, then $\mathfrak{x}$ has no branch points at all on I.

S. Hildebrandt

Evolutionary surfaces of prescribed mean curvature

We consider evolutionary surfaces of prescribed mean curvature, i.e. solutions of the parabolic quasilinear equation

$$\dot{u} + Au + H(x,u) = 0 \quad \text{in } \Omega \times (0,T)$$
$$u(0) = u_0 \quad \text{in } \Omega$$
$$\frac{\partial H}{\partial u} \geq -\bar{\varkappa}, \quad \bar{\varkappa} > 0$$

where $Au = -D_i\left(\frac{D_i u}{\sqrt{1+|Du|^2}}\right)$ is the minimal surface operator.

It is an interesting fact that solutions of this equation show a quite similar behaviour to solutions of capillary type equations of the form:

$$Au + H(x,u) = 0$$
$$\frac{\partial H}{\partial u} \geq \varkappa > 0$$

in fact for both equations the following results hold

$$|Du(x_0)| \leq c \cdot d^{-2}$$

where $d = \text{dist}(x_0, \partial\Omega)$ and the constant depends on the data. Furthermore one can prove Höldercontinuity ~~of~~ solutions to the corresponding Dirichlet problems where the Hölderexponent is independent of the data.

K. Ecker (Heidelberg)

Periodic solutions of large norm of Hamiltonian systems

Let $H: \mathbb{R}^{2n} \to \mathbb{R}$, $p, q \in \mathbb{R}^n$, $z = (p, q)$, + $J = \begin{pmatrix} 0 & -id \\ id & 0 \end{pmatrix}$ where id is the $n \times n$ identity matrix. Consider the Hamiltonian system of ordinary differential equations:

$$(HS) \qquad \dot{z} = J H_z(z)$$

<u>Theorem</u>: Suppose $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$ and there are constants $\mu > 2$ and $r > 0$ such that

$$0 < \mu H(z) \le z \cdot H_z(z) \qquad \forall\ |z| \ge r$$

Then for any $T, R > 0$, (HS) possesses a T-periodic solution z with $\|z\|_{L^\infty} > R$.

The proof involves obtaining solutions of (HS) as critical points of

$$I(z) = \int_0^T [p \cdot \dot{q} - H(z)]\, dt$$

Critical points of I are obtained by minimax arguments which rely on an S^1 symmetry that I possesses ($I(z(t)) = I(z(t+\theta)) \quad \forall\, \theta \in \mathbb{R}$).

Paul H. Rabinowitz (Madison)

Ein eindimensionales Variationsproblem aus der nichtparametrischen Statistik

[Gemeinsame Arbeit ~~von~~ mit W. Sauermann]

Es wird das Problem betrachtet, den Verlust an Fisher Information beim Übergang von einer nichtparametrischen Verteilungsfamilie zu Multinomialverteilungen asymptotisch zu minimieren. Das analytische Problem ist: finde h_0 mit

$$\sup_{g \in D} \int \left(\frac{g'}{h_0}\right)^2 f = \inf_{h \geq 0,\ \int h = 1} \ \sup_{g \in D} \int \left(\frac{g'}{h}\right)^2 f$$

$$D = \{ g\sqrt{f} \in L^2 \mid 0 \leq g' \leq M,\ \int g^2 f = \text{const} \}$$

Das ist zur Differentialgleichung mit Nebenbedingung:

$$-\left(\frac{1}{f} v'\right)' - \inf\left(M,\ S f^{1/2} v^{-3/2}\right) = 0$$

$$v(\pm\infty) = 0, \quad \int \frac{1}{f} (v')^2 = \text{const}$$

äquivalent.

Es wird gezeigt, daß v, S von der Nebenbedingung monoton abhängen.

Stephan Luckhaus

Recent Results on Harmonic Maps

1) Thm (Jost-Schoen): Suppose M_1, M_2 are 2-dim. Riem. mfs. of the same pos. genus, $\phi: M_1 \to M_2$ a diffeomorphism. Then there exits a harmonic diffeomorphism homotopic to ϕ.

2) Existence of the harmonic diffeomorphisms, if the image is a compact Kähler mf. of the form $D \times \dots \times D / \Gamma$, D = hyperbolic disc, Γ discrete subgroup of $Aut(D^n)$. Applications to deformations of complex structures (Jost-Yau)

3) Existence of solutions of the Dirichlet problem for harmonic maps, if the image has a strictly convex boundary and supports a strictly convex function.

4) $C^{2+\alpha}$ a-priori estimates for harmonic maps, depending only on curvature bounds, injectivity radii and dimensions, if the image is a ball $B_M(p)$, disjoint to the cut locus of p, with radius $M < \pi/2\sqrt{\kappa}$, $\kappa \geq 0$ upper curvature bound (Jost-Karcher)

Jürgen Jost

Smoothness of the free boundary of a minimal surface

Take a minimal surface $x = x(u,v) = (x_1(u,v), \dots, x_N(u,v))$

$$\Delta x = 0, \quad x_u x_v = 0, \quad |x_u| = |x_v|$$

defined on $Q^+ = \{(u,v) \mid |u| < 1, 0 < v < 1\}$ which meets a supporting surface $S \in C^{1,1}$ orthogonally in some weak sense. Then $x \in C^{1,\nu}$ up to the free boundary ($v = 0$), for each exponent $\nu \in (0,1)$. The trace $\{x(u,0) \mid |u| < 1\}$ is a smooth curve. This had been proved by W. Jäger in 1970 for minimal surfaces which minimize Dirichlet's integral $D(x)$. We prove regularity under the assumption that the distance function $\mathrm{dist}(x,S)$ is continuous up to $v = 0$ and that x meets S orthogonally (i.e. x is a stationary point of the functional $D(x)$). Higher regularity is proved: if $S \in C^{k,\mu}$ ($k \geq 2$, $0 \leq \mu \leq 1$), then $x \in C^{k,\mu}$ up to $v = 0$, if $0 < \mu < 1$. The cases where $\mu = 0$ or $\mu = 1$ are as usual.

G. Dziuk

A new existence theorem for capillary surfaces without gravity

Given a cylindrical container Z with base Ω, one asks whether it can be partly filled with liquid, in stable equilibrium in the absence of gravity, and whose surface simply covers the base Ω. One is led to seek a surface $u(x,y)$ of constant mean curvature $H = \left|\frac{\partial\Omega}{\Omega}\right| \cos\gamma$, that makes with Z a prescribed constant angle γ.

Solutions do not in general exist, and explicit conditions for existence have been given only in special situations. Here the problem is reduced to a subsidiary variational problem (S) in one lower dimension. It is shown that the nonexistence of a solution to (S) is a sufficient condition for existence of a solution to the original problem. As an application, it is shown that if Ω is convex with boundary curvature κ satisfying $0 \le \kappa_m \le \kappa \le \kappa_M < \infty$, then a solution exists if either $H > \kappa_M$ or if

$$-\min\left\{\frac{H\sin^2\gamma}{\kappa_M\cos\gamma - H\cos^2\gamma}\,,\ 1\right\} + \left(1 - \frac{\kappa_m}{H\cos\gamma}\right) \le 0 .$$

R Finn

Eine Variationsmethode für elliptische Differentialoperatoren mit strengen Nichtlinearitäten.

Es geht um die Frage, wann ein $u \in W_0^{m,1}(\Omega)$, das das Integralfunktional $I(u) := \int_\Omega F(x, D^{(0,m)}u)\,dx$ minimiert, eine schwache Lösung der zugehörigen Eulergleichung ist (mit $\Omega \subset \mathbb{R}^N$, offen, $0 \leq F : \Omega \times \mathbb{R}^S \to \mathbb{R}$, $D^{(0,m)}u :=$ S-Vektor der $D^\alpha u$ mit $|\alpha| \leq m$). Dabei werden an F keine polynomiellen Wachstumsbedingungen der Art $|F(x,\xi)| \leq c(1+|\xi|)^r$ gestellt, so daß man i.a. aus $I(u) < \infty$ nicht mehr auf $I(u+t\varphi) < \infty$, $\forall \varphi \in C_0^\infty(\Omega)$, schließen kann. Es zeigt sich jedoch, daß man durch Wahl geeigneter Testfunktionen (nämlich φ-au an Stelle von φ, mit $0 < a < 1/2$) in Verbindung mit dem Lemma von Fatou zum Ziel kommt, falls F einer gewissen Strukturbedingung genügt. Diese ist z. Bp. erfüllt, wenn $F(x,\xi)$ in ξ konvex ist oder die Gestalt $F(x,\xi) = \sum_{|\alpha| \leq m} p_\alpha(x)\, G_\alpha(\xi_\alpha)$ hat, mit $0 \leq p_\alpha \in L^1_{loc}(\Omega)$, $0 \leq G_\alpha \in C^1(\mathbb{R},\mathbb{R})$, $G'_\alpha(t)\cdot t \geq 0$, $\forall t \in \mathbb{R}$.

Rainer Hempel
(Universität München)

Tori of prescribed mean curvature and the rotating drop.

A one-parameter family is constructed of rotationally symmetric surfaces S_γ of the type of the torus, having prescribed mean curvature r^2+k for constant $k=k(\gamma)$. Here r is the distance to the axis of rotation. After rescaling, these tori are equilibrium surfaces for the problem corresponding to a drop of oil rotating with angular velocity ω in a body of water, having any positive volume less than C_0/ω^2. Here C_0 is in proportion to the constant of surface tension. As $\gamma \to 0$ the surfaces approach a spheroid, described by elliptic integrals, which is tangent to itself on the axis. The surfaces S_γ are unstable for large and for small values of γ. However, Plateau's experiments lead one to conjecture that S_γ is stable for an intermediate range of values for γ. A similar existence theorem holds for tori of prescribed mean curvature $g(r)+k$, where g is assumed to satisfy the condition that $r\frac{d^2}{dr^2}(rg(r))$ has a positive lower bound. In contrast, if g is monotone decreasing then no such tori can exist.

Robert Gulliver (Minneapolis)

Unstable Critical Points of Certain Functionals in the Calculus of Variations

For a wide class of the variational problems considered in the book of Ladyzhenskaya - Uraltseva a theorem about the existence of unstable solutions of Euler's equations can be proved. The theorem is rather similar to the ones known for Plateau's problem. In addition the second variation of the unstable solutions given is not positive definite.

The idea of the proof is to consider the problem with the artificially imposed constraint

$$\|u\|_{H^2_{r_2}} \leq K$$

with $r_2 > n$, which makes the functional more regular. Then one proves that this constraint does not do any harm, if K is chosen large enough.

Gerhard Strüwe

Minima and quasi-minima of variational integrals.

I consider regular functionals of the Calculus of Variations:

$$F(u;\Omega) = \int_\Omega f(x,u,Du)\,dx$$

where $\Omega \subset \mathbb{R}^n$ and f satisfies

$$|p|^m - B|u|^\gamma - g(x) \le f(x,u,p) \le A|p|^m + B|u|^\gamma + g(x)$$

with $1 < m < n$, $0 \le \gamma < m^* = nm/(n-m)$, $g \in L^r(\Omega)$, $r > n/m$.
A Q-minimum of F is a function $u \in H^{1,m}_{loc}(\Omega;\mathbb{R}^N)$ such that for every $\varphi \in H^{1,m}(\Omega;\mathbb{R}^N)$ with $\operatorname{supp}\varphi \subset \Omega$:

$$F(u;\operatorname{supp}\varphi) \le Q\,F(u+\varphi;\operatorname{supp}\varphi).$$

The minima of F are of course 1-minima. Other examples of Q-minima are

i) solutions of linear elliptic equations with L^∞-coefficients:

$$\int_\Omega a_{ij}(x)\,u_{x_j}\,\varphi_{x_i}\,dx = 0 \qquad \forall \varphi \in C_0^\infty(\Omega)$$

are Q-minima for the Dirichlet integral $\int |Du|^2 dx$. Similar results hold for solutions of nonlinear elliptic equations and systems in divergence form.

ii) Quasi-conformal mappings $u:\Omega \to \mathbb{R}^n$ (i.e. functions $u:\Omega\to\mathbb{R}^n$ such that $|Du|^n \le c \det(Du)$) are Q-minima of $\int |Du|^n dx$.

