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Cohomologie der Gruppen

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Calculating cohomology from local subgroups

Theorem If $G = PSL(3,q)$, q odd, then

$$\sum_{n=0}^{\infty} t^n \dim H^n(G, \mathbb{F}_2) = \frac{1+t^5}{(1-t^3)(1-t^4)}$$

If M is any finitely generated $\mathbb{Z}G$-module then

$$H^n(G,M)_2 = H^n(C,M)_2 + H^n(N,M)_2 - H^n(P,M)_2$$

where $C = C_G(t)$, $t^2 = 1 \neq t$

$N = N_G(C_2 \times C_2)$, $P = C_{q-1} \times C_{q-1} \rtimes C_2$

The equation for the 2-parts of the cohomology groups in the above theorem is to be understood in the Grothendieck group of finite abelian groups with respect to direct sums. The method of proof is an example of a general technique for reducing the cohomology of a finite group to that of certain subgroups of local subgroups. This technique makes use of the same double coset information as the classical method of stable elements, but the double cosets do not appear explicitly.

Peter Webb

Homological properties of locally indicable groups

J. Howie and H. R. Schneebeli

Notation $\underline{LI}(R)$ (where $R = \mathbb{Z}$ or $\mathbb{F}_p$) is the class of locally R-indicable gps ($\forall$ f.g. subgp $H \neq 1$ $\exists$ $H \twoheadrightarrow R$).

$\underline{C}(A)$ (A an abelian gp) is the class of groups conservative over A (for any regular covering $\tilde{X} \to X$ of 2-complexes with covering transformation group G, $H_2(X, A) = 0 \Rightarrow H_2(\tilde{X}, A) = 0$)

$\underline{D}(R)$ (R comm. ring with 1) is the class of groups satisfying the following property: if $\varphi : M \to N$ is a map between projective RG-modules such that $1 \otimes \varphi : R \otimes_{RG} M \to R \otimes_{RG} N$ is injective, then so is φ.

These were studied for various reasons by Higman, Adams, Strebel respectively.

Theorem ($R = \mathbb{Z}$ or $\mathbb{F}_p$) $\underline{LI}(R) = \underline{C}(R) = \underline{D}(R)$

Theorem
$$\underline{C}(A) = \begin{cases} \text{all gps}, & \text{if } A = 0 \\ \bigcap_{p \in \Pi} \underline{LI}(\mathbb{F}_p) & \text{if } A\ \Pi\text{-torsion gp } (\Pi \text{ finite set of primes}) \\ \underline{LI}(\mathbb{Z}) & \text{otherwise} \end{cases}$$

Theorem
$$\underline{D}(R) = \begin{cases} \bigcap_{p \mid n} \underline{LI}(\mathbb{F}_p) & \text{if char } R = n > 0 \\ \underline{LI} & \text{if char } R = 0 \end{cases}$$

The classes $\bigcap_{p \in \Pi} \underline{LI}(\mathbb{F}_p)$ are distinct (for various finite sets Π of primes) (but $\bigcap_{p \in \Pi} \underline{LI}(\mathbb{F}_p) = \underline{LI}$ whenever Π infinite)

Applications I) New light is shed on Adams' work on Whitehead's problem about aspherical 2-complexes.

II) Suppose $P \neq 0$ is a projective $\mathbb{Z}G$-module, such that $\mathbb{Z} \otimes_{\mathbb{Z}G} P = 0$ then there exists a subgp $H \neq 1$ in G minimal w.r.t. the property that $\mathbb{Z} \otimes_{\mathbb{Z}H} P = 0$, and no proper hom. image of H is $\underline{LI}(\mathbb{F}_p)$ for any p (in particular H perfect.)

Note added in proof (Due to P.A. Linnell) Conversely, if G has a f.g. perfect subgp then $\exists$ such a module.

Ends of groups and graphs

M.J. Dunwoody

Stallings proved that if G is a finitely generated group and $H^1(G, \mathbb{Z}G) \neq 0$, then G splits over a finite subgroup (i.e $G = A *_C B$, C finite $C \neq A, C \neq B$ or $G = A*_C$)

Two generalizations of this theorem are presented.

Suppose the group G acts on the graph X. Let $C^0(X) = \mathrm{Hom}(VX, \mathbb{Z})$ and let $Q(X) = \{\alpha \in C^0(X) \mid \delta\alpha \text{ has finite support}\}$.

THEOREM 1. If Δ is a finite subset of $Q(X)$ then there exists a G-tree T and a G-map $\omega : VX \to VT$ such that

$$\Delta \subset \omega^* Q(T) \subset Q(X).$$

Let $Q_f(X) \subset Q(X)$ be the subset consist of those α with finite support. Let $C(X)$ be the set of constant maps $VX \to \mathbb{Z}$.

<u>Corollary</u>. If $Q(X) \neq Q_f(X) + C(X)$ and $G \backslash X$ is finite, then G splits over a subgroup K which contains the stabilizer of an edge of X as a subgroup of finite index.

Let T be a <u>right</u> G-tree and if $u, v \in VT$ let $T(u,v) = e_1 + e_2 + \dots + e_n \in C_1(T)$ where $e_1, e_2, \dots, e_n$ are the directed edges of a geodesic going from u to v. Let $v_0 \in VT$ be a fixed vertex. Let $d_T : G \to C_1(T)$, $d_T(g) = T(v_0, v_0 g)$. Then d_T is a derivation; d_T is inner if and only if $G = G_v$ for some $v \in VT$. If $d : G \to \mathbb{Z}G$ is a derivation, then (T, v_0) is said to admit d if d factors through d_T.

THEOREM 2. Let $d_1, d_2, \dots, d_k$ be a finite set of derivations $d_i : G \to \mathbb{Z}G$. Here G is an arbitrary group. There exists a G-tree T with $v_0 \in VT$ such that (T, v_0) admits each of $d_1, \dots, d_k$.

<u>Corollary</u>. ~~If~~ If G is any group, $H^1(G, \mathbb{Z}G) \neq 0$ if and only if either G is countably infinite and locally finite or G splits over a finite subgroup.

The Cohomology Ring of a Module

by Jon F. Carlson

Let G be a finite p-group and let K be an algebraically closed field of characteristic $p>0$. Let $E = \langle x_1, \dots, x_n \rangle$ be an elementary abelian group of order p^n. For any $\alpha = (\alpha_1, \dots, \alpha_n) \in K^n$ let $u_\alpha = 1 + \sum \alpha_i (x_i - 1)$. Then u_α is a unit of order p in KG. Let M be a finitely generated KE-module. The variety of M is defined to be $V_E(M) = \{0\} \cup \{\alpha \in K^n \,/\, \alpha \neq 0$ and $M_{\langle u_\alpha \rangle}$ is not a free module$\}$. Then $V_E(M)$ is an affine variety in K^n.

<u>Theorem</u>. An element $\zeta \in \mathcal{E}xt^t_{KE}(M,M)$ is nilpotent if and only if its restriction to every $\mathcal{E}xt^+_{K\langle u_\alpha \rangle}(M,M)$ is a nilpotent element.

It was known previously that an element $\zeta \in \mathcal{E}xt^t_{KG}(M,M)$ is nilpotent if and only if its restriction to every elementary abelian p-subgroup is nilpotent. By combining these two results we can determine the radical of $\mathcal{E}xt^*_{KG}(M,M)$ in terms of the inverse images of radicals under the restriction maps. The radical of $\mathcal{E}xt^*_{KG}(M,M)$ is a nilpotent ideal. This can also be used to give an independent proof of the Avrunin-Scott result that the given variety coincides with a certain cohomological variety.

Groups and Poincaré Duality

Beno Eckmann

(Survey lecture)

1. Poincaré duality for closed manifolds M of dim. n:

$$H^i(M;A) \cong H_{n-i}(M;\tilde{\mathbb{Z}}\otimes A)$$

for all $\pi_1(M)$-modules A and all $i\in\mathbb{Z}$, $\tilde{\mathbb{Z}}$ being the orientation module (i.e., trivial for orientable, non-trivial for non-orientable manifolds). By analogy one defines:

A group G is a Poincaré duality group of dim. n (in short a PD^n-group) if there is a G-module structure on $\mathbb{Z}$ such that

(1) $$H^i(G;A) \cong H_{n-i}(G;\tilde{\mathbb{Z}}\otimes A)$$

for all $i\in\mathbb{Z}$ and all G-modules A. If $G=\pi_1(M)$ as above with $\tilde{M}$ contractible then G is a PD^n-gp; the converse is not known (for $n=2$ see below).

Consequences: ~~It~~ is of type (FP) with $cd\,G=n$, and $H^i(G;\mathbb{Z}G)=0$ for all i except for $i=n$ where $H^n(G;\mathbb{Z}G)=\tilde{\mathbb{Z}}$.

2. More general type of duality:

(2) $$H^i(G;A) \cong H_{n-i}(G;\underline{D}\otimes A)$$

with a dualizing (non-negative) G-complex $\underline{D}$. This again implies type (FP), and – if $\underline{D}$ exists – the homology of $\underline{D}$ is determined by G, namely $H^i(G;\mathbb{Z}G)=H_{n-i}(\underline{D})$. Conversely, if G is of type (FP), then $\underline{D}=\underline{P}^* = \mathrm{Hom}_G(\underline{P},\mathbb{Z}G)$ is a dualizing module for any FP-resolution $\underline{P}\to\mathbb{Z}$ over $\mathbb{Z}G$.

Special case: $H_i(\underline{D})=0$ except for $i=0$, where $H_0(\underline{D})=C$ is $\mathbb{Z}$-torsion free; i.e., $H^i(G;\mathbb{Z}G)=\begin{cases}0 & i\neq n\\ C \text{ torsion free} & i=n\end{cases}$. Then (2) becomes

(3) $$H^i(G;A) \cong H_{n-i}(G;C\otimes A)$$

Such groups are called duality groups of dim. n (D^n-gps). So G is a D^n-gp iff G is of type (FP) and $H^i(G;\mathbb{Z}G)=0$ for all i except for $i=n$, where $H^n(G;\mathbb{Z}G)=C$ is torsion free. The case $C=\mathbb{Z}$

yields PD^n-groups.

3. This criterion for PD^n or D^n in terms of the "endpoint groups" $H^i(G;\mathbb{Z}G)$ & a finiteness condition can be applied

a) to geometrically given groups G (operating freely and properly in $\mathbb{R}^n$ (e.g., torsionfree discrete subgroups of real Lie groups.

b) to groups admitting a finite CW-complex $X = K(G,1)$ – which can be chosen to be a compact ∂-manifold. Then the $H^i(G;\mathbb{Z}G)$ are expressed as $H_j(\partial\tilde{X};\mathbb{Z})$, $\tilde{X}$ = universal cover of X.

c) to extension theorems, e.g.: A torsion-free group G_1 containing a PD^n-gp G (a D^n-gp G) as subgroup of finite index is itself a PD^n-gp (a D^n-gp).

etc.

4. Are PD^n-groups fundamental groups of closed manifolds with contractible universal cover?

Almost complete answer for $n=2$:

Theorem (B.E – H. Müller). A PD^2-group G with $\beta_1(G) > 0$ is $\pi_1(\Sigma_g)$ where Σ_g is a closed surface of positive genus g.

The proof relies on splitting theorems for groups and subgroups (H. Müller) and on the theory of PD^2-pairs (R. Bieri – BE).

If G is of type (FF) then the condition $\beta_1(G) > 0$ is fulfilled.

Application: A finite torsionfree extension G_1 of $G = \pi_1(\Sigma_g)$ is again a "surface group" $\pi_1(\Sigma_{g_1})$. From this one obtains special positive answers of the Nielsen realization problem.

5. More generally: A virtual PD^2-gp with $\beta_1(G) > 0$ is a motion group of the Euclidean or hyperbolic plane with compact fundamental domain (BE – H. Müller, to appear in Invent. Math.)

Accessibility of Groups

P.A. Linnell.

Let G be a finitely generated group and suppose $H^1(G, \mathbb{Z}G) \neq 0$. Then a celebrated theorem of Stallings states that G splits over a finite subgroup; that is we may write $G = A *_F B$ $(A \neq F \neq B)$ or $G = A *_F$ (HNN extension) with F finite. If $H^1(A, \mathbb{Z}A) \neq 0$, then A itself splits over a finite subgroup. I was concerned with the ~~process~~ conjecture of accessibility; that is whether this process of splitting must always come to a stop after a finite number of steps, and a proof of the following theorem was sketched.

<u>Theorem</u> Let G be a finitely generated group. If the subgroups of G have bounded order, then G is accessible.

The proof of this theorem is derived from an inequality for the number of generators of the rational augmentation ideal of G when G is expressed as the fundamental group of a finite graph of groups, which in turn depends on some analysis.

Complexity of modules

J. L. Alperin

A survey of the results and applications of the notion of complexity of a module is given. Let k be a field of characteristic p, G a finite group. If

$$\cdots \to P_1 \to M \to 0$$

is the minimal projective resolution of M then

$$\dim P_n = a_r(n)\, n^r + \cdots + a_0(n) + \varphi(n)$$

where the a_i are periodic, $a_r \neq 0$, φ has finite support and

$$c_G(M) = r + 1$$

(with $c_G(M) = 0$ if the "polynomial" is zero) is the complexity of M.

Quillen Stratification for Modules
George S. Avrunin

Let G be a finite group and k a fixed algebraically closed field of characteristic $p>0$. If p is odd, let H_G be the subring of $H^*(G,k)$ consisting of elements of even degree; take $H_G = H^*(G,k)$ if $p=2$. H_G is a finitely generated commutative k-algebra, and we let V_G denote the associated affine variety $\operatorname{Max} H_G$. If M is any finitely generated kG-module, the <u>cohomology variety</u> $V_G(M)$ may be defined as the largest support of $H^*(G, L\otimes M)$ in V_G, where L is any finitely generated kG-module. A module L with each irreducible kG-module as a direct summand will serve.

D. Quillen proved a number of beautiful results relating V_G to the varieties V_E associated with the various elementary abelian p-subgroups E of G, culminating in his stratification theorem. This theorem gives a piecewise description of V_G in terms of the subgroups E and their normalizers in G. Leonard Scott and I have proved an analogous stratification theorem for the cohomology variety $V_G(M)$. A key step in our argument is the proof of a conjecture of Jon Carlson's regarding the varieties $V_E(M)$ for E elementary. We are also able to generalize several of Quillen's other results to the module case.

The Cohomology of Soluble Groups
Dion Gildenhuys

I reported on my joint work with Ralph Strebel on the cohomological dimension $cd_R G$ of a soluble group G over a commutative ring R with $1\neq 0$. It is well known that if G is torsion-free and $hG<\infty$, then
$hG = hd_{\mathbb{Z}} G = hd_{\mathbb{Q}} G \leq cd_{\mathbb{Q}} G \leq cd_{\mathbb{Z}} G \leq hG+1$.

We have proved that if G is countable and its

Hirsch number hG equals $cd_{\mathbb{Q}} G$, then $h\bar{G} = cd_{\mathbb{Q}} \bar{G}$ for every homomorphic image $\bar{G}$ of G. If $hG = cd_{\mathbb{Q}} G < \infty$ and G is nilpotent-by-abelian, then G is finitely generated. We conjecture that if G is torsion-free and $hG = cd_{\mathbb{Q}} G < \infty$, then G must be constructible. We have succeeded in reducing this conjecture to the case where G is a semi-direct product of a torsion-free abelian group A of finite rank $d-1$ and a free abelian group $\langle t_1, \dots, t_n \rangle$ on a finite number of generators $t_1, \dots, t_n$. We derived, for this case, an explicit formula for $H^{n+d}(G, X)$, where $X = \mathbb{Q}(G/\langle t_1, \dots, t_n \rangle)$ and $R = \mathbb{Q}$, by means of which we prove that $cd_{\mathbb{Q}} G > hG$ if G is Abels' finitely presented group, or if G is nilpotent-by-infinite cyclic and not constructible

The spectral sequence of a group extension,
F. Rudolf Beyl

It was indicated why the Cartan-Leray (-Serre), the Hochschild-Serre, and the Grothendieck spectral sequences are isomorphic. The first two spectral sequences are isomorphic as spectral rings, not only as bigraded modules. For the details see: Bull. Sci. Math. Sér. (2) 105 (1981), 417-434.

The cohomology of $GL(\mathbb{F}_q)$ and other classical groups
Johannes Huebschmann

Let $q = p^n$ be a prime power, let $\mathbb{F}_q$ denote the field with q elements and denote by $GL(\mathbb{F}_q)$ the infinite general linear group. By results of Quillen there is a map $BGL(\mathbb{F}_q) \to F\Psi^q$ inducing an isomorphism in (co)homology (with suitable coefficients). Here Ψ^q is the q'th Adams operation in complex K-theory and

$\Psi^q - 1: BU \to BU$ denotes a map that represents the operation denoted by the same symbol. The purpose of the talk was to present a detailed calculation of the additive structure of the cohomology of $F\Psi^q$, and hence of $GL(\mathbb{F}_q)$, for arbitrary coefficient rings R, and to obtain information about the multiplicative structure also. The methods involve differential homological algebra, in particular the description of the (co)homology of an induced fibre space by means of a differential TOR- or COTOR-functor due to Eilenberg and Moore.

Using results of Friedlander and Fiedorowicz ~~&~~ Priddy, similar results can be obtained for the other infinite classical groups such as $SL(\mathbb{F}_q)$, $Sp(\mathbb{F}_q)$, $U(\mathbb{F}_q)$, $SU(\mathbb{F}_q)$ and various orthogonal groups.

A geometric invariant for modules over an abelian group (by R. Strebel)

In 1980 Robert Bieri (Frankfort) and I introduced a geometric invariant Σ_A; here A is a finitely generated $\mathbb{Z}Q$-module and Q a finitely generated abelian group of torsion-free rank n. The invariant Σ_A is an open subset of the unit sphere S^{n-1} of $\mathbb{R}^n$.

In the lecture I explained how Robert and I were led in our work on finitely presented soluble groups to introduce this invariant, gave the characterization of finitely presented metabelian groups in terms of the invariant and surveyed its other properties. I also sketched the connection with G. Bergman's solution of a conjecture of Zalesskii's.

Cohomology rings of infinitesimal groups (Leonard Scott)

This is work toward the Ph.D of my student Ronny Crane.

Let U be the group of $n\times n$ uppertriangular unipotent matrices over an algebraically closed field of characteristic $p>0$, and let U_1 be its first infinitesimal subgroup (equivalently one may use its Lie p-algebra, which is just the strictly uppertriangular matrices.) Then for $p=2$ the ring $H^*(U_1,k)$ is shown to be the homology of a polynomial algebra $k[e_\alpha \mid \alpha \text{ a pos. root}]$ with differential $\partial e_\alpha = \sum_{\beta+\gamma=\alpha} e_\beta e_\gamma$. The method involves constructing explicit small resolutions, reminiscent of those of Peter May, but formulated in terms of the normal subgroup structure of U. Preliminary indications are that U acts on this resolution, extending the action of U_1 and that there is an additive factorization $H^*(U,k) = H^*(U/U_1,k)\otimes H^*(U_1,k)$.

Maximal subgroups and nonabelian cohomology (Leonard Scott)

The theorem discussed is part of joint work with Michael aimed at understanding maximal subgroups of finite groups. Let G be any group, finite or infinite, containing a normal subgroup $D=\prod_{i\in I} L_i$ where G permutes the L_i's according to a transitive action of G on the (finite) index set I. A full diagonal subgroup of D is one which projects isomorphically onto each factor L_i. Fix one of the L_i's and call it L.

Theorem. If there are any such subgroups A with the additional property $G=N_G(A)D$ then the set of G-conjugacy classes of all such with this property is bijective with $\mathrm{Hom}_c(G,\mathrm{Out}\,L)$, the set of homomorphisms from G to the outer automorphism group $\mathrm{Out}\,L$ of L which restrict to the conjugation map $c: N_G(L)\to \mathrm{Out}\,L$ on $N_G(L)$.

The proof is obtained from some new results in nonabelian cohomology. For example, we prove an analogue of the Eckmann–Shapiro lemma which gives the cohomology of an induced module in terms of the subgroup and module from which it was induced. This is eventually applied to the action of G on $\prod_{i\in I}\mathrm{Out}\,L_i$ in the proof of the Theorem.

On the cokernel of res: $H^1(G;\mathbb{Z}G) \to H^1(\underline{S};\mathbb{Z}G)$

Let G be a finitely generated infinite group, $\underline{S} = \{S_1, \dots, S_m\}$ a finite family of finitely generated infinite subgroups of G, and let C denote the cokernel of the restriction map res: $H^1(G;\mathbb{Z}G) \to H^1(\underline{S};\mathbb{Z}G)$.

Theorem. If G is accessible then C is a free-Abelian group of rank $\operatorname{rk} C = 0$, ~~≤~~ 1, or ∞ — except in the special case when G is infinite-cyclic-by finite in which case $\operatorname{rk} C$ can attain any positive integer.

(The freeness of C is due to Heinz Müller)

One can obtain a complete classification of the three cases $\operatorname{rk} C = 0, 1, \infty$ in terms of the structure of the pair $(G, \underline{S})$

Robert Bieri (Frankfurt)

K-theory of classifying spaces of arithmetic groups.

Let Γ be an arithmetic group of finite representation type (c.f. H. Bass), for example $SL(n,\mathbb{Z})$ or $Sp(2n,\mathbb{Z})$. We consider the completion with respect to the I-adic topology of the flat bundle homomorphism $\alpha : R_f(\Gamma) \longrightarrow K_{compact}(B\Gamma)$, where $R_f(\Gamma)$ is the ring of finitedimensional complex representations and $K_{compact}$ is defined to be $\varprojlim_{r} K(B\Gamma^r)$. Then $\alpha^\wedge$ is a monomorphism with finite cokernel (Theorem 1). This result implies that "up to finite extension" $K_{compact}(BSL(n,\mathbb{Z}))$, $n \geq 3$, is generated by the class of the flat bundle associated to the natural representation of $SL(n,\mathbb{Z})$ in $SL(n,\mathbb{C})$ together with classes inflated up from $K(BSL(n,\mathbb{Z}/p^t))$.
Another approach to the study of $K^*(B\Gamma)$ is provided by the Tate-Farrell ring $\hat{K}^*(B\Gamma) = \varinjlim_{\mathfrak{C}(\Gamma)} K^*(B\Delta)$, where the objects of the (finite) category $\mathfrak{C}(\Gamma)$

an equivalence classes of finite subgroups of Γ, and the morphisms are inclusions twisted by conjugation (c.f. D. Quillen). As one might expect there is a relation between the skeletal filtrations on $\hat{K}^*(B\Gamma)$ and $K^*_{\text{compact}}(B\Gamma)$, analogous to the relation between the gradings in ordinary cohomology. At present one can prove that for $k > k_0$ and with $\mathbb{F}_p$-coefficients $F_k/F_{k+2} = \hat{F}_k/\hat{F}_{k+2}$ (Theorem 2).

As an example one has that $\hat{K}^*(BSL(n, \mathbb{Z}[q_i^{-1}]), \mathbb{Z}[p_j^{-1}]) \cong K^*(BA_n, \mathbb{Z}[p_j^{-1}])$, where $p_j = 2, 3, \ldots$, largest prime less than $\frac{n}{2}+1$, and q_i = some (hard to determine) family of primes. (The role of the primes q_j is to provide unique conjugacy classes of maximal p_i-subgroups in $SL(n, \mathbb{Z}[\cdot])$.)

C. B. Thomas
(Cambridge & ETH, Zürich)

Cohomology of algebraic and related finite groups
(by Brian Parshall)

Let G be a semisimple, simply connected algebraic group defined and split over $k_0 = GF(p)$. If V is a rational G-module, let $V^{(r)}$ be the rational G-module obtained from V by making G act first through σ^r, where $\sigma : G \to G$ is the Frobenius morphism. Let h be the Coxeter number of G and assume $\operatorname{char} k \geq 2h$. Also, for a dominant weight λ, let $S(\lambda)$ denote the irreducible rational G-module of high weight λ.

After a survey of the basic cohomology theory of G, we discussed the following

<u>Theorem</u> (Friedlander-Parshall) Let the dominant weight λ lie in the bottom p-alcove. For $0 \leq n \leq \frac{1}{4}(p-2h+2)$ and $r \geq 1$,

$$\dim H^n(G, S(\lambda)^{(r)}) = \begin{cases} 0 & n \text{ odd} \\ \sum_{w \in W} \det w \, \underline{P}_m(w(\lambda+\rho)-\rho) & n = 2m \text{ even} \end{cases}$$

In the above ρ denotes the sum of the fundamental dominant weights, and, for a weight ψ, $\mathcal{P}_m(\psi)$ is the number of ways ψ can be written as a sum of m positive roots. In the indicated range the above theorem calculates the "generic" cohomology of G and hence calculates certain finite group cohomologies. The proof of the above theorem involves infinitesimal methods. We indicated the connections with the coordinate ring of the "null cone" of G.

Linear Groups of Finite Cohomological Dimension

Serre has shown that a finitely generated subgroup of $GL_n(\mathbb{Q})$ has finite virtual cohomological dimension. Using a "finiteness theorem" for finitely generated rings: A a finitely generated ring of characteristic zero, F its quotient field, then there are discrete valuations $v_1, \dots, v_m$ of F so that

$$A \cap \bigcap_{i=1}^{m} \mathcal{O}_{v_i} \subset \mathcal{O}$$

where $\mathcal{O}_{v_i}$ are the respective valuation rings and $\mathcal{O}$ is the ring of integers in the algebraic closure of $\mathbb{Q}$ in F; we obtain then by actions on suitable Tits' buildings the following result

Theorem: $\Gamma \subset GL_n(A)$ has finite virtual cohomological dimension iff there is a finite upper bound on the vcd of unipotent subgroups of Γ.

By dint of Merzljakov's Theorem: If a torsion free polycyclic group has its abelian subgroups (f.g.) of bounded rank r then the hirsch rank of the group is bounded by a suitable function of r we can restate the theorem.

Theorem: $\Gamma \subset GL_n(A)$ has finite virtual cohomological dimension iff there is a finite upper bound on the ranks of its finitely generated abelian unipotent subgroups.

As a sample application: a finitely generated group of unitary matrices has finite virtual cohomological dimension

Roger Alperin (Norman)

<u>Relative Cohomology of Group Extensions</u>

by Nathan Hobegger (Geneva)

Let $1 \to N \to G \to Q \to 1$ be an extension of groups, A a G-module, A^N the set fixed by N. If $C^*(G;A)$ denotes the normalized inhomogeneous cochain complex, then there is an inclusion $C^*(Q,A^N) \subset C^*(G;A)$ and we study the quotient complex $\dfrac{C^*(G,A)}{C^*(Q,A^N)}$ whose homology, denoted by $H^*(Q,G;A)$, sits in a long exact sequence with $H^*(Q,A^N)$ and $H^*(G;A)$.

<u>Theorem</u> There is a spectral sequence

$$E_2^{p,q} = \begin{cases} H^p(Q, H^q(N,A)) & q>0 \\ 0 & q=0 \end{cases} \Longrightarrow H^{p+q}(Q,G;A)$$

One gets $H^0(Q,G;A)=0$, $H^1(Q;G;A)=H^1(N,A)^Q$ and
$0\to H^1(Q,H^1(N,A))\to H^2(Q,G;A)\to H^2(N,A)^Q\to H^2(Q,H^1(N,A))\to H^3(Q,G;A)$
is exact.

Let $C^q(N,A)$ be a G-module via $(g\alpha)(n_1 \dots n_q) = g\alpha(g^{-1}n_1 g, \dots, g^{-1}n_s g)$. Hochschild and Serre, Cohomology of group extensions, TAMS, 1953, define a map of complexes:

$$l: C^*(G,A) \longrightarrow \bigoplus_{p+q=n} C^p(G, C^q(N,A))$$ using a shuffling operation.

Since $l(C^n(Q,A^N)) \subset C(G,A^N)(\subset C^n(G,C^0(N,A)))$, l passes to quotients giving $\bar{l}: \dfrac{C^n(G,A)}{C^n(Q,A^N)} \to \dfrac{\bigoplus_{p+q=n} C^p(G,C^q(N,A))}{C^n(G,A^N)}$.

Let $L^*(G,N;A) = $ image $\bar{l}$. This subcomplex may also be defined by $\binom{n}{2}$ equations involving "partial" shuffles.

Theorem The map $\dfrac{C^*(G,A)}{C^*(Q,A^N)} \to L^*(G,N;A)$ induces an isomorphism on homology.

The above theorem was motivated by work of John Ratcliffe and Johannes Huebschmann who studied the notion of crossed extension. A crossed extension naturally gives rise to a 2 cocycle of the complex L^* and one has
$$XExt_G(N,A) \cong H^2(L^*(G,N;A)) \cong H^2(Q,G;A).$$

Endlichkeitseigenschaften S-arithmetischer Gruppen

Sei k ein globaler Funktionenkörper, $O_S \subset k$ ein S-arithmetischer Ring, G eine reduktive algebraische k-Matrizengruppe und $G(O_S)$ die zugehörige S-arithmetische Gruppe. Thema des Vortrags ist die endliche Präsentierbarkeit von $G(O_S)$ bzw. die $(FP)_n$-Eigenschaften.

Ergebnisse:

$Sl_2(O_S)$ endlich präsentiert $\Leftrightarrow$ $|S| \geq 3$

$n \geq 4 \Rightarrow GL_n(O_S)$ endlich präsentiert $(|S|=1)$

G eine einfache Chevalleygruppe, nicht vom Typ G_2

Rang $G \geq 3 \Rightarrow G(\mathbb{F}_q[t])$ endlich präsentiert

Rang $G \geq 2 \Rightarrow G(\mathbb{F}_q[t,t^{-1}])$ endlich präsentiert

Rang $G = 2 \Rightarrow G(\mathbb{F}_q[t])$ nicht endlich präsentiert

Sei $S = \{w\}$ $(|S|=1)$ und k_w die Komplettierung von k an der Stelle w. Sei G klassisch und $\mathrm{Rang}_k G = \mathrm{Rang}_{k_w} G = 2$ (ferner char $k \neq 2$ in manchen Fällen) $\Rightarrow$

$G(O_S)$ ist nicht endlich präsentiert

In bezug auf die $(FP)_m$-Eigenschaften:

$PGL_2(O_S)$ ist vom Typ $(FP)_{|S|-1}$, aber nicht vom Typ $(FP)_{|S|}$

$SL_3(\mathbb{F}_q[t])$ ist vom Typ $(FP)_1$ ($\Leftrightarrow$ endlich erzeugt) aber nicht vom Typ $(FP)_2$, falls $\mathbb{F}_q$ die Dimension $\nu \geq 2$ über seinem Primkörper hat.

Graeme McHardy

Discrete Cohomology of Topological Groups

We present a series of conjectures concerning the $\mathbb{Z}/n$ cohomology of various large groups. For a Lie groups G, the conjecture is that

$$G^\delta \to G \quad \text{induces} \quad H^*(BG,\mathbb{Z}/n) \xrightarrow{\simeq} H^*(K(G^\delta,1),\mathbb{Z}/n)$$

This is easily seen to be true for G solvable and J. Milnor has shown that the above map is a split inclusion whenever G has finitely many components.

The motivation for this conjecture for Lie groups arose from an investigation of algebraic groups over $\overline{\mathbb{F}}_p$. For a wide class of algebraic groups,

$G(\overline{\mathbb{F}}_p) \to G(\overline{\mathbb{F}}_p)_{et}$ induces $H^*(BG(\overline{\mathbb{F}}_p)_{et}, \mathbb{Z}/n) \xrightarrow{\simeq} H^*(K(G(\overline{\mathbb{F}}_p), 1), \mathbb{Z}/n)$ for $(p,n)=1$ where $G(\overline{\mathbb{F}}_p)_{et}$ is the etale homotopy type of the algebraic group. We conjecture for any algebraic group G (perhaps only affine) and any algebraically closed field k,

$G(k) \to G(k)_{et}$ induces $H^*(BG(k)_{et}, \mathbb{Z}/n) \xrightarrow{\simeq} H^*(K(G(k), 1), \mathbb{Z}/n)$.

In the "special case" $G = Gl_\infty$, this is the well-known Lichtenbaum-Quillen conjecture.

Following a point of view developed jointly with Bill Dwyer, we may generalize this last conjecture further. For a k-algebra A, we define

$G(A)_{et} = Hom.((\operatorname{Spec} A)_{et}, G_{et})$

the function complex associated to etale homotopy type. More generally, for a $\mathbb{Z}[1/n]$ algebra A, we define

$$G(A)_{et} = \text{space of liftings} \left\{ \begin{array}{ccc} & & G_{et} \\ & \nearrow & \downarrow \\ (\operatorname{Spec} A)_{et} & \longrightarrow & (\operatorname{Spec} \mathbb{Z}[1/n])_{et} \end{array} \right\}$$

and conjecture

$G(A) \to G(A)_{et}$ induces $H^*(BG(A)_{et}, \mathbb{Z}/n) \xrightarrow{\simeq} H^*(K(G(A), 1), \mathbb{Z}/n)$

Partial results concerning the injectivity of this map in certain cases of interest have been obtained.

Metabelian groups with finitely generated integral homology

In recent years, work of Baumslag and others has shown that the relationship between finiteness conditions on a group and finiteness conditions on its integral homology is, in the more general cases, tenuous. For metabelian groups, however, there is more hope of proving such relationships. In joint work with R. Bieri the conditions of $(FP)_n$ and $(FP)_\infty$ for metabelian groups were investigated. In particular, in the case of $(FP)_\infty$ it is possible to show that such groups are virtually (FP). In the talk, I discussed a continuation of these results - with weakened conditions.

<u>Theorem</u>. Let $1 \to A \to G \to Q \to 1$ be an exact sequence of groups with G finitely generated and A, Q abelian. Suppose, also, that the extension splits. If $H_n(G, \mathbb{Z})$ is finitely generated for all n then G has finite rank.

The restriction to split extensions is due to the nature of the proof and it seems possible that, with a better proof, it could be omitted.

John Groves (Melbourne)

<u>Isomorphisms and automorphisms of</u>
<u>integral group rings</u>

(A preliminary report on joint work with L.L. Scott)
Let G be a finite p-group and consider the following implications ($\hat{\mathbb{Z}}_p$, $\hat{\mathbb{Q}}_p$ p-adics)

IP: $\hat{\mathbb{Z}}_p G \cong \hat{\mathbb{Z}}_p H \Longrightarrow G \cong H$

ZC: $\alpha \in \mathrm{Aut}_{aug}(\hat{\mathbb{Z}}_p G) \Longrightarrow \alpha \in \mathrm{Autcent}(\hat{\mathbb{Q}}_p G) \cdot \mathrm{Aut}(G)$

SR: $\alpha \in \mathrm{Aut}_{aug}(\hat{\mathbb{Z}}_p G) \Longrightarrow \alpha \in \mathrm{Inn}(\hat{\mathbb{Z}}_p G) \cdot \mathrm{Aut}(G)$

<u>Proposition</u>: Given an short exact sequence

$$1 \to A \to G \to \bar{G} \to 1, \quad A \text{ abelian}.$$

Assume IP, ZC, SR hold for $\bar{G}$, then IP, ZC hold for G. (These hypotheses are satisfied for Dihedral 2-groups, Quaternion groups and $C_{p^n} \rtimes C_{p^m}$)

By induction it is shown, that to SR

it suffices to show that a certain map of nonabelian cohomology groups is trivial.

Conjecture 1 (official): SR hold for nilpotent class 2 groups

Conjecture 2 (unofficial): SR holds for all p-groups. (We have to make this official soon)

K.W. Roggenkamp (Stuttgart)

Bounds for the gap of a finite group

The gap of a finite group G is the difference between $d(G)$, the minimal number of generators of G, and $d_G(\mathfrak{g})$, the minimal number of generators of its integral augmentation ideal. Most of the known results depend on the Lyndon-Hochschild-Serre 5-term sequence. In contrast to this the following results are obtained using cohomology relative to arbitrary subgroups.

Denote by $d(G,H)$ the minimal number of elements, which is needed to generate G together with the subgroup H, and by $\pi(G)$ the set of primes dividing the order of G.

1) Assume that each nonabelian composition factor of G is generated by two of its 2-Sylow subgroups and that there exists an odd prime q such that $d_G(\mathfrak{g}) = d_G(\mathfrak{g}/q\mathfrak{g})$, then for $S \in Syl_2(G)$

$$d(G) - d(G,S) \geq gap(G) \geq d(G) - d(G,S) - 1.$$

2) Assume there exist $p, q \in \pi(G)$ such that each nonabelian composition factor of G is generated by two of its p-Sylow subgroups and generated by two of its q-Sylow-subgroups, then exists $T \in Syl_p(G) \cup Syl_q(G)$

such that
$$d(G)-d(G,T) \geqq gap(G) \geqq d(G)-d(G,T)-1.$$
By a recent result of M. Aschbacher & R. Guralnick there exist many nonabelian simple groups satisfying the composition factor assumptions in 1) or 2).

W. Kimmerle (Stuttgart)

Relation modules of finite groups

(Report on joint work with W. Kimmerle)

If G is a finite group and $\bar{R}_0$ a minimal relation module, if $\mathbb{Z}G$ is a direct summand of $\overset{n}{\underset{1}{\oplus}} \bar{R}_0$ for some n, then $\bar{R}_0$ is called a generator. Let $\mathcal{C}$ denote the class of all groups so that $\bar{R}_0$ is a generator. If $G \in \mathcal{C}$ and either $d(G) \geq 3$ or $\mathbb{Z}G$ satisfies the Eichler condition, then there is only one isomorphism class of minimal relation modules. This talk is concerned with finding groups in $\mathcal{C}$.

Theorem

If G is a finite simple group such that $G = \langle x_1, \dots x_m \mid x_1^2 = 1, \text{other relations}\rangle$ and G nonabelian, then $G \in \mathcal{C}$.

This result forms part of the basis under the additional assumption of the simple group classification of a theorem of Aschbacher - Guralnick, that any faithful irreducible module satisfies $|H^1(G,M)| < |M|$. An immediate corollary to this is that even if the simple group has no generating set as above it still lies in $\mathcal{C}$. These results can be extended to

Thm

Suppose $G \supset P \neq 1$, P perfect and normal in G, every abelian chief factor of P is cyclic and either $G/P \in \mathcal{C}$ or $d(G) > d(G/P)$ then $G \in \mathcal{C}$.

This result covers $Alt(n)$, $Sym(n)$, all finite Chevalley groups and all groups simple $\subseteq L \subseteq Aut(\text{simple})$ s.t. L/simple is cyclic. Note however the result of Kimmerle that there are infinitely many perfect groups not in $\mathcal{C}$.

J. Williams (Adelphi)

The cup coproduct and cup product of a combinatorially aspherical group.

It is shown that there exists a natural cup coproduct which gives the homology modules of certain groups with trivial coefficients in suitable cyclic groups the structure of a commutative graded co-ring. For combinatorially aspherical presentations this cup coproduct and the cup product are both calculated from a diagonal map constructed on the Lyndon resolution. The cup product and coproduct are computable in terms of first and second order Fox derivatives alone and so can be read off directly from the CA presentation.

Kathy Horadam
MELBOURNE UNIVERSITY

Finite presentability of S-arithmetic groups

The problem which S-arithmetic groups are finitely presentable is solved for arbitrary algebraic groups over a number field k by reducing it first to the question of compact presentability of $G(K)$ for certain local fields K over k (Kneser, Crelle 1964), then passing to a maximal solvable K-split subgroup (Borel - Tits IHES 1965) and finally giving necessary and sufficient conditions for this case as follows.

Theorem. Let $G = T \ltimes U$ be a K-split solvable algebraic group over a local field K of characteristic 0. Then $G(K)$ is compactly presentable iff the following two conditions hold

1) 0 is not a positive linear combination of any two roots of T on U^{ab}.
2) There is no K-split solvable algebraic group H such that

$$N \rightarrowtail H \twoheadrightarrow G$$

with $N \neq 0$ central in H and $H/H'' \xrightarrow{\sim} G/G''$.

Herbert Abels
Bielefeld

Finiteness properties of groups

Kenneth S. Brown

A group G is of <u>type FP_n</u> if the $\mathbb{Z}G$-module $\mathbb{Z}$ admits a resolution (P_i) with P_i finitely generated and projective for $i<n$. The following result generalizes well-known criteria for G to be of type FP_n:

<u>Theorem</u>. Suppose G acts on a CW-complex X with $\tilde{H}_i(X)=0$ for $i<n$, and assume the isotropy groups G_σ ~~[illegible]~~ are of type $FP_{n-\dim\sigma}$ for all cells σ of X. Let X be the filtered union of G-invariant subcomplexes X_α such that each X_α has only finitely many cells mod G in dimensions $\leq n$. Then G is of type FP_n iff the direct system $\{\tilde{H}_i(X_\alpha)\}$ is essentially zero for $i<n$.

[This means that for each α $\exists \beta \geq \alpha$ such that the map $\tilde{H}_i(X_\alpha) \to \tilde{H}_i(X_\beta)$ is the zero map.]

In case $n=2$, this is a criterion for the FP_2 condition, which is closely related to finite presentability. The apparent difference between FP_2 and finite presentation is make clear by the following analogue of the theorem above:

<u>Theorem</u>. Suppose G acts on a simply-connected CW-complex X, and assume G_v is finitely presented for every vertex v and G_e is finitely generated for every edge e. Let X be the filtered union of G-invariant subcomplexes X_α with only finitely many cells mod G in dimensions ≤ 2. Then G is finitely presented iff $\{\pi_i(X_\alpha)\}$ is essentially zero for $i<2$.

This result appears to be an improvement of previously known results even in the case of compact quotient. [In this case one needs only the single $X_\alpha = X$, and the conclusion is then that G is necessarily finitely presented.]

Euler characteristics of 3-manifold groups and discrete subgroups of $SL_2(\mathbb{C})$ by John G. Ratcliffe (Madison)

Def: A gp G is of type FK if there is a finite $K(G,1)$ cell complex, and of type VFK if G has a subgp of finite index of type FK.

Thm 1 Every f.g. 3-manfd gp is of type VFK.

Def: If G is of type VFK, then $\chi(G) \equiv \chi(G_0)/[G:G_0]$ where G_0 is a subgp of type FK of finite index

Thm 2 Let G be a f.g. virtual 3-manfd gp, ~~then t.f.a.e.~~:
then $\chi(G)$ has the following properties:
(i) $\chi(G) \le 1$;
(ii) $\chi(G) = 1$ iff $G = 1$;
(iii) $\chi(G) > 0$ iff G is finite;
(iv) $\chi(G) = 0$ iff G is virtually the groups of an aspherical, compact, 3-manfd all of whose boundary components are tori.

Thm 3. Every f.g. discrete subgp of $PSL_2(\mathbb{C})$ is virtually a 3-manfd gp (well known)

Thm 4. Let Γ be a f.g. discrete subgp of $PSL_2(\mathbb{C})$
then $\chi(\Gamma)$ has the following properties:
(i) $\chi(\Gamma) \le 1$;
(ii) $\chi(\Gamma) = 1$ iff $\Gamma = 1$
(iii) $\chi(\Gamma) > 0$ iff Γ is finite;
(iv) $\chi(\Gamma) \ge 0$ iff Γ is virtually abelian or $\mathbb{H}^3/\Gamma$ has finite hyperbolic volume
(and $\mathbb{H}^3$ is hyperbolic 3-space).

On the cohomology of groups of p-length 1

Let $k = \mathbb{F}_p$, let G be a finite group with $p \mid |G|$, and let A be a simple kG-module lying in the principal block of kG, then:

$H^n(G,A) \neq 0$ for infinitely many $n \in \mathbb{N}$ if $G = O_{p'pp'p}G$ (in particular $\underset{p\text{-length } 1}{G = O_{p'pp'}G}$).

For proving this result, we have to analize the action of a p'-group on the cohomology ring of a p-group:

Let the p'-group Q act faithfully on the p-group P, then every simple kQ-module A is infinitely often a direct summand of $H^*(P,k)$.

Thomas Wilhelm
ETH Zürich

Relation modules of infinite groups.

We give a survey of recent results on the isomorphism and decomposition problems for relation modules of infinite groups. These include the existence of infinitely many non-isomorphic minimal relation modules for certain groups, the number of minimal relation modules for finitely generated abelian groups and the theorems that if the augmentation ideal decomposes G is finite or has more than one end, and for a polycyclic group which is nilpotent by free abelian minimal relation modules are indecomposable. These latter theorems are due to P.A. Linnell.

Peter Webb

FUNCTIONAL EQUATIONS

1.8 – 7.8.1982

On a functional equation connected with derivations

All solutions ~~of~~ $f, g : \mathbb{R}^2 \to \mathbb{R}$ of

$$\left[f\left(x+z, y+\tfrac{1}{z}\right) - f(x,y)\right] + z^2\left[g\left(x+z, y+\tfrac{1}{z}\right) - g(x,y)\right] = 0$$

$(\forall_{x,y,z \in \mathbb{R}}$ with $z \neq 0)$ are given by

$$f(x,y) = d(x) + \alpha x + \beta, \quad g(x,y) = d(y) - \alpha y + \gamma,$$

where α, β, γ are constants and where $d : \mathbb{R} \to \mathbb{R}$ is a derivation of $\mathbb{R}$. Applications on the characterization of Lorentz transformations in Hjelmslev's world are discussed.

Walter Benz

Principal solutions of Nörlund difference equations

Nörlund's principal solution $g_N(x) = \lim_{s \to 0+} \left[\int_1^\infty \varphi(t) e^{-st}\, dt - \sum_{n=0}^\infty \varphi(x+n) e^{-s(x+n)} \right]$

of (D) : $g(x+1) - g(x) = \varphi(x)$

exists, if the following assumptions on φ are satisfied:

$\varphi : [b,\infty) \to \mathbb{R}$ m times differentiable, $\varphi^{(m)}|_{[b,c]} \in L_1[b,c]$ for all $c \geq b$,

$\lim_{x\to\infty} \varphi^{(m)}(x) = 0$ and $\int_0^\infty P_m(t)\varphi^{(m)}(t+x)\,dt$ uniformly convergent for $x \in [b, b+1]$ as an improper Lebesgue integral. $\left(b \in (0,1),\ P_m(t) = B_m(t - [t]),\ B_m \text{ the } m\text{-th Bernoulli polynomial} \right)$

For the sequel we assume φ to satisfy the above conditions.

From (D) we obtain $g(N+1) = \sum_{\nu=1}^{N} \varphi(\nu)$ for $N \in \mathbb{N}$ and $g(1) = 0$.

Application of the Euler-Maclaurin sum formula in its standard version

$$(S_1) \quad \sum_{\nu=1}^{N} \varphi(\nu) = \int_1^N \varphi(t)\,dt + \frac{1}{2}[\varphi(N) + \varphi(1)] + \sum_{r=2}^{m} \frac{B_r}{r!}[\varphi^{(r-1)}(N) - \varphi^{(r-1)}(1)] + \frac{(-1)^{m+1}}{m!} \int_1^N P_m(t)\varphi^{(m)}(t)\,dt$$

and following replacement of $N \in \mathbb{N}$ by $x \in (0, \infty)$ yields

$$\tilde{g}(x+1) = \int_1^x \varphi(t)\,dt + \frac{1}{2}[\varphi(x) + \varphi(1)] + \cdots .$$

It can be shown that in general $\tilde{g}$ is not a solution of (D). We show:

If we use Euler-Maclaurin's sum formula in the following form

$$(S_2) \quad \sum_{\nu=1}^{N} \varphi(\nu) = \int_1^N \varphi(t)\,dt + \frac{1}{2}\varphi(N) + \sum_{r=2}^{m} \frac{B_r}{r!}\varphi^{(r-1)}(N) + \frac{(-1)^m}{m!} \int_N^\infty P_m(t-N)\varphi^{(m)}(t)\,dt - \gamma$$

where $\gamma = \frac{(-1)^m}{m!} \int_1^\infty P_m(t)\varphi^{(m)}(t)\,dt + \sum_{r=1}^{m} \frac{B_r}{r!}\varphi^{(r-1)}(1)$,

the above substitution procedure gives

$$\hat{g}(x+1) = \int_1^x \varphi(t)\,dt + \frac{1}{2}\varphi(x) + \sum_{r=2}^{m} \frac{B_r}{r!}\varphi^{(r-1)}(x) + \frac{(-1)^m}{m!} \int_x^\infty P_m(t-x)\varphi^{(m)}(t)\,dt - \gamma$$

and we have $\hat{g}(x) = g_N(x) - \gamma$.

Hans-Heinrich Kairies (Clausthal-Z.)

Nonhomogeneous functional equations usually have no continuous solutions

The functional equation

$$(*) \qquad \varphi[f(x)] = g(x)\varphi(x) + h(x),$$

where φ is an unknown function, usually is considered under the following assumptions:

(i) $f: I \to I$ is a continuous function, $I \subset \mathbb{R}$ is an interval and $0 < \frac{f(x)-\xi}{x-\xi} < 1$ for every $x \in I \setminus \{\xi\}$, where ξ is a point of I;

(ii) $g: I \to \mathbb{R}$ is a continuous function and $g(x) \neq 0$ for every $x \in I \setminus \{\xi\}$.

It is well known that if $|g(\xi)| \neq 1$ and the function f is strictly increasing in a neighbourhood of the point ξ then for every continuous function $h: I \to \mathbb{R}$ equation $(*)$ has a continuous solution $\varphi: I \to \mathbb{R}$. The case $|g(\xi)| = 1$ is called indeterminate. It turns out that in this case the existence of continuous solutions of $(*)$ is rather an exception. In order to see it let us consider the space C of all continuous mappings of the interval I into $\mathbb{R}$ with the topology of uniform convergence on all compact subsets of I, and its subspace C_0 of all functions from the space C vanishing at the point ξ. In both these spaces C and C_0 we have two σ-ideals; the σ-ideal of all subsets of the first category (let us denote them by $\mathcal{C}_1(C)$ and $\mathcal{C}_1(C_0)$, respectively) and the σ-ideal of all Christensen zero subsets (let us denote them by $\mathcal{C}_0(C)$ and $\mathcal{C}_0(C_0)$, respectively).

Putting

$$H = \{h \in C : \bigvee_{\varphi \in C} (\varphi \circ f = g\varphi + h)\}, \quad H_0 = \{h \in C_0 : \bigvee_{\varphi \in C} (\varphi \circ f = g\varphi + h)\}$$

we have the following theorem, the topological part of which has been proved by W. Jarczyk.

Theorem. Assume (i) and let $g: I \to \mathbb{R}$ be a continuous function.

If $g(\xi)=-1$, then $H \in \mathcal{C}_1(C) \cap \mathcal{C}_0(C)$ and $H_0 \in \mathcal{C}_1(C_0) \cap \mathcal{C}_0(C_0)$.

If $g(\xi)=1$, then $H = H_0$ and $H_0 \in \mathcal{C}_1(C_0) \cap \mathcal{C}_0(C_0)$.

Karol Baron (Katowice)

Euler-Frobenius-Polynome

Die nach Euler und Frobenius benannten Polynome $(p_m)_{m\geq 1}$ wurden von L. Euler 1755 eingeführt und von F. G. Frobenius im Jahre 1910 im Zusammenhang mit den Bernoulli-Zahlen näher untersucht. Frobenius stellte u. a. die Funktionalgleichung

$$\lambda^{m-1} p_m\left(\frac{1}{\lambda}\right) = p_m(\lambda) \qquad (\lambda \neq 0,\ m \geq 1)$$

fest. Ziel des Vortrags ist es, auf der Grundlage der Theorie der kardinalen Spline-Interpolation für die Polynome $(p_m)_{m\geq 1}$ eine komplexe Kurvenintegraldarstellung (mit nicht kompakten Integrationsweg) anzugeben und mit dieser einen neuen Zugang zu den Eigenschaften der Euler-Frobenius-Polynome zu eröffnen.

Walter Schempp (Siegen)

3 - Symmetric and α-recursive inset measures

Let B be a ring of sets,

$\Omega_n = \{(X_1, \dots, X_n) : X_i \in B,\ X_i \cap X_j = 0 \text{ if } i \neq j;\ i,j = 1,2,\dots,n\}$,

$\Gamma_n = \{(p_1,\dots,p_n) : \sum_{i=1}^{n} p_i = 1,\ p_i \geq 0,\ i,j = 1,2,\dots,n\}$, $n \geq 2$

$\Gamma_n^o = \{ (p_1, \dots, p_n) : \sum_{i=1}^n p_i = 1, \ p_i > 0, \ i,j = 1,2,\dots,n \}, \quad n \geq 2.$

The inset measure is a sequence

$$S_n : \mathcal{Q}_n^k \times \Gamma_n \times (\Gamma_n^o)^{m-1} \longrightarrow \mathbb{R} \qquad k = 1,2,\dots; \quad m,n = 2,3,\dots$$

We give the general form of all 3-symmetric, α-recursive inset measures, where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_m)$ with $\alpha_1 \neq 0$:

The result is (in the special case $k = 1$):

$$S_n \begin{pmatrix} x_1, \dots, x_n \\ p_{11}, \dots, p_{1n} \\ \vdots \\ p_{m1}, \dots, p_{mn} \end{pmatrix} = \begin{cases} g(\bigcup_{i=1}^n x_i) - \sum_{i=1}^n p_{1i}\, g(x_i) + \sum_{i=1}^n p_{1i} L(p_{1i}) + \sum_{i=1}^n p_{1i}\, \ell(p_{2i}, \dots, p_{mi}) & \text{if } \alpha_1 = 1, \ (\alpha_2, \dots, \alpha_m) = 0 \\ g(\bigcup_{i=1}^n x_i) - \sum_{i=1}^n p_{1i}^{\alpha_1} p_{2i}^{\alpha_2} \cdots p_{mi}^{\alpha_m} g(x_i) & \text{if } \alpha_1 = 1 \ (\alpha_2, \dots, \alpha_m) \neq 0 \\ & \text{or } \alpha_1 \neq 1 \end{cases}$$

Here $g : B \to \mathbb{R}$, $0^{\alpha_1} = 0$,

L and ℓ are arbitrary solutions of

$$L(u \cdot v) = L(u) + L(v), \qquad u,v \in (0,1]$$

$$0 \cdot L(0) = 0$$

$$\ell(u \cdot v) = \ell(u) + \ell(v), \qquad u,v \in (0,1)^{m-1}$$

Our result contains some recent results of the mixed theory of information, introduced by Aczél and Daróczy in 1978.

Wolfgang Sander (Braunschweig)

Some functional equations on ring of sets

Gy. Maksa, Debrecen

Let B be a ring of subsets of a given set. In this talk we present the general solutions of the following functional equations

$F(x) = F(y)$; $F(x,y) = F(x,z)$; $F(x \cup y, z) + F(x,y) = F(x \cup z, y) + F(x,z)$

holding for all pairwise disjoint $x, y, z \in B \setminus \{\emptyset\}$.

We apply these results for determining the symmetric and recursive inset entropies of all degrees excluding zero probabilities and empty sets.

Discontinuous solutions of an iterative functional equation.

Consider the functional equation

(1) $\varphi(f(x)) = g(x)\varphi(x)$

in an interval $I := (0, A)$ or $(0, A]$ such that $f, g : I^* \to \mathbb{R}$ are continuous in $I^* := I \cup \{0\}$; $0 < f(x) < x$ and $g(x) > 0$ in I. We investigate the asymptotic behaviour at the origin of those solutions $\varphi : I^* \to \mathbb{R}$ of equation (1) which are continuous in I but not necessarily in I^*. Results obtained improve slightly some contained in the paper by M. Kuczma and the speaker [Zeszyty Nauk. Uniw. Jagello. Prace Mat. 22 (1981), 119-123]. However, we assume more on asymptotic properties (as $x \to 0+$) of the given function g.

B. Choczewski (Kraków)

A completeness criterion for inner product spaces in terms of orthogonally additive functions

If $(X, \langle\cdot,\cdot\rangle)$ is a real inner product space, $\dim_{\mathbb{R}} X \geq 2$, a function $f: X \to \mathbb{R}$ is called orthogonally additive if

(*) $f(x_1+x_2) = f(x_1)+f(x_2)$ $(\forall x_1, x_2 \in X;\ x_1 \perp x_2)$.

$(X, \langle\cdot,\cdot\rangle)$ is a Hilbert space if and only if every solution f of (*) which is bounded below has a minimum on X.

Jürg Rätz (Bern)

On a Cauchy equation on restricted domain.

Consider the following functional equation

$$(*) \qquad f(x+f(x)) = f(x) + f(f(x)), \quad f:\mathbb{R}\to\mathbb{R}.$$

This equation can be viewed as a Cauchy equation on restricted domain: f must be additive on its graph.

The following theorem holds:

If f is a solution of $(*)$ continuous on $\mathbb{R}$ and differentiable at zero, then it is linear.

Using the same techniques as in the previous case, analogous results can be proved for the equation

$$f(x+h(x)) = f(x) + f(h(x)), \quad f:\mathbb{R}\to\mathbb{R}, \quad h:\mathbb{R}\to\mathbb{R},$$

where h is a given function.

Gian Luigi Forti (Milano).

On a summability theorem

With help of the theory of Cauchy functional equation the following statement is proved:

If the function $f:\mathbb{R}\to\mathbb{R}$ has the property, that $\sum f(x_n)$ is $(C,1)$ summable for every set $\{x_n\}$ for which $\sum x_n$ is $(C,1)$ summable, then $f(x)=cx$ (c is an arbitrary constant.)

István Fenyö (Budapest)

On the generalization of Lorentz transformation

Consider in the affine plane K^2 (K being a field) the Lorentz-Minkowski distance of any two points $P=(p_1,p_2)$, $Q=(q_1,q_2)$ defined by $\overline{PQ}:=(q_1-p_1)(q_2-p_2)$. Generalizing results of W. Benz the followings are obtained:

Theorem 1. Let $\operatorname{char} K \notin \{2,3,5\}$ or $K=GF(5^m)$, $m>1$. Then every map $\sigma: K^2 \to K^2$ satisfying the condition

(1) $\overline{PQ}=1 \Rightarrow \overline{P^\sigma Q^\sigma}=1$

is of the form $\sigma=\tau\circ\hat{f}$, where τ is an affine isometry of K^2 and $\hat{f}: K^2\to K^2$ is given by $(x_1,x_2)\mapsto(f(x_1),f(x_2))$ with f a monomorphism of K.

Theorem 2. If $\operatorname{char} K=5$, then a map σ with (1) and $\sigma(0,0)=(0,0)$ is an endomorphism of the vector space K^2 over the prime field K_0 of K.

F. Radó
(Cluj-Napoca)

On a problem concerning two differential equations

The following problem arose in connection with a problem in probability theory. Suppose that a function $f_1(t)$ satisfies a certain differential equation while a second function $f_2(t)$ satisfies a differential equation whose coefficients are close to the coefficients of the equation satisfied by $f_1(t)$. Are then the functions $f_1(t)$ and $f_2(t)$ close to each other? In the case considered $f_1(t)$ satisfies the equation (a) $tf_1''+2f_1'+tf_1=0$ while $f_2(t)$ satisfies (b) $(t-\varepsilon_1)f_2''+2(1-\varepsilon_2)f_2'+(t-\varepsilon_1)f_2=0$ where ε_1 and ε_2 are small.

E. Lukacs
(Washington, D.C.)

Directed graphs and the structure of chaos: work by U. Burkart (Marburg)

The pseudostochastic behaviour of iteration sequences of continuous functions that arises under suitable conditions has been termed chaos by Li and Yorke. If one looks more globally on this behaviour one finds that there exists a macroscopic structure in the chaos that can be obtained by considering directed functional graphs. A graph theoretical condition for chaos to arise is given and connections with other chaos conditions are discussed for continuous functions of $\mathbb{R}^n$ to $\mathbb{R}^n$

Presented by Gy. Targonski (Marburg)

An approximation theorem for operator cosine functions

Let X denote a Banach space and $B(X)$ the algebra of bounded linear operators on X. Let C denote a strongly continuous operator cosine function on $\mathbb{R}$ with values in $B(X)$ and let A be its infinitesimal generator with domain $D(A)$. Suppose further that $\|C(t)\| \leq 1$ for all $t \in \mathbb{R}$.

Define for $t \in \mathbb{R}$, $t \neq 0$, $C_m(t) \in B(X)$ by

$$C_m(t)x := \Big[\sum_{j=0}^{m} \binom{2m}{2j} \Big(\frac{t}{2m}\Big)^{2j} A^j\Big]\Big[\Big(\frac{2m}{t}\Big)^2 R\Big(\Big(\frac{2m}{t}\Big)^2, A\Big)\Big]^{2m} x, \quad x \in X.$$

Then

$$\lim C_m(t)x = C(t)x \quad \text{for all } x \in X, \ m \in \mathbb{N}$$

and
$$\|C_m(t)x - C(t)x\| \leq t^2 \cdot \frac{1}{\sqrt{m}} \|Ax\| \quad \text{for } x \in D(A), \ m \geq 2.$$

Dieter Lutz (Essen)

Commuting Analytic Functions on the Unit Disk

Let f and g be non-constant analytic mappings of the unit disk, D, into itself such that no iterate of either is the identity. We say <u>f and g commute</u> if $f(g(z)) = g(f(z))$ for all z in D. If g is an iterate of f or vice versa, the f and g trivially commute. In fact, the converse is almost true! We define <u>the local iteration semigroup of f</u>, a semigroup of analytic functions on D that are generalized fractional iterates of f, and obtain the following.

<u>Theorem</u>. If f and g (as above) commute, then both f and g are both in the local iteration semigroup of $f \circ g$.

Let a denote the distinguished fixed point of f in $\bar{D}$ at which $|f'(a)| \leq 1$.

<u>Cor. 1</u>. If f is univalent and $|f'(a)| < 1$, and g commutes with f, then g is also univalent.

<u>Cor. 2</u>. If $0 < |f'(a)| < 1$ and g_1 and g_2 (as above) commute with f, then g_1 and g_2 commute with each other.

Examples can be given with $f'(a) = 1$ in which the conclusions of the above corollaries are false. In certain cases, it can be shown that the fixed point set of g is the same as that of f.

Carl C. Cowen (West Lafayette, Ind)

J. Aczél: Aggregation theorems for allocation problems

(Joint work with C. Wagner, an alternative proof and generalization by C. T. Ng)

Each of n individuals ~~assigns allocates~~ (say, members of a committee) distributes an amount σ among m decision variables (say, grant application) allocating ζ_{ij} ($i=1,\dots,m$, $j=1,\dots,n$) to the i'th. These individual assignments are aggregated (~~…~~ say, by a chairman) in order to produce a 'consensual' value $\phi_i(\zeta_{i1},\dots,\zeta_{in})$. Thus

$$\sum_{i=1}^{m} \zeta_{ij} = \sigma \quad (j=1,\dots,n) \Longrightarrow \sum_{i=1}^{m} \phi_i(\zeta_{i1},\dots,\zeta_{in}).$$

has to be satisfied. With $\zeta_{ij} \in \langle \alpha, \delta \rangle$ (open or closed interval; $\alpha \geq 0$, $\delta + (m-1)\alpha \leq \sigma \leq \alpha + (m-1)\delta$; more general domains are also permitted) we find all solutions, all solution bounded from below on a set of positive measure, all solutions bounded by α from below and by δ from above and all solutions satisfying $\phi_i(\gamma,\dots,\gamma) = \gamma$ for at least one γ between α and δ ($\gamma = \sigma/m$ has a singular behaviour). Here σ and m may be fixed ($m > 2$ is more interesting for applications).

Dominates on equivalence classes of operations

Fix a partially ordered set S and let O be the set of all associative binary operations on S having e in S as an identity. For any H, G in O we say H dominates G and write $H \gg G$ if, for all a,b,c,d in S,

$$H(G(a,b), G(c,d)) \geq G(H(a,c), H(b,d)).$$

The set M of all order-preserving bijections $\alpha: S \to S$ with $\alpha(e)=e$ is a group under composition. For H in O and α in M, define $H_\alpha : S^2 \to S$ via

$$H_\alpha(a,b) = \alpha^{-1} H(\alpha(a), \alpha(b)).$$

Each H_α is in O. For H, G in O write $H \sim G$ if there is an α in M such that $G = H_\alpha$. Observe that $\sim$ is an equivalence relation and let $[H]$ denote the equivalence class determined by H.

Proposition. $H_\alpha \gg H_\beta$ if and only if $H_{\alpha\circ\gamma} \gg H_{\beta\circ\gamma}$.

Proposition. Dominates is transitive on $[H]$ if and only if, whenever $H_\alpha \gg H$ and $H_\beta \gg H$ then $H_{\alpha\circ\beta} \gg H$.

H. Sherwood (Univ of Central FL)

BILINEAR TRIANGLE FUNCTIONS

We have solved several functional equations in the space of probability distribution functions by determining the values of the unknown function on the dense subspace of step-functions.

Theorem 1. The unique continuous triangle function τ of L-class that satisfies the bilinear functional equation

$$\tau(aF+bG, H) = a\tau(F,H) + b\tau(G,H), \quad F,G,H \in \Delta^+, a,b \in [0,1], a+b \leq 1,$$

is $\tau = \sigma_{Prod,L}$, i.e., $\sigma_{Prod,L}(F,G)(x) = \iint_{L(u,v)<x} d(F(u)\cdot G(v))$.

This characterizes convolution as the only bilinear triangle function of L-class.

Theorem 2 A (T,L)-triangle function is distributive simultaneously respect to the lattice operation in Δ^+ if and only if $\tau = \tau_{T,L}$, i.e., $\tau(F,G)(x) = \sup\{T(F(u), G(v)) \mid L(u,v) = x\}$.

Further applications of this method have been used in characterizations of the truncation function (C. Alsina, Proceedings S.W.C.M.S.M., Las Palmas, 1982) and strong negations on Δ^+ (C. Alsina, Proceedings I$^{\text{r}}$ Congrés Català de Lógica Matemática, Barcelona, 1982).

