MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

# Tagungsbericht 14/1974

## Lineare Operatoren und Approximation II

30.3. - 6.4.1974

Die diesjährige Tagung über "Lineare Operatoren und Approximation" stand unter der Leitung von Prof. P.L. Butzer (Aachen) und Prof. B. Sz.-Nagy (Szeged, Ungarn). Sie war mit 54 Teilnehmern aus 15 Ländern international besetzt. Dabei haben sich die Tagungsleiter wieder von dem Prinzip leiten lassen, neben einer Reihe bedeutender Kapazitäten auf diesem Gebiet auch junge Mathematiker einzuladen, um auf diese Weise den Kontakt zwischen erfahrenen und jungen Forschern zu fördern. So kam es am Rande des offiziellen Programms zu vielen anregenden Diskussionen, die auch durch die herzliche Atmosphäre des Oberwolfacher Hauses begünstigt wurden.

Insgesamt wurden 43 Vorträge über Probleme der Approximationstheorie und Linearen Operatoren gehalten; ein Teil davon waren ausführliche Übersichtsvorträge. Die Palette der Themen reichte von der klassischen Approximationstheorie, der Theorie der Orthogonalentwicklungen, der divergenten Reihen, der Integraltransformationen über Fourier - Analysis, Interpolation insbesondere auf rearrangement-invarianten Räumen bis zu Halbgruppentheorie, Operatortheorie und Funktionen - algebren.

Die Ausarbeitungen der Vorträge werden als Band 25 der Serie ISNM im Birkhäuser Verlag, wie es inzwischen zur Tradition geworden ist, erscheinen.

Teilnehmer

R. Askey, Wisconsin
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J. Boman, Stockholm
G. Bragard, Aachen
S.D. Chatterji, Lausanne
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## Vortragsauszüge

### R.ASKEY: Some new positive operators

Some useful positive summability methods for Fourier series are the (C,1) means, the Abel means, the Weierstrass means and the la Vallee-Poussin means. These have all been extended to series of Jacobi polynomials and a number of interesting problems and results arise. The Abel and Weierstrass means extend with no difficulty. The de la Vallee-Poussin kernel was considered by Bateman in 1905 and his sum can be used to prove the variation diminishing property (Robert Horton) and also to obtain the addition formula for Jacobi polynomials (Tom Koornwinder). If $f(x)=\sum_{n=0}^{\infty} a_n P_n^{(\alpha,\beta)}(x)\geq 0$, it seems likely that the $(C,\alpha+\beta+2)$ means are positive when $\alpha,\beta\geq -1/2$. This is true for $\alpha=\beta\geq -1/2$ (Kogbetliantz); $|\alpha-\beta|\leq 1$ and $\alpha+\beta\geq 1$, or $|\alpha-\beta|\leq 2$, $\alpha+\beta\geq 3$ (Askey). A related positive sum is $\sum_{k=0}^{\infty} P_k^{(3/2-\ 1/2)}(x)/P_k^{(-1/2,3/2)}(1)$, $-1<x<1$, which is equivalent to

$$\frac{d}{d\theta}\sum_{k=1}^{n+1}\frac{\sin n\ \theta}{n\ \sin\ \theta/2} < 0, \quad 0<\theta<\pi.$$

### C.BENNETT: Banach Function Spaces and Interpolation Methods. II. Interpolation of Weak-type Operators.

The author recently introduced the $(\rho;j)$ and $(\rho;k)$ methods of interpolation which are based on the J- and K-methods of Peetre. Here $\rho$ is a rearrangement-invariant norm on $R^+=(0,\infty)$ and the j and k are simple modifications of the J- and K-functionals of Peetre. There are the usual theorems of interpolation, equivalence, stability and duality for the ρ-methods, formulated in terms of the Boyd indices of ρ.

In the present paper we give some applications of the abstract theory to various problems involving the interpolation of weak-type operators. In particular, we find a characterization of the weak-interpolation spaces between $L^p$ and $L^q$ in terms of the (ρ;k)-method, and we give interpolation-theoretic proofs of the theorems of Boyd and Fehér, Gaşpar and Johnen concerning the boundedness of the Hilbert transform and the conjugate operator on rearrangement-invariant spaces.

H.BERENS: Korovkin-type Theorems for Contractions and Positive Contractions on Banach Lattices

Let E be a Banach lattice, and let $\mathcal{T} = \{T_n : n = 1,2,\ldots\}$ be a sequence of linear contractions on E into itself. The convergence set $\mathcal{C}_{\mathcal{T}}$ for $\mathcal{T}$ is defined by

$$\mathcal{C}_{\mathcal{T}} = \{f \in E : \lim_n T_n f = f \text{ in } E\}.$$

For Banach lattices having a uniformly monotone norm (i.e., for any $0 < \varepsilon < 1$ there is a $\delta(\varepsilon) > 0$ such that for any $g, f \in E$ satisfying $0 \leqslant g \leqslant f$, $\|f\| = 1$, the condition $\|f - g\| \geqslant \varepsilon$ implies $\|g\| \leqslant 1 - \delta(\varepsilon)$) the convergence set for sequences of positive linear contractions on E is shown to be a vector sublattice of E. The norm of the Lebesgue space $L^p$, $1 \leqslant p < \infty$ is uniformly monotone, while the norm of $L^\infty$ obviously is not. For these spaces the convergence set for a sequence of linear contractions (not necessarily positive) can be characterized as the range of a contractive projection. This is a report on some common work of Professor G.G.Lorentz and the speaker.

J.BOMAN: Comparison of Approximation processes and division in measure algebras

For notation and background see H.S. Shapiro, Acta Math. 120(1968) and the book Smoothing and approximation of functions, Van Nostrand 1969, by the same author. For a bounded and regular measure $\sigma$ in $R^n$, a real number $t > 0$, and a bounded and continuous function f defined in $R^n$, one defines the generalized modulus of continuity $\omega_\sigma(f,t)$ of f with respect to $\sigma$ in terms of the supremum norms of the convolutions $\sigma_t * f$, where $\sigma_t$ is the so-called dilation of $\sigma$ with respect to t. Under certain assumptions on $\sigma$ and $\tau$ it was proved in the cited paper by H.S. Shapiro that there are constants C and B depending only on $\sigma$ and $\tau$ such that (A) $\omega_\tau(f,t) \leq C\int_0^t u^{-1}\omega_\sigma(f,Bu)du$. This result implies the classical theorems of Jackson and Bernstein. In the present note note it is studied which additional conditions have to be imposed on the measure $\sigma$ in order that (A) may be strengthened to $\omega_\tau(f,t) \leq C\omega_\sigma(f,Bt)$. The proofs depend on division on noncompact sets in the algbra $\hat{M}(R^n)$ of Fourier transforms of measures in $R^n$. This division is possible since a certain subalgebra of $M(R^n)$ is known to be a so-called Wiener algebra, i.e. without Wiener-Pitt phenomenon.