In a forthcoming paper with M. Giaquinta we prove:

(I) If $N=1$, every Q-minimum of F is holder-continuous in Ω.

(II) If $N \ge 1$, every Q-minimum of F has first derivatives in $L^{m+\varepsilon}_{loc}$, for some $\varepsilon > 0$.

Enrico Giusti
Università di Firenze

Differentiability of minima of non-differentiable functionals.

We consider functionals in the C. of V

$$F(u;\Omega) = \int_\Omega f(x,u,Du)\,dx$$

where $f(x,u,p): \Omega \times \mathbb{R}^N \times \mathbb{R}^{nN} \longrightarrow \mathbb{R}$, $\Omega \subset \mathbb{R}^n$, and

i) $\lambda|p|^2 - a \leq f(x,u,p) \leq \Lambda|p|^2 + a \qquad \lambda > 0$

ii) f is of class C^2 and convex in p, and moreover

$$(f_{pp}(x,u,p)\xi,\xi) \geq \nu|\xi|^2 \qquad \nu > 0$$

iii) $(1+|p|^2)^{-1} f(x,u,p)$ is Hölder-continuous in (x,u) uniformly with respect to p.

We prove

<u>Theorem</u> - Let u be a minimum for F. Then

a) If $N=1$, then u has Hölder-continuous first derivatives

b) If $N>1$, then u has Hölder-continuous first derivatives in an open set $\Omega_0 \subset \Omega$ and meas $(\Omega - \Omega_0) = 0$

Moreover

b_1) If

$$f(x,u,p) = A_{ij}^{\alpha\beta}(x,u)\, p_\alpha^i p_\beta^j$$

then $\Omega - \Omega_0$ has Hausdorff dimension less than $n-2$

b_2) If, further,

$$A_{ij}^{\alpha\beta}(x,u) = G^{\alpha\beta}(x)\, g_{ij}(x,u)$$

$|u| \leq M$, then the singular set $\Omega - \Omega_0$ is made of isolated points in dimension $n=3$, and in general has Hausdorff dimension not greater than $n-3$.

These results have been obtained jointly with E. Giusti.

Mariano Giaquinta
Università di Firenze.

On a weakened Palais-Smale condition and applications to nonlinear scalar field equations and quasilinear eigenvalue problems

For a functional E from a reflexive Banach space B into $\mathbb{R}$ and a sequence of "test-spaces" $\hookrightarrow T_l \hookrightarrow T_{l+1} \hookrightarrow$ with union lying dense in B the standard Palais-Smale condition

P.S.: If $\{u_m\} \subset B$ satisfies: $u_m \to u$ weakly in B and $\nabla E(u_m) \to 0$ strongly in B^* $(m \to \infty)$, then the sequence $\{u_m\}$ possesses a strongly convergent subsequence.

is replaced by the following criterion:

A^*: If $\{u_m\} \subset B$ satisfies: $u_m \to u$ weakly in B and if for all l $\nabla E(u_m) \in T_l^*$ converges to 0 strongly in T_l^*, then the sequence $\{u_m\}$ possess a strongly convergent subsequence.

For such functionals satisfying Criterion A^* the existence of critical points characterized by minimax- or mountain-pass-type conditions can be obtained along the lines of classical Ljusternik-Schnirelman theory.

As an application of this method a simple proof the existence of infinitely many radially symmetric solutions of nonlinear scalar field equations

$$\Delta u + g(u) = 0 \quad \text{in } \mathbb{R}^n,\ n > 2$$
$$u(x) \to 0 \quad (|x| \to \infty)$$

in the "zero-mass" case studied e.g. by Berestycki and P.L. Lions is derived. Moreover, for functionals $E, G: H_0^{1,2}(\Omega; \mathbb{R}^N) \to \mathbb{R}$, e.g.

$$E(u) = \frac{1}{2}\int a^{\alpha\beta}(x,u)\, \partial_\alpha u^i \partial_\beta u^i\, dx, \quad G(u) = \|u\|_{L^p}^p,$$

the existence of nontrivial solutions of the quasilinear eigenvalue problem: $G(u) = 1$; $-\partial_\alpha a^{\alpha\beta}(x,u)\partial_\beta u^i + \frac{1}{2}\partial_{u^i} a^{\alpha\beta}(x,u)\partial_\alpha u^j \partial_\beta u^j = \mu |u|^{p-2} u$ can be shown.

Michael Struwe, Universität Bonn

REGULARITY RESULTS FOR MINIMAL SURFACES AND H-SURFACES

We presented a method to prove various regularity results for weak surfaces in conformal parameters. The result concerning the interior regularity can be summarized as follows.
In a complete threedimensional Riemannian manifold (with a uniformity condition at infinity) any weak surface of bounded mean curvature which is conformally parametrized and which has finite area is regular.
For stationary minimal surfaces with a (partially) free boundary the same approach can be used to obtain the regularity of the free boundary (including the obstacle case).
Finally we give an application to a problem for stationary H-surfaces with a free boundary.

Michael Grüter 15-7-82
Universität Düsseldorf

STABILE TRENNFLÄCHEN KONVEXER GEBIETE

Es seien $\mathcal{B}$ ein konvexer Körper vom Volumen $m_3(\mathcal{B})$ und S eine das Innere von $\mathcal{B}$ überspannende Fläche vom Typ der Kreisscheibe, die $\mathcal{B}$ in zwei Teile $\mathcal{B}'$, $\mathcal{B}''$ vorgeschriebener Volumina $m_3(\mathcal{B}') = \sigma\, m_3(\mathcal{B})$, $m_3(\mathcal{B}'') = (1-\sigma)\, m_3(\mathcal{B})$ zerlegt $(0<\sigma<1)$. Wenn die Fläche S kleinsten oder stationären Inhalt besitzt, muß ihre mittlere Krümmung H konstant sein; auch muß S den Rand $\partial\mathcal{B}$ unter rechtem Winkel schneiden. Die (Rand-)Regularität ist durch jüngste Untersuchungen gesichert (Grüter-Hildebrandt-Nitsche, Dziuk). Für eine Kugel sind alle stationären Flächen Ebenen durch den Mittelpunkt ($\sigma = \frac{1}{2}$) oder Kugelkappen ($\sigma \neq \frac{1}{2}$). Beim Vorhandensein von Symmetrien existieren Kontinua stationärer Flächen S. Für das Ellipsoid $x^2+y^2+\frac{z^2}{c^2}=1$, $0<c<1$ (als konkretes Beispiel) ist der Kreis $S: \{x^2+y^2 \leq 1, z=0\}$ eine echt stationäre Lösung für den Fall $\sigma = \frac{1}{2}$. Es stellt sich heraus, daß S „isoliert" liegt, d.h. daß sich S nicht in eine Schar stationärer Flächen (mit möglicherweise wechselnder mittlerer Krümmung) einbetten läßt. Die Diskussion zweidimensionaler Analoga ist lehrreich.

Johannes C.C. Nitsche
Minneapolis

Variational approach of problems with hysteresis

Let Ω be a bounded open subset of $\mathbb{R}^n$ $(n \geq 1)$, A an elliptic operator on Ω, f a function $\mathbb{R} \to \mathbb{R}$.

Problem: Find u, w such that

$\frac{\partial w}{\partial t} + Au = g$ (g = datum) in $\Omega \times]0,T[$ $(T>0)$

$w = f(u)$ in $\Omega \times]0,T[$

+ boundary condition for u + initial condition for w.

For the corresponding variational formulation, well-posedness results are well-known for f monotone.

In a paper to appear on "Ann. Mat. Pura e Appl." and announced in "C.R. Acad. Sc. Paris, t. 293 (14 déc 1981)" I dealt with the case of a memory functional $f: C^0([0,T]) \to C^0([0,T])$ representing hysteresis, as it arises in ferromagnetism e.g.. I proved existence of at least one solution of the variational formulation, assuming a generalized monotonicity property for f in particular. Uniqueness of the solution is an open question.

Augusto Visintin
SFB 123, Heidelberg and Pavia - Italy

On harmonic maps from the n-dimensional ball into the n-dimensional sphere

Consider maps $u: B^n \to S^n \subset \mathbb{R}^{n+1}$ which are critical points of the energy functional $E(u) = \frac{1}{2}\int_{B^n} |\nabla u|^2 dx$
These are solutions of the p.d.e

$$\Delta u = -u\, |\nabla u|^2 .$$

The smallness condition used to show that there exists a unique, smooth solution of the Dirichlet-problem for harmonic maps restricts in this case to open half spheres. Studying radial symmetric solutions which are characterized by a pendulum equation

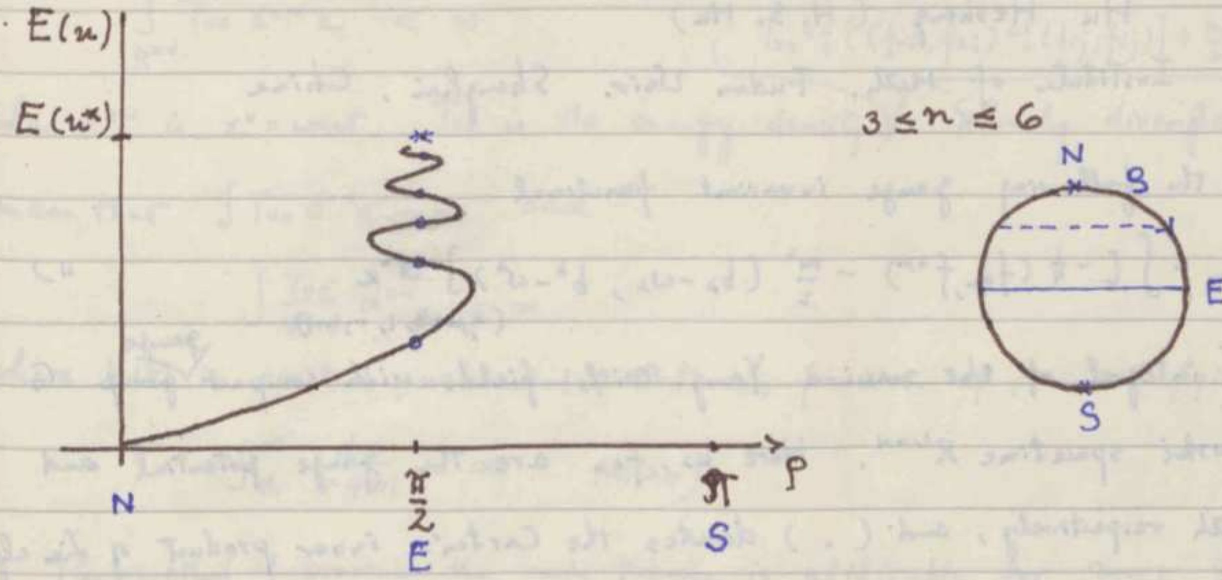
$$\psi'' + (n-2)\psi' - (n-1)\sin\psi = 0 \text{ in }]-\infty, 0[$$