Claudi Alsina (U.P.B., Barcelona, Spain)

Finding subgroups

Let G, F be two non empty sets and $*$ be an associative binary law on $\mathcal{G} = G \times F$. We look for subsemigroups H of $(\mathcal{G}, *)$ which possess a convenient parametrization ("faithful").

Thus our problem is reduced to the solving of a functional equation of the form

$$(f(z), z) = (f(x), x) * (f(y), y)$$

When $(\alpha, \beta) * (\alpha', \beta') = (\alpha\alpha', \alpha\beta' + \beta)$ we get for example

$$f(x + f(x)y) = f(x) f(y)$$

and we deduce all subsemigroups (with the convenient parametrization) of the group of all proper affine transformation on a topological linear space over the real field.

When $(\alpha, \beta) * (\alpha', \beta') = (\lambda\alpha\alpha', \alpha\beta' + \beta\alpha')$, λ a parameter, we get for example

$$f(x f(y) + f(x) y) = \lambda f(x) f(y)$$

and we deduce all subsemigroups (with the convenient parametrization) of the group of all matrices $\begin{pmatrix} \alpha & 0 \\ \beta & \alpha \end{pmatrix}$, $\alpha \in \mathbb{R} \setminus \{0\}$, $\beta \in \mathbb{R}$.

This method shows how little important is the set of parametrization but looks for subgroups of the cartesian product lying on special cartesian curves. To be more general (all cartesian curves) would require an extension of the field of functional equation, namely for example

Find F, in two variables, such that

$F(x,x')=0$, $F(y,y')=0$ implies $F(xy, xy'+x'y)=0$

It seems a promising field of investigation

Université de Nantes (France), J. Dhombres

Bedingungen, unter welchen $f(x)=x$ die einzige Lösung der Funktional-Ungleichung $f(x+y) \geq f(x)+f(y)$ ist.

Satz 1: Sei A ein Ring mit Einselement, und für $f: A \to \mathbb{R}$ gelte $f(x+y) \geq f(x)+f(y)$, $f(xy) \geq f(x)f(y)$. Dann ist f additiv und multiplikativ. – Zusatz (M. Rădulescu 1980): Im Falle $A=\mathbb{R}$ ist $f(x) \equiv 0$ oder $f(x)=x$. – Satz 2: Für $f: \mathbb{R} \to \mathbb{R}$ gelte $f(x+y) \geq f(x)+f(y)$, (*) $f(x^2) \geq f(x)^2$, $f(1)=1$. Dann ist $f(x)=x$. – Statt (*) genügt $f(g(|x|)) \geq g(|f(x)|)$ mit einer stetigen, injektiven Funktion $g: [0,\infty) \to [0,\infty)$, für welche $g(0)=0$, $g(1)=1$ gilt.

Peter Volkmann (Karlsruhe).

Some remarks on the equation $2h(x)=h(x+\varphi(x))+h(x-\varphi(x))$.

The equation

(1) $$h(x+\varphi(x))=h(x)+h(\varphi(x))$$

where $\varphi: [0,+\infty) \to [0,+\infty)$ is a homeomorphism was studied by Zdun in [1]. He proved that if $h'(0)$ exists then $h(x)=h'(0)x$. This result was improved by Dhombres in [2]. We consider the equation

(2) $$2h(x)=h(x+\varphi(x))+h(x-\varphi(x))$$

assuming moreover that $0<\varphi(x)<x$ for $x>0$. Although (2) is similar to (1) existence of $h'(0)$ is not sufficient for the uniqueness of solution and for every φ a convenient example can be constructed. However, we can prove the following

THEOREM. If $h'(0)$ and $\lim_{x\to 0}\frac{h(x)}{x}$ (finite or not) exist then solution h of (2) is of the form $h(x)=h'(0)x+h(0)$.

[1]. M.C. Zdun, On the uniqueness of solution of the functional equation $\varphi(x+f(x))=\varphi(x)+\varphi(f(x))$, Aeq. Math. 8 (1972), 229–232.

[2]. J. Dhombres, Some aspects of functional equations, Chulangkorn University, Bangkok 1979.

Maciej Sablik (joint with Joanna Cox)

Algebraic methods for the solution of an aggregation problem for demand functions:

The functional equation

$$(1)\quad \frac{f_1(x_1)}{\sum_{i=1}^{n} f_i(x_i)} = \lambda\,\frac{g_1(x_1)}{\sum_{i=1}^{n} g_i(x_i)} + (1-\lambda)\,\frac{h_1(x_1)}{\sum_{i=1}^{n} h_i(x_i)}$$

is important for solving an aggregation problem of economics. Let X_i be arbitrary sets, the functions $f_i, g_i, h_i : X_i \to F$, $i=1,\dots,n$, where F is a field. Of course, all denominators have to be different from zero. Equation (1) is solved for arbitrary $\lambda\in F$, but $\lambda\notin\{0,1\}$; $0\notin f_1(X_1)\cup\sum_{j=2}^{n} f_j(X_j)$, $|f_i(X_i)|\geq 4$ for $n=2$ and $|f_i(X_i)|\geq 3$ for $n\geq 3$, $i=1,\dots,n$.

Helmut Funke (Karlsruhe)

Functional equations of sum form

The functional equation

$$(1)\quad \sum_{i=1}^{k}\sum_{j=1}^{\ell}\Big[f_{ij}(p_iq_j)-\sum_{t=1}^{N} g_{it}(p_i)h_{jt}(q_j)\Big]=0 \quad \big((p_1,\dots,p_k)\in\Gamma_k,\ (q_1,\dots,q_\ell)\in\Gamma_\ell\big)$$

is investigated where $k,\ell\geq 3$ are fixed integers, $f_{ij}, g_{it}, h_{jt} : [0,1]\to R$ $(i=1,\dots,k;\ j=1,\dots,\ell;\ t=1,\dots,N)$ and

$$\Gamma_n=\{(p_1,\dots,p_n)\mid p_i\geq 0,\ \sum_{i=1}^{n} p_i=1\}.$$

If $f_{i1}, f_{1j}, g_{it}, h_{jt}$ are measurable for all possible values of i,j,t.

then (1) is satisfied if and only if

$$(2) \quad \bar{f}_{ij}(pq) - \sum_{t=1}^{N} \bar{g}_{it}(p)\bar{h}_{jt}(q) = 0 \qquad (p,q\in[0,1];\ i=1,\dots,k;\ j=1,\dots,\ell)$$

holds where

$$\bar{f}_{ij}(p) = f_{ij}(p) - f_{ij}(0) + p\sum_{r=1}^{k}\sum_{s=1}^{\ell} f_{rs}(0)$$

$$\bar{g}_{it}(p) = g_{it}(p) - g_{it}(0) + p\sum_{r=1}^{k} g_{rt}(0)$$

$$\bar{h}_{jt}(p) = h_{jt}(p) - h_{jt}(0) + p\sum_{s=1}^{\ell} h_{st}(0) \qquad (p\in[0,1];\ i=1,\dots,k;\ j=1,\dots,\ell;\ t=1,\dots,N).$$

Possibilities for solving equation (2) and a method for finding the general solution of (1) are also discussed.

L. Losonczi (Debrecen & Lagos)

On the continuous solutions of the multiplication formula $f\left(\frac{x}{k}\right)f\left(\frac{x+1}{k}\right)\cdots f\left(\frac{x+k-1}{k}\right) = f(x)$.

It is well-known that certain elementary and transcendental functions satisfy a so-called replicative formula

$$(1) \quad f\left(\frac{x}{k}\right) + f\left(\frac{x+1}{k}\right) + \dots + f\left(\frac{x+k-1}{k}\right) = a_k f(x) + b_k \qquad (k\in\mathbb{N},\ x\in(0,1)),$$

where (a_k) and (b_k) are sequences of real numbers depending on the function in question. Our aim is to determine the class $\mathcal{H}(a_k)$ of functions f which are measurable and locally summable on $(0,1)$ and satisfy (1) with a given sequence (a_k) and $b_k = 0$.

Theorem 1. If $(a_k) \neq (k^s)$ $(s=2,3,\dots)$ then there are functions $p, q \in \mathcal{H}(a_k)$ such that

$$\mathcal{H}(a_k) = \{ap + bq;\ a,b\in R\}.$$

This extends some results of H. H. Kairies, M. Yoder, and P. Schroth concerning solutions of (1) which are continuous, Riemann-integrable or summable on $[0,1]$.

Theorem 2. Let $f:(0,1)\to R$ be a continuous solution of the multiplication formula given in the title. Then either $f\equiv 0$ or there are real numbers a, b such that $a>0$ and

$$f(x) = a^{x-\frac{1}{2}} \cdot (2\sin\pi x)^{b} \quad \text{for every } x\in(0,1).$$

Miklós Laczkovich (Budapest)

Über algebraische Relationen zwischen additiven und multiplikativen Funktionen

Wir nennen eine Funktion f additiv, falls $f:\mathbb{C}\to\mathbb{C}$, und Lösung der Funktionalgleichung $f(u+v)=f(u)+f(v)$ ist, und eine Funktion g multiplikativ, falls $g:\mathbb{C}\to\mathbb{C}$, $g\neq 0$, und Lösung der Funktionalgleichung $g(u+v)=g(u)g(v)$ ist. Es seien $f_1,\dots,f_n$ $(n\geq 0)$, $g_1,\dots,g_m$ $(m\geq 0)$, $n+m>0$, feste additive bzw. multiplikative Funktionen. Im Vortrag werden mit Hilfe der linearen Relationen zwischen $f_1,\dots,f_n$ über $\mathbb{C}$ und des Systems der Relationen zwischen $g_1,\dots,g_m$ in der von ihnen erzeugten (abelschen) Gruppe (ebenfalls multiplikativer Funktionen) diejenigen Polynome $R(Y_1,\dots,Y_n,Z_1,\dots,Z_m)$ über $\mathbb{C}$ charakterisiert, für die $R(f_1,\dots,f_n,g_1,\dots,g_m)=0$ (d.h. $R(f_1(t),\dots,f_n(t),g_1(t),\dots,g_m(t))=0$, für alle t aus $\mathbb{C}$.

Speziell ist in diesen Sätzen enthalten:
1) Die Funktionen $f_1, \ldots, f_n, g_1, \ldots, g_m$ sind genau algebraisch unabhängig über $\mathbb{C}$, falls $f_1, \ldots, f_n$ über $\mathbb{C}$ linear unabhängig und $g_1, \ldots, g_m$ multiplikativ unabhängig sind in der von ihnen erzeugten abelschen Gruppe.
2) Die Funktionen $t \mapsto t$, $t \mapsto \exp(\lambda_1 t), \ldots, t \mapsto \exp(\lambda_n t)$ sind ~~linear~~ algebraisch unabhängig, falls aus jeder Relation $\alpha_1 \lambda_1 + \ldots + \alpha_n \lambda_n = 0$ ~~für~~ mit ganzen Zahlen α_i $\alpha_1 = \ldots = \alpha_n = 0$ folgt.

Ludwig Reich (Graz)

On the functional equations of the Dilogarithm

The Dilogarithm $Li(x) = \int_0^x -\frac{\ln|1-t|}{t}\,dt$ has recently reappeared in geometry (Gelfand-Gabrielov-Losik), and in number theory in connection with Apéry's proof of the irrationality of $\zeta(3)$.

Set $M(x) = Li(x) + \frac{1}{2}\ln|x|\ln|1-x|$. Then I show M satisfies $M(xy) = M(x) + M(y) - M\left(\frac{x-xy}{x-1}\right) + M\left(\frac{y-xy}{y-1}\right) - \lambda(x \circ y)\, M(2)$, where of course $x \neq 1, y \neq 1$; $\lambda(z) = \begin{cases} 2 & z > 1 \\ 0 & z < 1 \end{cases}$; $x \circ y = x + y - xy$. Taking suitable choices for rational substitutions one can deduce Abel's equation for Li, and the equation of Gelfand-MacPherson (Adv. Math 44). A general reference for the Dilogarithm is the book of Lewin.

Tom Davison (Hamilton)

~~Some functional equations arising in the mixed theory of information on the open domain.~~

~~The following problem arises in the characterization~~

On a formula connected with regular iteration semigroups

We can prove the following theorem closely connected with some results of L. Berg in 1960 and M. Kuczma in 1962.

Theorem: Let f be a continuous function defined in $\langle 0,a\rangle$, strictly increasing in $\langle 0,b\rangle$ with the absolute maximum at b and $0 < f(x) < x$ for $x \in (0,a\rangle$. If in a neighbourhood of zero f fulfils one of the following conditions:

1° f is convex or concave.

2° f is of class C^1 and $f'(x) = q + O(x^{\mu})$, $\mu > 0$

and $0 < f'(0) =: q < 1$, then there exists the limit

(1) $$\lim_{n\to\infty} f|_{\langle 0,b\rangle}^{-n}\left(q^t f^n(x)\right) =: f_t(x), \quad t>0,\ x \in \langle 0,a\rangle$$

and $\{f_t, t>0\}$ is a regular iteration semigroup.

By this theorem we get the following corollary.

If a function f of class C^1 fulfils the assumptions of the theorem and $f'>0$ in $\langle 0,b)$, then equation $g^N = f$ has a unique C^1-solution in $\langle 0,a\rangle$. This solution is given by the formula (1), where $t = \frac{1}{N}$.

Marek Cezary Zdun (Bielsko-Biała)

Some functional equations arising in the mixed theory of information on the open domain.

The following problem arises in the characterization of 3-symmetric, β-recursive inset measures of information on the open domain. Find all solutions of

(*) $F(x\cup y, z) + F(x,y) = F(y,z) + F(x, y\cup z)$, $\quad F(x,y) = F(y,x)$,

for $(x,y,z) \in D_3 := \{(x_1,x_2,x_3) \mid \emptyset \neq x_i \in B,\ x_i \cap x_j = \emptyset \text{ for } i\neq j;\ i,j = 1,2,3\}$, where B is a ring of sets and F is real-valued.

Theorem. The general solution of (*) is given by

$$F(x,y) = f(x) + f(y) - f(x\cup y)$$

for all $(x,y) \in D_2 := \{(x,y) \mid x,y \in B\setminus\{\emptyset\},\ x\cap y = \emptyset\}$, where f is an arbitrary real-valued function on $B\setminus\{\emptyset\}$.

This theorem can be used to find all 3-symmetric,

β-recursive inset entropies on the open domain, and to find all symmetric, branching inset entropies on the "classical" domain. The proof of the theorem relies on a result of K. Davidson and C. T. Ng.

Bruce R. Ebanks (Lubbock)

<u>The natural domain of definition of $\varphi(x^{\varphi(y)}y) = \varphi(x)\varphi(y)$.</u>

If T is a subset of a group G we call a map $\varphi: T \to G$ a <u>coupling</u> if:

$$x^{\varphi(y)}y \in T \text{ and } \varphi(x^{\varphi(y)}y) = \varphi(x)\varphi(y) \text{ for all } x, y \in T.$$

This functional equation is the multiplicative analogue of the equation of GOŁĄB and SCHINZEL. In the present context it was first considered by H. KARZEL (Arch. d. Math 1965) for the case where T is a group and G its holomorph.

<u>Theorem</u>: Let G be a group. Then there is a bijection between the following two families of objects:

a) pairs (H, η), where: H a subgroup of G, $\eta: H \to G$ an anti-homomorphism, $\eta(H) \cap H = 1$

b) pairs $(T, \varkappa)$, where: T a subset of G, $\varkappa: T \to G$ a coupling, $\varkappa(T) \cap T = 1$.

This bijection is given by $\Phi: (H, \eta) \mapsto (T, \varkappa)$ with $T = \{\eta(h)h \mid h \in H\}$ and $\varkappa(t) = \eta(h)^{-1}$ if $t \in T$ has the representation $t = \eta(h)h$, $h \in H$

Peter Plaumann (Erlangen)

<u>Über gewichtbare Mittelwerte</u>

Die Folge von Funktionen $M_n: I^n \to I$ ($n \in \mathbb{N}$, $I \subseteq \mathbb{R}$ Intervall) wird ein diskreter Mittelwert genannt, wenn sie symmetrisch und intern ist. Ein diskreter Mittelwert M_n ist $\mathbb{Q}$-gewichtbar, wenn die Gleichung

$$M_{kn}(\underbrace{x_1, \ldots, x_1}_{k}; \ldots; \underbrace{x_n, \ldots, x_n}_{k}) = M_n(x_1, \ldots, x_n)$$

für alle $k, n \in \mathbb{N}$ und $(x_1, \ldots, x_n) \in I^n$ gültig ist. Sind $r_i = \frac{k_i}{k}$ ($k_i \geqq 0$ ganze Zahlen mit $\sum_{i=1}^{n} k_i > 0$

und $k \in \mathbb{N}$) rationale Zahlen, so definieren wir die Funktion

$$\widetilde{M}_n(x_1,\dots,x_n; r_1,\dots,r_n) := M_{\sum_{i=1}^{n} k_i}(\underbrace{x_1,\dots,x_1}_{k_1};\dots;\underbrace{x_n,\dots,x_n}_{k_n}).$$

Hat $\widetilde{M}_n$ eine stetige Fortsetzung $\overline{M}_n$ bezüglich der Gewichte r_i, so nennt man M_n gewichtbar. Es sei $\Delta_n := \{(p_1,\dots,p_n) \mid p_i \geq 0, \sum_{i=1}^{n} p_i > 0\}$. Dann die Folge $\overline{M}_n : I^n \times \Delta_n \to I$ wird monoton genannt, wenn die Funktion

$$t \to M_n(\underline{x}; t\underline{a} + \underline{b})$$

($\underline{a}, \underline{b} \in \mathbb{R}^n$ sind beliebig) auf $J_{\underline{a},\underline{b}} := \{t \mid t \in \mathbb{R}, t\underline{a}+\underline{b} \in \Delta_n\}$ streng monoton oder konstant ist. Es gilt der folgende <u>Satz</u>. Ist M_n ein gewichtbarer, monotoner diskreter Mittelwert, dann existiert eine Abweichung A, so daß die Darstellung

$$M_n(\underline{x}) = \mathfrak{M}_{A,n}(\underline{x})$$

für alle n und $\underline{x} \in I^n$ erfüllt ist.

Zoltán Daróczy (Debrecen, Ungarn)

<u>On some functional equations from additive and nonadditive measures – V.</u>

The 'sum form' functional equations

$$(1) \quad \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij}(p_i q_j) = \sum h_i(p_i) + \sum f_j(q_j) \quad \sum p_i = 1 = \sum q_j.$$

$$(2) \quad \sum_{i=1}^{m}\sum_{j=1}^{n} g(p_i q_j, r_i s_j) = \sum_i g(p_i, r_i) + \sum g(q_j, s_j)$$

are studied. (1) is solved when $g_{ij}, h_i, f_j : [0,1] \to \mathbb{R}$ measurable and when (1) holds for $m \geq 2$, $n \geq 3$. (2) is solved when g is measurable and when (2) holds for $m \geq 2$, $n \geq 3$.

P. L. Kannappan (Waterloo, Canada)

Exponential polynomials

Exponential polynomials on Abelian groups are investigated and a number of regularity theorems concerning them are proved. One typical result states that on a locally compact Abelian group generated by any neighborhood of the origin the set od all zeros of a nonidentically zero exponential polynomial has measure zero.

Lénló Székelyhidi (Debrecen, Hungary)

Stability conditions for special linear difference equation systems

A theorem is proved which gives a necessary and sufficient condition for the (global) stability of the solutions of the system

$$p(t+1) = Ap(t) + b \qquad (t=0,1,2,\dots),$$

where $p(t) \in \mathbb{R}^n$, $b \in \mathbb{R}^n$, $A=(a_{ij})$ a real $n\times n$- matrix ~~where~~ with entries of the form

$$(M) \qquad \begin{cases} a_{ij} = d_i \cdot e_j & (i \neq j) \\ a_{ii} = s_i & (i=1,\dots,n). \end{cases}$$

The theorem is a generalization of results of Kosomatsu (1969) and Neudecker (1970) (Review of Economic Studies).

Frank Stehling (Karlsruhe, West Germany)

The sine functional equation for groups

We consider the functional equation

(1) $f: G \to K, \; f(xy)f(xy^{-1}) = f^2(x) - f^2(y)$

where G is an group and K is a field. Clearly the fonctions of the form

$$(2) \qquad f(x) = \frac{g(x) - [g(x)]^{-1}}{2b}$$

where g is a homomorphism of G into the multiplicative group of K, satisfy the equation (1).

Let $K(b)$ denote the extention of the field K by the element b and let $K^*(b) = K(b) \setminus \{0\}$.

We prove the following theorem

Theorem. Let G be a group whose elements are of odd order, K a skewfield of $\operatorname{char} K \neq 2$ and $f: G \to K$ a non zero solution of the equation (1). Then f has the form (2) where g is a homomorphism from G into the multiplicative group of $K^*(b)$, or f is a homomorphism from G into the additive group of K.

J. Corovei (Cluj-Napoca, Romania)

A Model of Optimal Economic Growth Yielding a Functional Equation

The arithmetic means of the investment ratios $\alpha_t = I_t / Y_t$ (= investment in capital goods of year t divided by the GNP of year t) of the years 1976 to 1980 were very different in different countries. Examples: 33% in Japan; 22 to 24% in F, I, D; 20% in GB; 18% in USA. Problem: Find sequences of optimal α_t's with respect to maximization of consumption. Model:

(i) $F_t : \mathbb{R}_+ \to \mathbb{R}_+$ (production function of year t); $Y_t = F_t(K_{t-1})$, K_t = capital stock of year t.

(ii) $K_t = \alpha_t F_t(K_{t-1}) + q_t K_{t-1}$ (q_t = depreciation factor).

(iii) Given a planning horizon n_t and a capital stock K_{t-1} at ~~...~~ (the beginning of) year t, find investment ratios $\alpha^*_{t+1}, \dots, \alpha^*_{t+n_t}$ that maximize the discounted consumption

$$\sum_{k=1}^{n_t} r_t^{(k)} (1-\alpha_{t+k}) F_{t+k}(K_{t+k-1}), \text{ where } r_t^{(k)} := r_{t+1} r_{t+2} \cdots r_{t+k} \text{ (discounting factors)}$$

Theorem: Let F_t be twice differentiable and concave. Let F_t' be invertible in a neighbourhood of $1/r$ and $(1-r_t q_t)/r_t$ and let $F_t''(a_t) < 0$, $F_t''(b_t) < 0$ at points a_t, b_t (whose values shall not be specified here). Then there exist optimal investment ratios α^*_{t+k} (formulas omitted here), and these depend only on F_{t+k}, F_{t+k+1} and K_{t-1}, the initial capital stock. Beginning with K_{t-1} one can calculate the optimal capital stocks K^*_{t+k-1} ($k = 1, \dots, n_t$).

Proof: Bellman's method of backward dynamic programming using Bellman's functional equation.

Specialization: $r_t = r$, $q_t = q$, $F_t(K) = F(K) = cK^\gamma$ ($\gamma < 1$, c positive reals) for all $t = 1, 2, \dots$, planning horizon infinite, suitable convergence conditions. Then $(\alpha_1^*, \alpha_2^*, \alpha_3^*, \dots) = (\alpha_1^*, \alpha^*, \alpha^*, \dots)$ and $\alpha^* = (1-q) r \gamma / (1 - rq)$. ⊗ The 1980 investment ratio 0.229 of D is obtained for, e.g., $q = 0.9$, $r = 0.97$, $\gamma = 0.3$ which are quite realistic values. ⊗ Note that our specialization implies zero growth.

Wolfgang Eichhorn (Karlsruhe)

Functional equations in probability theory

Let $\bar{X} = (\bar{X}_1, \dots, \bar{X}_n)$ be a n-dimensional random variable. Let $\underline{f} : \mathbb{R}^n \to \mathbb{R}^n$ be a one-to-one transformation. Let us define a new random variable by $\bar{Y} = \underline{f}(\bar{X})$. To give a proof of a characterization theorem we must solve the functional equation

$$q(\underline{y}) = p(\underline{g}(\underline{y}))\, J(\underline{y}) \qquad (\underline{y} \in \mathbb{R}^n),$$

where p, q are the density functions of $\bar{X}$ and $\bar{Y}$ respectively, $\underline{g}$ is the inverse of transformation $\underline{f}$ and ~~g~~ J is the Jacobian of $\underline{g}$.
In the lecture we give a characterization of normal and gamma distribution.

K. Lajkó (Debrecen, Hungary)

Two functional equations for power series

We consider the two functional equations $\phi(x, \phi(y, z)) = \phi(x \cdot y, z)$ and $\phi(x, \phi(y, z)) = \phi(x + y, z)$ as identities for $\phi \in K[[x, z]]^n$, the ~~ring of~~ cartesian product of the ring of formal power series in the $n+1$ variables x and $z = (z_1, \dots, z_n)$ with coefficients of the field K. Some aspects of the solutions are discussed

D. Gronau (Graz)

Certaines équations fonctionnelles de la recherche opérationnelle

Soit le processus défini par les événements
$A_n(t,u) \iff$ (à $(t, t+u) \to n$; $(t,u)\in\mathbb{R}_+^2$, $n\in\mathbb{N}$)
En notant $P(A_n(t,u)) = P_n(t,u)$ (probabilité de $A_n(t,u)$)
le processus est modélé par le système

$$(1)\quad \begin{cases} P_0(t,u+v) = P_0(t,u)\cdot P_0(t+u,v) \\ P_n(t,u+v) = \sum_{k=0}^{n} P_k(t,u)\, P_{n-k}(t+u,v) \end{cases}$$

$(t,u)\in\mathbb{R}_+^2$, $n\in\mathbb{N}^*$

On détermine la solution générale de l'équation (2) $F(t,u+v) = F(t,u)\cdot F(t+u,v)$ exprimée à l'aide d'une fonction $h: X\to\mathbb{R}$, $X\in\mathbb{R}_+$, $h(t)\neq 0$, $\forall t\in X$.

En utilisant une solution H de l'équation de la convolution $H(f*g) = H(f)\cdot H(g)$, $H: \mathcal{N}\to\mathbb{C}^{\mathbb{C}}$ ($\mathcal{N}^0 = \{f:\mathbb{N}\to\mathbb{R} / f(0)\geq 0\}$.) on obtient l'équation fonctionnelle

$$(2')\quad F(t,u+v,z) = F(t,u,z)\cdot F(t+u,v,z)$$

$F: \mathbb{R}_+\times\mathbb{R}_+\times\mathbb{C}\to \mathbb{R}_+\times\mathbb{R}_+\times\mathbb{C}$ et $F(t,u,z) = H(P(t,u))$, $P(t,u) = (P_n(t,u))_0^\infty$. La solution de (1) s'exprime est sous la forme: $(P_n(t,u))_0^\infty = H^{-1}\left(\frac{h(t+u,z)}{h(t,z)}\right)$

$h(t,z)$ - arbitraire; $h(t,z)\neq 0$ $(t,z)\in X\times\mathbb{C}$

On caractérise en suite les processus: stationaires, amortisés, etc, avec la fonction „génératrice" h.

Opriș D. (Cluj-Napoca).

Les prolongements des homomorphismes et des solutions de l'équation de translation

On fait quelques remarques au sujet des théorèmes sur les prolongements des homomorphismes d'un groupe à un surgroupe donné par A. Grząślewicz (Aeq. Math. 17, 1978) et on compare ces théorèmes avec le théorème du prolongement de la solution de l'équation de translation (Z. Moszner, Tensor, 26, 1972). Ces sont les résultats de K. Danielewicz et de moi-même, qui sont sous presse dans Rocznik Naukowo-Dydaktyczny WSP w Krakowie.

5 VIII 1982 Z. Moszner (Cracovie, Pologne)

JORDAN-ALGEBREN UND
IHRE ANWENDUNGEN
8. Aug. — 14. Aug. 1982

Bounded symmetric domains and Jordan structures

Mit Jordantheoretischen Methoden kann eine Art Riemannscher Abbildungssatz für beschränkte symmetrische Gebiete bewiesen werden: Die beschränkten symmetrischen Gebiete in komplexen Banachräumen sind (bis auf Biholomorphie) genau diejenigen offenen Einheitskugeln komplexer Banachräume, die homogen unter der Gruppe aller biholomorphen Automorphismen sind. Beschränkte symmetrische Gebiete sind insbesondere symmetrische komplexe Banachmannigfaltigkeiten und können daher durch gewisse hermitesche Jordansche Tripelsystem (kurz J^*-Tripel) algebraisch charakterisiert werden. Definitionsgemäß ist ein J^*-Tripel ein komplexer Banachraum U zusammen mit einem stetigen, sesquilinearen, operatorwertigen Produkt

$$U \times U \ni (x,y) \longmapsto x \square y^* \in \mathcal{L}(U),$$

so daß für alle $x,y,z,w \in U$ gilt:

(i) Das Tripelprodukt $\{xy^*z\} := x \square y^*(z)$ ist symmetrisch in den äußeren Variablen x, z.

(ii) $[x \square y^*, z \square w^*] = \{xy^*z\} \square w^* - z \square \{wx^*y\}$ (Jordan-Tripel-Identität)

(iii) $z \square z^*$ ist ein hermitescher Operator auf U.

Die beschränkten symmetrischen Gebiete sind (bis auf Biholomorphie) genau die offenen Einheitskugeln von JB^*-Tripeln, das sind J^*-Tripel, für die zusätzlich gilt:

(iv) $z \square z^* \geq 0$ (d.h. hat Spektrum ≥ 0) und

(v) $\|z \square z^*\| = \|z\|^2$ (C^*-Bedingung).

8. Aug. 1982 W. Kaup.

Toeplitz operators on bounded symmetric domains

Using the algebraic description of bounded symmetric domains $D \subset \mathbb{C}^n$ in terms of Jordan algebras and Jordan triple systems, we study Toeplitz operators

$$T_f h := \pi(fh) \quad \forall\, h \in H^2(S)$$

on the Hardy space $H^2(S)$ associated with the Shilov boundary S of D. Here $\pi : L^2(S) \to H^2(S)$ denotes the Szegö projection. Our main results are as follows:

(1) Let $N_1, \dots, N_r$ denote the norm functions associated with a frame $\{e_1, \dots, e_r\}$ of the Jordan triple system $Z = \mathbb{C}^n$. Then the irreducible $\mathrm{Aut}(Z)$-modules E_m associated with the conical polynomials

$$N_1^{m_1 - m_2}\, N_2^{m_2 - m_3} \cdots N_r^{m_r}$$

for integers $m_1 \geq \dots \geq m_r \geq 0$ constitute an explicit Peter-Weyl decomposition of $H^2(S)$.

(2) Using the explicit determination of the differential and integral scalar products on E_m, the Toeplitz operators T_ℓ and $T_\ell{}^*$ associated with linear forms ℓ on Z can be expressed in terms of Jordan differential operators $\{z u^* z\}\frac{\partial}{\partial z}$ and $u\frac{\partial}{\partial z}$.

(3) The Toeplitz C^*-algebra

$$\mathcal{T} := C^*(T_f : f \in \mathcal{C}(S))$$

generated by all Toeplitz operators T_f with continuous symbol functions f is solvable of length $r := \mathrm{rank}(D)$ with spectral components

$$S_k := \{\text{tripotents of rank } k\}$$

for $0 \leq k \leq r$ in the sense of Dynin.

8. August 1982 Harald Upmeier

Structure and Representation of Noncommutative Jordan Algebras

Noncommutative Jordan algebras appear naturally in physiks (Gell-Man quark model) and functional analysis (Vidav-Palmer characterization). We give the definition of noncommutative JB*-algebra, -factor, -factor representation, JW* algebra, rank rk A of A and a sketch of proof for the following theorems.

Thm 1 A unital noncommutative JB*-algebra A has a faithful family of factor representation, namely that of A with Jordan product $x \circ y = \frac{1}{2}(xy + yx)$

Thm 2 Let A be a n.c. JB*-factor. Then

1) $A = \mathbb{C}\mathbb{1}$ if $\mathrm{rk}\, A = 1$

2) A is quadratic if $\mathrm{rk}\, A = 2$

3) A is commutative or quasiassociative if $3 \leq \mathrm{rk}\, A < \infty$

4) $\mathbb{1}$ is the sum of four orthogonal selfadjoint strongly connected idempotents if $\mathrm{rk}\, A \in \{0, \infty\}$.

Thm 3 (Coordinatization) Let A be a noncommutative JB*-algebra, whose unit element is a finite sum of $n \geq 4$ strongly connected nonzero selfadjoint idempotents $e_1, e_2, \ldots, e_n$. Let $A = \bigoplus_{1 \leq r \leq s \leq n} A_{rs}$ be the Peirce decomposition. Then the following three conditions are equivalent

1) $A \cong \mathcal{H}_n(D)^+$ or $A \cong M_n(D)^{(\lambda)}$ for an appropriate associative algebra D with an involution and $\lambda \neq \frac{1}{2}$

($\mathcal{H}_n(D) := \{$selfadjoint $n \times n$ matrices$\}$, $M_n(D) := \{ n \times n$ matrices $\}$
$A^{(\lambda)}$ is the λ-mutation, that is the vector space A with multiplication
$x \cdot y = \lambda xy + (1-\lambda) yx$)

2) A is commutative or quasiassociative

3) The number of elements in the spectrum of the left multiplication $L_{e_1} : A_{12} \to A_{12}$ is bounded by two.

8. August 1982 Robert Braun

Normed non-commutative Jordan-Algebras

A non-commutative (n.c.) Jordan-alg. is an algebra satisfying $(xy)x = x(yx)$ and $(x^2y)x = x^2(yx)$. If $\mathcal{J}$ is a n.c. J.a. define $a \times_\lambda b := \lambda ab + (1-\lambda) ba$ ($\lambda \in$ ground field) then this new alg $\mathcal{J}^\lambda$ is again a n.c. J.A.

A NJB-algebra is a n.c. J.a. $\mathcal{J}$ with unit 1 over $\mathbb{R}$ and a Banach-space with $\|ab\| \le \|a\| \cdot \|b\|$; $\|a^2\| = \|a\|^2$; $\|a^2\| \le \|a^2 + b^2\|$. A NJC*-algebra is a n.c. J.a $\mathcal{J}$ over $\mathbb{C}$ with unit 1 and involution * ~~s.t.~~ and a Banach-space s.t. $\|ab\| \le \|a\| \|b\|$; $\|a^*\| = \|a\|$; $\| P(a) a^* \| = \|a\|^3$.

A NJW- resp. NJW*-algebra is a NJB- resp. NJC*-algebra ~~and~~ that is the dualspace of a Banach-space. A NJW- resp. NJW*-factor of type I has as central projections only 0 and 1 and there exist minimal projections.

Then you can prove the following results:

Every NJW-factor of type I is in fact commutative, so a well known so called JBW-factor of type I.

The NJW*-factors of type I are: quadratic, commutative or $(\mathcal{B}(L))^\lambda$; L a complex Hilbert-space. From this one get's $z^* z \ge 0$ $\forall z \in \mathcal{J}$ in a NJW*-factor of type I.

10. August 1982 Klaus Alvermann

Classification of central simple Jordan superalgebras

V.G. Kac classified in his paper „Classification of simple Z-graded Lie superalgebras and simple Jordan superalgebras" (Communications in Algebra 5(13)) the simple Jordan superalgebras over an algebraically closed field of characteristic zero by means of a generalized Koecher-Tits-construction.

In the following let K be a field with char(K) $\neq 2,3$. A superalgebra is a $\mathbb{Z}/2\mathbb{Z}$-graded algebra. For an example let $V = V_0 \oplus V_1$ be a vectorspace over K, which is the direct of the subspaces V_0, V_1. Define $\mathrm{End}(V, V_0, V_1)_\alpha := \{ f \in \mathrm{End}(V) \mid f(V_\beta) \subset V_{\alpha+\beta} \}$. Then End(V) becomes an associative superalgebra. For $f \in \mathrm{End}(V, V_0, V_1)_\alpha$

$g \in \mathrm{End}(V; V_0, V_1)_\beta$ define $\langle f, g\rangle := fg - (-1)^{\alpha\beta} gf$. Let $\mathcal{A}$ be a superalgebra. $\mathcal{A}$ is a Jordan-superalgebra, if for all $a \in \mathcal{A}_\alpha$, $b \in \mathcal{A}_\beta$, $c \in \mathcal{A}_\gamma$:

(1) $ab = (-1)^{\alpha\beta} ba$

(2) $(-1)^{\alpha\gamma} \langle L(ab), L(c)\rangle + (-1)^{\beta\alpha} \langle L(bc), L(a)\rangle + (-1)^{\gamma\beta} \langle L(ca), L(b)\rangle = 0$

is true.

$\mathcal{A}$ is called semi-simple, if $\mathcal{A}$ has no nonzero solvable ideal. In general $\mathcal{A}$ semi simple does not imply that $\mathcal{A}_0$ is semi simple, too. This is the case under certain conditions, which are always satisfied if the characteristic is zero. ~~Jordan~~ It is not true, that a semi simple Jordan superalgebra $\mathcal{A}$ with $\mathcal{A}_0$ separable, is the direct sum of simple one's. But such algebras can be described by simple super algebras.

<u>Theorem</u>: Let $\mathcal{A}$ be a finite dimensional central simple Jordan superalgebra over K and assume $\mathcal{A}_0$ is separable. Then $\mathcal{A}$ is either a central simple exceptional Jordan algebra or $\mathcal{A}$ is isomorphic to $\mathcal{H}(\mathcal{B}, J)$ where $(\mathcal{B}, J)$ is a central simple associativ superalgebra with superinvolution J or $\mathcal{A}$ is „exceptional".

There are only three types of „exceptional" Jordan superalgebras with dimensions 3, 4 and 10.

10.08.1982 Oda Kühn

Automorphism groups of exceptional Jordan algebras over local rings,

Let (R, m) be a local ring containing $1/2$. Let $\mathcal{O}$ be an octonion R-algebra, that is, a unital R-algebra which is a free R-module of rank 8 and which has a nondegenerate quadratic form permitting composition. Assume that $\mathcal{O}$ is split in the sense that there is $a \in \mathcal{O} - m\mathcal{O}$ such that $n(a) = 0$. Let $J = \mathcal{H}(\mathcal{O}_3)$ be the

Jordan algebra of 3-by-3 Hermitian matrices over $\mathcal{O}$. Let $\mathrm{Aut}(\mathcal{J})$ be the automorphism group of $\mathcal{J}$. If I is an ideal of R, let $\mathrm{Aut}(\mathcal{J}, I)$ be the kernel of the canonical homomorphism from $\mathrm{Aut}(\mathcal{J})$ to $\mathrm{Aut}(\mathcal{J}/I\mathcal{J})$. It is proved that there is a one-to-one correspondence between the ideals I of R and the normal subgroups N of $\mathrm{Aut}(\mathcal{J})$ given by $N = \mathrm{Aut}(\mathcal{J}, I)$.

Robert Bix

Central simple Jordan Triple Systems over fields

As Loos has shown, there exists a 1-1-correspondance between the category of Jordan Triple Systems and of Jordan Pairs with involution. [illegible] a [illegible] Triple system, To classify all c.s. JTS, one uses the fact, that they corresponds either to a c.s. J.P $(W \oplus W^{op}, \varepsilon)$, where W is a c.s. J.P. and ε is the exchange involution, or to an simple Jordan Pair, possessing an involution of 1st or 2nd kind. Only the later type is interesting. If the Jordan Pair comes from a Jordan Algebra, the involutions are given by (may be semilinear) elements η of the structure group, satisfying $\eta^{\#} = \eta$. Examples:

I.: $V = (M_{p,q}(\vartheta), M_{q,p}(\vartheta))$; ϑ a division algebra. If $p = q$, then the involutions are given by (i) $L_c \circ \varphi$, φ an automorphism of $M_p(\vartheta)$, $\varphi^2 = \mathrm{Inn}_c$, $c \in M_p(\vartheta)$. Otherwise one have to determine the automorphism group of V, getting

$$\mathrm{Aut}(V) = \{ (X \mapsto A X^{\varphi} B,\ Y \mapsto B^{-1} Y^{\varphi} A^{-1} \mid A \in M_p^{\times}(\vartheta),\ B \in M_q^{\times}(\vartheta),\ \varphi \text{ an assoc. auto of } \vartheta \}.$$

Therefore, an involution of V exists iff ϑ possesses one. II.: If $V = (W, q)$ is the JP of a quadratic form, the involutions of V are the orthogonal semisimilitudes in the sense of Dieudonné.

11.8.82 T. Schwarz

Contractive Projections on Operator Algebras

If A is a unital C^*-algebra and $P: A \to A$ is a contractive projection, that is, $P^2 = P$ and $\|P\| = 1$, we present the conjecture that $P(A)$, the image of A under P, is a Jordan triple system under the triple product

$\{a,b,c\} = \frac{1}{2}P(ab^*c + cb^*a)$. An indication of how this conjecture arose and why it may be true is given by verifying the result if $P(1) = 1$ and (more generally) if $P(u) = u$ for some unitary u. Some further cases in which the conjecture is true under additional hypotheses on P or A are described.

Martin A. Youngson

A Trace Formula and Derivations of simple Jordan Pairs.

The trace of the product of two left multiplications in an absolutely, finite dimensional Jordan pair over a field of characteristic $\neq 2$ is expressed in terms of the reduced trace. The resulting (non trivial) formula is used to determine all derivations and the dimensions of the derivation algebras of such pairs.

Kurt Meyberg

Strukture and tensor products of JC-algebras.

A JC-algebra A is called universally reversible if, for each representation $\pi : A \to \mathcal{B}(H)_{sa}$ (H - Hilbertspace), $a_1, \dots, a_n \in \pi(A) \Rightarrow a_1 a_2 \cdots a_n + a_n \cdots a_2 a_1 \in \pi(A)$. A is universally reversible iff it has no spin factor quotient of dimension ≥ 5. For any JC-algebra A there is a universally reversible ideal I such that A/I is a "spin algebra", i.e. it has a faithful family of spin factor representations. Any universally reversible JC-algebra is of the form $\{x \in \mathfrak{A} : x = x^* = \Phi(x)\}$ where $\mathfrak{A}$ is a C^*-algebra and Φ is an involutory $*$-antiautomorphism of $\mathfrak{A}$ (a "flip").

To any JC-algebra A is associated a "universal"

C^*-algebra $C^*(A)$, such that any homomorphism of A into the self-adjoint part of a C^*-algebra $\mathcal{B}$ factors through $C^*(A)$.

$$\begin{array}{ccc} C^*(A) & \dashrightarrow & \mathcal{B} \\ \uparrow & & \uparrow \\ A & \longrightarrow & \mathcal{B}_{sa} \end{array}$$

For two JC-algebras A and B, we define their universal tensor product as the JC-algebra $A \tilde{\otimes} B$ generated by $A \otimes B$ in $C^*(A) \underset{max}{\otimes} C^*(B)$. This is characterized by a universal property, and is easily computed in many cases.

Harald Hanche-Olsen

Idempotent maps on C^*-algebras and Jordan algebras.

Let A be a unital C^*-algebra with self-adjoint part A_{sa} acting on a Hilbert space H. Let $P: A \to A$ be a positive linear map such that $P(1) = 1$ and $P^2 = P$. Assume for simplicity that P is faithful, i.e. $x \geq 0$, $P(x) = 0 \Rightarrow x = 0$. Then $P(A_{sa})$ is a Jordan subalgebra of A_{sa}. Call P decomposable if $\exists$ a Hilbert space K, a Jordan representation π of A on K and a bounded linear operator $V: H \to K$ such that $P(x) = V^* \pi(x) V$, $x \in A$. Then P is decomposable iff $P(A_{sa})$ is a reversible Jordan subalgebra of A_{sa}. If ι denotes the identity map of A onto itself then we have that $\iota - P$ is contractive iff $P = \frac{1}{2}(\iota + \theta)$ with θ a Jordan automorphism of A of order 2. In many interesting cases θ is an anti-automorphism, in which case the null space $P^{-1}(0)$ of P is a Lie algebra. For example if, $A = M_n(\mathbb{C})$, $P^{-1}(0)$ will be one of the classical Lie algebras B_n, C_n or D_n

Erling Størmer

Coordinatization of Moufang-Veldkamp Planes

A plane is Veldkamp if it has 2 relations $P I l$, $P \approx l$, and if $P \approx Q$ is defined by $P \approx l$ for all $Q I l$ satisfying

(VP1) $P I l \Rightarrow P \approx l$

(VP2) $P \not\approx Q \Rightarrow \exists!\ l = P \vee Q,\ l I Q,\ l I P$

(VP2') the dual of (VP2)

(VP3) $l \not\approx m,\ P I l,\ P \not\approx l \wedge m \Rightarrow P \not\approx m$

(VP4) given l $\exists$ P with $P I l$

(VP5) given P, Q $\exists$ l with $P \not\approx l \not\approx Q$

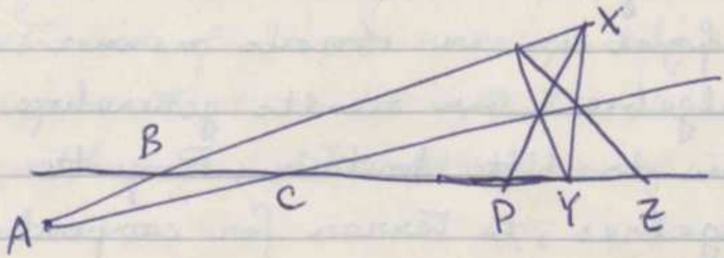
(VP6) $\exists$ l.

Theorem A Moufang-Veldkamp plane is coordinatized by an alternative ring of stable range two.

A Veldkamp plane is Moufang if

(1) for all $C I l$ & $B \not\approx l \not\approx B'$ with $B' I B \vee C$ there is a collineation φ fixing $P I l$ and $m I C$ with $\varphi(B) = B'$

(2) Given A, B, C with $A \not\approx B \vee C$, $Y, Z \in B \vee C$, $Y \not\approx C$, $Z \not\approx B$ there is $P \in B \vee C$, $P \not\approx C$ with $X * P = ((X * Y) * Z) * Y$ for all $X \in A \vee B$, $X \not\approx B$ where $X * P = (X \vee P) \wedge (A \vee C)$ etc.



Stable range two means $ax + y$ left invertible implies $x + by$ left invertible for some $b \in \mathfrak{A}$. The coordinatization is accomplished by constructing the groups analogous to the 2×2 and 3×3 groups generated by elementary matrices and making heavy use of Jordan pair theory.

John R. Faulkner

Generic Reduction of Jordan Pairs

Let k be an arbitrary base field and $\mathfrak{V}$ an absolutely simple Jordan pair of finite dimension over k. If $\mathfrak{V}$ contains a maximal orthogonal system of absolutely primitive idempotents it is said to be <u>reduced</u>. A field extension L/k is called a ~~generic~~ <u>reducing field of</u> $\mathfrak{V}$ in case the extended Jordan pair $\mathfrak{V}_L$ is reduced over L. In this lecture we are concerned with the construction of various reducing fields K/k which are <u>generic</u> in the sense that, whenever we are given any reducing field L/k of $\mathfrak{V}$, there exists a k-place from K to L and conversely. More specifically, the following field extensions K/k turn out to be generic reducing fields of $\mathfrak{V}$.

(i) $K = k^{red}(\mathfrak{V}) = \mathrm{Quot}\, k[T,X,Y]/(m(T,X,Y))$, the quotient field of the polynomial ring over k in the right number of variables, reduced modulo the generic minimum polynomial of $\mathfrak{V}$.

(ii) $K = k^0(\mathfrak{V})$, the field of rational functions defined over k of the variety of singular elements in the Jordan algebra $J(e)$ associated with any maximal idempotent e of $\mathfrak{V}$.

(iii) $K = k^1(\mathfrak{V})$, the field of rational functions defined over k of the variety of rank one elements in $J(e)$.

Also, $\mathfrak{V}$ is reduced iff either one of the above fields is purely transcendental over k. Finally, we indicate the construction of a <u>generic splitting field</u>, defined in an obvious manner, for an exceptional simple Jordan algebra. Our results generalize, at least in part, earlier ones due to Witt, Amitsur, Roquette, Heuser, Saltman for associative algebras, to Ferrar for composition algebras and to Knebusch for quadratic forms.

Holger P. Petersson

The Tits Process and Jordan Algebras of Degree 3.

Let $(\Phi, *)$ be a unital commutative associative ring with involution, Φ_0 a unital ample subring of $\mathcal{H}(\Phi, *)$, $(\mathcal{B}, *)$ a unital alternative algebra with involution over $(\Phi, *)$. Suppose that $\mathcal{B}^+ = J(N, \#, 1)$ for a suitably chosen cubic norm form, adjoint and base point and that it remains so under scalar extensions. Assume moreover that $N(vw) = N(v)N(w)$, $N(v^*) = N(v)^*$. Let $\mathfrak{A}$ be a unital ample Jordan Φ_0-subalgebra of $\mathcal{H}(\mathcal{B}, *)$ contained in the nucleus of $\mathcal{B}$ and that (i) $N(\mathfrak{A}) \subset \Phi_0$, (ii) $\mathfrak{A}^\# \subset \mathfrak{A}$. Given an invertible element $u \in \mathfrak{A}$, $\mu \in \Phi^\times$ with $N(u) = \mu\mu^*$, then

(1) $N(x) = N(a) + \mu N(b) + \mu^* N(b^*) - T(a\, b u b^*)$,

(2) $x^\# = (a^\# - b u b^*,\ \mu^* b^{*\#} u^{-1} - ab)$,

(3) $1 = (1, 0)$

define a cubic form with adjoint and base point on $J = \mathfrak{A} \oplus \mathcal{B}$ which induce a Jordan structure on J. We have called this process the Tits process. It contains the two Tits constructions of Jordan algebras of degree 3.

In fact if J is generically algebraic of degree 3 over a field F and $F1 \subsetneq J$ then J is a Tits process algebra unless $J = E^+$, E a separable cubic field extension of char 3 or a purely inseparable field extension of exponent 1 and infinite dimension over F of char 3. Further results concerning this process were indicated

Michel L. Racine

The ideal structure of enveloping algebras

Let k be an algebraically closed field of characteristic $p>0$, $\mathcal{L}$ a finite-dimensional restricted Lie algebra over k, $\mathcal{U}=(U(\mathcal{L}), j)$ the universal enveloping algebra of $\mathcal{L}$ and $\mathcal{O}_1$ the subalgebra of $\mathcal{U}$ generated by $\{ j(x)^p - j(x^{[p]}) \mid x \in \mathcal{L} \} \cup \{ j(z), z \in \mathrm{Cent}(\mathcal{L})\}$. Besides other things we prove the following:

1) The contraction map $\pi: \mathrm{Spec}\,\mathcal{U} \to \mathrm{Spec}\,\mathcal{O}_1$ is open and closed.
2) If $\mathcal{L}$ is nilpotent, then π is a homeomorphism.
3) If $\mathcal{L}$ is nilpotent, the following are equivalent
   a) There exists a maximal ideal m of $\mathcal{O}_1$ s.t. $\mathcal{U}/m\mathcal{U}$ is semisimple
   b) $\mathrm{Center}(\mathcal{U}) = \mathcal{O}_1$
   c) There exists $l \in \mathcal{L}^*$ s.t. the attached skew-symmetric bilinear form $B_l: \mathcal{L}/\mathrm{Centre}(\mathcal{L}) \times \mathcal{L}/\mathrm{Centre}(\mathcal{L}) \to k$, $B_l(\bar{x},\bar{y}) := l(xy)$ is nondegenerate.

Helmut Strade

Homomorphisms of projective planes

Klingenberg, in 1956, showed that all homomorphisms of Desarguesian projective planes are induced by places between their coordinate rings. We show (jointly with J. R. Faulkner) that the same result holds for Moufang (little Desarguesian) planes. Partial classification of places from octonion algebras allows us to conclude that, up to conjugation by collineations, there is at most one surjective homomorphism from an octonion plane to a Moufang plane. We also establish the existence of proper homomorphisms between octonion planes and of homomorphisms from octonion planes to Desarguesian planes.

Joseph C. Ferrar

Noncommutative Jordan-Banach Algebras.

Let N be a NJC^*-algebra in the terminology of Alvermann. Then we have the important inequality

$$0 \leq x^* x \quad \text{for all } x \text{ in } N.$$

This follows from the simple observation that N has a faithful family of type I factor representations (here we use that N'' is a NJW^*-algebra) and the theorem of Alvermann for NJW^*-factors of type I.

To handle the real case we must get information about the bidual. Call a real or complex Banach algebra $(N, \circ)$ admissible if $(J, \cdot) := (N, \circ)^{(1/2)}$ is a JB- or a JC^*-algebra. Theorem: If N is admissible and if N'' is equipped with the Arens product, then N'' is admissible and $(N'')^{(1/2)} = J''$. The product $x \circ y$ is separate $\sigma(N'', N')$-continuous and $(x, y) \mapsto x \circ y$ is jointly s^*-continuous on bounded subsets of $N'' \times N''$. Every identity for N is an identity for N'', especially, if N is a NJB- or a NJC^*-algebra, then the same is true for N''.

This again gives a faithful family of type I factor representations for a NJB-algebra. Combined with Alvermanns result for NJW-factors of type I we obtain: Every NJB-algebra is commutative, hence, a JB-algebra.

S. Jhm

Algebras with given symmetries.

Example: The space of O-invariant algebras on $\mathbb{R}^3$, O the octaeder-group ($\leq SO(3, \mathbb{R})$), has dimension 1.

Theorem 1: In any Aut J-invariant commutative algebra on J (J a simple formally real or simple complex f.d. Jordan algebra with reduced trace λ) the square of an element $x \in J$ is given by $\alpha \lambda(x^2) e + \beta \lambda(x)^2 e + \gamma \lambda(x) x + \delta x^2$, where $\alpha, \beta, \gamma, \delta$ are constants.

Theorem 2: For any subgroup $G \leq O(n, \mathbb{R})$ acting irreducibly on $\mathbb{R}^n$ and for which the space of G-invariant algebras on $\mathbb{R}^n$ has dimension 1 (let it be $\mathbb{R}L$) $(x,y) \mapsto t + L(x)L(y)$ is associative and ε-definite if $L = \varepsilon R$ ($\varepsilon = \pm$, R right-multiplication for L). L is direct sum of isomorphic simple ideals. If G has no normal subgroup $\neq G$ of finite index then L is central simple. The irreducible representations of $SO(3,\mathbb{R})$ give interesting examples for Theorem 2.

Ulrich Hirzebruch

Central separable Jordan algebras over the integers

A unital quadratic Jordan algebra J over the integers is defined to be central separable if the following holds:

(1) J is a finitely generated free $\mathbb{Z}$-module

(2) J/pJ is central simple over $\mathbb{Z}/p\mathbb{Z}$ for all primes.

Then we get that J is a maximal order in $J_{\mathbb{Q}} = J \otimes_{\mathbb{Z}} \mathbb{Q}$ and $J_{\mathbb{Q}}$ is central simple over $\mathbb{Q}$. If $J_{\mathbb{Q}}$ is of generic degree 2 we have $J = \mathrm{Jord}(q,1)$, the Jordan algebra of a non-degenerate quadratic form q with base point and $\dim J = 2m \Rightarrow |\det q| = 1$, J selfdual, $\dim J = 2m+1 \Rightarrow |\det q| = 2$.

If $J_{\mathbb{Q}}$ is of generic degree ≥ 3 and $J_{\mathbb{Q}} = A^+$, A central simple associative, then we get that $J \subset \mathrm{Mat}_m(\mathbb{Q})^+$ selfdual and $J \sim \mathrm{Mat}_m(\mathbb{Z})^+$. To have a look at the case $J_{\mathbb{Q}} = H(A,*)$, we proof for $(A,*)$ ass. with involution $*$ over a field k, char $k \neq 2$, and A generated by the symmetric elements the following: $H(A,*)$ simple $\Rightarrow$ $(A,*)$ simple.

K. Thilo

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Cartan subalgebras in Lie algebras of characteristic p

Let F be an algebraically closed field of characteristic $p > 7$ with P the prime field of F. Assume that L is a simple finite-dimensional Lie algebra over F, and that K is a Cartan subalgebra of L with $L = K + L_\alpha + \ldots + L_\beta$ the corresponding root space decomposition. The general classification problem for such algebras is still open, however some major results have been obtained by studying the subalgebras $L^{(\alpha)} = K + \sum_{i=1}^{p-1} L_{i\alpha}$ and how they act on L. If $M^{(\alpha)}$ is a maximal ideal of $L^{(\alpha)}$, then by a result of Wilson, $L^{(\alpha)}/M^{(\alpha)}$ is 1-dimensional, $sl(2)$ or simple of generalized Cartan type $W(1:\underline{n})$ or $H(2:\underline{n}:\Phi)^{(2)}$. Moreover, $K+M^{(\alpha)}/M^{(\alpha)}$ is a Cartan subalgebra of $L^{(\alpha)}/M^{(\alpha)}$ and has toral rank one when $L^{(\alpha)}/M^{(\alpha)}$ is simple (where the toral rank is the P-dimension of the P-vector space spanned by the roots). In this work we examine such Cartan subalgebras of these algebras.

We present a uniform method for constructing certain Cartan subalgebras in Lie algebras of generalized Cartan type using the filtrations on these algebras, which we then use to prove

Theorem. If $L = H(2:\underline{n}:\Phi)^{(2)}$ and if K is a Cartan subalgebra of L of toral rank one, then $\dim K = p^{n_1+n_2-1} - \varepsilon$ where $\underline{n} = (n_1, n_2)$, $\dim L = p^{n_1+n_2} - \varepsilon$, and $\varepsilon = 0, 1$ or 2 depending on Φ.

Since every Cartan subalgebra of $sl(2)$ is 1-dimensional, and since every toral rank one Cartan subalgebra of $W(1:\underline{n})$ has dimension p^{n_1-1} where $\underline{n} = n_1$, we obtain as a consequence

Theorem. If L is an arbitrary finite-dimensional simple Lie algebra of characteristic $p > 7$, and if K is a Cartan subalgebra of dimension $\leq p-3$, then for each root α, $L^{(\alpha)}/M^{(\alpha)}$ is 1-dimensional, $sl(2)$ or $W(1:\underline{1})$ (the Witt algebra).

G. Benkart

Lie Algebras of Rank One and Beyond

We are reporting on joint work with Georgia Benkart. Let L be a simple finite-dimensional Lie algebra over an algebraically closed field Φ of characteristic $p > 3$, and let H be a Cartan subalgebra of L. By the rank of L we mean the dimension of H. We have proved that if L has rank one then it is either $sl(2)$ or an Albert-Zassenhaus algebra. L is called an <u>Albert-Zassenhaus</u> algebra if there exists a finite additive subgroup G of Φ and a basis $\{e_\alpha \mid \alpha \in G\}$ of L such that

$$[e_\alpha, e_\beta] = \{\alpha \Psi(\beta) - \beta \Psi(\alpha) + \beta - \alpha\} e_{\alpha+\beta}$$

for some additive map Ψ of G into Φ.

The proof of this result involves looking at L as a module for the different subalgebras $\Upsilon(\alpha)$ where, for each root α, $\Upsilon(\alpha) = H + \sum_{i=1}^{p-1} L_{i\alpha}$. In case $\Upsilon(\alpha)$ contains elements e, f such that $[e,f] = h$, the algebra $\Upsilon(\alpha)$ fits into a class of algebras which have been classified by Yermolaev.

We have made some progress using the same method toward the objective of classifying Lie algebras of rank two. This time the subalgebras $\Upsilon(\alpha)$ are more complicated and fit into four types. Our results so far in this case concern the question of which types of algebras $\Upsilon(\alpha)$ can occur together in the same simple Lie algebra.

J. Marshall Osborn

A Cayley-Dickson Process for a Class of Nonassociative Algebras with Involution

The results described are joint work with John Faulkner. Let k be a field of characteristic $\neq 2$ or 3. A unital algebra with involution $(\mathcal{A}, {}^-)$ is said to be structurable if $[V_{x,y}, V_{z,w}] = V_{V_{x,y}z,\, w} - V_{z,\, V_{y,x}w}$ for all $x, y, z, w \in \mathcal{A}$,

where $V_{x,y} z = (x\bar{y})z + (z\bar{y})x - (z\bar{x})y$. These algebras are used in constructions of Lie algebras due to Kantor. We consider algebras $(\mathcal{A}, {}^-)$ satisfying

Hypothesis F: $(\mathcal{a}, {}^{-})$ is a finite dimensional central simple structurable algebra such that $\dim_k \mathcal{S}(\mathcal{a}, {}^{-}) = 1$, where $\mathcal{S}(\mathcal{a}, {}^{-}) = \{ s \in \mathcal{a} \mid \bar{s} = -s \}$.

Every algebra satisfying (F) is a form of a 2×2 matrix algebra with coefficients taken from a Jordan algebra. We give necessary and sufficient conditions for an algebra satisfying (F) to be isomorphic or isotopic to a 2×2 matrix algebra. We next describe a Cayley-Dickson process which when applied to a separable Jordan algebra of degree 4 gives an algebra satisfying (F). We finally give necessary and sufficient conditions for the process to give a 2×2 matrix algebra as well as sufficient conditions for the process to give a division algebra.

Bruce N. Allison

The radicals of Freudenthal-Kantor triple systems

For $\varepsilon = \pm 1$, a Freudenthal-Kantor triple system (F-K.t.s.) is a vector space U with a trilinear product $(\; , \; , \;)$ satisfying

$$(U1) \qquad [L(a,b), L(c,d)] = L((abc),d) + \varepsilon L(c,(bad))$$

$$(U2) \qquad K(K(a,b)c,d) - L(d,c)K(a,b) + \varepsilon K(a,b)L(c,d) = 0$$

where $L(a,b)x = (abx)$, $K(a,b)x = (axb) - (bxa)$. When $\varepsilon = -1$, U is the generalized Jordan triple system of second order. When $\varepsilon = +1$ and $K(a,b) = L(b,a) - L(a,b)$, U is the J-ternary algebra. Let A_1 and A_2 be ideals of F-K.t.s. U. We put $A_3 = U$ and define $A_1 * A_2 := \sum_{\pi \in S_3} A_{\pi(1)} A_{\pi(2)} A_{\pi(3)}$. Then we can define a radical of F-K.t.s. U, denote it by $R(U)$. U is called semisimple if $R(U) = 0$. To construct a Lie triple system T from a F-K.t.s. U, put $T = U \oplus U$ and define a triple product in T by

$$\left[\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix}\right] := \begin{pmatrix} L(a_1, b_2) - L(a_2, b_1) & K(a_1, a_2) \\ -\varepsilon K(b_1, b_2) & \varepsilon L(b_2, a_1) - \varepsilon L(b_1, a_2) \end{pmatrix}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix}$$

Then it is shown that T is a Lie triple system with respect to this product, denote it by T_U and $J\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ -\varepsilon a \end{pmatrix}$ is an ε-involution. i.e. $J^2 = -\varepsilon \, Id$. In the case of J-ternary algebra, since Nijenhuis operator $= 0$, it has a complex structure.

Theorem. F-K.t.s U is semisimple if and only if T_U is semisimple and this is so if and only if L_U is semisimple. Where L_U is the standard enveloping Lie algebra of T_U.

Theorem. If F-K.t.s. U is semisimple, then U is a direct sum of simple ideals.

Noriaki Kamiya

On the classification of Lie and Jordan triple systems

a) <u>the grid-approach to the classification of Jordan triple systems</u>: It is shown, that

1) every Jordan triple system V with enough tripotent (e.g. V finite-dimensional semisimple over an algebraically closed field of char $\neq 2$) contains a "grid", i.e. a multiplicatively closed family of tripotents,
2) every grid is the disjoint union of connected grids,
3) there are 7 types of connected grids,
4) the space covered by a connected grid can be coordinatized

b) the connection between Lie and Jordan triple systems: To every Jordan triple V one associates a Lie triple V^- by alternating the Jordan product in the first 2 variables. It is shown, that in case V does not come from a Jordan algebra and is not $\mathrm{Rat}(4,4;K)$ or $\mathrm{Alt}(8;K)$ the Lie triple and Jordan triple k-forms of V resp. V^- coincide. A similiar theorem holds for Jordan algebras. One can apply this theorem to obtain results for the classification of exception Jordan triple systems.

E. Neher
(Münster)

Some Applications of Jordan Norms to Associative Algebras

Let $\mathfrak{a}$ be a central simple associative algebra over a field Φ such that $\{\mathfrak{a}\}^2 = 1$ in $Br(\Phi)$. By a theorem of Albert, $\mathfrak{a}$ is involutorial. We have two types of involutions in $\mathfrak{a}$: orthogonal and symplectic. An involution J defines the Jordan algebra of symmetric elements $H(\mathfrak{a}, J)$ and if char $\Phi = 2$ we also have the Jordan algebra $H(\mathfrak{a}, J)'$, the outer ideal generated by 1 in $H(\mathfrak{a}, J)$. Denote the generic norm in $H(\mathfrak{a}, J)$ ($H(\mathfrak{a}, J)'$) by N_J (N_J')

Theorem 1 N_J (N_J') is anisotropic $\Leftrightarrow$ either $\mathfrak{a}$ is a division algebra or $\mathfrak{a} = M_2(\Phi)$ and J is of symplectic type.

Theorem 2. Let $\mathfrak{a}_i$, $i = 1, 2$, be involutorial central simple, J_i an involution in $\mathfrak{a}_i$ of orthogonal (symplectic) type. Then

(i) If char $\Phi \neq 2$ and $n > 2$ in the symplectic case then $\mathfrak{a}_1 \cong \mathfrak{a}_2 \Leftrightarrow N_{J_1} \sim N_{J_2}$ (equivalence up to a multiplier)

(ii) If char $\Phi = 2$ assume $n > 2$ and J_i is of symplectic type. Then $\mathfrak{a}_1 \cong \mathfrak{a}_2 \Leftrightarrow N_{J_1}' \sim N_{J_2}'$.

