

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 10/1974

EUROMECH COLLOQUIUM 46

Investigations of the Stability of Boundary Layers

4.3. bis 8.3.1974

The European Mechanics Colloquium 46 was held at the Mathematisches Forschungsinstitut in Oberwolfach (Black Forest) under the joint chairmanship of Prof.Dr.R.Eppler and Prof.Dr.F.X. Wortmann of the University of Stuttgart.

The colloquium was attended by 47 researchers from nine countries: Denmark (1), France (6), Germany (20), The Netherlands (3), Norway (2), Sweden (1), Switzerland (1), United Kingdom (8), U.S.A. (5).

Altogether 29 papers including one survey lecture were presented in five morning and three afternoon sessions. Wednesday afternoon was reserved for an excursion to the Black Forest Museum in Hausach. The Friday afternoon session was reserved for a general discussion. The morning sessions usually lasted three hours without a break and the afternoon sessions were from 2 p.m. until 6 p.m. with a coffee or tea break in between. The presentations were approximately 20 to 40 minutes and there was adequate time for thorough discussion of the material presented.

The colloquium was divided into three main topics:

1. Stability Theory, 2. Experimental and Empirical Investigations, 3. Free Jets - Free Shear Layers. Of course some contributions dealt with problems touching more than one of these topics; therefore the classification of the papers was sometimes rather arbitrary.

Most of the contributions (17) were in Stability Theory, including the survey lecture by Prof.Criminale who gave a

thorough review on the most recent results in linear stability theory. The presentations in this field gave evidence that substantial progress has been made in recent years and is presently being made.

Eight papers were included in the second topic, Experimental and Empirical Investigations, all of which aroused considerable interest. The experimental techniques to investigate phenomena involved in laminar-turbulent transition seem to have become rather sophisticated. However it was also demonstrated that sometimes impressive results can be obtained with relatively simple experimental arrangements.

Five contributions dealt with the third topic, Free Jets - Free Shear Layers.

The colloquium was concluded with a general discussion on Friday afternoon. There, the many new ideas and the progress that became evident during the colloquium were discussed and evaluated.

The following conclusions can be stated:

1. There is considerable progress in stability theory of boundary layers, mainly in the field of secondary instabilities and in finite difference methods for unstable boundary layers.
2. Some new results show new points of view for the understanding of transition, although there is a long way to a complete description of this phenomenon.
3. Experimental results suggest, that there is not only one way to turbulence but many.
4. Free stream disturbances even being very small, are just as important as the amplification of controlled perturbations.
5. It was adequate to have as a topic only the stability of boundary layers. Maybe in a few years a meeting with the more ambitious topic "Boundary Layer Transition" should be organized.

In general most participants agreed that the location and the facilities of the Mathematisches Forschungsinstitut were very proper for a colloquium of this type. The rather unique tradition at the Institute of changing the seating arrangement for a very meal, the fact that all participants were resident in the Institute itself and the remote setting of the Institute in the Black Forest greatly helped to facilitate the development of personal contacts and encouraged informal discussions which usually lasted until late at night.

Prof.Dr.Richard Eppler (Stuttgart)

Participants

F.Bark, Stockholm	S.Haaland, Trondheim (Norway)
D.Beichert, Berlin	H.Hassler, Freiburg
B. van den Berg, Amsterdam	Th.Herbert, Freiburg
H.Bestek, Stuttgart	E.H.Hirschel, Porz-Wahn
H.Bippes, Freiburg	D.J.R.Houston, Edinburgh
Bouthier, Paris	J.L. van Ingen, Delft
C.A.Brebbia, Hampshire	G.Iooss, Orsay (France)
G.J. de Bruin, Enschede	R.Jordinson, Edinburgh
W.O.Criminale, Göteborg	R.E.Kelly, London
Dallmann, Porz-Wahn	Chr. von Kerczek, Baltimore
A.Davey, Newcastle	P.Krogmann, Göttingen
H.G.Elrod, London	H.Kümmerer, Stuttgart
R.Eppler, Stuttgart	L.M.Mack, Pasadena
H.M.S.Fasel, Stuttgart	F.Martin, Meudon
H.Fernholz, Berlin	R.Michel, Toulouse
A.E.Forest, Derby	U.Michel, Berlin
P.Gouyat, Meudon	E.Pfizenmaier, Berlin
Ph.Hall, London	P.Plaschko, Berlin

