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ORDERS AND THEIR APPLICATIONS
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Hereditary Orders

Theorem (joint with E.L. Green). A (classical) order is hereditary if and only if all of its artinian factor rings are of finite representation type.

The proof depends on an independently interesting theorem on artin algebras.

W. H. Gustafson
Lubbock, Texas

Grothendieck Groups of Dihedral and Quaternion Group Rings

Let R be a ring with 1, G a finite group, ~~Methods~~ $G_0(RG)$ the Grothendieck group of finitely-generated RG-modules relative to short-exact sequences. Methods used by H. Lenstra for the case of Abelian G are applied to compute $G_0(\mathbb{Z}G)$ for certain non-Abelian groups, e.g., the dihedral groups D_{2n}, the quaternion groups Q_{4m}, etc. For example, $\tilde{G}_0(\mathbb{Z}D_{2n}) \cong \bigoplus_{\substack{d \mid n \\ d > 2}} \mathrm{Pic}(\mathbb{Z}[\zeta_d, \frac{1}{d}]_+)$, where + denotes the maximal real subring. A similar formula is obtained for quaternion groups, by more elaborate calculations. Finally, let G be a finite group in which element has prime-power order, and let $\mathfrak{M} \subseteq \mathbb{Q}G$ be a maximal order containing $\mathbb{Z}G$. Then the transfer map $G_0(\mathfrak{M}) \to G_0(\mathbb{Z}G)$ is an isomorphism. The proof combines Lenstra's techniques with induction techniques. Similar calculations for the functors G_n, $n>0$, are possible.

David L. Webb

Auslander-Reiten quivers of local Gorenstein orders of finite type

We derive necessary conditions for the configurations of Gorenstein orders Λ of finite lattice type. In particular these conditions give rise to a complete list of possible finite Auslander-Reiten quivers of local Gorenstein orders. Concrete examples show that these conditions are also suffitient in this case.
In general if Λ is an R-order in $A = \prod_{i=1}^{s} (D_i)_{n_i}$, D_i skewfields, then $s, n_1, \dots, n_s$ are already determined by the combinatorial structure of the Auslander-Reiten quiver of Λ (Λ of finite type). Then for a local Gorenstein order Λ of finite type all n_i's are 1 and the valuation of all arrows in the AR-quiver is $(1,1)$ if and only if Λ has the "same" AR-quiver as a simple plane curve singularity over $\mathbb{C}$.

Alfred Wiedemann (Stuttgart)

The Schur Group of a Commutative Ring

We define the *Schur subgroup* $S(R)$ (for a commutative ring R with identity), of the Brauer group $B(R)$, to consist of those classes having a representative A such that there exists a finite group G and an R-algebra epimorphism $f: RG \twoheadrightarrow A$. If R is a commutative ring of non-zero characteristic then $S(R) = (0)$. On the other hand, any finite Abelian group is the Schur group of a commutative ring which is finitely generated as an algebra over the rational integers. We generalize several standard facts about the Schur group of a field to commutative rings with finitely many idempotents. We also investigate two subgroups of $S(R)$, one generated by cyclotomic algebras and the other by homomorphic images of separable group algebras.

Richard A. Mollin
(joint work with Frank De Meyer)

<u>Zeta and L-functions of orders</u>. Irving Reiner (joint with Colin Bushnell)

Let Λ be an R-order in a f.d. semisimple K-algebra A, where $R = \text{alg. int.}\{K\}$, $\dim_{\mathbb{Q}} K$ finite. L. Solomon defined a zeta function $\zeta_\Lambda(s) = \sum_X (\mathfrak{N}X)^{-s}$, where X ranges over all left ideals of Λ of finite norm $\mathfrak{N}X = (\Lambda : X)$. Let $g(\Lambda)$ = genus of Λ = set of locally free ideals X, and let $Cl\,\Lambda$ be the locally free class group of Λ, whose elements are stable isomorphism classes $[M]$, $M \in g(\Lambda)$.
For each linear character $\chi : Cl\,\Lambda \to \mathbb{C}^\times$, we introduce an L-function

$$L_\Lambda(s,\chi) = \sum_{X \in g(\Lambda)} \chi(X)(\mathfrak{N}X)^{-s}, \quad \mathrm{Re}\, s > 1.$$

There are Euler products

$$\zeta_\Lambda(s) = \prod_P \zeta_{\Lambda_P}(s), \quad L_\Lambda(s,\chi) = \prod_P L_{\Lambda_P}(s,\chi_P),$$

where P ranges over all maximal ideals of R, and $\chi_P : A_P^\times \to \mathbb{C}^\times$.