We specialize to involutorial central simple algebras of degree 4. By a theorem of Albert such an algebra $\mathfrak{a} = \mathfrak{a}_1 \otimes \mathfrak{a}_2$ where $\mathfrak{a}_i$ is a quaternion algebra. For such a factorization we define an Albert quadratic form Q on $\mathfrak{a}' = \mathfrak{a}_1' \oplus \mathfrak{a}_2'$

where $a_i' = \{a_i \in a_i \mid T(a_i) = 0\}$ by $Q(a_1 + a_2) = N(a_1) - N(a_2)$, $a_i \in a_i'$.

Theorem 3 (i) Any two Albert quadratic forms for an involutorial central simple algebra of degree 4 are similar ($\sim$). (ii) Two such algebras are isomorphic $\Leftrightarrow$ their Albert quadratic forms are similar. (iii) If Q is an Albert quadratic form for A then the (Schur) index of A is 4, 2 or 1 according as the (Witt) index of Q is 0, 1, or 3.

The second part of this paper deals with generic norm fields. Let $(u_1, \ldots, u_m)$ be a base for $H(A, J)$, $\xi_1, \ldots, \xi_m$ indeterminates. Then $\Phi[\xi_1, \ldots]/N_J(x)$, $x = \sum \xi_i u_i$, is a domain and its field of fractions $\Phi(\xi_1, \ldots)$ is called a generic norm field for A. If J is of orthogonal type this field is a generic splitting field for A in the sense of Amitsur. A similar result involving a concept of $\frac{1}{2}$-splitting fields holds if J is of symplectic type

N Jacobson

Toward a classification of Lie triple systems

Let k be a field of characteristic 0, $\bar{k}$ its algebraic closure, G the Galois group of $\bar{k}/k$, $(\mathcal{L}, \sigma)$ the standard imbedding of a simple Lie triple system over $\bar{k}$, and $\mathrm{Aut}(\mathcal{L}, \sigma)$ the group of automorphisms of $\mathcal{L}$ which commute with σ. Then if $\mathcal{L}^-$ is a simple Lie triple system, $\mathrm{Aut}(\mathcal{L}, \sigma)$ is the group of Lie triple systems automorphisms of $\mathcal{L}^-$, hence $H^1(G, \mathrm{Aut}(\mathcal{L}, \sigma))$ corresponds bijectively with the k-forms of $\mathcal{L}^-$. Calling $\mathcal{L}_1^-/k$ scalar isomorphic to $\mathcal{L}_2^-/k$ if they are isomorphic up to a scalar in k^*, we have a new equivalence relation on k-forms of $\mathcal{L}^-$. By computing cohomology sequences of short exact sequences of groups, these equivalence classes are shown to correspond bijectively with $H^1(G, \mathrm{Aut}(\bar{\mathcal{L}}, \sigma))$ where $\bar{\mathcal{L}}$ is the split null extension of $\mathcal{L}^+$ by $\mathcal{L}^-$ if we assume $\mathcal{L}^-$ is an irreducible $\mathcal{L}^+$-module. $H^1(G, \mathrm{Aut}(\bar{\mathcal{L}}, \sigma))$ is then related to $H^1(G, \mathrm{Aut}(\mathcal{L}^+))$, which gives the k-forms of $\mathcal{L}^+$.

Nora Hopkins $_{KSU}$

TOPOLOGIE

15 - 8 - 1982 — 21 - 8 - 1982

Some Analytic Functors and their Taylor Series

I explained what I mean by an analytic functor; it is a homotopy functor from spaces to spaces which also satisfies certain "n-th order Blakers-Massey conditions" (or "approximate excision conditions") for all $n \geq 1$. I stated a theorem to the effect that every such functor has a convergent "Taylor series."

The example which led to this general point of view is Waldhausen's functor A (algebraic K-theory of topological spaces). In this case the "Taylor series" leads to calculations. The identity functor is also a very interesting example in principle, but it is not clear whether this point of view will lead to any new calculations in that example.

For the functor A we have the following result:

<u>Theorem</u> For any pair of spaces (X, Y) there is a natural tower of fibrations

$$\dots \to (\Gamma_d A)(X,Y) \xrightarrow{p_d} (\Gamma_{d-1} A)(X,Y) \to \dots \to (\Gamma_0 A)(X,Y)$$

and a natural map $\quad A(Y) \xrightarrow{\gamma} \varprojlim (\Gamma_d A)(X,Y)$.

If the pair (X,Y) is k-connected, $k \geq 2$, then

(a) γ is a homotopy equivalence, and

(b) p_d is $(d(k-1)+1)$-connected.

Moreover, the layers in the tower are computable:

$(\Gamma_0 A)(X,Y) = A(X)$, and if we denote the fiber of p_d by $(\mathcal{D}_d A)(X,Y)$ then $(\mathcal{D}_d A)(X,-)$ is a certain "homogeneous functor of degree d" from {spaces over X} to spaces. In the simplest case, when X is contractible and Y is the suspension of a pointed space Z, we have:

$$\Omega(\mathcal{D}_d A)(*, SZ) \simeq \Omega^\infty S^\infty\left(\frac{Z^{\wedge d} \wedge E(\mathbb{Z}/d)_+}{\mathbb{Z}/d}\right)$$

(Here $Z^{\wedge d}$ is the d-fold self-smash-product of Z, a pointed space on which $\mathbb{Z}/d$ acts by cyclic permutation. $E(\mathbb{Z}/d)_+$ is a free ~~pointed~~ contractible $(\mathbb{Z}/d)$-space with added base point.)

Thomas G Goodwillie

Geometric operations in complex cobordism.

Some internal and stable operations $U^* \to U^*$ are defined, which lift the Steenrod squares Sq^{2j}. They are described in a geometric way, using singularities of vectorbundle morphisms. The properties of those operations are proved in a geometric way too, and the study of their action on the coefficient ring $U^*(pt)$ allows one to make geometric observations, The easiest consequence is an elementary (i.e. without computations with Chern numbers or Landweber-Novikov operations) proof of Mischenko's theorem (coefficients of the logarithm of the formal group law on $U^*(pt)\otimes\mathbb{Q}$).
The algebraic way to define those operations generalizes in an easy way to construct, for each prime p, operations Θ_p^j:

$$U^*(-) \longrightarrow U^*(-;A)$$

(A a ring such that $\frac{1}{(p-1)!} \in A$), which lift the Steenrod powers P^j, if $A = \mathbb{Z}/p$ or $\mathbb{Z}_{(p)}$: namely the diagram:

$$\begin{array}{ccc} U^*(-) & \xrightarrow{\Theta_p^j} & U^{*+2j(p-1)}(-;A) \\ \downarrow & & \downarrow \\ H^*(-;\mathbb{Z}/p) & \xrightarrow{p^j} & H^{*+2j(p-1)}(-;\mathbb{Z}/p) \end{array}$$

is commutative

Michèle Audin (Orsay).

Bridge presentations of the unknot.

Let K be a knot in S^3. A n-bridge presentation of K is a 2-sphere Σ embedded in S^3, transversal to K, which cuts S^3 in two balls B_1, B_2 such that $K \cap B_i$ is the union of n unknotted arcs in B_i. We show the following.

Th Up to isotopy respecting K, there exist an unique n-bridge presentations of the unknot K.

The proof consits in simplifying the intersection of a n-bridge presentation Σ with a disc D bounded by K.

Jean Pierre Otal (Orsay)

~~Unstable~~

Unstable algebras over the Steenrod algebra.

Let X be a 1-connected space with ΩX a finite complex; ~~eg~~ the classical example is $X = BG$ where G is a compact and connected Lie group.

Problem: What is the algebraic structure of $H^*(X;F_p)$ where F_p is the finite field with p elements, p is a prime number?

The result when p is sufficiently large has been obtained by Adams and Wilkerson (Annals of Maths 111 ~~(1980)~~ (1980) pp. 95 - 143). For small p, D. Rector suggested a plausible extension of the programe of Adams and Wilkerson in which one should seek to generalize a certain result of Quillen (Annals of Maths 94 (1971) pp. 549 - 572, Theorem 7.1).

<u>Theorem</u> Let H^* be an unstable algebra over the mod p Steenrod algebra s.t.

(i) H^* has finite transcendence degree over $\mathbb{F}_p$;

(ii) H^* has a finite number of minimal primes ideals.

Then there exists a finite category $\mathcal{C}$ s.t.

(a) ob $\mathcal{C}$ consists of elementary abelian p-groups;

(b) morphisms in $\mathcal{C}$ are group homomorphisms;

(c) there is an induced algebra homomorphism

$$H^* \longrightarrow \varprojlim_{\mathcal{C}} H^*(BV; \mathbb{F}_p)/\mathrm{Nil}^*$$

which is an inseparable isogeny.

Remark: (i) When H^* is a finitely generated algebra over $\mathbb{F}_p$, the result was first obtained by Rector, and independently by myself. The general result has been obtained by Adams and myself.

(ii) In the application, C.W. Wilkerson has an argument which shows that $H^*(X; \mathbb{F}_p)$ has finite transcendence degree over $\mathbb{F}_p$ (where X is a space as in the first paragraph). We are still unable to verify the other condition on the number of minimal primes in such application. However, we can always apply the theorem to $H^*(X; \mathbb{F}_p)/P^*$ where P^* is an invariant prime ideal of $H^*(X;\mathbb{F}_p)$. The next question is how to glue these partial results together ~~into~~ to get a global result.

Siu-Por Lam (Cambridge)

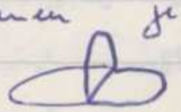
Simple homotopy & group presentations (Andrews-Curtis & Zeeman Conjectures)

Wenn die kompakten Komplexe K^n und L^n denselben einfachen Homotopietyp haben, läßt sich für $n \neq 2$ stets eine Kollabierfolge $K^n \overset{n+1}{\nearrow\searrow} L^n$ angeben, bei der die Dimension $n+1$ nicht überschritten wird (Wall, J.H.C. Whitehead). Die Andrews-Curtis-Vermutung besagt, daß das analoge Resultat auch für $n=2$ gilt. Besonders wichtig ist dabei der Fall $K^2 \simeq *$ (und die Frage $K^2 \overset{3}{\nearrow\searrow} *$). Zwei 2 dimensionale Polyeder lassen sich genau dann durch $\overset{3}{\nearrow\searrow}$ ineinander überführen, wenn für abgelesene Präsentationen von π_1 gilt, daß sie durch eine endliche Folge der Elementartransformationen:
I) freie Transf. + Konj. der Relationen, II) freie Erzeugendenwechsel
III Verlängern um eine Erz. und eine neue Relation, welche die zusätzliche Erzeugende trivialisiert
auseinander hervorgehen (P. Wright).

Die Zeeman Vermutung besagt, daß für $K^2 \simeq 0$: $K^2 \times \{0\}$ kollabiert, und ist ebenfalls noch offen.

Folgende Implikationen sind bekannt: 1) Die Zeeman Vermutung (ZV) impliziert die 3-dimensionale Poincaré-Vermutung (P).
2) (ZV) $\Rightarrow$ AC (= Andrews-Curtis Vermutung für $K^2 \simeq 0$)

Obwohl namentlich die Zeeman Vermutung i. allg. angezweifelt wird, sind in letzter Zeit die beiden Implikationen in gewissem Sinn zu Äquivalenzen ausgebaut worden:

I (ZV') $\Longleftrightarrow$ (P), wobei ZV' bedeutet,
daß man sich auf „standard spines" von Homotopie-3-Bällen beschränkt, das heißt, Kollabierretrakte, bei denen jede Kante höchstens 3 anstoßende 2Zellen hat  und jede Ecke als link das 1 Gerüst eines Tetraeders (Gillman – Rolfsen)

II (ZV'') $\Longleftrightarrow$ (AC), wobei ZV'' bedeutet, daß man, um $K \times I \searrow *$ zu erhalten, den gegebenen 2-Komplex nur um Zellen der Dimension ≤ 2 erweitern darf (Kreher – Metzler).

Eine direkte Beziehung zwischen (AC) und (P) ergibt sich ferner daraus, daß ein in eine 3-Mannigfaltigkeit einbettbarer 2-Komplex $K^2 \simeq 0$ mit $K^2 \overset{3}{\not\searrow} *$ ein Gegenbeispiel für (P) ergäbe.

In dem Vortrag wurde dann über einige Strategien zur Klärung der Andrews-Curtis-Vermutung berichtet:

(i) mittels Identitäten von Präsentationen (ergibt Charakterisierung)

(ii) mittels Gleichungen in Automorphismengruppen freier Gruppen

(iii) mittels der Untersuchung 3-dimensionaler Aufdickungen von fake surfaces (= zweidimensionale Polyeder, die die oben angegebenen lokalen Bedingungen erfüllen), welche 3-Mannigfaltigkeiten mit Singularitäten sind (d.h.: endlich viele Punkte dürfen die reelle projektive Ebene als link haben) (Frank Quinn)

(iv) eine geometrische Strategie, um (ohne algebraische Topologie, welche üblicherweise die Einschränkung $n \neq 2$ ergibt) eine Transformation $K^n \overset{N}{\searrow\!\nearrow} L^n$ von $N = n+2$ auf $N = n+1$ zu reduzieren.

Diese Strategie hat unter einer gewissen Zusatzannahme bereits eine positive Entscheidung (Metzler) $K^n \overset{n+1}{\nearrow\!\searrow} L^n$ ergeben (für alle n). Der allgemeine Fall ist jedoch weiterhin offen. Sollten sich dabei Invarianten ergeben, wären diese jedoch gegenüber simplizialen Unterteilungen „unempfindlich“, wogegen die algebraischen Ansätze, für (i): kombinatorische Gruppentheorie mit Operatoren (Dunk-Metzler), (ii): ganzzahlige Darstellungen von $\mathrm{Aut}\,F_n$ mit „kleinem“ Kern (Lustig-Metzler) zum Schluß die Stabilisierung durch III erfordern.

Ferner wurde auf Querverbindungen zum Whiteheadschen Asphärizitätsproblem hingewiesen, sowie auf die Frage, welche Elemente von $Wh(\pi)$ sich durch zweidimensionale Erweiterungen eines CW-Komplexes realisieren lassen.

W. Metzler

Higher order Whitehead torsion and automorphisms of manifolds (D. Burghelea O.S.U.-Columbus U.S.A.)

Let A denote one of the geometric categories Diff or Top and $A(M^n)$ ($\tilde{A}(M^n)$) the group of diffeomorphisms resp homeomorphisms (block diffeomorphisms resp block homeomorphisms) of M which restrict to the identity on ∂M. ~~and~~ Let $H(X)$ be the topological monoid of selfhomotopy equivalences of X.

The concordance theory permits to define two homotopy functors Wh^{Diff} and Wh^{Top} and a natural transformation $Wh^{Diff} \rightsquigarrow Wh^{Top}$. It should be noticed that $\pi_1(Wh^A(X)) = Wh_1(\pi_1(X))$.

<u>Theorem</u>: For X a finite complex there exists a homotopy commutative diagram

$$\begin{tikzcd} \Omega Wh^{Diff}(X) \arrow[r] \arrow[d] & B\mathcal{F}^{Diff}(X) \arrow[r] \arrow[d] & BH(X) \arrow[d, equal] & (*) \\ \Omega Wh^{Top}(X) \arrow[r] & B\mathcal{F}^{Top}(X) \arrow[r] & BH(X) & (**) \end{tikzcd}$$

whose horizontal lines are fibrations and the holonomy

$$\begin{array}{ccc} & & \Omega Wh^{Diff}(X) \\ \mathcal{H}(X) & \nearrow & \downarrow \\ & \searrow & \Omega Wh^{Top}(X) \end{array}$$

induces for π_0 the usual Whitehead torsion (The fibration (**) was first defined by Hatcher and (*) is implicit in the work of Burghelea Lashof)

The space $B\mathcal{H}(X)$ classifies the Hurewicz fibration with fibres homotopy equivalent to X and $B\mathcal{F}^A(X)$ classifies the "equivalence classes" of A-bundles with fibres manifolds simple homotopy equivalent to X via the following equivalence relation: $E_1 \to B$ and $E_2 \to B$ are equivalent iff there exist the disc (A-) bundles $\tilde{E}_1 \to E_1$ and $\tilde{E}_2 \to E_2$ so that $\tilde{E}_1 \to B$ and $\tilde{E}_2 \to B$ are isomorphic

The geometric and homotopic interpretation of the holonomy in these fibrations (called also higher order Whitehead torsion) has the following consequences.

1) If $\varphi(n)$ denotes the stability range for concordances

$$\left(\frac{\mathcal{H}(M^n;\partial M^n)}{A(M^n)}\right)_{odd} \overset{\varphi(n)}{\sim} \left(\frac{\mathcal{H}(M^n;\partial M^n)}{\hat{A}(M^n)}\right)_{odd} \times \left(\frac{\hat{A}(M)}{A(M)}\right)_{odd}$$

2) If for a given K ($K \leq \varphi(n)$) there exists a $(K+1)$ connected manifold M with $\chi(M) \equiv 1 \bmod 2$ then

$$\left(\frac{\tilde{A}(M)}{A(M)}\right)_{[K]} \quad \text{and} \quad \left(\frac{\mathcal{H}(M^n;\partial M^n)}{A(M^n)}\right)_{[K]} \quad \text{depends}$$

only on the homotopy type of M and of the stable spherical class of ~~the~~ the tangent bundle $\tau(M^n)$ and $(X)_{[k]}$ the k-Postnikov term of X.

3) The Hatcher Waldhausen map

$$G/O \longrightarrow \Omega Wh^{Diff}(*) \text{ is a rational}$$

equivalence ~~of~~

$$SO_{(n+1)} \longrightarrow \frac{Diff(S^n \times D^N)}{Maps(S^n; SO_N)} \quad \text{for } N \gg n$$

~~[illegible]~~ is injective for rational homotopy groups in

dimension $<$ ~~[illegible]~~ $n-2$

Dan Burghelea

On Second Homology of Groups of Diffeomorphisms of $\mathbb{R}^n$

Let $Diff_c^r(\mathbb{R}^n)$ be the group of diffeomorphisms of $\mathbb{R}^n$ with compact support with C^r-topology. Let $Diff_c^r(\mathbb{R}^n)_\delta$ be the same group but with the discrete topology. The identity $Diff_c^r(\mathbb{R}^n)_\delta \longrightarrow Diff_c^r(\mathbb{R}^n)$ induces a map between corresponding classifying spaces

$$B\,Diff_c^r(\mathbb{R}^n)_\delta \longrightarrow B\,Diff_c^r(\mathbb{R}^n).$$

Let $B\overline{Diff}_c^r(\mathbb{R}^n)$ denote the homotopy theoretic fiber of this map. Then we have

Theorem. $H_2(B\overline{Diff}_c^r(\mathbb{R}^n); Z) = 0$ if $2r \leq n$ or $r = n = 1$.

By results of Mather and Thurston, this implies that

$B\bar{\Gamma}_n^r$ is $(n+2)$-connected, where $B\bar{\Gamma}_n^r$ is the classifying space for Haefliger's Γ_n^r-structures with trivial normal bundle.

Takashi Tsuboi (Tokyo / Genève)

On Homotopy spheres bounding highly connected manifolds

General problem: given a $(k-1)$-connected m-dimensional manifold M with ∂M a homotopy sphere, determine ∂M in the group of homotopyspheres (for example in terms of the invariants of Wall in the cases $m=2k, 2k+1$).

A partial answer is the following result: Suppose $k \geq 113$, $m = 2k, \dots, 2k+5$, and exclude the cases $m = 2k+1$, $k \equiv 1 \bmod 8$ and $m = 2k+3$ $k \equiv 0,4 \bmod 8$. Suppose moreover that the Pontrjagin numbers of M vanish. Then ∂M lies in bP_m, the group of homotopy spheres bounding stable parallelisable manifolds, and is explicitly given by

$$\partial M = \begin{cases} \frac{1}{8}(\operatorname{sign} M - \langle L(M,\alpha), [M,\partial M]\rangle)\Sigma \in bP_m & \text{if } m \equiv 0(4) \\ K(M)\cdot \Sigma \in bP_m & \text{if } m \equiv 2(4) \end{cases}$$

Here Σ is the generator of bP_m, $L(M,\alpha)$ the L-polynomial, α a framing of ∂M with vanishing e-invariant, and $K(M)$ the Kervaire-invariant of M (as a Wu-manifold).

The proof uses surgery + Pontrjagin–Thom-construction to reduce the question to stable homotopy theory. Then Adams-Spectral-Sequence methods + Mahowald 1/5 vanishing line give the result. Stephan Stolz (Mainz)

Simply connected 4-manifolds with boundary

Let M be a simply connected 4-manifold with connected boundary $\partial M = V$. Then $H_2(M)$ is torsion free and M is a Moore space of type $M(H_2(M), 2)$. Moreover the restriction map $H^2(M) \to H^2(V)$ is surjective.

Now suppose that L is a free $\mathbb{Z}$-module and that f is an epimorphism from L to $H_1(V)$. We will classify up to homeomorphism the manifolds M such that $H^2(M) \simeq L$ and the composite map $H^2(M) \to H^2(V) \xrightarrow{\sim} H_1(V)$ is f.

Let X be the Moore space of type $M(L^*, 2)$. Denote by $\mathcal{C}(V, X)_f$ the set of maps from V to X inducing f on H^2, and by $[V, X]_f$ the set of homotopy classes of maps in $\mathcal{C}(V,X)_f$.

Denote by $T_1(V)$ the torsion of $H_1(V)$, by λ the linking form from $T_1(V) \otimes T_1(V)$ to $\mathbb{Q}/\mathbb{Z}$ and by μ_σ the quadratic linking form from $T_1(V)$ to $\mathbb{Q}/2\mathbb{Z}$ associated to any spin structure $\sigma \in \operatorname{Spin} V$.

Denote by K the kernel of f and by K' the preimage $f^{-1}(T_1(V))$.

If φ lies in $\mathcal{C}(V, X)_f$ we may consider φ as an inclusion and there exists a unique element $[X]_\varphi$ in $H_4(X, V)$ with boundary $[V]$. If x and y are elements in K we lift y in $y' \in H^2(X, V)$ and we set: $b(x,y) = \langle x \smile y', [X]_\varphi \rangle$.

So we get a symmetric bilinear form $b : K \otimes K \to \mathbb{Z}$ that we extend to a rational form b over K'.

On the other hand if σ is a spin structure on V the inclusion $\varphi : V \hookrightarrow X$ induces an element in $\Omega_3^{\mathrm{spin}}(X) \simeq H_2(X, \mathbb{Z}/2) \simeq \mathrm{Hom}(L, \mathbb{Z}/2)$.

So we get a map $\gamma_\sigma : L \to \mathbb{Z}/2$

Proposition

We have the following properties:

(P_1) $\forall x, y \in K'$ $\quad b(x,y) \equiv -\lambda(f(x), f(y))$ in $\mathbb{Q}/\mathbb{Z}$

(P_2) $\forall x \in K'$ $\forall \sigma \in \operatorname{Spin} V$ $\quad b(x,x) \equiv -\mu_\sigma(f(x)) + \gamma_\sigma(x)$ in $\mathbb{Q}/2\mathbb{Z}$

(P_3) $\forall x \in L$ $\forall \sigma \in \operatorname{Spin} V$ $\forall u \in H^1(V, \mathbb{Z}/2)$ $\quad \gamma_{\sigma+u}(x) = \gamma_\sigma(x) + \langle u, x \rangle$ in $\mathbb{Q}/2\mathbb{Z}$

Theorem

Let $\mathcal{A}(L,f)$ be the set of (b,γ) satisfying (P_1) (P_2) and (P_3). Then the above correspondance gives a bijection between $[V,X]_f$ and $\mathcal{A}(L,f)$

Definition

$(b,\gamma) \in \mathcal{A}(L,f)$ is said to be non singular if b induces a non singular pairing from $K \otimes K'$ to $\mathbb{Z}$.

Theorem

The correspondance between $[V,X]_f$ and $\mathcal{A}(L,f)$ induces a 1-1 correspondance between the set of elements in $[V,X]_f$ such that the pair (X,V) is Poincaré, to the set of non singular couples in $\mathcal{A}(L,f)$

Theorem

Let (b,γ) be a non singular couple in $\mathcal{A}(L,f)$ and k be an element in $\mathbb{Z}/2$. Suppose ~~that~~, in the case where b is even on K, that we have:

$$\operatorname{sgn} b \equiv \rho(V,\sigma) + 8k \mod 16$$

where σ is the unique spin structure on V such that $\gamma_\sigma = 0$, and ρ is the Rohlin's invariant.

Then there exists a top. manifold M with boundary V, unique up to homeomorphism such that:

- $\pi_1(M) = 0$
- $H^2(M) = L$ and the composite map $H^2(M) \to H^2(V) \simeq V$ is f
- the inclusion $V \subset M$ gives the couple (b,γ) in $\mathcal{A}(L,f)$
- the Kirby-Siebenmann invariant of M is k.

Pierre VOGEL (NANTES)

Algebraic K-theory of forms and formations in exact categories.

Quillen has developed a very effective higher algebraic K-theory for exact categories. The object of this lecture is to describe a higher unitary algebraic K-theory for exact categories (+ extra structure) corresponding to and extending in a natural way Quillen's algebraic K-theory. The basic approach taken here is to express the Hermitian K-theory in terms of the fixed point theory of a certain involution on the Quillen theory. In addition to the Q-construction and Waldhausen's multi-Q-construction generalization, a good looping of the Q-construction for arbitrary exact categories is needed (the $S^{-1}S$-construction, which works only for "split exact categories", does not at all suffice). This is provided by the following K-construction, which associates to an exact category $\underline{\underline{M}}$ a new category $K\underline{\underline{M}}$, whose objects are diagrams $(K \rightarrowtail M \twoheadleftarrow L)$ in $\underline{\underline{M}}$, and in which a morphism $(K' \rightarrowtail M' \twoheadleftarrow L') \longrightarrow (K \rightarrowtail M \twoheadleftarrow L)$ is an isomorphism class of commutative diagrams of the form shown at the right with upper squares bicartesian in $\underline{\underline{M}}$. There is an evident rule of composition so that taking the middle column of the diagram defines a functor $K\underline{\underline{M}} \longrightarrow Q\underline{\underline{M}}$. Now $K\underline{\underline{M}}$

$$\begin{tikzcd}
K \arrow[r, tail] & M & L \arrow[l, two heads] \\
K' \arrow[u, tail] \arrow[r, tail] \arrow[ur, phantom, "\square"] & M_0 \arrow[u, tail] \arrow[d, two heads] \arrow[ur, phantom, "\square"] & L' \arrow[l, two heads] \arrow[u, tail] \\
K' \arrow[u, equal] \arrow[r, tail] & M' & L' \arrow[l, two heads] \arrow[u, equal]
\end{tikzcd}$$

may be regarded as the fiber over $0 \in Q\underline{\underline{M}}$ of a fibered category $E\underline{\underline{M}} \xrightarrow[p]{} Q\underline{\underline{M}}$ over $Q\underline{\underline{M}}$, where $p^{-1}(V)$ is the category with objects diagrams $(K \rightarrowtail M \twoheadleftarrow L \twoheadrightarrow V)$ and morphisms much the same as for $K\underline{\underline{M}}$, except the L's all map consistently onto V. There is an evident base change for morphisms $V' \longrightarrow V$ in $Q\underline{\underline{M}}$, taking $p^{-1}V \longrightarrow p^{-1}V'$, and $K\underline{\underline{M}} = p^{-1}0$.

THEOREM 1. ⓐ $E\underline{\underline{M}}$ is contractible, and ⓑ base change functors are homotopy equivalences, so ⓒ $K\underline{\underline{M}} \to E\underline{\underline{M}} \to Q\underline{\underline{M}}$ is a homotopy fibration and $K\underline{\underline{M}} \simeq \Omega Q\underline{\underline{M}}$.

Now, a unitary structure (D, δ) on the exact category $\underline{\underline{M}}$ consists of an exact contravariant functor $D: \underline{\underline{M}} \longrightarrow \underline{\underline{M}}$ and a natural isomorphism $\delta: 1_{\underline{\underline{M}}} \xrightarrow{\cong} D^2$ such that $(D\delta)\cdot(\delta D) = 1_D$. One usually has D as a "dualizing functor" and δ as a "canonical isomorphism with the double dual." Let $Q_D\underline{\underline{M}}$ be the category whose objects are ~~pairs~~ triples $(M, N; h: M \xrightarrow{\cong} DN)$, and for which a morphism $(M', N'; h') \longrightarrow (M, N; h)$ is a pair of morphisms $(M' \twoheadleftarrow M_0 \rightarrowtail M,\ N' \twoheadleftarrow N_0 \rightarrowtail N)$ in $Q\underline{\underline{M}} \times Q\underline{\underline{M}}$ such that the diagram at the right is bicartesian. Composition is defined so that there is an evident functor $Q_D\underline{\underline{M}} \longrightarrow Q\underline{\underline{M}} \times Q\underline{\underline{M}}$. It is easy to show, using $\delta: 1_{\underline{\underline{M}}} \cong D^2$, that following this functor with

$$\begin{tikzcd}
 & M \arrow[r, "h", "\cong"'] & DN \arrow[dr, two heads] & \\
M_0 \arrow[ur, tail] \arrow[dr, two heads] & & & DN_0 \\
 & M' \arrow[r, "h'", "\cong"'] & DN' \arrow[ur, tail] &
\end{tikzcd}$$

either projection to $Q\underline{\underline{M}}$ provides an <u>equivalence of categories</u> $Q_D\underline{\underline{M}} \cong Q\underline{\underline{M}}$. There is an involution $T = T_\delta$ on $Q_D\underline{\underline{M}}$ defined on objects by $T(M,N;h) = (N,M;th)$ where th is the composition $N \xrightarrow[\delta N]{} D^2N \xrightarrow[Dh]{} DM$ and on morphisms by swapping components. (That $T^2 =$ identity requires $(D\delta)(\delta D) = 1_D$). The fixed point subcategory $\underline{\underline{WH}}(\underline{\underline{M}},D,\delta)$ of T on $Q_D\underline{\underline{M}}$ is called the <u>Witt category</u> of non-singular Hermitian forms over $(\underline{\underline{M}},D,\delta)$, for $\pi_0\underline{\underline{WH}}(\underline{\underline{M}},D,\delta)$ is easily seen to be the Witt group of such forms. (After all, a fixed object looks like $(M,M;h=th)$.) In a similar way two categories with involution, $K'_D\underline{\underline{M}}$, $K''_D\underline{\underline{M}}$ both equivalent to $K\underline{\underline{M}}$ are defined. For $K'_D\underline{\underline{M}}$, objects are

$$\left(K \rightarrowtail M \twoheadleftarrow L,\ K' \rightarrowtail M' \twoheadleftarrow L';\quad \begin{array}{ccccc} K & \rightarrowtail & M & \twoheadleftarrow & L \\ \wr\wr & & h\downarrow\wr\wr & & \wr\wr \\ D(M'/K') & \rightarrowtail & DM & \twoheadleftarrow & D(M'/L') \end{array}\right),$$

and for $K''_D\underline{\underline{M}}$, they are

$$\left(K \rightarrowtail M \twoheadleftarrow L,\ K' \rightarrowtail M' \twoheadleftarrow L';\quad \begin{array}{ccccc} K & \rightarrowtail & M & \twoheadleftarrow & L \\ \wr\wr & & h\downarrow\wr\wr & & \wr\wr \\ D(M'/L') & \rightarrowtail & DM' & \twoheadleftarrow & D(M'/K') \end{array}\right),$$

(and morphisms)

The involution T swaps the objects of $K\underline{\underline{M}}$, and is th in the middle. The fixed point subcategory of T on $K'_D\underline{\underline{M}}$ is the <u>non-singular Hermitian formation category</u> $\underline{\underline{UH}}(\underline{\underline{M}},D,\delta)$ (its objects are $M, h=th: M \rightrightarrows DM$, with two "Lagrangians" $K=K^\perp, L=L^\perp$), and the fixed point subcategory of T on $K''_D\underline{\underline{M}}$ is the <u>Hermitian K-construction</u> on $(\underline{\underline{M}},D,\delta)$, called $\underline{\underline{KH}}(\underline{\underline{M}},D,\delta)$. (In split exact cases, it is $\simeq$ to definition via $+$-construction or $S^{-1}S$-construction).

THEOREM 2. There is an exact fibration sequence (up to homotopy)

$$\underline{\underline{UH}}(\underline{\underline{M}},D,\delta) \to K\underline{\underline{M}} \to \underline{\underline{KH}}(\underline{\underline{M}},D,\delta) \to \underline{\underline{WH}}(\underline{\underline{M}},D,\delta) \to Q\underline{\underline{M}}.$$

In particular,

COROLLARY 3. $\underline{\underline{UH}}(\underline{\underline{M}},D,\delta) \simeq \Omega\, \underline{\underline{WH}}(\underline{\underline{M}},D,\delta)$.

Similar games may be played with multi-Q-constructions.

— Charles H. Giffen (Charlottesville).

Some James numbers

(joint work with M.C. Crabb)

Let $W_{n,k}$ be the complex Stiefel manifold of k-frames in $\mathbb{C}^n$ and $p: W_{n,k} \longrightarrow S^{2n-1}$ be the canonical projection map. James defined the number $\bar{U}(n,k)$ (= unstable James number) to be the index of $p_* \pi_{2n-1}(W_{n,k})$ in $\pi_{2n-1}(S^{2n-1}) = \mathbb{Z}$. Stable James numbers $U(n,k)$ are defined ~~as~~ to be the index of $\pi^S_{2n-2}(P_{n-1}\mathbb{C}/P_{n-k-1}\mathbb{C})$ in $H_{2n-2}(P_{n-1}\mathbb{C}/P_{n-k-1}\mathbb{C})$ under the Hurewicz map. By approximating $W_{n,k}$ as a suspension of stunted complex projective spaces $P_{n-1}\mathbb{C}/P_{n-k-1}\mathbb{C}$, one sees $U(n,k) = \bar{U}(n,k)$ for $n \geq 2k-1$. James numbers appear in a number of other problems, e.g. in computing homotopy groups of the unitary group $U(m)$, computing e-invariants of Cartan obstruction classes, determining the image of the generalized J-homomorphism $J: \pi_i^S(W_{n,k}) \to \pi_{i-2n+2k}(S^0)$, in vector field problems and immersion problems. For fixed k, the stable homotopy typ of stunded projective spaces is periodic in n, so one can define $U(n,k)$ for all $n \in \mathbb{Z}$. For fixed k, there is a number M_k such that $U(n,k) = U(n+tM_k, k)$, that is the numbers $U(n,k)$ are periodic in n. For $U(n,k)$ one has lower bounds given by K-Theory. Our main result is, that in the cases $n \geq -1$, and $k \geq n-2$ these lower bounds give the correct value. Another result is, that if one knows for a given pair of n and k that the K-Theory lower bound is the correct value for $U(n,k)$, one can conclude this to be true for a lot of other values of n (same k).

K. Knapp (Wuppertal)

Realising exotic classes in $H^*(BG)$ by Poincaré spaces

According to results of Browder, Liulevicius and Peterson the cohomology ring $H^*(BG)$ of the classifying space for spherical fibrations splits into two sub-Hopf-algebras namely $H^*(BG) \cong H^*(BO) \otimes C$, where BO is the classifying space for spherical bundles and C is some exterior algebra. The elements of positive degree in C are called exotic classes. We prove: (Coefficients $\mathbb{Z}_2$ understood everywhere!)

Theorem: Let $C_0 \subset C$ be an (as an algebra) finitely generated subalgebra of C. Then there exists an orientable Poincaré space P such that $\nu_p^*: H^*(BG) \to H^*(P)$ maps C_0 monomorphically where $\nu_p: P \to BG$ is the classifying map for the spherical normal fibration of P.

By construction P is the total space of some spherical fibration over a differentiable manifold.

Cornelia Wissemann-Hartmann (Bochum)

Periodic flows on open 3-manifolds

Can $\mathbb{R}^3$ be foliated by circles? This question was asked by D.B.A. Epstein in 1976. With regard to this question the following 2 results were presented:

(1) Let N be a 3-manifold with $\tilde{H}_*(N;\mathbb{Z}) = 0$, and let $\mathcal{F}$ be a periodic flow on a 3-manifold $M \subset N$. If $\tilde{H}_0(M) = H_2(M) = 0$, then $H_1(M) \cong \mathbb{Z}$, provided that the length of the Epstein hierarchy of $\mathcal{F}$ is at most 1.

(2) For any ordinal $\alpha \leq \omega$ there is a connected 3-manifold M_α and a periodic flow $\mathcal{F}_\alpha$ on M_α such that the length of the Epstein hierarchy of $\mathcal{F}_\alpha$ is α.

(Remark: For the examples (M_α) constructed to prove (2), one has rank $H_1(M_\alpha;\mathbb{Z}) = \alpha$)

Elmar Vogt (Berlin)

How to define matrix rings for A_∞-ring spaces

Let $End^*(TOP_*)$ be the topological monoidal category of functors $F: TOP_* \circlearrowleft$ with a unit $Id \to F$ and with composition as tensorproduct. $\mathcal{E}_F^\circ(n) = \underline{Hom}(\underbrace{F \circ \cdots \circ F}_{n}, F)$ defines a non-Σ operad. This enables us to define actions by operads on functors. Let $\mathcal{G}$ be a fixed A_∞ non Σ operad (i.e. $\mathcal{G}(n) \sim pt$ for all n). Thm The passage from spectra to functors given by $\{E_W, \ldots\} \longmapsto \operatorname{colim}_W \Omega^W(E_W \wedge -)$ takes $\mathcal{G}$-ring spectra to $\mathcal{G}$-functors. The evaluation map which takes a functor F to $F(S^0)$ takes $\mathcal{G}$-functors to $\mathcal{G}$-spaces ($\mathcal{G}$-ring spaces if F is a reduced homology theory). — We define the $(n \times n)$-matrix functor of a functor F as $M_n(F)(-) := F(\underline{n} \wedge -)^{\underline{n}}$ where $\underline{n} = \{1, 2, \ldots, n\}_+$. Lemma The natural transformations $F \to M_n(F)$ and $F \to F^S = \operatorname{colim} \Omega^n F(S^n \wedge -)$ are maps of $\mathcal{G}$-functors if F is a $\mathcal{G}$-functor. We define a ring up to homotopy as a $\mathcal{G}$-functor which is a reduced homology theory (i.e. $\pi_*(F(-))$ is a generalized reduced homology theory.) These are the functors which are obtained by passing from $\mathcal{G}$-ring spaces to $\mathcal{G}$-ring spectra to $\mathcal{G}$-functors, up to natural w.e.'s they correspond to grouplike A_∞ ring spaces. By considering monads (i.e. $\mathcal{G}(n) = pt$ for all n) we motivate why $M_n(F)(S^0)$ is a good model for the matrix ring. The main conclusion is that it's multiplication is given by an action by the original operad.

T.E.W. Gunnarsson (Göteborg)

Limits of Thom Spectra

Let $D_2(Y) = (S^\infty)^+ \wedge_{\mathbb{Z}/2} (Y \wedge Y)$ be the quadratic construction on the pointed CW-complex Y. In a recent paper J. Jones and S. Wegmann constructed maps $S^{-k} D_2(S^k Y) \to S^{-k-1} D_2(S^{k+1} Y)$, and showed that the inverse limit of homotopy groups in this directed system is naturally isomorphic to the completed homotopy groups of $\Sigma^{-1} X$.

In this talk a conceptual proof of a generalization of these results was presented. Let G be a finite group, and $V = \mathbb{R}G/\mathbb{R}$ the reduced regular representation of G. Define, for a finite pointed G-CW-complex X, the spectrum $\tilde{P}_k(X)$ as the Thom spectrum of bundle over $EG \times_G X$ corresponding to the representation $k \cdot V$. We put $P_k(X) = \tilde{P}_k(X)/\tilde{P}_k(x_0)$ where $x_0 \in X$ is the base point. Then we form an inverse limit as $k \to \infty$, among several possibilities we consider $\mathcal{L}(X) = \varprojlim_{k\to\infty} \pi_* P_{-k}(X)$.

<u>Prop. 1</u>: The inclusion $X^G \hookrightarrow X$ of the fixed point set induces an isomorphism $\mathcal{L}(X^G) \xrightarrow{\cong} \mathcal{L}(X)$.

This proposition relies on the following crucial properties of the functor $\mathcal{L}$: 1) it is G-homotopy invariant, 2) it is exact, i.e. G-cofibrations are sent to long exact sequences, 3) the inclusion $0 \hookrightarrow V \hookrightarrow V^c = V \cup \{\infty\}$ induces an isomorphism $\mathcal{L}(X) = \mathcal{L}(X \times 0) \to \mathcal{L}(X \wedge V^c)$. In fact, the proof of the Atiyah-Segal localization theorem shows: <u>Prop 2</u>: Every functor $\mathcal{L}$ with properties 1), 2), 3) leads to isomorphisms $\mathcal{L}(X^G) \xrightarrow{\cong} \mathcal{L}(X)$.

The result of Jones & Wegmann are obtained by taking $G = \mathbb{Z}/2$ and $X = Y \wedge Y$ with the obvious action.

These are analoguous results for compact Lie groups G and families of representations of G.

Erich Ossa (Wuppertal)

Elliptic structures on 3-dimensional manifolds.

Let Γ be a finite group which acts freely on S^3. Without loss of generality we may assume that the action is smooth, and it has long been conjectured that the inclusion of Γ in $\mathrm{Diff}^+ S^3$ is conjugate to ~~a~~ an inclusion in $SO(4)$. This conjecture is known to be true for groups of order $2^s 3^t$; $s>0$, $t \geq 0$, and in this case it follows that the orbit space S^3/Γ admits an elliptic structure. We outline more ~~pa~~ ~~rec~~ recent progress towards a solution of the problem.

Theorem 1. Let Γ be a finite solvable group, which acts freely on S^3. If the action of Γ, restricted to an arbitrary cyclic subgroup, is linear, then the action of Γ is linear. (This result is a specifically three dimensional result, since it is known to be false in dimensions $2q-1$, $q \geq 3$.)

Complement. If $\Gamma \cong \mathbb{Z}/u \times SL(2,5)$, where $(u,120)=1$, the orbit space S^3/Γ is simple homotopy equivalent to an elliptic manifold, under the cyclic subgroup hypothesis of Theorem 1.

For cyclic~~s~~ groups acting freely on S^3, we have a number of partial results. Perhaps the most interesting of these is:

Theorem 2. Let $\Gamma = \mathbb{Z}/p^t$ act freely on S^3. Then the classifying map $f: B\Gamma \to B\mathrm{Diff}^+ S^3$ is stably equivalent to the classifying map for a lens space.

The proof of Theorem 2 depends on A. Hatcher's proof of the Smale Conjecture together with work by the author & J. F. Adams on maps between classifying spaces. We conclude with the following question: is the Reidemeister torsion of S^3/Γ equal to that of a lens space? A positive answer, combined with Theorem 2 would at least lead to an embedding result for arbitrary smooth actions on S^3 into a linear action on S^7.

C. B. Thomas (Cambridge, England).

Results and problems on the additivity of the regular genus.

An (n+1)-coloured graph is a pair (Γ, γ), where Γ is a multigraph, regular of degree $n+1$, and γ is an edge coloration with $n+1$ colours ($0, 1, \dots, n$ say). To any such (Γ, γ) a pseudocomplex $K(\Gamma)$ is associated. A crystallization of a closed PL n-manifold M^n is an (n+1)-coloured graph (Γ, γ) such that $|K(\Gamma)| \overset{PL}{\cong} M^n$ and $K(\Gamma)$ has exactly $n+1$ vertices. Existence, characterization and equivalence theorems for crystallizations have been proved.

A 2-cell imbedding $i : |\Gamma| \to F$ (F a surface) is said to be regular if there exists a cyclic permutation ε of the colour set such that each region of i is bounded by edges coloured by consecutive colours of ε. $\rho(\Gamma)$ is the least possible genus of a surface into which (Γ, γ) regularly imbeds. The regular genus of an n-manifold M^n is the integer $\mathcal{G}(M) = \min \{\rho(\Gamma) \mid (\Gamma, \gamma) \text{ is a crystallization of } M^n\}$.

In dimension 2, $\mathcal{G}$ equals the classical genus. In dimension 3, $\mathcal{G}$ equals the Heegaard genus in the orientable case, and doubles it for non-orientable manifolds.

In dimension n, $\mathcal{G}(M^n) = 0 \iff M^n \overset{PL}{\cong} S^n$.

In view of Wall's theorem by which, if M^4 is a homotopy 4-sphere, then $M^4 \# h(S^2 \times S^2) \cong S^4 \# h(S^2 \times S^2)$ for a suitable integer h, the additivity of $\mathcal{G}$ with respect to connected sums would imply the 4-dimensional Poincaré conjecture.

Additivity holds trivially in dimension 2, and has been proved in dimension 3 (for the Heegaard genus) by W. Haken.

In general, a standard, simple construction proves subadditivity, i.e. $\mathcal{G}(M \# N) \le \mathcal{G}(M) + \mathcal{G}(N)$. In order to reverse the inequality, a development of crystallization techniques for bounded manifolds should be carried out. The principal obstacle to the proof seems to be the behaviour of the attaching sphere under the moves which lead from a "standard" crystallization of $M \# N$ to one of minimal imbedding.

M. Ferri and C. Gagliardi (Napoli, Italy).

Simply connected 4-manifolds; the situation after the work of Freedman and Donaldson

The topological and differentiable C^∞ (=smooth) theories of simply connected 4manifolds are diverging radically. Simon Donaldson of Oxford has announced that every closed smooth DIFF 4-manifold M^4 with $\pi_1 M = 0$ and an even definite, intersection form on $H_2(M;\mathbb{Z})$ bounds a compact orientable 5-manifold W^5; it follows that $\sigma(M) = 0$ and $H_2(M;\mathbb{Z}) = 0$. Further, W^5 is claimed to be a mild desingularization of the space of self dual (Yang-Mills) connections on the principal $Spin(3) = S^3$ bundle with euler class ± 1 over M^4. On the other hand Michael Freedman of San Diego and Berkeley will be publishing in the September issue of Differential Geometry deep and difficult topological arguments (using theories of A. Casson and R.H. Bing) proving:

Theorem. Closed simply connected topological manifolds are classified up to homeomorphism by their intersection form on H_2 and their stable smoothing obstruction $k \in \mathbb{Z}/2$ of Kirby-Siebenmann. All forms and values of k are realized, subject to the condition that for even forms α, one has $\frac{1}{8}$ signature$(\alpha) = k$ mod 2. A proviso in the quoted article that the manifolds be smoothable in the complement of a point is unnecessary (always satisfied) if we grant that a "controlled" version of Freedman's (loc. cit.)

non-compact h-cobordism theorem holds as F. Quinn has recently asserted. See P. Vogel's generalization of this classification to compact manifolds with a given 3-manifold as boundary. We discussed, among other things the somewhat alarming corollary: There exists an exotic smooth structure Σ on $\mathbb{R}^4$, so that $\mathbb{R}^4_\Sigma$ does not smoothly embed in any smooth homotopy 4-sphere but $\mathbb{R}^4_\Sigma$ does smoothly embed in $S^2 \times S^2$. One knows that $\mathbb{R}^n$, $n \neq 4$, has no exotic smooth structure, i.e. $\mathbb{R}^n_\Sigma \cong \mathbb{R}^n$.

L. Siebenmann (Orsay)

The canonical involution in concordance theory.

Given a compact manifold M, let $C(M) = \mathrm{Diff}(M \times I \text{ rel } M \times 0 \cup \partial M \times I)$ denote the concordance space. Multiplying with an interval defines a stabilization map $C(M) \to C(M \times I)$. Thus one defines the stabilized concordance space $\mathscr{C}(M) = \lim C(M \times I^k)$. $\mathscr{C}(M)$ is a homotopy functor. The concordance space^k carries a natural involution given by reflecting the interval at its midpoint. Such an involution gives a splitting up to homotopy of the concordance space into the ± 1-eigenspaces of the involution (at least away from the prime 2). The interest in the splitting comes from the fact that the factors have a meaning in their own. They are

related to diffeomorphism groups of manifolds. —
There is a homotopy fibration relating concordances with algebraic K-theory

$$B\mathcal{C}(M) \to h^{Diff}(M) \to A(M).$$

Here $B\mathcal{C}(M)$ denotes a certain deloop of the stabilized concordance space, $h^{Diff}(M)$ is a homology theory, and $A(M)$ is the algebraic K-theory of M. Each term in this fibration carries a natural involution, and the maps are compatible with it. The involution on the algebraic K-theory of M can be identified algebraically in the following sense. Given a (simplicial) ring R with an antiinvolution $\bar{\ }: R \to R$ one can define a natural involution on the K-theory of R. Given a space X one has the linearization map $A(X) \longrightarrow K(\mathbb{Z}[G(X)])$, where $\mathbb{Z}[G(X)]$ denotes the group ring of the loop group of X. The involution on $A(X)$ considered before and the involution on $K(\mathbb{Z}[G(X)])$ are compatible through this map. (if X is a manifold, say). This is proved by constructing an involution on $A(X)$ for any space X and showing that on the one hand it agrees with the involution considered before and on the other hand it gives, upon linearization, the correct one on $K(\mathbb{Z}[G(X)])$. The construction of this involution on $A(X)$ depends on a given (stable) spherical fibration over X, and it uses an appropriate concept of equivariant Spanier-Whitehead duality in the category of spaces over X.

Wolrad Vogell (Bielefeld)

Planefields with finite singularities

Let α be a k-plane bundle over a closed connected smooth n-dimensional manifold M, and let $u: \alpha \to TM$ be a monomorphism with finite singularity set $S \subset M$, i.e. over $M - S$ u realizes α as a planefield tangent to M. Given local orientations θ of α and M at S, we define as usual $\mathrm{index}(u, \theta) \in \pi_{n-1}(V_{n,k})$, where $V_{n,k}$ denotes the Stiefel manifold of k-frames in $\mathbb{R}^n$.

Theorem. For $n \geq k \geq 1$ every element in the homotopy group $\pi_{n-1}(V_{n,k})$ can be realized as the index of some planefield with finite singularities on some suitable manifold M.

In contrast, in the setting of frame fields with finite singularities ($\alpha = M \times \mathbb{R}^k$), possibly occurring indices are subjected to strong restrictions: e.g. for $n > 2k+1$ they have to lie in the image of $\pi_{n-1}(V_{n,k+1}) \xrightarrow{p_*} \pi_{n-1}(V_{n,k})$, where p is the obvious projection.

As another striking difference, note that for non-trivial α and for even n and k the $\mathbb{Z}$-index

$$pr_*(\mathrm{index}(u,\theta)) \in \pi_{n-1}(S^{n-1}) = \mathbb{Z} \qquad (pr: V_{n,k} \to S^{n-1} \text{ the obvious projection})$$

is in general not determined by M and α alone, but depends also on the monomorphism u with finite singularities. For fixed M and α such that $n > 2k$ we give a formula describing the set of possibly occurring such $\mathbb{Z}$-indices. This vastly generalizes a result of E. Thomas (concerning the special case $n = 2$, M and α oriented) which was a key step in his solution of some existence problems for 2-planefields without singularities. Our formula follows from the existence and classification theory for monomorphisms, given by the singularity method.

U. Koschorke (Siegen)

TRANSFORMATIONSGRUPPEN

22.8. - 28.8.82

Almost extremal torus actions on homotopy spheres

If the torus T^r acts smoothly and effectively on a homotopy sphere Σ^n, then the fixed point set is at most $(n-2r)$-dimensional. If the dimension is exactly $(n-2r)$ — the extremal case — then there are results which imply that Σ^n is diffeomorphic to S^n. If the dimension of the fixed point set is $(n-2r-2)$, then examples discovered in the 1960's imply that Σ^n need not be diffeomorphic to S^n; in fact, Σ^n need not even bound a π-manifold.

Theorem I (Case $r=1$) Let P be the map which assigns to each exotic n-sphere Σ^n its Pontrjagin-Thom invariant in $\pi_n/\mathrm{Im}\, J$. Then Σ^n admits an S^1 action* with $(n-4)$-dimensional fixed point set $\Leftrightarrow P(\Sigma)$ is expressible as a triple Toda bracket $\langle \alpha, \nu, \eta \rangle$, where $\nu \in \pi_3$ and $\eta \in \pi_1$ are the Hopf maps, and $\alpha \in \pi_{n-5}$ desuspends to S^3. [* smooth + effective!]

Theorem II ($r \geq 2$) If $r \geq 2$ and Σ^n admits a T^r action* with $(n-2r-2)$-dimensional fixed point set, then $P(\Sigma) \in \pi_{n-r+1} \cdot \eta^{r-1} \subseteq \pi_n$. Conversely, if $P(\Sigma) \in \pi_{n-r} \cdot \eta^r$ then $\Sigma \# (\text{bdy } \pi\text{-manifold})$

admits a smooth T^r action with $(n-2r-2)$-dimensional fixed point set.

Corollary In Thm. II, if $r \geq 5$ and Σ^n admits a T^r action as described, then $\Sigma \in bP_{n+1}$ (since $\eta^4 = 0$).

Reinhard Schultz (West Lafayette IN, USA)

"Finite Transformation Groups Of Bounded Compact Manifolds"

(Joint work with P. Vogel).

Let W^n be a compact manifold such that

$\partial W = \partial_+ W \cup \partial_- W$, $\partial_+ W \cap \partial_- W = \partial(\partial_+ W) = \partial(\partial_- W)$, and $\pi_1(\partial_- W) \xrightarrow{\cong} \pi_1(W)$. Suppose G is a finite group acting freely on $\partial_+ W$. The main result concerns the situations under which this G-action extends to a free G-action to W. Under the conditions that ① $H_*(W, \partial_+ W; \mathbb{Z}_{(q)}\pi)$ vanishes (where q = order of G and $\pi = \pi_1(W)$) and ② G acts trivially $H_*(\partial_+ W/G; \mathbb{Z}(\pi \times G) \otimes \mathbb{Z}(\frac{1}{q}))$, ③ $\exists$ a homomorphism $\pi_1(\partial_+ W/G) \longrightarrow \pi$ such that the following diagram commutes

$$\begin{tikzcd} \pi_1(\partial_+ W/G) \arrow[r] & \pi \\ \pi_1(\partial_+ W) \arrow[u] \arrow[ur] & \end{tikzcd}$$

④ then an obstruction is defined in a group $Wh^T(\pi \subset \pi \times G)$ which vanishes if and only if this extension (i.e. smooth action on W) exists. This obstruction group is related to the other algebraic invariants via an exact sequence

$$Wh(\pi \times G) \xrightarrow{T} Wh(\pi) \xrightarrow{\alpha} Wh^T(\pi \subset \pi \times G) \xrightarrow{\beta} \tilde{K}_0(\pi \times G) \to \tilde{K}_0(\pi)$$

and the obstruction is the image of a certain homomorphism γ from $K_1(\mathbb{Z}_{(q)}\pi)/\{\pm\pi\}$ which makes the following diagram

$$\begin{tikzcd} Wh(\pi) \arrow[rr, "\alpha"] \arrow[dr] & & Wh^T(\pi \subset \pi \times G) \\ & K_1(\mathbb{Z}_{(q)}\pi)/\{\pm\pi\} \arrow[ur, "\gamma"] & \end{tikzcd}$$

Amir H. Assadi (Charlottesville, VA, USA)

Proper transformation groups

A continuous action of a locally compact topological group G on a locally compact topological space X is called proper if, for any two compact subsets K, L of X, the subset $\{g \in G;\ gK \cap L \neq \emptyset\}$ is compact. Examples are groups of deck-transformations, of isometries, left translation action of G on G/K (K a compact subgroup of G), action of a finitely generated group on its Cayley diagram. A necessary condition for a space X to admit a proper action of some non-compact group G was given: The number of ends of X must be 1, 2 or infinite. The structure of G in the respective cases was discussed. One of the results generalizes Stallings' theorem describing the structure of finitely generated groups with infinitely many ends. For G almost connected (i.e. G modulo its connected component of e is compact) a classification of proper actions of G on a space X was given reducing it to two problems, namely to determine all spaces S with $S \times \mathbb{R}^n$ homeomorphic to X (here $\mathbb{R}^n$ is homeomorphic to G/K, K a maximal compact subgroup of G) and to determine all actions of K on S. Finally, some remarks concerning the concept of non-compact dimension of a group were made.

Herbert Abels (Bielefeld)

Symmetries on simply-connected manifolds

(joint work with Peter Löffler)

Let M^n be a closed n-dim. manifold, let G be a Lie group. A G-symmetry of M is a G-action on M such that $M^G \neq M$.
Question: Does there exist a G-symmetry on M for some G?
The general answer is no; there are manifolds M with large

$\pi_1(M)$ and no G-symmetry on M at all (R. Schultz). But we have the following positive answer:
If $\pi_1(M)=0$, $H_i(M^n;\mathbb{Q})=0$, $3i+1\leq n$ and, if n is even, additionally, $\chi(M)=0$, $\mathrm{ind}(M)=0$ $\sum_{i=0}^{n}|H_i(M;\mathbb{Q})|\equiv 0\,(4)$,
then there is a manifold M' homotopy-equivalent to M, admitting a free $\mathbb{Z}/p$-symmetry for almost all primes p. Such free $\mathbb{Z}/p$-symmetries do even exist on M itself, if a condition on the distribution of non-vanishing Pontryagin-classes of M holds.
The proof consists of the construction of a free S^1-operation on a manifold M'' rationally homotopy-equivalent to M, a method of pulling back and pushing forward free $\mathbb{Z}/p$-operations over rational homotopy-equivalences f for all primes p prime to ~~the~~ the $|H_i(f)|$, and finally surgery techniques to find $\mathbb{Z}/p$-operations on M itself.

Martin Raußen (Göttingen)

The equivariant triangulation theorem for actions of compact Lie groups

Let G be a compact Lie group acting smoothly on a smooth manifold M. More than a decade ago Takao Matumoto and the present author, independently of each other, showed how to lift a well-behaved triangulation of the orbit space M/G so that one gets a G-CW complex structure on M. In my version of this M was in fact given an equivariant triangulation, which then in particular provides M with a G-CW complex structure. By a well-behaved triangulation of M/G we mean a triangulation obtained by taking the first barycentric subdivision of a triangulation of M/G in which each open simplex lies in one orbit type. Although one in Takao Matumoto's version gets a G-CW complex structure on M, and in my version gets an equivariant triangulation of M, the method of proof we used is basically the same.
The history of the result that M/G admits a well-behaved triangulation has been somewhat confusing, and this has apparently also caused some

confusion concerning the question of what exactly Matumoto and I did prove more than a decade ago. In this talk we tried to clarify the situation by giving an account of the lifting procedure used by Matumoto and myself.

More precisely we do the following. First we define the notion of an equivariant n-simplex $\Delta_n(G; H_0, \dots, H_n)$ of type $(H_0, \dots, H_n)$, where $H_0 \supset H_1 \supset \dots \supset H_n$ are closed subgroups of G. The key result that we prove is then the following.

Theorem. Let X be a G-space with orbit space $X/G = \Delta_n$ such that the orbit type is constant in each of the sets $\Delta_m - \Delta_{m-1}$, $0 \le m \le n$. Then X is G-homeomorphic to an equivariant n-simplex $\Delta_n(G; H_0, \dots, H_n)$ for some closed subgroups $H_0 \supset H_1 \supset \dots \supset H_n$ of G.

The proof of this lemma uses the above mentioned lifting procedure. That any G-space whose orbit space admits a well-behaved triangulation can be given an equivariant triangulation is an immediate consequence of the above result. Combining this with the fact that the orbit space M/G of a smooth G-manifold M can be given a well-behaved triangulation one obtains the result that any smooth G-manifold admits an equivariant triangulation.

Sören Illman (Helsinki)

Pursell-Shanks type theorem for orbit spaces of G-manifolds

Let G be a compact Lie group and M a smooth connected G-manifold without boundary. Let $\bar{M}$ denote the orbit space of M. Then $\bar{M}$ is not always a smooth manifold, but $\bar{M}$ has a smooth functional structure induced from M. And we can defined a smooth vector field on $\bar{M}$. Let $\mathcal{D}(\bar{M})$ denote the set of all smooth vector field on $\bar{M}$ with compact support. $\bar{M}$ is a stratified set by the orbit type of M. Let $\mathfrak{X}(\bar{M})$ denote the set of all smooth vector field* which preserve each stratum of $\bar{M}$. (*with compact support).
Let G' be an another compact Lie group and N a smooth G'-manifold without boundary. Let $\bar{N}$ be the orbit space of N. Then we have:

Theorem (A). For any compact Lie groups G, G' and ~~no~~ smooth G-manifold M, G'-manifold as above, we have the following:
There exists a Lie algebra isomorphism $\Phi : \mathfrak{X}(\bar{M}) \xrightarrow{\cong} \mathfrak{X}(\bar{N})$ iff there exists a strata preserving diffeomorphism $\sigma : \bar{M} \xrightarrow{\cong} \bar{N}$ satisfying $\Phi = \sigma_*$.

(B) For any finite groups G, G', we have:
There exists a Lie algebra isomorphism $\Phi : \mathcal{D}(\bar{M}) \xrightarrow{\cong} \mathcal{D}(\bar{N})$ iff there exists a diffeomorphism $\sigma : \bar{M} \xrightarrow{\cong} \bar{N}$ satisfying $\Phi = \sigma_*$.

For absolute case (i.e. without smooth action), the above theorem is just the theorem of Pursell-Shanks. Theorem (B) should be proved for any compact Lie groups, but I can't till now.

孝順 阿部
Kōjun Abe (Shinshu, Japan)

Units in the Burnside ring.

The group of units $\Omega^*(G)$ in the Burnside ring of a finite group can be identified with the group $\mathcal{H}_G = \varinjlim_V \mathcal{H}_G(S(V))$ of stable G-homotopy selfequivalences of real representations of G. Here $\mathcal{H}_G(S(V))$ denotes the group of G-homotopy classes of G-homotopy equivalences $S(V) \to S(V)$. There is a homomorphism $\Delta : RO(G) \to \Omega^*(G)$ carrying $[V]$ into the unit corresponding to the antipodal map on $S(V)$.

Theorem I Δ is onto for every 2-group G.

Theorem II Let G be a 2-group and V a real representation of G. Then any G-homotopy equivalence $S(V) \to S(V)$ is G-homotopic to an isometry.

Note that theorem II is an unstable version of theorem I. It is deduced from theorem I using equivariant obstruction theory and some reprerentation theoretic ideas from the proof of theorem I.

Let $j: \Omega(G) \to R_{\mathbb{Q}}(G)$ be the ring homomorphism sending a finite G-set X into the permutation representation $\mathbb{Q}X$, and let $N(G)$ be the kernel j. For a subquotient K/H of G we let $\mathrm{Ind}_{K/H}^{G}: \Omega(K/H) \to \Omega(G)$ be the composite of pull back by the projection $K \to K/H$ with induction from K to G. The main tool in the proof of theorem I is the following induction theorem analogous to Blichfeldt's classical result for complex representations.

Induction theorem For every 2-group G

$$N(G) = \sum_{K/H} \mathrm{Ind}_{K/H}^{G} N(K/H)$$

where K/H runs through all dihedral subquotients (the Klein four group included).

This induction theorem combined with Segal's result that j is onto for every 2-group provides a description of $R_{\mathbb{Q}}(G)$ by generators and relations, which is used to construct an epimorphism $F: R_{\mathbb{Q}}(G) \to \mathrm{Hom}(\Omega^*(G), \{\pm 1\})$. Induction techniques for representations over $\mathbb{Q}$ are used to identify $\mathrm{Ker}\, F$, and theorem I follows by a variant of Frobenius reciprocity. The induction theorem uses further results on induction in rational representation rings and some combinatorial properties of subgroups of 2-groups.

Jørgen Tornehave (Aarhus)

The spectrum of an equivariant K-theory

Let G be a compact Lie group and X a compact G-ENR.
Let C be a family of topologically cyclic closed subgroups of G.
We define a category $C(G.X)$ which reveals the topological properties of the action:

Df: objects of $C(G.X)$ are pairs: $(S.c)$ where $S \in C$ and $c \subset X^S$ is a connected component.

We relate the properties of the prime ideal spectrum of $K_G(X)$, to properties of $C(G,X)$. The results and methods are similar to those of D. Quillen concerning equivariant Borel cohomology with $\mathbb{Z}_p$ coefficients. They provide also a different aproach to G. Segal's results on prime ideals of the representation ring of a compact Lie group.

Theorem 1: If X is a compact G-ENR then there exists a natural homeomorphism
$$\varinjlim_{C(G,X)} \operatorname{Spec} R(\cdot) \xrightarrow{\approx} \operatorname{Spec} K_G(X).$$

This description of $\operatorname{Spec} K_G(X)$ enables to prove the following:

Theorem 2: If X is a compact G-ENR, X' - a compact G'-ENR, $\theta: G \to G'$ a homomorphism & $f: X \to X'$ a map such that $\theta(g)f(x) = f(gx)$ then the following conditions are equivalent:

1) the induced map $(\theta, f)_*: \operatorname{Spec} K_G(X) \to \operatorname{Spec} K_{G'}(X')$ is a homeomorphism
2) the induced functor: $(\theta, f)_\#: C(G,X) \to C(G',X')$ is an equivalence of categories.

This theorem applied to the case $X = X' = pt$ yields the following:

Theorem 3: If $\theta: G \to G'$ is a homomorphism of compact Lie groups inducing a bijection $\theta_*: \operatorname{Spec} R(G) \to \operatorname{Spec} R(G')$ then:

1) θ is a monomorphism
2) the induced homomorphism $\bar\theta: G/G^0 \to G'/G'^0$ is an isomorphism, where G^0 is an identity component
3) if one of the groups is connected then θ is an isomorphism.

Agnieszka Bojanowska (Warsawa)

Introduction to Mackey Functors

Properly understood, a G-equivariant cohomology theory is $RO(G)$-graded and takes values in the category of Mackey functors - instead of just abelian groups. The $RO(G)$-grading is essential for the existence of transfers and duality. Also, the generators of the cohomology of an arbitrary G-space are quite likely to appear in the non-trivial grading. The Mackey functor structure can be essential for calculations (e.g. computing the $RO(G)$-graded ordinary cohomology of a point with Burnside ring coefficients – this is now done for $G = \mathbb{Z}/p^n\mathbb{Z}$). It is also possible for the generators of the cohomology of a space to lie in values of the Mackey functor at some proper subgroup of G. These observations motivate the topic of this talk: the structure of the category of Mackey functors.