N.Rott, Zürich	H.True, Lyngby (Denmark)
G.S.R.Sarma, Freiburg	M.Tveitereid, Oslo
O.Sawatzki, Karlsruhe	M.A.Weissmann, London
H.Schlichting, Göttingen	F.-X.Wortmann, Stuttgart
M.Sibony, Tours	J.Zierep, Karlsruhe
S.Smith, Hampshire	

Summaries

W.O.CRIMINALE: Linear stability theory for boundary layers:  
A review and assessment of progress

A great deal of effort has gone into the investigation of the stability of the laminar boundary layer since the theoretical basis was first presented. Originally, it was thought that the solutions would provide answers for transition to turbulence. It has taken a good many years, but it appears that this goal is now almost within reach. It is significant that this development has come about by use of linear theory. The purpose of this review is to trace the formulation from this point of view, clear up one by one the major shortcomings and difficulties that have been encountered, and to provide an outline with the proper guidelines for treating the problem. The center of interest is directed towards the incompressible laminar boundary layer without pressure gradients, but the salient results for the compressible case and boundary layers with more complexities are also discussed.

G.J. DE BRUIN: Stability of a layer of liquid flowing down an inclined plane

The stability of the flow down an inclined plane is studied for small angles of inclination. The same problem has been studied by S.P. Lin, however using an incorrect boundary condition. The correctly formulated eigenvalue-problem is solved by a numerical integration of the Orr-Sommerfeld equation employing the orthonormalization technique. It is shown that in the range  $3' < \beta < 1^0$  a decrease in the critical reynoldsnumber for the hard mode, which is a shear wave modified by the presence of the free surface. In that range the stability is still more or less governed by the stability of the soft waves, which are essentially gravity waves modified by the presence of shear.

The hard mode does exhibit instabilities for large values of  $\alpha R e$  only.

For values of  $\beta < 1'$  the stability is governed by the hard mode, contradictory to Lin's statements.

R.JORDINSON, M.GASTER: An investigation of a singularity in the eigenvalues of the Orr Sommerfeld equation for Blasius flow

The Orr Sommerfeld equation for Blasius mean flow has been solved for real  $R$  and complex values of  $\alpha$  and  $\beta$  i.e. assuming that a small disturbance in the boundary layer is subject to both spatial and temporal amplification. A square root singularity has been found in the relationship between  $\alpha$  and  $\beta$  (at fixed  $R$ ) and a brief account of a Fourier method to represent it is given.

M.A.WEISSMAN , P.M.EAGLES: On the stability of flow in a divergent channel

The "WKB" method has been adapted to study the stability of diverging channel flow. Waves are found to pass through an "unstable" region (actually, a region of growth) and then decay. In contrast to the familiar theory for parallel flows, the growth rate is shown to depend on the cross-stream coordinate and upon the flow quantity (i.e. stream function, velocity components, etc.) under consideration. Thus, the "corrected" neutral curves are not unique, but in general the "unstable" region is expanded and the "critical" Reynolds number lowered.

H.KÜMMERER: Some numerical results on the stability of laminar plane boundary layers

Solutions of the linear stability equations for boundary layers with pressure gradients including the effect of increasing boundary layer thickness have been obtained by a numerical method for solving Orr-Sommerfeld eigenvalue-problems. For the Blasius flow small perturbation-waves and the energy-exchange between perturbation flow and basic flow have been considered in more detail. As a side result we obtain that the evolution of Tollmien-Schlichting-waves can only be described numerically by integration of the complete Navier-Stokes-equations; simplified equations lead to "non well-posed" eigenvalue-problems of Orr-Sommerfeld-type.

