MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 11/1973

Reine Kombinatorik

25.3. bis 31.3.1973

Dieses Jahr wurde wieder in Oberwolfach eine Tagung über "Reine Kombinatorik" gehalten. Die Leiter waren Professor Dr.K.Jacobs (Erlangen) und Professor Dr.D.Foata (Strasbourg). Der Erfolg war von vornherein bestimmt, da Teilnehmer aus sieben Ländern (USA, England, Dänemark, Italien, Frankreich, Schweiz und natürlich Deutschland) vorgesehen waren.
Es wurden insgesamt einundzwanzig Vorträge gehalten; die Hauptthemen waren Ramseytheorie, Young Tableaux, die symmetrische Gruppe, Aufzählung und kombinatorische Identitäten sowie Zahlentheorie. Eine sehr interessante Sitzung über offene Probleme vervollständigte die Tagung.Das Vortragsprogramm wurde bereichert durch anregende Diskussionen und persönliche Gespräche sowie durch die traditionelle Schwarzwaldwanderung.

Teilnehmer:

M.Aigner, Tübingen
J.André, Saarbrücken
G.E.Andrews, University Park (USA)
E.S.Andersen, Kopenhagen
A.Barlotti, Bologna
E.A.Bender, Princeton
Th. Beth, Göttingen
Böhne, Aachen
L.Comtet, Orsay
J.Désarménien, Paris
W.Deuber, Bern
H.Dinges, Frankfurt
Dostert, Saarbrücken
D.Dumont, Besançon
D.Foata, Strasbourg
H.O.Foulkes, Swansea
J.Françon, Strasbourg
J.R.Goldman, Minnesota
H.Harborth, Braunschweig
Hilgers, Göttingen
Hischer, Saarbrücken
K.Jacobs, Erlangen
L.Jones, Göttingen
A.Kerber, Aachen
B.Kittel, Strasbourg
S.Lawrence, University Park
K.Leeb, Erlangen
G.Letac, Clermont
W.Oberschelp, Aachen
T.D.Parsons, University Park
G.Schmidt, München
Schützenberger, Neapel
S.Sherman, Bloomington
Soenoto, Neapel
Stamm, Zürich
R.P.Stanley, Berkeley
Steffens, Hannover
H.J.Stoß, Konstanz
V.Strehl, Erlangen
Ströhlein, München

Vortragsauszüge

J. ANDRE : On binomial coeffecients whose elements are ordinal numbers.

Let $(X,\leq)$ be a wellordered set whose ordinal number is $\|(X,\leq)\| = \alpha$. For any ordinal number β define

$$\binom{X}{\beta} := \{Y \subseteq X \mid \|(Y,\leq|_Y)\| = \beta\}.$$

We define an order $\prec$ in $\binom{X}{\beta}$ by the following lexicographic way :

$$Y \prec Y' \ :\Leftrightarrow \bigvee_{y\in Y} y < f(y) \wedge \bigwedge_{\substack{x<y\\ x\in Y}} x = f(x) \quad ,$$

where f is the unique order isomorphism from Y to Y'. Now define the binominal coefficient $\binom{\alpha}{\beta} := \| (\binom{X}{\beta}, \prec) \|$ as an order type.

The binomial coefficients for finite β are computed completely. If β is infinite, however, almost nothing is known about the value of the binomial coefficient.

G.E. ANDREWS : Partially ordered sets and the Rogers-Ramanujan identities.

In this work, certain partially ordered sets are associated with each partition of an integer n. In the simplest case, the associated partially ordered set is just the lattice of subsets of the set of different parts of the partition. An application of the inclusion-exclusion principle yields an identity of Sylvester

$$1 = \sum_{n=0}^{\infty} \frac{x^n (-1)^n q^{n(3n-1)/2} (1 - xq^{2n})}{(1-q)(1-q^2)\dots(1-q^n) \prod_{j=n}^{\infty} (1-xq^j)}$$

which implies Euler's Pentagonal Number Theorem. In other instances the partially ordered sets are more complicated ; however, they can still be used to prove the Rogers-Ramanujan identities and related results.

E.A. BENDER : Asymptotic methods in enumeration

Easily applied analytic tools were surveyed. The topics discussed included : sums with smoothly varying positive terms, sums with alternating terms (from Inclusion-Exclusion), generating functions with algebraic singularities (Darboux's Theorem), functional equations, entire generating functions (Hayman's Theorem), and some Tauberian Theorems. Areas where tools need development were discussed. These include recursions, multivariate generating functions, and group reduced distributions (E.G. Unlabelled Graphs).

