MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 18/1973

Ringe, Moduln und homologische Methoden

6.5. bis 12.5.1973

Die Tagung wurde wie seit mehreren Jahren von F.Kasch (München) und A.Rosenberg (Ithaca, USA) geleitet.

Mathematiker aus 15 Ländern konnten an dieser Tagung teilnehmen und über aktuelle Forschungsergebnisse berichten.

Die Vortragsthemen reichten von reiner Ringtheorie, Darstellungstheorie, kommutativer Algebra, homologischer Algebra bis zur algebraischen K-Theorie und algebraischen Geometrie.

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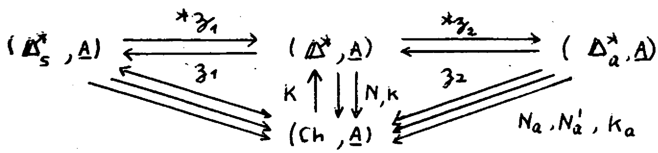
Vortragsauszüge

Stanislaw Balcerzyk: Three types of simplicial objects

Let Δ, Δ_s, Δ_a denote categories with the same set of objects consisting of sets $[n] = \{0,1,\dots,n\}$, for $n = 0,1,\dots$ and sets of morphisms $[m] \to [n]$ consisting of increasing, strictly increasing or all functions, respectively; thus $\Delta_s \subset \Delta \subset \Delta_a$. We consider three categories of simplicial objects $(\Delta_s^*, \underline{A})$, $(\Delta^*, \underline{A})$, $(\Delta_a^*, \underline{A})$ of all contravariant functors with values in an abelian category $\underline{A}$ and we present some results concerning functors in a diagram



where $\mathfrak{z}_1$, $\mathfrak{z}_2$ are forgetful functors, ${}^*\mathfrak{z}_1$, ${}^*\mathfrak{z}_2$ are left adjoint of $\mathfrak{z}_1$, $\mathfrak{z}_2$, N_s, N, N_a, N'_a are normalization functors, k_s, k, k_a are functors of "associated chain complex". We study also connections between homotopies in three categories of simplicial objects.

H. Bass: A finiteness theorem of Quillen in algebraic K-theory

Quillen's new definition of higher K-groups, K_iM, for an additive category M with exact sequences, is briefly described. For a ring A one puts $K_iA = K_iP(A)$, where P(A) is the category of finitely generated projective A-modules. Quillen's proof of the following theorem is then sketched. Theorem: If A is the ring of integers in a number field then K_iA is a finitely generated group for all $i \geq 0$. The proof makes use of the Salomon-Tits theorem on the homotopy type of a building, and of the spectral sequence for the homology of small categories.

P.M. Cohn: <u>The affine scheme of a general ring</u>

With every ring R a topological space $X_1(R)$ can be associated such that $R \mapsto X_1(R)$ is a contravariant functor generalizing the functor spec on commutative rings. Like the spectrum of a commutative ring, $X_1(R)$ satisfies Hochster's axioms for a spectral space and one can construct a sheaf of local rings $\tilde{R}$ on X_1 with a homomorphism $\gamma_1 : R \rightarrow \Gamma(X,\tilde{R})$ from R to the ring of global sections of $\tilde{R}$, but γ_1 will not usually be an isomorphism. To get a better representation one replaces X_1, which essentially consists of epimorphisms from R to skew fields, by X_n, the set of 'epimorphisms' from R to $n \times n$ matrix rings over skew fields. The X_n form a direct system with a map $X_m \rightarrow X_n$ whenever m/n, and $X = \varinjlim X_n$ is called the <u>total</u> spectrum. One can again define a sheaf $\tilde{R}$ over X, this time the stalks are matrix local rings, i.e. rings which modulo their Jacobson radical are $n \times n$ matrix rings over skew fields. There is again a homomorphism $\gamma : R \rightarrow \Gamma(X,\tilde{R})$ which is injective for a larger class of rings (and this allows one to think of $\Gamma(X,\tilde{R})$ as a sort of completion of R), but it seems difficult to determine when γ is surjective because X (unlike X_1) need not be compact.