F.BARK: Some aspects of the wall region of the turbulent boundary layer

The response of the wall layer in a turbulent shear flow due to compact sources is discussed. Two-dimensional spectra of the streamwise fluctuating velocity,  $S_{uu}(k_x^+, w^+)$  and  $S_{uu}(k_z^+, w^+)$ , at  $y^+ = 15$ , are computed from the solutions of an inhomogeneous Orr-Sommerfeld equation. The structure in the normal direction of the forcing term is taken from conditionally averaged measurements of the Reynolds stress during bursting.

Secondary instability prior to bursting short time averaged velocity distributions is computed and some analogies with the laminar-turbulent transition phenomenon are drawn.

Some effects of non Newtonian fluid properties are discussed.

M.SIBONY: Approximation de certaines équations paraboliques non linéaires

On se propose d'approcher la solution  $u$  de l'équation

$$(1) \quad \begin{cases} \frac{du}{dt} + Au = f & , \quad 0 < t < T < +\infty \\ u(0) = u_0 \end{cases}$$

pour  $f$  et  $u_0$  donnés,  $A$  étant un opérateur non nécessairement linéaire.

A l'aide d'une méthode de discréétisation des variables d'espace et de temps on se ramène à la résolution de systèmes algébriques non linéaires de la forme

$$(2) \quad \frac{1}{k}(u_{hk}^{(r+1)} - u_{hk}^{(r)}) + A_h(\theta u_{hk}^{(r+1)} + (1-\theta) u_{hk}^{(r)}) = f_{hk}^{(r+1)}$$

pour  $r = 0, \dots, s-1$ ,  $\theta \in [\frac{1}{2}, 1]$ ,  $h$  étant le pas de discréétisation d'espace et  $k$  le pas de discréétisation de temps.

Mais pratiquement il est impossible de déterminer la solution exacte des systèmes (2) : en effet, si pour résoudre (2) on utilise une méthode itérative, il faut s'arrêter à une itération  $m = m(h, k, r+1)$  et au lieu de la détermination de  $u_{hk}^{(r+1)}$  on obtient une certaine approximation  $u_*^{(r+1)}$ , elle-même solution approchée par les approximations de l'étape (r) d'un système perturbé de la forme

$$(3) \quad \frac{1}{k}(u_{hk}^{(r+1)} - u_*^{(r)}) + A_h(\theta u_{hk}^{(r+1)} + (1-\theta) u_*^{(r)}) = f_{hk}^{(r+1)}$$

Le problème "naturel" consiste alors en la détermination d'un nombre optimal  $m(h, k, r, \theta)$  d'itérations nécessaires à l'inversion numérique des systèmes (3) et qui assure a priori la convergence de l'ensemble des approximations (discrétisations et itérations sur des systèmes perturbés) du problème initial.

Le plus souvent ce problème a été touché comme ici, en même temps que celui de la majoration a priori des erreurs d'approximations.

F.MARTIN, P.GOUGAT: Existence d'un harmonique des instabilités naturelles dans une couche limite laminaire

L'étude a permis de mettre en évidence un harmonique des instabilités naturelles qui se propagent dans la couche limite laminaire au voisinage d'une paroi déformable.

Nous avons pu caractériser l'évolution de cette fréquence et nous pouvons dire qu'elle est due à une perturbation venant se greffer sur les bouffées instables du fondamental.

Il est remarquable que, bien que l'énergie de cet harmonique soit fonction de l'amplitude de la déformation de paroi, il n'existe pas d'amplitude critique pour son existence.

Par ailleurs, une exploration de la couche limite avec une sonde à double film, nous a permis de constater que ce phénomène se localise très près de la paroi, là où la vitesse semble avoir une incidence négative par rapport à l'horizontale.

Nous avons fait l'hypothèse que cet harmonique est dû à la présence d'un micro-décollement probablement intermittent que nous n'avons pas encore pu mettre en évidence.