G.BRAGARD: Teilbarkeitssätze in Banachalgebren

Am Beispiel der klassischen Teilbarkeitssätze (Shapiro 1968) wird untersucht, wie Probleme aus der Approximationstheorie in den Rahmen abstrakter Banachalgebren eingefügt werden können. Sei X eine kommutative, halbeinfache B.A. mit Identität und L eine reguläre (komm., halbeinf.) B.A. ohne Identität, so daß L mit einem Ideal $X_L$ in X identifizierbar ist. Wir fordern weiter die Existenz einer normalisier ten Familie gleichmäßig beschränkter Automorphismen auf X, die auf einer Teilmenge $N \subset L$ approximierende Identitäten $f_t$ erzeugt (entsprechend den klassischen Fejêr-Typ-Verfahren), und stellen Bedingungen an den max. Idealraum von X. Dann erhält man einen Satz über lokale Teilbarkeit: Falls g ein Element $f \in X$ lokal teilt und einige technische Voraussetzungen erfüllt sind, so gilt für $h \in L$

$$\|h\, f_t\| \leq \text{const}\, \|h\, g_t\| + \text{const} \sum_{j=0}^{\infty} \|hg_{t_0 b^j t}\| .$$

S.D.CHATTERJI: On a theorem of Banach and Saks

The theorem in question is that given $f_n \in L^p$ $(1 < p < \infty)$ such that $f_n \to 0$ weakly, there exists a subsequence $\{n(k)\}$ such that $\lim_{N\to\infty} \|N^{-1} \sum_{k=1}^{N} f_{n(k)}\|_p = 0$ . Various sharpenings of this result for $L^p$ are given e.g. replacing arithmetic averages by very general summability methods. This is done by proving some inqualities using probability theory (Burkholder inqualities of martingale theory) which are valid for any subsequence of a suitably chosen subsequence of $\{f_n\}$. It is also pointed out that the result of Banach and Saks is also true for p=1, contrary to a statement in their original paper (Studia Math.2, 1930). A discussion of the various generalizations of the property proved by Banach and Saks in $L^p$ (Banach-Saks property) is discussed in general Banach Spaces. Its relationship to uniform convexity and reflexivity in particular is discussed.

J.L.B.COOPER: Validity of Integral Transform methods for solution of differential equations

The classical use of Fourier, Laplace, Bessel and other transforms leaves open the question of whether the formal solutions found by these methods actually are solutions. It can be shown that even for ordinary differential equations the formal solutions need not satisfy the initial conditions, but that they are solutions in some weak sense.

Various generalized integral transforms have been proposed as a means of overcoming restrictions on growth of the solutions. It will be shown that these restrictions can be avoided while using the ordinary transforms.

R.A.DEVORE: On the degree of monotone approximation

If f is monotone on $[-1,1]$ let $E_n^*(f)$ denote the error in approximating f in $L^\infty[-1,1]$ by polynomials of degree less than or equal to n, which are monotone on $[-1,1]$. $E_n^*(f)$ is also called the degree of monotone approximation. We study the question of how $E_n^*(f)$ compares with $E_n(f)$, the degree of ordinary approximation by polynomials of degree n. When comparison is not possible, we ask if Jackson type estimates of the same order as for unresticted approximation still hold. This is known to be the case for $\omega(f)$ and $\omega(f')$ by results of Lorentz and Zeller.

We have two main results. First, we give estimates for $E_n^*(t^{n+1})$ which show that this order is different from $E_n(t^{n+1})$. Second, we give Jackson type estimates for $E_n^*(f)$ when $f^{(r)}$ is of bounded variation. These estimates are of the same order as the unrestricted case. We also illustrate a new technique for proving the Lorentz - Zeller theorems.

G.FREUD: On onesided weighted polynomial approximation

Es sei $Q(x)$ $(-\infty < x < \infty)$ eine gerade konvexe Funktion, $Q(x)$ sei differenzierbar und zunehmend für $x > 0$. Es sei $q_n > 0$ bestimmt durch die Gleichung $q_n Q'(q_n) = n$. Ferner sei $f(x)$ eine r-te Integralfunktion einer Funktion $f^{[r]}$ von lokal beschränkter Schwankung und

$$V_Q(f^{[r]}) \overset{\text{Def}}{===} \int_{-\infty}^{\infty} e^{-Q(x)} |df^{[r]}(x)| < \infty .$$

Wir setzen weiter voraus, daß

$$|f(x)| \leqslant A+Bx^{2m} \qquad (-\infty < x < \infty)$$

für eine natürliche Zahl m und zwei geeignete positive Zahlen A,B gültig ist.

Unter diesen Voraussetzungen gibt es zwei Polynomfolgen $\{p_n(x)\}$ und $\{P_n(x)\}$, so daß der Grad von $p_n$ und $P_n$ nicht n übersteigt, und es gelten die Ungleichungen

$$p_n(x) \leqslant f(x) \leqslant P_n(x) \quad (-\infty < x < \infty;\ n = 2m+r,\ 2m+r+1,\ \ldots)$$

und

$$\int_{-\infty}^{\infty}[P_n(x) - p_n(x)]\, e^{-Q(x)}dx \leq C(r,Q)\, V_Q(f^{[r]})\left(\frac{q_n}{n}\right)^{r+1} +$$

$$+ K(r,Q,m)(A + B)\, e^{-\alpha(r,Q,m)\, n},$$

wobei $C(r,Q)$ eine außschließlich von $r$ und $Q$ abhängende positive Zahl ist und $K(r,Q,m)$, $\alpha(r,Q,m)$ nur von $r,Q$ und $m$ abhängende positive Zahlen sind.

D. GAŞPAR: Eine Klasse rearrangement invarianter Banachräume

Es wird die Klasse jener rearrangement invarianten Banachräume untersucht, deren Elemente sich aus der Fouriertransformierten wiedergewinnen lassen (durch Norm-Konvergenz von Fourierreihen bzw. von Fourierintegralen). Mit $\mathcal{F}_{2\pi}$ bezeichnet man diese Klasse im Falle der $2\pi$ - periodischen Funktionen, und einfach mit $\mathcal{F}$, wenn die auf der ganzen Geraden definierten Funktionen in Frage kommen. $L^\lambda_{2\pi} \in \mathcal{F}_{2\pi}$ ist zu jeder der folgenden Aussagen äquivalent:

(a) $H \in [L^\lambda_{2\pi}]$, und $C_{2\pi}$ ist dicht in $L^\lambda_{2\pi}$, wobei $H$ der übliche Konjugiertenoperator ist.