In the local case, $L_{\Lambda_P}(s,\chi_P)$ can be expressed as a zeta integral $\int_{A_P^\times} \Phi(x)\|x\|^s \chi(x)\, d^\times x$ over the locally compact group $A_P^\times$, with Φ a locally constant function of compact support, $\|x\|$ = P-adic absolute value, $d^\times x$ = Haar measure.
A key theorem shows the existence of a "common denominator" $L_{A_P}(s,\chi_P)$ for all zeta integrals (as Φ varies). When $\chi = 1$ on $(\Lambda_P')^\times$, where Λ_P' = maximal order, $L_A(s,\chi_P) = L_{\Lambda_P'}(s,\chi_P)$.

The above implies that $\zeta_{\Lambda_P}(s)/\zeta_{\Lambda_P'}(s) \in \mathbb{Z}[p^{-s}]$, with analogous results for L-functions. Combining these facts with the Euler products, we obtain analytic continuation of zeta and L-functions in the global case, as well as their behavior at $s=1$.
Consequences: 1) Given a left ideal M of Λ, the number of $X \subset \Lambda$ with $X \cong M$ and $\mathfrak{N}X \le T$ is asymptotic (as $T \to \infty$) to $\frac{1}{(n-1)!}\frac{(M^{-1}:\Lambda)}{(\Lambda':M)}(\Lambda'^\times : \mathrm{Aut}_\Lambda M)\, b_{\Lambda'}\, T(\log T)^{n-1}$, where n = number of simple components of A, Λ' = maximal order, and $M^{-1} = \{x \in A : Mx \subset \Lambda\}$.
Here, $b_{\Lambda'} > 0$ depends on Λ' but not on M or Λ.

2) Given a class $c \in Cl\,\Lambda$ and an integer $k \ge 1$, the number of $X \in g(\Lambda)$ with $X \in c$, composition length $\ell(\Lambda/X) = k$, $\mathfrak{N}X \le T$ is asymptotic to $\frac{N(k,c)}{(k-1)!\,h_\Lambda}\frac{T}{\log T}(\log\log T)^{k-1}$ as $T \to \infty$, where $h_\Lambda = |Cl\,\Lambda|$, and $N(k,c)$ = non-negative integer. (Further, $N(k,c) = 0$ if and only if there are <u>no</u> $X \in c$ with $\ell(\Lambda/X) = k$.)

3) Given $M \in g(\Lambda)$, let $M_1, \dots, M_h$ represent the isomorphism classes in the stable class $[M]$. Then $\sum_i (\Lambda^\times : \mathrm{Aut}_\Lambda M_i)$ is an explicitly computable constant, independent of the choice of $M \in g(\Lambda)$.

Isomorphisms of p-adic group rings I, II

Klaus Roggenkamp and Leonard Scott

Let G be a finite p-group. Our main result is that there is only one conjugacy class of subgroups of order $|G|$ in the group of normalized units (augmentation 1) of the p-adic group ring $\hat{\mathbb{Z}}_p$. As a consequence we obtain a positive answer to the isomorphism problem for group rings over $\mathbb{Z}$ of finite nilpotent groups, as well as any extension of a finite abelian group by a finite p-group. In the nilpotent case a conjecture of Zassenhaus, that the isomorphism may be achieved by a group automorphism followed by conjugation in the group ring over $\mathbb{Q}$, is verified.

The main result holds also with $\hat{\mathbb{Z}}_p$ replaced by $\mathbb{Z}_p$ or $\mathbb{Z}_\pi$ where π is a finite set of primes containing p. The consequences above also hold with $\mathbb{Z}$ replaced by $\mathbb{Z}_\pi$, if π contains each prime divisor of the group order.

For such a $\mathbb{Z}_\pi$ with G finite nilpotent, we have $\operatorname{Picent} \mathbb{Z}_\pi G = \prod_{i,j} R_{ij} P_i \supseteq \prod_i \operatorname{Outcent}(P_i)^{n_i}$ where P_i is a Sylow p-subgroup and R_{ij} is the center of a component of $\mathbb{Z}_\pi P_i'$, where $G = P_i \times P_i'$. As a consequence we show that the analogue of our main result for $\mathbb{Z}_\pi G$ need not hold, and that there are non-isomorphic groups E, E', extensions of G by abelian groups A, A' (for some G), with isomorphic "small" group rings

$$\mathbb{Z}_\pi E / I(A) I(E) \simeq \mathbb{Z}_\pi E' / I(A) I(E')$$

Leonard Scott
Charlottesville

Klaus Roggenkamp
Stuttgart

K-theory of group-rings of finite groups over maximal orders in division algebras – A.O. Kuku (Ibadan)