BOUTHIER: Stabilité des écoulements presque parallèles

On propose une étude de la stabilité linéaire et non linéaire des écoulements fluides presque parallèles (c'est-à-dire des écoulements qui ne sont pas strictement unidimensionnels et qui présentent des variations sur deux dimensions d'espace - temps dont le rapport des longueurs caractéristiques est très grand). Elle comprend une partie théorique où l'on utilise essentiellement une méthode de double échelle. On obtient ainsi une équation linéaire qui décrit l'influence du non-parallélisme des lignes de courant sur l'évolution des ondes de Tollmien - Schlichting. Une étude séparée est faite pour les écoulements de couche limite et divers cas particuliers sont traités numériquement (couche limite de Blasius, écoulement de Jeffery Hamel, etc.). Enfin on montre qu'il y a découplage entre les interactions non linéaires (avec évidemment l'approximation faiblement non linéaire) et les phénomènes engendrés par le non-parallélisme de l'écoulement de base.

C.A.BREBBIA, S.L.SMITH: Finite element solution of Navier-Stokes equations for transient two-dimensional incompressible flow

A method is described for the solution of transient, incompressible viscoelastic flow in two dimensions. The dependent variables - stream function and vorticity - were approximated over each triangular element using linear interpolation functions. This approximation reduces the problem to a set of matrix equations whose term involving derivatives of time is the mass matrix. The lumping of this matrix together, with the application of Ruge-Kutta integration scheme produces an efficient method of solution. Once the real values of the stream function are known the velocities and pressure can be computed.

The method has been checked by comparison with known theoretical results. As an application a study of the vortex street development behind a rectangular obstruction is described. The flow has been impulsively accelerated to a constant speed in a channel of finite width. The Reynolds number range investigated is between 20 and 100.

H.FASEL: Stability investigation of laminar boundary layer flows by numerical integration of the Navier-Stokes equations

Stability and transition phenomena of laminar, two-dimensional incompressible boundary layer flows are investigated by introducing forced, time-dependent perturbations into the steady flow field along a semi-infinite flat plate; the reaction of the flow is then directly determined by numerical solution of the unsteady Navier-Stokes equations using an implicit finite difference method.

In contrast to the linear stability theory which is limited to sinusoidal disturbances of small amplitudes, this approach contains no restriction in respect to form or intensity of the perturbations as long as the calculated flow is physically meaningful, i. e., essentially two-dimensional.

Stability analysis of laminar boundary layer flows requires numerical experimentation with Reynolds numbers that are large enough to allow physical instability and thus amplification of the introduced forced disturbances, i.e., Reynolds numbers larger than the critical Reynolds number. Therefore for the development of the numerical method special care had to be taken to avoid oscillations caused by numerical instabilities or by the built-in boundary conditions; in the unstable region such oscillations might become amplified just like the physically meaningful perturbations and thus the results might become distorted.

The main aspects of the numerical method will be described and some results discussed. For periodic perturbations of small amplitudes the numerical calculations will be compared with results of the linear stability theory and measurements of the experiments by Schubauer and Skramstad or Ross.

G.IO OSS, H.TRUE: The nonlinear stability of the stationary incompressible Ekman-layer

We consider a rotating flow of a viscous incompressible fluid near a plane boundary. The flow satisfies the Navier-Stokes equations including the additional Coriolis force term. The boundary layer (Ekman layer) becomes unstable when the Ekman number increases as linearized stability theory shows. This corresponds to the fact, that the two conjugated eigenvalues associated with the most unstable eigenmode of the operator from the linearized theory pass the imaginary axis. It is then proved, that when the boundary layer flow becomes unstable, a new stable time-periodic flow develops with period close to  $\frac{2\pi}{n_0}$ .

M.TVEITEREID: On the stability of thermally stratified plane poiseuille flow

The stability of thermally stratified plane Poiseuille flow is considered with respect to small disturbances. The interaction between the usual Tollmien-Schlichting type of instability and the thermal type of (in)stability is analysed for various values of the Prandtl number. The problem is solved as an eigenvalue problem. The most important features from the results are that the critical Rayleigh number is found to be nearly linearly dependent of the Prandtl number, and that a critical Reynolds number always exists, no matter how great the fluid is stabilized by a linear temperature profile.