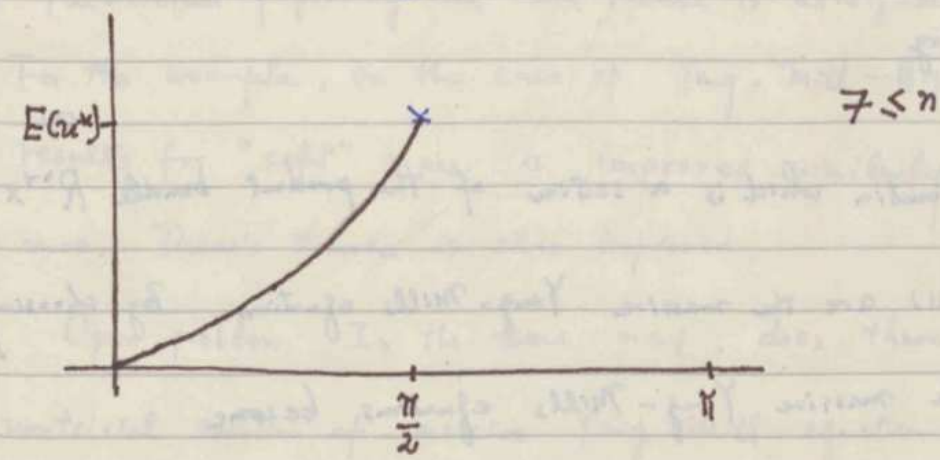
$$\psi(t) \to 0, \ \psi'(t) \to 0 \text{ for } t \to -\infty$$

it is shown that the 'singular' harmonic map $u^*(x) = (\frac{x}{|x|}, 0)$ is unstable in dimensions $3 \leq n \leq 6$ and absolute minimum for $n \geq 7$ in the class of all maps, mapping ∂B^n trivially onto the equator. This shows that for $n \geq 7$ the regularity results cannot be improved, whereas for $n \leq 6$ a positive conjecture is allowed. If one considers maps with boundary data

$$u(x) = (x \sin\rho, \cos\rho) \quad \text{(s. sketch)}$$

one gets the following branch of radial symmetric solutions in the energy-ρ-plane





This results are joint work of Kaul and J.

Willi Jäger (Heidelberg)

On the static solutions of massive Yang-Mills equations

Hu Hesheng (H. S. Hu)
Institute of Math. Fudan Univ. Shanghai, China

Consider the following gauge invariant functional

$$L_m = \int [-\tfrac{1}{4}(f_{\lambda\mu}, f^{\lambda\mu}) - \tfrac{m^2}{2}(b_\lambda - \omega_\lambda, b^\lambda - \omega^\lambda)]\, d^n x \quad (1)$$
$$(\lambda, \mu = 0, 1, \cdots, n-1)$$

as the action integral of the massive Yang-Mills fields with compact gauge group G over Minkowski spacetime $R^{1,n-1}$. Here b_λ, $f_{\lambda\mu}$ are the gauge potential and field strength respectively, and (,) denotes the Cartan's inner product of Lie algebra g of G. ω_λ is defined by

$$\omega_\lambda = U^{-1}\partial_\lambda U \quad (2)$$

where U is a G-valued function which is a section of the product bundle $R^{1,n-1}\times G$.

The Euler equations of (1) are the massive Yang-Mills equations. By choosing the Lorentz gauge, the massive Yang-Mills equations become

$$J_\lambda - m^2 b_\lambda = 0 \quad (3)$$

For a static gauge field, b_λ is independent of x^0. The energy momentum tensor $T_{\alpha\beta}$ obeys the conservation law

$$\frac{\partial T^\beta_\alpha}{\partial x^\alpha} = 0 \quad (4)$$

Main theorem: In n dimensional spacetime $R^{1,n-1}$ with $n \neq 4$, the compact group Yang-Mills field with real mass does not ~~not~~ posses any non-trivial static solution which is free of singularities and has finite or "slowly divergent" energy.

Finite energy means that

$$\int_{R^{n-1}} T_{00}\, d^{n-1}x < \infty$$

$$T_{00} = \frac{1}{2}\left[(f_{0i}, f_{0i}) + \frac{1}{2}(f_{ij}, f_{ij})\right] + \frac{m^2}{2}(b_0, b_0) + \frac{m^2}{2}(b_i, b_i) \qquad i, j = 1, 2, \cdots, n-1$$

when R^{n-1} is $x^0 = \text{const}$. T_{00} is the energy density. "Slowly divergent energy" mean that $\int T_{00}\, d^{n-1}x = \infty$ and

$$\int \frac{T_{00}}{\psi(r)}\, d^{n-1}x < \infty$$

where $\psi(r)$ is positive, unbounded, continuous function of r satisfying

$$\int_R^\infty \frac{dr}{r\psi(r)} = \infty \qquad (R > 0)$$

The method of proving the main theorem is utilizable for more general case. For example, in the case of Yang-Mill-Higgs-Kibble field, the results for "soft" mass is improved similarly. For the massless case $m=0$. Deser's theorem is also improved.

Open problem. In the case $n=4$, does there exist a static regular nontrivial solution of massive Yang-Mill equation with finite energy or "slowly divergent" energy?

Hu Hesheng
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Über die eindeutige Lösbarkeit der Eulergleichung gewisser Variationsprobleme

Ist $T \in (W_0^{m,p}(\Omega))^*$ $m \in \mathbb{N}$, $1 < p < \infty$ und $g \in L^1(\Omega)$ gegeben, sodaß $T(\varphi) = \int_\Omega g \cdot \varphi \, dx$ für alle $\varphi \in C_0^\infty(\Omega)$, gilt für $v \in W_0^{m,p}(\Omega) \cap L^1(\Omega)$ noch $g \cdot v \in L^1(\Omega)$, so haben Brezis und Browder gezeigt, daß $T(v) = \int_\Omega g \cdot v \, dx$.

Eine Verallgemeinerung dieser Problemstellung lautet:
Ist $T \in (W_0^{m,p}(\Omega))^*$ gegeben durch $T(\varphi) = \int_\Omega \sum_{|\alpha| \le m} g_\alpha D^\alpha(\varphi) \, dx$ für alle $\varphi \in C_0^\infty(\Omega)$ mit $g_\alpha \in L^1(\Omega)$, gilt dann für solche $v \in W_0^{m,p}(\Omega)$ für die $\sum_{|\alpha| \le m} g_\alpha D^\alpha(v) \in L^1(\Omega)$ auch

$$T(v) = \int_\Omega \sum_{|\alpha| \le m} g_\alpha D^\alpha(v) \, dx \; ?$$

Wir zeigen, daß eine Lösung dieses Problems äquivalent ist zur eindeutigen Lösbarkeit der Eulergleichung gewisser Variationsprobleme.

R. Landes (Bayreuth)

Harmonic maps of indefinite metrics and non-linear wave equations

Let $R^{1,1}$ be the Minkowski plane $\{(t,x)\}$ and N a Riemannian manifold. In local coordinates the equations of harmonic maps are

* $\phi^\alpha_{tt} - \phi^\alpha_{xx} + \Gamma^\alpha_{\beta\gamma}(\phi)(\phi^\beta_t \phi^\gamma_t - \phi^\beta_x \phi^\gamma_x) = 0$ ($\Gamma^\alpha_{\beta\gamma}$ — the Christoffel symbols of N)

Theorem. If N is a complete Riemannian manifold, for arbitrary regular initial conditions the Cauchy problem for the harmonic maps from $R^{1,1}$ to N admits a global solution uniquely.

The physical meaning is that non-linear σ models over $R^{1,1}$

are fields free of singularities.

Let $\xi = \frac{t+x}{2}$, $\eta = \frac{t-x}{2}$ and $S^{1,1}$ be the "sphere" of radius 1 in $R^{2,1}$:

$l^2 = l_1^2 + l_2^2 - l_3^2 = 1$.

Theorem. The Cauchy problem for the harmonic maps from $R^{1,1}$ to $S^{1,1}$ admits a global solution if the initial condition satisfies $l_\xi^2 > 0$, $l_\eta^2 > 0$, $l_\xi \cdot l_\eta \geq 0$ or $l_\xi^2 < 0$, $l_\eta^2 < 0$, $l_\xi \cdot l_\eta \geq 0$. If these conditions do not hold, the solution may blow up at finite time.

The construction of these harmonic maps is reduced to solving the equations

$$\alpha_{tt} - \alpha_{xx} = \pm \sinh\alpha \quad \text{or} \quad \alpha_{tt} - \alpha_{xx} = \cosh\alpha$$

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Zur speziellen Struktur in der Nähe von kritischen Randpunkten der Lösungen des Dirichletproblems zu quasilinearen Gleichungen

Es wird aufgezeigt, daß die Lösungen u von

$$-\operatorname{div}\{|\nabla u|^{p-2}\nabla u\} = f, \text{ in } \Omega, \qquad \left(\Omega \subset \mathbb{R}^n, \; p > 1\right)$$
$$u = g, \text{ auf } \partial\Omega,$$

die nicht "wesentlich" ihr Vorzeichen in der Nähe eines kritischen Randpunktes $x_0 \in \partial\Omega$ ändern, dort entweder so glatt sind, wie man es auf Grund der Daten f und g erwartet, oder daß sie sich dort wie $r^\lambda \cdot \varphi(\sigma)$ ($r = |x - x_0|$, $\sigma \in S \subset S^{n-1}$) verhalten. Hierbei ist $\lambda > 0$ eindeutig bestimmt und φ bis auf skalare Vielfache auch. Die spezielle Struktur der Lösung wird aus Invarianzeigenschaften und qualitativen Aussagen wie Maximumprinzip, Vergleichsprinzip, Regularitätssätzen und Harnack'schen Ungleichungen hergeleitet.

Peter Tolksdorf
Universität Bonn

Liouville theorems, partial regularity and Hölder continuity of weak solutions of quasilinear elliptic systems

We investigate the connections between Liouville type theorems and regularity results for quasilinear elliptic systems of the form

$$(*) \quad -D_\alpha \left(a_{ik}^{\alpha\beta}(x,u)\, D_\beta u^k \right) = f^i(x,u,\nabla u) \qquad (i=1,\dots,N),$$

the right hand side of which grows quadratically with respect to ∇u. The case of a linear growth has been investigated by Giaquinta-Nečas, and by Kawohl. We present an example of a two-dimensional system for which a Liouville theorem holds and which also possesses a discontinuous bounded weak solution. It is proved that a certain Liouville property implies the Hölder continuity of those bounded solutions whose gradients belong to the class $L_{2+\varepsilon,loc}$ and satisfy a reverse Hölder inequality. Furthermore, if there exist nonconstant entire solutions (i.e solutions on all of $\mathbb{R}^n$) we construct discontinuous solutions of the same system. The proofs rest upon partial regularity results due to Giaquinta and Giusti, and a blow-up method is used.

Michael Meier
Universität Bonn

Über eine Methode zur lokalen Analysis des Plateau-Proble[ms] für H-Flächen

Bei der Behandlung der Gleichungen für das Plateau-Probl[em] für H-Flächen zeigt es sich, daß die Einführung eines angepaßt[en] Koordinatensystems („moving frame") diese Gleichungen in solche überführen, deren Struktur wesentlich einfacher und über= schaubarer ist. Das ursprüngliche System zerfällt in ~~ein~~ 2 Ant[eile] einem solchen vom Riemann-Hilbert'schen Typ und einer Laplace-Beltrami-Gleichung für reelle Funktionen. Die Gleichungsstruktur gestattet es, Jacobifelder explicit zu berechnen. Deren genaue Kenntnis wiederum ermöglicht es relativ einfach, Isoliertheit und Stabilität der Lösungen des Plateau-Problems unter sehr allgemeinen Voraussetzungen zu beweisen. Darüberhinaus erlauben es die Gleichung in "angepaßter Koordinaten"-Schreibweise, zahlreiche Betrachtungen über Surjektivität, bzw. konstanter Rang anzustellen. Man gewinnt somit sehr schnell und bequem die Mannigfaltigkeitsstruktur der jeweiligen Objekte. Dies gilt insbesondere auch beim Vorhandensein von Verzweigungspunkten und bei mehrfachem Zusammenhang. Schließlich ermöglicht es diese Methode unter anderem, einen Indexsatz für H-Flächen gemäß dem Tromba'schen Modell im Minimalflächenfall zu beweisen, wobei sich der Beweis ebenfalls ganz erheblich vereinfacht, obwohl die zu betrachtenden Gleichungen nichtlinear sind

Karlheinz Schüffler
Universität Düsseldorf

Regularity for a Minimum Problem with a Free Boundary.