(b) Es existiert eine Zahl $p$ $(1 < p < 2)$, so daß für $(1/p) + (1/p') = 1$, $L^\lambda_{2\pi}$ Interpolationsraum von $(L^{p'}, L^p)$ und $L^{p'}$ dicht im $L_2$ ist.

Für den r.i. Banachraum $L^\lambda \subset L^1 + L^2$ ist $L^\lambda \in \mathcal{F}$ durch jede der folgenden Aussagen charakterisiert:

(a') $H \in [L^\lambda]$, und die einfachen Funktionen sind dicht in $L^\lambda$; hier bezeichnet H die Hilbert Transformierte.

(b') Es existiert eine Zahl $p \in (1,2)$, so daß $L^{\lambda}$ Interpolationsraum von $(L^2, L^p)$ ist.

J.E. GILBERT: Interpolation Space Theory and Harmonic Analysis

Results from harmonic analysis are used to give some positive and some negative results in interpolation space theory. Problems of interpolation between operator ideals, closed subspaces and quotient spaces are considered. Both real and complex interpolation methods can be used.

If $\mathbb{T}$ denotes the circle group and $Cv^p(\mathbb{T})$ the $L^p$-convolution operators, $1 \leq p \leq 2$, then

Theorem: (i) For each $\theta$, $0 < \theta < 1$, $(Cv^2(\mathbb{T}), Cv^1(\mathbb{T}))_{\theta,1} \leq Cv^p(\mathbb{T})$, $\theta = (2-p)/p$.

(ii) For each $\varepsilon > 0$

$$\bigcap_{1+\varepsilon \leq p \leq 2} Cv^p(\mathbb{T}) \not\subseteq (Cv^2(\mathbb{T}), Cv^1(\mathbb{T}))_{\theta,q}$$

for any choice of $\theta$, $0 < \theta < 1$, and $q$, $1 \leq q \leq \infty$.

The proof of (ii) uses $\Lambda(p)$-sets and Sidon sets. The theorem is used in two ways. In the first convolution operators are obtained from the space $\mathcal{L}(L^p(\mathbb{T}))$ of all bounded linear operators on $L^p(\mathbb{T})$ via the Marcinkiewicz projection P. The second method obtains convolution operators from absolutely p-summing operators $T: \mathcal{C}(\mathbb{T}) \to M(\mathbb{T})$. Deep tensor product techniques are used together with the Varopoulos theory.

R.P. GILBERT: Reproducing Kernels for Elliptic Systems

The kernel function method as presented in the book by Bergman and Schiffer has had an important impact on approximation theory and the numerical treatment of elliptic differential scalar equations. With this as motivation, we have developed a kernel function theory for elliptic systems of differential equations. We treat here the case of the self-adjoint system:

$$\Delta U = C(x,y)U$$

where U and C are complex valued $n \times n$-matrices and $\Delta$ is the Laplace operator in two dimensions $\Delta \equiv (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$. However, many of our results can be extended to more general self-adjoint systems, and also to higher order, and higher dimensional cases.

The system will be considered on a bounded regular region D in the Euclidean plane and for simplicity D, the boundary of D, will be assumed to be anylytic. In D the matrix C will be assumed to be positive definite and Hermetian: $C = C^*$, where C is the conjugate -transpose of C; also, C will be assumed to belong to $\mathscr{E}'(D)$.

E.GÖRLICH: On a conjecture of M.Golomb concerning asymptotically optimal approximation processes

Let $C_0^r$ be the class of functions $f \in C_{2\pi}$ for which $\|f^{(r)}\|_{C_{2\pi}} \leq 1$, $r \in \mathbb{N}$ = {naturals} and $A_0^\alpha$ the class of functions f of a complex variable $z = x + iy$ which are $2\pi$-periodic in x, real for $y = 0$, analytic for $|y| < \alpha$, continuous for $|y| \leq \alpha$, and satisfy $\sup \{|\mathrm{Re}\, f(z)|; -\infty < x < \infty, |y| \leq \alpha\} \leq 1$, $\alpha > 0$. In joint work with W.Dahmen (Aachen), the following conjectures of M. Golomb [Approximation of functions, ed. by H.L. Garabedian, Proceedings Conference Detroit 1964] are established.

(A) There does not exist a sequence of bounded linear polynomial operators (BLPO) $\{U_n\}$ which is asymptotically optimal (with respect to the rate of best approximation) for all classes $C_0^r$, $A_0^\alpha$, i.e. which satisfies $\|U_n f - f\|_{C_{2\pi}} = \mathcal{O}(n^{-r})$, $n \to \infty$, for all $f \in C_0^r$ and all $r \in \mathbb{N}$ as well as $\|U_n f - f\|_{C_{2\pi}} = \mathcal{O}(e^{-\alpha n})$, $n \to \infty$, for all $f \in A_0^\alpha$ and all $\alpha > 0$.

(B) There does not exist a sequence $\{U_n\}$ of BLPO which is optimal for all classes $C_0^r$, i.e. for which $\|U_n f - f\|_{C_{2\pi}} \leq \mu_r (n+1)^{-r}$ holds for all $f \in C_0^r$, $n, r \in \mathbb{N}$, ($\mu_r$ = Favard - Achieser - Krein constant).

For the proof, an inequality of Hardy - Littlewood - Sidon and a device of Marcinkiewicz are used. The results generalize the Nikolaev - Harsiladze - Lozinskiĭ theorem.

H.GÜNZLER: Linear functionals which are integrals

If L is a vector lattice of real - valued functions f on X such that with f also min (1,f) $\in$ L, if $\mathfrak{U}$ is any system of sets from X such that $U \cap V$ and $U \cup V \in \mathfrak{U}$ if $U,V \in \mathfrak{U}$, and if $\varphi$: L $\to$ reals R is linear and $\varphi(f) \geq 0$ if $f \geq 0$, then necessary and sufficient conditions on L, $\varphi$, $\mathfrak{U}$ can be given such that there is a "regular" additive non-negative set-function $\mu$ on the ring generated by $\mathfrak{U}$ for which all $f \in L$ are $\mu$-integrable with $\varphi(f) = \int f d\mu$; under additional assumptions $\mu$ is a measure. This representation theorem is applied to $L = C_0(X,R)$ = space of continuous $f: X \to R$ such that to each f there is a compact $K \subset X$ with $f \equiv 0$ outeside K, X arbitrary topological space, yielding a generalization of Riesz' theorem : All linear non-negative $\varphi : C_0(X,R) \to R$ are of the form $\int \ldots d\mu$, $\mu$ measure in X. Similar results hold for the space of all continuous, or all bounded continuous, functions on X. Abstract versions and extensions to $\varphi$ not $\geq 0$ are indicated.