T.HERBERT: Berechnung der Amplitude neutraler Störungen  
der Plattengrenzschicht

Die Berechnung endlicher Amplituden periodischer Geschwindigkeitsschwankungen in der Grenzschicht baut auf der Methode der Landau-Konstanten auf. Durch Betrachtung der stationären Lösungen der Amplitudengleichung werden bisherige Einschränkungen umgangen. Der Einfluß der anwachsenden Grenzschichtdicke wird in den Gleichungen für die mittlere Geschwindigkeit berücksichtigt.

Anhand numerischer Ergebnisse wird die neutrale Fläche für Tollmien-Schlichting-Wellen endlicher Amplitude diskutiert. Bei dem Versuch, den Gültigkeitsbereich der Ergebnisse abzuschätzen und auf größere Amplituden auszudehnen, zeigen sich erneut Grenzen für die Anwendbarkeit der Methode der Landau-Konstanten. Bei einem abschließenden Vergleich mit Meßwerten werden die Aussagen der Theorie weitgehend bestätigt.

R.E.KELLY: Instability of the free convection boundary  
layer on an inclined plate

When a heated plate is inclined relative to the vertical, observations using dye have indicated that the free convection boundary layer can become unstable in two distinct ways. For angles of inclination less than  $14^\circ$ , two-dimensional waves which travel in the flow direction predominate, whereas for angles greater than  $17^\circ$ , longitudinal vortices, whose axes are in the flow direction, predominate the flow pattern.

A linear stability analysis is presented here in order to compare theory with experiment. It is found that each mode of instability first becomes unstable at the same location along the plate at an angle of only  $4^\circ$ . However, the amplification rates of the unstable travelling waves are relatively higher. By calculating the total amplification of each disturbance mode from the predicted point of onset of instability to the point of observed instability for various angles of inclination, satisfactory correlation between theory and experiment is obtained.

P.HALL: The effect of modulation on the stability of viscous fluid flows

In this lecture we describe methods which can be used to determine the stability characteristics of some oscillatory viscous flows. In particular we discuss the stability of the flow between concentric cylinders when the outer one is at rest and the inner one has angular velocity  $\Omega_0(1 + \epsilon \cos \omega t)$ . When  $\omega$  is small we seek a solution by letting  $\epsilon$  tend to zero with  $\epsilon/\omega$  fixed. When  $\omega$  is large the Stokes layer associated with the motion of the inner cylinder is thin compared to the separation of the cylinders and it is found that the disturbance velocity field also has a Stokes layer at the outer cylinder with a central 'inviscid' region between these layers. We expand in powers of  $\omega^{-\frac{1}{2}}$  in each region and 'match' where different regions meet. We also describe how non-linear effects modify the solution in each case, and mention other problems to which the methods can be applied.

C. VON KERCZEK, S.H.DAVIS: Linear stability theory of oscillatory Stokes layers

The stability of oscillatory Stokes boundary layers is examined using two quasi-static linear theories and an integration of the full time-dependent linearized disturbance equations. The quasi-static theories involve the numerical solution of the Orr-Sommerfeld equation whereas the full time-dependent theory involves a partial differential equation in space and time whose coefficients are periodic in time. The time-dependent linear theory disturbance equation is reduced, using Galerkin's method, to equations a system of ordinary differential with time-periodic coefficients. On the basis of Floquet theory, the eigenvalues of the fundamental matrix of this system, which is obtained by numerical integration, determines the stability or instability of the flow.

The full theory predicts absolute stability within the investigated range and perhaps for all Reynolds numbers. In fact, the Stokes layers are found to be more stable than the motionless state (zero Reynolds number). The least stable disturbance is found to decay monotonically with time. The quasi-static theories predict strong inflexional instabilities. The failure of the quasi-static theories is discussed. Relevant experimental work is also discussed and compared with the theoretical predictions.

H.HASSLER: Experimentelle Untersuchung von Instabilitätserscheinungen im Staupunktgebiet eines Kreiskegels

Es wurden visuelle Untersuchungen und Hitzdrahtmessungen im Strömungsfeld eines Kreiskegels bei turbulenter Anströmung in einem kleinen Wasserschleppkanal durchgeführt.

Im Staupunktgebiet geben beide Untersuchungsmethoden einen Hinweis auf das Vorkommen von längswirbelartigen Instabilitätserscheinungen.