Let π be a finite group, R a regular ring, $\mathcal{P}(R)$ the category of finitely generated projective R-modules, $[\pi, \mathcal{P}(R)]$ the category of π-representations in $\mathcal{P}(R)$, and $K_0(\pi, \mathcal{P}(R))$ the Grothendieck group of $[\pi, \mathcal{P}(R)]$. Swan proved that if R is a commutative semilocal Dedekind domain with field of quotients F, then the canonical map $K_0(\pi, \mathcal{P}(R)) \longrightarrow K_0(\pi, \mathcal{P}(F))$ is an isomorphism. The question arises if this is true if R is non-commutative. We prove in this paper that if R is a p-adic field with quotient field F and A a maximal R-order in a central division F-algebra D, then $K_0(\pi, \mathcal{P}(A) \longrightarrow K_0(\pi, \mathcal{P}(D))$ is not injective.

Since $K_n(\pi, \mathcal{P}(A)) \simeq G_n(A\pi)$ $\forall n \geq 0$, and $A\pi$ is an ~~separable~~ R-order in the separable F-algebra $D\pi$, we prove in general that if R is the ring of integers in a number field F, Λ any R-order in a separable F-algebra Σ, then (i) $\forall n \geq 1$, $G_n(\Lambda)$ is finitely generated (ii) $SG_{2n}(\Lambda) = 0$, $SG_{2n-1}(\Lambda)$ is finite (iii). $G_{2n-1}(\Lambda_{\mathfrak{p}})$ is finitely generated if $\mathfrak{p}$ is a maximal ideal of R and ~~$\Lambda_{\mathfrak{p}}$~~ is $\Lambda_{\mathfrak{p}} = R_{\mathfrak{p}} \otimes_R \Lambda$. We then deduce that ~~for~~ if A is a maximal R-order in a central division algebra D over F, then

($\forall n \geq 1$ (i) $K_n(\pi, \mathcal{P}(A))$ is finitely generated ~~$\forall n \geq 1$~~.

(ii) $K_{2n-1}(\pi, \mathcal{P}(A_{\mathfrak{p}}))$ is finitely generated

(iii) $SK_{2n}(\pi, \mathcal{P}(A)) = SK_{2n}(\pi, \mathcal{P}(A_{\mathfrak{p}}) = SK_{2n}(\pi, \mathcal{P}(A)) = 0$

(iv) $SK_{2n-1}(\pi, \mathcal{P}(A_{\mathfrak{p}}), SK_{2n-1}(\pi, \mathcal{P}(\hat{A}_{\mathfrak{p}}))$ are finite groups of order relatively prime to the rational prime p lying below $\mathfrak{p}$

(iii) $SK_{2n-1}(\pi, \mathcal{P}(A))$ is ~~fi~~ a finite group.

Almost split sequences and Isolated singularities

Let R be a fixed complete regular local ring. An R-algebra Λ which is a finitely generated free R-module is called an isolated singularity if for all nonmaximal prime ideals $\mathfrak{p} \subset R$ we have that $\mathrm{gl.dim}\, \Lambda_{\mathfrak{p}} = \mathrm{gl.dim}\, R_{\mathfrak{p}} = \dim R_{\mathfrak{p}}$. Suppose Λ is an R-algebra which is finitely generated free R-module and let $\underline{\underline{P}}_R(\Lambda)$ be the category of Λ-modules which are free R-modules. Then we have the following theorem: Λ is an isolated singularity if and only if $\underline{\underline{P}}_R(\Lambda)$ has almost split sequences.

M. Auslander

Galois modules and elliptic functions

Let K be an imaginary quadratic number field in which 2 splits. We let $\mathcal{O}_K$ denote the ring of integers of K and we fix $\pi \in 1 + 4\mathcal{O}_K$ such that $(\pi, \bar{\pi}) = 1$. For $\alpha \in \mathcal{O}_K$ we let $K(\alpha)$ denote the ray classfield of K with conductor $\alpha\mathcal{O}_K$. We then construct an elliptic function f and a $4\pi^2$ division point for the complex torus $\mathbb{C}/\mathcal{O}_K$ with the property the $f(\alpha)$ generates the ring of integers of $K(4\pi^2)$ as a Galois module over the associated order for the extension $K(4\pi^2)/K(4\pi)$.

Martin Taylor

Trinity College.
Cambridge - UK

Representations of orders, and module valuations

We show that representations of orders can be viewed as functions on a module with values in a modular lattice on which a certain algebra operates. These functions satisfy the formal properties of an ultrametric norm on a vectorspace and are therefore called 'module valuations'. We demonstrate that module valuations give rise to an equivalent approach to the representation theory of orders provided that the ground ring R is a complete discrete valuation ring, and that the Jordan–Zassenhaus theorem holds. As for the global case we remark that an equivalence between representations and module valuations still holds if a restricted class of valuations is considered.