R. Keith Dennis: <u>The non-triviality of $SK_1(\mathbb{Z}\pi)$</u>

The following describes joint work with R. C. Alperin and M.R. Stein. Let $\mathbb{Z}\pi$ denote the integral group ring of a finite abelian group π and let $SK_1(\mathbb{Z}\pi)$ denote the kernel of the determinant homomorphism from $K_1(\mathbb{Z}\pi)$ to the unit group of $\mathbb{Z}\pi$. A Mayer-Vietoris sequence together with previous work of Dennis and Stein on K_2 of local rings yields an explicit computation of the group $SK_1(\mathbb{Z}\pi)$ in a number of cases. In particular, it can be shown that $SK_1(\mathbb{Z}\pi)$ is an elementary abelian p-group of rank

$$\frac{p^k-1}{p-1} - \binom{p+k-1}{p} \quad \text{(which is never 0 if } k \geq 3)$$

in case π is an elementary p-group of rank k $(p \neq 2)$. As a surjection of finite abelian groups induces a surjection on the SK_1's, this shows that $SK_1(\mathbb{Z}\pi)$ is almost never trivial.

David Eisenbud: <u>Abstract of "Ideals + Resolutions"</u>

Let R be commutative ring, and let

$$\mathbb{P} : \quad \dots P_2 \xrightarrow{d} P_1 \xrightarrow{d} R$$

be a projective resolution of a cyclic R-module.
It is easy to show that $\mathbb{P}$ can be given the structure of a strictly anti-commutative, homotopy-associative, differential graded R-algebra. (It is not known whether the algebra structure can be chosen to be associative, or under what circumstances it will be unique). One can apply this construction to prove the following structure theorem for Gorenstein ideals of height 3 which is joint work with David A. Buchsbaum.
Recall, first, that if φ is an alternating $n \times n$ matrix with coefficients in a domain R, then rank (φ) is even, and, if k is even, then there is an element $Pf(\varphi)$ of R, called the Pfaffian of φ, with $(Pf(\varphi))^2 = \det \varphi$. If n is an odd number we will write $Pf_{n-1}(\varphi)$ for the ideal of R generated by the Pfaffians of the n alternating $(n-1) \times (n-1)$ matrices obtained from φ by deleting, for each i in turn, the i^{th} row and column.

<u>Theorem</u>: Let R be a regular local ring, with maximal ideal J.

a) Let $n \geq 3$ be an odd number, and let φ be an $n \times n$ alternating matrix with entries in J. If the height of the ideal $Pf_{n-1}(\varphi)$ is ≥ 3 then it is $= 3$, $R/Pf_{n-1}(\varphi)$ is a Gorenstein ring, and $Pf_{n-1}(\varphi)$ is minimally generated by n elements.

b) Every ideal $I \subset R$ of height 3, such that R/I is a Gorenstein ring, arises as in a).

J. M. Gersten: K-groups of regular local rings

A conjectural resolution of the sheaf of K-groups, $\underline{K}_n$, on a regular scheme is introduced. The conjecture is valid for X of finite type over a field and in certain one dimensional cases. As a result, the n-component of the Chow ring $A^n(X)$ can be identified with the group $H^n(X,\underline{K}_n)$. A consequence is an algebraic description of the cycle map from singular algebraic cycles on X to Hodge cohomology.

A. W. Goldie: Ore Extensions and polycyclic group rings

Let R be a ring and $S=R[x,\alpha]$ where $xa=a^{\alpha}x$, α being an automorphism of R. Then
Theorem 1 Suppose that R is a Jacobson ring with max-r then S is a Jacobson ring with max-r.
Theorem 2 Let P be a prime ideal of S such that $P\cap R$ is fixed. Then $(P\cap R)S$ is the unique minimal prime with this property and in the set of such primes P with $P\cap R$ fixed ascending chains are finite of bounded length.
By Jacobson ring it is meant that every prime ideal is an intersection of primitive ideals. Theorem 1 implies that the group ring of a finitely generated polycyclic group over a Jacobson ring with max-r has the same properties. This result was obtained by J.E. Roseblade when the ground ring is commutative and the group is polycyclicly finite.