F.HOLLAND: Square-summable positive-definite functions on the real line.

We are concerned with locally square-summable functions f on the real line R that are positive-definite in the sense that

$$\iint f(x-y)\, g(x)\, \bar{g}(y)\, dx\, dy \geq 0$$

for all $g \in L^2(R)$ with compact supports. It is shown that such functions are "nearly-bounded" and can be represented as Fourier transforms of positive measures. A complete description of the positive measures that generate them and the manner in which they are generated is presented. It turns out that each such f can be written as f = g + h, where g is pseudo almost periodic and

$$\int_{-T}^{T} |h(t)|^2 \, dt = o(T) \quad (T \to \infty).$$

R.A. HUNT: <u>Almost everywhere convergence of Fourier series</u>

Some recent results on the a.e. convergence of trigonometric and Walsh Fourier series will be presented. Emphasis will be given to the role played by the Hardy-Littlewood maximal function and its relation to the Hilbert transform. The a.e. convergence of certain subsequences and rearrangements of Fourier series will be considered.

J.P. KAHANE: <u>Les opérateurs de Toeplitz et la meilleure approximation</u>

Soit $e_n(t) = e^{2\pi int}$, P l'opérateur défini sur les distributions par

$$P\left(\sum_{-\infty}^{\infty} a_n e_n\right) = \sum_{0}^{\infty} a_n e_n,$$

$\varphi$ une fonction de la classe de Hardy $H^\infty$, $\varphi \sim \sum_0^\infty \hat{\varphi}_n e_n$, et $T_\varphi$ l'opérateur de $L^1$ dans $\mathcal{D}$ défini par

$$T_\varphi f = P(\bar{\varphi} f).$$

$T_\varphi$ (qu' on restreint habituellement à $H^2$) s'appelle un opérateur de Toeplitz.

On montre, entre autres, que $T_\varphi$ conserve les classes $\Lambda_\alpha$ ( $0 < \alpha < 1$)

et $\Lambda_*$ (de Zygmund).

Comme conséquence, la projection métrique de $L^1$ sur $H^1$ conserve aussi les classes $\Lambda_\alpha$ et $\Lambda_*$.

D.KERSHAW: Generalized Bernstein - Rogosinski Operators

The Bernstein - Rogosinski operator $R_n: C_{2\pi} \to C_{2\pi}$ is defined by

$$R_n f(x) = (1/2)\, a_0 + \sum_{k=1}^{n} \cos\, (k\pi/2n+1)\, [a_k \cos kx + b_k \sin kx]\,,$$

$n = 0, 1, \ldots$

where $\{a_k\}$ and $\{b_k\}$ are the Fourier coefficients of f. It is known that $R_n f \to f$ uniformly for all $f \in C_{2\pi}$. The operator $R_n$ is not monotone, however it can be decomposed into the sum of two monotone operators each of which satisfies the conditions of Korovkin' s theorem (i.e. if $\{T_n\}$ a sequence of monotone operators $C_{2\pi} \to C_{2\pi}$, and if $T_n f \to \lambda f$ uniformly for $f = 1, \cos, \sin$, then $T_n f \to \lambda f$ $\forall f \in C_{2\pi}$). This approach is used to invesitigate the convergence of the generalized Bernstein - Rogosinski operator $\tilde{R}_n$, defined by

$$\tilde{R}_n f(x) = (1/2)\, a_0 + \sum_{k=1}^{n} \beta_k^{(n)}\, [a_k \cos kx + b_k \sin kx]\,, \quad n = 0, 1, \ldots$$

where $\beta_k^{(n)}$ are real numbers. Sufficient conditions are stated to ensure that $\tilde{R}_n f \to f$ uniformly for all $f \in C_{2\pi}$. Some applications are given in the theory of the summability of Fourier series.

L.LEINDLER: On Approximation of Fourier Series

Among others the following theorem was presented: Suppose that $f(x) \in \mathrm{Lip}\, \gamma$ , $0 < \gamma < 1$, $\alpha > 1/2$, $p > 0$ and $p(1 - \alpha) < 1$.

Then

$$\{(1/A_n^{\beta})\sum_{\nu=0}^{n} A_{n-\nu}^{\beta-1}\,|\sigma_\nu^{\alpha-1}(x) - f(x)|^p\}^{1/p} = \begin{cases} O(n^{-\gamma}) & \text{if } p\gamma < 1 \\ O(n^{-\gamma}(\log n)^{1/p}) & \text{if } p\gamma = 1 \\ O(n^{-1/p}) & \text{if } p\gamma > 1 \end{cases}$$

for arbitrary $\beta > \max(0, p(1-\alpha))$.

It was shown that these estimations are best possible. Similar results can be proved if $f^{(r)}(x) \in \mathrm{Lip}\,\gamma$.

There is an open problem in this subject: Does $\sum |s_n(x) - f(x)|^p \leq K$, $0 < p < 1$, imply that $f(x) \in \mathrm{Lip}\,1$? Here $s_n(x)$ and $\sigma_n^\alpha(x)$ denote the partial sums and $(C,\alpha)$-means of Fourier expansion of $f(x)$, respectively.

D.LEVIATAN: On Müntz - Jackson approximation

Given a finite sequence $0 = \lambda_0 < \lambda_1 < \ldots < \lambda_n$, we seek to find how well do the combinations $\sum_{i=0}^{n} a_i x^{\lambda_i}$ approximate functions $f$ in $L_p[0,1]$, $1 \leq p < \infty$, or $C[0,1]$. Let $\omega_p(f,\delta) = \sup_{|h|\leq\delta} \|f(x+h) - f(x)\|_p$ and denote $\varepsilon_p = \sup_{\mathrm{Re}\, z=1} |B_p(z)/z|$, where

$$B_p(z) = \prod_{i=1}^{n} \frac{z - (\lambda_i + 1/p)}{z + (\lambda_i + 1/p)}.$$

Then we have Theorem. Let $1 \leq p \leq \infty$ (where $L_\infty[0,1]$ denotes $C[0,1]$). Then for every $f \in L_p[0,1]$ there exists $P(x) = \sum_{i=1}^{n} a_i x^{\lambda_i}$ such that $\|f - P\| \leq C\,\omega_p(f,\varepsilon_p)$, $C$ an absolute constant independent of $f$. Furthermore this is best possible in the sense that there exists an $f \in L_p[0,1]$ for which for any $P$, $\|f - P\|_p \geq D\omega_p(f,\varepsilon_p)$ for some fixed $D$.