In the case of tiled orders, the range Γ of the corresponding valuations can be described easily, and the operation on Γ mentioned above reduces to a natural operation of the infinite cyclic group on Γ. Moreover, the domain of such a valuation is a vector space in this case.

As an example for the application of valuations in the tiled order case, we show how irreducible representations can be split off by means of a valuation theoretic criterion which makes use of the concept of a dual valuation.

Wolfgang Rump (Eichstätt)

The Merkurjev - Suslin Theorem

Let n be a positive integer, F a field so that $1/n \in F$. The Merkurjev-Suslin Theorem states that the Galois symbol

$$\alpha_F : K_2(F) / n K_2(F) \longrightarrow H^2(F, \mu_n^{\otimes 2})$$

is an isomorphism.

See Math. USSR Izvestiya Vol 21, 1983, 307-340. Discussed was Merkurjev's more elementary proof of this theorem, based on Hilbert 90 for K_2 and specialization arguments similar to those used in his original proof for the case n = 2.

Wilberd van der Kallen

Utrecht

<u>Applications of ring theory to number theoretic algorithms.</u> – H. W. Lenstra, Jr (Amsterdam)

In this lecture it is shown how Galois theory for finite rings underlies most practical primality testing methods. Let A be a Galois extension of $\mathbb{Z}/n\mathbb{Z}$ with group $\mathcal{G}$; i.e., A is a $\mathbb{Z}/n\mathbb{Z}$-algebra, commutative, that is f.g. free as a $\mathbb{Z}/n\mathbb{Z}$-module, and $A \otimes A \to \prod_{\sigma \in \mathcal{G}} A$, $a \otimes b \mapsto (a\sigma(b))_{\sigma \in \mathcal{G}}$ is an isomorphism. Assume $\mathcal{G}$ is abelian. Then for every prime r dividing n there is a unique $\varphi_r \in \mathcal{G}$ (the <u>Artin symbol</u>) with $\forall x \in A$: $\varphi_r(x) \equiv x^r \bmod rA$. Extend this definition to <u>all</u> $r|n$ by $\varphi_{rr'} = \varphi_r \varphi_{r'}$. The decomposition group $D \subset \mathcal{G}$ is defined to be the subgroup of $\mathcal{G}$ generated by all φ_r, $r|n$. Clearly $\langle \varphi_n \rangle \subset D$, with equality if n is prime. Many primality testing methods can be interpreted as attempting to show that $\langle \varphi_n \rangle = D$. For example, if there is a ring homomorphism $A^{\langle \varphi_n \rangle} \to \mathbb{Z}/n\mathbb{Z}$ (mapping 1 to 1) then we must have $D = \langle \varphi_n \rangle$. Applying this to $A = \mathbb{Z}[\zeta_s]/(n)$ (cyclotomic, with $\gcd(s,n)=1$) with $\mathcal{G} \cong (\mathbb{Z}/s\mathbb{Z})^*$ this leads, if n passes certain tests, to the information that $\forall r|n: \exists i \;\; r \equiv n^i \bmod s$. If s is large and $\#\langle n \bmod s \rangle$ is small this can be used to check whether n is prime. The best methods used nowadays rely on the same ideas but are somewhat more involved. For $n \leq 10^{100}$ one can use $s = 2 \cdot 5040 \cdot \prod_{q \text{ prime},\, q-1|5040} = 2^6 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 29 \cdot 31 \cdots 1009 \cdot 2521 \approx 1.5 \cdot 10^{52}$; then $\#\langle n \bmod s \rangle \leq 5040$, for $\gcd(n,s)=1$. The resulting test runs in approximately 45 seconds.

Hendrik Lenstra (Amsterdam)

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<u>Principle orders and arithmetic</u>

The basic arithmetic properties of a principle order $\mathfrak{O}$ (where $\mathfrak{p}_{\mathfrak{O}}$ (its Jacobson radical) is left (hence right) principal) were noted and the congruence Gauss sums for admissible representations of the normaliser, of $\mathfrak{O}$, $\mathcal{G} = \mathcal{G}(\mathfrak{O})$ were introduced. The significance of these in connection and comparison with Galois Gauss sums was discussed

A. Fröhlich (Bordeaux & London)

Principal orders & arithmetic II

The relation between the Gauss sum $\tau(\rho)$ attached to a representation ρ of the normalizer $\mathcal{G}(\mathfrak{A})$ of a principal order $\mathfrak{A}$ in a p-adic simple algebra A, and the Godement-Jacquet local constant $\varepsilon(\pi, s)$ attached to a representation π of $A^\times$ was given:

$$\varepsilon(\pi, s) = (-1)^{m(d-1)} \; N_{\mathfrak{A}}(\mathcal{D}_{\mathfrak{A}} \, f(\rho))^{(1/2 - s)/n} \, \frac{\tau(\check{\rho})}{\sqrt{N f(\rho)}}$$

$$n^2 = \dim_{Z(A)}(A), \quad A \cong M_m(D), \; D \text{ a division algebra}$$

Cf. Bushnell.