I. D. Ion: Neat and coneat homomorphisms with respect to a torsion theory

Let $\tau=(\mathcal{T},\mathcal{F})$ be a hereditary torsion theory in the category Mod A of left modules over a ring A with unity and let $\mathbb{F}$ be the filter of left ideals of A associated to τ.

Definition: We say that a module homomorphism $f: M \to N$ is τ-neat if for any module G and any homomorphism $\xi: H \to M$ from a submodule H of G such that $G/H \in \mathcal{T}$, $f \circ \xi$ has a proper extension in G if and only if ξ has this property.

If τ is the Goldie torsion theory our results are exactly those obtained by Bowe (Pacif. J. Math. 40(1972)).

A module homomorphism $f: M \to N$ is τ-neat if and only if there are no proper extensions of f in the $\mathbb{F}$-injective envelope of M.

We characterize those hereditary torsion theory τ such that the class of τ-neat homomorphisms is closed under countable direct sums, under directed limits or under arbitrary direct products.

Dually we define the τ-coneat homomorphisms. Satisfatory results we obtain if A is right perfect and left cogenerator ring.

H. Imself: The Immortality of Rosenbuzz [1]

In order to generalize the notion of a ring, we introduced the buzz. The buzz was so timely that it appeared in each subsequent talk. A further development of the buzz was the Chow gong, extending the well known Chow ring. This was, if anything, an even greater and more immediate success. The Chow gong was applied to the central harmonic problem.[2] We observed that the first Chow gong induces Pavlovian faithful descent and the problem was thus reduced to a search of tables.

H. Imself [3]

1) Changed title
2) We are grateful that the triviality of this problem was publicly pointed out to us near the end of our talk.
3) Altered Author.

M. Knebusch: Real closures of commutative rings

We consider pairs (A,σ) consisting of a connected commutative ring A and a signature σ of A, i. e. a homomorphism $\sigma : W(A) \rightarrow \mathbb{Z}$. Here W(A) means the Witt ring of inner product spaces over A. If A is a field, then the signatures of A correspond uniquely to the orderings of A (Harrison, Leicht-Lorenz). There is an obvious notion of morphism between pairs. A morphism $\varphi : (A,\sigma) \rightarrow (R,\rho)$ is called a covering if $\varphi : A \rightarrow R$ is a covering in the sense of Galois theory, and φ- or (R,ρ) - is called a real closure of (A,σ), if in addition (R,ρ) has no coverings except isomorphisms.

Let $\tilde{A}$ denote the universal covering of A and (R,ρ) denote a real closure of (A,σ) (which always exists).

Th.1. Any other real closure of (A,σ) is isomorphic to (R,ρ) over A.

Th.2. $[\tilde{A}:R] \leq 2$. If 2 is a unit in A or if A is semi-local, then $[\tilde{A}:R] = 2$. In the first case further $\tilde{A} = R[\sqrt{-1}]$.

Th.3. If A is semi-local, then ρ is the unique signature of R. Furthermore $W(\tilde{A},J) = \mathbb{Z}$ with J the involution of $\tilde{A}$ over R.

Th.4. If A is local, then the signatures of A correspond uniquely to the conjugacy classes of the involutions in the Galois group $G(\tilde{A}/A)$. (This probably remains true for A semilocal).

The basic techniques to prove these theorems are provided by two joint papers with A. Rosenberg and R. Ware (Amer. J. 1972, Pacific J. 1973), and by the following theorem, recently proved by A. Dress: For any minimal prime ideal $\mathfrak{p}$ of W(A) there exist a maximal ideal m of A and a minimal prime ideal $\mathfrak{q}$ of $W(A_m)$ such that $\mathfrak{p}$ is the pre-image of $\mathfrak{q}$ under the canonical map from W(A) to $W(A_m)$.