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G.G. LORENTZ: Birkhoff interpolation problem and expansion of determinants

The determinant $D_E(X)$ of a Birkhoff interpolation problem, where $X = (x_1,\ldots,x_m)$ are the knots, and E is the incidence matrix, is expanded into powers of one of the knots, when another knot is possibly equal to zero. We determine: the lowest non-zero term (when the system of functions is arbitrary), the highest term (when the system is $x^k$, $k = 0,\ldots,n$). There are several applications: two types of coalescence theorems (two rows of E are coalesced); results that $D_E(X)$ is not identically zero if E satisfies some kind of Pólya condition ; theorem that E is singular if it has odd supported sequences, and only one row has more than one entry.

J.T. MARTI: Approximation mit Polynomoperatoren

Es sei X eine beschränkte Teilmenge eines Hilbertraumes E über $\mathbb{R}$ oder $\mathbb{C}$. Da es in E eine Familie $\{Q_a\}$ von orthogonalen Projektionen mit endlichdimensionalem Wertebereich gibt so, daß die Vereinigung der $Q_a(E)$ in E dicht ist, ist E ein $\pi$-Raum. C sei die Menge der kompakten und auf den beschränkten Mengen von E gleichmäßig schwach stetigen (nicht notwendigerweise linearen) Operatoren in E. Mit der durch die Halbnorm $\sup\{\|f(x)\| : x \in X\}$, $f \in C$, auf C definierten Topologie wird C ein lokalkonvexer topologischer Vektorraum. Ferner sei P die Algebra der Polynomoperatoren auf E, d.h. die Menge der Linearkombinationen von stetigen k-linearen Operatoren mit endlichdimensionalem Wertebereich in E (mit der auf der Hand liegenden Verallgemeinerung falls E nicht reell sondern komplex ist). Unter Verwendung der Eigenschaft, daß E ein $\pi$-Raum ist, wird schließlich gezeigt, daß die Algebra P der Polynomoperatoren dicht ist in C, d.h., daß sich jeder

Operator in C auf X gleichmäßig durch Polynomoperatoren approximieren läßt.

P.MASANI: The multiplication operator in $L_2$ over a localizable space and Bochner's theorem.

Let $\Gamma$ be a l.c.a. group with Haar measure m on the $\sigma$-ring $\mathcal{B}$ generated by the open sets. Let $\mathcal{B}_m = \{B : B \in \mathcal{B} \ \& \ m(B) < \infty\}$ and $\mathcal{A} = \{A : A \subseteq \Gamma \ \& \ \forall B \in \mathcal{B}_m, A \cap B \in \mathcal{B}\}$. Then $\mathcal{B} \subseteq \mathcal{A}$. Let $\mathbb{C}$ = complex number field, $\mathcal{L}_2 = L_2(\Gamma, \mathcal{B}, m; \mathbb{C})$ and $\hat{\Gamma}, \hat{\mathcal{B}}, \hat{m}, \hat{\mathcal{A}}, \hat{\mathcal{L}}_2$ be the duals of $\Gamma, \mathcal{B}, m, \mathcal{A}, \mathcal{L}_2$. For any $\mathcal{A}$-measurable $\emptyset$ on $\Gamma$ to $\mathbb{C}$, we consider the multiplication operator $M_\emptyset = \{(f; \emptyset \cdot f) : f \ \& \ \emptyset \cdot f \in \mathcal{L}_2\}$.

Theorem 1. (a) If $\emptyset$ is $\mathcal{A}$-measurable on $\Gamma$ to $\mathbb{C}$, then $M_\emptyset$ is a closed linear operator from $\mathcal{L}_2$ to $\mathcal{L}_2$ with e.d. domain such that $\forall \lambda \in \hat{\Gamma}$, $M_\lambda M_\emptyset = M_\emptyset M_\lambda$. (b) If T is a closed linear operator from $\mathcal{L}_2$ to $\mathcal{L}_2$ with e.d. domain such that $\forall \lambda \in \hat{\Gamma}$, $M_\lambda T = T M_\lambda$, then there exists an $\mathcal{A}$-measurable $\emptyset$ on $\Gamma$ to $\mathbb{C}$ such that $T = M_\emptyset$. (c) For $\sigma$-compact $\Gamma$, the $\emptyset$ in (b) is $\mathcal{B}$-measurable. This theorem yields, upon applying the Fourier-Plancherel transformation $\mathcal{F}$ on $\mathcal{L}_2$ onto $\hat{\mathcal{L}}_2$, the following extension of Bochner's Theorem.

Theorem 2. Let S be a closed linear operator from $\mathcal{L}_2$ to $\mathcal{L}_2$ with e.d. domain such that $\forall t \in \Gamma$, $\tau_t S = S \tau_t$, where $\tau_t$ = translation through t. Then there exists an $\hat{\mathcal{A}}$- measurable $\Psi$ on $\hat{\Gamma}$ to $\mathbb{C}$ such that $\mathcal{F} S \mathcal{F}^{-1} = M_\Psi$. For $\sigma$-compact $\hat{\Gamma}$, $\Psi$ is $\hat{\ }$-measurable.

W.MEYER-KÖNIG: Über das Verträglichkeitsproblem bei den Kreisverfahren der Limitierungstheorie

Es handelt sich um die Taylor-Verfahren $T_p$ und $S_p$, die Euler-Knopp-Verfahren $E_p$, und die Borel-Verfahren $B_p$ (als Matrixverfahren in der

Reihe-Reihe-Form, p reell). Es ist - sofern man sich auf reguläre Summierbarkeit beschränkt - bekannt, ob zwei dieser Verfahren verträglich sind oder nicht; siehe dazu Ishiguro/Meyer-König/Strasser, Math. Zeitschr. 120, 107-123 (1971). In der Zwischenzeit wurden durch Ishiguro und Meyer-König auch die restlichen Fälle, in denen also singuläre Summierbarkeit beteiligt ist, untersucht. Im Vortrag wird insbesondere die Verträglichkeitsfrage bei den Paaren $(T_p^S, T_q^S)$ und $(T_p^R, T_q^S)$ beantwortet (R: regulärer Fall, S: singulärer Fall). Das Hilfsmittel bei den Beweisen ist ein Kriterium von Eidelheit und Pólya für die Auflösbarkeit eines unendlichen linearen Gleichungssystems.