We consider the minimum problem for,
$J(u) = \int_G \frac{|\nabla u|^2}{2} + u^\gamma dx$ with γ fixed $0 < \gamma < 2$, where we minimize in the set $\mathbb{K} := \{v \in H^{1,2}(G) \mid v \geq 0,\ v = u^0|_{\partial G}\}$ for $u^0(x) \geq 0$ fixed.

If for a minimum, $u(x)$, $\{x \mid u(x) = 0\} \neq \emptyset$ the Euler equation degenerates and a loss of regularity occurs across $\partial\{u=0\} =: F$. We show this phenomenon occurs in a precise Hölder way, $u(x) \in C^\beta_{loc}(G)$, $\beta = \frac{2}{2-\gamma}$, $1 < \beta < \infty$. Next we study the free boundary F, deriving a number of Hausdorff measure estimates for F.

Finally we discuss further regularity near flat points of

F (the reduced boundary). In particular for $1 \le \gamma < 2$ in a nbd. of such a pt. F is a $C^{1,\alpha}$ graph. This result was obtained previously for $\gamma = 1$ by L. Caffarelli and for $\gamma = 0$ ($u^0 = \chi(\{u > 0\})$) by H. Alt. and L. Caffarelli.

Daniel Phillips
Purdue University

Bifurcation problems of variational inequalities

Let K be a closed convex cone with its vertex at the origin in a real Hilbert space H. Let $A : H \to H$ be a linear completely continuous operator, $N : \mathbb{R} \times H \to H$ a nonlinear completely continuous mapping satisfying the assumption

$$\lim_{\|v\| \to 0} \frac{N(\mu, v)}{\|v\|} = 0 \text{ uniformly on bounded } \mu\text{-intervals.}$$

The bifurcation problem for the variational inequality

(I) $\quad v \in K,$

(II) $\quad \langle v - \mu A v + N(\mu, v), w - v \rangle \geqq 0$ for all $w \in K$

is considered.

Suppose that there exists an operator $\beta : H \to H$ (a penalty operator) which is completely continuous, monotone ($\langle \beta u - \beta v, u - v \rangle \geq 0$ for all $u, v \in H$) positive homogeneous ($\beta(tu) = t\beta u$ for $t > 0, u \in H$) and such that $\beta u = 0$ for all $u \in K$, $\langle \beta u, u \rangle > 0$ for all $u \notin K$, $\langle \beta v, u \rangle < 0$ for all $v \notin K$, $u \in K^0$ (the interior of K). Suppose that there is no characteristic value of A in $(\mu^{(0)}, \mu^{(1)})$ ~~with~~ having an eigenvector in K. Here $\mu^{(0)}, \mu^{(1)}$ are given simple characteristic values of A with the corresponding eigenvectors $u_0, u_1 \in K^0$. Then there exists a bifurcation point $[\mu_B, 0]$ of (I) (II) with $\mu_B \in (\mu^{(0)}, \mu^{(1)})$. The bifurcating solutions are obtained from the branch of solutions of the equation with the penalty

$$v - \mu A v + \frac{\varepsilon}{1+\varepsilon} N(\mu, v) + \varepsilon \beta v = 0$$

satisfying the norm condition $\|v\|^2 = \frac{\sigma \varepsilon}{1+\varepsilon}$. More precisely, for an arbitrary sufficiently small $\delta > 0$ there exists an unbounded (in ε) closed connected ~~subset~~ set C_δ of solutions of the penalty equation satisfying the norm condition. This set contains $[\mu^{(0)}, 0, 0]$, lies in $(\mu^{(0)}, \mu^{(1)})$ (in μ) and outside of K (in v). By the limiting process $\varepsilon \to +\infty$ along this set the solution $\mu(\delta)$, $v(\delta)$ of (I), (II) satisfying $\mu(\delta) \in (\mu^{(0)}, \mu^{(1)})$, $v(\delta) \in \partial K$, $\|v(\delta)\|^2 = \delta$ is obtained.

In the case $N \equiv 0$, an analogous result for multiple characteristic values is proved.

In some cases the method ensures the existence of an infinite sequence of bifurcation points of (I), (II).

Mila Kučera (Praha)

KONVEXE KÖRPER

19 July – 23 July 1982

Connectivity and freely rolling convex bodies

If C and K are convex bodies in E^d we say that C slides freely in K if for every $\underline{x} \in \partial K$ there is a $\underline{t} \in E^d$ such that

$$\underline{x} \in C + \underline{t} \subset K.$$

Some results of Wolfgang Weil show that this is equivalent to saying that C is a summand of K. We say that C rolls freely in K if every rotation of C slides freely in K. The purpose of the talk is to show that if $(\text{int}\, C + \underline{t}) \cap \partial K$ is a topological ball for all translates $\underline{t} \in E^d$ then C slides freely in K. This provides an answer to a problem posed by Bill Firey at the previous Konvexe Körper meeting. The main tool in the proof is a geometrical adaptation of the Alexander Duality theorem in combinatorial topology. A corollary of the result is the fact that K is a geometric ball if and only if $\text{int}\, gK \cap \partial K$ is a topological ball for all rigid motions g of E^d.

Paul Goodey (London).

THE BRIANCHON–GRAM THEOREM AND GENERALIZATIONS

A theorem attributed (for dimension $d = 3$) to Gram (1874), but in fact originally proved by Brianchon (1837), states that, if $\alpha(F,P)$ is the interior angle of the d-polytope P ($d \geq 1$) at its face F, then

$\Sigma_F (-1)^{\dim F} \alpha(F,P) = 0$. Similarly, Sommerville (1927) showed that, if P is a polyhedral d-cone with apex o, then $\Sigma_F (-1)^{\dim F} \alpha(F,P) = (-1)^d \alpha(o,P)$. The subject of this talk is a common generalization to a general d-dimensional polyhedral set P. The proof is in the spirit of that of Shephard (1967) of the Brianchon-Gram theorem, and so the result is better expressed in terms of equidissectability of polyhedral cones. Thus, if $A(F,P) = \text{pos}(P-x)$, where $x \in \text{relint} F$, $a(F,P)$ is the representative in the spherical polytope group Σ^d of the equivalence class of $A(F,P)$ under orthogonal transformations and equidissectability, and $\text{rec} P$ is the recession cone of P, then

$$\Sigma_F (-1)^{\dim F} a(F,P) = (-1)^d a(o, \text{rec} P).$$

The right side of this is 0 if $\dim(\text{rec} P) < d$, but there is a variant which detects $\text{rec} P$, involving analogues in Σ^d of the spherical quermass-integrals. The angle sum relations of McMullen (1975) can similarly be generalized to equidissectability theorems for polyhedral cones.

Peter McMullen (London)

EQUIDISTANT CONVEX SETS

Let $N(n,\varepsilon)$ denote the maximal number of convex subsets of the n-dimensional Euclidean unit-ball, such that the HAUSDORFF-distance of each pair of these sets is equal to $\varepsilon > 0$. Then $N(1,\varepsilon) = 4$ and for $n \geq 2$ the following inequalities hold:

$$2^{c(n,\varepsilon)} \leq N(n,\varepsilon) \leq 2^{h_n(\frac{\varepsilon}{2})},$$

where $c(n,\varepsilon)$ denotes the maximal number of points on the $(n-1)$-dimensional unit-sphere, the pairwise geodesic distance of which is not less than $\arccos(1-\varepsilon)$, and $h_n(\eta)$ with $\eta > 0$ denotes the η-entropy of the n-dimensional Euclidean unit-ball [BRONSTEIN, Sib. Math. J. 17 (1976), 393-398]. Using an upper bound of Bronstein for $h_n(\frac{\varepsilon}{2})$ and a lower bound for $c(n,\varepsilon)$ one obtains $\log_2 N(n,\varepsilon) = O(\varepsilon^{(\frac{1-n}{2})})$ as $\varepsilon \to 0$.

Günter Lettl (Graz)

SOME PROPERTIES OF SETS OF CONSTANT WIDTH

In this talk we prove that every compact convex set in Euclidean space $\mathbb{R}^n$ is contained in the set of constant width of the same diameter which itself is contained in the circum-ball of the given set. Moreover, we prove that it could be required for the set of constant width also to have the same intersection as the given set with its circumsphere. As the consequence of this theorem we prove that ~~any~~ for every closed subset C of the sphere S which contains no pair of antipodal points and whose convex hull contains the centre of S, there is a set of constant width B such that $B \subset \operatorname{conv} S$, $\operatorname{diam} B = \operatorname{diam} C$ and $B \cap S = C$. Also, as the consequence, we get for every compact, convex set C the inequalities $w \leq r + R \leq D$, where w, D, r and R are the width, diameter, the radii of inscribed and circumscribed ball respectively.

Siniša Vrećica (Beograd)

FINITE SETS WHICH CONTAIN THEIR RADON POINTS

A <u>Radon-partition</u> in a set S is a pair A, B of disjoint subsets of S such that $\operatorname{conv} A \cap \operatorname{conv} B \neq \emptyset$. The partition is <u>primitive</u> if no proper subset of $A \cup B$ has a Radon-partition, and in that case $\operatorname{conv} A \cap \operatorname{conv} B$ consists of a single point, a <u>Radon-point</u> of S. This talk describes a complete characterization of the <u>stable</u> sets, i.e. those finite sets in E^d which contain all their Radon points.

These sets are classified according to their <u>core</u>:
core $S = S \cap \text{relint conv} S$.

1) A set is stable with an empty core iff it is a <u>free union</u> of ~~sets~~ stable sets with nonempty cores, i.e. $S = \bigcup_{T\in\mathcal{T}} T$ with $|\mathcal{T}|>1$, and for every selection of affinely independent ~~sub~~sets B_T in T, the set $\bigcup_{T\in\mathcal{T}} B_T$ is affinely independent.

2) A set S is stable with full dimensional core (i.e. dim core S = dim S) iff it is a ~~g~~ <u>pinwheel</u>, which is the proper d-dimensional generalization of the 6-point set:  in the plane.

3) A set S is stable with $0 \leq$ dim core $S <$ dim S iff it is obtained from a free union by adding one point, with certain restrictions about that free union, and about the location of the additional point.

Michael Kallay (Norman)

<u>Sattelpunkte der Distanzfunktion eines konvexen Körpers.</u>

Zu einem konvexen Körper K im euklidischen Raum E^n $(n \geq 3)$ ist ein Punkt p gesucht, durch den möglichst viele Normalen gehen. Dazu werden die kritischen (stationären) Stellen von

$$f_p: \partial K \longrightarrow \mathbb{R}, \quad f_p(x) = \|x-p\|^2,$$

gesucht. Sei μ_i die Anzahl der kritischen Punkte mit Index i. Aus den Morse-Relationen ergibt sich für die Gesamtzahl der Normalen durch p

$$\sigma = \sum_{j=0}^{n-1} \mu_j \geq 2(\mu_0 + \mu_{n-1} - 1).$$

Anwendungen:

Satz: In int K existiert p mit $\sigma \geq 6$ (Vermutung $\sigma \geq 2n$).