For any finite group G, there is a small additive category $\mathcal{B}$ such that a Mackey functor is a contravariant additive functor from $\mathcal{B}$ to abelian groups. Thus, the category of Mackey functors is an abelian category with all the properties needed for doing homological algebra. There is a tensor product-like operation $M \square N$ defined on a pair of Mackey functors M and N such that a pairing $(M,N) \to L$ of Mackey functors in the sense of Dress is just a map $M \square N \to L$. The product functor $\square$ has a right adjoint which should be thought of as a Mackey functor ~~of~~ valued hom-set. (The adjunction is just like the hom-tensor adjunction for abelian groups). It follows that $\square$ and the hom functor have derived functors which behave like Tor and Ext.

One can define the Mackey functor-valued singular chains of a G-space X and thereby obtain the Mackey-functor valued homology ~~H~~ $H_*(X; M)$ and cohomology $H^*(X; M)$ of X with coefficients in any Mackey functor M. Universal coefficient and Kunneth theorems exist for these theories.

The main difficulty with this theory is that the category of Mackey functors is that it has infinite homological dimension. Thus, the universal coefficient and Kunneth theorems yield spectral sequences instead of short exact sequences. This difficulty can be circumvented by ~~to~~ working with coefficients in a Mackey-functor field. The structure of these fields has a simple description in terms of induction theory and Galois theory. Of particular interest are the "prime fields" coming from the ~~of Mackey functor~~ prime ideals of the Burnside very Mackey functor. The structure of these Mackey functor ideals has important implications for the study of torsion in Mackey functors at primes dividing the order of the groups

T. Hauna Zewsf (Syracuse University).

Multiplicative structures in equivariant surgery groups.

Let M be a G-manifold, G a finite group. We denote by $\dot M^H$ the subset of M^H which is the closure of the set of points in M with stabilizer $H \subset G$. Let α enumerate the connected components $\{\dot M^H_\alpha\}$ of $\dot M^H$. We denote by $\Psi_\alpha(H)$ the natural H-representation in the fibers of the normal bundle $\nu_\alpha^H = \nu(\dot M_\alpha^H, M)$. For any representation Ψ of H $\rho_\Psi(M)$ will be the subset $\{\alpha\} \subset \pi_0(\dot M^H)$ such that $\Psi_\alpha(H)$ is isomorphic to Ψ.

Now we are able to give a definition wich is inspired by Tom Dieck's definition of Burnside ring.

Let us say that two G-manifolds M_1 and M_2 are $\mathcal{L}$-equivalent if for every subgroup $H \subset G$ and for every representation Ψ of H the two sums

$$\sum_{\alpha \in \rho_\Psi(M_i)} \operatorname{Ind}_{N_\alpha H}^{NH} \left(\operatorname{Sign}[N_\alpha H, (\dot M_i)^H_\alpha] \right), \qquad (i = 1,2)$$

are equal. Here NH is the normalizer of H in G and $N_\alpha H$ is the maximal subgroup which normalizes the component $\dot M^H_\alpha$.

It turns out that this equivalence relation is compartible with the operations of disjoint union and cartesian product (with diagonal G-action) on closed G-manifolds. So the $\mathcal{L}$-equivalence classes form a commutative ring $\mathcal{L}\Theta(G)$. We also have an alternative definition of $\mathcal{L}\Theta(G)$, which is purely algebraic. We prove that the groups of G-equivariant surgery ${}_G L_*$ become an $\mathcal{L}\Theta(G)$-module (after tenzoring with $\mathbb{Z}[1/2]$). The subgroups of all elements in the

G-surgery groups which are realizable by closed G-manifolds (after tenzoring by $\mathbb{Z}[1/2]$) is isomorphic to some subgroup in $\mathcal{L}\Theta(G)$. Therefore it has a nice ring structure.

Gabriel Katz (Tel-Aviv Univ.)

Induction theorems in equivariant surgery.

The purpose of this talk is to prove an induction theorem for equivariant surgery. This is, we show that (under appropriate assumptions, in particular the acting group is nilpotent of odd order) the surgery obstruction $\sigma(\omega_0)$ of a normal map ω_0 vanishes if it does so after restricting the action of G on ω_0 to actions of groups in a class of subgroups of G (not containing G, here we have to use the class of subgroups which extend a p group by a cyclic or a cyclic by a p group). This theorem should be compared with Dress's result that Wall groups satisfy 2-hyperelementary induction. The main applications are Petrie's actions on homotopy spheres with exactly one fixed point and those actions on homotopy sphere with two fixed pts having distinct slice representations at these points. The theorem is joint work with Ted Petrie and will appear in Amer. J. of Math.

Karl Heinz Dovermann (Purdue Univ)

Actions of unitary groups whose number of orbit types is not too "astronomical".

In the study of transf. groups it is a fundamental problem to determine what are the possible orbit structures for a given group G and space X. This talk is based upon joint work with Wu-yi Hsiang where we study the possibilities of geometrical behavior for (classical) groups acting on homology spheres or acyclic spaces, and here we specialize to SU(n) groups. Let

$$F(\alpha) = \{\text{orbit types of } \alpha\}, \quad \mu(\alpha) = \# I(\alpha)$$

The linear case is already complicated from a transf. group view point. In fact, $\mu(\varphi)$ (and the orbit structure) is only known for a few linear representations of $SU(n)$, namely

$$\varphi = k\mu_n,\ \Lambda^2\mu_n,\ S^2\mu_n,\ Ad_G,\ \Lambda^2\mu_n + k\mu_n,\ S^2\mu_n + k\mu_n,\ Ad_G + k\mu_n$$

and one case extra for $n = 4, 6, 8$ ($n \geq 4$)

An action ψ is orthogonally modelled on the repr. φ if, by definition,

(1) $I(\varphi) = I(\psi)$

(2) φ, ψ have the same slice representations, $\forall G_x$.

Theorem Let ψ be a smooth action of $SU(n)$ on $X \underset{\mathbb{Z}}{\sim} S^N, \mathbb{R}^N$, and assume $\mu(\psi) \leq p(n)$ = partition function of n ($= \mu(Ad_G)$) Then ψ is orthog. modelled on φ + (trivial repr.), where φ is one of the repr. mentioned above.

The basic ideas of proof is to show, by different techniques and combinatorial methods, that unless the (geometric) weight system $\Omega'(\psi)$ is quite simple, the number of conjugacy classes of max. tori of isotropy groups is already too large. Hence, one can show $\Omega'(\varphi) = \Omega'(\psi)$ for some well know linear repr. φ. From this it follows that $\varphi + k\cdot 1$ must be the linear (orthog.) model. Essentially, the theorem also says that, if φ is more complicated than Ad_G, then we don't know the orbit structure, and the linear and non-linear case behave practically in the same way in the region where orbit structures can be determined.

Eldar Straume (Univ. of Tromsø, Norway)

Deformations of fat points in the cohomology theory of transformation groups

Let $G = T^r$ a torus, X a sufficiently nice space and let G act on X with finitely many orbit types: What information do we have on the fixed space $X^G = F(G;X)$? (P.A. Smith – Question)

Ex.: 1. $X \sim_{\mathbb{Q}} pt$ $(X \sim_k Y \Leftrightarrow H^*(X;k) \cong H^*(Y;k))$
$\Rightarrow X^G \sim_{\mathbb{Q}} pt$. (Very classical)

2. $X \sim_{\mathbb{Q}} S^n \Rightarrow X^G \sim_{\mathbb{Q}} S^m$, $n - m \equiv 0(2)$ (very classical)

3. $X \sim_{\mathbb{Q}} \mathbb{C}P^n \Rightarrow X^G \sim_{\mathbb{Q}} \sum \mathbb{C}P^{n_i}$, $\sum(n_i+1) = n+1$
(classical) (P.A. Smith, Floyd, Borel, Su, Bredon, Hsiang)

We consider special case: The Serre-s.s. of $X \to E_G \times_G X \to B_G$ collapses. Then we have a commutative diagram

$$\begin{array}{ccc} H^*_G(X) & \xrightarrow{i^*} & H^*(X) \\ \uparrow{\scriptstyle \pi^*} & & \uparrow \\ H^*(B_G) & \longrightarrow & k \end{array}$$

s.t. $H^*_G(X)$ is a flat (in fact free) $R_G = H^*(B_G)$-module. Moreover by Eilenberg-Moore it follows
$$H^*(X) \cong H^*_G(X) \otimes_{R_G} k \quad (\operatorname{char} k = 0)$$
This shows $H^*_G(X)$ to be a deformation of the k-algebra $H^*(X)$, ($H^*(X)$ is an artinian k-algebra for finite-dimensional reasonable spaces, i.e., a fat point!).

Now, we ask, which deformations over the line (= Spec $H^*(B_{S^1}, k)$) can arise as equivariant cohomology rings. We call a deformation $\xi = \begin{pmatrix} X_0 & \to & X \\ \downarrow & & \downarrow \\ * & \to & V \end{pmatrix}$

~~The~~

geometric ($V = \mathbb{A}^r_k$) $\Leftrightarrow$ $X = \bigcup_{i=1}^{s} X_i$, $X_i \cong \mathbb{A}^r_k \times_k Y_i$, Y_i a fat point for all i. Then we show:

Theorem 1
A G_m-equivariant (i.e. graded) deformation ξ of A_0 over the line (G_m acts on the line with weight 2) is geometric $\Leftrightarrow$ ξ gives an equivariant cohomology ring of an S^1-action on a finite CW-complex X with $H^*(X;k) \cong A_0$.

The above theorem can then be used to construct in a purely algebraic way new actions from known ones, for example using a graded version of the concept of openess of versality.

We show the following:

Theorem 2:
Let X a space s.t. $H^*(X;k) = k[x_1, \dots, x_n]/I_0$, $|x_i| = 2$, a complete intersection and let $\varphi: S^1 \times X \to X$ a circle action with fixed space $X^G = F_1 + F_2 + \dots + F_s$.
Let $\varphi_i: S^1 \times F_i \to F_i$ a family of circle actions with corresponding fixed spaces $F_{i1} + F_{i2} + \dots + F_{is_i}$. ($= F_i^{S^1}$)
Then $\exists$ a finite S^1-CW-complex X', $H^*(X';k) \cong H^*(X;k)$ s.t. $(X')^G = \sum_{i=1}^{s} \sum_{j=1}^{s_i} F_{ij}$.

If we consider the case $X = U(3)/T^3$ we get the following possibilities for the fixed sets $3 \cdot S^2$, $2 \cdot S^2 + 2p$, $S^2 + 4p$, $6p$ (Hakama, Kono, Okayama Journal). In this case this are all possibilities. In the case $X = U(4)/T^4$ we get 72 possibilities using Theorem 2 applied to linear actions. But it is not clear whether we get all possibilities by the above procedure, probably not!

Volker Hauschild (Univ. Konstanz)

Equivariant algebraic K-theory.

We define our equivariant algebraic K-theory along the line of classical ~~K~~ algebraic K-theory and investigate its fundamental properties. Let G be a group and Λ be a ring with G-action (called G-action) and Λ' be its G-subring. Then we define a notion of $\Lambda(\Lambda')G$-projective modules. By the direct sum operations, the set of finitely generated $\Lambda(\Lambda')G$-projective modules forms a semi-group. Then we define our equivariant algebraic K-theory $K_0^G(\Lambda(\Lambda'))$ as its Grothendieck group. We now list some results "Equivariant Swan theorem", The Grothendieck group of the set of f.g. $\Lambda(\Lambda)G$-free modules is denoted by $R(G, \Lambda)$. Then we have an isomorphism,

$$R(G,\Lambda) \cong \mathrm{Groth.}\Big(\coprod_{n\geq 0} H^1(G, GL(n,\Lambda))\Big)$$

where H^1 denotes the cohomology of group with non-abelian group coefficient and G acts on $GL(n,\Lambda)$ obviously and Groth. means the Grothendieck group of the semi-group $(\coprod_{n\geq 0} H^1(\cdot))$. When Λ is a field, principal ideal domain or local ring,

$$K_0^G(\Lambda) \cong R(G,\Lambda) \cong \mathrm{Groth.}\Big(\coprod_{n\geq 0} H^1(G, GL(n,\Lambda))\Big).$$

An interesting example is the following. Let K/k be a Galois extension, $G = \mathrm{Gal}(K/k)$. Then K is a G-ring and $K_0^G(K) \cong \mathbb{Z}$. This follows from vanishing theorem of Galois cohomology and from the above theorem. A notion of "Induction" is defined and we show Mackey and Frobenius properties. Accordingly Dress induction theorem is applicable. These are generalized to the case of sheaf theory with group actions. Brauer theory is developed in the case of valuation ring with group actions. $K_1^G(\Lambda)$ and $K_2^G(\Lambda)$ are also defined and studied.

川久保勝夫(大阪大学) K. KAWAKUBO OSAKA JAPAN

1982年, 8月27日.

On some problems in compact transformation groups

Let G be a compact Lie group. The question of the equivalence of the representations at two fixed points induced by smooth actions of G on disks, spheres, and euclidean spaces, and the weaker question of the equality of the dimensions of two fixed point set connected components of smooth actions of G on those spaces go back to P.A. Smith and G.E. Bredon. We show that except for the distinguished case of smooth actions of G on spheres with exactly two fixed points, the answer to these questions is affirmative if and only if each element of the quotient group G/G_0 has prime power order, where G_0 denotes the identity connected component of G.

T. Petrie has announced that he knows of no smooth non-abelian group actions on disks, spheres, or euclidean spaces with isolated fixed points at which the induced representations are distinct. For some finite non-abelian groups G, we obtain such examples of actions by showing what sets of complex representations of G can be realized (stably) at isolated fixed points of smooth actions of G on disks.

We also deal with Problems 1, 2, 3, and 4 posed on page 205 in the Bredon's book: Introduction to compact transformation groups. In particular, we completely solve all four problems in the case of smooth finite cyclic group actions.

Krzysztof Pawałowski, Poznań, Poland
(talk given by Bob Oliver, Århus, Denmark)

Normal Fibrations and Normal Maps of Poincaré G complexes

The homotopy theoretic analogue of a smooth G manifold is a Poincaré G complex which we define so that, roughly speaking, it is a G complex X with each fixed set $f(H,X)$ being a Poincaré complex together with a Poincaré embedding in X with normal fibration denoted $t(H)$. Our first theorem is analogous to the well known result of Spivak on the normal fibration of a Poincaré complex:

Theorem 1 : Each Poincaré G complex X admits a fibration $n = (E, p\,X)$ with the following properties: 1) E is a G space, p is a G map and $f(H,E) \xrightarrow{f(H,p)} f(H,X)$ is a spherical fibration 2) $E \mid f(H,X)$ comes with a canonical splitting of the form: $E \mid f(H,X) = f(H,E) \oplus e(H,E)$ where $e(H,E) \xrightarrow{p} X$ is also a spherical fibration

3) There is a G map $c : S^V \longrightarrow (X)^n$ ← Thom complex which has degree 1 on each fixed set, where V is some G-module.

4) There is a fiber homotopy trivialization $F(H) : e(H,E) \oplus t(H) \longrightarrow V/f(H,V)$ for each H. Write $e(H,V)$ for $V/f(H,V)$

Addendum: There is a canonical choice for $F(H)$, as it turns out. To understand this let $k(H) : X \longrightarrow f(H,X)^{t(H)}$ be the collapse map. Then, there are 2 collapse maps from ~~V~~ S^V onto $f(H,X)^{t(H)\oplus n}$: one is given by the composite $S^V \xrightarrow{c} X^n \xrightarrow{k(H)^n} X^{t(H)\oplus n}$. The other is $S^V = S^{e(H,V)} \wedge S^{f(H,V)} \xrightarrow{1 \wedge f(H,c)} S^{e(H,V)} \wedge f(H,X)^{f(H,n)} \xrightarrow{F(H)\oplus 1} f(H,X)^{t(H)\oplus e(H,n)\oplus f(H,n)} = f(H,X)^{t(H)\oplus n}$. There is precisely one choice of $F(H)$ – stably – which makes these 2 collapse maps equal.

All of this is meant to emphasize that the normal fibration information implicit in a Poincaré G-complex consists of $t(*)$, n, and $F(*)$ where * runs over the various subgroups H. This is important, for Sullivan-Browder proved that the linearizations of the normal fibration of a Poincaré complex are in bicorrespondence with its normal maps. We prove:

Theorem 2 Suppose given a linearization of $(t(*), n, \text{and } F(*))$ say $(\tau(*), \nu, \Phi(*))$ where $\tau(H)$ is a $N(H)$ vector bundle, ν is a G vector bundle, $\Phi(H)$ is a vector bundle trivialization. This linearization defines – up to normal cobordism, uniquely – a C^∞ G manifold M, a degree 1 G-map $M \xrightarrow{f} X$, a covering bundle map $b : \nu(M,V) \longrightarrow \nu$, and cove for each H a covering bundle map $a(H) : (\tau(M)|M^H)/\tau M^H \longrightarrow \tau(H)$ so that $\Phi(H) \circ a(H) \oplus b(H)$ is the natural trivialization defined by the embedding of M in V.

This is a notion of normal G-map which is stronger than has been in common use. It should be emphasized that it ~~ariges~~ arises directly from the homotopy theory and bundle theory implicit in a Poincaré G-complex

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Komplexe Analysis

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Geometric Fourier transform.

I discussed the work of Bernstein, Kashiwara, Mebkhout, etc. on holonomic $\mathcal{D}$-modules and Riemann-Hilbert correspondence. Let $\mathcal{D}_X$ (= $\mathcal{D}$) be the sheaf of linear differential operators on a complex analytic manifold X, and let M be a "differential system on X", i.e. a left coherent $\mathcal{D}$-module. One says that M is "holonomic" if ~~the~~ its characteristic variety $\operatorname{ch} M \subset T^*X$ has the minimal dimension, i.e. $\dim X$; furthermore, one says that M has regular singularities if it admits a "good filtration" [locally or globally, this is equivalent, and this equivalence is a deep theorem by Kashiwara-Kawai] such that $\operatorname{gr} M$ is annihilated by the ideal $\mathcal{I}(\operatorname{ch} M)$ of elements of $\operatorname{gr}\mathcal{D}$ vanishing on $\operatorname{ch} M$.

Let $D_{b,cons}(\mathbb{C})$ the category of complexes of $\mathbb{C}$-modules over X with bounded and constructible cohomology (in the sense of the theory of derived categories).

Def. $G \in D_{b,cons}(\mathbb{C})$ is called "a perverse sheaf" if the following conditions are satisfied

1) $\underline{H}^i(G) = 0$, $i<0$ 2) Codim supp $\underline{H}^i(G) \geq i$, $i \geq 0$ 3) The dual DG in the sense of Verdier satisfies 1) and 2). The main result are the following

Th. 1. If M is holonomic, then $R\mathcal{H}om_{\mathcal{D}}(M, \mathcal{O})$ is perverse ($\mathcal{O} = \mathcal{O}_X$, sheaf of holomorphic functions on X)

Th 2. The ~~correspondence $M \mapsto R\mathcal{H}$~~ functor $M \mapsto R\mathcal{H}om_{\mathcal{D}}(M, \mathcal{O})$ is an equivalence of the category of holonomic $\mathcal{D}$-modules with r. s. and the category of perverse sheaves.

Th 1. is due to Kashiwara, Th 2 to Kashiwara-Mebkhout (under slightly different forms; the introduction of perverse sheaves is due to Deligne).

I discussed also briefly some examples and some aspects of this correspondence: intersection homology, relationship between vanishing cycles and microlocalisation, relation with Fourier transform.

B. Malgrange

(Institut Fourier Université de Grenoble 1)

Numerical Positivity of Ample Vector Bundles (~~[illegible]~~).

I ~~described~~ discussed a joint theorem with Fulton describing all numerically positive polynomials in the Chern classes of an ample vector bundle.

One says that a polynomial $P \in \mathbb{Q}[c_1, \dots, c_e]$ of (weighted) degree n is num. pos. for ample v.b.'s if for every ample v.b. E of rank e on a proj variety X of dim n, the Chern number $\int_X P(c_1(E), \dots, c_e(E))$ is > 0.

Let $\Lambda(n,e)$ denote the set of all partitions of n by non-negative integers $\leq e$: so an element $\lambda \in \Lambda(n,e)$ is determined by giving $e \geq \lambda_1 \geq \dots \geq \lambda_n \geq 0$ with $\sum \lambda_i = n$. For $\lambda \in \Lambda(n,e)$ define the Schur polynomial P_λ as the determinant

$$P_\lambda = \begin{vmatrix} c_{\lambda_1} & c_{\lambda_1+1} & \dots & \\ c_{\lambda_2-1} & c_{\lambda_2} & \dots & \\ & & & \vdots \\ & \dots & & \dots c_{\lambda_n} \end{vmatrix},$$

where $c_0 = 1$, and $c_i = 0$ if $i \notin [0,e]$. The main result is that $P = \sum a_\lambda(P) \cdot P_\lambda$ is numerically positive for ample vector bundles if and only if each of the Schur coefficients $a_\lambda(P)$ is ≥ 0. (N.B.: the P_λ form a basis for the $\mathbb{Q}$-vector space of weighted homogeneous polys of degree n in e variables.). The proof uses on the one hand the fact that the P_λ give the ~~expression~~ basic Schubert cycles on a Grassmannian, and on the other hand a basic technical theorem on the positivity of certain "cone classes". We also discussed the relations with some earlier work and conjectures of Griffiths.

R. Lazarsfeld
(Harvard Univ, Cambridge, MA.)

Double sextics and singular K-3 surfaces

For any compact (complex) surface X one can define its Picard number ρ as $\rho = \mathrm{rk}_{\mathbb{Z}} NS(X)$, where $NS(X)$ is the Neron-Severi group of divisors mod hom. equivalence. By transzendental theory it is known that $NS(X) \cong H^2(X,\mathbb{Z}) \cap H^{1,1}$ thus we have the apriori bound $0 \leq \rho \leq h^{1,1}$.

If $p_g = 0$ (e.g. if X is rational) then always $\rho = h^{1,1} = h^2$, otherwise not very much can be said about ρ, and its value seems almost impossible to determine in general for a given surface.

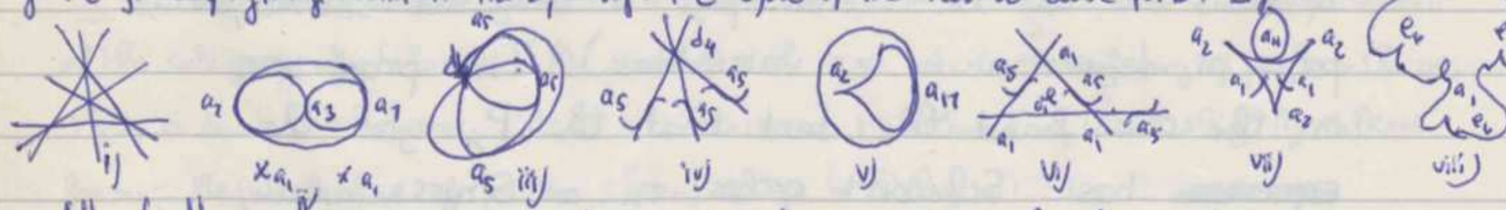
Restricting our attention to the simplest objects like hypersurfaces in $\mathbb{P}^3$ or double planes, not even then much is known.

For hypersurfaces (discarding the trivial cases $n=1,2\,\&\,3$) examples of surfaces with maximal Picard number (i.e. $= h^{1,1}$) are only known for $n=4$ (classical) $n=6$ (Beauville, Shioda) where the Fermat surfaces work. In particular no one knows about a quintic with $\rho = 45 = h^{1,1}$.

On the other extreme examples of generic ($\rho=1$) ("transzendental") hypersurfaces have been written down by Shioda ($w^m + xy^{m-1} + yz^{m-1} + zx^{m-1}$) for $m \geq 5$. But there is no generic example known of a quartic with $\rho = 1$.

For double planes the branchcurves $XY((X^n+Y^n+Z^n)^2 - 4((XY)^n + (YZ)^n + (ZX)^n))$ yield maximal Picard for all n, but on the other hand no explicit examples of generic are known.

In the special case of sextics it is quite easy to write down systematically a whole slew of examples with simple singularities, using ad-hoc constructions of elementary geometry. E.g. giving maximal Picard, through the cycles of the resolved double points. E.g.



All of those give examples of singular K-3 surfaces (i.e. K-3 surfaces with maximal Picard number $= 20$).

There is a general theory of Shioda, who establishes a 1-1 correspondence between singular K3 surfaces and elements of $\mathcal{M}/SL(2,\mathbb{Z})$ where $\mathcal{M}$ denotes the even, integral pos. def 2×2 matrices. (The correspondence is given by associating to each singular K-3 surface the restriction of the intersection form to the orthogonal complement of the Néron-Severi group.) Shioda also shows that each singular K-3 surface X admits a special elliptic fibration (with given types of singular fibers out of the Kodaira list) and an associated involution σ such that the quotient $X/\sigma = \mathrm{Km}(A)$ where $\mathrm{Km}(A)$ is the Kummer surface associated with the Abelian surface A, and such that A is singular and such that its intersection form on the transcendental part equals that of X.

The intriguing question is how to relate the above sextic configuration with the general theory, and in particular to find the magic elliptic fibrations and the corresponding involutions.

This has so far only been possible in the case v) corresponding to $A = C_\omega \times C_\omega$ ($C_\omega = \mathbb{C}/\mathbb{Z}+\omega\mathbb{Z}$ $\omega^3=1, \omega\neq 1$) with matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

There is also a purely geometrical-combinatorial aspect. To get some order in the profusion of examples. To classify and list them all (finite # ?), to determine which determine the same K-3 surface. (e.g. vi) and viii) being Cremona transformations of each other). There are also some mysterious connections between (configurations of curves of deg ≤ 2) and (configurations with a triangle of lines), by associating a net of conics together with its discriminant cubic. Furthermore any sextic configuration containing a cubic, naturally defines an elliptic rational pencil, which turns out to be degenerate in the sense of having only a finite group of sections etc...

One outstanding problem would be to give explicit examples (preferably defined over $\mathbb{Q}$) with ρ ranging from 1 to 20. As to the knowledge of the lecturer, no explicit examples of K-3 surfaces with $\rho \leq 15$ are known.

U. Persson
(Inst. Mittag-Leffler, Djursholm, Schweden)

Strongly pseudoconvex Kähler manifolds

In this talk algebraization and embedding theorems for 1-convex manifolds were discussed. Introducting the following notation: "a 1-convex Kähler manifold satisfies the condition (*), iff: there exists an open neighborhood U of the exceptional set A of X, a Kähler form ω on U and a Moišezon manifold $\tilde{X}$ such that:

a) A is deformation retract of U

b) U is open in $\tilde{X}$

c) $\int_b \omega = 0$ for all analytic homology classes b on U"

we have the following theorem:

Every 1-convex Kähler manifold satisfying (*) has an open neighborhood which is embeddable into a space $\mathbb{C}^n \times \mathbb{P}_m$.

There are topological conditions for the singularity obtained by blowing down A, such that (*) is satisfied.

Furthermore one has the

Theorem: Every 1-convex manifold which is embeddable into $\mathbb{C}^n \times \mathbb{P}_m$ has a neighborhood of the exceptional set which is open in a quasi-projective ~~(neighborhood)~~ 1-convex manifold with the same exceptional set. This theorem is proved by using the following proposition: let X be a 1-convex manifold which is embeddable into $\mathbb{C}^n \times \mathbb{P}_m$ and $\mathcal{E}$ a locally free sheaf on X. Then there exists an open neighborhood U of the exceptional set of X, an 1-convex ~~qu~~ manifold X' with the same exceptional set and a projective manifold such that: U is open in X', X' is Zariski-open in $\tilde{X}$ and there is a locally free sheaf $\mathcal{F}$ on $\tilde{X}$ such that $\mathcal{F}|U \simeq \mathcal{E}|U$.

This proposition also gives a method to prove vanishing theorems on 1-convex manifolds using ~~1-co~~ vanishing theorems on projective manifolds.

Th. Peternell

(Universität Münster)

Umbilical hypersurfaces and uniformization of circular domains

In this talk I discussed an approach concerning the characterization of certain special types of domains. The main result is as follows.

Theorem 1 Let D be a bounded strictly pseudoconvex domain with smooth boundary in a Stein manifold M. Assume that $D = \{\tau < 1\}$ is defined by an exhaustion function $\tau : M \to [0, R)$ with the properties that (1) τ is continuous on M and C^∞ on $M_* = M - \{\tau = 0\}$ (2) $dd^c\tau > 0$ on M_* and (3) $(dd^c \log \tau)^n \equiv 0$ on M_* where $n = \dim M$. Then

(i) D is biholomorphic to a complete generalized weighted circular domain in $\mathbb{C}^n$ iff the foliation defined by $\ker \partial\bar\partial \log \tau$ is holomorphic;

(ii) D is biholomorphic to a complete weighted circular domain in $\mathbb{C}^n$ iff the associated foliation is holomorphic and the leaves of the foliation are closed;

(iii) D is biholomorphic to a complete circular domain in $\mathbb{C}^n$ iff the foliation is holomorphic, all leaves are closed and $\int_{L \cap \partial D} d^c \log \tau$ is a constant independent of the leaf L of the foliation.

The condition (3) in the above theorem is the homogeneous complex Monge-Ampère equation. Holomorphicity of the foliation ~~may~~ can be understood also in a different way. For example let S be a compact real hypersurface in $\mathbb{C}^n$ such is given by $S = \{\tau = 1\}$ for some τ satisfying (2) and (3) on an open neighborhood of S. Such function τ always exists when S is a real analytic strictly pseudoconvex hypersurface.

Theorem 2 S is umbilical with respect to the structure induced by the Kähler metric $dd^c\tau$ iff the $J\nu$ is pseudo-conformal where ν is the unit normal to S and J is the complex structure on $\mathbb{C}^n$

Notice that when the foliation defined by $\overbrace{\partial\bar\partial \log \tau}^{\ker}$ is holomorphic then $J\nu$ is pseudoconformal. Thus the boundary of the domains in theorem 1 are examples of umbilical hypersurfaces. In the ~~space~~ special case where $\tau = |z|^2$, it is well-known that the only umbilical hypersurface w.r.t. $dd^c|z|^2$ the standard Euclidean metric is the sphere (we also note that $(dd^c \log |z|^2)^n \equiv 0$). Thus theorem 2 is a generalization of this classical theorem.

Finally I will mention that there are many different circular domains in $\mathbb{C}^n$. in fact

Theorem 3 Let $\mathcal{D}$ = biholomorphism classes of bounded complete circular domains in $\mathbb{C}^n$ with smooth boundaries and $\mathcal{D}^+ \subset \mathcal{D}$ be those that are strictly pseudo-convex. Let $[\Omega] = \{\omega \,|\,$ smooth real $(1,1)$ forms on $\mathbb{C}P^{n-1}$ such that $\omega = \Omega + d\chi\}$ where Ω is the Fubini-Study metric and $[\Omega]^+ \subset [\Omega]$ those that are positive ~~matrix~~ definite. Then $\mathcal{D} \simeq [\Omega]/\mathrm{Aut}\,\mathbb{C}P^{n-1}$ and $\mathcal{D}^+ \simeq [\Omega]^+/\mathrm{Aut}\,\mathbb{C}P^{n-1}$. In particular we see that $\mathcal{D}^+$ (hence also $\mathcal{D}$) are infinite dimensional $(n \geq 2)$.

During the talk I mentioned that Webster had proved that a strictly convex domain is biholomorphic to the ball iff it is also a Reinhardt domain. This is wrong, actually Webster proved that an ellipsoid is biholomorphic to the ball iff it is also a Reinhardt domain. I apologized for misquoting Webster.

Pit-Mann Wong
(University of Notre Dame)

Smooth Boundary Values Along Totally Real Submanifolds

Given a domain D in $\mathbb{C}$, bD a simple closed curve, and given $f\in A(D)=\mathcal{O}(D)\cap\mathcal{C}(\bar{D})$ with $|f|=1$ on an arc $\lambda\subset bD$, $|f|<1$ on D, one knows classically that $f|\lambda$ is essentially as smooth as λ: If χ is a conformal map from the unit disc Δ to D and if $l\subset b\Delta$ is the arc that corresponds to λ, then $f\circ\chi$ continues analytically across λ, and the equation $f=(f\circ\chi)\circ\chi^{-1}$ shows that f and χ are equally smooth. ~~aton~~ NB. If λ is of class $\mathcal{C}^1$, χ will, in general, not be of class $\mathcal{C}^1$ but only of class $\mathcal{C}^{0,\alpha}$ for all $\alpha\in(0,1)$.

We have an N-dimensional analogue of this result: THEOREM. Let $D\subset\mathbb{C}^N$ be a strongly pseudoconvex domain ~~of~~ with bD of class $\mathcal{C}^k$, $3\leq k$. Let $\Sigma\subset bD$ be an N-dimensional totally real submanifold of class $\mathcal{C}^r$, $3\leq r\leq k$, and let $f\in A(D)$ satisfy $|f|=1$ on Σ, $|f|<1$ on D. Then $f_\Sigma=f|\Sigma$ belongs to the class $\mathcal{C}^{r-0}(\Sigma)$ if $r<k$ and to class $\mathcal{C}^{k-1}(\Sigma)$ if $r=k$. Here $\mathcal{C}^{r-0}$ is the class of functions whose $(r-1)^{st}$ order derivatives exist and satisfy Hölder conditions of order α for all $\alpha\in(0,1)$.

The proof depends on the theory of peak-interpolation manifolds, the Frobenius theorem, the construction à la Hill and Taiani of parameterized families of discs in D that abut Σ and, finally, on a theorem of Chirka on the boundary smoothness of maps from Δ to $\mathbb{C}^N$ which take $b\Delta$ to a totally real manifold

Edgar Lee Stout
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Homogeneous-rational manifolds and unique factorization

Let $X = G/H$ be a homogeneous-rational manifold, where G is a connected simply-connected semisimple complex Lie group and H a proper parabolic subgroup of G. A holomorphic embedding $f: X \to \mathbb{P}_N$ is called homogeneously normal if there is a holomorphic representation $\varphi_f: G \to SL(N+1, \mathbb{C})$ such that $\varphi_f(g)(f(x)) = f(g(x))$ for all $x \in X, g \in X$ (this definition is independent of G). A homogeneously normal embedding $f: X \to \mathbb{P}_N$ is called homogeneously minimal if N is minimal (such homogeneously minimal do always exist, and they are uniquely determined). Furthermore, the rank of X is defined as $rk(X) := \dim_{\mathbb{C}} H/H'$, where H' denotes the commutator group of H. We prove:

Theorem 1: The following statements about a holomorphic embedding $f: X \to \mathbb{P}_N$ of a homogeneous-rational manifold X are equivalent:

(i) The homogeneous coordinate ring of $f(X)$ is factorial.

(ii) a) $rk(X) = 1$ and

b) there is a k-plane $\mathbb{P}_k \subset \mathbb{P}_N$ such that $f(X) \subset \mathbb{P}_k$ and $f: X \to \mathbb{P}_k$ is homogeneously minimal.

Crucial for the proof of Theorem 1 is

Theorem 2: The following statements about a holomorphic embedding $f: X \to \mathbb{P}_N$ of a homogeneous-rational manifold X are equivalent:

(i) $f(X)$ is projectively normal.

(ii) f is homogeneously normal.

Let $f: X \to \mathbb{P}_N$ be a homogeneously minimal embedding of a homogeneous-rational manifold X. An affine kernel X_a of X is defined to be the complement of a general hyperplane section in $f(X)$. We have:

Theorem 3: The divisor class group $Cl(X_a)$ of X_a is isomorphic to $\mathbb{Z}^{rk(X)-1}$.

Finally we note the following application of Theorem 1: Let $f: X \to \mathbb{P}_N$ be a homogeneously minimal embedding of a rank-1-homogeneous-rational manifold X. Then the homogeneous coordinate ring of $f(X)$ is a Gorenstein ring.

Manfred Steinsiek (Univ. Münster)

Réduction d'Albanese d'un morphisme propre et faiblement Kählérien.

Th : • Soient : • $\varphi : X \to S$ un morphisme propre et faiblement Kählérien, lisse et à fibres irréductibles au-dessus de l'ouvert de Zariski dense S^* de S.

• $\tau^* : T^* \to S^*$ un morphisme lisse, propre dont les fibres sont des tores.

• $\alpha^* : X^* = \varphi^{-1}(S^*) \to T^*$ un morphisme au-dessus de S^* tel qu'existe $s_0 \in S^*$ tel que $\alpha^*(X_{s_0})$ engendre le tore T_{s_0}.

Il existe alors une compactification propre $\tau : T \to S$ de T^* au-dessus de S et une application méromorphe $\alpha : X \to T$ au-dessus de S qui prolonge α^*. Le couple (τ, α) est unique à équivalence biméromorphe près, et $\tau : T \to S$ est faiblement Kählérien propre •

On déduit de ce résultat l'existence d'une réduction d'Albanese relative pour φ, compactification de celle de φ^*, restriction de φ au-dessus de S^*, qui peut être obtenue facilement par quotient. Cette réduction d'Albanese relative est un outil important pour classifier les variétés Kählériennes compactes.

La démonstration du théorème précédent repose, d'une part sur la stabilité des morphismes propres et faiblement Kählériens par passage à l'espace des cycles relatifs, d'autre part sur une réalisation canonique dans un espace de cycles convenable pour tout morphisme $\alpha : X \to T$ d'une variété complexe compacte et connexe X dans un tore T tel que $\alpha(X)$ engendre T. Une telle réalisation repose sur les 3 propriétés suivantes : si $0 \in T$ est un élément nul, et $+$ l'addition qu'il définit, on note pour tout $n \in \mathbb{N}^*$, $\alpha_n : X^n \to T$ le morphisme défini par $\alpha_n(x_1, \dots, x_n) = \alpha(x_1) + \dots + \alpha(x_n)$, on a :

1. Pour $n \geq q = [b_1(X)/2]$, α_n est surjectif.
2. Pour $n \geq q^2$, les fibres de α_n sont toutes de dimension pure $d_n = n.\dim(X) - \dim(T)$.
3. Soit $T_n \subset \mathcal{C}(X^n)$ la famille des fibres de α_n : elle ne dépend pas de $0 \in T$, et le cycle de $\mathcal{C}(X^n)$ égal à $(1 . T_n)$ est isolé. Cette propriété est très particulière aux morphismes à valeurs dans un tore.

F. Campana
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Un Théorème d'Algébricité intermédiaire

Nous donnons une généralisation du théorème classique suivant

Théorème (Kodaira, Grauert-Moisezon ...)

Soit V une variété compacte (Kählerienne) et soit $F \xrightarrow{\pi} V$ un fibré vectoriel sur V fortement o-convexe (i.e. admettant une fonction d'exhaustion φ C^2, fortement psh en dehors d'un compact). Alors V est algébrique projective

De manière précise on a le résultat suivant

Théorème

Soit V une variété compacte Kählerienne ; soit $F \xrightarrow{\pi} V$ un fibré vectoriel sur V, que l'on suppose fortement n-convexe. On suppose de plus que par tout point $x \in F$ passe un sous ensemble analytique compact de dimension pure n. Alors si $a(V)$ est le degré transcendance sur $\mathbb{C}$ du corps des fonctions meromorphes sur V on a : $a(V) \geq \dim V - n$

L'ingrédient essentiel de la preuve est une version renforcée du Théorème 5 de [1] :

Théorème 5 renforcé :

Soit Z un space analytique complexe fortement n-convexe de compact exceptionnel K. On suppose qu'il existe un voisinage ouvert U Kählerien de K. Alors si Γ est une composante irréductible de $\mathcal{C}_n(Z)$ l'space des cycles compacts de dimension pure n de Z dans laquelle il existe $X_0 \in \Gamma$ tel qu'aucune surface irréductible de $|X_0|$ ne soit contenue dans U, alors Γ (qui est holomorphiquement convexe d'après le Th. 5) est modification propre de sa réduction de Stein-Remmert

[1] Convexité de l'espace des cycles Bull Soc Math (1978) t 106

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Some geometric aspects of elliptic curves

It was the aim of this talk to point out a connection between some classical results of Bianchi (Math. Ann. 17, 1880) and a problem concerning the normal bundle of elliptic space curves of degree 5. Starting with the Weierstraß σ-function one can construct an embedding of an elliptic curve C as a linearly normal curve $C_n \subseteq \mathbb{P}_{n-1}$ in such a way that the canonical operation of the n-torsion points on C_n lifts to the standard representation of the Heisenberg group H_n in dimension n, and that the involution on C lifts to $\iota: \mathbb{C}^n \to \mathbb{C}^n$, $e_i \mapsto e_{-i}$. If $n = p \geq 3$ is a prime number one can associate to H_p and ι an abstract configuration of hyperplanes and subspaces of dimension $\frac{1}{2}(p-1)$ which is of type $(p^2{}_{p+1}, p(p+1)_p)$. If $p = 3$ this is just the "Wendepunktskonfiguration" of a plane cubic. If $p = 5$ one gets a configuration of 30 hyperplanes and 25 skew lines. The 30 hyperplanes form 6 so-called fundamental pentahedra.
Recently Auslander and Lazarsfeld classified all possible normal bundles of elliptic space curves of degree 5. Any such curve is

the projection of a normal curve $C_5 \subseteq \mathbb{P}_4$, and the normal bundle varies with the centre of projection. The result of Ellingsrud and Laksov is that any point $P \in \mathbb{P}_4 - \mathrm{Sec}\, C_5$ is on a quintic hypersurface Y_M where $M \in \mathrm{Pic}^0 C$ is unique up to involution and that the normal bundle of the ~~involution~~ projection can be computed in terms of M. They give, however, no interpretation of the quintics Y_M. The main result of this talk was that the quintic hypersurfaces Y_M form a linear family and that they are all linear combinations of the 6 fundamental pentahedra. The common intersection of this family can also be described and the 25 skew lines of the above configuration can be identified in this picture.

Klaus Hulek
(Erlangen)

Rationality of the Prym moduli space $\mathcal{R}_g$ for curves of genus $g = 3, 4$.

Let X be a compact Riemann surface of genus g.
The datum of $\tilde{X} \xrightarrow{\pi} X$, a double unramified cover by a connected $\tilde{X}$, classically considered by Prym and Wirtinger, is equivalent to the datum of a pair (X, η) where $\eta \in \mathrm{Pic}_2(X) - \{0\}$.
$\mathcal{M}_g$ denotes the coarse moduli space representing isomorphism classes of X as above, $\mathcal{R}_g$ the coarse moduli space representing isomorphism classes of pairs (X, η) as above.
$\mathcal{R}_g$ is thus a finite cover of $\mathcal{M}_g$ of degree $2^{2g} - 1$.
One of the basic problems in curve theory is: what can be said about the birational structure of $\mathcal{M}_g$, $\mathcal{R}_g$?
For $g = 1$ both spaces are classically known to be rational, for $g = 2$ this was proven in the sixties by Igusa. For $3 \leq g \leq 10$ Severi proved unirationality of $\mathcal{M}_g$, whereas this year Harris and Mumford proved that $\mathcal{M}_g$ is of general type for g odd, $g \geq 25$.
For $\mathcal{R}_g$, recently unirationality was proved by Clemens for $g = 5$,

and by Donagi for $g = 6$.
A basic problem is: when is R_g, M_g rational?
The object of the talk was to illustrate recent results of the author, by which R_g is rational for $g = 3, 4$: we remark that, beyond the quoted results no more is known up to now, though it is conjectured that M_3, M_4 are rational.
In the lecture I illustrated the proof for $g = 3$.
The idea, in both cases, is to use concrete geometry of a general curve of genus 3, 4, in order to exhibit a Galois (finite) cover of R_g by a rational variety : the problem is then reduced to compute explicitly the invariant subfield for the action of the Galois group.
In the case of $g = 3$, when X is canonically embedded as a plane quartic curve, the datum of (X, η) is equivalent to giving the equation of X as $\det\begin{pmatrix} Q_0 & Q_2 \\ Q_2 & Q_1 \end{pmatrix}$, where the Q_i's are quadratic functions in 3 variables $(x_0, x_1, x_2) = X$, up to the action of $h \in GL(2)$ by $Q = \begin{pmatrix} Q_0 & Q_2 \\ Q_2 & Q_1 \end{pmatrix} \longmapsto {}^t h \, Q \, h$.
To this Q we associate the quadratic function $z \in \mathbb{C}^2 \longmapsto {}^t z \, Q \, z \in Sym^2(\mathbb{C}^3)$. So (X, η) corresponds to a conic $\Gamma \subset W = \mathbb{P}(Sym^2(\mathbb{C}^3))$. Two pairs are isomorphic if $\exists$ $g \in PGL(3)$ s.t. $Sym^2(g)$ takes Γ to Γ'.
So $R_3 = \{\text{space of conics in } W\}/G$, where $G = Sym^2(PGL(3))$.
Since the space of conics in W is a fibre bundle on the Grassmann variety of 2-dim. projective subspaces of W, $Gr(2, W)$, we have a fibration $R_3 \longrightarrow Gr(2, W)/G$.
Now $Gr(2, W)/G \cong R_1$ a rational curve, since to $\Pi \in Gr(2, W)$ we associate the pair $(\Pi \cap \Delta, \vartheta|_{\Pi})$ where, W being the space of symmetric 3×3 matrices (a_{ij}), Δ is the cubic hypersurface $\Delta = \{(a_{ij}) \,/\, \det(a_{ij}) = 0\}$, and ϑ is the cokernel of $0 \to \mathcal{O}_W(-2)^3 \xrightarrow{(a_{ij})} \mathcal{O}_W(-1)^3 \to \vartheta \to 0$.
Using some isomorphisms, plus the fact that any pair $(C, \eta') \in R^1$ can be represented by a matrix M_λ of the form $\begin{pmatrix} \lambda x & z & y \\ z & \lambda y & x \\ y & x & \lambda z \end{pmatrix}$ (this is the so called Hesse normal

form of cubics), we see that

I) $R_3 \xrightarrow{p} R_1$ is a fibration which is birationally a product

II) The fibre of p is the quotient of $\{$space of conics in $\mathbb{P}^2\}$ by the action of the following group H which is a semidirect product of $\mathbb{Z}/3$, acting by $\begin{cases} y_0 \mapsto y_0 \\ y_1 \mapsto \varepsilon y_1 \\ y_2 \mapsto \varepsilon^{-1} y_2 \end{cases}$ where (y_0, y_1, y_2)

are homogeneous coordinates of on $\mathbb{P}^2$, $\varepsilon^3 = 1$, by the group S_3 acting on $\mathbb{P}^2$ by permutation of the 3 coordinates.

The difficulty lies in the fact that this is only a projective representation, not liftable to a linear one.

Finally I only mentioned that for R_4 the geometric part was based on some old investigations of Wirtinger, concerning canonical choices of generators for the ideal of the canonical curve of genus 4 in $\mathbb{P}^3$, and that the problem of invariants was reduced to compute the quotient of $\{$quadrics in $\mathbb{P}^3\} / S_4$, where S_4 acts on $\mathbb{P}^3$ by permutation of the 4 homogeneous coordinates.

Fabrizio CATANESE (Università di PISA, ITALIA)

Hermite - Einstein - Vektorbündel

Wir verallgemeinern den Begriff der Einstein - Metriken auf Kählermannigfaltigkeiten wie folgt: Ist M eine kompakte komplexe Mannigfaltigkeit und $E \to M$ ein holomorphes Vektorbündel, so heißt eine hermitesche Metrik h in E Hermite - Einstein - Metrik (HE-Metrik; E heißt dann HE-Bündel), wenn der assoziierte Ricci - Tensor bezüglich einer geeigneten hermiteschen Metrik auf M proportional zu h ist.

Man sieht leicht, daß alle Geradenbündel sowie Tensorprodukte und duale von HE-Bündeln HE-Bündel sind.

Weitere Beispiele sind:

i) Das Tangentialbündel des komplex-projektiven Raumes $\mathbb{P}_n$ (Fubini-Study-Metrik) und das kanonische Geradenbündel $\mathcal{O}_{\mathbb{P}_n}(-1)$ (mit der Standardmetrik, die durch $0 \to \mathcal{O}_{\mathbb{P}_n}(-1) \hookrightarrow \mathcal{O}_{\mathbb{P}_n}^{\oplus(n+1)}$ induziert wird). Durch Tensorieren und Dualisieren erhält man auch eine HE-Metrik in $\Omega^1_{\mathbb{P}_n}(1)$.

ii) Das Nullkorrelationsbündel N auf $\mathbb{P}_3$ hat eine Metrik, die durch die definierende Sequenz

$$0 \to \mathcal{O}_{\mathbb{P}_3}(-1) \to \Omega^1_{\mathbb{P}_3}(1) \to N \to 0$$

induziert wird; man kann nachrechnen, daß dieses eine HE-Metrik ist.

iii) Unitäre Bündel besitzen eine lokal euklidische Metrik; da diese verschwindende Krümmung hat, ist sie eine HE-Metrik. (Unitär sind z.B. alle stabilen Bündel vom Grad 0 auf kompakten Riemannschen Flächen vom Geschlecht $g \geq 2$.)

Alle diese Bündel sind stabil oder semistabil; dies ist auch nicht anders möglich, da gilt

<u>Satz (Kobayashi)</u>: Ein HE-Bündel ist nicht instabil im Sinne von Bogomolov.

Bogomolov hat gezeigt, daß die ersten zwei Chernklassen eines nicht instabilen Bündels über einer algebraischen Fläche der Ungleichung $(r-1)c_1^2 - 2rc_2 \leq 0$ genügen ($r = \mathrm{rg}\, E$).

Für HE-Bündel gilt allgemeiner der

<u>Satz</u>: Die ersten zwei Chernformen eines HE-Bündels vom Rang r über einer kompakten komplexen Mannigfaltigkeit der Dimension n genügen der Ungleichung $((r-1)c_1^2 - 2rc_2) \wedge \omega^{n-2} \leq 0$, wobei ω die zu der Metrik in M assoziierte 2-Form ist.

Noch offen ist die Frage, ob jedes (semi-)stabile Bündel eine HE-Metrik besitzt.

Martin Lübke (Bayreuth)

Strongly pseudoconvex (spc) manifold and spc domain

I call an spc manifold X an spc domain, if X is realized as a sublevel set of the exhaustion function in another spc manifold. I wanted to discuss a good sufficient condition for an spc manifold (X, ψ) to be an spc ~~#~~ domain.

I couldn't give a complete proof. I proposed a working hypothesis that

If (X, ψ) is an spc manifold with the condition:

(A0) ψ is bounded above

(A1) X has a Hermitean metric for which the metric tensor is given by $g_{\alpha\bar\beta} = \frac{\partial^2\psi}{\partial z^\alpha \partial \bar z^\beta}$ outside a compact set, and X has a finite volume.

(A2) Tensor fields "constructed" by successive covariant derivatives of ψ are, in the pointwise norm defined by the metric, ~~are~~ bounded.

(A3) $\exists\ k_1 > 0$ such that

$$\sum g_{\alpha\bar\beta} u^\alpha \bar u^\beta \geq k_1 \cdot \left|\sum \frac{\partial\psi}{\partial z^\alpha} u^\alpha\right|^2 \quad \text{for } {}^\forall u \in \mathbb{C}^n$$

outside a compact set.

(B) $\exists\ k_2 > 0$ (const) such that

$$\sum g^{\bar\beta\alpha} \frac{\partial\psi}{\partial z^\alpha} \frac{\partial\psi}{\partial \bar z^\beta} \geq k_2,$$

then X will be an spc domain, (possibly when provided ~~another~~ further suitable conditions.)

I gave supporting evidences for the statement.

Shigeo Nakano (Kyoto, Japan)

Positivity properties of the direct image of powers of dualizing sheaves

Let $f: V \to W$ be a fibre space, i.e. a surjective morphism of projective non-singular varieties (over $\mathbb{C}$) with a connected general fibre $V_\omega = V \times_W \operatorname{Spec}(\overline{\mathbb{C}(W)})$. Let L be a minimal field of definition for V_ω (up to birational equivalence) containing $\mathbb{C}$, and $\operatorname{Var}(f) = \operatorname{trdeg}_{\mathbb{C}}(L)$.

We consider the following question:

Q: If $\operatorname{Var}(f) = \dim W$, does there exist numbers $\mu, \gamma > 0$, an ample invertible sheaf $\mathcal{M}$ and an inclusion $\bigoplus^{r} \mathcal{M} \hookrightarrow \hat{S}^{\gamma}(f_* \omega^{\mu}_{V/W})$, where $r = \operatorname{rk}(\hat{S}^{\gamma}(f_* \omega^{\mu}_{V/W}))$ and $\hat{S}^{\gamma}(\ )$ = "the reflexive hull of the symmetric product"

The answer is "yes" in each of the following cases:

a) If for some $k > 0$. $f_* \omega^{k}_{V/W}$ contains an ample invertible sheaf.

b) If $\kappa(\det(f'_* \omega^{k}_{V'/W'})) = \dim W$ for all fibre spaces $f: V' \to W'$ with $\overline{\mathbb{C}(W)} = \overline{\mathbb{C}(W')}$ and $V'_\omega \simeq V_\omega$

c) If V_ω is a curve

d) If V_ω is a surface and W a curve

e) If $\kappa(V_\omega) = 0$ and $\mathcal{O}_{V_\omega} = \omega^{\mu}_{V_\omega}$ for some $\mu > 0$

f) If $\kappa(V_\omega) = \dim V_\omega$ and $\omega^{\mu}_{V_\omega}$ generated by its global sections for some $\mu > 0$

Eckart Viehweg (Mannheim)

Differential geometry of projective space curves

The only known explicit formulas for the sectional curvature of subvarieties $M \subset \mathbb{P}_n(\mathbb{C})$ concern hypersurfaces (Vitter 1974, Linda Ness 1977). Here we treat the case of a curve $M \subset \mathbb{P}_n$.

There is a moving frame associating to every $p \in M$ a basis of $\mathbb{C}^{n+1}$ and a sequence of determinants $\Delta_0, \ldots, \Delta_n$ with nonnegative real values.

For $n=3$ we have expressions

$$\sigma = \frac{\Delta_1}{\Delta_0^2} \quad \text{"surface factor"}$$

$$\kappa = \frac{\Delta_0^3}{\Delta_1^3} \Delta_2 \quad \text{"curvature"}$$

$$\tau = \frac{\Delta_0^2}{\Delta_2^2} \Delta_3 \quad \text{"torsion"}.$$

Results: 1) $1-\kappa$ = gaussian curvature of the real surface

2) The orders of zero of κ and τ correspond to the local numerical invariants of the curve

3) σ, κ and τ (functions with values in $\mathbb{R}_+$) determine the curve up to an isometry of $\mathbb{P}_3$.

Gerd Fischer (Düsseldorf)

An application of the Calabi-Yau-theorem

According to a theorem of W. Fischer and H. Grauert, a family of isomorphic compact complex manifolds is locally trivial with respect to the base. The result was generalised by H.W. Schuster to families of compact complex spaces over a reduced base. The following problem is known: Given two families $f: X \to S$, $g: Y \to S$ (S a reduced space) of compact complex manifolds with isomorphic fibres: $X_s \xrightarrow{\sim} Y_s$ for all $s \in S$; are X and Y locally isomorphic over S? Examples show that one has to impose the additional assumption $\dim \mathrm{Aut}(X_s) = \text{const}$. The problem was solved by J. Wehler for families of compact tori or compact manifolds with strongly negative (sectional) curvature. By a theorem of V. Palamodov about the universality of a versal deformation with respect to deformations with $\dim \mathrm{Aut}(X_s) = \text{const}$. and the representability of the functor $\mathrm{Isom}_S(X,Y)$ (Schuster) one shows that it suffices to consider families over smooth curves. By a theorem of Fujiki (properness of the components of the relative Barlet-space) in the case of Kähler mappings one may assume the existence of a bimeromorphic mapping, which is biholomorphic everywhere except for a distinguished fibre. We prove:

Theorem 1: Let $f: X \to S$ and $g: Y \to S$ be families of compact complex manifolds with negative first Chern-class. Then X and Y are locally isomorphic over S.

Theorem 2: Let $f: X \to S$ and $g: Y \to S$ be Kähler families of compact complex manifolds with first Chern-class equal to zero and $\dim \mathrm{Aut}(X_s) = \text{const}$. We assume the existence of volume-preserving isomorphisms $X_s \xrightarrow{\sim} Y_s$ for all s. Then X and Y are locally isomorphic over S.

The theorems of Calabi-Yau yield families of Kähler metrics such that the constructed meromorphic mappings are isometries outside the distinguished fibres. The proof then follows from:

Theorem 3: In a family of compact Kähler manifolds the solutions of the Calabi-Yau problems depend continuously on the parameter.

An example of families of K3 surfaces by Atiyah (communicated by F. Catanese) shows the necessity of the additional assumptions in thm 2.

Corollary: In the situation of theorem 1 and theorem 2 the components of $\mathrm{Aut}(X/S) \longrightarrow S$ are proper over S.

Georg Schumacher (Münster)

Reflexive sheaves on $\mathbb{P}^n$

(joint work with Heinz Spindler) We proved the following theorems

Theorem 1: E loc. free sheaf on $\mathbb{P}^2$ with splitting type $(a_1, \dots, a_s; r_1, \dots, r_s)$. N graded $k[X_0, X_1, X_2]$ submodule of $\bigoplus H^1(E(l))$, R graded quotient with $n_l = \dim_k N_l$, $r_l = \dim_k R_l$. Then we have

i) $n_l \leq n_{l+1}$ $\quad l \leq -a_s - 2$ $\qquad$ ii) $n_l < n_{l+1}$ $\quad l \leq -a_s - 3$ $\quad n_l \neq 0$

i') $r_{l+1} \leq r_l$ $\quad l \geq -a_1 - 2$ $\qquad$ ii') $r_{l+1} < r_l$ $\quad l \geq -a_1 - 1$ $\quad r_{l+1} \neq 0$

Theorem 1 is used to prove

Theorem 2: F reflexive on $\mathbb{P}^3$ with splitting type $(a_1, \dots, a_s; r_1, \dots, r_s)$, then there exists a uniquely determined sequence $k_F = (k_1, \dots, k_m)$

$$k_1 \leq k_2 \leq \dots \leq k_{m'} \leq a_1 \leq a_s \leq k_{m'+1} \leq \dots \leq k_m$$

with i) $h^1(F(l)) = h^0(\bigoplus \mathcal{O}_{\mathbb{P}^1}(k_i + l + 1))$ $\quad l \leq -a_s - 1$

ii) $h^2(F(l)) = h^1(\bigoplus \mathcal{O}_{\mathbb{P}^1}(k_i + l + 1))$ $\quad l \geq -a_1 - 3$

We call k_F the spectrum of F. It has the following properties:

i) F loc. free: $k_{F^\vee} = (-k_m, \dots, -k_1)$

ii) k_F is connected

iii) k_F is bounded

iv) for $a_s - a_1 \leq 2$ we can calculate $\sum k_i$ in terms of num. invariants.

Using these techniques one can prove

Theorem 3: For every $d \geq 2$ there exists a stable reflexive rk-3-sheaf F on $\mathbb{P}^n$, $n \geq 3$ with $c_1 = -1$, $c_2 = d$, $c_3 = d^2 - 2d + 2$. It has $hd(F) \leq 1$, $c_t(F) = \dfrac{(1-t)^4 (1 + (1-d)t)}{(1-2t)(1-dt)}$. The moduli scheme ${}^{st}M^3_{\mathbb{P}^n}(-1, d, d^2 - 2d + 2, \dots)$ is irreducible and contains no locally free points for $n > 3$. For $n = 3$ locally free points are dense.

Chr. Okonek

Classifying singularities via deformation theory

We work in the category of germs of complex analytic spaces.
Let $X \subseteq \mathbb{C}^n$ be a germ. Then X is said to be <u>of type</u> $Y \subseteq \mathbb{C}^N$, if there is a map $\varphi: \mathbb{C}^n \to \mathbb{C}^N$ s.t. $X = Y \times_{\mathbb{C}^N} \mathbb{C}^n$ and φ is <u>transversal</u> to the inclusion of $Y \subseteq \mathbb{C}^N$, i.e. $\mathrm{Tor}_i^{\mathcal{O}_{\mathbb{C}^N}}(\mathcal{O}_Y, \mathcal{O}_{\mathbb{C}^n}) = \begin{cases} \mathcal{O}_X & i = 0 \\ 0 & i > 0 \end{cases}$.
(In case Y is Cohen-Macaulay, transversality is equivalent to $\mathrm{codim}_{\mathbb{C}^n} X = \mathrm{codim}_{\mathbb{C}^N} Y$).

We want to study those deformations of X, $\tilde{X} \to T$, which are flat and of type Y, i.e. $\exists$ a comm. diagram

$$\begin{tikzcd} & Y \arrow[dd] & \\ \tilde{X} \arrow[ur] \arrow[dd] & & X \arrow[ll] \arrow[ul] \arrow[dd] \\ & \mathbb{C}^N & \\ \mathbb{C}^n \times T \arrow[ur] & & \mathbb{C}^n \arrow[ll] \arrow[ul, "\varphi"'] \end{tikzcd}$$

whose faces are cartesian.

Let $\psi: Y \times \mathbb{C}^n \xrightarrow{\mathrm{incl.} \times \varphi} \mathbb{C}^N \times \mathbb{C}^N \xrightarrow{\text{"}-\text{"}} \mathbb{C}^N$ be the <u>diagonalization</u> of φ.
Then we show:

<u>Theorem</u>: If X is a singularity of type Y, the following assertions are equivalent:

(α) Each first-order deformation of X is of type Y

(β) Each deformation of X is of type Y

(γ) Y is rigid & X is unobstructed (i.e. there are no obstructions to lift deformation of X whatsoever)

(δ) $T^1_Y(\mathcal{O}_X) = 0$

(ε) Y is rigid & the natural restriction $N_{Y/\mathbb{C}^N} \to N_{X/\mathbb{C}^n}$ is surjective

(η) Y is rigid & φ is transversal to D_Y, the <u>Auslander-module</u> of Y
(If $F_1 \xrightarrow{\chi} F_0 \to M \to 0$ is a finitely free repr. $\mathcal{O}_Y$-module, $D_Y(M) = \mathrm{Coker}\, \chi^*$;
$D_Y = D_Y(I_Y/I_Y^2)$, I_Y the ideal of Y in $\mathbb{C}^N$. φ transversal to $D_Y \Leftrightarrow \mathrm{Tor}_i^{\mathcal{O}_Y}(\mathcal{O}_X, D_Y) = 0_i\ i>0$)

If $\dim_{\mathbb{C}} T^1_X < \infty$, the foregoing conditions are equivalent to

(θ) $\exists$ an unfolding of ψ, which is a versal deformation of X.

As a Corollary we get: A red. singularity X has $T^2_X = 0$ iff X and any "Cartier-divisor" $(t=0) = X_t \subseteq X$ is unobstructed.
($\Leftrightarrow$ t a NZD in $\mathcal{O}_X$)

At the end we discuss shortly the consequences of the theorem for classification theorems for singularities and show that the existing results can be interpreted as obtaining classes of singularities Y, s.t. for every specialization of the resulting sing. X of type Y satisfies the theorem. Such singularities Y are called very rigid and we give some characterizations in terms of the cohomology of the cotangent-complex $\mathbb{L}_Y^\cdot$.

Ragnar-Olaf Buchweitz
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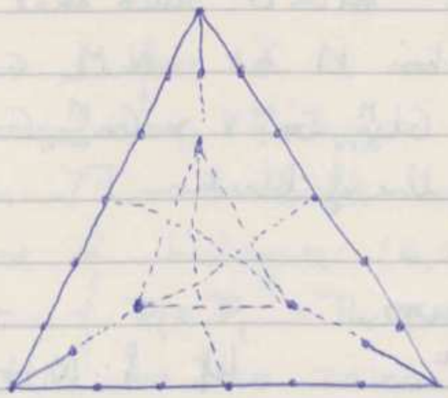
A variant of the Baily-Borel construction pertaining to K3 surfaces

Consider the moduli space M_d of primitively polarized K3 surfaces of degree $2d$ (we allow rational double points). One knows that M_d can be naturally identified with an orbit space of the form Ω/Γ, where Ω is a symmetric domain of type IV and Γ is an arithmetic group acting on it. The K3's for which the polarization does not define an embedding in $\mathbb{P}^{d+1}$ $(d>1)$ or a finite map to $\mathbb{P}^2$ $(d=1)$ form a Cartier divisor D on M_d. The pre-image D in Ω under the composite $\Omega \to \Omega/\Gamma \cong M_d$ is a union of 'hyperplanes' Σ in Ω. For $d=1,2$ Jayant Shah has constructed a compactification $\hat{M}_d$ of M_d by means of geometric invariant theory ~~with~~ has the property that the closure of D in $\hat{M}_d$ is a Cartier divisor.

The aim of the talk you give an 'arithmetic' description of Shah's compactification. This description is solely in terms of Ω, Γ and the union of hyperplanes Σ and makes sense in a more general content (in particular it makes sense for all d). The resulting compactification has (always) the property that the closure of D ~~which~~ is Cartier - presumably it is obtained from the Baily-Borel compactification by blowing up the closure of D in it. The verification that our compactification coincides with Shah's requires a detailed analysis of the groups Γ involved. For the case $d=1$ (which is considerably less involved than the case $d=2$), the following Dynkin diagram appears to play an important rôle



– it describes all the degenerate K3 surfaces of degree 2 in the boundary of M_1

Eduard Looijenga
Nijmegen – Niederlande

The degeneration of the variation of mixed Hodge structures ~~a~~ (V of MHS) is diagonal.