Modules over Dedekind-like Rings

All finitely generated modules — and their direct-sum behavior — are described over a class of rings called Dedekind-like. These include the group ring $\mathbb{Z}G_n$ (G_n cyclic of square-free order), some rings of algebraic integers that are not integrally closed in their field of fractions, and many subrings of $\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}$.

Lawrence Levy

Algebraic geometry of quaternion orders

Let R be a Dedekind ring with field of quotients K and A a central simple K-algebra of dimension n^2. Each R-order Λ in A defines an $\operatorname{Spec} R$-scheme X_Λ. The set $X_\Lambda(R')$ of R'-rational points of X_Λ, for a commutative R-algebra R', is the set of left Λ'-ideals I' of R'-rank n and such that Λ'/I' is R'-projective, $\Lambda' = \Lambda \otimes_R R'$. We look at the schemes X_Λ in the particular case of quaternion algebras A. In this case, X_Λ is integral iff Λ is Gorenstein, normal iff Λ is Bass and regular iff Λ is hereditary. Each Gorenstein order Λ defines in a natural way a Bass order $B(\Lambda)$ such that $X_{B(\Lambda)}$ is the normalization of X_Λ. For each Bass order Λ there is a chain $\Lambda = \Lambda_0 \subset \Lambda_1 \subset \ldots \subset \Lambda_n$ such that $X_{\Lambda_{i+1}}$ is an elementary transform of X_{Λ_i} at singular points in one of its fibers (a suitable blowing-up followed by a suitable contraction) and X_{Λ_n} is regular.

J. Brzezinski

Nilpotent elements in the Green ring.

Let G be a p-group and R be an integral domain in which p is not a unit. Let $a(RG)$ be the Green ring of RG lattices. The speaker and D. Benson have found a new method for finding nilpotent elements in $a(RG)$ for many p-groups G. The technique improves on that of Zemanek in that it gives an infinite number of examples and it substitutes a cohomology calculation for the more difficult tensor product calculation.

Let G be any finite group and let K be a field of characteristic $p>0$. Benson has shown that, for M, N indecomposable KG-modules with M absolutely indecomposable, K is a direct summand of $M \otimes N$ if and only if $N \cong M^*$ and p does not divide the dimension of M. Let $a(KG, p)$ be the subgroup of $a(KG)$ generated by all $[M]$ such that p divides the dimension of every component of $K' \otimes M$ for any extension K' of K. Then $a(KG, p)$ is an ideal in $a(KG)$ and $a(KG)/a(KG,p)$ has no nilpotent elements. The speaker, working with M. Auslander has discovered a proof of Benson's theorem that appears to extend these results to $a(RG)$ for R a complete DVR.

Jon F. Carlson (Athens, Ga. and Essen BRD)

Decomposition of relation cores of non-solvable groups.

The existence of non-solvable groups with decomposable relation cores was shown.

A) Let G be a symmetric group of degree n. Then relation cores of G decompose if, and only if, $n=p$ or $n=p+1$, where p is an odd rational prime.

B) The following finite groups have decomposable relation cores. The alternating groups of degree p or $p+1$, p a prime ≥ 5; any Zassenhaus group; any insoluble primitive permutation group of degree p, p a prime.

Wolfgang Kimmerle

On a Zassenhaus conjecture on units in group rings (joint work with S.K. Sehgal)

For a unit u of finite order in the integral group ring $\mathbb{Z}G$ of a finite group G one of the Zassenhaus conjectures states the existence of a group element g such that $u = \alpha g \alpha^{-1}$ with some invertible $\alpha \in \mathbb{Q}G$. It is shown that this conjecture is true if $G = \langle a \rangle \rtimes \langle x \rangle$ is split metacyclic with either $(\operatorname{ord} a, \operatorname{ord} x) = 1$ or $\operatorname{ord} a = p^m$, $\operatorname{ord} x = pq$, p and $q \mid p-1$ being prime numbers here. Moreover, if G is a nilpotent class 2 group or a metacyclic p-group, then actually $u = \alpha g \alpha^{-1}$ when $u \equiv g \bmod W$, where W is any Whitcomb ideal in $\mathbb{Z}G$.