W.MLAK: Operator valued representations of function algebras

Let $A \subset C(X)$ be a function algebra. Denote by L(H) the algebra of all linear bounded operators on the complex Hilbert space H.The linear multiplicative and bounded mapping $T: A \to L(H)$ is called a representation of A. Using the abstract Riesz-brothers theorem on measures orthogonal to A, one proves in a dilation free way several decomposition theorems for T, the parts of the decomposition being absolutely continuos with respect to Gleason parts of A.With minor technical modifications one proves similar theorems related to the Bishop's decomposition of the spectrum of A. There are available several applications to spectral sets and polynomially bounded operators. Using the Wilken's results one proves the absolute continuity of pure representations related to spectral sets. It is shown that Foguel type decomposition, known for contradictions, holds true for polynomially bounded operators.

B. MUCKENHOUPT: <u>Weighted Norm Inequalities for Classical Operators</u>

The general problem is that of determining all pairs of non-negative weight functions, U and V, such that $\int |Sf(x)|^p\, U(x)\, dx \leq C\int |Tf(x)|^p\, V(x)\, dx$ where S and T are given classical operators and C is a constant independent of f. Typical operators used for S and T are the identity operator, Hardy-Littlewood maximal function, Hilbert transform, fractional integrals and Littlewood - Paley operators. Most of the results currently known are in the case when T is the identity and U = V. The results when T is not the identity are interesting since the size of the set of weight functions gives a measure of how similar or dissimilar the operators S and T are. The results when U is not the same as V are surprisingly different from those when U equals V and are important in obtaining weighted mean convergence results for partial sums and Cesàro sums of various orthogonal series. Various partial results will be described.

A.OSTROWSKI: <u>Partielle Differentiation singulärer Integrale</u>

Wird ein (mehrdimensionales) singuläres Integral durch

$$A_\alpha(f)(P_x) := \int_E f(P_\xi)K(P_x,P_\xi)dP_\xi$$

definiert, so gilt definitionsgemäß

$$A_\alpha(f)(P_x) \to f(P_x) \qquad (\alpha \to \infty).$$

Das Problem ist, ein singuläres Integral zu konstruieren, für das auch unter minimalen Annahmen über die Existenz von $f'_{x_1}(P_x)$, die Formel gilt

$$\frac{\partial}{\partial x_1} A_\alpha(f)(P_x) \to \frac{\partial}{\partial x_1} f(P_x) \qquad (\alpha \to \infty).$$

Ein so konstruiertes singuläres Integral kann zum Nachweis von "Klammerformeln" in der Theorie der Differentialoperatoren unter "natürlichen Differenzierbarkeitsvoraussetzungen" verwendet werden.

A.PEŁCZYNSKI: <u>On approximation problem and bases in Banach spaces</u>

The following properties of Banach spaces have been discussed:

(a) The existence of a total, fundamental and bounded biorthogonal system,

(b) the approximation property,

(c) the bounded approximation property,

(d) the existence of a basis.

Recent results of Enflo, A.M.Davie, Kwapien, Figiel and Johnson, Johnson, Pełczynski and Ovsepjan have been reviewed.

Contrary to Enflo's example of a Banach space for which the approximation property fails, every separable Banach space admits a biorthogonal system satisfying (a)(Ovsepjan-Pełczynski, to appear in Studia Math.).

G.M.PETERSEN: <u>Topologie of summability fields and matrix singularities</u>

Let $\{A^i\}$, $(1 \leqslant i \leqslant N)$ be a finite set of regular summability matrices and $\mathcal{A}^i$ $(1 \leqslant i \leqslant N)$ the bounded sequences limited by the matrices. The matrices are said to have a singularity $S_1(s)$, if for every $\varepsilon$, $\varepsilon > 0$, there exists $r(\varepsilon)$, $M(\varepsilon)$ and $x^i(\varepsilon)$, $x^i(\varepsilon) \in \mathcal{A}^i$ $(1 \leqslant i \leqslant N)$ such that

$$\sum_{i=1}^{N} ||x^i(\varepsilon)|| \leq M(\varepsilon), \quad |\sum_{i=1}^{N} x_n^i (\varepsilon) - S_n| < \varepsilon, \; n > r(\varepsilon)$$

and

$$\sum_{i=1}^{N} | A - \lim x^i(\varepsilon) | < \varepsilon,$$

where

$$||x|| = \lim\sup |x_n|.$$

The space $\mathcal{L} = \sum_{i=1}^{N} \mathcal{A}^i$ is studied as a topological space when the matrices leave a singularity $S_1(s)$.

G. DA PRATO: Sums of infinitesimal generator of semigroups

Let X be a Banach space, let A and B be linear closed operators in X. We give conditions for the solvability of the equation

$$\lambda x - Ax - Bx = y$$

and some application to abstract evolution equations.

P.O.RUNCK: Über 1-positive lineare Operatoren

Es wird über Ergebnisse berichtet, die Herr Stadler, Düsseldorf, über 1-positive lineare Operatoren 1973/74 erzielt hat. Man vergleiche hierzu das von Prof. G.G.Lorentz auf der Approximationstheorie-tagung 1971 in Oberwolfach gestellte Problem (ISNM Vol. 20, S.497, Nr. 6). Ein linearer Operator $L: X \to X$, $X \subset C[a,b]$ heißt 1-positiv:$\Leftrightarrow$ jede monoton wachsende Funktion $f \in X$ wird in eine monoton wachsende Funktion $Lf \in X$ abgebildet. Untersucht werden Folgen solcher Operatoren in den Räumen $Z^p := \{f \in C[a,b] \,|\, f$ absolut stetig, $f' \in L^p[a,b]\}$, $1 \leq p < \infty$, mit der Norm $\|f\|_{Z^p} := \max\{f(a), \|f'\|_{L^p[a,b]}\}$, und $C^1[a,b]$. Analog zur bekannten Theorie positiver Operatoren wird gezeigt, daß für eine Folge linearer 1-positiver Operatoren $(L_n)_{n\in\mathbb{N}}$ $L_n: Z^p \to Z^p$ die Bedingungen I i) $\|L_n\|_{Z^p} \leq C$, $C > 0$, $n \in \mathbb{N}$, ii) $(L_n f)^{(a)} \underset{n\to\infty}{\to} \hat{f}(a)$ II) $(\forall f \in \{\hat{t}, \hat{t}^2, \hat{t}^3\})$ $\|L_n f - f\|_{Z^p} \underset{n\to\infty}{\to} 0$ mit der Bedingung $(\forall f \in Z^p)$ $\|L_n f - f\|_{Z^p} \underset{n\to\infty}{\to} 0$ gleichwertig sind. Ähnliche Aussagen gelten für $C^1[a,b]$. Es werden weiter Beispiele angegeben und Fragen der Fortsetzung von $Z^1$ auf $C[a,b]$ und Fragen der maximalen Konvergenzordnung behandelt.