L. COMTET : Sur quelques formules explicites

1- Avec la formule de Lagrange : On cite l'exemple bien connu de la formule explicite la plus "courte" pour les nombres de Stirling de première espèce $s(n,k)$. Cette méthode cependant échoue dans le cas des nombres Eulériens inverses.

2- Relations par coupes : Soit $P(n,r)$ le nombre de relations (matrices de 0 et 1) sur $[n]^2$ dont les sommes par lignes et colonnes valent r, $Q(n,r)$ le nombre de déploiements (matrices de 0,1,2,3,...) sur $[n]^2$ ayant la même propriété, et $G(n,r)$ le nombre de graphes sur $[n]$ réguliers d'ordre r, c'est-à-dire tels que r arêtes exactement aboutissent en chaque sommet. On donne pour chacune de ces suites des formules explicites.

3- Quelques systèmes plus difficiles : Le cas des sytèmes de Schröder $S \subset \mathcal{P}(N)$ (tels que $B,B' \in S \Rightarrow B \subset B'$ ou $B' \subset B$ ou $B \cap B' = \emptyset$) est bien connu (Comtet, C.R., 271(1970)913-6). Le cas des systèmes de Sperner S (tels que $B,B' \in S \Rightarrow B \not\subset B'$ et $B' \not\subset B$) est déjà fort difficile. Enfin, le cas des relations d'ordre, dont le nombre n'est connu que pour $n \leq 8$, paraît plus difficile encore (cf. Comtet, C.R., 262(1966)1091-4) et Kleitman, Rotschild, Proc. A.M.S., 25(1970)276-82).

4- Fonctions implicites : On peut donner une formule explicite pour la $n^{\text{ème}}$ dérivée d'une fonction implicite (Comtet, C.R.,267(1968)457-60), Une simplification très notable apparaissant dans le cas des fonctions algébriques : les coefficients de Taylor satisfont à une récurrence linéaire à coefficients polynômes en n (Comtet, Ens.Math.,10(1964) 267-70).

5- Développements asymptotiques : Deux exemples sont donnés : un complètement traité concernant les entiers $g(n)$ générés par $(\sum_{n\geq 0} n!\, t^n)^{-1} = 1 - \sum_{n\geq 1} f(n)\, t^n$ (Comtet, C.R.,275(72)569-72) et l'autre, non résolu, concernant le nombre de birecouvrement sur un ensemble fini (Comtet, Studia Math Sc Hungaricae, 3(1968)137-52).

W. DEUBER :

Eine Kategorie sei Ramsey, falls zu jedem Paar (X,Y) von Objekten ein Objekt Z existiert mit folgenden Eigenschaften : Wird die Klasse der Y-Subobjekte von Z irgendwie in 2 Klassen partitioniert, so gibt es ein X-Subobjekt von Z, dessen Y-Subobjekte alle in derselben Klasse sind.

Ein Baum ist ein Paar $(A,<)$, wo A eine Menge und $<$ eine teilweise Ordnung auf A ist, die folgenden Bedingungen genügt : [i] $(A,<)$ hat ein kleinstes Element, [ii] für alle $x \in A$ ist die Menge der Vorgänger total geordnet.

Wir benötigen

[1] $(A,<)$ ist n-regulärer Baum, falls jedes nichtmaximale Element genau n direkte Nachfolger hat ($n < \omega$).

[2] $(A,<)$ ist n-beschränkter Baum, falls jedes Element höchstens n direkte Nachfolger hat ($n < \omega$).

[3] $(A,<)$ ist Baum der Länge m, falls jede maximale Kette Ordungstyp $1+m$ hat ($m < \omega$).

[4] eine eineindeutige Abbildung $f : (A,<) \rightarrow (B,<')$ ist Baummorphismus, falls für alle $x, y \in A$ gilt $f(\inf(x,y)) = \inf(f(x), f(y))$.

[5] ein Baummorphismus ist stark ($f: A \longrightarrow B$), falls für nichtmaximalen Elemente $x \in A$ gilt : Anzahl direkte Nachfolger von x = Anzahl direkte Nachfolger von $f(x)$.