Es lassen sich je nach Modellkonfiguration und Anströmgeschwindigkeit die Wellenlängen der längswirbelartigen Störungen feststellen. Weiterhin werden die Geschwindigkeitsprofile in den verschiedenen radialen Querschnitten des Kreiskegels und ihre Änderungen mit der Lauflänge untersucht.

J.ZIEREP, O.SAWATZKI: Instabilities on a rotating sphere and in a gap between two rotating spheres

In the transition region between laminar and turbulent flow on a rotating sphere vortex lines can be observed. The experiment shows that all vortex lines rotate in the same direction and are inclined to the streamlines. Investigations, done in our institute gave an explanation to this phenomena.

In the gap between two spheres, with the inner sphere rotating a number of different fluid instabilities, all at the same Reynolds number, are observable. The observed flow fields, especially the development of the flow configurations depending on the initial conditions (acceleration of angular velocity) will be discussed. A survey will be given about the theoretical considerations concerning the laminar flow without vorticities.

Instabilitäten auf der rotierenden Kugel und im Spalt zwischen zwei rotierenden Kugeln

Auf der rotierenden Einzelkugel treten im Übergangsgebiet zwischen laminarer und turbulenten Strömung gleichsinnig drehende Wirbel auf, deren Achsen schräg zu den Stromlinien verlaufen. Untersuchungen im hiesigen Institut haben dieses Phänomen geklärt.

Im Spalt zwischen zwei rotierenden Kugeln sind eine ganze Anzahl verschiedener Strömungsinstabilitäten möglich. Die zugehörigen Strömungsfelder lassen sich experimentell bei gleicher Reynolds-Zahl realisieren. Die unterschiedlichen Strömungen werden besprochen und ihr Einsetzen in Abhängigkeit von den Anfangsbedingungen (Winkelbeschleunigung) diskutiert. Abschließend wird ein Überblick über die bisher erfolgten theoretischen Betrachtungen gegeben, die den laminaren Fall ohne Wirbel betrafen.

H.BIPPES: Experimente zur Entstehung von sekundären Instabilitäten in Grenzschichten an konkaven Wänden

Die laminare Grenzschicht an parallel angestromten konkaven Wänden wird unter gewissen Bedingungen instabil gegen gegenläufig rotierende Wirbelpaare mit Achsen in Anströmungsrichtung. Während ihrer Anfachung stromabwärts nimmt diese Instabilität, auch Görtler-Wirbel genannt, eine Schlangelbewegung an, welche sich ebenfalls anfacht. Gleichzeitig bildet sich in den instabilen Längszonen des gestörten Strömungsfeldes, wo die Grenzschichtdicke aufgrund der Längswirbel angewachsen ist, eine sekundäre Instabilität. Sie zeigt sich durch periodisch auftretende Konzentrationen der Querwirbelstärke, welche der Längswirbelstörung überlagert sind.

In diesem Beitrag werden die Ergebnisse einiger Experimente beschrieben, in welchen die Entstehung dieser sekundären Instabilität und ihre Wirkung auf den laminar-turbulenten Umschlag untersucht wurde.

F.X.WORTMANN: Sekundäre und tertiäre Instabilitäten  
bei Görtler-Wirbeln

There exist a special interest to investigate the details of the transition process downstream of longitudinal Görtler vortices. Today it is well known, that these vortices can develop an extremely amplified oscillation, which leads to turbulence within one or two wavelength's downstream. In an earlier investigation the autor found between the ordinary Görtler vortices and the oscillating regime a strongly distorted form of the Görtler vortices, which he considered a second instability mode. In contrast Bippes in a recently published work could not observe this intermediate type of perturbation mode. It is hoped that a new experimental investigation can clarify this discrepancy.

N.ROTT: Recent observations of instability of oscillating boundary-layer-flow at the ETH in Zürich

Experiments with oscillating boundary layers were carried out at the ETH by Merkli and Thomann, by use of air-filled piston-driven "resonance tubes". Instability observations were made by hot-wires and by flow visualization with the help of smoke addition. A critical Reynolds number based on the thickness of the Stokes boundary layer and the velocity amplitude was found; its magnitude turned out to be essentially in agreement with earlier observations of similar flows. The character of the initial instability can be described as a "burst" which occurs regularly at a certain phase and lasts over a certain fraction of the flow oscillation.