Y. SAGHER: Lions-Peetre interpolation and norm inequalities on Fourier coefficients

A survey of mutual influences of the two circles of ideas. Norm inequalities for Fourier coefficients serving as a source of examples simple enough to be well understood, yet sufficiently rich to motivate interpolation theory.

1. Interpolation of $\rho$-normed semi-groups.
2. Relation between intersection and interpolation.

Examples of applications of interpolation results to norm inequalities of Fourier coefficients.

1. Using Gilbert's characterization of $(X,X_W)_{\theta,q}$ we prove

$$\sum_{k=1}^{\infty} k^{-\theta} \sup_{k\leq n} \frac{|a_n|}{n} \leq C_\theta \int_0^\pi x^{-\theta} f(x)dx \quad (a_n = \int_0^\pi (1-\cos nx)f(x)dx \quad 0<\theta<1).$$

2. (Joint work with Riviere) Using the recent interpolation result $(H^1, C_W)_{\theta,q} = L(p,q)$, we prove $\|\{\hat{f}(n)\}\|_{l^{2,2}(l^{p',p})} \leq c_p \|f\|_p$, $1 < p < 2$. $l^{2,2}(l^{p',p})$ are the mixed norm spaces for binary blocks.

E.SCHOCK: Approximation Hölder-stetiger Funktionen

Am Beispiel der Approximation Hölder-stetiger Funktionen $f\colon \Omega \to \mathbb{R}^k$, $\Omega$ kompakte Teilmenge von $\mathbb{R}^m$, wird demonstriert, daß es mitunter von Vorteil ist, Approximationstheorie in nicht normierbaren lokalkonvexen Räumen zu betreiben.

A.SCHÖNHAGE: Optimal quadrature formulae for periodic functions

We consider $\tilde{H}_\beta := \{f \mid 2\pi\text{-periodic, holomorphic in a strip of width } 2\beta,$ real on $\mathbb{R}$ with $|f|_\beta := \sup_{|y|<\beta} |\mathrm{Re}\, f(x+iy)| < \infty\}$ and $\tilde{H}^1_\beta = \{f \in \tilde{H}_\beta \mid |f|_\beta \leq 1\}$ as a compact subset of $C_{2\pi}$. For fixed n, $x_j = j(2\pi/n)$ the norms of $R_n$ and $R_\alpha$ are discussed, where

$$R_\alpha(f) = \frac{1}{2\pi}\int_0^{2\pi} f(t)dt - \sum_{\nu=1}^{n} \alpha_\nu f(x_\nu),\ \alpha = (\alpha_1,\ldots,\alpha_n)$$

and $R_n$ denotes the interpolatory quadrature $\alpha_\nu = 1/n$. On $C_{2\pi}$ we have $|R_n f| \leq 2\,\tilde{E}_{n-1}(f)$, and here the factor 2 is optimal. On $H^1_\beta$ however, the optimal $R_\alpha$ is obtained by $\alpha_j = \alpha/n$, where $\alpha$ minimizes $\int_0^{2\pi} |1 - \alpha\, K_{n\beta}(t)|dt$, with $K_\tau(u) = 1 + 2\sum_{m=1}^{\infty} \frac{\cos\,(m\,u)}{\cosh\,(m\,\tau)}$. In general $\alpha \neq 1$, but asymptotically $|R_n|_{H^1_\beta} \sim \tilde{E}_{n-1}(\tilde{H}^1_\beta) \sim (8/\pi)\, e^{-n\beta}$ for $n\beta \to \infty$.

L.L.SCHUMAKER: Local Spline Approximation Methods

Local spline approximation methods based on B-splines and defined on classes of continuous or integrable functions are considered. The methods are designed to reproduce polynomials so that they provide high order (as good as best spline approximation) approximations to smooth functions. Multidimensional analogs are also considered for approximation of functions in Sobolev spaces on arbitrary sets in higher dimensions.

R.SHARPLEY: Interpolation theorems for $M_\phi$ spaces

An obvious problem in interpolation theory is to find necessary and sufficient conditions for interpolation properties to hold among spaces of the same type. For example, the Riesz-Thorin theorem characterizes those pairs of Lebesgue spaces $(L^p, L^q)$ so that each operator which is bounded from $L^{p_1}$ to $L^{q_1}$ and $L^{p_2}$ to $L^{q_2}$ has a unique extension to a bounded operator from $L^p$ to $L^q$. Lorentz and Shimogaki have characterized interpolation for $\Lambda_\phi$ spaces in terms of a simple expression involving the functions $\phi$ of the six spaces in question. Here we provide with a straight-forward proof the solution for $M_\phi$ spaces (dual to $\Lambda_\phi$). These spaces appear strategically in classical interpolation theory (Marcinkiewicz, Stein-Weiss) as well as in the lattice structure of the class of rearrangement-invariant Banach function spaces.

I. SUCIU: Functional models for operator valued maps on C(X)

Let X be a compact space, H a complex Hilbert space and $\mu$ a positive map from C(X) into B(H) such that $\mu(1) = I$. According to the Naimark dilation theorem from H into K and a representation $\pi$ of C(X) in B(K) such that for any $f \in C(X)$ we have $\mu(f) = V^*\pi(f)V$. Let A be a function algebra on X and m a representing measure for A. Suppose that $\mu/A$ is a representation of A in B(H). Define $K_+ = \bigvee_{f \in A} \pi_+(f)VH$ and $\pi_+(f) = \pi(f)|K_+$. Then $\pi_+$ is a representation of A on $K_+$. Some special conditions on A, m and $\pi_+$ permit us to give a functional model for $\mu$, following the way in which B. Sz.-Nagy and C. Foiaş obtained the functional model for contractions.