Iulius Gy. Maurer: La source commune de certaines topologies définies dans des modules et dans des anneaux

On introduit une topologie quasi-uniforme dans un ensemble quelconque, basée sur un système de certaines relations de préordre définies dans cet ensemble. Cette topologie fait possible l'étude unitaire de plusieurs topologies introduites dans de différentes structures algébriques. Du point de vue des modules et des anneaux, la topologie linéaire définie pour des Ω - groupes forme un cas particulier important de la topologie considérée. On réussit de donner une méthode de décomposition des Ω - groupes, munis par une topologie linéaire. La notion introduite - dénommée produit direct intérieur complet - est une généralisation de la notion de produit intérieur discret - c'est-à - dire de la décomposition directe dans le sens habituel - et elle est liée à la notion de produit direct extérieur complet d'une manière analogue de la liaison qu'il existe entre les notions de produit direct intérieur discret et produit direct extérieur discret.

G. Michler: Uniserial group algebras

A review was given on the known results on uniserial group algebras from a point of view of block theory. Using this method easy proofs for these results were given.

Barry Mitchell: Exact Colimits

The colimit functor colim: $Ab^{\mathbb{C}} \to Ab$ is right exact, but is not in general exact. It is wellknown, however that if the category $\mathbb{C}$ has filtered components, then colim is exact, and the converse was an open question for some time. The converse turns out to be false, the counterexample being a well known category in topology, namely, the category of finite totally ordered sets and order-preserving injections. The correct necessary and sufficient condition for exactness of colim can be given in terms of a certain category aff $\mathbb{C}$ which is constructed by first making $\mathbb{C}$ preadditive and then taking the subcategory of morphisms whose integer coefficients sum to one.

Susan Montgomery: Jordan rings of quotients for symmetric elements

The problem of the existence of quotient rings for Jordan rings with the common multiple property will be studied. In particular, when R is an associative ring with involution, and the symmetric elements S of R have this property, then a quotient exists when S is semi-simple and every $s \in S$ is nilpotent or regular.

Robert A. Morris: Formal groups over rings

Jointly with B. Pareigis we have generalized formal groups as described by Cartier "Groupes formels associés aux anneaux de Witt généralisés"[C.R. Acad. Sci. Paris, t 265 (1967), 49-52] to include both finite formal groups and formal Lie groups simultaneously. For this purpose we consider functors from the category k-Alg of commutative k-algebras over a commutative ring k to Sets which are described by certain representable subfunctors. By a category anti-equivalence we associate to them topological algebras of functions, which occur as certain projective limits of discrete algebras, and by a perfect duality we establish a category equivalence with a category of cocommutative coalgebras described by certain axioms. If k is a field this category is the category of all cocommutative coalgebras. Frobenius, F, and Verschiebung, V, maps are described and a setting is proposed for Dieudonné theory. Our results also recover some standard observations about V operating on sequences of divided powers in Hopf algebras in case k is a field.

Constantin Nita: Sur les S-anneaux

Nous donnerons quelques résultats sur les S-anneaux. Ainsi, on étend un résultat de Morita (donné pour les S-anneaux en sens de Kasch) aux S-anneaux sans condition de chaîne, precisement "Soit M un générateur à gauche de Mod A et $B=\mathrm{Hom}_A(M,M)$ l'anneau des endomorphismes de M. Alors les assertions suivants sont équivalentes: A est un S-anneau à gauche et M est un A-module projectif de type fini $\Leftrightarrow$ B est un S-anneau". Puis, on étudie certains S-anneaux particuliers. Ainsi, on donne, à l'aide de la classe de modules à gauche sans torsion et de la classe de modules à gauche dont le dual est nul, des caractérisations des anneaux dont l'enveloppe injective est sans torsion; en particulier on en déduit des propriétés analogues des anneaux cogénérateurs et des P.F-anneaux.