E.H.HIRSCHEL: Stability and transition in three-dimensional boundary layers - application and evaluation of existing criteria

Existing stability and transition criteria are applied to numerically calculated boundary-layer flows on swept wings. The different criteria, and especially their underlying assumptions are discussed. Although the understanding of the transition phenomena in general three-dimensional boundary layers is poor, certain results can be obtained. The crossflow instability seems to be the dominant mechanism in creating turbulence on swept wings.

P.KROGMANN: Ermittlung des Umschlags laminar-turbulent bei Überschallgeschwindigkeiten mittels Messung des örtlichen Wärmeübergangs

Der Rohrwindkanal (Ludwieg Tube) der DFVLR/AVA Göttingen wird seit einiger Zeit zu systematischen Messungen des örtlichen Wärmeübergangs an dünnwandigen Modellen eingesetzt. Diese Meßmethode gestattet neben der Messung des örtlichen Wärmeübergangs auch eine genaue Ermittlung des laminar-turbulenten Umschlags der Grenzschicht sowie die Ermittlung von Strömungsablösung, da die Strömung am Modell durch keinerlei Sonden beeinflußt wird.

An einem spitzen, schlanken Kegel (Halbwinkel  $\Theta = 5^\circ$ ) wurde an zehn Stellen längs einer Mantellinie der Wärmeübergang bei  $Ma_\infty = 5$  im REYNOLDSzahlbereich  $Re_\infty/cm = 1 \cdot 10^5$  bis  $4,3 \cdot 10^5$  gemessen. Dabei wurde bei den höheren REYNOLDSzahlen natürlicher Umschlag der Grenzschicht von laminar zu turbulent festgestellt. Experimentell ermittelte Umschlags-REYNOLDSzahlen sind von verschiedenen Parametern abhängig. Unter anderem wurde von verschiedenen Autoren eine starke Abhängigkeit von der Anström-REYNOLDSzahl ("unit Reynolds number effect") festgestellt, d. h. mit zunehmender Anström-REYNOLDSzahl wächst auch die Umschlags-REYNOLDSzahl.

Dies wird in erster Linie auf Störungen zurückgeführt, die von der turbulenten Grenzschicht an den Düsenwänden abgestrahlt werden. Die im Rohrwindkanal möglichen Anström-REYNOLDSzahlen liegen beträchtlich über denen vergleichbarer Windkanäle. Die hier ermittelten Umschlags-REYNOLDSzahlen zeigen einen weniger starken "unit Reynolds number effect", was die Vermutung nahelegt, daß dieser Effekt bei entsprechend hohen Anström-REYNOLDSzahlen verschwindet. Der Einfluß des Anstellwinkels auf die Lage des Umschlagpunktes an der Leeseite des Modells wurde für verschiedene REYNOLDSzahlen im Anstellwinkelbereich  $\alpha = 0^\circ$  bis  $16^\circ$  untersucht, und es wurde eine sehr starke Anstellwinkelabhängigkeit festgestellt. Außerdem wurden für drei Anstellwinkel  $\alpha = 5^\circ$ ,  $10^\circ$  und  $15^\circ$  und konstant gehaltener REYNOLDSzahl die Wärmeübergangsverteilungen auf dem Kegelumfang gemessen.

A.E.FOREST: Problems of boundary layer stability on  
turbine aerofoils

The purpose of this short paper is to stimulate interest in the important problem of predicting the location and extent of the laminar-turbulent transition region of a turbine aerofoil boundary layer. Knowledge of the transition region is vital to the satisfactory prediction of heat transfer and aerodynamic loss and hence to the component design. The general characteristics of the boundary layer are reviewed with particular attention to those parameters known to affect the transition process. It is shown that the most difficult area is the prediction of transition in the pressure (concave) surface of the aerofoil, where the effect of very large acceleration in delaying transition is opposed by the effects of curvature and free-stream unsteadiness. The necessity for more experimental and theoretical work on interactive effects on boundary layer stability is emphasised.