SATZ. Die folgenden 12 Kategorien sind Ramsey

Objekte : Bäume, die folgendem genügen				Morphismen :	
endlich	n-reg	n-besch	länge m	starke	Baummorphismen
*	*			*	*
*	*				*
*	*		*	*	*
*	*		*		*
*		*		*	*
*		*			*
*		*	*	*	*
*		*	*		*
*			*	*	*
*			*		*
*				*	*
*					* (Leeb)

Bemerkung : Als Korollar ergibt sich der endliche Fall des Satzes von Ramsey. Der abzählbare Fall desselben kann nicht auf abzählbare Bäume übertragen werden.

D. FOATA : Rearrangement groups

The Euler coefficients are defined by the Taylor expansion of $D(u) = \tan u + \sec u = 1 + \sum_{n\geq 1} (u^n/n!)\, D_n$. They are also uniquely defined by the recurrence formula

$$2\, D_{n+1} = \sum_{0\leq i\leq n} \binom{n}{i} D_i\, D_{n-i} \quad (n \geq 0)\ ,\ D_0 = 1.$$

Here we make them appear as the numbers of orbits of groups $(G_n)_{n>0}$, the group G_n acting on the symmetric group $\mathfrak{S}_n$. These groups were already described in a paper read during the last meeting at Oberwolfach on "angewandte Kombinatorik" (Oktober 1972). We further establish the following result. Let $p(\sigma)$ denote the number of peaks, i.e of local maxima, of a permutation σ. Then the following formula holds

$$D_n = \sum \{2^{-n+p(\sigma)} : \sigma \in \mathfrak{S}_n\}.$$

Moreover, other results about the distributions of several statistics on permutations can be established by a close study of the orbits of the groups G_n.

H. O. FOULKES : Generalisation of a result of Kreweras on Young's Lattice.

A result attributed to Kreweras by Berge (Principes de Combinatoire, p. 59) relates the enumeration of paths between two partitions in Young's lattice to an enumeration connected with standard Young tableaux. Here some generalisations of Young's lattice are given, and the analogous enumerations of paths in these generalised structures are given explicitly in terms of characters of the symmetric group.

J. FRANÇON : Preuves combinatoires des identités d'Abel

De nombreuses identités du type de celle d'Abel (Abel, Hürwitz, Cauchy, Abel-Rothe) sont démontrées de façon purement combinatoire et leurs connexions avec des dénombrements de familles d'applications d'un ensemble fini dans lui-même sont mises en évidence.

J.R. GOLDMAN : Formal Languages in Enumeration

A review of recent developments by the speaker and others on the application of formal languages and the corresponding theory of generating functions in non-commuting variables. Applications include enumeration of planar graphs, random walks, and the representation of combinatorial objects by rooted planar trees. The relation between formal language solutions of enumeration problems and the analytic inversion formulas required in these solutions will be discussed.

H. HARBORTH : Ein Gitterpunkt-Problem

Mit f(r, d) sei die kleinste Anzahl beliebiger Gitterpunkte eines d-dimensionalen Gitters bezeichnet, so dass mindestens von einem r-Tupel dieser f(r, d) Punkte der Schwerpunkt selbst Gitterpunkt ist. Es gilt $f(2^n, d) = (2^n - 1)2^d + 1$. Ein Ergebnis der additiven Gruppentheorie von Erdös, Ginsburg und Ziv (1961) besagt $f(r, 1) = 2r - 1$. Es wird weiter $f(3, 2) = 9$ und $f(3, 3) = 19$ gezeigt. Weitere Zahlenwerte scheinen ähnlich schwer wie Ramsey-Zahlen zu bestimmen zu sein. Auf eine ähnliche Fragestellung für Blockpläne wird hingewiesen.