In WEIL II, Publ IHES, 1981, Deligne ~~ask~~ prove the following proposition. Let V be an object of an abelian ~~variety~~ category, W a finite increasing filtration on V, N a nilpotent endomorphism respecting W, then there exists at most one finite increasing filtration M such that $N M_i \subset M_{i-2}$ and N^b induce an isomorphism $Gr^M_{a+b} Gr^W_i V \simeq Gr^M_{a-b} Gr^W_i V$.

Then define a V of MHS on an analytic variety S by the following:

1) $\mathbb{V}_{\mathbb{Z}}$ a local system of free $\mathbb{Z}$-modules on S
2) an increasing filtration finite $\mathbb{W}$ on $\mathbb{V}_{\mathbb{Q}} = \mathbb{V}_{\mathbb{Z}} \otimes \mathbb{Q}$ by sub-local systems of $\mathbb{Q}$ vector spaces.
3) A decreasing filtration $\mathbb{F}$ on $\mathbb{V} = \mathbb{V}_{\mathbb{Z}} \otimes_{\mathbb{Z}} \mathcal{O}_S$ by sub-bundles s.t $\nabla \mathbb{F}^i \subset \Omega^1_S \otimes \mathbb{F}^{i-1}$ and for each point $s \in S$ the fibers of $\mathbb{W}$ and $\mathbb{F}$ define a MHS.

Consider a V of MHS : $(\mathbb{V}, \mathbb{W}, \mathbb{F})$ on $D^* = D - \{0\}$ a punctured disc in $\mathbb{C}$. V the fiber of $\mathbb{V}$ at $t \in D^*$, T = the monodromy on V. Suppose T unipotent, then $N = \operatorname{Log} T$ is nilpotent.

Deligne asks the following Pb (WEIL II) :
Define a class of good V of MHS ~~st~~ on D^* such that for V good and unipotent there exists a filtration M s.t $N M_a \subset M_{a-2}$ and N^b induce an isomorphism : $Gr^M_{a+b} Gr^W_i V \simeq Gr^M_{a-b} Gr^W_i V$
Study also the behaviour of the filtration F.

We solve the problem as follows
Let

$$\begin{array}{ccccc} Y_\bullet & \longrightarrow & X_\bullet & \longleftarrow & \bar{X}^*_\bullet \\ \downarrow & & \downarrow \pi & & \downarrow \\ Y & \longrightarrow & X & \longleftarrow & \bar{X}^* \\ \downarrow & & \downarrow & & \downarrow \\ 0 & \longrightarrow & D & \longleftarrow & \bar{D}^* \end{array}$$

Let $f : X \longrightarrow D$ be a family of proper algebraic varieties and $X_\bullet$ be a simplicial resolution of X.

Then ~~$\bar{X}^*_\bullet \to$~~ $X_\bullet \to D$ is a simplicial ~~resolution~~ Variations of Hodge structures in the classical sense. By Steenbrink's construction we get a complex $(A^\bullet_{X_i}, W_i, F)$ for each X_i
Then we consider

$$A^\bullet_X = R\Pi_* A^\bullet_{X_\bullet} = \oplus_i R\Pi_* A^\bullet_{X_i} \quad , \quad F/A^\bullet_X = \oplus_i F/A^\bullet_{X_i}$$

$$W^l/A^\bullet_X = \oplus_i W_{n+i}/A^\bullet_{X_i} \quad \left(W^l = \delta(W, L) \text{ in Hodge III by Deligne is the diagonal filtration} \right)$$

Let W^f the increasing filtration on $A^\bullet_X$ s.t.

$$W^f_{-i} A^\bullet_X = L^i A^\bullet_X = \oplus_{i' \geq i} R\Pi_* A^\bullet_{X_{i'}}$$

Then $(A^\bullet_X, W^l, F, W^f)$ satisfy axioms we call for Limit Mixed Hodge Complex. $H^n(Y, A^\bullet_X) \simeq H^n(\bar{X}^*, \mathbb{C})$.
and we deduce filtrations W^l, W^f, F on cohomology $H^n(\bar{X}^*, \mathbb{C})$.

Theorem: Let $(H^n(\bar{X}^*, \mathbb{C}), W^l, W^f, F)$ be as above, then
1) (W^l, F) is a mixed Hodge structure (MHS). It is the limit one.
2) W^f is the filtration corresponding to the trivialisation on $H^n(\bar{X}^*, \mathbb{C})$ of the weight filtrations on the fibers $H^n(X_t, \mathbb{C})$ for $t \in D^*$
It is a filtration by sub-MHS.
3) Consider the complex $Gr^W_j H^i(\bar{X}^*_\bullet, \mathbb{C})$ equal in degree m to $Gr^W_j H^i(\bar{X}^*_m, \mathbb{C})$
then we have (*) $Gr^{W^l}_j Gr^{W^f}_i H^n(\bar{X}^*, \mathbb{C}) \simeq H^{n-i}(Gr^W_j H^i(\bar{X}^*_\bullet, \mathbb{C}))$
4) We deduce from (*) that N^b induce an isomorphism

$$Gr^{W^l}_{a+b} Gr^{W^f}_a H^n(\bar{X}^*, \mathbb{C}) \simeq Gr^{W^l}_{a-b} Gr^{W^f}_a H^n(\bar{X}^*, \mathbb{C})$$

The above case include the case of a Normal Crossing divisor, its dual is the case of smooth varieties, then we get the case of quasi-projective morphism.

Fouad ELZEIN, univers. Paris VII - France

Automorphisms of Enriques surfaces

An Enriques surface X is a quotient of some K3-surface Y by an involution without fixed points. In Horikawa's representation Y is a double cover of $\mathbb{P}_1 \times \mathbb{P}_1$ branched along a curve of bidegree $(4,4)$ invariant under an involution τ of $\mathbb{P}_1 \times \mathbb{P}_1$. This representation is used to compute $\mathrm{Aut}(X)$ in two cases. For general X one finds that $\mathrm{Aut}(X)$ equals the 2-congruence subgroup of the orthogonal group of its cohomology lattice $H^2(X,\mathbb{Z}) = \mathbb{Z}^{10} \times \mathbb{Z}_2$. For a special 2-dimensional family of surfaces the result is $\mathrm{Aut}(X) = \mathbb{Z}_2 \times D_\infty$, D_∞ the infinite dihedral group. So there is no semi-continuity for automorphisms of surfaces. As an application one proves: The general Enriques surface admits 129 515 520 different representations as sextic surface in $\mathbb{P}_3$ passing doubly through the edges of a tetrahedron.

Wolf Barth (Erlangen)

Fibrés vectoriels holomorphes sur $\mathbb{P}_2$ et $\mathbb{P}_3$, topologiquement triviaux

On a présenté les résultats suivants:
- Tout fibré vectoriel holomorphe de rang 2 sur $\mathbb{P}_2$, topologiquement trivial et ayant splitage générique $(1,-1)$ ou $(2,-2)$, est déformation (locale) du fibré trivial.
- Il existe sur $\mathbb{P}_3$ un fibré vectoriel holomorphe de rang 2, topologiquement trivial, qui n'est pas déformation (même globale, avec paramètre lisse) du fibré trivial.
- La classification des fibrés de rang 2 sur $\mathbb{P}_3$, topologiquement triviaux et avec splitage $(2,-2)$ (le type $(1,-1)$ n'est pas possible).

Constantin Bănică (Bucarest)

Boundary behavior of finite holomorphic mappings

The following theorem independently proved by Bell/Catlin and Diederich/Fornæss was explained

Theorem. Let $\Omega_1, \Omega_2 \subset\subset \mathbb{C}^n$ be pseudoconvex domains with smooth boundaries and such that Ω_1 satisfies condition R for it's Bergman projection. Then every finite holomorphic mapping $f: \Omega_1 \to \Omega_2$ extends smoothly up to the boundary.

As a consequence one obtains non-existence theorems for branching.

Klas Diederich (Wuppertal)

Locally free resolutions of sheaves on surfaces

It is an open problem if each coherent sheaf on a compact complex space X is quotient of a holomorphic vector bundle (i.e. a locally free coherent sheaf). This is known to be true if X is projective algebraic or algebraic and nonsingular. Here the case of a smooth analytic surface X is treated.

Theorem Every coherent sheaf on a surface is quotient of a holomorphic vector bundle.

The proof makes use of Kodaira's classification of surfaces

Hans Werner Schuster (München)

Geometrie

5.9. – 11.9. 1982

Diskrete Spiegelungsgruppen in höherdimensionalen hyperbolischen Räumen

Eine Spiegelungsgruppe des sphärischen, euklidischen, oder hyperbolischen Raumes ist eine von den Spiegelungen an endlich vielen Hyperebenen des entsprechenden Raumes erzeugte Gruppe. Die diskreten Spiegelungsgruppen der Sphären und euklidischen Räume wurden von Coxeter untersucht und klassifiziert. Eine allgemeine Theorie diskreter Spiegelungsgruppen der hyperbolischen Räume sowie der zugehörigen Fundamentalpolyeder wurde von Vinberg entwickelt, aber eine Klassifikation dieser Gruppen ist gegenwärtig ausser Reichweite.

Wir versuchen, möglichst viele Beispiele von diskreten Spiegelungsgruppen des hyperbolischen Raumes H^n mit Fundamentalpolyedern endlichen Volumens zu finden. Eine Klasse solcher Gruppen ist von Coxeter betrachtet worden. Es sind die Symmetriegruppen gewisser regulärer Tessellationen des H^n; ihre Fundamentalpolyeder sind Orthoscheme (d.h. rechtwinklige Simplices) mit eventuell unendlichen Hauptecken. Lässt man ein Orthoschem so gross werden, dass eine oder beide seiner Hauptecken ideale Punkte werden, so entstehen Polyeder unendlichen Volumens. Endliches Volumen wird erreicht durch Abschneiden der unendlichen Teile mit Hilfe der zu den Hauptecken polaren Hyperebenen. Es entstehen so Polyeder, die durch $n+2$ oder $n+3$ Hyperebenen begrenzt sind. In einigen Fällen (für $n \leq 9$) gelingt es, Polyeder endlichen Volumens zu konstruieren, deren Diederwinkel von der Form $\frac{\pi}{p}$, $n \in \mathbb{N}$, $n \geq 2$, sind. Diese Polyeder führen zu Spiegelungsgruppen von der gesuchten Art.

Hans-Christoph Im Hof (Bonn)

Minimalflächen in $\mathbb{C}P^2$

Es wurde eine lokale Kennzeichnung von minimalen Immersionen $f: M^2 \to \mathbb{C}P^2$ (komplex-projektive Ebene) gegeben mit Hilfe von drei Funktionen auf der Fläche M: Krümmung, Normalkrümmung und Kählerform / Volumenform. Insbesondere lassen sich diejenigen Metriken auf M kennzeichnen, die eine minimale isometrische Immersion in $\mathbb{C}P^2$ gestatten. Als globale Anwendung erhält man für kompakte Flächen u.a. Beziehungen zwischen dem Abbildungsgrad von f sowie den Euler-zahlen von Tangential- und Normalbündel.

J.H. Eschenburg

Univ. Münster

Riemannsche Mannigfaltigkeiten mit lokal beschränkter Krümmung

Definition. Die Schnittkrümmung einer riemannschen Mannigfaltigkeit M heisst lokal δ-beschränkt falls eine positive Funktion $A: M \to \mathbb{R}$ existiert sodass für jedes $x \in M$ gilt $\delta A(x) < K < A(x)$ (K = Krümmung einer Ebene in $x \in M$).

Das bekannte "δ-pinching" unterscheidet sich von dem oben definierten "local δ-pinching" dadurch, dass dort die Funktion $A: M \to \mathbb{R}$ konstant ist. Der Zweck dieser Definition ist eine qualitative Version des bekannten Lemmas von Schur zu formulieren.

Satz. $\exists\, \delta = \delta(n) < 1$ sodass für jede kompakte riemannsche Mannigfaltigkeit mit lokal δ-beschränkter Krümmung und Dimension $n \geq 3$ gilt: M ist diffeomorph zu einer sphärischen Raumform $\Gamma\backslash S^n$.

Der Beweis stützt sich auf die Bianchi-Gleichung sowie die bekannte Ungleichung von Calderón-Zygmund aus der Analysis. Die Idee ist den Levi-Civita Zusammenhang so zu deformieren, dass die Voraussetzungen der Vergleichssätze von Min-Oo-Ruh erfüllt sind.

Ernst Ruh (Bonn)

Ein elementarer Beweis des Splitting-Satzes von Cheeger-Gromoll

Der Satz von Cheeger und Gromoll lautet: Ist M vollständige Riemannsche Mannigfaltigkeit mit $\mathrm{Ric} \geq 0$ und enthält M eine Gerade (d.h. eine Geodätische, die den Abstand je zweier auf ihr liegender Punkte realisiert) so spaltet M isometrisch als $M' \times R$.

Der gemeinsam mit J. Eschenburg gefundene Beweis geht folgendermaßen vor:

1. Lemma M vollst., $\mathrm{Ric} \geq 0$, $f \in C^2(M)$ mit $\|\mathrm{grad} f\| = 1$ $\Rightarrow$ M spaltet als $M' \times \mathbb{R}$ isometrisch.
2. Die zu einer Geraden gehörenden Busemannfunktionen $b_\pm$ besitzen C^∞-Stützfunktionen $b^\pm_{p,\varepsilon}$ mit $\Delta b^\pm_{p,\varepsilon} \geq -\varepsilon$
3. Aus $b_+ + b_- \leq 0$ und $b_+ + b_- = 0$ auf der Geraden folgt mit einer Verallgemeinerung des Hopfschen Maximumprinzips $b_+ + b_- \equiv 0$ und damit $b_\pm$ C^1 und $\|\mathrm{grad}\, b_\pm\| = 1$
4. $b_\pm$ sind C^2

E. Heintze (Münster)

Verallgemeinerte Regelflächen mit geradlinigen Leitkurven

$(k+1)$-Regelflächen Φ werden erzeugt von einer ein=parametrigen Schar k-dimensionaler Teilräume $E_k(t)$ des n-dimensionalen euklidischen Raums E_n. Die Erzeugenden $E_k(t)$ besitzen entweder einen $(k-m)$-dimensionalen Zentral=raum oder einen $(k-m)$-dimensionalen Kehlraum. Fallen die Kehlräume nicht zusammen, so bilden sie die Kehlregelfläche Π von Φ. Die Zentralräume bilden die Zentralregelfläche Ω. Weitere Begleitregelflächen von Φ sind die Hauptregelflächen Λ_y. Sie besitzen die Leit=kurve $y(t)$ und als Erzeugende das orthogonale Komplement in $E_k(t)$ des Zentralraums beziehungsweise des Kehlraums von $E_k(t)$.
Es wurden unter anderem Kriterien angegeben für die Existenz geradliniger Orthogonaltrajektorien (der Erzeugenden) von Φ. Ferner wurden geradlinige Leitkurven $y(t)$ untersucht, welche Orthogonaltrajektorien der Haupt=regelfläche Λ_y sind, sowie geradlinige Isogonal=trajektorien von Φ. Neben anderen wurde folgender Satz bewiesen: Ist Φ eine $(k+1)$-Regelfläche mit einer k-dimensionalen Zentralregelfläche Ω, so gilt: Eine geradlinige Leitkurve $y(t)$ von Φ ist Isogonaltrajektorie von Φ genau dann, wenn sie Leitkurve von Ω oder Orthogonaltrajektorie der Hauptregelfläche Λ_y ist.

Günter Aumann (München)

Über zwei altbekannte Klassen von Raumkurven

Trägt man über den Punkten X einer ebenen Kurve k' ein konstantes Vielfaches der vom Ortsvektor $\vec{OX}$ überstrichenen Fläche, gemessen von einer Anfangslage OX_0 aus, jeweils senkrecht zur Kurvenebene auf, so erhält man die Punkte einer Raumkurve k, die als Gewindekurve (S. Lie) mit dem Grundriß k' nachgewiesen wird. Die gleiche Erzeugungsweise führt zu einer Böschungslinie, wenn man ein konstantes Vielfaches der Bogenlänge des von X durchlaufenen Bogenstückes von k' abträgt. – Ist die Ausgangskurve k' geschlossen, so gelangt man in beiden Fällen zu Integralinvarianten, die als Ganghöhen der erzeugten Gewindekurve bzw. Böschungslinie anzusprechen sind und bis auf einen konstanten Faktor durch den orientierten Flächeninhalt bzw. den Umfang von k' dargestellt werden.

Die kinematische Erzeugung von k' als Bahnkurve eines Punktes bzw. Hüllbahn einer Geraden der Gangebene (bei einem geschlossenen Zwanglauf in der Grundrißebene) führt zu linearen Beziehungen zwischen den Ganghöhen der erzeugten Raumkurven, wenn man im Fall der Gewindekurven von drei kollinearen Punkten ausgeht und, auf der Steinerschen Formel fußend, Beziehungen zwischen den Bahninhalten heranzieht, bzw. im Fall der Böschungslinien auf entsprechende Relationen zwischen den Hüllbahn-Umfängen dreier Geraden der Gangebene zurückgreift.

Hans Robert Müller
(Braunschweig)

Verallgemeinerte Regelflächen und ihre Kommerellhyperflächen

Mit den nichttotalgeodätischen Untermannigfaltigkeiten des E^n sind algebraische Hyperflächen in den Normalenräumen invariant verbunden, die in Verallgemeinerung der Kommerellkegelschnitte für Flächen der Codimension 2 definiert sind. Diese algebraischen Hyperflächen werden als Kommerellhyperflächen bezeichnet. Sie sind für $(k+1)$-Regelflächen entweder Hyperebenen oder Quadriken.

Satz: Eine nichtzylindrische $(k+1)$-Regelfläche Φ besitzt genau dann Zentralräume als ausgezeichnete Teilräume, wenn in jedem Normalenraum von Φ ein Teilraum existiert, der aus der jeweiligen Kommerellquadrik ein m-dim. Ellipsoid ausschneidet.

Satz: Die Kommerellquadriken einer nichtzylindrischen $(k+1)$-Regelfläche Φ haben genau dann den jeweiligen Flächenpunkt als Mittelpunkt, wenn Φ minimal in E^n ist.

Hans Hagen (Dortmund)

Über die Gauß-Gleichung und die isometrische Immersierbarkeit Riemannscher Räume in $\mathbb{R}^n$

Es wird ein Algorithmus angegeben, mit dem man die Gauß-Gleichung
$R = \Lambda^2 h_1 + \dots + \Lambda^2 h_l$ $\left(\Lambda^2 h_i(X\wedge Y, Z\wedge W) := h_i(X,Z)h_i(Y,W) - h_i(Y,Z)h_i(X,W)\right)$
algebraisch und mit $l = \binom{n}{2}$ (vgl. Schlaefli-Janet-Burstin-Cartan), $n=\dim(V)$, lösen kann; dabei bezeichnet V einen n-dimensionalen reellen Vektorraum und R eine symmetrische Bilinearform auf $\Lambda^2 V$, welche der 1. Bianchi-Identität genügt; die zu bestimmenden h_i sind symmetrische Bilinearformen auf V. Der Schlüssel zur Lösung liegt

in der exakten Sequenz $0 \to \mathcal{S} \to \mathcal{T} \xrightarrow{\vartheta} \mathcal{B} \to 0$, wobei $\mathcal{B} \subset L(V^4;\mathbb{R})$ den Raum der Krümmungstensoren bezeichnet, $\mathcal{T} := \{\phi \in L(V^4;\mathbb{R});\ \phi(X_1,X_2,X_3,X_4) = \phi(X_2,X_1,X_3,X_4) = \phi(X_3,X_4,X_1,X_2)\}$, $\mathcal{S} := \{\phi \in L(V^4;\mathbb{R});\ \phi(X_{\sigma_1},X_{\sigma_2},X_{\sigma_3},X_{\sigma_4}) = \phi(X_1,X_2,X_3,X_4)\ \forall \sigma \in \mathcal{S}_4\}$ und ϑ durch $(\vartheta\phi)(X_1,X_2,X_3,X_4) := \phi(X_1,X_3,X_2,X_4) - \phi(X_2,X_3,X_1,X_4)$ definiert ist.

S. Steiner (Stuttgart)
E. Teufel (Stuttgart)

Ein lineares Konstruktionsprinzip in der Raumstatik

In Verallgemeinerung eines Satzes von A. Möbius wird zunächst gezeigt, daß es für Kräfte $\mathfrak{f}_1, \mathfrak{f}_2, \ldots, \mathfrak{f}_j$ $(2 \leqq j \leqq 6)$ im dreidimensionalen euklidischen Raum E_3 genau 35 inäquivalente Gleichgewichtsfälle gibt. Bei der konstruktiven Lösung verschiedener Fragestellungen wird unter Anwendung einer Raumkollineation eine Gerade des Trägers der entsprechenden Komplexmannigfaltigkeit auf eine Ferngerade abgebildet, sodaß man Methoden aus der Theorie des Flaggenraumes anwenden kann. Mit Hilfe der ersten und zweiten Schließungsbedingung für vollisotrope und isotrope Gewinde ergeben sich lineare Konstruktionen für die betrachteten Gleichgewichtsfälle. So kann man z.B. zu 4 Kräften auf windschiefen Geraden $g_1,\ldots,g_4$ jene durch einen Punkt P verlaufende Kraft bestimmen, die mit $g_1,\ldots,g_4$ ein Gleichgewichtssystem bildet, auch wenn $g_1,\ldots,g_4$ in einem elliptischen Netz liegen.

H. Sachs (LEOBEN)

Schmiegquadriken, vom Standpunkt der mehrdimensionalen projektiven Geometrie aus behandelt.

Zu Beginn wird darüber berichtet, wie die Geometer seit Plücker (1829) sich mit Fragen der Berührung einer Quadrik Q_2 und Fläche 3. Grades F_2^3 in einem Punkt A_0 beschäftigt haben. Nach Plücker sind hier zu nennen Hesse, Cayley, Lie, Darboux, Gosteaux, Scheffers, Blaschke und Bol. Man gelangte dabei zu dem Begriff der Lie-Quadrik eines Punktes A_0 auf einer beliebigen Fläche F_2, mit der sich fast alle Geometer befasst haben. Die doppelt gezählte Tangentialebene $\not\approx A_2$ ($T_2(A_0)$ bestimmt zusammen mit der Liequadrik $Q_2(A_0)$ das sog. Darboux-Büschel, dem noch weitere ausgezeichnete Quadriken angehören die man Schmiegquadrik und Quadr. von Wilczynski nennt. Der Vortragende hat sich nun vorgenommen, dies Büschel a ~~m~~ für den Fall wo F_2 eine kubische Fläche F_2^3 ist, synthetisch zu erklären. Dazu müssen zuvor einige Tatsachen über die Veronescschen

$V_2^2 \subset P_5$, V_2^3 und $V_3^2 \subset P_9$ und $V_3^3 \subset P_{19}$,

ihre Schmiegräume und Oskulanten sowie damit verbundene Projektionen erklärt werden. Das Darboux-Büschel a wird dann durch ein Hyperebenenbüschel im Schmiegraum $T_9^2(A_0^3)$ für $A_0^3 \in V_3^3$ festgelegt. Dies Büschel wird durch 2 P_8 aufgespannt wovon die eine P_8 die Doppel-tangentialebene der F_2^3 im fraglichen Punkt A_0, die andere die Polarquadrik $F_2^2(A_0)$ von A_0 in Bezug auf F_2^3 definiert. Diese $F_2^2(A_0)$ ergibt sich bei diesem Aufbau leichter als die viel untersuchte Lie-Quadrik. (es ist $F_2^2(A_0)$ die Schmiegquadrik).

Werner Burau
Hamburg.

Schreiben von L. Fejes Toths Wurstvermutung

$B_1^d, \ldots, B_k^d$ seien k Translate (nicht überlappend) der Einheitskugel B^d des E^d (euklidisch); C_k sei die konvexe Hülle ihrer Mittelpunkte, S_k eine Strecke der Länge $2(k-1)$ und V sei das Volumen.
L. Fejes Toths Wurstvermutung von 1975 besagt, daß für $d \geq 5$ gilt:

$$V(S_k + B^d) \leq V(C_k + B^d) \qquad (*)$$

d.h. das Volumen der konvexen Hülle der k Kugeln ist minimal, wenn diese "wurstförmig" angeordnet sind.
In einer gemeinsamen Arbeit von Betke, Gritzmann, Wills (1982) wird gezeigt:
Th. 1 $(*)$ gilt für alle C_k mit $\dim C_k \leq \frac{7}{12}(d-1)$
Th. 2 $(*)$ " " " " $\dim C_k \leq d-1$ und $\dim C_k \leq 3$
In Th. 1 und 2 gilt ~~x~~ "=" in $(*)$ genau dann, wenn $C_k = S_k$.
Th. 2 wurde von Betke + Gritzmann auf $\dim C_k \leq 10$ verbessert.

Weitere Teilergebnisse, verwandte finite Probleme und das analoge finite Überdeckungsproblem werden angegeben.
Außerdem wird gezeigt, daß der Problemkreis dieser finiten Packungs- und Überdeckungsprobleme eine Brücke zwischen Diskreter und Konvex-Geometrie schlägt.

Jörg Wills (Siegen)

Die "tightness" symmetrischer Untermannigfaltigkeiten.

Eine Untermannigfaltigkeit $M^m \subset R^{m+k}$ heißt extrinsisch symmetrisch (ES), wenn sie bei der Spiegelung an jedem ihrer Normalenräume in sich übergeht. Es wurde ein direkter geometrischer Beweis dafür gegeben, daß kompakte, zusammenhängende ES-Untermannigfaltigkeiten tight im Sinne von Chern/Banchoff/Kuiper sind. Der Beweis benutzt Induktion über die Dimension m. Im Induktionsschritt benutzt man die Fixmannigfaltigkeiten einer Normalenspiegelung und einen Satz von Floyd über $\mathbb{Z}_2$-Mannigfaltigkeiten.

Dirk Ferus TU Berlin

Hyperflächen mit einer konstanten höheren mittleren Krümmungsfunktion

Hyperflächen dieser Art in euklidischen Räumen wurden intensiv studiert, zumeist unter Voraussetzung der Kompaktheit und Konvexität (vgl. die klassischen Liebmann/Süß-Kennzeichnungen der Sphären durch die Konstanz einer mittleren Krümmung H_r). Für den nichtkompakten, jedoch vollständigen Fall werden hier in Verallgemeinerung von Resultaten von Klotz/Osserman im $\mathbb{R}^3$ (1966) und Cheng/Yau im $\mathbb{R}^{m+1}$ (1975-77) für $r \leq 2$ die Hyperflächen M des $\mathbb{R}^{m+1}$ mit $H_r = \text{const.} \neq 0$ und $K \geqq 0$ (K = Schnittkrümmung von M) bestimmt. Dies führt auf eine globale Kennzeichnung der orthogonalen sphärischen Zylinder. Des weiteren werden kompakte Hyperflächen einer riemannschen Mannigfaltigkeit $\tilde{M}(c)$ konstanter Krümmung c betrachtet. Die Lösungen des Problems $H_r = \text{const.}$ sind hier unter den Voraussetzungen $K \geqq 0$, $\mathrm{II} \geqq 0$ (II = zweite Fundamentalform), $H_r \neq 0$ für ein $r \in \{2, \ldots, m\}$ bzw. $K \geqq 0$ für $r = 1$ isoparametrische Hyperflächen von $\tilde{M}(c)$ mit höchstens zwei verschiedenen Hauptkrümmungen. Die Beweise stützen sich einerseits auf Eigenwertbetrachtungen elliptischer Differentialgleichungen, zum anderen auf eine Weitzenböck-Formel für H_r, die 1972 von Münzner in Oberwolfach (ansonsten unpubliziert) angegeben wurde und hier in einer neuen Version vorgestellt wird. Beidemale sind gewisse elliptische Nichtstandardoperatoren, die auf K. Voss zurückgehen, involviert.

Rolf Walter (Dortmund)

Mean-value operators in Riemannian manifolds.

On an analytic Riemannian manifold, one can define two different spherical mean-value operators $\mathcal{M}$, $\mathcal{L}$ by the formulas

$$\mathcal{M}_m(r,f) = \int_{S_m(r)} f\,dv_{n-1} \Big/ \int_{S_m(r)} dv_{n-1}$$

$$\mathcal{L}_m(r,f) = \int_{S^{n-1}(1)} (f\circ \exp(ru))\,du \Big/ \int_{S^{n-1}(1)} du$$

where $S_m(r)$ is a geodesic sphere of M centered at m and $S^{n-1}(1)$ is a Euclidean sphere of the tangent space T_mM centered at 0. The formula "$\mathcal{M}_m(r,f) = f(m)$ for all harmonic functions" or "$\mathcal{L}_m(r,f) = f(m)$ for all harmonic functions" characterizes the harmonic spaces. A. Gray and T. J. Willmore have studied analytic Riemannian manifolds satisfying the formula "$\mathcal{M}_m(r,f) = f(m) + O(r^{2k})$ for harmonic functions as $r \to 0$"; they have solved the problem for $k = 2, 3, 4$. In the general case the problem remained open.

The complete solution of the problem (including a complete classification) is given now for locally symmetric spaces. In particular, a locally symmetric

space satisfying the formula $M_m(r,f) = f(m) + O(r^{18})$
for harmonic function must be a rank one ~~space~~
symmetric space or a Euclidean space.
Also, the so called <u>comparison theorem</u> was proved:
If $M_m(r,f) = Z_m(r,f) + O(r^{2k})$, $k \in \mathbb{N}$,
holds on an analytic Riemannian manifold for
<u>all harmonic functions</u>, then the conditions put
on the curvature are so restrictive that the above
formula holds <u>identically</u>.

Oldřich Kowalski (Prague)

"<u>Taut" Untermannigfaltigkeiten</u>

M^{2k} sei eine "taut" kompakte, substantielle Untermannigfaltigkeit in S^N, die $(k-1)$-zusammenhängend ist. Es wurde bewiesen, daß <u>entweder</u> $N = 2k+1$ ist und M^{2k} eine Sphäre oder Dupinsche Zyklide ist <u>oder</u> $N = 3k+1$ und M^{2k} ist homöomorph zu einer der projektiven Ebenen $P_2\mathbb{R}$, $P_2\mathbb{C}$, $P_2\mathbb{H}$ oder $P_2\mathbb{O}$.

G. Thorbergsson (Bonn)

Otto RÖSCHEL:
Ebene Schattengrenzen auf Flächen mit einparametriger projektiver Transformationsgruppe.

Zu einer gegebenen einparametrigen projektiven Trans - formationsgruppe T und einer festen Lichtquelle L existiert im dreidimensionalen projektiven Raum im allgemeinen eine Raumkurve dritter Ordnung k(L), deren Punkte bei T entweder Fixpunkte sind oder Bahntangenten durch L besitzen.

Es wird gezeigt, daß es im allgemeinen in jeder Ebene ε des zugrundegelegten Raumes eine einparametrige Schar von Kurven u mit der folgenden Eigenschaft gibt : u ist auf der Fläche T(u) bei Beleuchtung aus L ebene Schattengrenze. Diese Schar von ebenen Kurven kann auch als Bahnkurvenschar einer gewissen einparametrigen projektiven Transformationsgruppe B der Ebene ε erzeugt werden - es handelt sich daher um die bekannten ebenen W - Kurven Die Pole von B sind dabei im allgemeinen genau die in ε gelegenen Punkte von k(L).

otto wald (Leoben)

<u>Some remarks on the infinitesimal automorphisms of a vector bundle.</u>

Let $E \xrightarrow{\pi} M$ be a vector bundle and let $(\rho, \Gamma(E))$ be a representation of the algebra $\mathfrak{X}(M)$ of vector fields of M on the space $\Gamma(E)$ of sections of E; then $(\rho, \Gamma(E))$ is said to be of <u>connection-type</u> if

$$\rho_X(fs) = (X.f)s + f\rho_X(s), \quad \forall f \in C_\infty(M), \forall s \in \Gamma(E). \forall X \in \mathfrak{X}(M).$$

This notion generalizes the Lie derivatives of tensor-fields as well as the flat covariant derivatives.

Let h_E be the Chern-Weil homomorphism of E and let $i : \Lambda(M) \to \Lambda(\mathfrak{X}(M))$ be the natural inclusion of the de Rham complex of M into the Chevalley complex of $\mathfrak{X}(M)$ associated to the Lie derivative of smooth functions on M. Then :

Thm 1. (Shiga and Tsujishita) If E has a connection type representation, then $i_{\#} \circ h_E = 0$.

One can derive a new proof of this result using the properties of the Lie algebra $\mathcal{U}_E$ of infinitesimal automorphisms of E as follows.

Let us first recall the exact sequence

$$0 \to gl(E) \to \mathcal{U}_E \to \mathcal{X}(M) \to 0 \qquad (S).$$

where $gl(E) = \ker p_*$ is identified whith the algebra of linear endomorphisms of E and $\mathcal{X}(M) = \operatorname{im} p_*$. Then, one obtains easily the following

Lemma 2. There is a natural bijection between the connection type representations $(\rho, \Gamma(E))$ and the splittings of the sequence (S).

Now, let $0 \to I \to L \to L/I \to 0$ be any exact sequence of Lie algebras. Then for each representation (ρ, V) of L/I, one may define a subspace I_ρ of the space of all multilinear mappings $f: \underbrace{I \times \dots \times I}_{p} \to V$ $(p \in \mathbb{N})$ and a mapping $c_\rho : I_\rho \to H_\rho(L/I)$ [the Chevalley cohomology of L/I associated to (ρ, V)] in such a way that

Thm. 3 If $0 \to I \to L \to L/I \to 0$ splits, then $c_\rho = 0$ for each representation (ρ, V) of L/I.

In order to apply thm 3 to (S), we take $V = \Lambda^q(M)$, the space of q-forms on M, ρ = the Lie derivative on q-forms $(q = 0, 1, \dots)$ and we denote by c_q the corresponding c_ρ. Then, one can compute that

Thm 4. If $q > 0$, then $c_q = 0$. If $q = 0$, then $c_q = i_{\#} \circ h_E$.

This suffices to prove thm 1 of Shiga and Tsujishita.

Pierre Lecomte (Liège)

Klassifikation der symmetrischen Untermannigfaltigkeiten des hyperbolischen Raumes

Es wurde über die Klassifikation symmetrischer Untermannigfaltigkeiten der n-dim. Standardräume N^n_κ konstanter Krümmung κ berichtet. Das wesentliche neue Ergebnis betrifft den Fall des hyperbolischen Raumes $N^n_\kappa = H^n_\kappa$; es wurde gemeinsam von E. Backes und dem Vortragenden (unabhängig auch von Takeuchi) erzielt.

Das entscheidende Hilfsmittel sind sog. extrinsische Produkte von N_κ^n, das sind Untermannigfaltigkeiten, die isometrisch zu Riemannschen Produkten $M_1 \times \dots \times M_k$ sind und deren Gestalt (in N_κ^n) jeweils der Produktstruktur angepaßt ist. Grob gesagt sind die symmetrischen Untermannigfaltigkeiten von N_κ^n extrinsische Produkte, bei denen die M_i selbst Standardräume oder aber standard eingebettete symmetrische R-Räume euklidischer Sphären sind.

H. Reckziegel

Nachtrag zu einem Extremalproblem über konvexe Rotationskörper im R_3

Im Vortrag über dasselbe Extremalproblem an der Geometrie-Tagung 1980 konnte die Klasse U nicht einbezogen werden. Die Lücke wird nun hiermit ausgefüllt.

Zunächst zerschneidet man einen unsymmetrischen Kappenkörper längs der Aequatorebene und schiebt eine symmetrische Kugelschicht mit demselben Aequatorradius ein. Zum vornherein soll die Oberfläche des Gesamtkörpers auf einer Seite glatt sein. Die Extremaleigenschaft des Kugelkappenkörpers kann man nun nur dann ausnützen, wenn auch die andere Seite glatt ist. Demnach ist der unsymmetrische Kappenkörper der symmetrischen Linse extremal (Grösstes V bei gegebenem F u. M). 2 unsymmetrische Kappen können aber ohne Änderung der Masszahlen durch 2 kongruente ersetzt werden. Demnach erreichen ihre Bilder wohl den bisherigen inneren Rand, überschreiten ihn aber nicht!

Jetzt betrachtet man noch Körperklassen in einem festen Zylinder der Länge l, wobei Grund- und Deckfläche mindestens einpunktig, der Mantel mindestens in einem Kreis berührt wird! Lässt man sodann l das Intervall $0 \le l < \infty$ durchlaufen, so hat man alle zulässigen Körper erfasst. Zylinder und symmetrischer Doppelkegel zeichnen sich durch maximale bzw. minimale Masszahlen aus, die ausgezeichnete Schar der unsym. Doppelkegel besitzt noch minimales Volumen.

Bei fester Zylinderlänge erhält man als Bild im Blaschke-diagramm einen Bereich, dessen unterer Rand besonders interessiert. Dieser ist nun das Bild der Kegelstumpfschar, die zwischen Kegel und Zylinder interpoliert, dies mit der Einschränkung $l \geq \frac{1+\sqrt{3}}{2}$, denn so besitzen die erwähnten Körper bei festem F und M grösstes V. Sie sind nach Carathéodory <u>starre Extremalen</u> ohne Vergleichskörper. Der gefundene untere Rand überschreitet nun die Kurve der speziellen Kegelstümpfe, die jetzt nicht diskutiert zu werden braucht. (vermutlich ist sie konkav.) Jede von einem l induzierte Randkurve berührt einpunktig!

Weil jetzt noch bekannt ist, dass die Zylinder-Kegelstümpfe (Halbkörper), welche ebenfalls der Ungleichung II von Hadwiger genügen, auch bei kleinem l nicht gefährlich sind, ist das Problem nun vollständig gelöst.

Es verbleibt im allgemeinen Problem (keine Rotationssymmetrie!) das Problem, eine zwischen Kreisscheibe und Kugel interpolierende Körperschar zu finden und die entsprechende Ungleichung aufzustellen, was extrem schwer sein dürfte. Über die gut begründete Vermutung, dass es Kugelkreispolyeder sein werden, ist man bis heute nicht hinausgekommen.

Hans Bieri (CH Wabern.)

<u>Kaustiken und Katastrophen</u>

Die "elementare Katastrophentheorie" lehrt, daß Brennflächen von Normalenkongruenzen im 3-dimensionalen Raum generisch nur wenige Typen von Singularitäten haben können. Die Funktionen der Thom'schen Liste geben sogar Normalformen für die zugehörigen Wellen-funktionen. Diese wurden von Physikern in Bristol (Berry, Nye u. a.) benutzt, um die Einzelheiten der Interferenzfiguren zu untersuchen und mit beobachteten Kaustiken zu vergleichen. Auch die Singularitäten mit höherer Kodimension (mehr als 3 Parametern) geben Einsichten, indem 3 Parameter als Raumdimensionen, die übrigen als physikalische Parameter interpretiert werden. So werden häufig vorkommende Veränderungen und das Zusammenwirken von Singularitäten der Kaustik verständlich. An Hand von Dias wird über diese Ergebnisse aus Bristol berichtet.

Erhard Heil (Darmstadt)

Codazzi-Tensoren auf Räumen konstanter Krümmung

Sei (M,g) geschlossener, zshgd., or. Riemann-Raum konstanter Krümmung K, $f \in C^\infty(M)$. Ein symmetrischer (2.0)-Tensor A heißt Codazzitensor, wenn die kov. Ableitung ∇A ein totalsymm. (3.0)-Tensor ist. D. Ferus (Lect. Notes Math. 838) bemerkte: $H_K(f) := \mathrm{Hess}\, f + K \cdot f \cdot g \in \mathrm{Cod}(M,g) := \{A \mid A \text{ Cod-Tensor}\}$ und lokal kann jeder Cod-Tensor durch eine Funktion erzeugt werden. Für die globale Erzeugung gilt:

<u>SATZ</u>: Sei $\mathrm{Cod}(M,g) \xrightarrow{Tr} C^\infty(M)$ der Spuroperator u. $\bar{L} : C^\infty(M) \mid \mathrm{Kern}\, H_K \to C^\infty(M) \mid \mathrm{Kern}\, H_K$ derjenige lineare Operator, der $\varphi \in C^\infty(M) \mid \mathrm{Kern}\, H_K$ die Lösungsfunktion f von $\Delta f + nKf = \varphi$ zuordnet. Dann gilt: Die folgende Diagramme enthalten Isomorphismen.

$K > 0$:

$$\begin{tikzcd} & C^\infty(M) \mid \mathrm{Kern}\, H_K \arrow[dl, "\bar{H}_K"'] & \\ \mathrm{Cod}(M,g) \arrow[rr, "Tr"] & & C^\infty(M) \mid \mathrm{Kern}\, H_K \arrow[ul, "\bar{L}"'] \end{tikzcd}$$

wobei $\mathrm{Kern}\, H_K = E_1 = \{f \mid \Delta f + nKf = 0\}$ auf $S^n(K)$
$\mathrm{Kern}\, H_K = \{0\}$ sonst; $\bar{H}_K$ induzierte Abb.

$K < 0$:

$$\begin{tikzcd} & C^\infty(M) \arrow[dl, "H_K"'] & \\ \mathrm{im}\, H_K \arrow[rr, "Tr"] & & C^\infty(M) \arrow[ul, "\bar{L}"'] \end{tikzcd}$$

und $\mathrm{Cod}(M,g) \cong \mathrm{Kern}\, Tr \oplus \mathrm{image}\, H_K$

Für $K = 0$ gilt ein ähnliches Resultat.

<u>Bem.</u> Mit Hilfe des obigen Darstellungssatzes lassen sich die Sätze v. Alexandrov-Fenchel-Jessen verallgemeinern, wenn ein Cod-Tensor durch Funktionen erzeugt ist ($A \in \mathrm{im}\, H_K$)

<u>Satz</u>: (M,g) von konst. Krümmung $K \neq 0$, geschlossen. $A, \hat{A} \in \mathrm{Cod}(M,g)$ pos. definit, $A, \hat{A} \in \mathrm{im}\, H_K$. Für ein s seien die s-ten elem.-symm. Funktionen $P_s(A) = P_s(\hat{A}) \Rightarrow A = \hat{A}$.

Die entsprechenden Existenzsätze werden formuliert u. bewiesen.

Die Arbeit erscheint in Kooperation mit V. Oliker (Iowa)

U. Simon

Conformal area of surfaces

A survey lecture was given based on a pre-print of a paper by P. Li and S-T. Yau "A new conformal invariant and its applications to the Willmore conjecture and the first eigenvalue of compact surfaces".

Let M be an m-dimensional compact manifold which admits a conformal map ϕ into the n-dimensional unit sphere S^n. Let ds^2 denote the metric on M and let $d\bar{s}^2$ be the standard metric on S^n. Let G denote the group of conformal diffeomorphisms of S^n.

Definition The n-conformal volume of ϕ is given by

$$V_c(n,\phi) = \sup_{g\in G} \int_M dV_g$$

when dV_g is the volume element associated with the tensor $\phi^* g^* d\bar{s}^2$.

Definition. The n-conformal volume of M is given by

$$V_c(n,M) = \inf_{\phi} V_c(n,\phi)$$

when ϕ runs over all non-degenerate conformal mappings of M into ϕ.

The concept of conformal area is closely related to the spectrum of the Laplacian, to minimal surfaces of S^n and to the value of $\int H^2$ when H^2 is the square of the length of the mean curvature vector.

Result 1. Let M be a compact surface in $\mathbb{R}^n$. Then

$$\int_M H^2 \geq V_c(n,M).$$

Equality implies that M is the image of some minimal surface in S^n under some stereographic projection.

Result 2 Let M be a compact surface homeomorphic to $P^2(\mathbb{R})$. Then for any immersion of M into $\mathbb{R}^n$,

$$\int_M H^2 \geq 6\pi.$$

Equality implies that M must be ~~[illegible]~~ the image of a stereographic projection of some minimal surface in S^4 with $\lambda_1 = 2$.

Result 3 Let $M \to S^n$ be a minimal immersion of a compact surface M into S^n. Then $V(M) < 8\pi \Rightarrow \phi$ must be a minimal imbedding.

Tom Willmore (DURHAM, ENGLAND).

h-konvexe Kurven in der hyperbolischen Ebene

Satz: K sei eine h-konvexe Kurve in der hyperbolischen Ebene mit der Länge L und dem Flächeninhalt F. Dann gilt für das „isoperimetrische Defizit" $\Delta := L^2 - 4\pi F - F^2$ die Abschätzung

$$0 \leq \Delta < 4\pi^2 - 32 .$$

Hat K konstante Breite, dann gilt sogar

$$\Delta < 4\pi^2 - 36 .$$

Beide Konstanten können nicht verbessert werden.

Zum Beweis wird jeder h-konvexen Kurve in der hyperbolischen Ebene eine konvexe Kurve mit Mittelpunkt Null in der zentralaffinen Ebene zugeordnet. Die behaupteten Abschätzungen ergeben sich dann aus bekannten Sätzen der affinen Differentialgeometrie.

U. Pinkall (Freiburg)

Archimedische Polyeder der Rhombokuboktaeder Familie

Eine bessere Einsicht in die Menge der möglichen verallgemeinerten Keplerischen und affinen archimedischen Polyeder kann man dadurch gewinnen, dass man aus dieser Menge einige Familien herausnimmt und einzeln studiert. Eine solche Familie kann man z. B. auf solche Weise definieren, dass man im Eckencyklus eines klassischen archimedischen Polyeder, oder auch eines neuen noch unbekannten solchen Polyeders ein oder einige bestimmte Glieder des Zyklus als variable ganzzahlige Parameter betrachtet. Die Polyeder einer so gewonnenen Familie zeigen dann gewisse gemeinsame Eigenschaften, welche besser durchschaut werden können, und damit auch eine Einsicht in die Eigenschaften der ganzen Menge der verallgemeinerten archimedischen Polyeder ermöglichen.

Stanko Bilinski (Zagreb)

Über den Vektorraum der M-Strahlensysteme

Jedes Strahlensystem, dessen Mittenhüllfläche eine Minimalfläche ist, wird als M-Strahlensystem bezeichnet.

Gegeben sei eine Minimalfläche $\vec{OM} = M(u,v)$. Es bezeichne $I := \omega_{31}^2 + \omega_{32}^2$ die dritte Fundamentalform der Fläche, wobei ω_{31}, ω_{32} unabhängige Pfaffsche Formen sind. Die Bestimmung aller M-Strahlensysteme, die die gegebene Minimalfläche als Mittenhüllfläche haben, führt auf die Differentialgleichung

(1) $\quad d \wedge (b\,\omega_{31} - a\,\omega_{32}) = 0,$

wobei $a(u,v)$, $b(u,v)$ die unbekannten Funktionen sind. Die allgemeine Lösung von (1) ist

(2) $\quad a = -D_2\varphi(u,v) \quad , \quad b = D_1\varphi(u,v),$

wobei $\varphi(u,v)$ eine beliebige differenzierbare Funktion ist. Hierin sind D_1, D_2 die Pfaffschen Ableitungen in bezug auf ω_{31}, ω_{32}. Somit ist

(3) $\quad s := \{M(u,v);\ (-D_2\varphi, D_1\varphi)\}$

das allgemeinste M-Strahlensystem.

Es sei $\mathfrak{M}$ die Menge aller s. Sind $s_1, s_2 \in \mathfrak{M}$, also

(4) $\quad s_1 = \{M(u,v);\ (-D_2 f, D_1 f)\},\ s_2 = \{M(u,v);\ (-D_2 g, D_1 g)\},$

so definieren wir

(5) $\quad s_1 + s_2 := \{M(u,v);\ (-D_2 f - D_2 g,\ D_1 f + D_1 g)\}.$

Ferner definieren wir für jedes $\lambda \in \mathbb{R}$

(6) $\quad \lambda \cdot s := \{M(u,v);\ (-\lambda D_2\varphi, \lambda D_1\varphi)\}.$

Offenbar gilt $(s_1 + s_2) \in \mathfrak{M}$, $(\lambda \cdot s) \in \mathfrak{M}$.

<u>Satz 1</u>. $(\mathfrak{M}, \mathbb{R}, +, \cdot)$ ist ein reeller unendlichdimensionaler Vektorraum. Das Nullelement des Raumes ist das Normalensystem der Minimalfläche.

<u>Satz 2</u>. Die Menge der Normalen-M-Strahlensysteme ist ein Untervektorraum von $\mathfrak{M}$.

<u>Satz 3</u>. Sind s_1, s_2 isotrope Strahlensysteme mit gemeinsamer Mittenhüllfläche M und erfüllen ihre mittleren Krümmungen h_1, h_2 die Bedingung $h_1 = c h_2$, $c = \text{const.} \neq 0$, so existieren $\lambda, \mu \in \mathbb{R}$, derart, daß $\lambda s_1 + \mu s_2$ ein Normalen-M-Strahlensystem ist.

N. K. Stephanidis (Thessaloniki-Griechenland).

Flächenabbildungen bei Geradenkongruenzen.

Eine Geradenkongruenz $\Sigma \in C^2$ im E^3 mit der Leitfläche $\Phi: \mathfrak{x}(u,v)$ und dem regulären sphärischen Erzeugendenbild $\mathfrak{n}(u,v)$ $(\mathfrak{n}^2 \equiv 1,\ \mathfrak{n}_u \times \mathfrak{n}_v \neq \mathfrak{o})$ heißt eine F-Kongruenz (bzw. eine J-Kongruenz) mit der Symmetriefläche Φ, wenn die beiden Flächen $\Phi^w:\ \mathfrak{x}^w(u,v) = \mathfrak{x}(u,v) + w \cdot \mathfrak{n}(u,v)$ und Φ^{-w} durch die Kongruenzgeraden lokal flächentreu (bzw. isometrisch) aufeinander abgebildet werden, $\forall w \in \mathbb{R} \setminus \{0\}$.

Es wird u.a. gezeigt:

1) Die Symmetriefläche einer F-Kongruenz (speziell: einer J-Kongruenz) ist deren Mittelfläche. Die J-Kongruenzen sind genau die isotropen Kongruenzen.

2) Die Normalenkongruenz einer regulären C^2-Fläche Φ ist eine (reguläre) F-Kongruenz mit Φ als Symmetriefläche (Mittelfläche) $\Leftrightarrow$ Φ ist eine (nichtebene) Minimalfläche.

3) Die Mittelfläche einer J-Kongruenz (isotropen Kongruenz) gestattet mit
$$\mathfrak{n}(u,v) = \frac{1}{\cosh v}\{\cos u, \sin u, \sinh v\}\ ;\quad (u,v) \in \mathbb{R}^2$$
die Darstellung $\mathfrak{x}(u,v) = A \cdot \mathfrak{n}_u + B \cdot \mathfrak{n}_v + (B \cdot \tanh v - B_v)\,\mathfrak{n}$, wo $A(u,v)$, $B(u,v)$ beliebige konjugiert-harmonische Funktionen sind.
Für die Gaußsche Krümmung K von Φ gilt: $\underline{K = -\rho^4(\alpha_u^2 + \alpha_v^2) \leqq 0}$, wobei $\rho(u,v) := \left(\frac{A}{\cosh v}\right)_v$, $\alpha(u,v) := A + A_{uu}$; ferner: $K \equiv 0 \Leftrightarrow \Phi$ ist eben.
Die einzigen Minimalflächen Φ, welche als Mittelfläche einer J-Kongruenz fungieren können, sind die Wendelflächen und die Ebenen; man erhält sie (bis auf Bewegungen) für $A = -kv - c$, $B = ku$ (k, c reelle Konstanten).
Im Falle $k = 0$ ($\Leftrightarrow \Phi$ ist eben) sind beide Flächen Φ^w, Φ^{-w} identisch mit dem Drehellipsoid
$$\frac{x^2 + y^2}{w^2 + c^2} + \frac{z^2}{w^2} = 1,$$
und die Abbildung $\Phi^w \to \Phi^{-w}$ ist eine uneigentliche Bewegung, erzeugt durch eine Spiegelung an der Ebene $z = 0$ und einer Drehung um die z-Achse durch den Winkel $2 \arcsin \frac{w}{\sqrt{w^2 + c^2}}$.

R. Koch (TU München)

Homogeneous Structures on Riemannian Manifolds

The lecture is centered around the following theorem of Ambrose and Singer: Let (M,g) be a connected, complete and simply connected Riemannian manifold. Then (M,g) is homogeneous if and only if there exists a $(1,2)$-tensor field T on M such that, with $\tilde{\nabla} = \nabla - T$, we have:
i) $\tilde{\nabla}$ is metric;
ii) $\tilde{\nabla} R = 0$;
iii) $\tilde{\nabla} T = 0$.

A tensor T on a Riemannian manifold (M,g) which satisfies these three conditions is called a homogeneous Riemannian structure.

We propose a classification of homogeneous structures into eight classes and give some properties about it. For example we prove that "$T_{XX} = 0$ for all $X \in \mathfrak{X}(M)$" is equivalent to the existence of a naturally reductive structure.

We use this result to give some examples of homogeneous manifolds with volume-preserving geodesic symmetries which are not naturally reductive. These are the so-called generalized Heisenberg groups with center of dimension $\neq 1, 3$. In particular we discuss in more detail the remarkable geometry of the 6-dimensional case.

L. Vanhecke (Leuven)

Krümmungstreue Diffeomorphismen zwischen Riemannschen und pseudo-Riemannschen Räumen

A) Problem: Seien (M,g) & $(\bar{M},\bar{g})$ zwei (ps.-) Riem Mgf, $\dim M = \dim \bar{M} = n$, analytisch.
Es seien K bzw $\bar{K}$ die Schnittkrümmung von (M,g) bzw $(\bar{M},\bar{g})$.

Def: Ein Diffeomorphismus $f: (M,g) \to (\bar{M},\bar{g})$ heißt krümmungstreu, falls
$$K(\sigma) = \bar{K}(f_*\sigma)$$
für alle nicht-ausgearteten σ, für die $f_*\sigma$ nicht-ausgeartet ist.
(M,g) & $(\bar{M},\bar{g})$ heißen krümmungsgleich, falls ein krümmungstreuer Diffeomorphismus $f: M \to \bar{M}$ existiert

Frage: Ist ein krümmungstreuer Diffeomorphismus eine Isometrie?
Falls nicht, wie sieht er aus?
Vor: $n \geq 3$; $K \neq$ const

B) Resultate:

1) $n \geq 4$: $\exists!$ Klasse von Räumen (konform-flache, rekurrente Räume), welche nicht-isometr. krümmungstreue Diffeomorphismen erlaubt
(Dies widerspricht einem Satz von Kulkarni)

2) $n = 3$: (M,g) Riem: es existieren nur die Beispiele von Yau
(M,g) ps.-Riem: Man hat 3 Klassen von Beispielen, die man nach Art der EW des Ricci-Tensors von (M,g) unterscheidet
a) EW des Ricci-Tensors reell & verschieden: Bis auf Signaturänderung die Bsp. von Yau
b) Ricci-Tensor hat komplexe EW: es existiert genau eine Klasse von Beispielen
c) Ricci-Tensor hat 3-fachen EW (ist aber nicht diagonalisierbar): es existieren 3 Klassen von Beispielen.

Bernhard Ruh (Zürich)

Zur projektiven Liniengeometrie

Die PLÜCKER-Koordinaten $p_1, \dots, p_6$ einer Geraden des dreidimensionalen reellen projektiven Raumes P^3 erfüllen die Bedingung $p_1 p_4 + p_2 p_5 + p_3 p_6 = 0$. Interpretiert man diese PLÜCKER-Koordinaten als homogene Punktkoordinaten im P^5, so liegen die zugehörigen Punkte also auf einer vierdimensionalen Quadrik vom Rang 6 und vom Index 3, der PLÜCKER-Quadrik Q^4_{63}. Umgekehrt entspricht jedem Punkt der Q^4_{63} genau eine Gerade im P^3.

Will man für die stereographische Projektion σ der Q^4_{63} aus einem Nordpol $N \in Q^4_{63}$ auf eine Hyperebene $\Pi \subset P^5$ Bijektivität erreichen, so hat man sowohl Q^4_{63} als auch Π längs der Tangentenhyperebene Γ_N von Q^4_{63} in N zu schlitzen.

Zeichnet man in P^5 eine beliebige Hyperebene Λ aus, die Q^4_{63} nicht berührt und $\Pi \cap \Gamma_N$ enthält, und darin Q^4_{63} als Absolutquadrik, so wird P^5 zu einem pseudoeuklidischen Raum P^5_{1102} vom Index 2; die Hyperebene Π wird zu P^4_{1102}. Mit der Verabredung, daß 3-Ebenen zu den Sphären zählen, gilt dann der

Satz: σ ist sphärentreu. (N. KUIPER 1949)

Durch σ ist das Studium der Liniengeometrie in P^3 auf das Studium der Geometrie in P^4_{1102} zurückgeführt. Betreibt man in P^4_{1102} konform-pseudoeuklidische Geometrie, pseudoeuklidische Geometrie, zentro-pseudoeuklidische Geometrie oder diejenige Obergeometrie der pseudoeuklidischen Geometrie, die durch Auszeichnung je einer Erzeugenden der Absolutquadrik von P^4_{1102} entsteht, so entspricht dem in P^3 die Betrachtung der projektiven, der axialen, der biaxialen oder der zweifach isotropen Liniengeometrie. Zweck dieses Vorgehens ist es, solche Invarianten in einer oder mehreren dieser Liniengeometrien zu deuten, die in P^4_{1102} einfach zugänglich sind.

Johann Harth (TU München)

Bemerkungen zur Nichtstandard-Analysis.

Die Adjunktion (transzendenter Körpererweiterung) wird so verallgemeinert:
Man nehme eine mathematische Theorie, welche zumindest die elementare Zahlentheorie enthält. Eine solche Theorie läßt sich formulieren in einem Alphabeth, welches Zeichen enthält für Gleichheit (=), für Konstante ($1, -2, 0, \frac{1}{2}, \pi, \sin, <, \ldots$) für Variable ($x, z, R(.,.)$), für die logischen Junktoren und Quantoren ($\vee, \wedge, \neg, \Rightarrow, \Leftrightarrow, \bigvee, \bigwedge$) und für Interpunktionszeichen (z. B. Klammern).
Wir nehmen für dieser Theorie ein neues Zeichen Ω für eine Konstante hinzu, mit der Definition: Jede Aussageform $A(n)$, die in der gegebenen Theorie für fast alle natürlichen Zahlen n eine wahre Aussage gibt, liefert in der erweiterten Theorie eine wahre Aussage $A(\Omega)$. Relative Widerspruchsfreiheit durch Angabe von Modellen (Ultraprodukte).

Beispiele besonders von Euler.

Detlef Laugwitz (Darmstadt)

Zur konformen Transformation von Einstein-Räumen

Es werden konforme Diffeomorphismen zwischen zwei Riemannschen Einstein-Räumen betrachtet. Ausgehend von lokalen Resultaten von Brinkmann (Math. Ann. 94 (1925)) wird gezeigt, welche vollständigen Einstein-Räume eine globale konforme Transformation gestatten. Es zeigt sich, daß es außer den einfach zusammenhängenden Räumen konstanter Krümmung genau eine weitere Familie von Beispielen gibt, die konform äquivalent sind zu $(0,\infty)\times M_*$, wobei M_* ein beliebiger vollständiger Ricci-flacher Raum ist. Dies stellt einen inkorrekten Satz von S.T. Yau richtig (J. Diff. Geom. 8 (1973), Theorem 3 und Corollary 4.1).

W. Kühnel (TU Berlin)

Schnittorttheorie, Anwendungen und Beziehungen zu Arbeiten von R. Walter V. Bangert und N. Kleinjohann

Der Schnittort C_A bezüglich einer abgeschlossenen Menge A wird definiert als der Abschluß der Menge aller Punkte q, in denen sich Kürzeste von A nach q nicht über q hinaus minimal verlängern lassen, solche Punkte nennen wir non-extender. Weil im allgemeinen bezüglich einer beliebigen abgeschlossenen Menge A keine Exponentialabbildung zur Verfügung steht, werden Konjugierte Punkte im Schnittort imitiert. Vice versa, motiviert dies den Begriff des Lipschitz-punktes. Ein Punkt $q \in (M \setminus A)$ heißt Lipschitz Punkt, wenn eine Umgebung $U(q)$ von q und eine Zahl $L > 0$ existiert, so daß in einer beliebigen Karte $|\dot{c}_q - \dot{c}_{\bar{q}}| \leq L\, d(q, \bar{q})$ f. a. $\bar{q} \in U(q)$, $\dot{c}_{\bar{q}}$ ist hier Tangenvektor im Punkt $\bar{q}$ von einer Kürzesten, die $\bar{q}$ mit A verbindet. Die Norm $|\,.\,|$ bezieht sich auf eine beliebige Kartendarstellung von $U(q)$. Es gilt nun q ist extender genau dann, wenn q Lipschitz Punkt ist. Der Schnittort ist die maximale offene Menge in $(M \setminus A)$, wo die Distanzfunktion $d(A, .)$ C^1 glatt ist. Eine Menge A ist genau dann eine UFP Menge im Sinne von R. Walter und N. Kleinjohann wenn eine offene Umgebung $U(A)$ den Schnittort C_A nicht trifft. Ein Folgerung aus den vorgestellten Ergebnissen ist ein Satz von R. Walter und N. Kleinjohann über die lokale Lipschitz-Stetigkeit der metrischen Projektion bei einer UFP Menge. Es werden Beziehungen zwischen topologischen Eigenschaften des Schnittortes C_A und der top. Struktur von $(M \setminus A)$ gebracht. Eine Folgerung daraus: Sei M eine vollständige unberandete Riem. Mf, p_0 sei ein Punkt in M, wenn die Menge der Punkte wo die Funktion $d(p_0, .) : (M \setminus \{p_0\}) \to \mathbb{R}$ nicht diffbar ist, mindestens einen isolierten Punkt enthält dann ist M homöomorph zur Sphäre.

Franz-Erich Wolter
TV Berlin

Schwingungstheorie

12. - 18. September 1982

Anwendung von Verzweigungsdiagrammen auf die Modellbildung mechanischer Systeme.

Verzweigungsdiagramme versaler Systemfamilien erlauben es, strukturinstabile Systeme, d.h. solche, die bei kleinen Parameteränderungen eine qualitative Verhaltens=änderung erleiden, in systematischer Weise zu erkennen. Strukturell instabile Systeme sind oft solche, bei denen weniger Parameter vorhanden sind als eine universal entfaltete Systemfamilie erfordern würde. Für Familien von Potentialfunktionen, Differentialgleichungen und Matrizen sind Verzweigungsdiagramme mehr oder weniger gut untersucht. Der Fall des Dämpfungseinflusses auf ein konservativ und ein nicht konservativ belastetes Doppel=pendel wird als Beispiel studiert. Weiters wurde die Form von Stabilitätsgrenzen im Parameterraum in Hinsicht auf ihre strukturelle Stabilität einer kritischen Betrachtung unterworfen.

H. Troger (Wien)

Chaotische Bewegungen beim Duffing - Schwinger

Betrachtet werden die chaotischen Lösungen der Duffingschen Dgl.

$$\ddot{x} + \delta\dot{x} + \beta x + \alpha x^3 = B\Omega^2 \cos \Omega t .$$

Die aus der Literatur bekannten Ergebnisse werden zusammengestellt, ergänzt und diskutiert. Ziel sind Aussagen über Erscheinungsformen, Parameterabhängigkeiten und Entstehung chaotischer Bewegungen.

K. Popp (Hannover)

Schwingungen und Wellen in Strukturen mit Fügestellen

Die Dynamik zusammengesetzter Strukturen wird durch das nichtlineare Übertragungsverhalten der Fügestellen, wie z.B. Schraub-, Niet-, Klemmverbindungen, beeinflußt. Unvermeidbare Hysteresen verursachen häufig stärkere Energiedissipation als die Materialdämpfung.
Das gemessene Übertragungsverhalten isolierter Fügestellen wird in Ersatzmodellen abgebildet. Die Identifikation der Ersatzmodellparameter mit dem Konzept der äquivalenten Linearisierung beschreibt die nichtlinearen Eigenschaften befriedigend genau.
Für die Körperschallanregung durch Biege- und Dilatationswellen wird der Leistungsfluß berechnet, der an Fügestellen reflektiert, transmittiert und dissipiert wird. Ein Versuchsaufbau zur Ermittlung der Übertragungseigenschaften einer Fügestelle zwischen zwei Stäben wird beschrieben. Die Gegenüberstellung von Meß- und Rechenergebnissen beschließt den Vortrag.

L. Gaul (Hamburg)

Partnerschaftsvorhaben Rotordynamik

Im Rahmen eines Partnerschaftsvorhabens zwischen den Universitäten Stuttgart und Campinas, Brasilien, wird ein Rotorprüfstand entwickelt, der durch seine Gestalltung die Untersuchung verschiedener Probleme aus der Praxis ermöglicht. Dabei wird zunächst das Gebiet der Schwingungsanalyse an großen Wasserkraftmaschinen betrachtet. Es sollen Fragen der Parameteridentifizierung, wie z.B. einen theoretischen und experimentellen Vergleich heute bekannter Identifizierungsverfahren, untersucht werden. Zu diesem Zweck wurde ein Verfahren entwickelt das mit quadratischen Mittelwerten den Rückschluss auf Systemgrößen ermöglicht. Die dazu notwendigen Gleichungen sind über die Ljapunovsche Matrizengleichung zu bekommen, obwohl zu diesem Zweck die Nachschaltung eines Filters sich als günstig zeigte. Es wurden lineare, zeitinvariante Mehrkörpermodelle mit stochastischer Erregung zugrunde gelegt. Die prinzipielle Möglichkeiten der Methode wurden an einem System mit einem Freiheitsgrad durchgeführt und verifiziert. Die erzielten Genauigkeiten liegen ~~im~~ einem Bereich von 3%, wobei das bandbegrenzte Rauschen einfach durch weisses Rauschen simuliert worden ist. Will man größere Werte von Systemdämpfungen identifizieren zeigt es sich als günstig die selben experimentellen Daten mit ein farbigen Rauschmodell zu verarbeiten.