Nach T. Zamfirescu berührt die Umkugel der meisten K (im Sinne Bairescher Kategorie) in genau $n+1$ Punkten. Sei p der Mittelpunkt. Das ergibt

Satz: Für die meisten K existiert p mit $\sigma \geq 2n+2$. — In- und Umkugel eines Körpers K konstanter Breite berühren in mindestens 3 Punkten. Das ergibt für den gemeinsamen Mittelpunkt $\sigma \geq 10$. Welche Zahl ist hier optimal?

Es wird gezeigt, daß an ∂K zur Konvexität keine zusätzlichen Glattheitsforderungen gestellt werden müssen. Nach A.D. Alexandrov ist ∂K f.ü. 2x differenzierbar. Dies reicht, um zu garantieren, daß die Morse-Relationen gelten und keine kritischen Punkte zusammenfallen (degenerieren). Dabei wird an Stelle des üblicherweise benutzten Morse-Lemmas ein Beweis aus Seifert-Threlfall (Variationsrechnung im Großen) herangezogen.

Erhard Heil (Darmstadt)

Construction theorems for Polytopes

Let $Q \subset R^d$ be a d-polytope and let $x \in R^d$ be a point outside Q. x defines a unique partition $\mathcal{A}, \mathcal{B}, \mathcal{C}$ of the set of facets of Q, such that x lies beyond every $A \in \mathcal{A}$, beneath every $B \in \mathcal{B}$ and in the affine hull of every $C \in \mathcal{C}$. On the other hand, for a given partition $\mathcal{A}, \mathcal{B}, \mathcal{C}$ of the set of facets of a polytope Q, not always there exists such a point x. Sometimes we may help the situation by replacing Q by another polytope Q' combinatorially equivalent to Q, but often even this will not do.

We describe some families of pairs of types $(\mathcal{A}, \mathcal{C})$ such that for every such pair $(\mathcal{A}, \mathcal{C})$ and for every polytope Q which "contains" $(\mathcal{A}, \mathcal{C})$, there is a point x which lies beyond every $A \in \mathcal{A}$, in the affine hull of every $C \in \mathcal{C}$ and beneath all the other facets of Q, and $\mathrm{vert}(\mathrm{conv}(Q \cup \{x\})) = \mathrm{vert}\, Q \cup \{x\}$. We also describe a family of pairs $(\mathcal{A}, \mathcal{C})$ such that the above cannot be guaranteed for every polytope which "contains" such a pair $(\mathcal{A}, \mathcal{C})$, but for every such a polytope Q there is a polytope Q' projectively equivalent to Q (under φ) and there is a point x such that x lies beyond every $A \in \varphi(\mathcal{A})$, in the affine hull of every $C \in \varphi(\mathcal{C})$ and beneath all the other facets of Q', and $\mathrm{vert}(\mathrm{conv}(Q' \cup \{x\})) = \mathrm{vert}\, Q' \cup \{x\}$.

Amos Altshuler (Beer-Sheva)

Funktionale konvexer Polyeder

Im Rahmen einer axiomatischen Theorie der Funktionale konvexer Polyeder wird auf einen möglichen Zusammenhang zwischen dem Funktionalwert der Minkowskischen Summe und der Summe der Funktionalwerte der Summanden hingewiesen. Weiterhin wird eine Ungleichung im Raum bewiesen, die die Charakterisierungsaufgabe der bewegungsinvarianten, additiven, definiten bzw. beschränkten Funktionale unter einen gemeinsamen Zusammenhang stellt.

Wolfgang Spiegel (Wuppertal)

Knapsack polytopes have relatively few vertices.

If c_i, a_i, b are non-negative integers the knapsack problem is to solve $\max\ c_1x_1 + \dots + c_nx_n$ subject to the constraints that $\underset{\sim}{x}$ is a non-negative integer point i.e. $\underset{\sim}{x} \geq 0$, $\underset{\sim}{x} \in \mathbb{Z}^n$ and that $a_1x_1 + \dots + a_nx_n \leq b$. The convex hull of the points satisfying the constraints is a lattice polytope K and, of course, linear functionals such as $c_1x_1 + \dots + c_nx_n$ will have their maximum at a vertex of K. Here we prove that the number of vertices of K is at most $\prod_{i=1}^{n} \log_2 \frac{4b}{a_i}$ which answers affirmatively a problem of C. A. Rogers. This is joint work with Alan Hayes.

David Larman (London).

Convex Bodies and Algebraic Geometry

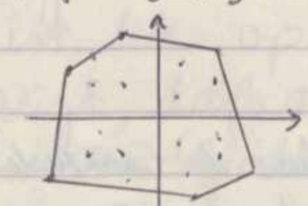
Since about 10 years interesting relationships have developed between theories of conv.-bod. and alg. geom.; we give a short survey about 4 lines of developement. In all cases the underlying idea is as follows:

Let $z \in \mathbb{C}^n$, $z = (z_1, \dots, z_n)$, $\varsigma \in \mathbb{Z}^n$, $\varsigma = (\varsigma_1, \dots, \varsigma_n)$; $z^\varsigma := z_1^{\varsigma_1} \cdots z_n^{\varsigma_n}$;

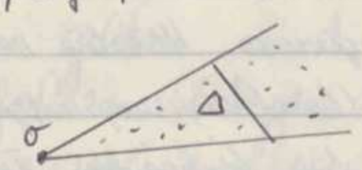
$f: \mathbb{C}^n \to \mathbb{C}$ $\quad f(z) = \sum_{\varsigma \in I} c_\varsigma z^\varsigma$, where f can be a polynomial, a Laurent polynomial, or a series. If f a Laurent pol.:

$$f \longrightarrow \operatorname{supp} f := \{\varsigma \mid c_\varsigma \neq 0\} \longrightarrow \operatorname{conv} \operatorname{supp} f = \mathcal{N}(f)$$

Newton-polytope of f (lattice polytope)



Many properties of such functions depend only on $\mathcal{N}(f)$, not on supp f. If Δ is a polytope s.th. $0 \notin \operatorname{aff} \Delta$, the cone $(\mathbb{R}_+ \Delta) \cap \mathbb{Z}^n$ is a semi-group of lattice pts. representing



a subring of $\mathbb{C}[z_1, \dots, z_n]$, $\mathbb{C}[z_1, z_1^{-1}, \dots, z_n, z_n^{-1}]$, or $\mathbb{C}[[z_1, \dots, z_n]]$.
This cone is an important tool.

1.) Knudsen and Mumford have shown that the following thm. plays an important role in problems of resolving singularities (Springer Lecture notes "Toroidal embeddings I" 1973).

Let P be a lattice polytope. There ex. $\nu \in \mathbb{N}$ and a subdivision of P into simplices T_j s.th. for all T_j:

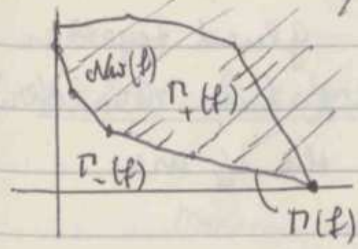
(1) vert $T_j \subset \frac{1}{\nu} \mathbb{Z}^n$

(2) vol $T_j = \frac{1}{\nu^n n!}$

The proof of this thm. together with refinements covers 55 pages. Maybe it can be simplified.

2.) Milnor's number: If a polynomial f has an isolated critical pt. (singularity) at 0, consider $V = \{z \mid f(z) = 0\}$, and for suff. small $\varepsilon > 0$ the $(2n-1)$-sphere S_ε about 0 with radius ε.

$K := S_\varepsilon \cap V$ is a (possibly knotted) $(2n-3)$-manifold. $S_\varepsilon \setminus K$ has a fibration under $\phi : S_\varepsilon \setminus K \to$ unit circle def. by $\phi(z) = \frac{f(z)}{|f(z)|}$.
The fibre is homotopically equivalent to a bouquet $S^{n-1} \vee \cdots \vee S^{n-1}$ of $\mu(f)$ spheres; $\mu(f)$ is called the Milnor number of f (Milnor 1967). Palamodov has shown (1967) that $\mu(f)$ can be calculated as $\mu(f) = \dim_{\mathbb{C}} \mathbb{C}[[z_1, \dots, z_n]] \big/ \left(\frac{\partial f}{\partial z_1}, \dots, \frac{\partial f}{\partial z_n}\right)$



Let $\Gamma_+(f) := \bigcup_{a \in Nw(f)} (\mathbb{R}^n_+ + a)$, $\Gamma_-(f) := \mathbb{R}^n_+ \setminus \operatorname{int} \Gamma_+(f)$, $\Gamma(f) = \Gamma_+(f) \cap \Gamma_-(f)$, where $\Gamma(f)$ is assumed to cut all coordinate axes (f permissible). f is called non-degenerate, if $z_1 \frac{\partial f}{\partial z_1}, \dots, z_n \frac{\partial f}{\partial z_n}$ restricted to $\Gamma(f)$ is $\neq 0$ in $(\mathbb{C} \setminus \{0\})^n$.
Let $V_n :=$ n-dim. vol. of $\Gamma_-(f)$; $V_k := \sum \operatorname{vol}_k (\Gamma(f) \cap U^k)$ where U^k is any coord. subspace of dim. k. Define
$\nu(f) := n!\, V_n - (n-1)!\, V_{n-1} \pm \cdots + (-1)^{n-1} 1!\, V_1 + (-1)^n$ Newton's number

Thm. (Kouchnirenko 1976) If f permissible, 0 isol. crit. pt., then
(1) $\mu(f) \geq \nu(f)$
(2) $\mu(f) = \nu(f)$ if f is non-degenerate

Similar thms. are shown for Laurent-pol. and about the sum of all Milnor numbers of f.

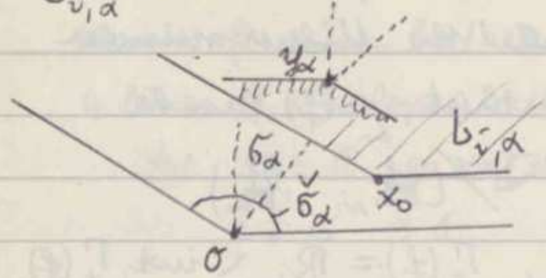
3.) Let $\left.\begin{array}{l} f_1(z_1, \dots, z_n) = 0 \\ \vdots \\ f_n(z_1, \dots, z_n) = 0 \end{array}\right\} (*)$ be a system of polynomial equ.

$\ell(f_1, \dots, f_n) := \#$ sol. of $(*)$ on $(\mathbb{C} \setminus \{0\})^n$ in case it is finite.

Thm. (D. I. Bernstein 1978)
$\ell(f_1, \dots, f_n) = n!\, V(Nw(f_1), \dots, Nw(f_n))$,
V denoting the mixed volume of the lattice polytopes $Nw(f_1), \dots, Nw(f_n)$.
Using this formula a new and short proof of the Alexandrov-Fenchel-Inequalities can be obtained: $V^2(K_1, \dots, K_n) \geq V(K_1, K_1, K_3, \dots, K_n) V(K_2, K_2, K_3, \dots, K_n)$, where $K_1, \dots, K_n$ are arbitrary convex bodies.

4.) Consider lattice polytopes $K_1, \dots, K_k \subset \mathbb{R}^n$. Let p_K denote the nearest point map of a compact set K.

If $y_\alpha \in \operatorname{vert}(K_1 + \dots + K_r)$, denote $\sigma_\alpha := \ell^{-1}_{K_1+\dots+K_r}(y_\alpha) - y_\alpha$, $\check{\sigma}_\alpha := \{x \in \mathbb{R}^n \mid x \cdot u \geq 0 \text{ for all } u \in \sigma_\alpha\}$. Let $x_0 \in \operatorname{vert} K_i$, and let $L_{i,\alpha}$ denote the sub-$\mathbb{C}[\check{\sigma}_\alpha \cap \mathbb{Z}^n]$-module generated by x_0.