A. KERBER : On the evaluation of cycle-indices

The following theorem has been proved :

Theorem (enumeration theorem for the exponentiation group, polynomial form)

The cycle index of the exponentiation group $[G]^H$ of $G \leq S_m$ and $H \leq S_n$ reads as follows

$$P_{[G]^H} = \frac{1}{|G|^n|H|} \sum_{(f;\pi) \in G \wr H} \prod_{i=1}^{m^n} x_i^{a_i(f;\pi)} ,$$

where $G \wr H = \{(f;\pi) \mid f : \{1,\ldots n\} \to G, \pi \in H\}$ denotes the wreath product of G and H, and where for $\pi \in H$ $\pi = \prod_{v=1}^{c(\pi)} (j_v \pi(j_v) \ldots \pi^{k-1}(j_v))$

denotes the usual cycle decomposition of π, where

$$a_i(f;\pi) = i^{-1} \sum_{r|i} \mu(r,i) a_1(f;\pi)^r$$

and

$$a_1(f;\pi)^r = \prod_{\nu=1}^{c(\pi)} a_1(g_\nu^{r/gcd(r,k_\nu)})^{gcd(r,k_\nu)}$$

and for an $f : \{1,\dots,n\} \to G$ and $\pi \in H$:

$$g_\nu = f(j_\nu)\, f(\pi^{-1}(j_\nu)) \dots f(\pi^{-k_\nu+1}(j_\nu)) \in G.$$

S. LAWRENCE : Ramsey Graph Theory

Let $r(A,B)$ be the least integer N such that any graph G on N vertices contains a subgraph of type A or its complement contains a subgraph of type B. If A and B are complete graphs, then $r(A,B)$ is the ramsey number $r(|A|, |B|)$.
Ramsey's theorem guarantees the existence of $r(A,B)$ for any A,B which are subgraphs of a complete graph. I will discuss what is currently known about $r(A,B)$.

K. LEEB

Pascal Theory, which was implicit in the work of Pascal, Nash-Williams (on trees 1963), Reiman (on vector spaces 1964) and Graham-Rothschild (on parametersets 1968), has made considerable progress over the last year. After Deuber proved the partition theorem for his Zmpc and a Ramsey for regular trees the way was open for new heredity results concerning the Ramsey property for labelled categories of
strong, weak tree morphisms, direct systems of ordinal morphisms (Leeb 1970), parametersets, choice functions (Leeb 1971). A more sophisticated version of labelled parametersets, called Deuber morphisms, also are Ramsey. We (Deuber + I) even think we now have a suitable Pascal theory for strong graph morphisms. This should soon result in a full Ramsey theorem for graphs, perhaps even respecting the Erdős-Hajnal degree condition.
I ask the question of giving a characterization (syntactic in some suitable language) of those properties of categories (as well and ramsey) which are preserved under the formation of finite sequences of objects with the Higman-morphisms. Once this is done one could study which categories with a forgetful functor into Ens could be substituted for Ord (inals) and still allow preservation of these properties.

G. LETAC : "p - adic behavior of multinomial coefficients"

Starting from the problem (first raised by J. Von Neumann) of building a random variable of given distribution with a coin with unknown probability of head and tail, we get first a combinatorial interpretation of the theorem of Lucas giving the rests moduls a prime number p of multinomial coefficients $c(x) = (x_1 + \ldots + x_n)\,!\,/\,(x_1\,!\,\ldots\,x_n\,!)$. Furthermore, we prove that $c(p^k x)$ has a p - adic limit (There is some connexion with B. Dwork results), and we get p - adic version of some well known theorems, as for example Dixon identity.

W. OBERSCHELP : Formulae for Rook Polynomials

Let a general staircase board K for forbidden positions of a permutation be given, with steps of broadth $k_i \geq 0$ $(1 \leq i \leq N)$ and total heighth l_i of the staircase K_i extending up to the i-th step. We introduce a position polynomial
$P(x,y_1,\ldots,y_N) = \Sigma_{i,j_1,\ldots,j_N} x^i y_1^{j_1} y_2^{j_2} \ldots y_N^{j_N}$, where the coefficient

is the number of systems with i admissible rooks and j_i rows in K_i unused by a rook.

Theorem 1 : $P(x,y_1,\ldots,y_N) = (1+x\,\partial_N)^{k_N} \ldots (1+x\,\partial_1 \frac{1}{y_2 \ldots y_N})[y_1^{l_1} \ldots y_N^{l_N}]$,

where ∂_i means partial differentiation with respect to y_i. The proof is by induction.

For the rook-polynomial of K we get, of course, $R(x) = P(x,1,\ldots,1)$.
This result generalizes the methods for calculating the rook-polynomials for several types of boards (e.g. trapezoidal, Simon Newcombs) from Riordan's first book.

There is an interesting specialization of this theorem and on the other hand a generalization, where a board is considered, which consits of complete rectangles in a rectangular pattern of the total tableau.