H. I. Weber (Campinas, Brasilien)

Die Entwicklung des Satzes von Andronov und Witt bis zur Gegenwart

Der Satz von Andronov und Witt besagt, daß eine periodische Lösung $p(t)$ eines n-dimensionalen autonomen Differentialgleichungssystems $\dot{x} = f(x)$ orbital asymptotisch stabil ist, falls $n-1$ charakteristische Exponenten der Jacobimatrix $f_x(p(t))$ negative Realteile besitzen. Dieser klassische Satz wird aus der Sicht der modernen Theorie der invarianten Mannigfaltigkeiten beleuchtet. Ferner werden Verallgemeinerungen und Erweiterungen angegeben, die dem derzeitigen Stand der Stabilitätstheorie (conditionelle Stabilität, center manifold etc.) entsprechen. Besonderes Augenmerk gilt der Verallgemeinerung des Satzes von Andronov und Witt auf Systeme mit Kontinua von periodischen Lösungen und der Frage, ob die Annäherung einer Lösung an ein solches Kontinuum bereits Konvergenz gegen eine bestimmte Lösung aus der zugehörigen Schar periodischer Lösungen impliziert.

Bernd Aulbach (Würzburg)

D'Alemberts Prinzip mit Quasigeschwindigkeiten für Vielkörpersysteme

Das d'Alembertsche Prinzip in der Lagrangeschen Fassung ermöglicht eine schnelle und einfache Formulierung von nichtlinearen Bewegungsgleichungen für beliebig aufgebaute Systeme von gelenkig gekoppelten starren Körpern. Für n Körper lautet es

$$\sum_{i=1}^{n}\left[\delta\vec{r}_i\cdot\left(m_i\ddot{\vec{r}}_i-\vec{F}_i\right)+\delta\vec{\pi}_i\cdot\left(J_i\cdot\dot{\vec{\omega}}_i+\vec{\omega}_i\times J_i\cdot\vec{\omega}_i-\vec{M}_i\right)\right]+\delta W=0$$

Als Variable in den Gleichungen können beliebige, geeignet erscheinende generalisierte Koordinaten verwendet werden. Wenn ein System so beschaffen ist, daß ein Körper relativ zum Inertialraum oder relativ zu einem Nachbarkörper drei Freiheitsgrade der Rotation besitzt, dann ist die Verwendung des Vektors einer absoluten oder relativen Winkelgeschwindigkeit als Quasigeschwindigkeit zu empfehlen, weil das d'Alembertsche Prinzip schon in derartigen Größen formuliert ist. Die Einführung von Eulerwinkeln oder anderen Koordinaten an Stelle von Quasigeschwindigkeiten würde die Gleichungen wesentlich schwieriger gestalten. Jeder Quasigeschwindigkeit ist eine kinematische Differentialgleichung zuzuordnen. Diese kinematischen Gleichungen sind voneinander entkoppelt.

Jens Wittenburg (Karlsruhe)

Komplizierte Phänomene in einfachen dynamischen Systemen

In zahlreichen Veröffentlichungen aus den letzten Jahren ist über mit numerischen Methoden ermitteltes, kompliziertes Lösungsverhalten (das gern als chaotisch bezeichnet wird) in nicht autonomen Systemen mit einem Freiheitsgrad berichtet worden. In vielen Fällen liegt der Grund in der Existenz von transversalen homo- und heteroklinen Punkten, deren Bedeutung schon von Poincaré erkannt worden ist. Erst Smale jedoch hat die nicht leicht zu durchdringende Komplexität solcher Systeme durch den Einbau des "Shifts" als Subsystem teilweise mathematisch erfaßt. Nach Anosov, Bowen, Conley, McGehee und Moser wird ein überraschend einfacher Zugang mittels eines "Shadowing-Lemmas" dargestellt, der z.B. den Nachweis von unendlich vielen periodischen und aperiodischen Lösungen mit vorgegebenem Verhalten gestattet.

Urs Kirchgraber (Zürich)

Kombinierte parametererregte, erzwungene und selbsterregte Schwingungen

Am Beispiel eines allgemeinen Systems mit einem Freiheitsgrad und nichtlinearer Dämpfung und Rückstellung wird das Zusammenwirken einer allgemeinen periodischen Zwangserregung, einer linearen und nichtlinearen Parametererregung und einer van der Polschen, verallgemeinerten van der Polschen oder einer harten Selbsterregung untersucht. Allgemeine Lösungsaussagen ergeben sich durch Einführung von Extremalkurven, die die Amplitudenextrema allgemein und übersichtlich in Abhängigkeit von Parameter- oder Zwangserregung, linearer oder nichtlinearer Dämpfung darstellen. Als wichtig erweist sich die Phasenbeziehung zwischen Zwangs- und Parametererregung. Eine geeignete Parametererregung kann erzwungene Schwingungen verkleinern. Vielfältige Möglichkeiten abgelöster Resonanzkurven mit relativ großen Schwingungsamplituden ergeben sich bei zusätzlicher Selbsterregung verschiedener Formen.

G. Schmidt (Berlin, DDR)

Zur Beschreibung der inneren Dämpfung mechanischer Schwinger mit inneren Variablen.

Die Beschreibung der inneren Dämpfung mechanischer Schwinger als viskose Dämpfung entspricht häufig nicht der Realität, und man führt in diesen Fällen frequenz- und amplitudenabhängige Dämpfungskonstanden ein. Durch die Einbeziehung von inneren Variablen (Freiheitgraden) und Vergrößerung der Zahl der Bewegungsgleichungen kann man die frequenz- und amplitudenabhängigen Konstanten umgehen. Im linearen Fall verläuft die weitere Rechnung analog zur viskosen Dämpfung und die Methode der finiten Elemente läßt sich (geringfügig abgeändert) anwenden. Im nichtlinearen Fall der statischen Hysterese bleibt auch bei schwacher Dämpfung ein Teil der Bewegungsgleichungen stark nichtlinear, wenn man das Ersatzmodell von Masing voraussetzt. Die Approximation durch lineare Gesetze kann zu großen Fehlern führen, was auch mit experimentellen Beobachtungen übereinstimmt.

D. Ottl (Braunschweig)

Berechnung der Verzweigungsgleichung bei n-dimensionalen dynamischen Systemen und Anwendung auf einen Sattelschleppzug

Bei n-dimensionalen dynamischen Systemen können bei Änderung von Parametern Verzweigungen von stabilem zu instabilem Verhalten auftreten. Die Theorie der zentralen Mannigfaltigkeit (Zentrumsmannigfaltigkeit) gibt an, wie das System im Verzweigungspunkt auf ein niedrigdimensionales System reduziert werden kann, das alle Information über die Stabilität des gesamten Systems

enthält. Die Ordnung des reduzierten Systems ist bei den beiden Kodimension c = 1 - Verzweigungen eins (Divergenzverzweigung) oder zwei (Hopfverzweigung).
Es wurde angegeben, wie das reduzierte System berechnet werden kann, und als Beispiel ein einfaches Modell eines Sattelschleppzuges untersucht.

Klaus Zeman (Wien)

Das nichtlineare Verhalten eines Eisenbahnfahrzeuges

Bei grösseren seitlichen Versetzungen eines Eisenbahnfahrzeugs muss der nichtlineare Einfluss der Spurkranzen in Betracht genommen werden. Es wird gezeigt, dass dieser Einfluss keine grosse Rechenschwierigketten verursacht, wenn man annimmt, dass die Normalkraft im Radaufstandspunkt konstant ist. Ein Verfahren wurde entwickelt, das für Kombinationen von willkürlichen Rad- und Schienenprofile geeignet ist. Numerische Ergebnisse werden gezeigt für einen Einzelradsatz. Zuletzt wird angegeben, wie der Fall eines vollständigen Fahrzeugs angefasst werden kann.

A. D. de Pater (TH Delft)

Parameternichtlinearitäten in nichtkonservativen nichtlinearen Systemen.

Schwingungen in modernen leichten und hochbeanspruchten Maschinenkonstruktionen werden häufig durch nichtlineare Bewegungsgleichungen beschrieben, die zeitlich variante Koeffizienten auch in nichtlinearen Gliedern – die sogenannten Parameternichtlinearitäten – besitzen. Die mit Hilfe der Integralgleichungsmethode durchgeführte Analyse des Kraftresonanzfalles zeigt insbesondere auf den beachtlichen qualitativen Einfluß dieser Parameternichtlinearitäten auf die dynamische Amplituden-Frequenzcharakteristik.

C. Höschl (Prag)

Identifikation schwingungsfähiger Systeme unter Berücksichtigung der Ergebnisse aus der Systemanalyse

Die direkte Identifikation schwingungsfähiger elastomechanischer Systeme verwendet a priori-Kenntnisse, die maximal die Modellstruktur beinhalten. Diese Parameteridentifikation verwendet jedoch nicht die Ergebnisse der Systemanalyse. Die indirekte Parameteridentifikation dagegen geht einen Schritt weiter und bezieht das Rechenmodell (der Systemanalyse) mit ein. Im einfachsten Fall sind die Parameterwerte des Rechenmodells lediglich Startwerte für ein aufgrund gemessener (identifizierter) Werte erstelltes Modell (korrigiertes Rechenmodell). Darüber hinaus können jedoch entsprechend der Konfidenz des Rechenmodells seine Parameterwerte mit in die Parameterschätzung eingehen (Bayes-Schätzung).

G. Natke (Hannover)

Identifikation der Parameter nichtkonservativer Mechanismen von Rotierenden Maschinen.

Das Schwingungsverhalten rotierender Maschinen wird durch verschiedene nichtkonservative Mechanismen wie Gleitlager, Dichtspalte usw. beeinflußt. Für die Vorausberechnung des dynamischen Verhaltens (Unwuchtschwingungen, Stabilität) muß der Konstrukteur die federnden und dämpfenden Eigenschaften dieser Mechanismen kennen. Es wird eine experimentelle Methode zur Ermittlung der Feder- und Dämpfungskonstanten von Gleitlagern vorgestellt. Dazu wird ein starrer Rotor in Gleitlagern durch Stoßkräfte angeregt und das Übertragungsverhalten gemessen. Durch Anpassung von analytischen Übertragungsfunktionen an gemessene Funktionen können die gesuchten Parameter bestimmt werden.

J. Nordmann (Kaiserslautern)

Instationäre, nichtlineare Zufallsschwingungen

Zur Berechnung instationärer, nichtlinearer Zufallsschwingungen stellt die Kovarianzanalyse in Verbindung mit der statistischen Linearisierung ein äußerst effizientes Näherungsverfahren dar. Ausgehend von der statistisch linearisierten Zustandsgleichung und den Formfilterbeziehungen für die Erregerprozesse erhält man durch Anwendung der Kovarianzanalyse Vektor- und Matrizendifferentialgleichungen, deren Integration auf den Mittelwertvektor und die Kovarianzmatrix des Lösungsprozesses führt. Die Anwendung der Kovarianzanalyse wird an Hand von nichtlinearen Fahrzeugschwingungen bei instationärer stochastischer Erregung veranschaulicht. Das abschließende Beispiel zeigt einen Vergleich zwischen stationärer und instationärer Betrachtungsweise.

G. Riel (Stuttgart)

<u>Koppelschwingungen eines Stabes in einem Flüssigkeitsbecken.</u>

Es wird über die Wechselwirkungen eines schwingenden Bernoulli-Euler-Stabes berichtet, der konzentrisch in einem kreisrunden Flüssigkeitsbecken mit der Füllhöhe H angeordnet ist und teilweise aus der Flüssigkeit herausragt. Das Fluid sei inkompressibel und reibungsfrei, an seiner Oberfläche sollen sich sog. Oberflächenwellen einstellen können. Eine orts- und zeitabhängige Druckverteilung oberhalb des Flüssigkeitsspiegels regt das gekoppelte Fluid-Festkörper-System zu erzwungenen Schwingungen an.
Nach der Herleitung des maßgebenden Randwertproblems werden zunächst die Eigenschwingungen systematisch untersucht, bevor dann abschließend in Kürze auch auf die eigentlichen Zwangsschwingungen eingegangen wird.

J. Wauer (Karlsruhe)

Anwendung von Schwingungsdämpfern in selbsterregten Systemen mit ~~ein~~ mehreren Freiheitsgraden

Gegenstand des Referates bildet eine Analyse der Wirkung eines Schwingungsdämpfers, der an ein selbsterregtes diskretes System mit mehreren Freiheitsgraden angeknüpft ist, wobei die einzelnen Systemmassen nur einen Freiheitsgrad haben. Es wird vorausgesetzt, daß es sich um eine weiche Selbsterregung des Grundsystems vom van der Polschen Typ handelt. Es wird die sogenannte Methode der Grenzkurven angewendet und solche Werte des gegebenen Parameters festgestellt, bei denen die Amplitude der Einfrequenten-schwingungen einer bestimmten Form zu Null konvergiert. Auf diese Weise wird die Stabilitätsgrenze der einer beliebigen Eigenschwingung des Systems entsprechenden Gleichgewichtslage bestimmt. Diese Methode erleichtert die Optimierung der Parameter des Schwingungsdämpfers.

A. Tondl (Prag)

Zur Theorie der Normalschwingungen in nicht-linearen mechanischen Systemen

Die Theorie der Normalschwingungen für lineare konservative Systeme ist in der Mechanik wohl-bekannt. Eine Erweiterung auf nichtlineare Systeme mit einer linear stabilen Gleichgewichts-lage wurde bereits von Liapunoff (1892) vorgenommen unter der Voraussetzung, dass das linearisierte

System keine Resonanzen aufweist. Die Frage der Normalschwingungen wurde in den letzten Jahrzehnten von verschiedenen Autoren untersucht, so zum Beispiel von Seifert (1947), Berger (1972), Roels (1971), Gordon (1971), Weinstein (1973/78), Moser (1976) und Rosenberg (1966). Während Seifert die Existenz von mindestens einer "Normalschwingung" für beliebig nichtlineare Systeme nachweisen konnte, gelang es Weinstein, den Liapunoffschen Satz auf Resonanzfälle zu erweitern. Sowohl Liapunoffs als auch Weinsteins Ergebnisse sind auf schwache Nichtlinearitäten beschränkt. Ohne Kenntnis von Seiferts Ergebnissen wurde von Rosenberg dieselbe Fragestellung unter eingeschränkten Bedingungen behandelt. Weinstein wiederum erweiterte Seiferts Resultat auf allgemeinere Hamiltonsche Systeme.

R. Weber

(Zürich)

Schwingungen aufeinander abrollender Walzen mit elastischer Beschichtung

Der Vortrag stellt Methoden zur Berechnung des Schwingungsverhaltens aufeinander abrollender Walzen mit elastischer Beschichtung und dazwischenliegendem Walzgutlos. Aufbauend auf der Modellbildung drehendes (Rayleigh-) Balken werden Starrkörper- und elastische Koordinaten definiert sowie die Berührgeometrie erklärt. Für vereinfachte Annahmen werden zwei Theorien für die Kontaktkräfte angegeben. Unter Verwendung bekannter Ergebnisse für elastische Rotoren kann der allgemeine Aufbau der Bewegungsgleichungen abgeleitet werden; aufgrund nichtlinearer Reibwert-Schlupf-Normalkraft-Verläufe sind selbsterregte Schwingungen zu erwarten. Die Systemordnung ist - im Vergleich zu reinen Starrkörpersystemen - relativ hoch. Eine Erweiterung der Theorie auf dünne Schalen ist einfach möglich. Wegen der nunmehr zweidimensionalen Verformungen ist mit deutlich größeren Systemen zu rechnen, die jedoch im Vergleich zu Standard-Finite-Element-Verfahren immer noch sehr günstig abschneiden.

A. Puchelbrodt (München)

Einfluß der Dämpfung auf die Stabilität elastischer Rotoren

Die Ursachen selbsterregter Schwingungen von Rotoren wie innere Dämpfung und die Unrundheit der Welle sollen näher untersucht werden. Die Bewegungsgleichungen werden über das Prinzip von Hamilton hergeleitet und dabei die Ortsfunktionen nach den Lösungen des runden ungedämpften Rotors entwickelt.
Für die innere Dämpfung werden 3 verschiedene Hypothesen betrachtet und die zugehörigen Stabilitäts=

grenzen in Abhängigkeit von der äußeren Dämpfung bestimmt.
Die Rotorunrundheit erzeugt parametererregte Schwingungen mit hier abzählbar unendlich vielen Instabilitätsbereichen, die durch die äußere Dämpfung stets verkleinert werden, durch innere Dämpfung aber auch vergrößert werden können. Im Fall einer anisotropen Rotorlagerung wirkt eine kleine innere Dämpfung im wesentlichen stabilisierend.

G. Müller (Darmstadt)

Stabilisierung linearer Systeme durch Rauschen

Die positive Eigenschaft des Rauschens zu stabilisieren wird untersucht.
Ein lineares System $\dot{x} = Ax$ kann ohne deterministischen Eingriff durch zero-mean-Rauschen stabilisiert werden, stets dann, aber auch nur dann, wenn die Systemmatrix verschwindende Spur hat.
Diese Art der Stabilisierung ist unabhängig vom Zustand des Systems (keine Messungen nötig) und die Konstruktion der stabilisierenden Vorrichtung ist bis auf die erforderliche Energie unabhängig vom System. Es können eine oder mehrere stationäre Rauschquellen benutzt werden. Allerdings bedarf es u.U. hoher Energie.
Die Verallgemeinerung auf stochastische parameter-erregte Systeme $\dot{x} = A(t,\omega)$ und auf nicht-lineare Systeme $\dot{x} = f(x,k)$ –via Linearisierung– ist möglich.

Volker Wihstutz (Bremen)

Transversale Schwingungen vom Balken regulärer einstöckiger Rahmen

Die Säulen solcher Rahmen haben die Funktion elastischer Federn, weil ihre Masse nicht berücksichtigt wurde. Hier ist das äquivalente Modell ein Durchlaufträger mit Rotationsfeder an den Lagern.
Die Befestigung des Balkens an seinen Enden ist beliebig, eins der beiden Enden ist aber unbeweglich in horizontalen Richtung.
Eine Rekursionsformel für die Amplituden von Drehwinkeln des Balkens an seinen Lagern wurde hergeleitet: ihre Lösung gibt die Eigenformen der Drehwinkel an. Die Struktur der Frequenzgleichung hängt von den Randbedingungen ab.
In gewissen Fällen gehören zu jeder Form der Drehwinkel unendlich viele Eigenformen, die an den Lagern eine gemeinsame Tangente haben.
Die Eigenformen der Drehwinkel sind zueinander orthogonal.

V. Korošić (ZAGREB)

FREE AND RANDOM VIBRATIONS OF ORTHOTROPIC PLATES

Method is presented for determination of the natural frequencies and mode shapes of rectangular plates. Method consists of representing the natural frequency / wave number relationship in conjunction with the solution of two auxiliary problems. An advantage of the method consists in possibility to find the natural frequency for any given wave numbers. For the cases capable of exact solution the method yields the results, coinciding with exact solution.

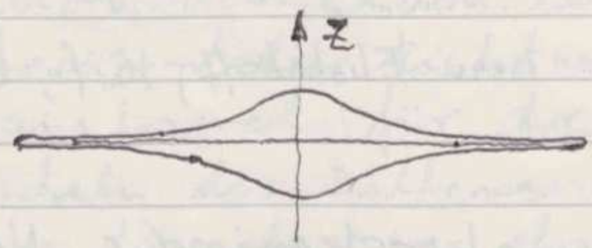
The normal modes and the natural frequencies obtained in the first part of the lecture, are utilized for the probabilistic analysis of the plates. The point driven force case is considered and number of interesting features discussed. The role of cross-correlations, which are usually neglected in the literature, is shown to be vital in estimating the probabilistic response.

Isaac Elishakoff, Haifa, ISRAEL.

Über einige makroskopische Verhaltensmuster nichtlinearer, dynamischer Kontinua mit stochastischer Mikrobewegung

Als Beispielsmedium für ein nichtlineares, dynamisches Kontinuum wird ein (meist) berandetes Plasma herangezogen, beschrieben durch die Navier-Stokes-Gleichung, die Kontinuitätsgleichung, die Maxwell-Gleichungen sowie die thermische Zustandsgleichung. Makroskopische Verhaltensmuster bzw. "Formen" kennzeichnen dabei Teilgebiete des Mediums, die sich infolge Selbstorganisation deutlich in ihren Merkmalen wie z.B. Temperatur, Dichte, Turbulenzgrad, Druck vom Rest des Mediums unterscheiden. Die Ausbildung charakteristischer Formen ist

an die Existenz hochturbulenter, stochastischer Mikrobewegungen gebunden, die in den Formen Re-Zahlen von 10^9 bis 10^{12} erreicht und mit steilen Gradienten zum Umgebungsplasma abfällt.
Eine typische Form ist nachstehend abgebildet; sie rotiert bei einem Durchmesser von 26,4 m mit einer Frequenz



von 27,5 Hz und pulst mit einem Rythmus von 14,3 Minuten. Verwandte Vorgänge sind aus der irdischen Atmosphäre bekannt.
An einem Laborplasma, experimentell realisiert im Plasmafokus Poseidon der Universität Stuttgart, wird die Existenz solcher Formen weiter belegt sowie auf ihre Bedeutung für die kontrollierte Kernfusion eingegangen.
Ein Film über eine Computersimulation zur Plasmadynamik derartiger Vorgänge schließt den Vortrag ab

A. Hayd, P. Meinke

P. Meinke

Analytische Methoden in der Mechanik von Mehr-Körper Systeme

Der Vortrag ist der dynamischen Analyse von Systemen starrer Körper gewidmet, die beliebig verbunden sind und wie offene, als auch geschlossene Ketten enthalten. Ein zur Anwendung der Digitalrechner orientierter mathematischer Apparat für kompaktes Aufschreiben der Bindungsgleichungen und Lösung der dynamischen Bewegungsgleichungen ist entwickelt. Die erfolgreiche Lösung der Bewegungsgleichungen stützt sich auf die Variationsprinzipien der Mechanik, insbesondere den Gaußschen Prinzip und auf die neulich entwickelten iterativen Methoden für bedingte und umbedingte Minimization eines Funktionels.

L. Lilov (Sofia)

Bebenerregte Schwingungen flüssigkeitsgefüllter Behälter.

Exemplarisch wird am stehenden kreiszylindrischen (oberirdischen) Tankbauwerk die Modellbildung erläutert und unter stochastischer Modellierung des erregenden Bebenprozesses die Standsicherheit und die Überschwappwahrscheinlichkeit untersucht. Neu ist die Berücksichtigung der (starren) Kippbewegung des Tanks am elastischen Untergrund und der Nachweis der immer noch möglichen modalen Analyse. Der schwingende Flüssigkeitskörper weist Oberflächenwellen an der freien Spiegelfläche auf, die allerdings nur in linearisierter Form berücksichtigt werden (steife Behälter). Wesentlich ist noch die Erwähnung der verwendeten Teilsystemtechnik: Flüssigkeit – fester Tank, die eine Auflösung des hybriden Systems gestattet.

F. Ziegler (TU-Wien)

Ein Beitrag zur Untersuchung strömungserregter deterministischer und stochastischer Schwingungen in Kontinua und diskreten Systemen

Zur Berechnung strömungserregter Schwingungen von räumlichen Doppelkörperpendeln in einem flüssigkeitsdurchströmten Kanal unter Berücksichtigung von Stößen an den Kanalwandungen werden ausgehend von allgemeinen Untersuchungen für die Formulierung von Übergangsbedingungen bei Stoßvorgängen an Schwingungssystemen mit n Freiheitsgraden und einer Erweiterung der in der Literatur vorliegenden Strömungsmodelle für die Erfassung von Strömungskräften an schwingenden Stäben in engen Kanälen charakteristische Frequenzen als Funktion der Durchsatzgeschwindigkeit bestimmt. Daran anschließend erfolgen Betrachtungen von durch turbulente Druckfelder erregten Zufallsschwingungen eindimensionaler Kontinua, insb. dabei die Bestimmung von Feldprozeßspektraldichten und Korrelationsfunktionen in Abhängigkeit der Strömungsgeschwindigkeit und Randbedingungen, die zur Erfassung von Ungenauigkeiten in der Lagerung durch elastische Einspannungen modelliert werden. Nach Bestimmung des Einflusses der zugehörigen Federkonstanten auf die charakteristischen Größen der Schwingungsreaktion im Rahmen der Korrelationstheorie werden abschließend Untersuchungen über die Empfindlichkeit des aufgestellten Modells gegenüber Störungen in den charakteristischen Parametern durchgeführt.

K. Hennig

Über die Zuverlässigkeit flexibler Strukturen unter Nicht-Gauß'schen Belastungen

Flexible Strukturen unter dynamisch wirkenden Zufallslasten können entweder im Zeit- oder auch im Frequenzbereich analysiert werden. Und zwar werden dabei die Zeitverlaufs- und die Leistungsspektralmethode verwendet. Es ist bekannt, daß bei Verwendung letzterer Methode eine statistische Aussage, d.h. eine Aussage über die Versagenswahrscheinlichkeit der Struktur, oder über die Erstüberschreitenswahrscheinlichkeit etc., nur unter der Annahme stationärer, Gauß'scher Eigenschaften des Eingangs- bzw. Lastprozesses möglich ist – lineare Systemeigenschaften sind selbstverständlich vorausgesetzt. Statistische Analysen von Aufzeichnungen stochastischer Belastungen wie z.B. Erdbebenbeschleunigungen, Windschriebe sowie Wellenhöhen, zeigen deutlich, daß Gauß'sche Eigenschaften nicht unbedingt vorausgesetzt werden können bzw. dürfen. Es wurde daher eine Methode vorgeschlagen, mit der die Wahrscheinlichkeitsverteilungen der Reaktion für nicht-normalverteilte, stationäre Eingangsfunktionen bei linearen Systemen berechnet werden können. Es werden dazu Reihendarstellungen der Wahrscheinlichkeitsverteilungen benützt, die die Berechnung der höheren statistischen Momente erfordern (Gram-Charlier sowie Edgeworth Reihe). Darüberhinaus erfordert die Methode auch die Berechnung der Korrelationen und Spektren höherer Ordnung

G. I. Schuëller

Stochastische Identifikation von Steifigkeits- und Dämpfungsmatrizen.

Der Vortrag behandelt die Meßbarkeit weiß erregter Systeme der Strukturmechanik. Bei Vorgabe der Massenmatrix und der Intensität der Erregung können die Koeffizienten der Steifigkeits- und Dämpfungsmatrizen aus den Quadratmittelwerten der gemessenen Schwingungsausschläge und -geschwindigkeiten berechnet werden, wenn die zugehörigen Identifikationsgleichungen regulär sind.

Beispiele für einen Rangabfall sind zusammenfallende Eigenkreisfrequenzen, Knotenpunktsanregungen oder der Verlust der asymptotischen Stabilität bei ungenügender Fixierung der elastischen Struktur. Solche Eigenschaften stimmen überein mit der Beobachtbarkeitsbedingung, die in der Regelungstechnik für deterministische Systeme erster Ordnung bereits hergeleitet wurde.

An einer Schwingerkette mit drei Freiheitsgraden werden erste Versuche durchgeführt, um die Streuung der Varianzmessungen zu zeigen. Wesentliche Verbesserungen so einer direkten Identifikation werden erreicht, indem man lineare Optimierungsverfahren nachschaltet, die bestimmte physikalische Eigenschaften wie Band-Struktur oder Homogenität der Matrizen ausnutzen.

W. Wedig

Risk Theory

19. 9. – 25. 9. 1982

Parametric Multiple Regression Risk Models

Parametric multiple regression models for the claim number and claim amount of an individual risk are introduced allowing explanatory variables to influence the mean claim number (amount). A model for an entire portfolio of risks is established on this basis. Then an extensive and in many cases efficient statistical analysis of this model is presented.

Peter Albrecht

Semi_Markov processes in risk theory.

We present the completely semi_Markov model in risk theory for which we give the interpretation and the comparison with classical models in terms of intensities of counting processes. Some useful particular cases are presented to show the possibility to get some computational results concerning ruin probabilities. Finally, we present shortly some other actuarial models in which the semi-Markov modelization is also used to obtain more judicious models than the classical ones.

Jacques Janssen.

Two premium calculation principles by bargaining

Using, as main tools, the classical risk exchange model by Borch and the bargaining models of Nash and Kalai-Smorodinsky, two new premium calculation principles are defined, whose main goal is to take explicitly into account the attitude towards risk of the policy-holders. Those principles are neither additive nor iterative, but they nevertheless possess several important properties: the premium is translation invariant, it does not depend neither on the reserves nor on the portfolio of the company; it takes into account all the moments of the claims distribution; it is independent of the policy-holder's wealth but increases with ~~his~~ his risk aversion.

Jean Lemaire

Bonus malus and premium calculation principles

Considering the well-known negative binomial model in automobile insurance, Bühlmann has shown that the bonus-malus system resulting from the expected value principle is financially balanced and minimizes the mean square error.

The same model is applied to the variance principle ~~and~~ and the exponential utility principle. It is shown that those two principles lead to merit rating system that ~~do~~ are not financially balanced: in both cases the ~~income ~~ income decreases with time. Then the bonus-malus systems that result from an absolute loss function, and from a 4th order loss function are computed: an example shows the lack of financial balance (increasing income, in our example)

Jean Lemaire

Practical Aspects of Stop-Loss Calculations

This paper examines various methods of "arithmetizing" the claim size distribution so that stop-loss premiums can be calculated recursively. Claim frequencies are assumed to be Poisson distributed. A decision strategy is developed for choosing a method when both error and computer costs are constrained.

Harry Panjer, Waterloo, Canada

Some Mathematical Aspects of Reinsurance.

The speaker considered a single risk for which the insurer had arranged some form of reinsurance, either proportional or excess of loss. In the spirit of two previous papers, Scan. Act. J. (1979) and ASTIN (1980), the effect on the insurer's safety, as measured by his adjustment coefficient considered as a function of the retention limit, as a result of varying the retention limit was examing. It was shown in the case of proportional reinsurance under very general conditions that the insurer's safety varied in an intuitively reasonable way, i.e. that the adjustment coefficient was always a unimodal function of the retention limit. The same result could be proved for excess of loss reinsurance only if some extra conditions were imposed.

Howard Waters.

On risk processes with the Markov property and with independent increments

Let $\{N_t : t \in \mathbb{R}_+\}$ be a counting process and let $\{X_n : n \in \mathbb{N}\}$ be a sequence of i.i.d. random variables, independent of $\{N_t : t \in \mathbb{R}_+\}$. Consider the risk process

$$S_t = \sum_{n \geq 1}^{N_t} X_n . \qquad (t \in \mathbb{R}_+)$$

We study the impact on the counting process $\{N_t : t \in \mathbb{R}_+\}$ of the Markov property and of the property of independent increments for the risk process $\{S_t : t \in \mathbb{R}_+\}$. We show that already the Markov property is very strong and actually implies that $\{N_t : t \in \mathbb{R}_+\}$ has "nearly" independent increments, which in its turn implies that $\{N_t : t \in \mathbb{R}_+\}$ is "nearly" a Poisson process

Jean Haezendonck
(Antwerpen - Belgium)

Industry-wide expense standards using random coefficient regression

A model establishing industry norms for the relationship between company activity levels and expected operating expenses can provide useful management information. For example, a company could substitute its activity levels into the model, receive the "industry standard" expected expenses, and compare its actual expenses with the "norm".

In this study, data from the annual statements of U.S. life insurance companies are used to derive a multiple regression model. Because of severe multicollinearity, ordinary least squares is an unreliable estimation technique, but, because the model contains many parameters, Bayesian methods prove to be effective. Some recent improvements on Bayesian hierarchical analysis are briefly discussed and illustrated. A predictive analysis confirms the superiority of Bayesian methods over ordinary least squares. Finally, practical questions regarding choice of variables and interpretation of coefficients are addressed.

Robert B. Miller
(Madison, Wisconsin USA)

Stop-loss dominance

In the theory of finance one studies stochastic dominance decision models for the choice among risky alternatives The present contribution deals with stop-loss dominance which to a certain extend represents a complement to stochastic dominance. The influence of both types of dominance to the ordering of claim size intensities is examined

F. De Vylder
J. Haezendonck
M. Goovaerts

Order statistics and largest claims reinsurance.

The lecture deals with the problem of calculating the net premium for the largest claims and ECOMOR reinsurance treaties. Already in 1964 Ammeter derived premium formulas, for which now an easier proof is given. In 1978 Benktander showed that the premium of the largest claims cover can be bounded by the premium of a corresponding excess of loss treaty plus a multiple of its priority. Now it is proved that under fairly weak assumptions Benktander's bound is asymptotically equivalent to the premium of the largest claims cover, when the size of the collective approaches infinity. Numerical calculations indicate that this equivalence is quite good already for comparable small collectives. A similar result is given for the ECOMOR-treaty. Finally simple distribution-free bounds are deduced for the net premiums of both covers, based only on the knowledge of the mean and standard deviation of the claims size distribution of the risks. The basic tool for proving the theorems is an important branch of mathematical statistics: the theory of order statistics.

E. Kremer

Asymptotic behavior of multivariate compound processes

For large t, the joint distribution of a general multivariate compound process at t is approximated by Edgeworth expansions. A generalized multivariate compound process is constructed with an independent sequence of multivariate claim amounts and a multivariate counting process which is allowed to be stochastically dependent on the claim amounts. The validity of the approximation is proved under a uniform Cramér condition. The terms of the Edgeworth expansion can be given explicitely.

To be submitted to Z.f. W theorie verw. Geb.

Christian Hipp

An analytic approach to claims reserving

The problem of analysis of non-life insurance claims experience is examined. An attempt is made to develop a model based, as far as possible, on analytic properties of the experience. Regression methods are used to fit the model to the data. The merits of regression relative to other estimation procedures are considered. The question of second moments of estimates of outstanding claims is considered, and the model currently under consideration applied to it.

G.C. Taylor

Maximization of a functional of a probability distribution.

We consider the problem

$$\sup_{F \in \mathcal{F}} \left(O(F) \,/\, C_1(F) = z_1, \ldots, C_n(F) = z_n \right)$$

where $\mathcal{F}$ is a family of probability distributions and O, $C_1, \ldots C_n$ functions on $\mathcal{F}$. Under the assumption of existence of a "simple basis" the problem is reduced to the determination of the convex hull of a $(n+1)$-dimensional set.

De Vylder F.

The General Economic Premium Principle

In a closed exchange economy the agents i are characterized by their

initial wealth W_i

risk variable X_i } on an arbitrary probability space $(\Omega, \mathcal{A}, \Pi)$

exchange variable Y_i }

utility function $u_i(x)$ $[u_i'(x) > 0,\ u_i''(x) \leq 0]$

Price equilibrium is characterized by

a) $E\left[u_i\left(W_i - X_i + \tilde{Y}_i - \int \tilde{Y}_i(\omega')\tilde{\varphi}(\omega')\,d\Pi(\omega')\right)\right] = \max$ among all possible choices of Y_i

b) $\sum_{i=1}^{n} \tilde{Y}_i(\omega) = 0$ for all ω

Let $X_i - Y_i = Z_i$ (and quite naturally $X_i - \tilde{Y}_i = \tilde{Z}_i$)

The $\tilde{Z}_i$ typically depend on ω only through $Z(\omega) = \xi$

One finds

$$*)\quad \tilde{Z}_i'(\xi) = \frac{\frac{1}{\rho_i(W_i - \tilde{Z}_i(\xi))}}{\frac{1}{\rho(\xi)}} \quad \text{where } \frac{1}{\rho(\xi)} = \sum_{i=1}^{n} \frac{1}{\rho_i(W_i - \tilde{Z}_i(\xi))}$$

Assume (H) : $\rho_i(x) > 0$ and continuous on $\mathbb{R}$

Then $*)$ is a system of differential equations with existence and uniqueness of the solution

$$\left(Z_i^T(\xi)\right)_{i=1,2,\dots n} \quad \text{where } Z_i^T(0) = -T_i$$

$T = (T_1, \dots, T_n)$ ~ initial transfer payments

Any solution of $*)$ is a Pareto Optimum. In order to be also an equilibrium initial transfer payment $\tilde{T}$ must be chosen such that

$$E\left[\tilde{\varphi}_{\tilde{T}}\, \tilde{Y}_i^{\tilde{T}}\right] = 0_{Z(\omega)} \quad \text{for all } i$$

$$\text{where } \tilde{\varphi}_T = \frac{e^{\int_0^{Z(\omega)} \rho_T(\xi)\,d\xi}}{E\left[e^{\int_0^{Z(\omega)} \rho_T(\xi)\,d\xi}\right]} \quad \text{for arbitrary } T \quad \left[Z(\omega) = \sum_{i=1}^{n} X_i(\omega)\right]$$

If (H) holds and $|X_i| \leq M$ for all $i = 1, 2, \dots n$ one can show the existence of $\tilde{T}$ by an application of Brouwer Fix Point Theorem.

Hans Bühlmann

Risk Exchange: Fairness and Relative Pareto Optimality

In the theory of risk exchange Bühlmann and Jewell (1978, 1979) have shown that fairness and Pareto optimality lead to a unique solution. Their model allows for several interpretations. If the risk belongs to a pool, which has to distribute it among its members the solution can be applied without any difficulty. In the case where all agents are endowed with an initial risk however, the risk exchange should not only be Pareto optimal and fair but also individually rational (i.e. after the risk exchange no agent has to be worse off). Bühlmann and Jewell (1979) have pointed out that these three conditions are not always compatible

Therefore it may be of some interest to look for a solution which is

- fair
- relative Pareto optimal (i.e. Pareto optimal relative to the set of all fair net trades)
- individually rational.

These requirements are satisfied by "uniform coupons equilibria", a notion borrowed from general equilibrium theory with quantity rationing (Drèze and Müller 1980). In the framework of risk exchange this notion has the following interpretation: An agent who accepts "undesired" parts of a risk portfolio obtains a bonus allowing him to purchase "desired" parts as well. Existence of uniform coupons equilibria can be shown under standard assumptions but no results about global uniqueness are known.
For an exact definition and existence see:
J. H. Drèze, H. Müller: "Optimality Properties of Rationing Schemes", Journal of Economic Theory, Vol. 23, N°2, pp. 131-149

Heinz Müller

Some Numerical Problems in Risk Theory

Die Berechnung der Gesamtschadenverteilung wird auf die Bestimmung gewisser Koeffizienten einer Rekursionsformel zurückgeführt, deren Berechnung für bestimmte Schadenanzahlverteilungen durchgeführt wurde. Treten auch negative Grundschäden auf, sind Rekursionsformeln in der Regel nicht mehr anwendbar. Für diesen Fall wird ein Verfahren beschrieben, das mit Hilfe der "Fast-Fourier-Transformation" die charakteristische Funktion des Gesamtschadens zurücktransformiert. Angewandt wurde dieses Verfahren für die Berechnung der Gesamtschadenverteilung einiger Gruppen-Rentenversicherungs-Bestände.

Die numerischen Ergebnisse werden mit der Normal-Power-Approximation für den Verteilung verglichen.

Jürgen Bertram / Manfred Feilmeier   Braunschweig

Forecasting of IBNR-claims

In early papers on the IBNR-problem the run-off scheme has been modelled by a multiplicative structure, each entry being a product of one factor for the accident year, another for the development period, and a third for the payment year (or possibly only one of the last two). The entries may be interpreted as the observable quantities or as their expected values; in both cases the analysis will essentially be the same as long as no further probability structure is added. Later works have given examples of how the model can be probabilized and how forecasts of IBNR-claims arise in a natural way from the model. The present work follows up this approach, starting from quite standard assumptions concerning the risk process. Exact (empirical) Bayes as well as credibility procedures are considered.

23.9.82   Ragnar Norberg.

Insurance premiums and optimal behavior of consumers and producers in risk situations.

I present a mathematical model of an economy for which I show that
- without insurance, the overall result of the economy becomes a maximum if the agents of the economy (consumers and producers) decide in risk situations according to the principle of expected values, i.e. choose the option which gives them the maximal expected value
- with total insurance [i.e. with insurance of all risks] the same result is obtained if insurance premiums are equal to the expected value of the claims.

The model assumes that the agents make their choice based on offers of the insurer for insuring the different options. It further assumes that the behavior of the agents is not influenced by the fact that they are insured. In reality this last assumption is not satisfied (moral hazard). Insurance therefore should be framed in such a way that the balance of

a) the positive effects of giving information about and taking over the risks

b) the negative effects of moral hazard
and

is maximized.
23. 9. 82 F. Rühsel

Markov chain models in insurance.
Nonparametric inference and analysis of selectional effects.

In recent papers by Aalen (1978, Ann. Statist, 6, 701-726), Aalen & Johansen (1978, Scand. J. Statist, 7, 161-171) and Andersen, Borgan, Gill & Keiding (1982, Int. Statist. Rev. 50, to appear) it has been shown how the theory of multivariate counting processes provides a general framework in which inhomogeneous Markov chain models may be analyzed, and that martingales and stochastic integrals are very useful tools in the study of nonparametric estimation and testing procedures. In the talk I tried to present some main ideas and results from the above mentioned works and to indicate how they might be of interest in applied actuarial work. A short comment concerning the possible bias introduced by retrospective studies was also given.

23/9-82 Ørnulf Borgan

"Models for claim frequencies in a portfolio of property insurance for single family houses and dwelling houses".

At the Lab. of Act. Math., Copenhagen, a research project concerning the insurance risk has just been started. The project is organized through the Research Committee of the Danish Actuarial Society, and it is based on an empirical investigation of a portfolio of property insurance delivered by a Danish non-life company. In an empirical project like this, numerous problems arise, and the talk will emphasize the various models which are available for the description of the occurrence of the claims. Hopefully, it will be possible to show theoretical as well as empirical results from the project.

23/9-82

Henrik Ramlau-Hansen

Ruin theory in the linear model

The probability of ruin is examined in a model where the annual gains of an insurance company are dependent random variables. The model used is the linear model (well known in time series analysis) which includes the auto regressive model and the moving weighted average model as special cases. It is also shown how a certain credibility model can be interpreted as a first-order model of the mixed type. The talk was based on a note in Insurance, Mathematics & Economics, 1 (1982), 177-184.

Hans U. Gerber (Lausanne)

Bounds for the optimal critical claimsize of a bonus system

The optimal critical claim size of a bonus system determines whether to file a claim with the insurance company after having an accident. The aim of the talk is to demonstrate, within the framework of a simple model, how bounds for the optimal critical claim size can be constructed when only incomplete information on the claim amount distribution is available.

M. De Pril - M. Goovaerts (Leuven)

Characterizations of Claim Distributions by Reliability Techniques

In the present lecture conceptions of reliability theory are used to characterize the dangerousness of claim distributions. Especially, an estimator of the failure rate is introduced which is appropriate to describe the skewness of sample distributions.

W.-R. Heilmann

Remarks on large claims.

Consider the classical situation of a compound Poisson risk process with claim size distribution B. Put T_x the time of ruin with initial reserve x. If B is exponentially bounded then both probabilities $P[T_x < \infty]$ and $P[t < T_x < \infty]$ can be estimated; the estimation are exponential both in time and in initial reserve. If B is of Pareto-type then also $P(T_x < \infty)$ and $P(t < T_x < \infty)$ are such, again both in time and in initial reserve.

The introduction of a retention in ~~a~~ distribution with Pareto-tail turns the dangerous behaviour of the risk process into an exponentially controlled process. This happens in practice with stop-loss reinsurance treaties.

Jef L Teugels

Credibility models allowing durational effects

In a classical credibility model it is assumed that the claim amounts of an insurance policy from different years are conditionally independent and identically distributed given an unknown, random risk parameter $\tilde{\Theta}$. In the present talk we introduce an additional random variable $\tilde{t}$, denoting the total time the policy stays in the portfolio. It is assumed that information about $\tilde{t}$ may say something about $\tilde{\Theta}$, and it should therefore be used in the rating scheme. In the first part of the talk, credibility estimators are deduced and discussed. Then we test whether $\tilde{t}$ says something about $\tilde{\Theta}$. We finally discuss estimation of structural parameters. A numerical example is given.

Björn Sundt

Asymptotic behaviour of compound distributions and stop-loss premiums.

We give some asymptotic results for the compound distributions of aggregate claims when the claim number distribution is negative binomial. The special case when the claim numbers are geometrically distributed, is treated separately.

Bjørn Sundt

Influence of inflation on insurance mortality

The real mortality rate of insured people depends of age $(x+t)$ and of duration t from entrance and might be written

$\theta_t \cdot \mu_{x+t}$, where θ_t is increasing from ca 0.3 to 1 in the first 15 years of the insurance. For reasons of simplicity this double-entrance mortality is often replaced by a single mortality function μ°_{x+t}, corresponding to the total population. This function is automatically too high in low durations and to high for big durations of the policies.

Since 1960 about, the fast inflation has caused, that the risk sums of new insurances are much higher than those of the old, why in ordinary death-risk insurance the payments are less than expected, creating a surplus that may be distributed as bonus. For annuity insurance, the same effect causes a loss in mortality payments, which is hidden by the large marge in nominal rate.

Jan Jung

On premium determination and solvency.

The total claims X_n during year n of a non life branch may be assumed to have an expectation

$$\cancel{\xi_n}\, E(X_n) = e_n \xi_n I_n \text{, where}$$

e_n is the exposure to risk, ξ_n is the expected real claim cost per unit of exposure and I_n is an index of claim cost, different from normal consumer price index.

The job of the actuary consists of predicting the ξ_n at the time, when in the best case $X_1 \cdots X_{n-2}$ are known.

Suppose he uses a linear predictor

$$\xi_n^* = \sum_2^k a_\nu X_{n-\nu} \quad \text{to determine } P_n.$$

Then the risk reserve will increase with the surplus $P_n - X_n$, and the resulting reserve U_n can be written

$$U_n = U_0 + \sum_0^\infty b_\nu X_{n-\nu}$$

How should the a_ν be chosen, in order to

a) make the premiums change slowly and

b) secure that the risk reserve does not vary too fast.

It seems to be impossible to obtain small variances for both P_n and U_n

Jan Jung

ON THE PROBLEM OF ESTIMATING ULTIMATE FREQUENCY

This problem originates in the field of reliability, where "reliability growth" occurs as systems undergo performance improvement during prototype testing, due to design changes, environmental modifications, and procedural revision. In many cases, only the epochs of the failures are available to the statistician, and there is great interest in using "early returns" to predict the ultimate failure rate. This paper constructs a general framework in which to analyze the problem, including many special model variations that have been previously proposed. Numerical trials indicate the difficulty of using classical MLE estimates, which is unstable for small testing intervals and a small number of systems on test; the MLE is even inconsistent for small intervals, with a unlimited number of systems on test! Bayesian procedures are recommended for implementation, as they can use data from any testing protocol.

William S. Jewell University of California, Berkeley.

Inference about parameters in empirical Credibility Decision problems. A sampling theoretic approach.

In empirical credibility problems the 1st and 2nd order moments of a structural distribution are estimated by linear techniques. The assumption of randomness of the design in the collateral data allows us to derive asymptotic results, and use these for inference: Testing, confidence estimation, etc. The methods are applied to a sample of reinsurance data and yield satisfactory results.

Walther Neuhaus.
Storebrand International
Reinsurance Co., Oslo.

Risk Models with Stochastic Discounting.

A discrete time model is proposed for a risk process with discounting factors given by a Markov chain. The process is best described ~~by~~ as a random walk with stochastic weights. The moments of the process at a given time (finite or infinite) are determined. In the case of exponentially distributed claims the probability of ruin for a finite time horizon can be computed recursively. In the general case an upper bound is provided for the probability of ruin.

A continuous time model is introduced. It consists in a discounted version of the compound poisson process. The interest rate is thereby a continuous time Markov chain. If the time horizon is infinite the above results still hold.

René Schnieper

Tariff construction: principles and methods.

Marc Hallin and Jean-François Ingenbleek. Université Libre de Bruxelles.

One of the most intensively treated subjects in the area of actuarial mathematics has certainly been, in the last few years, the problem of theoretical premium calculation. It has however very little influence on current tariff building. The reasons for such a gap between a highly developed theory and its practical applications are multiple; among them is the lack, somewhere between the abstract concept of a risk premium and the concrete amount of money which is collected from policyholders, of a coherent tariff theory. The concept of a "good tariff" itself remains quite fuzzy and undefined. Just as the fairness of a risk premium can be appraised only with reference to a given premium principle, the degree of excellence of a tariff cannot be evaluated without some elements of a tariff theory. It is our purpose to give here a brief outline of what this theory should be, to suggest some general principles, and to describe methods for putting them into practice.

The Swedish automobile portfolio will be treated as an example.

Marc Hallin.

PROBABILITY 0.5

PROBABILITY 0.5

Russian Roulette, Insurance and other Hazardous games.

The practical relevance of risk theory illustrated by some illustrations. Ragnar Norberg

Optimal Dividends

An Insurance Company receive premiums with intensity c and interests on their capital x. The main aim is to look for the optimal dividends to maximize the return on the capital x. For this purpose the quotient of the expected value of the sum of the discounted dividends over the capital x is to be maximized. For ~~sat~~ the sake of the existence of analytic solutions we assume that the claim process is mixed Poisson with exponential distributed claim amounts. In the case of premium income only out of insurance business and no interests on capital optimal dividends are easily found by a pocket calculator or by a rough approximation. In the case of interests on the capital x you pay this amount to the shareholders plus or minus a certain additional factor. The optimal factor may easily be computed by a pocket calculator. It is possible to give a bound for the premiums c to pay an additional positive amount to the interests on the capital x.

Wolfgang ETTL

On asymptotically deterministic random variables

The class of asymptotically deterministic random variables is introduced, which are determined by the fact, that one can reckon with their estimated values in two respects:

- one can expect that the in fact realized variables are sufficiently close to the estimated values
- one can calculate with them, that is, one can use the estimated values in order to derive from them other interesting values,

if a great amount of samples exist.

To estimate asymptotically deterministic random variables, deterministic simulations as well as stochastic ~~variables~~ simulations are suitable. With respect to the latter it is possible to show that a single realization may be relied on. Finally, it will be demonstrated by a generalization of the weak law of large numbers, that the proportional numbers (index values), generally used in actuarial practice, are asymptotically deterministic. Therefore, one may reckon with them in the sense above mentioned.

Edgar Neuburger

A practical application of optimal trimming in credibility

In Switzerland a group of mathematicians was charged with the task of elaborating a new tariffication system for the line "Salary compensation in case of Sickness". The new system should come nearer to the true individual risk premium of each contract. Bichsel & Straub proposed a credibility procedure for this situation. As underlying model they used the Bühlmann & Straub model (1970). The practitioners felt that the proposed credibility estimator has a great disadvantage concerning big claims. If such big claims are fully charged in the credibility formula, single big claims may be the cause of a precipitous rise in the estimated premium rate. Hence they arose the question how such big claims should be handled in the credibility formula. The right answer to this question is "Optimum trimming of Data in Credibility" (MUSVM, 1980 /3). In this practical application the optimal trimming point in dependence of the premium volumes and the corresponding parameters have been estimated out of the data. Afterwards some simplifications and approximations have been done to make the credibility formula with optimal trimming applicable to practical purposes. The proposed system is now discussed in Switzerland.

Alois Gisler

Risk Exchanges with Partial Information

This paper extends the risk exchange models of Borch, Gerber & Bühlmann and Jewell to the case where the participants recognize different possible events as outcomes. In addition each participant has different probability estimates of event from those of the other participants. An example using exponential utilities is given.

Charles Hachemeister

Inkorrekt gestellte Probleme und ihre numerische Behandlung – Improperly posed problems and their numerical treatment

26. Sept. – 2. Okt. 1982

The Inverse Scattering Problem for Acoustic Waves

We consider the problem of determining the shape of an obstacle in $\mathbb{R}^2$ from a knowledge of the far field pattern. We first examine the class of far field patterns corresponding to entire incoming waves. A model problem is then considered which shows the improperly posed nature of the problem and suggests sufficient criteria for stabilization. Stabilization criteria are then established through the use of a priori restrictions on the unknown obstacle, and the inverse scattering problem is reformulated as a nonlinear optimization problem with constraints. We conclude by discussing the numerical solution of this constrained optimization problem.

David Colton
(University of Delaware)

A stable marching scheme for an ill posed initial value problem.

We develop and analyze a marching procedure for the numerical computation of linear backwards parabolic equations with variable coefficients and noisy initial data. The scheme is stable but inconsistent and leads to error bounds of logarithmic convexity type, for t bounded away from the line $t = T$ where the solution is only of class L^2. The scheme is a two step procedure where the solution is appropriately filtered in the frequency domain, at every alternate step. The procedure assumes a constraint on the class of solutions which is stronger than the usual L^2 bound at $t = T$. This constraint is valid for variable coefficient problems where the coefficients are only small perturbations away from constants. Our analysis, and the error bounds we obtain, are only valid in that case. On the other hand, the scheme itself is applicable to problems where the coefficients vary appreciably and even to nonlinear problems. We demonstrate the robustness of this scheme with two interesting computational experiments, one of which involves BURGERS' equation.

Alfred Carasso 9/27/82

Comments on Morozov's Discrepancy Principle

The choice of regularization parameter by Morozov's principle is characterized in a new way and is related to another parameter choice strategy. An asymptotic order of accuracy is derived which is essentially best possible and a discrepancy principle is developed in a finite element context.

C.W. Groetsch
University of Cincinnati
27 September 1982

Regularisierungsverfahren für Gleichungen erster Art
mit selbstadjungierten Operatoren

Zur approximativen Lösung der Gleichung

$$Tx = y$$

mit einem selbstadjungierten injektiven (kompakten) Operator T, $y \in \text{Bild}\, T$, wurden

1. die Fehlerquadrat-Regularisierung
$Q_\alpha(w) = \|Tw - y\|^2 + \alpha \|w\|^2 = \min!$
2. die Ritz-Regularisierung für positive T
$R_\alpha(w) = (Tw, w) - 2(y, w) + \alpha(w, w) = \min!$
3. die Galerkin-Regularisierung
$(Tw + i\alpha w - y, v) = 0 \quad \forall v \in H$
in der Komplexifizierung von H

miteinander verglichen und unter Annahme von Glattheitsvoraussetzungen, z. B. $x \in \text{Bild}\, |T|^\rho$, $0 < \rho \leq 1$, die Konvergenzordnungen für $0 < \alpha \to 0$ bei exakten und bei fehlerhaften Daten angegeben.

Eberhard Schock
Universität Kaiserslautern

27. September 1982

Numerical solution of Fujita's equation

The Fujita equation is a special case of a Fredholm integral equation of the first kind with compact integral operator and as such is a typical example of a severely ill-posed problem. It is proved that a class of Fredholm integral equations of the first kind (containing the Fujita equation) has injective integral operators and thus solutions coinciding with the corresponding minimum norm solution. An application of an algorithm for a good choice of the regularization parameter gives satisfactory results for an example of Fujita's equation. Moreover, the use of Sobolev norms for partial regularization of the problem may improve these results considerably. The same example shows that the collocation method based on equidistant points is numerically extremely instable and inaccurate.

Jürg T. Marti
ETH Zürich
28. September 1982

On the convergence of regularization methods for ill-posed linear operator equations

Let X, Y be Hilbert spaces, $A \in L(X,Y)$ with a nonclosed range. We are interested in approximating the best-approximate solution A^+y of $Ax=y$ ($y \in D(A^+)$) and assume that instead of y, we know only y_δ with $\|y-y_\delta\| \le \delta$.
We first report results about necessary and sufficient conditions (on the choice of the regularization parameter $\alpha = \alpha(\delta)$) for strong and weak convergence of a class of methods of the form $U(\alpha, A^*A)A^*y_\delta$ with $U(\alpha,t) \to t^{-1}$ $(\alpha \to 0)$, which include e.g. Tychonoff regularization, truncated singular value expansion, and Landweber iteration. For Tychonoff regularization, we discuss asymptotically optimal choices of $\alpha(\delta)$, which depend on smoothness assumptions about the data.

In the second part of the talk, we discuss least-squares collocation for solving $Ax=y$, where now Y is a reproducing kernel space with continuous kernel. It turns out that the role of a regularization parameter is played by the smallest eigenvalue of $(Q(m_i, m_j))_{1\le i,j\le n}$ where the m_i are the observation functionals. We illustrate our results about the way λ_n and the error level have to be related in order to ensure convergence for numerical differentiation.
Finally, we use similar methods to identify a regularization parameter for projection methods of the form $\langle Ax, v_i\rangle = \langle y, v_i\rangle_{i=1}^{n}$, $x \in \mathrm{lin}\{A^* v_1, \dots, A^* v_n\}$. An asymptotically optimal choice for the v_i's turn out to be (for compact A) the singular vectors ψ_i ($AA^*\psi_i = \sigma_i^2 \psi_i$, where σ_i are the singular values of A).

Heinz W. Engl
Universität Linz (Austria)
9-27-82

Uniform Expansions for a Class of Finite Difference Schemes for Weakly Coupled Semilinear Systems

A class of finite difference schemes, due to H.-O. Kreiss, for weakly coupled mildly nonlinear elliptic systems of the type

$$-d_j(x)\Delta_j u_j(x) = f_j(x, u_1(x), \dots, u_m(x)) \qquad 1\le j\le m, \; x\in\Omega$$
$$u_j(x) = g_j(x) \qquad 1\le j\le m, \; x\in\Gamma$$

where Ω is a bounded region in $\mathbb{R}^n$ and $0 < \underline{d} \le d_j(x)$ $\forall\, 1\le j\le m$ $\forall\, x\in\Omega$, will be considered.
The schemes are the standard $(2n+1)$-point-approximation of the

Laplacian combined with polynomial extrapolation of degree k near the boundary. The FD-schemes thus obtained is neither symmetric nor of monotone type. No conditions regarding the definiteness or the sign-pattern of $\partial_u (f_1, \ldots, f_m)^T$ are imposed. The convergence of the FD-solutions to isolated solutions of the original problem and the existence of asymptotic error expansions are stated for $k \leq 4$. Finally we report on numerical tests, in which we exploited the asymptotic expansions by a modified deferred correction method.

Harry Mans 28. 9. 82
Universität Tübingen

Vorgelegt ist eine Erweiterung aus der nichtlinearen Tragflügeltheorie von K. GERSTEN. Sie resultiert aus der Tragflächengleichung für kompressible Medien im Unterschall, die für Anström-machzahl 1 den Fall verschwindender Flügelstreckung beschreibt. Betrachtet wird der Spezialfall des Rechteckflügels. Ein naives Verfahren liefert hochoszillatorische Lösungen. Eine Anwendung des Verfahrens von TIKHONOV in Form des Algorithmus von MARTI liefert Approximationen von Δc_p, die mit den Messungen akzeptabel korrespondieren. Der Auftriebsbeiwert c_A stimmt für Winkel bis 10° in Einklang mit den Messungen und den Werten aus dem Modell von GERSTEN.

U. H. Wichert 28. 9. 82
Universität [illegible].

Improperly posed problems and their numerical treatment

A large class of two-dimensional elliptic boundary value problems in acoustics, elasticity as well as hydrodynamics can be reduced to systems of boundary integral equations of the first kind with logarithmic singularities. This lecture concerns in particular the stability of the finite element method for treating such integral equations. It is shown that an optimal choice of the mesh size can be made so that one may obtain an asymptotic optimal rate of convergence of the approximate solutions. The results here are in consistence with those obtained by the Tikhonov regularization procedure.

George C. Hsiao
University of Delaware, U.S.A.
September 28, 1982

Über Lösungsmethoden mathematischer Probleme mit ungenauer Eingangsinformation und Automatisierung der Verarbeitung von Experimentaldaten.

Für die automatische Bearbeitung von Beobachtungen ist es notwendig stabile Algorithmen zu haben. Die Interpretation der Beobachtungsdaten führt zur Funktionalgleichung $Az=u$, wobei z - das Mathematische Modell des zu untersuchenden Objektes ist. Diese Aufgabe ist in der Regel unstabil. Damit entsteht das Problem, für eine instabile Aufgabe stabile, computergerechte Algorithmen auszuarbeiten.

Für ~~dieses~~ dieses Problem wird eine mathematische Aufgabestellung vorgelegt und die entsprechenden Algorithmen ausgearbeitet, wenn sowohl u wie A ungenau gegeben sind. Es werden Beispiele durchgehend automatisierter, problemorientierter Programmkomplexe diskutiert und ihre Anwendung auf Aufgaben der Kernspektroskopie und der Plasmadiagnostik gegeben.

А.Тихонов (Tikhonov)
Moskau, Keldysch Institut für angewandte Mathematik

28-IX-1982.

Verallgemeinerung des Oettli/Prager-Satzes auf hypernormierte Räume.

Der klassische Oettli/Prager-Satz gibt Auskunft darüber, ob eine Näherungslösung für
ein lineares Gleichungssystem $Ax=b$
aufgefaßt werden kann als exakte Lösung eines "leicht gestörten" linearen Gleichungssystems $\bar{A}x=\bar{b}$.
Dieser Satz wird verallgemeinert in zweifacher Hinsicht. Zum einen werden die im Oettli/Prager-Satz verwendeten Betragsfunktionen ersetzt durch Hypernormen. Zum anderen wird das lineare Gleichungssystem ersetzt durch
eine Operator-Gleichung $Ax=b$.
Dabei ist $A: X \to Y$ ein beschränkter linearer Operator (beschränkt bezüglich der verwendeten Hypernormen) und die Dimensionen von X und Y sind beliebig, so daß z.B. auch lineare Integral-Gleichungen erfaßt werden.

Herbert Fischer
28. Sept. 82, TU München

Some regularisation in control theory

The "initial state reconstruction" problem in linear control theory is : given dynamics, $\dot{z}=Az$, output equation, $y=Cz$, can one exactly reconstruct the initial state z_0 from the observations (A, C are linear maps ; $y(t)$ lies in the output space Y; $z(t)$ in the state space Z). Thus one considers the initial state-to-output map
* $H: z_0 \mapsto Ce^{At}z_0$ and asks whether a. it is injective, and if so, b. it has closed range. In infinite dimensional control theory a. is referred to as "initially observable" and b. as "continuously initially observable". Clearly the techniques of regularisation for

* by abuse of notation $e^{At} \equiv S(t)$ the semigroup

linear operators can be applied to H in cases where initial, but not continuous initial, observability obtains. Such is invariably the case with parabolic equations (cf Dolecki & Russell) and regularisation techniques have been used, more or less explicitly, by various authors (esp. Lions). Rather than survey these areas the talk proceeded to some partially baked considerations regarding semi linear control theory; viz. ... : given $\dot{z} = Az + f(z)$, $y = Cz$ in the case where Ce^{At} has closed range in natural output space the linear part can be used to create a fixed point formulation of the state reconstruction problem, viz., $\Phi : z \mapsto S(t)H^{-1}(y(\cdot) - C\int_0^{\cdot} S(\cdot - s)f(z(s))ds) + \int_0^t S(t-s)f(z(s))ds$ where Φ is a map whose fixed points are consistent with both the output equation and the dynamics. For hyperbolic equations local existence and uniqueness results can be found in Carmichael, Pritchard, Quinn; Applied Maths. and Opt., 1982. Can the same be done for equations with parabolic linear parts? This talk indicated a partial result in this direction; a complete treatment would have to study behaviour as $\varepsilon \to 0^+$.

Neil Carmichael
30.9.82

Ein freies Randwertproblem aus der physikalischen Geodäsie

Betrachtet wird das Problem, aus Meßdaten für die Anziehungskraft und ihr Potential auf der Erdoberfläche deren Gestalt in einem euklidischen, mit der Erde rotierenden Koordinatensystem zu bestimmen. Da die gesuchte Erdoberfläche annähernd bekannt ist, handelt es sich um eine lokale Fragestellung. Mittels der Legendre-Transformation läßt sich dieses freie Randwertproblem in ein nichtlineares Randwertproblem mit festem Rand überführen. Hierfür werden ein lokaler Existenz- und Eindeutigkeitssatz sowie ein

Regularitätssatz beweisen, woraus man lokale Existenz in $C^{1,2}$, lokale Eindeutigkeit in C^1 sowie Regularität des gesuchten freien Randes folgen kann.

Karl G. Kalin
Universität Essen - GHS

Nichtcharakteristische Cauchyprobleme für die Wärmeleitungsgleichung

Für nichtcharakteristische Cauchyprobleme für die Wärmeleitungsgleichung wird für gewisse a priori Informationen ~~an~~ für die Lösung, wie Normschranken und Positivität, die Stärke der Regularisierung, d.h. der Charakter des Stetigkeitsmoduls untersucht. Insbesondere wird für die Vorzeichenrestriktion in bestimmten Normen Hölderstetige Abhängigkeit gezeigt. Dies verschärft ein Resultat von Pucci (1959), wo keine Abschätzung des Stetigkeitsmoduls gelang, liefert eine mögliche Begründung für das in der Ingenieur-literatur gebräuchliche Konzept der 'zukünftigen Temperaturen' und zeigt, daß in diesem Problem die Regularisierung durch Vorzeichen-restriktion und durch spezielle Normschranken etwa gleich ist. Mit einem Defektminimierungsverfahren gewonnene numerische Resultate unterstreichen die Ergebnisse.

Peter Knabner
29. 9. 82, Universität Augsburg

Identifizierung des freien Randes beim Stefan - Problem aus Cauchy - Daten

Beim eindimensionalen Ein - Phasen - Stefan - Problem soll der freie Rand ohne Kenntniss der Anfangstemperatur -

verteilung $h(x)$ und der Anfangsposition b des freien Randes allein aus dem Wärmefluß und der Temperatur auf dem festen Rand bestimmt werden. Näherungslösungen werden gewonnen mit Hilfe eines nichtlinearen Tschebyscheff-Approximations-Problems über einem endlich dimensionalen Ansatz für $h(x)$ und b. Numerische Erfahrungen sind positiv.

Martin [signature]
29.9.82, Universität Augsburg

Maxwell equations with incident waves as a field source.