P.PLASCHKO: Die Stabilität eines elektrisch leitenden  
Freistrahles im longitudinalen Magnetfeld

Es wurde eine theoretische Untersuchung des Stabilitätsverhaltens eines Flüssigmetall-Freistrahles, umgeben von bewegten Gasen, in einem von außen angelegten parallelen Magnetfeld durchgeführt. Für den reibungsfreien Grenzfall werden die linearisierten Stabilitätsgleichungen unter Zulassung von Variationen des Grundprofiles und/oder der Stoffwerte senkrecht zu den Freistrahlrändern hergeleitet. Mit ihrer Hilfe gewinnt man exakt die Übergangsbedingungen an Grenzflächen zwischen Media differenter Stoffwerte oder Geschwindigkeitsprofile. Die Rechnungen wurden für räumliche Anfachungen eines ebenen Freistrahlers unter Berücksichtigung des Einflusses beider MHD-Kennzahlen (Wechselwirkungsparameter und magnetische Reynoldszahl) ausgeführt. Im Bereich kleiner magnetischer Reynoldszahlen ist das induzierte magnetische Feld proportional und das Strömungsfeld unabhängig von diesem Parameter, dessen Variation nur von geringem Einfluß auf das Stabilitätsverhalten ist. Durch das angelegte Magnetfeld kann der Freistahl partiell stabilisiert werden, wobei die Stabilisierungsraten bei Werten der Strouhalzahl bzw. des Wechselwirkungsparameters von 1 respektive 0,1 in der Größenordnung vom 10 Prozent liegen.

D.BECHERT, E.PFIZENMAIER: Experimental investigations on instability-waves in a free jet, travelling faster than the mean jet exit velocity

The phase velocity of disturbances in a free jet at small Strouhal-numbers is investigated experimentally. In different theoretical investigations Michalke [1] and also Crow and Champagne [2] have shown the ratio between phase velocity and mean velocity of the basic flow to be greater than one in a specific region of low Strouhal-numbers, not only for plane or cylindrical vortex sheets, but also for an axisymmetric free jet with finite ratio between free jet radius and free boundary layer thickness.

Such a region of small Strouhal-numbers for phase velocities greater than the basic mean velocity, first time in our measurements is proved experimentally and the changes downstream in the mean velocity profile can be shown to influence the development of phase velocity in an important way.

- [1] Michalke, A. Instabilität eines kompressiblen runden Freistrahls unter Berücksichtigung des Einflusses der Strahlgrenzschichtdicke  
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- [2] Crow, S.C. Orderly structure in jet turbulence  
Champagne, F.H. Boeing Sci. Res. Lab. Doc.  
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D.BECHERT, U.MICHEL: The control of a free shear layer starting from a semi-infinite plate by a pulsating monopole or dipole. - Some new analytical solutions obtained from incompressible theory

The stability and wave motion of a free shear layer with exterior forcing fields and limited rigid boundaries is considered. For the case of an incompressible medium and for symmetric boundary conditions with respect to the shear layer a general solution for the nonsteady shear layer motion is obtained. The resulting wave motion magnitudes depend on the exterior forcing field only. For the infinite free shear layer and for a shear layer starting from a semi-infinite plate the wave motion due to the external forcing fields of a monopole and a dipole is investigated. Some new analytical solutions are given.

D.J.R.HOUSTON: A numerical study of spatially growing disturbances in a free shear layer

Using the hyperbolic-tangent velocity profile the spatial development of disturbances has been investigated. As a first step, the inviscid linearised stability equation has been solved for complex wave-numbers and real frequencies. The matrix iterative method used gives the eigenvalue and eigenfunction simultaneously. The results obtained are in excellent agreement with those of Michalke (1965) who used an entirely different numerical method.

In the non-linear case the stability problem is far more difficult. However, by making certain assumptions about separability, it is possible to obtain a second order solution which gives the mean flow distortion as well as the first and second harmonics of the perturbation.