T. D. PARSONS : A Ramsey problem related to the Friendship Theorem.

Let f(n) be the least positive integer N such that, for any graph G with N vertices, either G contains a simple cycle of length 4 or the complementary graph $\bar{G}$ contains a vertex of degree at least n.

Then $f(n) < n + \sqrt{n} + 2$ for all $n \geqq 1$. For $n = g^2 + 1$, this gives $f(g^2 + 1) \leqq g^2 + g + 3$. We show that if $g \geqq 2$ is any integer such that $g \neq 5$ and $g \neq k^2 - 1$ for any odd k, then $f(g^2 + 1) \leqq g^2 + g + 2$. The proof involves graph theoretic arguments, plus consideration of eigenvalues of adjacency matrices of graphs.

We use a construction due to Erdős, Rényi and Sos (Stud. Math. Hungar. 1, 1966) to show that $f(g^2 + 1) > g^2 + g + 1$ if g is a prime power. This construction gives infinitely many examples of "non homogeneous friendship sets" defined by Skala (SIAM J. Appl. Math., 1972). The construction is based on finite projective planes.

The problem of f(n) is closely related to the "friendship theorem" (Longyear and Parsons, Nederl.Akad.v. Wetenschap.,1972) and to an extremal problem of Erdős (see the above reference). A suitably defined variant of the "friendship Theorem" is equivalent to the study of f(n).

M. SCHUTZENBERGER : "Groupe symétrique par la méthode de Specht".

On décrit quelques résultats élémentaires nouveaux sur les représentations naturelles du groupe symétrique obtenues par la méthode de Specht.

S. SHERMAN : Monotonicity and Magnetism.

One approach to qualitative results on phase transitions in ferromagnets is by way of a monotonicity theorem which states that correlations are monotone increasing functions of interactions for the d - dimensional classical Heisenberg ferromagnet (D = 1 corresponds to the ising model) this can be formulated as a conjecture (D = 1) on the count of "icings" on an arbitrary graph (Sherman) (Settled affirmatively by Dyson) ; (D > 1) on the count of multicolored threadings on an arbitrary graph (Lenard) (Settled affirmatively by Sherman in very special cases (Neighborhood of O - interaction)).

E. STAMM : Combinatorial aspects in differential calculus.

We give a new formulation of Faa di Bruno's Formula for the n_th derivative of the composition of two differentiable maps between open sets in Banach-spaces. The formula involves the set of partitions of the set $[1, 2, \dots, n]$. We then give a new formula for the n-th derivative for the composite of an arbitrary number of differentiable maps. For the formulation of this formula we use S-maps, these generalize the monomials occuring in the Bell polynomials.
Using this same formalism, we are able to state and prove a formula for the n-th derivative of the inverse of a diffeomorphism between open sets of a Banach space. In contrast to the classical methods for establishing the Lagrange-Bürmann series (Cauchy Integral, Residues) our proof is purely combinatorial and algebraic. Indeed the aforementioned tools are not available in the infinite dimensional case and any proof has to be formal. The key ingredient of the proof is a careful study of the relation between the set of chains of partitions on $[1, \dots, n]$ and the correponding set on $[1, \dots, n + 1]$. By specialisation to 1-dimensional Banach-spaces, a comparision of our formula with the known one due to Bödewadt-Ostrowski yields a purely combinatorial result, we get the number of a special type of trees with fixed number of end points but the total number of vertices is not fixed.

R. STANLEY : The Hilbert syzygy theorem and magic squares.

Let $H_n(r)$ (respectively $S_n(r)$) be the number of nxn matrices (respectively nxn symmetric matrices) of non-negative integers summing to r in every row and column. Anand, Dumir, and Gupta conjectured that $H_n(r)$ is a polynomial function of r of degree $(n-1)^2$ satisfying $H_n(-1) = H_n(-2) = \dots = H_n(-n+1) = 0$, $H_n(r) = (-1)^{n-1} H_n(-n-r)$. Carlitz conjectured that there exist polynomials $P_n(r)$ (of degree $\binom{n+1}{2}$) and $Q_n(r)$ such that $S_n(r) = P_n(r) + (-1)^r Q_n(r)$ for all $r \geq 0$. Proofs of these conjectures will be sketched, based on the Hilbert syzygy theorem of commutative algebra. If time permits, some generalizations will be discussed.

B. Kittel (Strasbourg)