The surfaces $\Gamma^+ = \{(x,y,z) \in R^3 : z = a\}$ and $\Gamma^- = \{(x,y,z) \in R^3 : z = 0\}$ of transition between the plasma and the vacuum of the plasmatic slab $\{(x,y,z) \in R^3 : 0 \leq z \leq a\}$ with the plasma density are excited by a laser ray. It is assumed that the waves of the corresponding light have the form $\exp\{i(\alpha x + \beta y)\}$, where $\beta p / 2\pi$ is an integer number, i.e. the waves are p-periodic as well as the density $\rho = \rho(x,y,z) = \rho(x,y) = \rho(x, y+p)$.

Our aim is to determine the corresponding components of the electric and magnetic fields $\mathbb{E} = (E_1, E_2, E_3)$ and $\mathbb{B} = (B_1, B_2, B_3)$ respectively. As a consequence of the above hypotheses we conclude that the fields are dependent on two variables x and y only and are p-periodic in y.

The existence of weak fields $\mathbb{E}$ and $\mathbb{B}$ is proved as well as the uniqueness. A relationship with the classical solution fields is obtained. The proof is constructive and offers effective algorithms for numerical solution. A particular algorithm is discussed and namely that whose one part is a non well posed problem (an initial value problem for an elliptic system). Some properties of this algorithm are discussed.

20.9.1982 Ivo Marek
Charles University, Prague

Local stability of the output least square Parameter Estimation Technique

We investigate the well-posedness of the OLS (output least square) settings of parameter estimation problems:

find $\hat{x} \in C$ which minimizes $\| \varphi(x) - z \|^2$ over C

where x is the parameter, C the convex of admissible parameters, z the measurement and φ the parameter $\to$ output mapping.

When $\| \varphi'(x)(y) \| \geq \alpha \|y\|$ with $\alpha > 0$ and C small enough compared to the "curvature" of φ, we obtain the (Lipschitz) continuity of the mapping $z \to \hat{x}$ on a neighborhood of $\varphi(C)$.

As "practical" application of this result, we get that for finite dimensional parameters, the OLS technique yields an estimated parameter converging towards the exact one when the model and measurement errors tends to zero, as soon as the derivative of the parameter → output mapping is injective for the exact parameter and the size of the admissible parameter set is small enough

30/9/82 Guy Chavent INRIA and U. Paris IX

Zur Mehrgittermethode bei Integralgleichungen 1. Art

Mehrgitterverfahren für Integralgleichungen 2. Art sind charakterisiert durch eine Kombination eines Glättungsschritts und eines Korrekturschritts. Die Korrektur wird mit Hilfe eines gröberen Gitters durchgeführt.
Wir diskutieren hier einen Versuch, die Mehrgittermethode für Integralgleichungen 1. Art zu formulieren. Bei der Übertragung der Methode auf Gleichungen 1. Art ersetzen wir den Glättungsschritt durch einen Iterationsschritt der Iteration nach Landweber. Wir formulieren das Verfahren rekursiv. Der Korrekturschritt wird wieder nach der Mehrgittermethode durchgeführt. Ein numerisches Beispiel illustriert Effektivität und Problematik des Verfahrens.

30.9.82 J. Baumeister / Frankfurt

On the order of regularization methods.

Let $A: L_2(\Omega) \to H$, H a Hilbert space, be a linear operator such that the norms $\|Af\|_H$, $\|f\|_{H^{-a}(\Omega)}$ are equivalent, where $H^k(\Omega)$ denotes the Sobolev space of order k. It is shown that the Tikhonov regularization for $Af = g$, i.e.

the minimizer of $\|Af-g\|^2+\alpha^2\|f\|^2_{H^p(\Omega)}$, is asymptotically optimal for any p, provided α is chosen optimally.

The same is true for the "coarse discretization regularization", i.e. the minimizer of $\|Af-g\|$ in a spline space $S_{h,p}$ of degree $p-1$, provided p is sufficiently large and h is chosen optimally.

Frank Natterer
Münster

"On the reconstruction of a star-shaped body from its "half-volumes" ".

The problem is the reconstruction of the shape of an object, whose shell is a surface star-shaped with respect to a point O, from the knowledge of the volume of every "half-object" obtained by taking any plane through O. Conditions for the existence and uniqueness of the solution are given. The main result consist in showing that any uniform a-priori bound on the mean curvature of the shell reestablishes continuous dependence on the data for bodies satisfying a certain symmetry condition.

30.9.82

Stefano Campi

"Ein Verfahren zur Lösung zweidimensionaler inverser Stefan-Probleme"

We are considering the single phase inverse Stefan problem in s space dimensions. Our main tool for a numerical approach to this problem is a complete family of solutions of the heat equation which has the benefit of reducing the dimension of the problem by one. Our idea is to minimize the maximal defect in the data under regularizing constraints on the finite subspace which is generated by the first n members of the complete family. It can be shown that the solutions of the arising linear programming problems tend to the solution of the inverse Stefan problem provided such a solution exists. For s = 2 examples of complete families of solutions can be given. Finally two numerical examples will be discussed.
This work had been done jointly with David Colton from the University of Delaware, USA.

Rembert Reemtsen

On ~~the~~ a constrained Fourier extrapolation method for numerical deconvolution.

This paper generalizes the method of Birand for solving Fredholm integral equations of the first kind of convolution type. The 'a priori' assumptions on the solution are (i) compact support and (ii) non-negativity. As an alternative to regularization by smoothing, this method exploits the analyticity of the Fourier transform of the solution. An optimal rectangular filter is applied to the raw transform to cut off the high frequency terms dominated by amplified noise, and an approximate method of numerical analytical continuation is then applied to the truncated transform to restore high frequency information.

The ill-posedness of the Fourier extrapolation problem is stabilized by "functional regularization" based on the non-negativity of the solution, and the algorithm involves the unconstrained minimization of a nonlinear least-squares functional. Numerical results are given.

30 Sept 82. Russell Davies.

The use of auto-correlation for pseudo-rank determination.

A method is proposed for pseudo-rank selection by considering auto-correlation properties of the residual vector. The method is based on one proposed by M.J.D. Powell for curve-fitting to data, and pre-supposes a given ordering of the linear equations. Pseudo-ranks determined by generalized cross-validation are used for comparison. A combination of the two methods increased the reliability of the calculated solutions.

1982-10-01 Louisa Baart.

Inverse Mapping Theorems and Ill-Posed Problem

In this lecture we present several new generalized implicit function theorems in Banach spaces when the derivative has no bounded inverse. The classical implicit function theorem in Banach spaces obtains a local solution of an operator equation $G(z)=0$, given $G(a)=0$, and some hypothesis on the derivatives, usually that the Fréchet derivative $M:=G'(a)$ is surjective, and thus right invertible. The distinctions between "local" and "global" and between "soft" and "hard" inverse mapping theorems are clarified via several new theorems. Related results are given for $G(z)\in S$, where S is a cone or a subspace.

It is shown that the hypothesis on M can be weakened to assume an approximate right inverse, with strong Fréchet, or strong Hadamard differentiability of G. The value of the derivative is required only at a single point instead of a neighborhood. The ill-posed nature of problems which require the use of "hard" implicit function theorems is emphasized and approximate regularizations methods for such theorems are given. The work continues earlier work of the author (1977) and Craven and the author (1982).

M. Zuhair Nashed
University of Delaware, Newark, DE, U.S.A.
September 30, 1982

Ill-Posed Problems with Convex Constraints

In this talk we report on some work of the thesis of W. Karrington which was completed with the author, and related results for (1) the convergence of the "discrepancy/defect" method applied to moment discretization of ill-posed problems with convex constraints; (2) convergence of a linear programming algorithm to extremal problems over the set of all solutions of a generalized moment problems; (3) convergence of a conjugate subgradient algorithm for the same problem as in (2).

M. Z. Nashed
University of Delaware, U.S.A.

Identification of Kinetical Parameters in Medical Receptor Analysis

The mathematical model of a harmone-receptor binding with one and two binding sites is established. The family of functions giving the bound part as a function of the total harmone concentration can be specified explicitely in the case of one binding site.

This family depends nonlinearly on two parameters, the dissociation constant and the receptor capacity. For the corresponding identification problem existence and uniqueness is proven. Numerical results computet with the Gauss-Newton procedure are presentet.

PETER JOCHUM
Universität München

ANALYTISCHE ZAHLENTHEORIE

03. – 09. Oktober 1982

Character sums and primitive roots in algebraic number fields

Let K be an algebraic number field of degree $n = r_1 + 2r_2$ (in the usual notation) over the rationals with discriminant d. $\mathbb{Z}_K$ will denote the ring of integers in K. Let $P_1, .., P_n$ be positive real numbers with $P_k = P_{k+r_2}$, $k = r_1+1, .., r_1+r_2$ and $P = P_1 \cdots P_n$.
Consider the set
$\mathcal{R} = \{ \alpha \in \mathbb{Z}_K ; \alpha \text{ totally positive}, |\alpha^{(k)}| \le P_k, k = 1, .., n \}$.

<u>Theorem 1</u>. Let χ be a non-principal character modulo a prime ideal $\mathfrak{p}$ of K. Let $r = r_1 + r_2 - 1$. Then we have for $\varepsilon > 0$ and $q \in \mathbb{N}$:

$$\sum_{\alpha \in \mathcal{R}} \chi(\alpha) \ll_{\varepsilon, q, K} \begin{cases} N\mathfrak{p}^{\frac{1}{2(r+2)}} \cdot P^{1 - \frac{1}{r+2} + \varepsilon} , & P \ge N\mathfrak{p}^{1/2} ; \\ N\mathfrak{p}^{\frac{q+1}{2q(2q+n)}} \cdot P^{1 - \frac{2}{2q+n}} \cdot \log N\mathfrak{p} , & P \le N\mathfrak{p}^{1/2} . \end{cases}$$

The first inequality can be considered as an extension of the Pólya – Vinogradov inequality to algebraic number fields. The proof requires results about Hecke's zeta-functions with Größencharacters and makes use of an identity given by Siegel and Grotz.
Generalizing to number fields in an appropriate way the method given by Burgess for rational character sums, we obtain the

the second estimate.

As an application of theorem 1 we can extend a well-known result about the distribution of primitive roots to algebraic number fields. Let $\mathcal{T}$ be the set of all totally positive primitive roots modulo a prime ideal $\mathfrak{p}$ of K.

<u>Theorem 2</u>. For large $N\mathfrak{p}$ and $P \geq N\mathfrak{p}^{1/4+\varepsilon}$, $\varepsilon > 0$, we have

$$\sum_{\alpha \in \mathcal{R} \cap \mathcal{T}} 1 = \frac{(\log 2)^{r_2}}{N\mathfrak{d}} \cdot \frac{\varphi(N\mathfrak{p}-1)}{N\mathfrak{p}-1} \cdot P \cdot \{ 1 + O((N\mathfrak{p})^{-a}) \},$$

where φ denotes Euler's function, and a is a positive real number depending on ε and the degree n.

This implies, in particular, that a least primitive root $\gamma_{\mathfrak{p}}^* \in \mathcal{T}$, least in the sense that its norm $N\gamma_{\mathfrak{p}}^*$ is minimal, satisfies

$$N\gamma_{\mathfrak{p}}^* \ll_{\varepsilon,K} N\mathfrak{p}^{1/4+\varepsilon}, \quad \varepsilon > 0.$$

<u>Theorem 3</u>. 1) For $\varepsilon > 0$ there exists a positive number $A = A(\varepsilon, n)$ such that

$$\operatorname{card}\{\mathfrak{p};\ N\mathfrak{p} \leq X,\ N\gamma_{\mathfrak{p}}^* > N\mathfrak{p}^{\varepsilon}\} \ll_{\varepsilon,K} (\log X)^{A}.$$

2) $$\sum_{N\mathfrak{p} \leq X} N\gamma_{\mathfrak{p}}^* \ll X \cdot \log X \cdot (\log\log X)^{2(r_1+r_2+1)}, \quad X \geq 3.$$

Jürgen Hinz, Marburg

About k-free numbers in totally real algebraic number fields

Let K denote a totally real algebraic number field with $[K:\mathbb{Q}]=n$ and discriminant d; let $\mathfrak{o}_K$ be the ring of integers of K. An element $\alpha\in\mathfrak{o}_K$ is called "k-free" ($k\geq 2$) if there is no integral ideal $\mathfrak{a}$ of K, $\mathfrak{a}\neq(1)$, s. t. $\mathfrak{a}^k|\alpha$.

Define $\tau_k(\xi):=\begin{cases}1, & \text{if } \xi\in\mathfrak{o}_K \text{ is k-free}\\ 0, & \text{otherwise}\end{cases}$. Then we have the

Theorem: Let $x_1,\dots,x_n$ denote positive real numbers satisfying the inequalities

$$c_2 X^{1/n}\leq x_1,\dots,x_n\leq c_1 X^{1/n},\quad X:=x_1\cdots x_n$$

with constants $0<c_2\leq c_1$. Let

$$a_k:=\frac{1}{k}-\frac{1}{4k-1},\quad k\geq 2.$$

Then the following asymptotic expansion holds true:

$$\sum_{\substack{x_i<\xi^{(i)}\leq x_i+x_i^{a_k+\frac{\log\log X}{\log X}}\\ i=1,\dots,n}}\tau_k(\xi)=\frac{X^{a_k}\log X}{\zeta_K(k)\sqrt{|d|}}\left\{1+O\left(\frac{1}{\log X}\right)\right\},\quad X\geq 3;$$

$\zeta_K(s)$ denotes the Dedekind ζ-function of K and the O-constant depends on k, K, c_1, c_2 only.

Cor. 1: For $X\geq X_0(k,K,c_1,c_2)$ there is a k-free $\alpha\in\mathfrak{o}_K$ satisfying

$$x_i<\alpha^{(i)}\leq x_i+x_i^{a_k+\frac{\log\log X}{\log X}},\quad i=1,\dots,n.$$

Cor. 2: If $\alpha\in\mathfrak{o}_K$ is a totally positive k-free number satisfying

$$c_2 N(\alpha)^{1/n}\leq\alpha^{(1)},\dots,\alpha^{(n)}\leq c_1 N(\alpha)^{1/n},\quad N(\alpha)\geq X_1(k,K,c_1,c_2)\geq 3,$$

then there exists a k-free $\beta\in\mathfrak{o}_K$ s.t. the distance r of the points $(\alpha^{(1)},\dots,\alpha^{(n)})$ and $(\beta^{(1)},\dots,\beta^{(n)})$ is less than

$$c_3\cdot N(\alpha)^{a_k/n}\,(2\log N(\alpha))^{1/n},\qquad c_3:=\sqrt{n}\,c_1^{1/k}.$$

For the proof of the theorem I apply a modification of a method given by Halberstam and Roth in their paper "On the gaps between consecutive k-free numbers" (J. London Math. Soc. 26 (1951), 268-273). Moreover, I use a lemma due to Nair in his paper " Power free values of polynomials II" (Proc. London Math. Soc. (3), 38 (1979), page 358).

Werner Schaal, Marburg/Lahn

A generalization of Siegel's summation formula

Let K be an algebraic number field of degree $[K:\mathbb{Q}] = n = r_1 + 2r_2$ (as usual) and let $r+1 = r_1 + r_2$, $e_p = 1$ for $p = 1,\dots,r_1$ and $e_p = 2$ for $p > r_1$. We denote integers in K by ν and their conjugates by $\nu^{(p)}$. Let $\eta_1,\dots,\eta_r$ be r independent units of K, $\mathcal{U} = \langle \eta_1,\dots,\eta_r \rangle$ the group generated by them, and $R(\mathcal{U})$ its regulator. Moreover, let f be defined on the numbers ν and have the invariance property $f(\eta\nu) = f(\nu)$ for all $\eta \in \mathcal{U}$. Finally, let $\Lambda(\nu) = \prod_{p=1}^{r+1} |\nu^{(p)}|^{-ie_p b_p}$ ($b_p \in \mathbb{R}$ arbitrary) be a generalized Hecke größencharacter, $\Phi : \mathbb{R}_+^{r+1} \to \mathbb{C}$ and $x \in \mathbb{R}_+^{r+1}$, where $\mathbb{R}_+^{r+1} = \{ u = (u_1,\dots,u_{r+1}) \in \mathbb{R}^{r+1} \,/\, u_p > 0 \text{ for } p = 1,\dots,r+1 \}$.
We consider the sum

$$F(x) = \sum_{\nu \neq 0} \Lambda(\nu) f(\nu) \Phi(|\nu^{(1)}| x_1^{-1}, \dots, |\nu^{(r+1)}| x_{r+1}^{-1}) .$$

Under certain (weak) conditions one has for $\varepsilon > 0$:

$$F(x) = F_\varepsilon(x) + R_\varepsilon(x),$$

where

$$F_\varepsilon(x) = \frac{2^{r_2}}{2\pi i\, R(\mathcal{U})} \sum_{m \in \mathbb{Z}^r} \int_{\sigma - i\infty}^{\sigma + i\infty} \psi(\dots, s - ib_p - iE_p(m), \dots) \prod_{p=1}^{r+1} x_p^{e_p(s - ib_p - iE_p(m))} \cdot$$

$$\cdot Z(s; f\lambda_m) \cdot \exp\{\varepsilon \sum_{p=1}^{r+1} e_p^2 (s - ib_p - iE_p(m))^2\}\, ds$$

with $\sum_{p=1}^{r+1} e_p E_p(m) = 0$, $\sum_{p=1}^{r+1} e_p E_p(m) \log|\eta_q^{(p)}| = 2\pi m_q \quad (m = (m_1,\dots,m_r))$,

$$\psi(s) = \int_0^\infty\!\!\cdots\!\int \phi(u) \prod_{p=1}^{r+1} u_p^{e_p s_p - 1}\, du \quad \text{(Mellin-transform)},$$

$$Z(s; f\lambda_m) = \sum_{(\nu)_U} \frac{f(\nu)\lambda_m(\nu)}{|N(\nu)|^s}, \quad \lambda_m(\nu) = \prod_{p=1}^{r+1} |\nu^{(p)}|^{i e_p E_p(m)}.$$

The remainder term is

$$R_\varepsilon(x) = -\int_{-\infty}^{\infty}\!\!\cdots\!\int \{ F(\dots, x_p e^{y_p}, \dots) - F(x) \}\, W(y,\varepsilon)\, dy,$$

where $W(y,\varepsilon) = (4\pi\varepsilon)^{-\frac{r+1}{2}} e^{-\frac{1}{4\varepsilon}(y_1^2+\dots+y_{r+1}^2)}$, the Gauss-Weierstrass kernel.

$R_\varepsilon(x) \to 0$ for $\varepsilon \to 0$, if x is a point of continuity of F. This is the case for instance, when $x = (1, \dots, 1)$ and ϕ is the characteristic function of some bounded domain in $\mathbb{R}_+^{r+1}$ with no vector $(\dots, |\nu^{(p)}|, \dots)$ on its boundary.

Putting $\phi(u) := \prod_{p=1}^{r+1} (1-u_p)^{k_p}$ if $0 < u_p < 1$ and 0 otherwise, one obtains SIEGEL's summation formula (1936) in the generalized version of SCHAAL (1965) and GROTZ (1981).

The formula can be extended to the case when ϕ is a function of the conjugates $\nu^{(p)}$ themselves, not only of their ~~conjugates~~ absolute values. There is also a version of this formula for rational number theory in several variables.

Ulrich Rausch, Marburg

Dirichlet series and holomorphic differential-difference equations

Let be $\mathfrak{D}$ the set of all Dirichlet series $f(s)=\sum_{n=1}^{\infty} a_n n^{-s}$ (with an abscissa of absolute convergence $\sigma_a(f)<\infty$) such that $\{n : a_n \neq 0\}$ contains infinitely many prime divisors. For given real numbers $h_0<h_1<\dots<h_\mu$ and $\nu_0,\nu_1,\dots,\nu_\mu \in \mathbb{N}_0$ one has the following

Theorem. Let be $f\in\mathfrak{D}$. If

$$\Phi\left(f(s+h_0), f'(s+h_0),\dots,f^{(\nu_0)}(s+h_0), f(s+h_1),\dots,f^{(\nu_1)}(s+h_1),\dots,f(s+h_\mu),\dots,f^{(\nu_\mu)}(s+h_\mu)\right)=0$$

holds for all $s\in\mathbb{C}$ s.t. $\operatorname{Re} s+h_0>\sigma_a(f)$ and Φ is holomorphic (more generally: Φ continuous and locally not trivial), then $\Phi=0$.

This gives (without additional assumptions) a full generalization of Ostrowski's dissertation in 1920 about hypertranscendental functions, where the theorem above is proved for exactly the same class $\mathfrak{D}$ of ordinary Dirichlet series in the case, that Φ is a polynomial. The method of proof is completely different to Ostrowski's method. In the special case of the Riemann zeta function there is an even stronger result as the theorem above.

Axel Reich (Göttingen)

LARGE VALUES OF THE ERROR TERMS IN DIVISOR PROBLEMS AND THE MEAN SQUARE OF THE ZETA-FUNCTION

Let $\Delta_k(x)=\sum_{n\le x} d_k(n) - \operatorname{Res}_{s=1} \zeta^k(s)x^s s^{-1}$ be the error term in the divisor problem, $E(T)=\int_0^T |\zeta(\frac12+it)|^2 dt - T\log\frac{T}{2\pi} - (2\gamma-1)T$, $P(x)=\sum_{n\le x} r(n) - \pi x$ be the error term in the circle problem. The best known order results for these functions, obtainable by G. Kolesnik's methods for estimating exponential sums of the form $\sum_{(x,y)\in D} e(f(x,y))$, are as follows:

$$\Delta_2(x)\ll x^{\frac{35}{108}}\log^2 x,\quad \Delta_3(x)\ll x^{\frac{43}{96}+\varepsilon},\quad E(T)\ll T^{\frac{35}{108}+\varepsilon},\quad P(x)\ll x^{\frac{35}{108}+\varepsilon}.$$

These results are rather far away from the known omega results for these functions and the conjectured values of the exponents, which are respectively $\frac{1}{4}+\varepsilon, \frac{1}{3}+\varepsilon, \frac{1}{4}+\varepsilon, \frac{1}{4}+\varepsilon$.

It seemed interesting to try to obtain mean power estimates which would in some sense support the conjectured values of the exponents. By using suitable explicit truncated formulas for the functions in questions, a large values estimate which depended on the use of the theory of exponential pairs and the Halász - Montgomery inequality, the following results have been obtained:

$$(1) \qquad \int_1^T |\Delta_2(t)|^A dt \ll T^{\frac{A+4+\varepsilon}{4}}, \qquad 0 \le A \le \tfrac{35}{4} = 8\tfrac{3}{4},$$

$$(2) \qquad \int_1^T |\Delta_2(t)|^A dt \ll T^{\frac{35A+38+\varepsilon}{108}}, \qquad A \ge \tfrac{35}{4},$$

$$(3) \qquad \int_1^T |\Delta_3(t)|^A dt \ll T^{\frac{106A+253+\varepsilon}{278}}, \qquad 2 \le A \le \tfrac{2237}{607} = 3.6853\ldots,$$

$$(4) \qquad \int_1^T |\Delta_3(t)|^A dt \ll T^{\frac{43A+63+\varepsilon}{96}}, \qquad A \ge \tfrac{2237}{607}.$$

Here A is an arbitrary fixed number, which does not have to be an integer. The estimates (1) and (2) hold also if $\Delta_2(t)$ is replaced by $E(t)$ or $P(t)$, and in view of the classical formula

$$\int_1^T \Delta^2(t)\,dt \sim \frac{1}{6\pi^2}\sum_{n=1}^{\infty} d^2(n)n^{-3/2}\cdot T^{3/2}$$

(and analogous formulas for $E(t)$ and $P(t)$) it is seen that (up to ε) the estimate (1) is best possible. The results are all new for $A > 2$, but although the method of proof works for general $\Delta_k(x)$, the results are sharp only for $k=2$ or $k=3$.

Aleksandar Ivić

Einige seltsame Identitäten mit π

Ausgehend von den Reihenentwicklungen von z.B. $e^x = \sum_{\nu=0}^{\infty} \frac{x^\nu}{\nu!}$ oder $\cos x = \sum_{\nu=0}^{\infty} \frac{x^{2\nu}}{(2\nu)!}(-1)^\nu$ kann man die Frage stellen, ob es bei solchen Reihen mit "größerer Schrittweite", wie z.B. für $1-\frac{x^6}{6!}+\frac{x^{12}}{12!}-\frac{x^{18}}{18!}+-\dots$, allg. für $a_n(x) := \sum_{\nu=0}^{\infty} \frac{x^{n\nu}}{(n\nu)!}(-1)^\nu$; $s_n(x) := \sum_{\nu=0}^{\infty} \frac{x^{n\nu}}{(n\nu)!}$, auch geschlossene Formeln gibt.

Nutzt man die Eigenschaften solcher Reihen aus, so gelangt man über ein System von Differentialgleichungen zu den eindeutigen Lösungen. Es ist aber auch nicht allzu schwer, die Lösung in der Form

$$a_n(x) = \frac{1}{n}\sum_{\mu=1}^{n} e^{x\cdot e^{\frac{2\pi i}{2n}(2\mu-1)}}$$

zu 'erraten' und auch gleich direkt zu bestätigen:

$$= \frac{1}{n}\sum_{\nu=0}^{\infty}\frac{x^\nu}{\nu!}\cdot\underbrace{\sum_{\mu=1}^{n} e^{\frac{2\pi i}{2n}\nu(2\mu-1)}} \qquad \text{(analog für } s_n\text{)}$$

$= n$ falls $2n\,|\,\nu$; $= -n$ falls $n\,|\,\nu$ aber $2n\nmid\nu$; $= 0$ sonst.

Für einige kleine n vereinfachen sich diese Formeln. z.B. hat man

$$a_4(x) = 1-\frac{x^4}{4!}+\frac{x^8}{8!}-+\dots = \cos\frac{\sqrt2}{2}x\cdot\cosh\frac{\sqrt2}{2}x \quad \text{sowie}$$

$$a_6(x) = 1-\frac{x^6}{6!}+\frac{x^{12}}{12!}-+\dots = \frac13\cos x+\frac23\cos\frac{x}{2}\cdot\cosh\frac{x}{2}\sqrt3 \quad \text{, etc.}$$

*

Wählt man nun x geeignet, z.B. gleich $\frac{\pi}{\sqrt2}$ in a_4 oder gleich π in a_6, so ergeben sich seltsame Identitäten:

$$\boxed{1-\frac{\pi^4}{2^2\cdot4!}+\frac{\pi^8}{2^4\cdot8!}-\frac{\pi^{12}}{2^6\,12!}+-\dots=0 \quad\text{sowie}\quad 1-\frac{\pi^6}{6!}+\frac{\pi^{12}}{12!}-\frac{\pi^{18}}{18!}+-\dots=-\frac13 \quad \text{usw.}}$$

*

Man kann durch Spielerei mit solchen Reihen (und ihren Ableitungen) auch besondere Teilreihen der Exponentialreihe gewinnen, wie z.B. $\sum_{\substack{\nu=0\\ \nu\equiv0,1 \bmod 4}}^{\infty} \frac{x^\nu}{\nu!} = \frac12(e^x+\cos x+\sin x)$. Die besondere Eigenschaft dieser Reihe liegt darin, daß sie mit den anderen drei verwandten $\sum_{\nu\equiv2,3\bmod 4}\dots$, $\sum_{\nu\equiv0,3\bmod 4}\dots$, $\sum_{\nu\equiv1,2\bmod 4}\dots$, die einzigen nichttrivialen Teilreihen von e^x sind, die für alle $x<0$ beschränkt sind. (Siehe L. Rubel und K. Stolarsky, Amer. Math. Monthly 87 (1980) p. 371 ff.).

*

Es ist aber auch überraschend, wie schnell sich aus obigen Identitäten numerische Näherungswerte für π ergeben. Ausgehend von $a_6(\pi)$: $\frac{\pi^6}{6!}-\frac{\pi^{12}}{12!}+\frac{\pi^{18}}{18!}-+\dots=\frac43$ ergibt sich mit dem ersten Glied allein bereits $\hat\pi^6=960$, was einem Fehler von 0,00075.. für π entspricht. Mit dem 2. Glied erhält man π mit einer Genauigkeit von $\approx 1{,}5\cdot10^{-7}$, usw.

*

Für Transzendenzfragen lassen sich diese π-Identitäten aufgrund ihrer Bauart und Herkunft leider nicht verwerten.

Hans-J. Bentz (Osnabrück)

On sums of primes

It is shown that every even natural number can be represented as a sum of at most eighteen primes. The previous best results of this kind are due to Vaughan (1977) and Deshouillers (1977) who have shown that every even number is the sum of at most twenty six primes and that every odd number is the sum of at most twenty five primes respectively.

The proof is divided into three parts. Let $N(x)$ denote the number of even numbers n not exceeding x for which n is the sum of at most two primes. Then it suffices to show that

$$N(x) > \frac{x}{18} \quad (x \geq 2),$$

for then the result will follow by a standard application of Mann's theorem.

The first case is when $\log x \geq 375$. This depends on obtaining accurate estimates for

$$\Psi = \sum_n \sum_{k=1}^{200} R^{(k)}(n) w(n)$$

where

$$R^{(k)}(n) = \text{card}\{(p,p') : p+p'=n, p \in I_k, p' \in I_k\},$$

$$I_k = (\tfrac{1}{2}ky, \tfrac{1}{2}ky + y),$$

$$w(n) = \prod_{\substack{p \mid n \\ p>2}} \frac{p-2}{p-1},$$

and this in turn depends on an accurate form of the two dimensional upper bound sieve.

The case $\log x \leq 27$ is trivial.

Finally the case $27 < \log x < 375$ is dealt with by an intricate combination of theoretical argument and calculation.

R. C. Vaughan.

Divisor problems.

Denote by $1=d_1<\ldots<d_{\tau(n)}=n$. The consecutive divisors of n. I conjectured more than 40 years ago that for almost all n $\min d_{i+1}/d_i<2$. Several recent results of Tenenbaum and myself seemed to point in the other direction, but I hear that Meier recently proved this conjecture and more.

Hall and I investigated the following function:

$$f(n)=\sum_{(d_i,d_{i+1})=1}1.$$

Recently Tenenbaum and I proved that for infinitely many n

$$(1)\qquad f(n)>\exp\left(c\,\log n/(\log\log n)^2\right).$$

We could not decide how close (1) is to being best possible.

I conjectured that for every $\varepsilon>0$ there is a constant c_ε so that for infinitely many n

$$(2)\qquad \sum_i\left(d_{i+1}/d_i-1\right)^{1+\varepsilon}<c_\varepsilon$$

This conjecture has recently been proved by M. ~~Wose~~ VOSE

Probably (2) is bounded for $n_k=k!$ or $n_k=2\,3\ldots p_k$ or if n_k is highly composite.

Paul Erdős

The Joint Distribution of the Binary Digits of Integer Multiples:

Let $B(n)$ denote the number of digits 1 in the binary representation of $n \in \mathbb{N}_0$. Then for different odd $k_1, k_2, \ldots, k_s \in \mathbb{N}$:

$$\# \{ 0 \le n < x, \ \forall \ \nu = 1, 2, \ldots, s : \ B(k_\nu n) = a_\nu \} =$$

$$= \frac{x}{\sqrt{2\pi \, ld\, x}^{\,s} \sqrt{\det V}} \exp\left(- \frac{1}{2\, ld\, x} \left(a - \frac{ld\, x}{2}\right) V^{-1} \left(a - \frac{ld\, x}{2}\right)' \right)$$

$$+ O\left(\frac{x}{\sqrt{ld\, x}^{\,s} \sqrt{ld\, x}} \right)$$

with a positive-definite matrix V. Herein, the O-constant does not depend on $a := (a_\nu)_{\nu=1}^{s} \in \mathbb{Z}^s$. ($ld :=$ logarithm to base 2).

Analogous results hold for $\# \{ 0 \le n < x, \ B(k_1 n) - B(k_2 n) = a \}$

and $\# \{ 0 \le n < x, \ \forall \nu : \ \dfrac{B(k_\nu n) - \frac{ld\, x}{2}}{\sqrt{ld\, x}} \le z_\nu \}$.

Idea of the Proof: In essence, by a one-step recursion the case $x = 2^m$ can be reduced to the case $x = 2^{m-1}$. Rewriting this recursion using the Fourier-transforms leads to the equivalent problem to estimate the m-th power of a matrix.

This, finally, is done via the estimation of the m-th power of the maximum modulus eigenvalue.

Johannes Schmid (Ulm)

L-functions associated with character sums over finite fields

Let χ be a nontrivial additive character of the finite field $\mathbb{F}_q$ and (y_n), $n=0,1,\ldots$, a kth order linear recurring sequence in $\mathbb{F}_q$ with irreducible characteristic polynomial and nonzero initial state vector. If τ is the least period of (y_n), we consider the character sum

$$S_1 = \sum_{n=0}^{\tau-1} \chi(y_n).$$

For any finite extension $\mathbb{F}_{q^r}$ of $\mathbb{F}_q$ we define a "lifted" character sum

$$S_r = \sum_{\text{period}} \chi^{(r)}(y_n^{(r)}),$$

where $\chi^{(r)}$ is the character of $\mathbb{F}_{q^r}$ obtained by lifting χ via the trace and $(y_n^{(r)})$ is a canonically lifted linear recurring sequence in $\mathbb{F}_{q^r}$. Then we set up an L-function in the sense of Weil and Grothendieck:

$$L(t) = \exp\left(\sum_{r=1}^{\infty} \frac{S_r}{r} t^r\right).$$

<u>Theorem 1.</u> For $h = \frac{q^k-1}{\tau}$ we have $L^h \in \mathbb{Z}[t]$ with $\deg(L^h) = h$.

<u>Theorem 2.</u> $L^h(t) = \prod_{j=1}^{h}(1-\alpha_j t)$, $\alpha_j \in \mathbb{C}$, with $\alpha_1 = 1$ and $|\alpha_j| = q^{k/2}$ for $2 \le j \le h$.

These results lead to a sharp estimate for S_r. By logarithmic differentiation of the two formulas for L^h above and comparison of coefficients we get

$$S_r = -\frac{1}{h}\sum_{j=1}^{h} \alpha_j^r \quad \forall r \in \mathbb{N}.$$

Thus the subsequent result follows.

<u>Theorem 3.</u> $|S_r| \le (1-\frac{1}{h})q^{kr/2} + \frac{1}{h}$, and this is best possible in the following sense: $\forall \varepsilon > 0$ $\exists r \in \mathbb{N}$ with $|S_r| > (1-\frac{1}{h}-\varepsilon)q^{kr/2}$.

Harald Niederreiter

Sums of Powers of Cusp Form Coefficients

Let $f(z) = \sum_{n=1}^{\infty} a(n) e^{2\pi i n z}$ $(\mathrm{Im}\, z > 0)$ be a cusp form of weight k belonging to the modular group $\Gamma(1)$. The estimates

$$\sum_{n \le x} |a(n)|^{2\beta} \le A_1 x^{\beta(k-1)+1} \quad (0<\beta\le 1), \qquad \sum_{n\le x} |a(n)|^{2\beta} \ge A_2 x^{\beta(k-1)+1} \quad (\beta > 1)$$

follow immediately since $\sum_{n \le x} |a(n)|^2 \sim A x^k$; here $A, A_1, A_2, \ldots$ are positive constants. In the opposite direction it is shown that

$$\sum_{n\le x} |a(n)|^{2\beta} \ge A_3 x^{\beta(k-1)+1} / (\log x)^{\gamma} \quad (0<\beta\le 1), \qquad \sum_{n\le x} |a(n)|^{2\beta} \le A_4 x^{\beta(k-1)+1} (\log x)^{\gamma} \quad (\beta>1),$$

where
$$\gamma = 2^{2(\beta-1)} - 1 .$$

Thus for Ramanujan's function $\tau(n)$ $(k=12)$ it follows, in particular, that

$$\sum_{n\le x} |\tau(n)| \gg x^{\frac{13}{2}} (\log x)^{-\frac{1}{2}}, \qquad \sum_{n\le x} |\tau(n)|^4 \ll x^{23} (\log x)^3 .$$

The latter result improves the estimate $\sum_{n\le x} |\tau(n)|^4 \ll x^{23}(\log x)^{15}$, obtainable by using Deligne's result that

$$|\tau(n)| \le n^{\frac{11}{2}} d(n) \qquad (n \in \mathbb{N}).$$

The proof uses the elementary inequalities:

$$|2\cos\theta|^{2\beta} \le 2^{2\beta-1}(1+\cos 2\theta) \quad (\beta>1), \qquad |2\cos\theta|^{2\beta} \ge 2^{2\beta-1}(1+\cos 2\theta) \quad (0<\beta<1),$$

the fact that

$$\zeta(s) \prod_p \left(1 - \tau(p) p^{-\frac{11}{2}-s} + p^{-2s}\right)^{-1}$$

is holomorphic and non-zero for $\sigma = \mathrm{Re}\, s \ge 1$, and Delange's extension of the Ikehara theorem. Similar results hold for cusp forms of level N and fixed character χ mod N. There are cases where it can be shown that the constant γ is not best possible; unfortunately not for $\tau(n)$, where no additional information is available.

R. A. Rankin

Gauss sums and Fourier analysis

Let $G(q)$ denote the multiplicative group of invertible elements in $\mathbb{Z}_q$, the ring of integers modulo q. Let H denote a multiplicative subgroup of $G(q)$ and let $f: \mathbb{Z}_q \to \mathbb{C}$ be supported in H. We show that any such f can be recovered from the values of $\hat{f}$ restricted to H if and only if the Gauss sums for H are non vanishing. (Here $\hat{f}$ is the Fourier transform of f with respect to the additive group $\mathbb{Z}_q$, and the Gauss sums are $\hat{\chi}(1)$ for each character χ of H.)

The main result is that the Gauss sums are all non vanishing if and only if

$$H \cap \{m \in \mathbb{Z}_q : m \equiv 1 \bmod v\} = \{1\},$$

where

$$v(q) = \begin{cases} \prod_{p \mid q} p & \text{if } 8 \nmid q \\ 2\prod_{p \mid q} p & \text{if } 8 \mid q. \end{cases}$$

If $H = G(q)$ for q square free, then f can be recovered from $\hat{f}$ restricted to $G(q)$ by a particularly elementary formula. This formula provides some inequalities and extremal functions.

This is joint work with F. Gerth and J. Vaaler.

Harold G. Diamond

A remark on the logarithmic derivative of Riemann's Zeta-function.

To obtain zero-free regions for the Riemann Zeta-function, it is usual to start from the inequality

$$\mathrm{Re}\,\frac{\zeta'}{\zeta}(\sigma+2it)+4\,\mathrm{Re}\,\frac{\zeta'}{\zeta}(\sigma+it)+3\,\frac{\zeta'}{\zeta}(\sigma)\leqslant 0 \qquad (\sigma>1,\ t\text{ arbitrary}).$$

t is taken equal to the imaginary part of a zero of ζ and σ is suitably chosen. A theorem on analytic functions of a complex variable provides lower bounds for the first two terms, depending on the real part of the zero, and a lower bound for that real part is derived.

A lower bound for $\frac{\zeta'}{\zeta}(\sigma)$ is needed.

Most authors (Landau, Walfisz, Prachar, Blanchard..) say that, since $\frac{\zeta'}{\zeta}(\sigma)\sim-\frac{1}{\sigma-1}$ as $\sigma\to 1$, given $\varepsilon>0$ there exits $\eta>0$ such that $\frac{\zeta'}{\zeta}(\sigma)\geqslant-\frac{1+\varepsilon}{\sigma-1}$ for $1<\sigma\leqslant 1+\eta$ (an $\varepsilon<\frac{1}{3}$ is needed).

In his book on multiplicative number theory, Davenport says that there exits a constant A such that $\frac{\zeta'}{\zeta}(\sigma)\geqslant-\frac{1}{\sigma-1}-A$ for $1<\sigma\leqslant 2$.

In the book by Ellison and Mendès-France, the same inequality is used, and, in an appendix to chapter V, it is proved that one can take $A=0$. The proof uses the formula

$$\frac{\zeta'}{\zeta}(s)=\log 2\pi-1-\frac{\gamma}{2}-\frac{1}{s-1}-\frac{1}{2}\frac{\Gamma'}{\Gamma}\left(\frac{s}{2}+1\right)+\sum_{\rho}\left(\frac{1}{s-\rho}+\frac{1}{\rho}\right)$$ where ρ runs through the set of non trivial zeros of ζ.

Much work is needed to obtain that formula.

I prove by a very simple calculation that, for every real $s>1$,

$$\frac{\zeta'}{\zeta}(s)>-\frac{1}{s-1} \qquad \left(\text{and even } >-\frac{1}{s-1}+\frac{1}{2s^2}\right).$$

Hubert Delange

Remarks on almost-even arithmetic functions.

Let f be an arithmetic function, $f'=f*\mu$.

It is known that, if $\sum_{n=1}^{\infty}\frac{|f'(n)|}{n}<\infty$, then f is almost-even B' and its "Ramanujan series" is $\cancel{\sum_{n=1}}\sum_{q=1}^{\infty}a_q c_q(n)$, where

$$a_q = \sum_{m=1}^{\infty} \frac{f'(qm)}{qm}.$$

Here, of course, the series is absolutely convergent.

I prove that, whenever f is almost-even-B', for every positive integer q, the series $\sum_{m=1}^{\infty} \frac{f'(qm)}{qm}$ is convergent and its sum is the coefficient of $c_q(n)$ in the Ramanujan series of f.

I had first found a rather complicated proof. Hildebrand gave me a simple proof of the following fact:

If $\sum_{n \leq x} |f(n)| = O(x)$ and if $M(f)$ exists, then the series $\sum_{n=1}^{\infty} \frac{f'(n)}{n}$ converges and its sum is $M(f)$.

By a suitable modification of his proof I obtain the following result: Let q be any positive integer. Suppose that

(i) $\sum_{n \leq x} |f(n)| = O(x)$,

(ii) for each divisor δ of q, $\lim_{x \to \infty} \frac{1}{x} \sum_{\substack{n \leq x \\ (n,q)=\delta}} f(n)$ exists (which obviously implies that $\lim_{x \to \infty} \frac{1}{x} \sum_{n \leq x} f(n) c_q(n)$ exists).

Then, the series $\sum_{m=1}^{q} \frac{f'(qm)}{qm}$ is convergent and its sum is $\frac{1}{\varphi(k)} \lim_{x \to \infty} \frac{1}{x} \sum_{n \leq x} f(n) c_q(n)$.

<u>Other remark</u>: Define a one to one correspondence between multiplicative and additive functions as follows:

To f multiplicative associate the additive function g determined by

$$g(p^r) = f'(p^r) \qquad \text{for every prime } p \text{ and every } r \geq 1.$$

The following two propositions are equivalent:

(a) f is almost-even B^λ with a not $\equiv 0$ Ramanujan series. $(\lambda \geq 1)$.

(b) g is almost-even B^λ.

Hubert Delange

Diameters of algebraic integers.

This talk reports results and progress made on a number of questions related to diameters of the set of conjugates of algebraic integers. If α is an algebraic integer, with conjugates $\alpha = \alpha_1, \alpha_2, \ldots, \alpha_n$ and $n \geq 2$, write

$$\mathrm{diam}(\alpha) = \max_{i,j} |\alpha_i - \alpha_j|,$$

and

$$D(\alpha) = \text{diameter of the circle of least diameter which encloses the conjugates } \alpha_1, \alpha_2, \ldots, \alpha_n.$$

Lower bounds for $\mathrm{diam}(\alpha)$ and $D(\alpha)$ have been investigated by J. Favard, R.M. Robinson, C.W. Lloyd-Smith and M.J. McAuley. The best possible bound is known in a number of special cases (e.g. if α is reciprocal, if $\alpha \in$ J-field, if α has small degree).

Recent joint work of Lloyd-Smith, McAuley and the speaker is discussed, including the following result, and possible extensions.

Theorem. There exist positive constants ε_0, n_0 such that for all algebraic integers α of degree exceeding n_0,

$$\mathrm{diam}(\alpha) > \sqrt{3} + \varepsilon_0 .$$

Peter Bundschuh
(Adelaide)

Ergodic properties of Brun's algorithm and of Selmer's algorithm.

Let $a = (a_0, a_1, \ldots, a_n) \in (\mathbb{R}^+)^{n+1}$. Put $a_s = \max_{0 \leq k \leq n} a_k$. Choose $t \neq s$ and form

$$\sigma a = (a_0, \ldots, a_{s-1}, a_s - a_t, a_{s+1}, \ldots, a_n).$$

The choice $a_t = \max_{\substack{0 \leq k \leq n \\ k \neq s}} a_k$ gives BRUN's algorithm and the choice $a_\ell = \min_{0 \le k \le n} a_k$ gives SELMER's algorithm. After a suitable projection one obtains corresponding mappings T_B, T_S on the set $\{x : 0 \le x_n \le \dots \le x_1 \le 1\}$, $x_i = \frac{a_i}{a_0}$.

Theorem B: (1) T_B is ergodic

(2) $h(x) = \int_0^\infty dy_1 \int_0^1 \dots \int_0^1 dy_2 \dots dy_n \left(1 + \sum_{i=1}^n x_i y_i\right)^{-n-1}$

is the density of an invariant measure.

Theorem S: (1) The transformation T_S splits into a transient and an ergodic part.

(2) $h(x) = \int_0^\infty \dots \int_0^\infty dy_1 \dots dy_n \left(1 + \sum_{i=1}^n x_i y_i\right)^{-n-1}$

is the density of an invariant measure.

F. Schweiger (Salzburg)

Exceptional zeros of Selberg zeta functions

Just as real zeros of $L(s,\chi)$ give poles of L'/L, so real zeros of Selberg's zeta function for the fundamental domain of $\Gamma^0(Q)$ give poles of

$$Z_{m,n}(s) = \sum_{q \equiv 0 \,(\mathrm{mod}\, Q)} \frac{S(m,n;q)}{q^{2s}},$$

the Kloosterman sum zeta function. Combinatorial geometry and numerical bounds show that there is no real zero $s = \beta$ in $\frac{1}{2} < \beta \le 1$ for levels up to 18, that is, for $Q \mid N^2$ for some integer $N \le 18$.

Martin Huxley (Cardiff)

Distribution of coefficients of ~~Dirichlet~~ Eisenstein series in residue classes.

The coefficients in question are simply equal to $\sigma_k(n)$. An algorithm was presented determining the set $\mathcal{M}(\sigma_k)$ of all N's such that $\sigma_k(n)$ is weakly uniformly distributed (mod N). This algorithm is applicable to all $k \geq 3$. The case $k=2$ needs a separate approach (see WN & F. Rayner, Mh. Math. in print).

For $k=3$ this algorithm can be performed by hand and leads to

$$\mathcal{M}(\sigma_3) = \{N : (N,14)=1\} \cup \{N : 2|N, 3\nmid N\}.$$

W. Narkiewicz
(Wrocław).

Recurrence Sequences (Taylor coefficients of rational functions)

Denote by R a finitely generated subring of a field of characteristic zero (one may think of R as "integers"). The following Hadamard problems have excited interest in the past: Suppose $\Sigma a_h X^h$, $\Sigma b_h X^h$ are rational functions, and $a_h/b_h \in R$ for all h; then? is $\Sigma \frac{a_h}{b_h} X^h$ again a rational function? If $\Sigma a_{m(h)} X^h$ is again rational, then is $m(h)$ indeed ("piecewise") a linear function up to unavoidable ambiguities? If $a_h \in R$ all h and $\Sigma a_h^3 X^h$ is rational, is $\Sigma a_h X^h$ again rational (for a suitable choice of the a_h)? All these questions have a positive answer. The methods are essentially those of naive p-adic analysis (see the 'poem' below) together with a corollary of the Roth-Schmidt-Schlickewei results which guarantees growth conditions for the recurrence sequences that occur. These methods yield an elegant criterion: if $\Sigma f_h X^h$ is a G-function defined over an algebraic number field and, for sufficiently many primes p: $\mathrm{ord}_p(\Delta^k_{p-1} f_r) \gtrsim ks$, some $s>0$, $0 \le r < p-1$ then $\Sigma f_h X^h$ is a rational function.

"Questions of specialisation
And Hadamard multiplication:
Were dealt with by Alf,
In an hour and a half,
By cunning p-adification."

A. J. van der Poorten
(Macquarie)

Intersections of Pjateckij-Shapiro-Sequences

The distribution of natural and prime numbers in $F_{c_1} \cap F_{c_2}$ was investigated, where $F_{c_i} = \{[n^{c_i}] : n \in \mathbb{N}\}$ and $1 < c_1 < c_2 < 2$. With the abbreviation $\gamma_i = \frac{1}{c_i}$ $(i=1,2)$ we got the following result.

Theorem 1. $\gamma_1 + \gamma_2 > \frac{5}{3} \Rightarrow \sum_{\substack{n \le x \\ n \in F_{c_1} \cap F_{c_2}}} 1 \sim \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1} x^{\gamma_1 + \gamma_2 - 1}$

Theorem 2. $\gamma_1 + \gamma_2 > \frac{43}{22} \Rightarrow \sum_{\substack{p \le x \\ p \in F_{c_1} \cap F_{c_2}}} 1 \sim \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1} \frac{x^{\gamma_1 + \gamma_2 - 1}}{\log x}$

The main tool is a non-trivial estimate for the exponential sum $S = \sum_{a < n \le b} e(\xi n^{\gamma_1} + \eta n^{\gamma_2})$, where $\xi \cdot \eta \neq 0$ and $3 \le a < b \le (1 + \frac{1}{\log a}) a$.

Dieter Leitmann (Clausthal-Zellerfeld)

Lattice points in many-dimensional ellipsoids

The proof of the following theorem was given:

Theorem. For $\rho > 0$, $\rho > \frac{p}{2} - \frac{1}{2}$

$$\int_0^x |P_\rho(y)|^2 dy = c x^{\frac{p}{2} + \rho + \frac{1}{2}} + O(x^{\frac{p}{2} + \rho}) + O_\varepsilon(x^{p+\varepsilon}),$$

where as usually

$$P(y) = \sum_{Q(m_j+b_j)\le y} 1 - \frac{\pi^{r/2} y^{r/2}}{\sqrt{D}\,\Gamma(\frac{r}{2}+1)} \qquad (D = \det Q)$$

is the lattice remainder term, Q positive definite quadratic form with integral coefficients, $b_1, b_2, \ldots, b_r$ real numbers,

$$P_\rho(y) = \frac{1}{\Gamma(\rho)} \int_0^y P(t)(y-t)^{\rho-1}\,dt \quad , \rho > 0,$$

and γ denotes some constant, for which the following estimate holds

$$\sum_{m\le x} |b_m|^2 = O(x^\gamma),$$

where

$$b_m = \sum_{\tilde{Q}(m_j)=c_1 m} e^{2\pi i \sum_{j=1}^{r} b_j m_j}.$$

(See also Czechoslovak Math. J. 23 (1973), 467-482 or Colloquia Math. Soc. J. Bolyai 13 (1974), 233-243.)

B. NOVÁK (Praha)

Difference sets and positive trigonometrical polynomials

This is a midway report on some investigations that started in the previous Oberwolfach conference, when M. Mendès France asked whether deleting a single number a from a van der Corput set H it still remains one. H is a v.d.C set if ~~for~~ every sequence u_n of reals for which $u_{n+k}-u_n$ is uniformly distributed for all $k\in H$ is itself u.d. modulo 1. The answer is positive, in fact if $H = H_1 \cup H_2$ and H is v.d.C., then so is at least one of H_1 and H_2. This is closely connected with the characterization of vdC sets via positive trig. polynomials and measures. Call H correlative if

$$\limsup x^{-1}\sum_{n\le x} |y_n|^2 \le 1 \quad \text{and} \quad x^{-1}\sum_{n\le x} y_{n+h}\bar{y}_n \to 0 \quad (h\in H)$$

implies $\sum_{n\le x} y_n = o(x)$ $(y_n\in\mathbb{C})$. We have v.d.C $\Leftrightarrow$ correlative

$\Leftrightarrow$ for every $\varepsilon > 0$ there is a trig. pol.

$$f(t) = \varepsilon + \sum_{\lambda \in H} a_\lambda \cos \lambda t \geq 0 \quad , \; f(0) = 1$$

with exponents exclusively from H $\Leftrightarrow$ $\Lambda(\{0\}) = 0$ for all measures Λ on $[-\pi, \pi)$ such that

$$\int \cos \lambda t \, d\Lambda = 0 \qquad (\lambda \in H).$$ (Kamae, Mendès France and I.)

It is also shown by the work of Kamae and Mendès France that every correlative set intersects $A - A$ whenever $d(A) > 0$ – this property will be called intersectivity. Problem: intersective $\Leftrightarrow$? correlative.

For $\varepsilon > 0$ and $\underline{u} = (u_1, \ldots u_k), u_j \in \mathbb{R}$, put

$$S(\underline{u}, \varepsilon) = \{n : n \in \mathbb{Z}, \| n u_j \| < \varepsilon, j = 1, \ldots k\}.$$

Since $S(\underline{u}, \varepsilon) \supset S(\underline{u}, \varepsilon/2) - S(\underline{u}, \varepsilon/2)$, an intersective sets intersect every $S(\underline{u}, \varepsilon)$. We call the property of intersecting every $S(\underline{u}, \varepsilon)$ the approximativity of H. Main problem: intersective $\Leftrightarrow$ approximative? Equivalently: $A - A \supset$ some $S(\underline{u}, \varepsilon)$ whenever $d(A) > 0$?. This has been asked by P. Flor. The sets $S(\underline{u}, \varepsilon)$ form the basis of neighbourhoods of 0 in the so called Bohr topology. Bogolyubov's theorem from 1939 asserts that $A + A - A - A$, i.e. the second diff. set, contains an $S(\underline{u}, \varepsilon)$. I can reduce 4 to 3, by showing that $A + A - A - a \supset S(\underline{u}, \varepsilon)$ for suitable $a \in A$ ($A + A - A$ need not).

Imre Ruzsa,
Budapest

On a sieve identity method

There are two recently discovered ways to investigate certain sums over primes. The first is due to R. C. Vaughan and the second to R. Heath-Brown. The lecture gives a simplified version of the latter and showes an application in which it is more efficient then the former. The main result is

THEOREM: Let $\frac{1}{3} \leq \theta < 1$ be a fixed number. If $N^{-1/4+25\varepsilon} \leq \Delta \leq 1$, then

$$(*) \quad \#\{N \leq p < 2N, \{p^\theta\} < \Delta\} = \Delta \cdot \#\{N \leq p < 2N\} + O(N^{\frac{1+\theta}{2}+20\varepsilon} + \Delta N^{1-\varepsilon}),$$

and if $N^{-3/10} \leq \Delta \leq 1$ then

$$\#\{N \leq p < 2N, \{p^\theta\} < \Delta\} \gg \Delta \cdot \#\{N \leq p < 2N\} + O(N^{\frac{1+\theta}{2}+20\varepsilon} + \Delta N^{1-\varepsilon}).$$

The implied constants depend at most on ε and Δ.

COROLLARY: If $\frac{2}{5} \leq \theta < 1$ then there exists infinitely many primes p with

$$\{p^\theta\} < p^{-\frac{1-\theta}{2}+\varepsilon}.$$

This was known earlier only as a consequence of Riemann Hypothesis.
The statement of the Theorem remains true if we confine ourselves to primes in a given arithmetic progression $p \equiv a(q)$, $(a,q)=1$. It has a quite interesting feature. When q is about $N^{\varepsilon/10}$

no correct asymptotic formula is known for the right hand side of (*). Even no good lower bound on the expected order of magnitude is known. It depends sharply on the existence of Siegel zeros. But the Theorem shows that the irregularities of $\#\{N \leq p < 2N,\ p \equiv a\,(q),\ \{p^{\Theta}\} < \Delta\}$ are equal to the irregularities of $\#\{N \leq p < 2N,\ p \equiv a\,(q)\}$. Probably there are no Siegel zeros and both of these amounts are well distributed.

Antal Balog (Budapest)

PROBLEM:

Let $A \subset \mathbb{N}$ be a sequence with positive density. We say A has p-property for a prime p if

i) for almost all $a \in A$ $p \cdot a \in A$ is also true

ii) for almost all $a \in A$, $p|a$ $\frac{a}{p} \in A$ is also true.

Examples: $A = \{$even numbers$\}$, A has p-property for all prime p except 2. For $0 < \alpha < 1$ $A_\alpha = \{n \in \mathbb{N}$, the greatest prime factor of $n > n^\alpha\}$, A has ~~prime~~ p-property for all primes p.

It is easy to see that if A has 2-property, then $d(A) > \frac{1}{3}$ implies the existence of infinitely many pairs $a \in A$, $a+1 \in A$. Similarly if A has 2,3-property, then the same conclusion follows from $d(A) > \frac{3}{10}$.

Prove or disprove: If A has p-property for all primes p, then $d(A) > 0$ implies the existence of infinitely many pairs $a \in A$, $a+1 \in A$.

Antal Balog (Budapest)

PROBLEM:

Let be $\zeta(s)$ the Riemann zeta function. Define the discrete mean values

$$A_\sigma = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} |\zeta(\sigma+in)|^2$$

$$B_\sigma(Q) = \lim_{N\to\infty} \frac{1}{N} \sum_{1}^{N} |\zeta(\sigma+iQ(n))|^2, \quad Q \in \mathbb{Z}[x],$$

$$C_\sigma = \lim_{N\to\infty} \frac{1}{N} \sum_{1}^{N} |\zeta(\sigma+ip_n)|^2, \quad p_n \text{ the sequence of primes.}$$

One can prove, that for $\sigma > 1$

$$\sum_{1}^{\infty} \frac{1}{n^{2\sigma}} = A_\sigma = B_\sigma(Q) = C_\sigma \qquad \text{for all } Q \in \mathbb{Z}[x], \deg Q > 0.$$

In the case $\frac{1}{2} < \sigma < 1$ one can prove also that

$$\sum_{1}^{\infty} \frac{1}{n^{2\sigma}} = A_\sigma = B_\sigma(Q) \quad \text{for all } Q \in \mathbb{Z}[x], \deg Q = 1.$$

Is it true, that for say $Q_2(n) = n^2$

$B_\sigma^{(Q_2)}, C_\sigma$ exists also for $\frac{1}{2} < \sigma \le 1$,

and if so, is it true that again

$$\sum_{1}^{\infty} \frac{1}{n^{2\sigma}} = A_\sigma = B_\sigma(Q_2) = C_\sigma \qquad \text{for } \frac{1}{2} < \sigma \le 1\,?$$

Axel Reich (Göttingen)

Dyson's lemma in diophantine approximation.

In 1947 F. Dyson improved the Thue-Siegel theorem by showing that if α is an algebraic number of degree $r \geqslant 2$ and if $\varepsilon > 0$ then $|\alpha - \frac{p}{q}| > q^{-\sqrt{2r}-\varepsilon}$ for all $q \geqslant q_0(\alpha, \varepsilon)$ (Acta Math. 79, 225-240). The constant $q_0(\alpha, \varepsilon)$ in the above result is not effectively computable. This was an intermediate step between the Liouville and the Roth exponents. The main tool in Dyson's proof is a technical lemma, where he uses the theory of generalized Wronskians to relate the vanishing of the derivatives of a polynomial in two variables at certain points with the degrees of the polynomial itself. Unlike Siegel's and Roth's approaches (where one needs rational approximations $\frac{p}{q}$ to α with q large compared with the height of α), Dyson's method is purely algebraic in the sense that it is free from any consideration of heights. As Bombieri has recently shown, Dyson's lemma can be used to obtain some <u>effective</u> results of Thue's type. Using the theory of singularities for algebraic curves I give a new approach (and an improvement) to Dyson's lemma.

Carlo Viola (Pisa)

The greatest prime factor of a polynomial

Let for $f \in \mathbb{Z}[x]$ $P(f(x))$ denote the greatest prime factor of the number $f(x)$. The survey has been given of research concerning the three following problems.

(i) How large can $P(f(x))$ be for infinitely many integers x?

(ii) How large is $P(f(x))$ for all $x > x_f$?

(iii) How small can $P(f(x))$ be for infinitely many integers x?

(i) The results of Tchebicheff (1895), Nagell (1921-22), Ricci (1934), Erdös (1951), Hooley (1967), ~~Iwa~~ Deshouillers and Iwaniec (1981) have been presented and a proof has been outlined that

$$\limsup \frac{P(f(x))}{x \log x} > 0$$

if f has an irrational zero.

(ii) The results of Störmer (1897), Polya (1918), Siegel (1921), Mahler (1935), Kotov and Sprindžuk (1973), Shorey and Tijdeman (1976) have been presented and a proof has been outlined that

$$P(f(x)) \gg \log\log x$$

if f has at least two distinct zeros.

(iii) The results of the speaker (1967) have been presented and a proof outlined that

$$P(x^2+1) < x^{\frac{c}{\log\log\log x}}$$

for infinitely many x.

Problem. Is $P((2x+1)^2+8) < \sqrt{x}$ for infinitely many x?

Andrzej Schinzel (Warszawa)

Introducing the real numbers via formal continued fractions

Write every $\alpha \in \mathbb{Q}$ as a finite continued fraction $\alpha = \langle a_0, a_1, \dots, a_n \rangle := a_0 + \cfrac{1}{a_1 + \ddots + \cfrac{1}{a_n}}$ where $a_0 \in \mathbb{Z}$, $0 \le n \in \mathbb{Z}$, and where $a_j \in \mathbb{N}$ $(0 < j \le n)$, $a_n > 1$ in case $n > 0$. For the symbol $\omega \notin \mathbb{Q}$ we assume $x < \omega$ $(x \in \mathbb{Q})$. For convenience write $\langle a_0, \dots, a_n \rangle = \langle a_0, \dots, a_n, \omega, \omega, \dots \rangle$. We extend $\mathbb{Q}$ to K by adjoining as new elements all infinite sequences $\alpha = \langle a_0, a_1, \dots \rangle$ ("formal continued fractions") with $a_0 \in \mathbb{Z}$, $a_j \in \mathbb{N}$ $(j > 0)$. Extending the order of $\mathbb{Q}$ we define the order in K by $\langle a_0, \dots, a_{k-1}, a_k, a_{k+1}, \dots \rangle < \langle a_0, \dots, a_{k-1}, b_k, b_{k+1}, \dots \rangle$ in case $a_k < b_k$ and $2 \mid k$ or in case $a_k > b_k$ and $2 \nmid k$. For every $M \subset K$, $M \neq \emptyset$ the supremum can be constructed. For $\alpha \in K$, $j \in \mathbb{Z}$, $j \geq 0$ let $\alpha^{(j)} := \langle a_0, a_1, \dots, a_j \rangle$. For $\alpha \in \mathbb{Q}$, $\beta \in \mathbb{Q}$ we have $\alpha + \beta = \sup(\alpha^{(2j)} + \beta^{(2j)} : j \geq 0)$. This we use as Definition of $\alpha + \beta$ for $\alpha \in K$, $\beta \in K$, $\alpha \notin \mathbb{Q} \vee \beta \notin \mathbb{Q}$. We prove that K is an ordered group. Similarly we handle multiplication, and K is a complete ordered field containing $\mathbb{Q}$. Now we may write $\mathbb{R}$ instead of K. We have also

$$\lim_{n \to \infty} a_0 + \cfrac{1}{a_1 + \ddots + \cfrac{1}{a_n}} = \langle a_0, a_1, \dots \rangle .$$

(This appears in DEDEKIND - Gedenkband, Braunschweigische Wiss. Ges., 1981).

G. J. Rieger, Hannover

On the explicit formula of Riemann-von Mangoldt

The following error term estimations for the classical formula of Riemann-von Mangoldt are given.

I. Let $4 \le T \le \frac{x}{2}$. There exists a $\tau \in [\frac{T}{2}, T]$ s.th.

$$\psi_0(x) = \frac{1}{2}\Big(\sum_{n \le x^-} \Lambda(n) + \sum_{n \le x^+} \Lambda(n)\Big)$$

$$= x - \sum_{\rho = \beta + i\gamma,\ |\gamma| \le \tau} \frac{x^\rho}{\rho} + O\Big(\frac{x}{T} \frac{\ln x}{\ln \frac{x}{T}}\Big).$$

II. ~~Under the~~ Let $\ln x < T \le \frac{x}{\ln x}$, $1 \le V \le x$. There exists a $\tau \in [\frac{T}{2}, T]$ s.th.

$$\frac{1}{x} \int_x^{2x} \Big| \psi\big(y(1+\tfrac{1}{V})\big) - \psi(y) - \Big(\frac{y}{V} - \sum_{\rho, |\gamma| \le \tau} \frac{y^\rho}{\rho}\Big(\big(1+\tfrac{1}{V}\big)^\rho - 1\Big)\Big) \Big| \, dy$$

$$\ll \frac{x}{T}.$$

A further improvement, which would have consequences in several theorems in prime number theory, seems not to be possible.

D. Wolke, Freiburg

P. Süsz and B. Volkmann (speaker)

On numbers containing each block infinitely often — A problem of Mahler.

A number $\alpha \in [0,1]$ is called N-CEBIO if, for given $N \in \mathbb{N}$ and $g \geqq 2$, its g-adic expansion $\alpha = {}_g.\, a_1 a_2 \ldots$ contains each block of length N infinitely often.

Mahler proved in 1973 that, in this terminology, there exists a constant $C_1 = C_1(N,g)$ such that for any irrational α there is a multiple $X\alpha$ with $X \in \{1, 2, \ldots, C_1\}$ for which $\{X\alpha\}$ is N-CEBIO. The bound which Mahler's approach yields turns out to be

$$C_1(N,g) = g^{2g^N + 2N - 1}.$$

The authors have sharpened this bound to

$$C_2(N,g) = 12\, g^{g^N + N}.$$

The proof uses a combination of methods from the theories of digit expansions (notably properties of the set $\Lambda(\alpha)$ of all limit points of the sequence $\alpha, \{g\alpha\}, \{g^2\alpha\}, \ldots$) and of continued fractions, showing that, for given irrational α, there exists a point $t \in \Lambda(\alpha)$ for which a suitable convergent $\frac{A_\nu(t)}{B_\nu(t)}$ of its continued fraction has the property that any interval of length $\frac{1}{C_2}$ will contain a point $\{Xt\}$, $1 \leq X \leq C_2$. The result can then be derived by applying this proposition to the interval I_K corresponding to a g-adic block K (of length $g^N + N - 1$) which involves each block of length N exactly once.