J.SZABADOS: Convergence and saturation problems of discrete linear operators

Convergence and saturation problems for convolution-type linear operators are well known and widely investigated. The use of such operators for practical purposes is rather restricted by the fact that their explicite evaluation involves the calculation of integrals. Therefore it is interesting to consider the discrete version of convolution operators, i.e. to replace them by properly chosen quadrature sums. Without assuming the positivity of the kernel, we prove a general result for the rate of convergence of convolution and discrete operators. Furtheron, we present a relation between the convergence of convolution- and discrete-type operators. This implies a general rule for solving saturation problems of discrete linear operators. When this rule does not apply special considerations are needed. These will be shown through the detailed investigation of the various discrete versions of the Fejér operator. Finally, some partial results ($\sigma$-theorems of the saturation) and open problems are stated.

B.SZ.-NAGY and C.FOIAŞ: Injection of shifts

Let S be the (simple) unilateral shift operator, defined on the Hardy-Hilbert space $H^2$ for the unit disc by $(Sx)(\lambda) = \lambda\, x(\lambda)$, and for any value $\alpha = 1, 2, \dots, \infty (= \aleph_0)$ let $S^{(\alpha)}$ be the unilateral shift of multiplicity $\alpha$, i.e.

$$S^{(\alpha)} = S \oplus S \oplus \dots \quad (\alpha \text{ terms}).$$

By a quasi-affinity from a Hilbert space H into a Hilbert space H' we mean a (continuous, linear) operator $X: H \to H'$ such that X is one-to-one and has its range dense in H'.

Lemma. For any complex number c, $0 < |c| < 1$, and any value of $\alpha$, there exists a quasi affinity X from the space of $S^{(\alpha)}$ into the space of S such that

$$(cS)\cdot X = X\cdot S^{(\alpha)}.$$

Hence we can also derive that for any strict contraction T of an infinite dimensional Hilbert space (i.e. $\|T\| < 1$), which has a cyclic vector, there exists a quasi-affinity X such that

$$TX = X\, S^{(\alpha)}.$$

As a further consequence we get that there exists, for any choice of $\alpha,\beta$ (=1, 2,..., $\infty$), a quasi-affinity X such that

$$S^{(\alpha)*}X = X\, S^{(\beta)}.$$

K.TANDORI: Über die Konvergenz der Funktionenreihen

Es sei $(X,A,\mu)$ ein Maßraum und $\Omega$ eine Klasse der Funktionenfolgen $\{f_n(x)\}_1^\infty$ wobei $f_n \in L^p_\mu$, $\|f_n\|_{L^p_\mu} \leq 1$ $(n = 1, 2,\dots; 1\leq p < \infty)$. Für eine Folge von Zahlen $\{a_n\}_{n=1}^\infty$ wird

$$\|\{a_n\}, \Omega\|_p = \sup_{\{f_n\}\in\Omega} \left\{ \int_X \sup_{\substack{i,j \\ 1\leq i\leq j<\infty}} |a_i f_i(x) + \dots + a_j f_j(x)|^p \, d\mu\right\}^{1/p}$$

gesetzt. Mit $M_p(\Omega)$ wird die Klasse der Folgen $\{a_n\}_{n=1}^{\infty}$, mit $\| \{a_n\};\Omega \|_p < \infty$ bezeichnet. Man sagt, daß der Raum $M_p(\Omega)$ die Eigenschaft $K_p$ besitzt, wenn aus $\{a_n\} \in M_p(\Omega)$ folgt, daß die Reihe $\sum a_n f_n(x)$ bei jeder Folge $\{f_n(x)\} \in \Omega$ $\mu$-fast überall konvergiert.

Verschiedene Bedingungen werden angegeben dafür, daß $M_p(\Omega)$ diese Eigenschaft besitzt.

H.F.TROTTER: Approximation of semigroups of operators

A survey of results on approximation of semi-groups of operators. Let $T_n^t$ be a sequence of semi-groups with infinitesimal generators $A_n$. The central question concerns conditions under which convergence of the $A_n$ implies convergence of the $T_n^t$. Some related questions about perturbations of infinitesimal generators will also be considered.

U.WESTPHAL: Über gebrochene Potenzen von Operatoren

Für den infinitesimalen Erzeuger A einer gleichmäßig beschränkten Halbgruppe $\{T(t);\ t \geq 0\}$ von Operatoren der Klasse $(C_0)$ auf einem Banachraum X wird die gebrochene Potenz $(-A)^{\alpha}$ $(\alpha > 0)$ mit Definitionsbereich $D((-A)^{\alpha})$ durch

$$(-A)^{\alpha} f = \underset{t \to 0+}{\text{s-lim}}\ t^{-\alpha} [I - T(t)]^{\alpha} f$$

definiert, wobei $[I - T(t)]^{\alpha} := \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} T(jt)$ ist. Ausgehend von dieser Definition, die ihren Ursprung in einer Arbeit über gebrochene Differentiation von Liouville (1832) hat, wird mit Hilfe der Laplacetransformation eine Methode angegeben, wünschenswerte Eigenschaften der Potenzoperatoren zu beweisen, wie

$\overline{D((-A)^{\alpha})} = X$, $D((-A)^{\alpha}) \subset D((-A)^{\beta})$ $(\beta < \alpha)$, Abgeschlossenheit von $(-A)^{\alpha}$, Potenzregel $(-A)^{\alpha}(-A)^{\beta} = (-A)^{\alpha+\beta}$. Folgende Formel (mit Verallgemeinerungen) liegt dabei zugrunde:

$$t^{-\alpha}[I - T(t)]^{\alpha} f = (-A)^{\alpha} \int_0^{\infty} \frac{1}{t}\, p_{\alpha}\left(\frac{u}{t}\right) T(u)\, f\, du \qquad (f \in X,\ t > 0),$$

wobei $p_{\alpha}(u) := [\Gamma(\alpha)]^{-1} \sum_{0 \leq j < u} (-1)^j \binom{\alpha}{j} (u - j)^{\alpha-1}$ eine skalarwertige Funktion aus $L^1(0,\infty)$ ist.

K.ZELLER (und W.BEEKMANN): Positive Operatoren in der Limitierung

Positive Operatoren spielen in der Numerik und in der Approximationstheorie eine große Rolle. Auch in der Limitierung hat man Positivitätseigenschaften ausgiebig verwendet. Allerdings wurden die Grundlagen nicht so stark herausgearbeitet; bei manchen Verallgemeinerungen wurden sie sogar verdeckt. Der Vortrag stellt daher einige Positivitätsprinzipien zusammen, wobei insbesondere folgende Themenkreise erörtert werden:

Permanenzfragen, Mercersätze, Abschnittspositivität, Vergleichssätze, Summierbarkeitsfaktoren.

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