C. Procesi: Positive Definite Rational Functions of Matrices

I. Reiner: A Mayer-Vietoris Sequence and Ideal Class Groups (joint work with S. Ullom)

Let R be a Dedekind ring whose quotient field K is an algebraic number field, and let Λ be an R-order in a semisimple finite dimensional K-algebra A. A locally free (rank one) Λ-lattice is a left Λ-submodule X in A, such that X is finitely generated and torsionfree as R-module, and such that for each prime ideal p of R, there is a left Λ_p-isomorphism $X_p \cong \Lambda_p$. Here, the subscript p denotes localization (or completion). Two locally free Λ-lattices X,Y are called stably isomorphic if $X \dotplus \Lambda^{(k)} \cong Y \dotplus \Lambda^{(k)}$ for some k. Given any two locally free Λ-lattices X,X', it is known (Swan) that there exists a locally free Λ-lattice X'' such that

(1) $$X \dotplus X' \cong \Lambda \dotplus X''.$$

Define the (locally free) <u>class group</u> $\mathrm{Cl}\Lambda$ as the additive group generated by symbols $[X]$, one for each stable isomorphism class of locally free Λ-lattices X, with addition given by the formula $[X]+[X'] = [X'']$ whenever (1) holds. When $\Lambda = R$, the group $\mathrm{Cl}\Lambda$ is the usual ideal class group of R.
If Λ' denotes a maximal R-order in A containing Λ, then there is a surjection $\mathrm{Cl}\Lambda \rightarrow \mathrm{Cl}\Lambda'$, with kernel $D(\Lambda)$, say. Since the calculation of $\mathrm{Cl}\,\Lambda'$ reduces to the calculation of ray class groups in the center of Λ', an arithmetic problem, it is of interest to determine the kernel $D(\Lambda)$. There are explicit formulas for $\mathrm{Cl}\Lambda$ and $D(\Lambda)$, due to Jacobinski, but these formulas are not well suited for calculations.
In many cases, it is more convenient to use an analogue of Milnor's Mayer-Vietoris sequence. Suppose that we are given a fibre product diagram

$$(2) \qquad \begin{array}{ccc} \Lambda & \longrightarrow & \Lambda_1 \\ \downarrow & & \downarrow \varphi_1 \\ \Lambda_2 & \xrightarrow{\varphi_2} & \bar{\Lambda} \end{array}$$

in which all maps are R-algebra homomorphisms, and where either φ_1, or φ_2 is surjective. Let Λ, Λ_1 and Λ_2 be R-orders in semisimple K-algebras, and suppose that $\bar{\Lambda}$ is a finite ring. Assume further that $K\Lambda$ satisfies the Eichler condition, that no simple component of $K\Lambda$ is a totally definite quaternion algebra. Then there are exact sequences

$$(3) \quad u^*(\Lambda_1)\cdot u^*(\Lambda_2) \rightarrow u(\bar{\Lambda}) \xrightarrow{\partial} \mathrm{Cl}\Lambda \rightarrow \mathrm{Cl}\Lambda_1 \dotplus \mathrm{Cl}\Lambda_2 \rightarrow 0,$$

$$u^*(\Lambda_1)\cdot u^*(\Lambda_2) \rightarrow u(\bar{\Lambda}) \rightarrow D(\Lambda) \rightarrow D(\Lambda_1) \dotplus D(\Lambda_2) \rightarrow 0.$$

Here, $u(\bar{\Lambda})$ denotes the group of units of $\bar{\Lambda}$, and $u^*(\Lambda_i) = \varphi_i\{u(\Lambda_i)\}$, $i=1,2$. The homomorphism ∂ is defined as follows: $\partial u = [\Lambda u]$ for $u \in u(\bar{\Lambda})$, where

$$\Lambda u = \{(\lambda_1,\lambda_2) : \lambda_i \in \Lambda_i,\ (\varphi_1\lambda_1)\,u = \varphi_2\lambda_2\}.$$