A counter-example shows that the bound C_2 is "close" to being best possible.

8.10.82 B. Volkmann

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Remarks on Ramanujan's function τ

Ramanujan's function τ is defined as follows:

$$\sum_{n=1}^{\infty} \tau(n)\, x^n = x \prod_{k=1}^{\infty} (1-x^k)^{24}, \quad x = e^{2\pi i z}, \quad \operatorname{Im} z > 0.$$

The following results have been given:

1) $$\sum_{\substack{p \le x \\ p \equiv \ell(q)}} \frac{\tau^2(p)}{p^{11}} \log p \sim \frac{x}{\varphi(q)}, \quad (\ell, q) = 1.$$

2) $B > 0$, $(\ell, q) = 1$, $x \geq 2$. Then there is a constant $c > 0$:

is $$\sum_{\substack{p \le x \\ p \equiv \ell(q)}} \log p - \frac{x}{\varphi(q)} = O\left(x \exp\{-c\sqrt{\log x}\}\right)$$

holds uniformly with respect to $1 \le q \le \exp\{B\sqrt{\log x}\}$,

or

is $$\sum_{\substack{p \le x \\ p \equiv \ell(q)}} \frac{\tau(p)}{p^{11/2}} \log p = O\left(x \exp\{-c\sqrt{\log x}\}\right)$$

holds uniformly with respect to $1 \le q \le \exp\{B\sqrt{\log x}\}$

F. Grupp (Ulm)

On the pointwise convergence of Ramanujan expansions of arithmetical functions

The Ramanujan sums

$$c_q(n) := \sum_{\substack{1 \le a \le q \\ (a,q)=1}} e^{2\pi i \frac{a}{q} n}$$

satisfy the orthogonality relation

$$M(c_q c_{q'}) = \begin{cases} 0 & \text{if } q \neq q', \\ \varphi(q) & \text{if } q = q', \end{cases}$$

where $M(f) := \lim_{x \to \infty} \frac{1}{x} \sum_{n \le x} f(n)$ denotes the mean value of an arithmetical function $f: \mathbb{N} \to \mathbb{C}$ (if it exists).
Because of this property one might try to expand an arithmetical function f into a Ramanujan series $f \sim \sum_{q \ge 1} a_q(f) c_q$

with the coefficients $a_q(f) = \frac{1}{\varphi(q)} M(f c_q)$ (provided these mean values exist).
The problem of the pointwise convergence of such a Ramanujan expansion has been studied in the case of multiplicative functions by Schwarz, Delange, Tuttas and Warlimont and in the case of additive functions by Spilker and the speaker.
Without the assumption of additivity or multiplicativity we have the following results:
Theorem 1: For every ~~natural~~ natural number k there exists a constant $c(k)$ such that for every function $f: \mathbb{N} \to \mathbb{C}$ with existing Ramanujan coefficients $a_q(f)$ $(q \in \mathbb{N})$ we have

$$\sup_{Q \ge 1} \left| \sum_{q \le Q} a_q(f) c_q(k) \right| \le c(k) \, \|f\|_\infty ,$$

where $\|f\|_\infty := \sup_{n \in \mathbb{N}} |f(n)|$.

Theorem 2: Every uniformly almost even function f is represented pointwise by its Ramanujan expansion $\sum_{q\geq 1} a_q(f)c_q$.

(A function $f: \mathbb{N} \to \mathbb{C}$ is called uniformly almost even if it can be approximated in the norm $\|\cdot\|_\infty$ by even functions, i.e. by finite linear combinations of the functions c_q, $q \in \mathbb{N}$)

Theorem 1 can be formulated in terms of a "finite" inequality which seems to be of independent interest:

Theorem 3: For every finite sequence $(a_q)_{q\leq Q_0}$ of complex numbers and every natural number k the inequality

$$\max_{Q\leq Q_0} \Big| \sum_{q\leq Q} a_q c_q(k) \Big| \leq C(k) \max_{n\in\mathbb{N}} \Big| \sum_{q\leq Q_0} a_q c_q(n) \Big|$$

holds.

All three theorems can be shown to be equivalent. Their proof is based on estimates for the functions $M(n,z) := \sum_{\substack{d|n \\ d\leq z}} \mu(d)$ and $M_1(n,z) := \int_1^z \frac{M(n,u)}{u}\,du$ $(z \geq 1)$.

Adolf Hildebrand

Small zeros of quadratic form

Let $f(x_1,\dots,x_n)$ be a quadratic form with integral coefficients. Let F be the maximum of the moduli of the coefficients of f. Cassels (1956) proved that if f represents zero nontrivially at integral points, then it has an integral zero $\underline{x}$ satisfying $0 < |\underline{x}| \ll F^{\frac{n-1}{2}}$, where $|\underline{x}| = \max_i |x_i|$ $(\underline{x} = (x_1,\dots,x_n))$. For forms of signature $(1, n-1)$ this was shown by Kneser to be best possible.

As for indefinite forms of signature (r,s) Watson (1958) showed, that the smallest nontrivial integral zero $\underline{\underline{x_0}}$ satisfies $|\underline{\underline{x_0}}| \gg F^{\frac{1}{2}[\frac{r}{s}]}$ if $r \geq s$.

I conjecture that any nondegenerate indefinite form f in at least n_0 variables, of signature (r,s), $r \geq s$, has a nontrivial integral zero $\underline{\underline{x}}$ satisfying $|\underline{\underline{x}}| \ll F^{\frac{1}{2}\frac{r}{s}}$.

We are not able to prove this conjecture, however we obtain the

<u>Theorem</u> Let $f(x_1, \dots, x_n)$ be an indefinite, nondegenerate quadratic form with integral coefficients and suppose $n \geq 5$. Let (r,s) with $r \geq s$ be the signature of f. Then f has a nontrivial integral zero $\underline{\underline{x}}$ which satisfies $|\underline{\underline{x}}| \ll F^{\frac{5}{2} + \frac{1}{2}\frac{r}{s}}$, where the constant implied by $\ll$ depends only upon n.

A nondiscrete version of the Theorem is given; moreover, we have a result of the existence of n linearly independent zeros of about the size as in the Theorem.

Hans Peter Schlickewei (Ulm)

<u>On sieves of Greaves' type</u> (joint work with H. Halberstam)

Report on more general sieves of the type investigated first by G. Greaves (Acta Arith. 40 (1982)) for dimensions $\frac{1}{2} \leq \kappa \leq 1$. In case of the linear sieve ($\kappa = 1$) the remainder term can be transformed into a bilinear form of Iwaniec' type. As to applications we mention the interval problem: For $r \in \mathbb{N}$ let $\theta_r = \inf \theta$ such that $[x - x^{\theta}, x)$ contains for $x \geq x_0$ a P_r, i.e. a number consisting of at most r prime factors. Then

$$\theta_2 \leq 0.4454\ldots, \quad \theta_3 \leq 0.3257\ldots, \quad \theta_4 \leq 0.2496\ldots, \quad \theta_5 \leq 0.202\ldots.$$

H.-E. Richert, Ulm

Exponential sums in several variables and L-functions of Artin

Let p be a prime number, $q = p^{\alpha}$ and

$$K_p \subseteq K_q \subseteq K_{q^s}$$

be finite fields consisting from p, q and q^s elements respectively.

Denote by $tr(x)$ the trace of element $x \in K_q$, by $tr_s(z)$ the relative trace of element $z \in K_{q^s}$ and by $tr(tr_s z)$ the absolute trace of element $z \in K_{q^s}$.

Let

$$f(x_1, \dots, x_n) = \sum_{0 \le i_1 + \dots + i_n \le m} a_{i_1 \dots i_n} x_1^{i_1} \dots x_n^{i_n}$$

be a polynomial of degree m and with coefficients $a_{i_1 \dots i_n}$ from the field K_q.

Put

$$T_{s,n} = \sum_{x_1, \dots, x_n \in K_{q^s}} e^{2\pi i \frac{tr(tr_s f(x_1, \dots, x_n))}{p}}$$

and

$$L_n(t, f) = \exp\left(\sum_{s=1}^{\infty} \frac{T_{s,n}}{s} t^s \right)$$

Call the polynomial $f(x_1, \dots, x_n)$ „general" one, if it's leading form is nonsingular in $n-1$-dimensional projective space $\mathbb{P}_{n-1}$.

over algebraic closure $\bar{k}_p$ of the field k_p.

I give arithmetic proof of the following theorem:

Theorem. Let $f(x_1, \dots, x_n)$ be „general" polynomial of degree $m \geq 1$ and let $p > m$. Let N be smallest integer for which is valid the inequality

$$nN \geq \binom{m+n}{n} - 1.$$

Then L-function of Artin $L_n(t, f)$ have the form

$$L_n(t, f) = P(t)^{(-1)^{n-1}},$$

where $P(t) = 1 + \alpha_1 t + \dots + \alpha_d t^d$ is polynomial of degree

$$d \leq (N-1)^n$$

with coefficients α_i from the field $Q(e^{2\pi i/p})$. Further, if

$$P(t) = \prod_{i=1}^{d} (1 - \omega_i t)$$

then

$$T_{s,n} = (-1)^n \sum_{i=1}^{d} \omega_i^s$$

for all $s = 1, 2, \dots$.

If now we use Bombieri's result [Amer. J. of Math., 88 (1966), 71-105], then for degree of $P(t)$ we obtain more precise estimate

<u>Corollary</u>

$$d = \deg P \le \begin{cases} m-1, & \text{if } n=1 \\ m^n - n(m-1)-1, & \text{if } n \ge 2. \end{cases}$$

Analogous results are valid for the sum

$$T^*_{s,n} = \sum_{x_1,\dots,x_n \in K_{q^s}} e^{2\pi i \frac{tr(tr_s f(x_1,\dots,x_n))}{p}}$$

and for L-function

$$L^*_n(t,f) = \exp\left(\sum_{s=1}^{\infty} \frac{T^*_{s,n}}{s} t^s\right).$$

(S. Stepanov)

ASPECT GÉOMÉTRIQUE DES SOMMES DE WEYL.

J.-M. DESHOUILLERS

fig. 1

Les figures ont été tracées avec l'aide de Michel Pallard, informaticien bordelais.

L'idée de représenter graphiquement les sommes de Weyl est dûe à Dekking et Mendès-France (cf. un récent numéro du Journal de la reine). Pour une fonction arithmétique réelle f (c'est-à-dire une suite de réels $(f(n))_{n\in\mathbb{N}}$), ils considèrent la suite de points du plan complexe $S_n = \sum_{m=0}^{n} e^{2\pi i f(n)}$ en joignant par un segment de droite les points S_{n+1} et S_n.

La première figure correspond à la fonction $f(n) = \frac{100}{1001} n^2$, la seconde à $f(n) = \frac{4}{3} n^{\frac{3}{2}}$ et la troisième à $f(n) = \frac{1}{3} n^{\frac{3}{2}}$.

Le but de l'exposé était de mettre en évidence les points suivants :

① Une application grossière de la formule sommatoire de Poisson permet d'expliquer le comportement global de sommes de Weyl : expansion dans la direction de la première bissectrice pour la fig. 2, promenade autour de l'origine pour la fig. 3

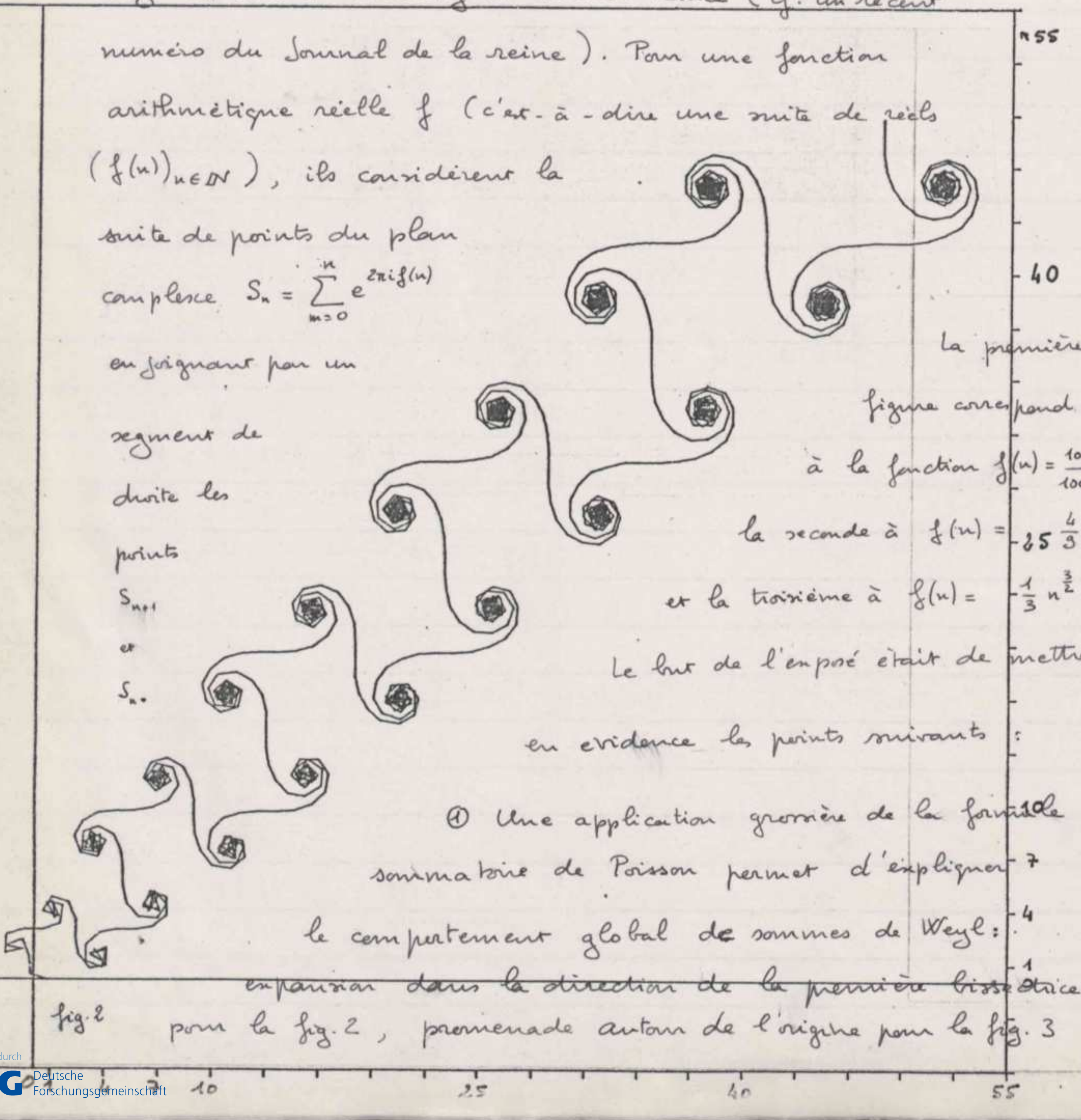
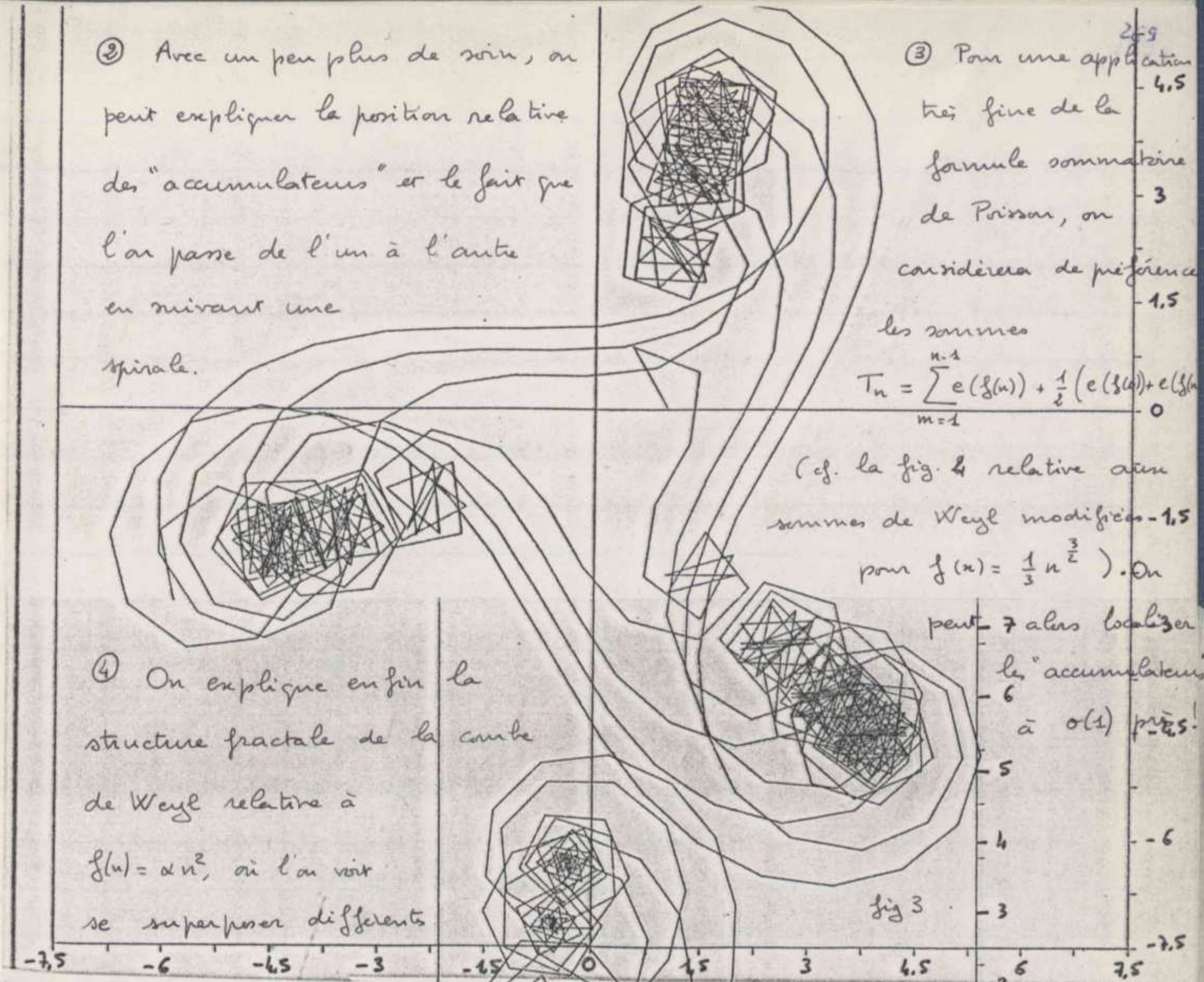


fig. 2



② Avec un peu plus de soin, on peut expliquer la position relative des "accumulateurs" et le fait que l'on passe de l'un à l'autre en suivant une spirale.

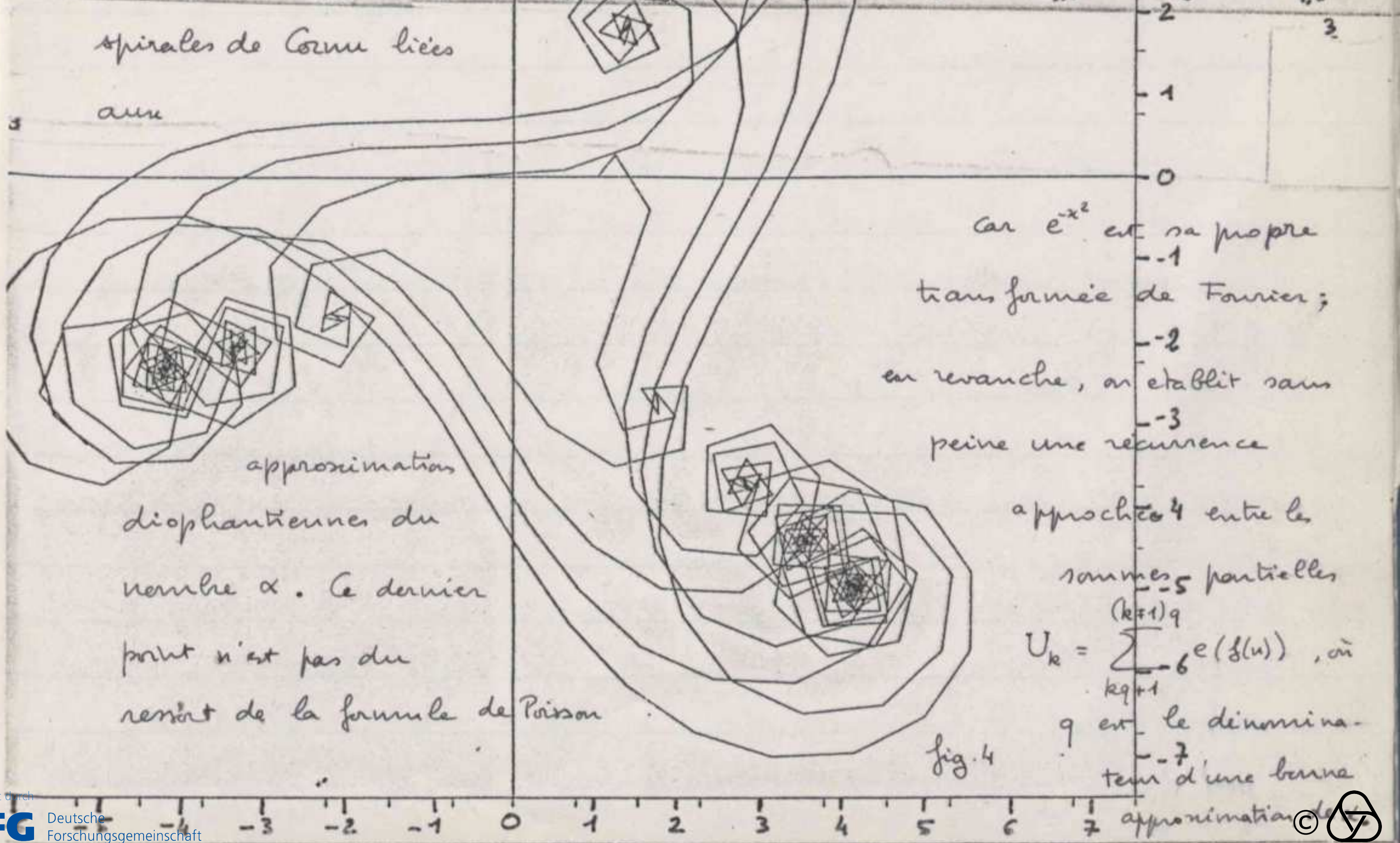
③ Pour une application très fine de la formule sommatoire de Poisson, on considèrera de préférence les sommes

$$T_n = \sum_{m=1}^{n-1} e(f(m)) + \frac{1}{2}\left(e(f(0)) + e(f(n))\right)$$

(cf. la fig. 4 relative aux sommes de Weyl modifiées pour $f(x) = \frac{1}{3} x^{\frac{3}{2}}$). On peut alors localiser les "accumulateurs" à $o(1)$ près!

fig 3

④ On explique enfin la structure fractale de la courbe de Weyl relative à $f(n) = \alpha n^2$, où l'on voit se superposer différents spirales de Cornu liées aux approximations diophantiennes du nombre α. Ce dernier point n'est pas du ressort de la formule de Poisson



car e^{-x^2} est sa propre transformée de Fourier ; en revanche, on établit sans peine une récurrence approchée entre les sommes partielles

$$U_k = \sum_{kq+1}^{(k+1)q} e(f(n)),$$

où q est le dénominateur d'une bonne approximation

fig. 4

Discrepancy theory : number theory or combinatorics

An analogy is drawn between three groups of theorems :

A) Van der Waerden's, Roth's, Szemerédi's etc. theorems for arithmetic progressions

B) Roth's, Schmidt's etc. classical discrepancy theorems for sequences

C) Ramsey, Turán, Erdős - Spencer, Erdős - Stone etc theorems for graphs.

In this setting e.g. Schmidt's theorem for discrepancy of sequence in $[0,1]$ can be formulated as a discrepancy theorem for bipartite graphs.

The general character of these results is the following :

Let X be a finite (or infinite) set, $\mathcal{A} \subset 2^X$. We consider

(1) arbitrary finite partitions ~~of~~ (colorings) of X,

or (2) partitions of given ratio, but otherwise arbitrary

or (3) X' subsets of X of given size, " " ".

Under certain (quantitative or structural) restrictions on $\mathcal{A}$ we can find (a) a monochromatic $A \in \mathcal{A}$

(b) an $A \in \mathcal{A}$ in which one colorclass has a certain preponderance

c) an $A \subset X'$, $A \in \mathcal{A}$.

E.g van der Waerden theorem for AP is of type (1) -(a), Szemerédi's theorem (3) - (c) and Roth's theorem (2) -(b).

No general structural theorems of Erdős - Stone ⊛ type (which (3)-(c)) are known for $X = [1, N]$ or for $X = [0,1]^k$.

E.g a special problem is the following : Let $r_4(a,b,c;n)$ be the maximal number of integers in $[1,n]$ without containing four numbers of form $(x, x+ay, x+by, x+cy)$. ($a=1, b=2, c=3$ is the arithm. progr. case). For which values of a, b, c is $r_4(a,b,c;n) \gtreqless r_4(1,2,3;n)$?

⊛ Let $f(n;H)$ be the least integer with the property that if a graph on n vertices has more than $f(n;H)$ edges then it must contain a subgraph isomorphic to H. Erdős - Stone th. says, that $f(n;H)$ assimptotically depends only on the chromatic number of H : $f(n;H) \sim \frac{1}{2} \frac{\chi(H)-2}{\chi(H)-1} n^2$.

Vera T. Sós

(Budapest

Hilbert's inequality

Schur established the sharp form of Hilbert's inequality, namely that

$$\Bigg|\sum_{\substack{m,n\\m\neq n}} \frac{a_m\overline{a_n}}{m-n}\Bigg| < \pi \sum |a_n|^2 .$$

The constant, though not attained, is best possible. In recent years, a generalized form of this inequality (established by myself and Vaughan, with help from Selberg) has played a useful role in treating Dirichlet series and the large sieve. For example, it has been shown that

$$\Bigg|\sum_{m\neq n} \frac{a_m\overline{a_n}}{\lambda_m-\lambda_n}\Bigg| < \tfrac{3}{2}\pi \sum_n |a_n|^2\delta_n^{-1},$$

where $\delta_n = \min\limits_{\substack{m\\m\neq n}} |\lambda_m-\lambda_n|$. Selberg (unpublished) has reduced the constant here from $\frac{3}{2}\pi$ to 3.2, although the best constant is presumably π. In order to obtain this best possible constant it would suffice to show that

$$\sum_{m\neq n} \frac{a_m\overline{a_n}}{(\lambda_m-\lambda_n)^2}(\delta_m+\delta_n) \leq \frac{2\pi^2}{3}\sum_n |a_n|^2\delta_n^{-1}.$$

With further possible applications in mind, we generalize the inequality further by showing that if $s_m = \sigma_m + i t_m$ with $\sigma_m \geq 0$ and $\delta_m = \min\limits_{n\neq m} |t_m - t_n|$, then

$$\Bigg|\sum_{\substack{m,n\\m\neq n}} \frac{a_m\overline{a_n}}{s_m+\overline{s_n}}\Bigg| < C \sum_n |a_n|^2\delta_n^{-1}.$$

(The case when $\sigma_m = 0$ is the former inequality.) The approach is now different, and employs the Hardy–Littlewood maximal inequality and the theory of H^2 functions in a half-plane. Hugh L. Montgomery (Ann Arbor)

Arbeitstagung Geyer-Harder
über das Eisensteinideal

1. Einführung in und Übersicht über die Theorie der quasi-endlichen flachen Gruppenschemata. (nach Mazur, "Modular curves and the Eisenstein ideal", Publ. Math. IHES 47 (1978), Kap. I, §1.)

Es wurden die quasiendlichen flachen kommutativen Gruppenschemata G über $S = \operatorname{Spec} \mathbb{Z}$ diskutiert, die p-Potenz-Ordnung haben und für eine Primzahl $N \neq p$ auf $S' := S \setminus \{N\}$ endlich sind. $G|S'$ ist durch den $\mathrm{Gal}(\overline{\mathbb{Q}}|\mathbb{Q})$-Modul $G(\overline{\mathbb{Q}})$ festgelegt und $G|S\setminus\{p\}$ ist durch die Injektion

$$i_G : G(\overline{\mathbb{F}}_N) \longrightarrow G(\overline{\mathbb{Q}})^{I_N}$$

von $\mathrm{Gal}(\overline{\mathbb{F}}_N|\mathbb{F}_N)$-Moduln ($I_N$ = Trägheitsgruppe zu N) und den $\mathrm{Gal}(\overline{\mathbb{Q}}|\mathbb{Q})$-Modul $G(\overline{\mathbb{Q}})$ eindeutig bestimmt. Zu jedem $\mathrm{Gal}(\overline{\mathbb{Q}}|\mathbb{Q})$-Untermodul M von $G(\overline{\mathbb{Q}})$ ist der schematheoretische Abschluß H_M von M in G wieder ein Gruppenschema der hier betrachteten Art. ~~Speziell~~ Ist G endlich über S und $(G:1) = p$, so ist $G \cong (\mathbb{Z}/p)_S$ (konstante Gruppe) oder $G \cong (\mu_p)_S$ (Oort-Tate). Abschließend wurden die sogenannten zulässigen Gruppen ($G|S'$ hat Filtrierung mit Faktoren $(\mathbb{Z}/p)_{S'}$, $(\mu_p)_{S'}$) betrachtet. Diese haben über S eine Filtrierung, in der als Faktoren nur 4 Bausteine $\mathbb{Z}/p$, $(\mathbb{Z}/p)^b$, μ_p, $(\mu_p)^b$ vorkommen.

Manfred Knebusch
(Regensburg)

2. Stufe-N-Struktur auf elliptischen Kurven, Modulkurven der Stufe N über $\mathbb{Z}[\frac{1}{N}]$, Tate-Kurve

Zunächst wurde in die klassische komplexe Theorie der Modulformen auf der oberen Halbebene $\mathfrak{H}$, der klassischen Modulkurven $\Gamma \backslash \mathfrak{H}$ und über Komplettierungen durch Adjunktion von Spitzen eingeführt. Über Stufe-N-Strukturen elliptischer Kurven wurde eine algebraische Konstruktion dieser Modulkurven über $\mathbb{Q}$ gegeben, und auf die analoge Konstruktion von Igusa über $\mathbb{F}_p$, $p \nmid N$ hingewiesen. Die universelle ~~für~~ elliptische Kurve mit Stufe-N-Struktur über der Modulkurve $Y(N)$ bzw. $X(N)$ wurde über $\mathbb{C}$ konstruiert, die allgemeinen Resultate über das feine Modulschema über $\mathbb{Z}[\frac{1}{N}]$ nach Deligne-Rapoport zitiert. Die Umgebung der Spitzen wurde durch Betrachtung der Tate-Kurve und deren kanonischer Stufe-N-Struktur parametrisiert.

W.-D. Geyer (Erlangen)

3 Néron-Modelle von Kurven und abelschen Varietäten; ein Satz von Raynaud.

Für eine glatte geometrisch irreduzible und eigentliche Kurve C über einem diskret bewerteten Körper K (mit Bewertungsring R) wird der Begriff des Minimalmodells $\mathcal{C}$ definiert und an Beispielen für Geschlecht 1 erläutert. Für abelsche Varietäten über K (mit vollkommenem Restklassenkörper) existiert nach Néron eine glatte Fortsetzung zu einem kommutativen Gruppenschema über R ("das Néronmodell"). Für die Jacobische Varietät einer Kurve C läßt sich dies Néronmodell J mit Hilfe eines Satzes von Raynaud (Publ. Math. IHES 38) mit Hilfe des Picardfunktors des Minimalmodells $\mathcal{C}$ der Kurve C beschreiben:

$$J \cong \mathrm{Pic}^{[0]}_{\mathcal{C}/S}/E$$

Die hierfür nötigen Begriffe wurden referiert.

R. Berndt (Hamburg)

4. Die spezielle Faser von $X_0(N)$

Für eine Primzahl N wurde die Reduktion modulo N der Hecke'schen Modulkurve $X_0(N)$ untersucht. Es wurde gezeigt, daß die spezielle Faser zwei rationale Kurven als irreduzible Komponenten besitzt, die sich transversal in den supersingulären Punkten schneiden.

E. Gekeler (Bonn)

5. Die Resultate des vorangehenden Vortrags wurden ausgenutzt, um zu zeigen, daß das Neron-Modell J der Jacobischen von $X_0(N)$ multiplikative Reduktion hat. Die Gruppe

$$J/J^0 \simeq \mathbb{Z}/n\mathbb{Z}$$

wobei $n =$ Zähler $\left(\frac{N-1}{12}\right)$ und diese zyklische Gruppe wird von $(0)-(\infty)$ erzeugt.

S. Hartl

6. Die Hecke-Operatoren T_ℓ können klassisch durch Summation über Doppel-Nebenklassen definiert werden. Dann haben sie Eigenwerte mit Multiplizität 1 auf $S_2(\Gamma_0(N))$ (N prim). Außerdem können sie als Korrespondenzen erklärt werden. Dann erhält man durch Reduktion mod ℓ, daß $T_\ell = F_\ell + F_\ell^t$ (Eichler-Shimura).
Als Korollar ergibt sich

$$\mathrm{End}(J_0(N)/\mathbb{Q}) \otimes_{\mathbb{Z}} \mathbb{Q} = \mathbb{Q}[T_\ell \mid \ell \nmid N]$$

G. Faltings

7. Modulformen mod m (Mazur, Kap. II, §§4 und 5)

Satz: Sei $N \geq 5$ prim, $\sigma'(m) := \sum_{\substack{d \mid m \\ N \nmid d}} d$, und $\delta := \sum_{m \in \mathbb{N}} \sigma'(m)\, q^m$

Dann ist δ eine Spitzenform mod m zu $\Gamma_0(N)$ vom Gewicht 2 genau dann wenn $m \mid$ Zähler$\left(\frac{N-1}{12}\right)$.

Das Problem beim Beweis dieses Satzes besteht darin, daß für $m = N$ und $(m,N)=1$ unterschiedliche Interpretationen des Begriffs „Modulform mod m" verwendet werden (die von Serre / Swinnerton-Dyer bzw. die von Deligne / Rapoport / Katz). Für $m = N$ wurde eine etwas ausführlichere Darstellung des Beweises gegeben; für $(m,N)=1$ der vergebliche Versuch unternommen, den nötigen Begriffsapparat zu umreißen und die Querverbindungen beider Definitionen darzustellen.

J. Wolfart (Frankfurt a. M.)

8. Die Involution w und die Heckeoperatoren T_ℓ werden auf dem groben Modulschema $M_0(N)$ für elliptische Kurven mit Hecke-V-Struktur definiert, desgleichen die Operation von w und T_ℓ auf $H^0(M_0(N), \Omega)$.

G. Tamme (Regensburg)

9. Es werden zwei Sätze aus der Arbeit von Ribet "Endomorphisms of semi-stable abelian varieties over number fields" (Ann. of math. 101) bewiesen. Angewendet auf die Jacobische $J_0(N)$ der Kurve $X_0(N)$ erhält man damit folgendes Resultat: (vgl. Vortrag Nr. 6)

I $\quad \mathrm{End}_{\mathbb{C}}\, J_0(N) = \mathrm{End}_{\mathbb{Q}}\, J_0(N)$

II $\quad \mathrm{End}_{\mathbb{C}}\, J_0(N) \otimes \mathbb{Q} = \mathbb{T} \otimes \mathbb{Q}$, $\mathbb{T} = \langle T_\ell ; \ell \nmid N \rangle$ die Hecke-Algebra

Das Ergebnis wird benutzt um zu zeigen, daß der im folgenden definierte Eisensteinquotient $\tilde{J}$ nicht trivial ist für $g(X_0(N)) > 0$.

Volker Diekert (Regensburg)

Gefördert durch DFG Deutsche Forschungsgemeinschaft

10. Sei $\mathfrak{m}$ ein maximales Ideal der Hecke Algebra T mit Restklassenkörper $k_{\mathfrak{m}}$ und $\operatorname{char} k_{\mathfrak{m}} = p$.

Sei $B^0[\mathbb{F}_p]$ der Raum der Spitzenformen vom Gewicht 2 mod. p.

Dann gilt:

Satz 1: $B^0(\mathbb{F}_p)[\mathfrak{m}]$ hat die Dimension 1 über $k_{\mathfrak{m}}$.

Dieser Satz wurde benutzt, um die beiden folgenden Sätze zu zeigen:

Satz 2: $H^1(X_0(N)/_{\mathbb{Z}}, \mathcal{O})$ ist ein lokal-freier T-Modul vom Rang 1.

und:

Satz 3: $T = \operatorname{End}(J/_{\mathbb{Q}}) = \operatorname{End}(J/_{\mathbb{C}})$.

R. Silhol (Regensburg).

11. Für ein Ideal $\mathfrak{a}$ der Hecke-Algebra T ist $J[\mathfrak{a}]/_{\mathbb{Q}}$ der Kern von $\mathfrak{a}$ in $J/_{\mathbb{Q}}$ $(J = J_0(N))$ und $J[\mathfrak{a}]/_{\mathbb{Z}}$ der Zariski-Abschluß in $J/_{\mathbb{Z}}$. Die $\mathfrak{a}$-adische Komplettierung $T_{\mathfrak{a}}$ der Hecke-Algebra operiert auf der Barsotti-Tate-Gruppe $\varinjlim_m J[\mathfrak{a}^m]/_{S'} =: J_{\mathfrak{a}}/_{S'}$ $(S' = \operatorname{Spec}(\mathbb{Z}[\tfrac{1}{N}]))$ und auf den zugehörigen Tate-Moduln $\mathrm{Ta}(J_{\mathfrak{a}}/_{S'}(k))$ für gewisse Körper k. Es wird gezeigt:

$\mathrm{Ta}(J_{\mathfrak{a}}(\overline{\mathbb{F}}_N)) \otimes \mathbb{Q}$ ist vom Rang 1 über $T_{\mathfrak{a}} \otimes \mathbb{Q}$,

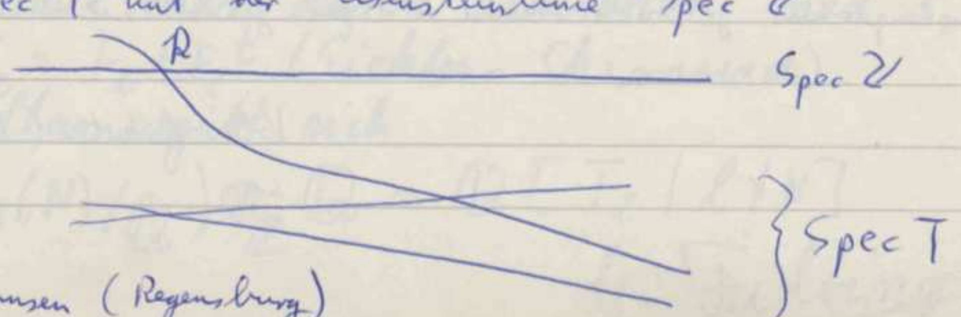
$\mathrm{Ta}(J_{\mathfrak{a}}(\overline{\mathbb{Q}}_N)) \otimes \mathbb{Q}$ ist vom Rang 2 über $T_{\mathfrak{a}} \otimes \mathbb{Q}$,

$\mathrm{Ta}(J_{\mathfrak{a}}(\overline{\mathbb{F}}_p)) \otimes \mathbb{Q}$ ist vom Rang 1 über $T_{\mathfrak{a}} \otimes \mathbb{Q}$, falls $p \in \mathfrak{a}$ und $J_{\mathfrak{a}}/_{\mathbb{Z}_p}$ ordinäre Barsotti-Tate-Gruppe ist.

Weiter wird das Eisensteinideal I definiert:

$$I = (T_\ell - (1+\ell),\ (\ell \nmid N),\ w+1)$$

Als erstes Resultat erhält man $T/I \cong \mathbb{Z}/n$, wobei $n = \text{Zähler}\left(\frac{N-1}{12}\right)$; insbesondere: $I \neq T \Leftrightarrow n > 1 \Leftrightarrow g > 0$, g das Geschlecht von $X_0(N)$. Ein Primideal $\mathfrak{P} \supset I$ ist von der Form $\mathfrak{P} = (I, p)$ für ein $p \mid n$ und heißt dann p-Eisensteinideal. Bedeutung: Die p-Eisenstein-Primideale sind die Durchschnitte von $\operatorname{Spec} T$ mit der Eisensteinlinie $\operatorname{Spec} \mathbb{Z}$



Uwe Jannsen (Regensburg)

12. Es wurde die Bildung von Quotienten $J^{(\alpha)}_{/\mathbb{Q}}$ der Jacobischen $J_{/\mathbb{Q}}$ für ein Ideal α der Hecke-Algebra beschrieben und der Eisensteinquotient bzw. p-Eisensteinquotient

$$\tilde{J} := J^{(\mathfrak{I})} \text{ bzw. } \tilde{J}^{(p)} := J^{(\mathfrak{P})}, \quad \mathfrak{P} = (p, \mathfrak{I}),$$

definiert. Der zweite Teil des Vortrags beinhaltete den Nachweis, daß ein Eisenstein-Primideal $\mathfrak{P}$ nicht supersingulär ~~ist~~ und das kommutative Gruppenschema $J[\mathfrak{P}]/\mathbb{Z}$ p-zulässig ist, d.h. es besitzt über $\mathbb{Z}[\frac{1}{N}]$ eine Filtrierung mit Faktoren $\mathbb{Z}/p$ oder μ_p.

Kay Wingberg (Regensburg)

13. Es wurde die Endlichkeit von $\tilde{J}(\mathbb{Q})$ gezeigt. Dies wird durch „descente infinie" zurückgeführt auf die Aussage, daß die Ordnung der flachen Cohomologiegruppe $H^1(S, \tilde{J}^0[p^m]_{\mathfrak{P}})$ für jedes Eisenstein-Primideal unabhängig von m nach oben beschränkt ist. Dies wiederum beruht wesentlich auf der im vorangehenden Vortrag gezeigten Tatsache, daß Eisenstein-Primideale zulässig sind. Der Beweis bedient sich der in Vortrag 1 bzw. 11 dargestellten Ergebnisse. Eine einfache Folgerung aus diesem Hauptsatz ist die Endlichkeit von $X_0(N)(\mathbb{Q})$ für $N \neq 2, 3, 5, 7, 13$ (d.h. Geschlecht$(X_0(N)) > 0$). Schließlich wurde erwähnt, daß man folgende präzisere Information hat: $\tilde{J}(\mathbb{Q})$ ist zyklisch von der Ordnung $n = \mathrm{num}\left(\frac{N-1}{12}\right)$ erzeugt vom Bild des Spitzendivisors $(0)-(\infty)$.

Peter Schneider, Regensburg

15. Im ersten Teil des Vortrags wurde Mazur's Beweis der Vermutung von Ogg gegeben, daß die $\mathbb{Q}$-rationalen Torsionspunkte der Jacobischen J von $X_0(N)$ [für N prim] durch die vom Spitzendivisor $(0)-(\infty)$ erzeugte zyklische Gruppe der Ordnung $n = \text{Zähler}\left(\frac{N-1}{12}\right)$ erschöpft wird.

Im zweiten Teil des Vortrags wurde ein Beweis für den folgenden Satz von Ogg gegeben (siehe Math. Ann. 228 (1977), ~~[illegible]~~ 279–29?)

Satz: Ist N quadratfrei und $N \neq 37$, so ist $\mathrm{Aut}\, X_0(N) = W$, die Gruppe der Atkin-Lehner Involutionen auf $X_0(N)$.

Der ursprüngliche Beweis von Ogg wurde – unter Benutzung der gleichen Argumente – dadurch wesentlich vereinfacht, daß das stärkere Resultat über $X_0(N)(\mathbb{Q})$ (für N prim) aus Mazur's Arbeit in Inventiones 44 systematisch verwendet wurde.

Norbert Schappacher, Göttingen.

16. Basierend auf den Resultaten von Mazur über $X_0(N), X_1(N)$ für Primzahlen N, wurden sämtliche rationalen Punkte von $X_0(N)$ und $X_1(N)$ für beliebiges N bestimmt.

Satz 1: Die affine Modulkurve $Y_1(N)$ hat unendlich viele rationale Punkte über $\mathbb{Q}$ für $N \leq 10$ oder $N = 12$, und sonst keine.

Satz 2: Die affine Modulkurve $Y_0(N)$ hat unendlich viele rationale Punkte für $N \leq 10$ oder $N = 12, 13, 16, 18, 25$; sie hat sonst keine rationalen Punkte außer in den Fällen:

N	11	14	15	17	19	21	27	37	43	67	163
Zahl der rat. Pkt.	3	2	4	2	1	4	1	2	1	1	1

Aus Satz 1 leitet man ab

Satz 3: Ist E eine elliptische Kurve über $\mathbb{Q}$, so ist die Torsionsgruppe $E(\mathbb{Q})_{tor}$ der rationalen Punkte von E eine der folgenden 15 Gruppen:

$\mathbb{Z}/m$ $(m \leq 10$ od. $m = 12)$ oder $\mathbb{Z}/2 \times \mathbb{Z}/m$ $(m = 2, 4, 6, 8.)$

H. Klingen (Köln)

17. A brief report on new work of R. Greenberg on the conjecture of Birch and Swinnerton-Dyer for elliptic curves with complex multiplication.

J. Coates (Paris).

18. Der Vortrag fing mit einigen allgemeinen Bemerkungen über diophantische Probleme an. Dann wurden die merkwürdigen Eigenschaften der $X_0(N)$ nochmals betont, und mögliche Zusammenhänge und Kontraste mit Fermat-Kurven und Shimura-Kurven erwähnt. Weitere Folgen der Reduktion modulo N von $X_0(N)$ wurden gegeben, z.B. die Formel $2g^+$ für die Anzahl der supersingulären j-Werte in $\mathbb{F}_{N^2}-\mathbb{F}_N$, und die Tatsache, daß die Spitzen von $X_0(N)$ — und etliche anderen Punkte — keine Weierstraß-Punkte sind.

A.P. Ogg (Bonn)

19. Es wurde ein Überblick über die wichtigsten Resultate der Mazurschen Arbeit (auch so weit sie nicht in den vorhergehenden Vorträgen abgedeckt wurden), und Perspektiven für ~~al~~ Verallgemeinerungen der Mazurschen Methoden aufgezeigt.

Bruce Jordan (Göttingen)

14. Die Ogg-Vermutung.

Theorem 1: Besitzt eine elliptische Kurve E über $\mathbb{Q}$ einen rationalen Punkt der Primzahlordnung N, so ist $N \leq 7$

Theorem 2: Besitzt eine elliptische Kurve E über $\mathbb{Q}$ einen rationalen Punkt der Ordnung m, so ist $m \leq 10$ oder $m = 12$.

Genauer $$E(\mathbb{Q})_{tor} = \begin{cases} \mathbb{Z}/m, & m \leq 10 \text{ oder } m = 12 \\ \mathbb{Z}/2 \times \mathbb{Z}/2m, & m \leq 4. \end{cases}$$

Theorem 3 $X_0(N)(\mathbb{Q}) = \{0, \infty\}$ für alle Primzahlen $N \neq 2, 3, 5, 7, 11, 13, 17, 19, 37, ?, ?, ?, 163\}$

Jürgen Neukirch

GRUNDLAGEN DER GEOMETRIE

17. Okt. – 23. Okt. 1982

1. Products of Reflections in Hyperlines

F. Bachmann characterizes the group of motions of a plane as the group O_3^+ which is generated by reflections. In his book "Aufbau der Geometrie aus dem Spiegelungsbegriff" he states that the characterization problem for motion groups of geometries of higher dimensions will be supported by first solving the length problem. We shall solve the length problem for the proper orthogonal groups $O^+(V)$ where V has any dimension. This group is generated by reflections in hyperlines. We prove the following theorem:
Assume $\dim V/R \geq 3$, $K \neq GF(3)$, $\pi \in O^+(V)$, and $\dim F(\pi)^\perp/R \geq \dim B(\pi)$.
Then there are half-turns (i.e. reflections in hyperlines) $\eta_1, \ldots, \eta_s$ such that $\pi = \eta_1 \cdots \eta_s$ and $2s = \dim B(\pi) + \dim(B(\pi) \cap R)$ or
$2s = \dim B(\pi) + \dim(B(\pi) \cap R) + 2$ if $\dim F(\pi)^\perp/\operatorname{rad} F(\pi)^\perp \geq 2$;
$2s = \dim B(\pi) + \dim(B(\pi) \cap R) + 2$ or $2s = \dim B(\pi) + \dim(B(\pi) \cap R) + 4$
if $\dim F(\pi)^\perp/\operatorname{rad} F(\pi)^\perp \leq 1$.
For $\dim V$ finite and V regular, this theorem is due to H. Ishibashi.

E. W. Ellers (Toronto)

2. Maximal partial spreads and translation nets of small deficiency

Bruen was the first to construct maximal nets of small deficiency (small meaning $0 < d \leq \sqrt{s}$, where s is the order and d the deficiency). He used maximal partial spreads in $PG(3,p)$, p a prime, to give examples for $s = p^2$. We determine the exact deficiency of these nets, and we generalize Bruen's result to the case $s=q^2$, q a prime power. This answers two questions of Bruen. We also construct a maximal partial spread of deficiency $q-1$ in $PG(3,q)$ such that the corresponding net is not maximal (and may in fact be completed to a translation plane), whenever q is a prime power but not a prime. Finally, we construct series of maximal partial t-spreads ($t \geqslant 3$ odd) with new parameters.

D. Jungnickel (Gießen)

3. Some results on the completeness of $(nq+n-q-2, n)$-arcs in finite projective planes of order q, when $q \equiv 0 \pmod{n}$, q even.

A (k,n)-arc K in a projective plane of finite order q is a set of k points such that n is the greatest number of collinear points in K. K is complete if it is not a subset of any $(k+1,n)$-arc and K is maximal if $k = nq+n-q$.
A necessary, but not sufficient condition for the existence of a maximal arc is that $q \equiv 0 \pmod{n}$. It is also known that any $(nq+n-q-1, n)$-arc is incomplete and is contained in a unique $(nq+n-q, n)$-arc.
The question arises as to whether, in this case, every $(nq+n-q-2, n)$-arc is also incomplete. Partial results make an affirmative answer a likely conjecture.

Lise Stein (London)

4. Über eine Kennzeichnung zweifach transitiver Permutationsgruppen von H. Wielandt

H. Wielandt bewies folgenden Satz (Permutation groups through relations and invariant functions, Ohio State Univ. Lecture Notes 1969, Theorem 5.17, p. 18): Eine auf X transitive (endliche) Gruppe G ist genau dann 2-fach transitiv auf X, wenn gilt:
(i) G ist primitiv auf X. (ii) Es gibt $x, y \in X$, $x \neq y$, so daß gilt: (a) $gx = y$, $gy = x$ für ein $g \in G$. (b) Zu jedem $z \in X \setminus \{x, y\}$ gibt es $h \in G_z$ mit $hx = y$. Durch Bildung des Gruppenraumes $V(G) = (X, \sqcup, \parallel)$ mit $x \sqcup y := \{x\} \cup G_x y$ und $x \sqcup y \parallel x' \sqcup y' :\Leftrightarrow x' \sqcup y' = g(x \sqcup y)$ für ein $g \in G$ kann dieser Satz in die Sprache der nichtkommutativen Geometrie übersetzt werden: Sei $S = (X, \sqcup, \parallel)$ Schiefaffin mit zwei verschiedenen Punkten x, y, so daß gilt: (1) $x \triangledown y := \{x, y\} \cup \{z \in X / z \sqcup x = z \sqcup y\} = X$; (2) $x \sqcup y \parallel y \sqcup x$. Dann ist $x \sqcup y = y \sqcup x$ eine Gerade. Der Beweis dieses Satzes ist elementar. Mit ihm kann man den Wielandtschen Satz, auch für unendliche G, sowie eine Reihe von Abschwächungen und Abwandlungen beweisen. Beispiel: G ist genau dann 2-fach transitiv auf X, wenn für alle $x, y \in X$ mit $x \neq y$ gilt: (a) $gx = y$, $gy = x$ für ein $g \in G$. (b) Zu jedem $z \in X \setminus \{x, y\}$ gibt es $h \in G_z$ mit $hx = y$.

Johannes André (Saarbrücken)

5. Über Automorphismengruppen von Kollineationsgruppen

Nach d'Angelo ist in einer desarguesschen affinen Ebene der char $\neq 2$ die volle Kollineationsgruppe isomorph zur Automorphismengruppe der von den affinen Spiegelungen erzeugten Untergruppe, und nach Yale ist die Gruppe der Ähnlichkeitsabbildungen des $\mathbb{R}^3$ isomorph zu ihrer eigenen Automorphismengruppe. Diese Ergebnisse werden auf ihren gruppentheoretischen Hintergrund hin untersucht. Es zeigt sich:
(G, P) sei eine Permutationsgruppe, und $T < G$ operiere

transitiv und fixpunktfrei auf P. Ist $T \trianglelefteq G$ und $U < G$ mit $U \cap T = \{id\}$, so gibt es eine Einbettung von U in $\mathrm{Aut}\, T$. Ist $H \trianglelefteq G$ mit $T \subsetneqq H$, so gibt es eine Einbettung $G \to \mathrm{Aut}\, H$. Diese Einbettung ist ein Isomorphismus, wenn G vollständige Gruppe und $G = H \cdot K$ ein semidirektes Produkt ist.

H. Hotje (Hannover)

6. Zweispiegeligkeit der engeren orthogonalen und symplektischen Gruppen

Eine Gruppe G heißt zweispiegelig, wenn sich jedes Element aus G als Produkt von zwei Involutionen darstellen läßt.

Sei V ein endlichdimensionaler K-Vektorraum, auf dem eine quadratische Form definiert ist und sei $O(V)$ die zugehörige orthogonale Gruppe. Die Gruppe $O^*(V) := \{\pi \in O(V) : x\pi = x \quad \forall x \in \mathrm{Rad}\, V\}$ heißt die engere orthogonale Gruppe. Zu einer alternierenden Bilinearform auf V wird entsprechend die engere symplektische Gruppe $Sp^*(V)$ definiert. Es gilt:

<u>Satz</u> (Ellers, Frank, Nolte 1982). Die Gruppe $O^*(V)$ ist zweispiegelig, ebenso bei Char. $K = 2$ die Gruppe $Sp^*(V)$.

Die Gruppe $Sp^*(V)$ ist bei Char. $K \neq 2$ nicht zweispiegelig.

Der Beweis des Satzes benutzt Resultate von Wonenburger und Ellers, Nolte bei regulärem V.

W. Nolte (Darmstadt).

7. Full four-absolute geometry - results due to Marek Kordos.

It was an old problem: to find a geometry which is absolute for three: Euclidean, Bolyai-Lobatchevski, and elliptic geometries. Marek Kordos tried to solve this problem looking for an elementary axiom system in terms of incidence relation and perpendicularity. His idea was to extend planes under consideration, i.e. Euclidean, hyperbolic, and elliptic planes, so that (for a fixed field) the universe is the same; moreover, incidence structures are the same, only perpendicularities are different. The result is stronger than was expected: the obtained theory is absolute for four: three mentioned above and Minkowski geometry. This theory is called full four-absolute geometry, FFAG.

$$FFAG := Cn(FPPG \cup \{AM1 - AM8\})$$

where FPPG = Fano Pappian projective geometry (as a theory of incidence structures (U_1, U_2, I))

and axioms AM1-AM8 („metric" axioms) are the following:

**) AM1 $A \perp B \Rightarrow B \perp A$

AM2 $\forall AB\, \exists C\, (A \neq B \Rightarrow A \not\perp C \vee B \not\perp C)$ (there is at most one singular line)*)

AM3 $\forall aA\, \exists B\;\; aIB \wedge B \perp A$

AM4 $p | ABC \wedge P \perp A,B \Rightarrow A = B \vee P \perp C$ (a line perp. to two distinct lines of a pencil is perp. to every line of this pencil)

AM5 $p | A,B \wedge P \perp A,B,C \Rightarrow A = B \vee p | C \vee p \perp D$ (all the lines perp. to a non-singular line belong to one pencil)

---

*) A is singular $:\Leftrightarrow A \perp B$ for every line B

**) Universal quantifiers in front of sentences are omitted

AM6 $\forall p, ABC\ \exists D\ \forall P\ (\ne(ABC) \wedge p|A,B,C \wedge A\perp A \wedge B\perp B \wedge C\perp C \Rightarrow p|D \wedge D\perp P)$

(if there are 3 isotropic lines in a pencil, then there is also a singular line in this pencil)

AM8 $\forall AB\ \exists CD\ (A\perp B \wedge A\not\perp A \wedge B\not\perp B \Rightarrow H(ABCD) \wedge (C\perp C \wedge D\perp D) \vee C\perp D)$

(every pair of perp. non-isotropic lines has a bisector)

AM7 $\forall p A_i B_i\ \exists C_i\ (\ne(A_1A_2A_3) \wedge \ne(B_1B_2B_3) \wedge (A_i \perp B_i \wedge p|A_iB_i)_{i=1,2,3} \Rightarrow$

$\begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \\ A_3 & B_3 \end{pmatrix} \vee \ne(C_1C_2C_3) \wedge p|C_i \perp C_i)$.

<u>Th</u>. There is a singular line $\Leftrightarrow$ there is a rectangle.

Omitting AM8 we get PMG = <u>projective metric geometry</u> := Cn(FPPG ∪ {AM1-AM8})

So FFAG = Cn(PMG ∪ {AM8}).

Let <u>special projective metric geometry</u> = SPMG = Cn(PMG ∪ R)

and <u>general</u> " " " = GPMG = Cn(PMG ∪ ~R)

where

R : There exists a rectangle.

<u>Representation Theorem</u>. A structure $(U_1, U_2, |, \perp)$ is a model of SPMG (GPMG) iff it isomorphic to $(F^3 - \{(0,0,0)\}/_\sim, F^3 - \{(0,0,0)\}/_\sim, |_F, \perp_F)$ for some field F with char. $\ne 2$, where

$[x_0x_1x_2] |_F [y_0y_1y_2] :\Leftrightarrow x_0y_0 + x_1y_1 + x_2y_2 = 0$ and $[x_0x_1x_2] \perp_F [y_0y_1y_2] :\Leftrightarrow \alpha x_0y_0 + x_1y_1 + \beta x_2y_2 = 0$

with $\beta \ne 0$ and $\alpha = 0$ $(\alpha \ne 0)$

Let now

I : There is an isotropic line.

We have

FFAG	R	~R	β
I	FMinG	FB-LG	−1
~I	FEuclG	FEllG	1
α	0	1	

where FMinG = Full Minkowski Geometry
FEuclG = " Euclidean "
FB-LG = " Bolyai-Lob. "
FEllG = " Elliptic "

and FFAG is of specific types for the four specific geometries.

Maria Moszyńska (Warszawa)

8. Euclidean geometry on the universe of circles.

A natural notion in metric (congruence) geometry is the notion of circle. It is possible to use circles as individuals in constructing plane Euclidean geometry. Namely the system of points, circles and incidence forms a sufficient system; also circles and lines or even circles alone together with tangency relation form a sufficient system of primitive notions.

K. Prazmowski (Warszawa)

9. Topologie projektiver Ebenen: Zu einer Vermutung von H. Freudenthal

Die genannte Vermutung stammt aus dem Jahre 1957 und besagt, daß die Geraden einer lokalkompakten zusammenhängenden projektiven Ebene homöomorph zu Sphären seien. Inzwischen weiß man aufgrund eines Satzes von Adams (1960), daß dann ihre topologische Dimension $l = 2^k \le 8$ sein muß. Wir nehmen hier an, daß $l < \infty$ sei und beweisen dann die Vermutung bis auf Homotopie, genauer: die Geraden sind Homologiemannigfaltigkeiten und homotopieäquivalent zu Sphären, und obige Dimensionsaussage gilt. Der Beweis benutzt garbentheoretische Methoden, insbesondere eine Charakterisierung der Homologiemannigfaltigkeiten von Bredon (1967). Die Resultate bleiben sinngemäß richtig in stabilen Ebenen (wo Geraden sich nicht immer schneiden). Die Ergebnisse haben Bedeutung für das von Salzmann initiierte Programm, Ebenen nach der Größe (Dimension) ihrer Automorphismengruppe zu klassifizieren, denn die Dimension der Gruppe muß in Relation zu der der Ebene bewertet werden.

R. Löwen (Tübingen)

10. Zur Fortsetzung affiner Ordnungsfunktionen

Es sei $(P,\mathfrak{G})$ eine affine oder projektive Ebene und $(\mathfrak{G}\times P\times P)' := \{(G,a,b)\in \mathfrak{G}\times P\times P \mid a,b\notin G\}$. Eine Abbildung $\omega: (\mathfrak{G}\times P\times P)' \to \{-1,1\}$, $(G,a,b)\mapsto (G|a,b)$ heißt nach E. Sperner

Ordnungsfunktion, wenn gilt:

O1 $(G|a,b)(G|b,c) = (G|a,c)$

O2 Es seien a,b,c kollinear und C, C' Geraden durch c mit $a,b \notin C, C'$. Dann gilt $(C|a,b) = (C'|a,b)$.

Wenn $(P,\mathfrak{G})$ eine Translationsebene ist, deren Kern mindestens 4 Elemente enthält, so läßt sich jede Ordnungsfunktion von $(P,\mathfrak{G})$ fortsetzen zu einer Ordnungsfunktion des projektiven Abschlusses von $(P,\mathfrak{G})$.

H.-J. Kroll (TU München)

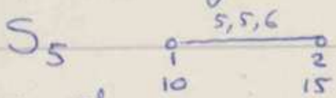
11. Endliche euklidische Ebenen

Es wurde gezeigt, daß sich endliche euklidische Ebenen durch drei einfache Eigenschaften der Kongruenzrelation kennzeichnen lassen. Dabei braucht nicht vorausgesetzt zu werden, daß ihre Inzidenzstruktur die einer affinen Ebene ist.

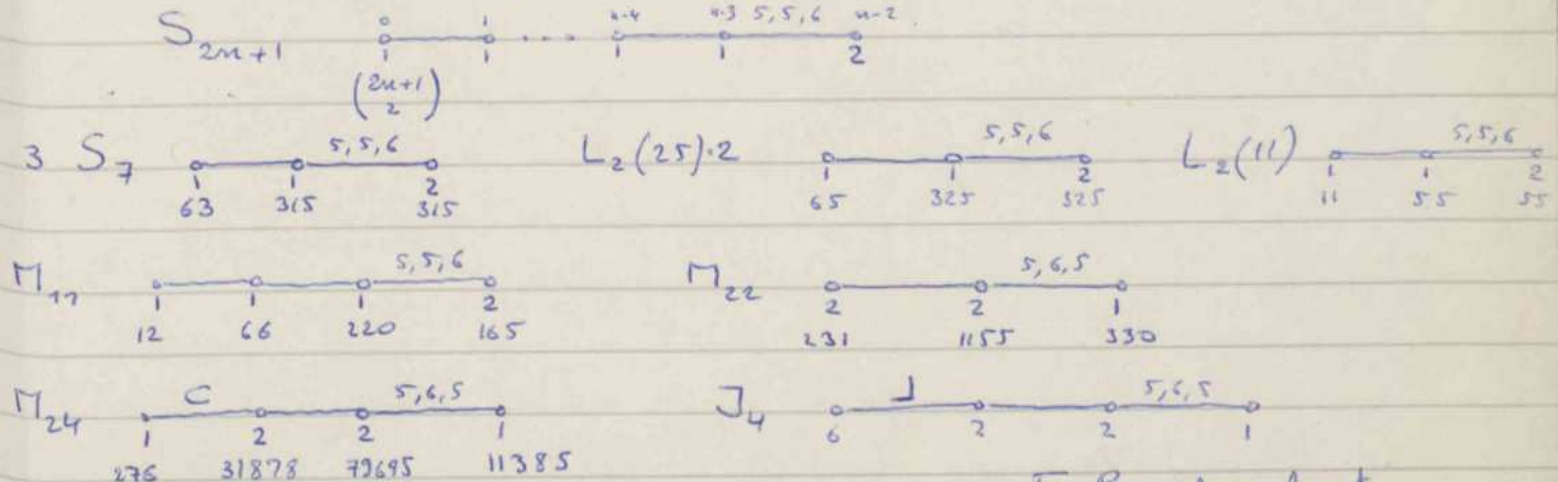
K. Sörensen (T.U. München)

12. Some relatives of the Petersen graph arising from sporadic groups

The Petersen graph of 10 vertices and 15 edges can be conveniently described by the diagram



Here are some of its relatives



F. Buekenhout (Brussels)

13. Bemerkungen zur Theorie der Inzidenzgruppen

Nach G. KIST ist jede gefaserte projektive Inzidenzgruppe bereits ein kinematischer Raum und nach M. MARCHI gibt es gefaserte subaffine Inzidenzgruppen, die nicht kinematisch ~~ist~~ sind. Im Falle einer gefaserten affinen Inzidenzgruppe, weiß man noch nicht, ob sie kinematisch ist. Um diese Frage zu beantworten, muß man Konstruktionsmethoden für affine Inzidenzgruppen entwickeln. Hier läßt sich die aus der Theorie der Fastkörper bekannte Methode der Kopplung übernehmen: Jede affine Inzidenzgruppe läßt sich durch Kopplung aus einem Vektorraum gewinnen. Bedingungen für φ die Kopplung φ damit die abgeleitete Inzidenzgruppe gefasert bzw. zweiseitig ist, werden angegeben.

H. Karzel, TU München