We assume the element of $\operatorname{Spec}\mathbb{C}[\check{\sigma}_\alpha \cap \mathbb{Z}^n]$ glued together to a variety X. Furthermore, we assume the $L_{i,\alpha}$ for fixed i glued together to a sheaf L_i. Let χ = Euler characteristic.

In $\chi(X, L_1^{\nu_1} \otimes \dots \otimes L_r^{\nu_r}) = \sum_{\substack{\alpha \in \mathbb{N}^r \\ |\alpha| = n}} V_\alpha \nu_1^{\alpha_1} \cdots \nu_r^{\alpha_r} + (\deg < n)$ the V_α are the mixed volumes of $K_1, \dots, K_r$: $V(\underbrace{K_1, \dots, K_1}_{\alpha_1}, \dots, \underbrace{K_r, \dots, K_r}_{\alpha_r})$

By using Hodge's index theorem a further proof of the Alexandrov-Fenchel inequality is obtained.

Günter Ewald

Peakless functions on G-spaces

Convex functions on noncompact complete Riemann spaces have been studied extensively in recent years. A report by R. Walter, Konvexität in riemannschen Mannigfaltigkeiten, is found in Jahresber. DMV 83 (1981)

The lecture represents joint work with B. B. Phadke and its purpose is to show that in this theory both the Riemannian character of the metric and the convexity of the function can often be replaced by very much weaker conditions. The Riemann space may be a G-space (H. Busemann, The Geometry of Geodesics, New York 1955) which include the Finslerspaces, but may not be differentiable, and the convexity may be replaced by peaklessness. $f(t)$ is peakless if continuous and $t_1 < t_2 < t_3$ implies $f(t_2) \leq \max(f(t_1), f(t_3))$ with equality only when $f(t_1) = f(t_2) = f(t_3)$ (The noncompactness enters because a nonconstant peakless

function on a compact space is constant and hence at least integral)
Often results on the general case yield stronger results than the original ones on Riemann spaces, because a peakless function on a Riemann space need not be convex.

Herbert Busemann

X-RAY PROBLEMS.

Modern methods in computerized tomography involve the use of the Radon transform to reconstruct a density function from its X-rays pictures in different directions. Exact reconstruction from a finite number of pictures is impossible, but approximate reconstructions can be obtained.

When the density function is the characteristic function of a compact convex body in the plane, the problem of reconstruction becomes that of P.C. Hammer (AMS Symposium in Convexity, 1961). The question of when a given set of X-ray pictures determine this convex body uniquely (and here, X-ray pictures may essentially be identified with Steiner symmetrals) was solved in R.J.G. & P.McMullen, J.L.M.S. 1980. It turns out that certain sets of 4 directions exist so that the corresponding pictures determine the convex body uniquely. The problem of actually reconstructing the body is still unsolved.

When the pictures are taken from point sources, the situation is still unresolved. It may be true that point X-rays taken from any 2 points will determine shape uniquely, but this has only been proved in certain cases, independently by R.J.G. & K.J. Falconer.

Finally, when the convex body is prescribed in advance and it is asked: how many X-ray pictures (Steiner symmetrals) must be taken in order to be unique to that body, we have Giering's problem. O. Giering showed the answer to be 3, and R.J.G. has a short proof of this fact.

Richard J. Gardner.

The $(e_1, \ldots, e_q)$ - convex sets and their applications

The idea behind the notion of an $(e_1, \ldots, e_q)$ - convex set is to analyze the behaviour at the infinity of the sections of a convex set A by nested affine varieties through a fixed point in the relative interior of A. This notion has a simple dual version in terms of barrier-cones and cohypercones. We use it to characterize dually similar convex sets (i.e. convex sets having the same barrier-cone), answering a problem of Valentine. The $(e_1, \ldots, e_q)$ - convex sets are the frame to present the strong intersection property and some generalizations. Moreover, descriptions of various maximal filters of convex sets are given.

René Fourneau

The minimal ellipsoid of a typical convex body

In the sense of Baire category most convex bodies touch the boundary of their minimal ellipsoid in exactly $d(d+3)/2$ points where d denote the dimension. For most convex bodies the group of affinities consists of the identity only.

Peter Gruber (Wien)

The toroidal analogue to Eberhard's theorem

A polyhedral 2-manifold M is a geometric cell-complex (whose facets are planar convex polygons) such that the underlying point-set is a closed connected 2-manifold in some euclidean space. Let $p_k(M)$, $v_k(M)$ denote the number of k-gonal facets of M, or the number of k-valent vertices, respectively.

Then the following analogue to Eberhard's theorem holds:
Let s, p_k $(k \geq 3, k \neq 6)$ be non-negative integers. There exists a polyhedral torus T in $\mathbb{E}^3$ such that $p_k(T) = p_k$ $(k \neq 6)$ and $\sum_{k \geq 3} (k-3) v_k(T) = s$ if and only if $\sum_{k \geq 3} (6-k) p_k = 2s$ and $s \geq 6$.

Peter Gritzmann (Siegen)

Volume and circumradius of simplicial polytopes

Let V be the volume and R the circumradius of a simplicial 4-polytope with n facets. It is conjectured that $V/R^4 \leq n \cdot v(2\pi^2/n)$, where $v(\tau)$ denotes the volume of a 4-simplex $OABCD$, such that O is the center of $S^3 = \{x \in \mathbb{R}^4 : \|x\| = 1\}$ and $ABCD$ are the vertices of a regular spherical simplex on S^3 with volume τ.

L. Fejes Tóth has proved an analogous inequality for the inradius in 1955 for arbitrary dimension.
Here a proof of the above inequality for $n \leq 8$ and $n \geq 16$ is discussed, which used e.g. the spherical Steiner symmetrization.

As a consequence we get for every dimension d: The regular simplicial d-polytopes have the greatest volume among all simplicial d-polytopes with the same number of facets and the same circumradius

Johann Linhart (Salzburg)

Spheres with small valences

By a simplicial 3-sphere S we mean an abstract simplicial complex whose body is a topological 3-sphere. S is called polytopal if it is isomorphic to the boundary complex of some 4-polytope. We prove the following result of joint work together with G. Ewald: Let S be a simplicial 3-sphere with: (1) The vertices of S are at least 5-valent and at most 9-valent. (2) There is a 3-cell $F \in S$ such that the sum of the valences of the vertices of F is ≤ 26. Then S is polytopal.

Christoph Schulz (Hagen)

On a conjecture of Fejes Tóth

Let k non-overlapping translates B_i of the unit ball $B^d \subset E^d$ be given and let C_k denotes the convex hull of their centers. In 1975 Fejes Tóth conjectured that for $d \geq 5$ the volume of $\bigcup_{i=1}^{k} B_i = C_k + B^d$ is minimal, if C_k is a segment of length $2(k-1)$. Because $S_k + B^d$ forms a sausage this is called the "sausage conjecture". In a first Theorem it is shown that the sausage is locally minimal or more precisely $V(C_k + B^d) \geq V(S_k + B^d)$ if $C_k \subset g + \frac{\sqrt{d-1}}{\sqrt{d-1+2\pi}} \cdot B^d$ for a line g. Then it is proved that the assertion holds if the centers of the balls lie in a lower dimensional subspace i.e. if $\dim C_k$ is small enough.

Ulr. Tietze[?]

Zum Problem einer algorithmischen Lösung des Steinitz-Problems

n points in $\mathbb{R}^d$, $p_1, \dots, p_n$, induce a map $OR : T^n_{d+1} \to \{-1, 0, 1\}$,

$$\operatorname{conv}\{p_{j_1}, \dots, p_{j_{d+1}}\} \longmapsto \operatorname{sign} \begin{vmatrix} 1 & p^1_{j_1} & \cdots & p^d_{j_1} \\ \vdots & \vdots & & \vdots \\ 1 & p^1_{j_{d+1}} & \cdots & p^d_{j_{d+1}} \end{vmatrix}, \quad j_1 < \dots < j_{d+1},$$

on the set T^n_{d+1} of all d-simplices (having d+1 of the points as vertices) in $\{-1,0,1\}$. This leads to a vector of orientations $OR(p_1,\dots,p_n) \in \{-1,0,1\}^{\binom{n}{d+1}} =: M$.

Realisation-problem: Given any $A \in M$. Do there exist points $p_1,\dots,p_n \in \mathbb{R}^d$ such that $OR(p_1,\dots,p_n) = A$?

- A method using Graßmann-Plücker-relations was discussed geometrically.
- All simplicial nonpolytopal spheres ($d=4$, $n=8,9$) were found again.
- Combinatorial automorphisms are not always metrical realisable (J.B./G. Ewald/P. Kleinschmidt) – To decide the problem in $\mathbb{R}^d$ it suffices to decide a corresponding problem in $\mathbb{R}^{n-d-2}$. – All neighborly spheres ($d=6$, $n=10$) are classified in polytopal and nonpolytopal spheres (J.B./I. Shemer).

Jürgen Bokowski (Darmstadt)

Kombinatorische Analoga regulärer Polytope

Das Konzept der regulären Inzidenzkomplexe verallgemeinert die klassische Theorie der regulären Polytope kombinatorisch und gruppentheoretisch. Es umfaßt außer den regulären Polytopen auch die regulären komplexen Polytope, projektive und andere Räume und zahlreiche bekannte Konfigurationen.

Unter anderem wurde eine Konstruktion angegeben, die jeden endlichen und nicht-ausgearteten d-dimensionalen regulären Inzidenzkomplex $\mathcal{K}$ als Facette eines endlichen und nicht-ausgearteten (d+1)-dimensionalen regulären Inzidenzkomplex $\mathcal{L}$ realisiert. Dabei werden die Automorphismen von $\mathcal{K}$ zu solchen von $\mathcal{L}$ fortgesetzt. Besitzt $\mathcal{K}$ genau m Facetten, so ist die Automorphismengruppe von $\mathcal{L}$ das direkte Produkt der symmetrischen Gruppe S_{m+1} und der Automorphismengruppe von $\mathcal{K}$.

Egon Schulte

László Fejes Tóth's six circle problem

The problem in the title is this. Suppose we are given a packing in the plane so that each circle is touched by at least six others. Let $\{r_i\}_{i=1}^{\infty}$ be the set of radii of the circles. Then either $\inf r_i = 0$ or $\sup r_i = 0$ or $r_1 = r_2 = r_3 = \ldots$
In a joint work with N. Dolbilin, Z. Füredi and J. Pach we prove this conjecture, and even a bit more than that, namely, under the above conditions either $\inf r_i = 0$ or $r_1 = r_2 = r_3 = \ldots$

Imre Bárány

Many endpoints and few interior points of geodesics

"Most" signifie "all, except those in a set of first Baire category". Alors, voilà nos résultats:

1. On most convex surfaces in $\mathbb{R}^n$, most points are endpoints.

2. On most convex surfaces in $\mathbb{R}^3$, at every point, most tangent directions are singular.

Pour apprécier les théorèmes il est util de savoir que:

Un "endpoint" est un point qui n'est intérieur à aucune géodesique de la surface;

une direction tangente est "singulière" s'il n'y a pas de géodesique dans cette direction (d'après Aleksandrov).