If the Eichler condition fails to hold, the sequences in (3) are exact when each u( ) is replaced by GL_2 ( ).
The sequences in (3) can be used to calculate the class group $Cl(\mathbb{Z}G)$, for the case where G is a dihedral group of order 2p (p=odd prime), and also for a metacyclic group G of order pq, where p is an odd prime, and q divides p-1. This latter calculation, due to Galovich-Reiner-Ullom, reduces the problem to the determination of $Cl(\mathbb{Z}H)$, where H is cyclic of order q. For q prime, it is known that $D(\mathbb{Z}H)=0$ and $Cl(\mathbb{Z}H) \simeq Cl(\mathbb{Z}[\sqrt[q]{1}])$. However, if q is a prime power, the calculation of $Cl(\mathbb{Z}H)$ is a difficult problem (see Kervaire-Murthy).

C.M. Ringel: Algebras of finite representation type

The following results were obtained in collaboration with V. Dlab. Let K be a commutative field, and A a finite dimensional K-algebra. Then A is said to be of finite type provided there is only a finite number of indecomposable left A-modules. If either A is hereditary or $(\text{Rad } A)^2=0$, then A is of finite type if and only if a certain diagram derived from A is a Dynkin diagram A_n, B_n, C_n, D_n, E_6, E_7, E_8, F_4 or G_2. Also, under these assumptions, if A is not of finite type and K is infinite, then there are infinitely many natural numbers d_i such that to every d_i there are infinitely many indecomposable A-modules of dimension d_i. This theorem extends previous results of Yoshii, Gabriel and Krugliak to arbitrary base fields K. For the proof, a generalized version of the characterization of structures of finite type as given by Kleiner, Nazarova and Raiter is needed.

J. Roseblade: Modules for Polycyclic Groups

M. Schacher: The Schur Subgroup of a Number Field

Let k be an abelian extension of the rational field Q. The Schur group S(k) is the subgroup of the Brauer group B(k) generated by classes [A] where A is a central simple algebra over k occuring as a simple component in the group ring Q[G] for some finite group G.
S(Q) was determined by M. Benard and K. Fields to be the group generated by all quaternion algebras over Q. M. Schacher and M. Benard have proved: S(k) has an element of order m $\Leftrightarrow$ k contains a primitive m-th root of unity. Using this information the Schur group of any cyclotomic field can be determined.

L. Small: Local Finiteness in PI-Rings

Moss Sweedler: A construction of simple algebras

Let A be a commutative ring and consider End A as an $A\otimes A$-module by $[(a\otimes b)\cdot f](c) = af(bc)$, $a,b,c \in A$, $f \in \text{End } A$. Let $\{L_\alpha\}$ be ideals in $A\otimes A$ satisfying:

1. given L_α and L_β there is $L_\gamma \subset L_\alpha \cap L_\beta$,
2. $(A\otimes A)/L_\alpha$ is a finite projective left A-module for each L_α,
3. given L_α and L_β there is L_γ with $e(L_\gamma) \subset A\otimes L_\alpha + L_\beta \otimes A$ where $e\colon A\otimes A \to A\otimes A\otimes A$, $a\otimes b \mapsto a\otimes 1\otimes b$,
4. given a proper ideal $0 \neq I \subset A$ there is an L_α with $A\otimes I \not\subset I\otimes A + L_\alpha$,
5. given L_α there is L_β with twist $(L_\beta) \subset L_\alpha$ where twist: $A\otimes A \to A\otimes A$, $a\otimes b \mapsto b\otimes a$,
6. there is an $L_\delta \subset \text{Ker}(A\otimes A \xrightarrow{\text{mult}} A)$.

If $C \subset \text{End } A$ is defined as $\{f \in \text{End } A \mid L_\alpha \cdot f = 0 \text{ for some } \alpha\}$ then C is a simple ring.