J. Ritter S.K. Sehgal

Computer calculation of units in modular group rings

Let G be a finite p-group, I the augmentation ideal of $\mathbb{F}_pG$, $V = 1 + I$ the Sylow p-subgroup of the group of units of $\mathbb{F}_pG$. Using Fortran programs to calculate in $\mathbb{F}_pG$ (for $|G| \leq 27$), I have obtained workable presentations for V. Group theoretic properties of V are then investigated by use of the software packages CAYLEY (from Sydney) and SOGOS (from Aachen). Such experimentation has suggested new theoretical such as:

Theorem If G is of nilpotency class 2 and has elementary abelian commutator subgroup, then i) V has a normal complement to G and ii) G is determined by $\mathbb{F}_pG$.

Robert Sandling

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Isomorphism of modules under ring extension

Let D be a Dedekind domain and R a module finite D-algebra. If M is a finitely generated R-module, there is an equivalence of categories between the classes of modules which are summands of $M^{(e)}$ for some e and finitely generated projective modules over $R' \oplus R''$ where R' is an order in a semisimple D-algebra and R'' is an artinian semisimple ring. In particular, this equivalence preserves the genus. Hence for D the ring of algebraic integers in a number field, one obtains a generalization of Jacobinski's cancellation theorem and a variation of his extension of base ring theorem (M and N are in the same genus $\Leftrightarrow$ $M \otimes_D D' \cong N \otimes_D D'$ for some larger ring of algebraic integers D'). We also discuss another proof of the extension theorem which depends essentially only on the fact that one is in the stable range of the ring of all algebraic integers. The proof generalizes to a larger class of integral domains

Robert M Guralnick (Los Angeles)

A survey of $K_0(\mathbb{Z}G)$

The current state of knowledge about the $D(\mathbb{Z}G) \subseteq Cl(\mathbb{Z}G) \subseteq K_0(\mathbb{Z}G)$ for finite G was summarized. The groups

$D(\mathbb{Z}G)$	G a 2-group
$D(\mathbb{Z}G)^-$	G a p-group, p odd
$D(\mathbb{Z}G)^+$	G a p-group, p odd and regular

are now fairly well understood: formulas have been derived for their order, and relatively simple algorithms for computing their structure are known. Also, Steve Ullom has results which describe $D(\mathbb{Z}G)^+$ in many cases when G is a cyclic p-group and p an irregular prime.

The difficult problem is thus to understand the kernel groups $D(\mathbb{Z}G)$ for G not of prime power order. Results on this problem worth mentioning include:

(1) Matchett has computed $|D(\mathbb{Z}C_n)|$ when n is squarefree

(2) Martin Taylor has described $D(\mathbb{Z}S_n)$ $(=Cl(\mathbb{Z}S_n))$: at least modulo 2-torsion

(3) Milgram has made computations in the $D(\mathbb{Z}G)$ for certain semidirect products $G \cong C_{pq} \rtimes Q(8)$ ($p \neq q$ odd primes); and succeeded in determining whether or not certain projective modules arising topologically are stably free.

Bob Oliver

Presentations of Grothendieck groups.

We introduce the concept of coherent pair $(\underline{A}, \underline{B})$ of additive categories over a commutative ring R. We use Quillens long exact sequence of K-groups to study the Grothendieck group $K_0(\mathrm{mod}\,\underline{B})$, where $\mathrm{mod}\,\underline{B}$ is the category of finitely presented contravariant functors from $\underline{B}$ to Mod R. We show that $K_0(\mathrm{mod}\,\underline{A}/\underline{B}) \to K_0(\mathrm{mod}\,\underline{A})$ is a monomorphism if $\mathrm{mod}\,\underline{B}$ is regular or if every object in $\mathrm{mod}\,\underline{B}$ has finite length, or if $\underline{B} = \mathrm{mod}\,\Lambda$ where Λ is a classical order of finite lattice type in a simple algebra. We further show that if G is a finite subgroup of $GL(n, \mathbb{C})$ acting naturally on $\mathbb{C}[[X_1, \dots, X_n]]$, and the action of G on $V \setminus \{0\}$ (V = corresponding n-dim. vector space) is free, then $K_0(\mathrm{mod}\,R) = \mathbb{Z} \oplus$ finite group, where $R = \mathbb{C}[[X_1, \dots, X_n]]^G$. (Joint work with Maurice)

Idun Reiten

<u>Finite unimodular groups of prime degree and circulants</u>

The maximal finite irreducible subgroups of $GL(p,\mathbb{Z})$ for prime dimensions p are classified up to conjugacy. Due to the fact that the p-th cyclotomic field has class number 1 for $p \le 19$ these groups can be represented as automorphism groups of quadratic forms whose Gram matrix is a circulant. The additional cases in dimension 23 are related to the Leech lattice. For dimensions $p \le 11$ and $p = 23$ all groups are essentially reflection groups. For dimensions 13, 17, and 19 it was necessary to compute the integral automorphism groups of ~~m~~quadratic forms by machine.