Pas seulement ça: voici d'autres choses fascinantes qui se passent sur la plupart des surfaces convexes:

3. Most geodesics are not extendable.

4. Most circles (bezüglich der inneren Metrik) have no smooth arc.

5. Most pairs of points are joined by a unique shortest path (sic!).

Et, plus généralement, trotz 5.,

6. Most facts are strange.

T. Zamfirescu

The Mean Quermassintegral of Simplices Circumscribed about a convex body

Let P be a simplex circumscribed about the unit ball in E^n and K any convex body. For each rotation of P, consider the simplex circumscribed about K with facets parallel to the facets of this rotated simplex. We shall establish a lower bound for the mean value of the q-th quermassintegral, $q = 0, \dots, n-1$, of these circumscribed simplices. Equality will hold for all K, if $q = n-1$; and for $q = 0, \dots, n-2$, equality will hold if and only if K is a ball

J.R. Sangwine-Yager

Some remarks on Eckhoff's conjecture on Radon numbers

In 1966, H. Tverberg gave a far reaching generalization of the classical theorem of J. Radon; he namely showed that any set S of $(d+1)k-d$ points in $\mathbb{R}^d$ can be partitioned into k components such that the convex hulls have a nonempty intersection, i.e. S has a k-Radon partition (he gave a new proof in 1981). This theorem was generalized by Doignon-Valette to any affine space over an ordered division ring. All this led J. Eckhoff [1978] to the conjecture that for any convexity structure $(X, \mathcal{C})$, i.e. a set together with a collection of subsets closed under intersections, $r_k \le (r_2-1)(k-1)+1$, r_k being the k-Radon number, i.e. the least natural number n such that for each subset A of X with $\# A \ge n$, A has a k-Radon partition. If Eckhoff's conjecture is true, there is a complete combinatorial proof of Tverberg's theorem.
All examples of convexity structures for which the k-Radon numbers have been calculated do not falsify Eckhoff's conjecture (see e.g. Jamison + Waldner). The k-Radon numbers are still not calculated for the so-called "Gaussian integers" in $\mathbb{R}^d$ with $d \ge 3$ (the case $d=2$ was settled by J-P. Doignon).

In 1981 R. Jamison-Waldner proved Eckhoff's conjecture to be true in case $r_2 = 2$ or $r_2 = 3$; in 1982 G. Sierksma settled the case for $k = 3$ and $\# X = 2r_2 - 1$ or $\# X = 2r_2$.

Let's hope the problem will be solved for the next meeting on "Konvexe Körper" in Oberwolfach

Eric Degreef (Brussels).

Approximation of convex domains by inscribed polygons

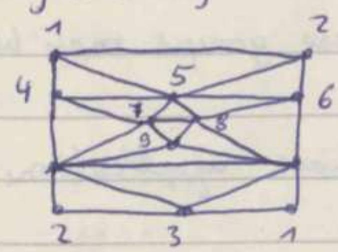
For a convex domain C in the plane let $f(k, C)$ denote the maximum of the area of a k-gon inscribed into C. Let $\mathcal{C}(a, p)$ be the class of convex domains in the plane with area $\geq a$ and perimeter $\leq p$. We are looking for a convex domain $C \in \mathcal{C}(a, p)$ wich can be approximated the best by inscribed k-gons, that is for which $f(k, C)$ is maximal. It is shown that for $\frac{p^2}{a} \leq \frac{4k}{\cos\frac{\pi}{k}}$ the convex domain $C \in \mathcal{C}(a, p)$ for which $f(k, C)$ attains its maximum $\bar{f}(k, a, p)$ is a regular arc-sided k-gon with area a and perimeter p. (A regular arc-sided k-gon is a convex domain with k-fold rotatory symmetry bounded by k congruent circular arcs.) The result above is used to show that the density of a covering with convex domains from $\mathcal{C}(a, p)$ not crossing each other is not less than $\frac{a}{\bar{f}(6, a, p)}$.

It is also shown that for $\frac{p^2}{a} \leq \frac{4k}{\cos\frac{\pi}{k}}$ the regular arc-sided k-gon with area a and perimeter p is the element of $\mathcal{C}(a, p)$ which can be approximated the best by inscribed k-gons in the sense that (i) the perimeter of a k-gon of maximal perimeter inscribed into it is maximal, or that (ii) the minimum of the area-deviation of an inscribed k-gon from it is minimal, or that (iii) the minimum of the perimeter-deviation of an inscribed k-gon from it is minimal.

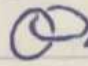
Gábor Fejes Tóth

A nonpolyhedral triangulated Möbius strip
A pair of linked Császár tori

Theorem: There exists a triangulated Möbius strip with 9 vertices which cannot be embedded in $\mathbb{R}^3$ such that all combinatorially given edges are straight lines.



The triangles 1231 and 4564 have to wind at least twice around one another for topological reasons, which is impossible for triangles with straight lines.

Theorem: There exists a pair of polyhedral tori with 7 vertices each ("Császár-tori") which are linked. They can be chosen such that each of them has an axis of symmetry, a pair of nonconvex quadrangles as faces and such that one can rotate in the other. 

Ulrich Brehm

Applications of convexity to problems of traffic control

The minimisation of delay at a junction with traffic signals is discussed. It is common to control traffic by repeating a fixed cycle of periods of green for the different streams of traffic at the junction. This gives rise to a convex programming problem. The problem is considered of how to structure suitable control sequences for junctions with n arms. The junction is represented by a convex polygon with 'origin' and 'destination' vertices. Streams of traffic are represented by line segments within this polygon. Using this model a method is given for constructing suitable control sequences.

f- vectors of d-representable complexes.

We prove a conjecture of Eckhoff concerning f-vectors of d-representable complex. The proof uses exterior algebra techniques. In particular, we a introduce a notion of generalized homology groups $H_k^{p,q}(C)$ for a simplicial complex C. These groups may be of some independent interest. The method used have applications to other problems in geometry and combinatorics. Gil Kalai (Jerusalem)

TURÁN-TYPE PROBLEMS FOR PLANAR SEGMENT GRAPHS

A GEOMETRIC GRAPH (gg) IS A PAIR $G = \langle V, E \rangle$, WHERE V (VERTICES) IS A FINITE SET OF POINTS IN THE PLANE, E (EDGES) IS A SET OF NONDEGENERATE CLOSED STRAIGHT LINE SEGMENTS WITH ENDPOINTS IN V, AND NO EDGE CONTAINS A VERTEX IN ITS RELATIVE INTERIOR. G IS A CONVEX gg (= cgg) IF V IS THE SET OF VERTICES OF A CONVEX POLYGON (OR $|V| \leq 2$). G IS SIMPLE IF THE RELATIVE INTERIORS OF THE EDGES ARE PAIRWISE DISJOINT. G IS COMPLETE IF $|E| = \frac{1}{2}|V|(|V|-1)$.

THEOREM 1 LET Γ BE AN (ABSTRACT) GRAPH WITH k VERTICES ($k \geq 3$). THEN THE FOLLOWING ASSERTIONS ARE EQUIVALENT:

(a) EVERY COMPLETE cgg WITH k VERTICES HAS A SIMPLE SUB-gg ISOMORPHIC TO Γ.

(b) Γ IS ISOMORPHIC TO A SPANNING SUBGRAPH OF THE GRAPH OF A TRIANGULATION OF A CONVEX k-GON.

(c) Γ IS AN OUTERPLANAR GRAPH.

(d) EVERY BLOCK OF Γ IS EITHER A K_2 (= ┃) OR A CIRCUIT WITH (POSSIBLY) SOME NON-CROSSING DIAGONALS.

(e) EVERY COMPLETE gg WITH k VERTICES (NOT NECESSARILY CONVEX) HAS A SIMPLE SUB-gg ISOMORPHIC TO Γ.

<u>THEOREM 2</u> EVERY gg WITH n VERTICES AND MORE THAN $\binom{n-1}{2}$ EDGES HAS A SIMPLE SPANNING SUBTREE.

FOR A GRAPH Γ, DEFINE:

$T(\Gamma, n) = \max\{e$: THERE EXISTS A gg WITH n VERTICES AND e EDGES, WHICH HAS NO SIMPLE SUB-gg ISOMORPHIC TO Γ.$\}$.

$T_c(\Gamma, n)$ IS DEFINED IN THE SAME WAY, WITH gg REPLACED BY cgg.

<u>THEOREM 3</u> IF Γ IS 2-CONNECTED AND SATISFIES THE ASSERTIONS LISTED IN THEOREM 1, THEN $T_c(\Gamma, n) = T(\Gamma, n) = T(k, n)$, WHERE $T(k, n)$ IS THE CLASSICAL (GRAPH THEORETIC) TURÁN NUMBER.

<u>DEFINITION</u> LET T BE A TREE. THE <u>DERIVED TREE</u> T' IS OBTAINED BY REMOVING THE ENDPOINTS OF T AND THE ADJACENT EDGES. T IS A <u>CATERPILLAR</u> IF T' IS A PATH (OR EMPTY).

<u>THEOREM 4</u> IF Γ IS A CATERPILLAR WITH k VERTICES, $k \geq 2$, THEN $T_c(\Gamma, n) = [\frac{1}{2} n(k-2)]$ FOR $n > k$. (IF $n = k$, REPLACE "=" BY "$\leq$".)

<u>DEFINITION</u> SUPPOSE Γ HAS k VERTICES AND SATISFIES THE ASSERTIONS OF THEOREM 1. LET CK_k BE A COMPLETE cgg WITH k VERTICES. DEFINE $\ell(\Gamma)$ TO BE THE SMALLEST POSSIBLE NUMBER OF BOUNDARY EDGES OF CK_k USED BY ANY SUB-gg OF CK_k ISOMORPHIC TO Γ. IF Γ IS A TREE OF DIAMETER ≥ 3, THEN $\ell(\Gamma)$ IS JUST THE NUMBER OF ENDPOINTS OF THE DERIVED TREE T'.

<u>THEOREM 5</u> SUPPOSE Γ SATISFIES THE ASSERTIONS OF THEOREM 1. THEN

$$\lim_{n \to \infty} \frac{T_c(\Gamma, n)}{n^2/2} = 1 - \frac{1}{\ell(\Gamma) - 1}.$$

מיכה א פרלס Micha A. Perles (Jerusalem)

Neighborly families of convex polytopes.

A family $\{P_1 \ldots P_n\}$ of convex d-polytopes in E^d is called neighborly if the mutual intersections are all $(d-1)$-dimensional. Let $f(d,k)$ $[f(d,*)]$ denote the maximal number of convex d-polytopes in a neighborly family in E^d, all having at most k facets [all satisfying property $*$]. Clearly $f(2,*) \leq 4$. $f(3, \_) = \infty$ was shown by Tietze (1905) and Besicovitch (1947). $8 \leq f(3,4) \leq 17$ is due to Bagemihl, improved by Baston (1964) to $8 \leq f(3,4) \leq 9$. We showed $f(d,k) \leq \frac{3}{2} k!$ and $f(d,d+1) \leq \frac{2}{3}(d+1)!$ (1979), and $f(d,d+1) \geq 2^d$ (1981). M. Perles improved the upper bound down to $f(d,d+1) \leq 2^{d+1}$.

Our recent results are as follows: (1) $f(d, \text{parallelotopes}) = 2^d$, $\forall d \geq 3$. (2) a simple proof that $f(3,4) \leq 12$. (3) $f(d, \text{symmetric})$ is unbounded $\forall d \geq 3$ [this answers a problem of B. Grünbaum dated back to 1966]. (4) (with G. Kalai & N. Linial) $f(d, \text{translates of } d\text{-cube}) = d+1$.

In my talk I proved that $f(3, \text{cubes}) = 6$ and $f(4, \text{cubes}) \leq 15$. The first result was mentioned by Baston (1964) without a proof; the second improves the upper bound $f(d, \text{cubes}) \leq 2^d$, which follows from M. Perles' result, for $d = 4$.