L. Szpiro: $\mathbb{P}^1 \times \mathbb{P}^2$ n'est pas ensemblistement intersection complete dans $\mathbb{P}^5$

On construit pour toute famille d'entiers positifs $m \geq 2$, $n_1 \dots n_{m-1}$, $d_1 \dots d_m$ tels que $\Sigma n_i = \Sigma d_j$ une famille semi-universelle nonvide de courbes arithmétiquement normales de $\mathbb{P}^3$, de degré $\frac{1}{2}(\Sigma n_i^2 - \Sigma d_j^2)$, et dont une résolution projective est

$$0 \to \sum_1^{m-1} \mathcal{O}_{\mathbb{P}^3}(-n_j) \to \sum_1^{m} \mathcal{O}_{\mathbb{P}^3}(d_i) \to \mathcal{O}_{\mathbb{P}^3} \to \mathcal{O}_c \to 0.$$

Une telle courbe c avec m = 3 fournit un morphisme $\mathbb{P}^3 \xrightarrow{f_c} \mathbb{P}^5$ tel que $f_c^{-1}(\mathbb{P}^1 \times \mathbb{P}^2) = c$. On trouve ensuite un nombre infini de telles courbes de degré premier et on conclue.

Les techniques principales sont:

- une utilisation intensive du théorme de Bertini pour les systèmes linéaire
- la liaison des variétés algébriques.

(d'après C. Peskine et l'auteur)

O.E. Villamayor: Taylor series and Wronskian

Let K be a commutative ring with 1, A a commutative K-algebra with 1 and B a commutative A-algebra. Then, a map $\tau : A \to B$ will be called a Taylor map if: 1) τ is K-linear, 2) $\tau(1)=0$, 3) $\tau(xy) = x\,\tau(y) + y\,\tau(x) + \tau(x)\,\tau(y)$.

Consider the exact sequence $0 \to I \to A \otimes_K A \xrightarrow{\mu} A \to 0$ where μ is the multiplication map, then the map $T: A \to I$ defined by $T(a) = 1 \otimes a - a \otimes 1$ is a Taylor map which is universal in the sense that, for every Taylor map $\tau : A \to B$ there is an A-algebra homomorphism $\lambda : I \to B$ such that $\tau = \lambda \circ T$. T is called the canonical K-Taylor series development. If M is an A-module, an n^{th} order derivation $A \xrightarrow{d} M$ is a map satisfying: 1) d is K-linear, 2) d(1) = 0, 3) $d(a_0, \dots, a_n) = \Sigma_{s=1}^{n} (-1)^{s-1} \sum_{i_1 < \dots < i_s}$

$x_{i_1} \dots x_{i_s} d(x_0, \dots, \hat{x}_{i_1} \dots \hat{x}_{i_s} \dots x_n)$

then the map $T:A \rightarrow I$ composed with the canonical homomorphism $I \rightarrow I/I^{n+1}$ is an n^{th} order derivation called T^n which is universal, i.e., $Der^n(A,M) \simeq Hom_A(I/I^{n+1},M)$. I/I^{n+1} is the module of n^{th} order differentials.

Wronskian theorem If $f_1,..,f_r \in A$ are K-linearly independent, there exists an N such that $(1+T^N)f_1,..,(1+T^N)f_r$ are A-linearly independent.

If V is an algebraic variety with a line bundle L with sufficient sections, these define a map of V into a projective space. The singularities of this map will be called the Weierstrass points of L.

In particular, if V is a non singular complete curve of genus $g > 1$ and K is an algebraically closed field these points coincide with the classical Weierstrass points if char K = 0 and with the Weierstrass-Boseck-Schmidt points if char K $\neq$ 0.

Richard Wiegandt: On N-radicals

Sands and Jaegerman have shown that some axiomatically defined radical properties, the so-called N-radicals, satisfy conditions described by means of Morita contexts. Examples for N-radicals are the lower, the locally nilpotent and the Jacobson radical properties.

The author has characterized the N-radicals also in the terms of upper radicals of suitable classes. It turned out that N-radicals are comparatively rare radical properties, e.g. there is no N-radical property between the Brown-McCoy radical and the upper radical of all fields.

H.-J. Schneider (München)