W. Plesken (Aachen)

<u>On A-group rings</u>

The Brauer conjecture ($\mathbb{Z}G \simeq \mathbb{Z}H$; G, H finite groups; $\Rightarrow G \simeq H$) seems to be intimately connected with the conjecture of the <u>Brauer invariance</u> of the group ring $\mathbb{Z}G$ of a finite group G (For any automorphism α of $\mathbb{Z}G$ there is $x \in U\mathbb{Z}G$ such that $\alpha G = xGx^{-1}$) as is demonstrated in the case that G is an A-Sylowtower group ($G = G_0 > G_1 > \dots > G_s = 1$, $(G_{i-1}, G_{i-1}) \le G_i$, $G_{i-1} : G_i = p_i^{n_i} > 1$ (p_i; $1 \le i \le s$), $p_1, \dots, p_s$ distinct prime numbers). It is shown that such groups are both Brauer invariant and affirmative for the Brauer conjecture. The methods seem to be suitable for showing the same thing for A-Groups (Taunt 1947) which, it is suggested, are simply defined as finite groups in which every Sylowsubgroup is abelian (equiv. every nilpotent subgroup is ~~nilp~~abelian).

The case $s = 1$ is dealt with by D.G. Higman's thesis – Induction over s. Applying a theorem of Schur-Zassenhaus one uses the induction argument to the proof of the following theorem: Let $G = A \rtimes B$, $(A,A) = 1$, $|A| = p^n > 1$, $p \nmid |B|$, p prime, $\alpha \in Aut_{\mathbb{Z}_p}(\mathbb{Z}_p G)$, $\alpha(b) = b$ $(b \in B)$, $\alpha a \equiv a \pmod{W_p}$ $(a \in A)$ where $W_p = \Delta_{\mathbb{Z}_p} B \Delta_{\mathbb{Z}_p} A \dotplus \Delta^2_{\mathbb{Z}_p} A$ is the Whitcomb ideal; let it also be known that α merely permutes the classsums C_i $(1 \le i \le g)$ over the G-conjugacy classes. Then $\alpha(C_i) = C_i$. – Use of lattice theory; tensoring.

H. Zassenhaus (Essen)

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Torsion Galois modules and duality

Let K/k be a finite Galois extension of algebraic number fields with Galois group Γ, and let $\mathcal{O}$ be the ring of integers of K, $\mathfrak{o}$, the ring of integers of k. The trace map gives an $\mathfrak{o}\Gamma$-isomorphism between C, the codifferent of the extension K/k, and $\mathrm{Hom}_{\mathfrak{o}}(\mathcal{O},\mathfrak{o})$, the dual of $\mathcal{O}$, enabling one to use the torsion module $T = C/\mathcal{O}$ to "measure" the difference between $\mathcal{O}$ and its dual. More precisely, if S is a fixed set of primes of $\mathfrak{o}$, let $G^S_{\oplus}(\mathfrak{o}\Gamma)$ (respectively $K^S_0(\mathfrak{o}\Gamma)$) denote the Grothendieck group corresponding to the category of finitely generated $\mathfrak{o}$-torsion free $\mathfrak{o}\Gamma$-modules (respectively also locally projective at the primes of $\mathfrak{o}$ outside S), with the relations arising from short exact sequences splitting at the primes of $\mathfrak{o}$ outside S. By computing the class of T in a suitably chosen Grothendieck group one obtains a purely algebraic proof of:

Theorem (Cassou-Noguès, Queyrut): If $S_{\mathbb{Z}}$ contains all the rational primes with a divisor in $\mathfrak{o}$ wildly ramified in K, then $[\mathcal{O}] = [\mathrm{Hom}_{\mathfrak{o}}(\mathcal{O},\mathfrak{o})]$ in $K_0^{S_{\mathbb{Z}}}(\mathbb{Z}\Gamma)$;

as well as another theorem computing the difference $[\mathrm{Hom}_{\mathfrak{o}}(\mathcal{O},\mathfrak{o})] - [\mathcal{O}]$ in $G^S_{\oplus}(\mathfrak{o}\Gamma)$, where S contains the primes of $\mathfrak{o}$ wildly ramified in K. This difference is seen to depend only on the ramification groups $\Gamma_{\mathfrak{P},0}$ and $\Gamma_{\mathfrak{P},1}$, and on the class of $\mathfrak{p}$ in the ideal class group of $\mathfrak{o}$, where $\mathfrak{p}$ runs through the primes of $\mathfrak{o}$ ramified in K, $\mathfrak{P}$ is a prime of $\mathcal{O}$ above $\mathfrak{p}$, and $\Gamma_{\mathfrak{P},i} = \{\gamma \in \Gamma$ such that $\gamma(x) - x \in \mathfrak{P}^{i+1}$ for all $x \in \mathcal{O}\}$. In particular, $[\mathcal{O}] = [\mathrm{Hom}_{\mathfrak{o}}(\mathcal{O},\mathfrak{o})]$ in $G^S_{\oplus}(\mathfrak{o}\Gamma)$ if all the primes of $\mathfrak{o}$ ramified in K are principal.