יוסף זקס Joseph Zaks (Haifa, ISRAEL)

Characterizations of zonoids

Zonoids are convex bodies which can be approximated by finite sums of line segments (zonotopes). For zonotopes, simple geometric and analytic characterizations exist. After a survey on characterization results for zonoids with negative answers to questions of Blaschke, Choquet, and Assouad, a characterization principle is derived by a simple application of Hahn-Banach's theorem to the cone of zonoids and the "dual" cone. As consequences, the following results are obtained: (i) A centrally symmetric (c.s.) body Z is a zonoid, iff $V(Z, \underbrace{L, \dots, L}_{k}, B, \dots, B) \leq c_d \cdot B(Z) \max_{u \in \Omega} v_k(L; u)$ for all c.s. bodies L. Here V is mixed volume, B the unit ball, c_d a dimensional constant, $B(Z)$ mean width, $v_k(L; u)$ quermassintegral of the projection of L orthogonal to u, and $k \in \{1, \dots, d-1\}$ is fixed. (ii) A c.s. body Z is a zonoid, iff $\sum_{i=1}^{k} H_Z(x_i) \leq b_d \cdot B(Z) \cdot \max_{u \in \Omega} \sum_{i=1}^{k} |\langle x_i, u \rangle|$ for all $k \in \mathbb{N}$, $x_1, \dots, x_k \in \mathbb{R}^d$. Here again b_d is a dimensional constant, H_Z is the support function of Z and $\langle \cdot, \cdot \rangle$ the scalar product. Finally, a geometric interpretation of "minimal" characterization by pairs of zonoids (zonotopes) is given which implies several open problems.

W. Weil (Karlsruhe)

Slices of L. Fejes Toth's sausage conjecture.

Let k non-overlapping translates of the unit-ball $B^d \subset E^d$ be given, let C_k be the convex hull of their centers, let S_k be a segment of length $2(k-1)$ and let V denote the volume.

L. Fejes Toth's sausage conjecture (1975) says that for $d \geq 5$

$$V(S_k + B^d) \leq V(C_k + B^d) \qquad (1)$$

In a common paper Betke, Gritzmann and Wills proved:

Th. 1 (1) holds for all C_k with $\dim C_k \leq \frac{7}{12}(d-1)$

Th. 2 (1) holds for all C_k with $\dim C_k \leq d-1$ and $d \leq 3$

In Th. 1 and 2 equality in (1) holds iff $C_k = S_k$.

In Th. 2 $\dim C_k \leq d-1$ cannot be replaced by $\dim C_k \leq d$. In an additional

paper Betke + Gritzmann could replace dim $C_k \le 3$ by dim $C_k \le 9$ in Th. 2.
Besides V other quermassintegrals (but not all) have sausage properties.
So Th. 1 and 2 are also partial results for the "sausage-skin-conjecture" for the surface area F:
Does for $d \ge 7$ and dim $C_k \le d-1$ always hold

$$F(S_k + B^d) \le F(C_k + B^d) \quad ?$$

J. M. Wills (Siegen)

Tilings in $\mathbb{E}^3$

A tiling $\mathcal{T} = \{T_1, T_2, \ldots\}$ of $\mathbb{E}^3$ by convex polyhedral tiles, each of the same combinatorial type as a given polyhedron P, is said to be monotypic of type P. The talk was concerned with locally-finite tilings of $\mathbb{E}^3$ which are monotypic of type P.

A tiling is face-to-face if the intersection of any two tiles is either empty or is a vertex, edge or face of each. A tiling is normal if there exist parameters u, U such that $u \le i(P) \le c(P) \le U$ for all tiles P. (Here $i(P)$ and $c(P)$ are the inradius and circumradius of P, respectively.)

In 1975, L. Danzer asked whether for every 3-dimensional polyhedron P there exists a monotypic tiling of $\mathbb{E}^3$ by polyhedra of type P. This question is still unanswered but proofs of the following two partial results were sketched.

1) There exists a polyhedron P with 45 edges such that there exists no face-to-face normal monotypic tiling of type P.

2) For every simplicial polyhedron P (ie polyhedron with triangular faces) there exists a face-to-face

monotypic tiling of $\mathbb{E}^3$ of type P. (In general, such tilings will be non-normal.)

G. C. Shephard
(Norwich)

COVERING PROBLEMS

Łoś raised the following question: Does there exist a (continuous) Peano curve $f: [0,1] \to [0,1] \times [0,1]$ such that $f([\alpha, \beta])$ is convex for every $\alpha, \beta \in [0,1]$?

If such a function existed, then for every $n \in \mathbb{N}$ one could find a sequence $C_1^n, C_2^n, \dots$ of convex sets in $\mathbb{R}^2$ so that

(i) $\operatorname{diam} C_i^n \le 1 \qquad (\forall i)$

(ii) $\bigcup_{r=i}^{j} C_r^n$ is convex $(\forall i, j)$

(iii) $\bigcup_{r=1}^{\infty} C_r^n$ contains a ball of radius n

Surprisingly enough with C. A. Rogers we managed to construct sequences satisfying these conditions. Nevertheless, we are unable to answer the original question to the affirmative. We can prove only that the following is true.

<u>Theorem</u>. There exists a Peano curve $f: [0,1] \to [0,1]^n$ whose all initial and final segments are convex sets.

Finally, solving a problem of Groemer we give a necessary and sufficient condition for a family of convex sets to permit a covering of $\mathbb{R}^n$. This a part of a joint work with E. Makai.

János Pach
(Budapest, London)

Affine Quermassintegrals

For a convex body K in R^n we define $n+1$ affine quermassintegrals $\Phi_0(A), \Phi_1(A), \ldots, \Phi_n(A)$ by letting $\Phi_0(A) = V(A)$, the volume of A, $\Phi_n(A) = \omega_n$, the volume of the unit n-ball, and for $0<i<n$

$$\frac{\omega_i}{\omega_n} \Phi_{n-i}(A) = \left[\frac{\omega_{n-i}}{\omega_n c_{in}} \int V_i(K|E_i)^{-n} d\bar{\bar{E}}_i \right]^{-1/n}.$$

Here V_i denotes i-dimensional volume, E_i is a freely rotating i-dimensional flat through the origin while $K|E_i$ is the projection of K onto E_i and $d\bar{\bar{E}}_i$ is the rotation density normalized so that

$$\frac{\omega_{n-i}}{\omega_n c_{in}} \int d\bar{\bar{E}}_i = 1.$$

Jensen's inequality leads to the following inequality between the affine quermassintegrals Φ_i, the harmonic quermassintegrals $\tilde{W}_i$ of Hadwiger, and the quermassintegrals W_i:

$$\Phi_i(K) \leq \tilde{W}_i(K) \leq W_i(K),$$

with equality iff the projections of K onto $(n-i)$-dimensional flats have constant $(n-i)$-dimensional volume. Inequalities of Santaló and Petty can be rewritten as the following strengthened forms of the classical isoperimetric inequalities

$$\omega_n^{n-1} V(K) \leq \Phi_{n-1}(K)^n \quad \text{and} \quad \omega_n V(K)^{n-1} \leq \Phi_1(K)^n.$$

For suf. smooth K, there is equality iff K is an ellipsoid

Erwin Lutwak (Brooklyn)

Remark on approximation

A convex body K of constant width in E^d can be approximated, in the Hausdorff metric, by convex bodies of constant width with algebraic support functions and having the same symmetries as K. The classical approximation methods for convex bodies do not yield this, and so far only weaker results for $d=2$ appear in the literature (S. Tanno 1976, B. Wegner 1977). A proof for the general case is sketched which uses first an appropriate kind of convolution, applied to the support function, and then expansion of the latter into a series of spherical harmonics.

Rolf Schneider

Schnittpunktzahl konvexer Kurven

Der Gegenstand dieses Vortrags ist die Berechnung der zu erwartenden Anzahl der in einer jordan-meßbaren Teilmenge der Ebene liegenden Schnittpunkte der Elementen eines translations-invarianten Poisson-Prozesses einfach geschlossener konvexer Kurven mit gleichmäßig beschränkter Länge. In diesem Zusammenhang werden einige ergodische Eigenschaften solcher Poisson-Prozesse angesprochen. Es zeigt sich, daß der genannte Erwartungswert bei vorgegebener mittlerer Länge der Kurven unter anderem dann maximal wird, wenn der Prozeß auch bewegungsinvariant ist. Die Grundidee des Beweises besteht darin, dieses Extremalproblem für Poisson-Prozesse auf das isoperimetrische Problem zurückzuführen.

J.A. Wieacker

Collapsing nerves

In this talk we discuss the well-known (and rather hopeless) problem of determining the intersection pattern realized by finite families of convex sets in R^d. These patterns are, by definition, the nerve complexes of the families (up to isomorphism). The finite abstract simplicial complexes arising in this way are called d-representable.

We add a new condition to the list of known necessary conditions for a complex to be d-representable, that of "strong d-collapsibility" (to be explained in the talk). This is the strongest necessary condition known so far. It may be used to decide the question of d-representability in certain open cases (as will be demonstrated by examples). It has also been used in our recent proof of the Upper-Bound Theorem for f-vectors of d-representable complexes (a different proof of which was found by G. Kalai in Jerusalem).

J. Eckhoff

Lattice polytopes and the inclusion-exclusion principle

Let $\mathcal{S}^d$ be a family of sets in $\mathbb{E}^d$. A valuation $\varphi : \mathcal{S}^d \to \mathbb{R}$ is a function for which
$\varphi(P_1) + \varphi(P_2) = \varphi(P_1 \cup P_2) + \varphi(P_1 \cap P_2)$ whenever
$P_1, P_2, P_1 \cap P_2, P_1 \cup P_2 \in \mathcal{S}^d$.
The inclusion-exclusion-principle (IEP) holds, if for $P, P_1, \ldots, P_n \in \mathcal{S}^d$, $\bigcap P_{i_j} \in \mathcal{S}^d$, $1 \le i_1 < \ldots < i_k \le n$, $\bigcup P_i = P$

$$\varphi(P) = \sum_{i=1}^{n} (-1)^{i+1} \sum_{1 \le j_1 < \ldots < j_i \le n} \varphi(\bigcap P_{i_j}).$$

The validity of the IEP is a consequence of the fact that $\mathcal{S}^d$ is sufficiently rich. While the validity of the IEP for $\mathcal{P}^d$ (the family of all polytopes) was shown by Sallee (1968), we here prove it for the smaller family of lattice polytopes $\mathcal{P}^d_{\mathbb{Z}}$

U. Betke (Siegen)

Stochastische Approximation konvexer Polygone

Es sei K ein konvexes Polygon mit dem Flächeninhalt F und m Seiten der Länge c_i, welche Winkel δ_i $(0 < \delta_i < \pi;\ i = 1, \dots, m)$ einschließen. Bezeichnet H_n die konvexe Hülle von n in K unabhängig und gleichverteilt gewählten Punkten, so gilt für die mathematische Erwartung des Umfanges L_n von H_n für beliebig kleines $\varepsilon > 0$

$$E(L_n) = \sum_{i=1}^{m} c_i - \frac{1}{4}\sqrt{\frac{2\pi F}{n}} \sum_{i=1}^{m} \left(\frac{2}{\sqrt{\sin \delta_i}} - I(\delta_i) \right) + o\left(\frac{1}{n^{1-\varepsilon}}\right),$$

wobei

$$I(\delta) = \int_0^{1/\sin\delta} \frac{\sqrt{1 + (-u + \cot\delta)^2} - \sqrt{1 + \cot^2\delta}}{u^{3/2}}\, du .$$

Der Spezialfall der zufälligen Approximation eines Quadrates wurde von A. Rényi und R. Sulanke behandelt ("Über die konvexe Hülle von n zufällig gewählten Punkten II", Z. Wahrsch. verw. Geb. 3 (1964), 138 - 147), der allgemeine Fall konnte von den beiden Autoren nicht gelöst werden.

Christian Buchta (Wien)