Hayne Desrochers Cambridge

The Auslander - Reiten quiver of a non-domestic tame group ring

Let R be a complete discrete valuation ring (with valuation v and parameter π) (and assume that $v(3) = 4$), C_3 the cyclic group in 3 elements, $\Lambda = RC_3$ the corresponding group ring. Then the stable Auslander - Reiten quiver of Λ, $\mathfrak{A}_s(\Lambda)$, can be described as follows:

(i) All components of $\mathfrak{A}_s(\Lambda)$ are regular tubes of rank 1 or 2

(ii) All tubes occur in $\mathcal{F}$-tubular series, where $\mathcal{F} = \{$ monic irreducible polynomials in $(R/\pi R)[X]\} \cup \{\infty\}$.

(iii) The tubular type of each $\mathcal{F}$-tubular series is $\begin{cases} \widetilde{\mathbb{CD}}_3 & \text{if } R/\pi R \text{ is not a splitting field for } C_3 \\ \widetilde{\mathbb{D}}_4 & \text{if } R/\pi R \text{ is a splitting field for } C_3 \end{cases}$

(iv) $\mathfrak{A}_s(\Lambda)$ is given by a $\mathbb{P}_1(\mathbb{Q})$-family of $\mathcal{F}$-tubular series : $\mathfrak{A}_s(\Lambda) = \bigcup_{\frac{\beta}{\alpha} \in \mathbb{P}_1(\mathbb{Q})} \dot{\bigcup}_{\lambda \in \mathcal{F}} \mathcal{T}_{\beta:\alpha}^{(\lambda)}$

(v) For any pair of integers β, α which are relatively prime and not both equal to zero, the dimension-type of the primitive one-parameter series of $\mathcal{T}_{\beta:\alpha}$ is equal to $|\beta|\, b + |\alpha|\, a$, where $b = (2,1,0,1)$ and $a = (1,0,1,0)$.

Ernst Dieterich.

Stickelberger ideals, monoid rings, and Galois module structure

Let K be a number field with ring of integers $\mathfrak{o}$, and G a finite abelian group. To G, one can associate a certain commutative monoid E and a Stickelberger submodule S^* of the dual $\mathbb{Z}E^* = \mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}E, \mathbb{Z})$ such that
(i) if $G \sim (\ell^n, \ldots, \ell^n)$, ℓ an odd prime, then $|Cl(\mathbb{Z}G)^-| = [\mathbb{Z}E^{*-} : S^{*-}]$ and
(ii) if G is any abelian ℓ-group (ℓ odd), then $|{}^{o}D(\mathbb{Z}G)| = |(\mathbb{Z}E^{*+}/S^{*+})_{tors}|$ or more generally, if G has odd order or has cyclic 2-primary component, then $|(\mathbb{Z}E^{*+}/S^{*+})_{tor}| = |A_{\mathbb{Q}}(G)|$ ($A_{\mathbb{Q}}(G)$ = the cokernel of Artin induction).
If $R(\mathfrak{o}G)$ (resp., $R_d(\mathfrak{o}G)$) is the sub~~group~~set of $Cl(\mathfrak{o}G)$ consisting of the Galois module classes of tame (resp., domestic) extensions L/K with $Gal(L/K) \cong G$, then one can define an action of $\mathbb{Z}E^*$ on the group I' of $\mathfrak{o}G$-ideals relatively prime to $|G|$, in terms of which one can characterize the elements of $R_d(\mathfrak{o}G)$. Moreover, one can show that $R_d(\mathfrak{o}G) \supseteq$ the image of $(I')^{S^*}$ under the natural surjection $I' \twoheadrightarrow Cl(\mathfrak{o}G)$. In particular $Cl(\mathbb{Z}G)^{S^*} = (1)$.
One can also show $R_d(\mathfrak{o}G)$ is a group.

Leon McCulloh

Crossed Product Orders

Let K/k be a finite Galois extension of local fields (with $[k:\mathbb{Q}_p]$ finite) with Galois group G, and rings of integers $\mathcal{O}$, $\mathfrak{o}$ (resp.) Let $A = (K/k, \rho)$ be a crossed product algebra where ρ is a factor set on G with values in $\mathcal{O}^\times$, and let $\Lambda = (\mathcal{O}/\mathfrak{o}, \rho)$ be the crossed product order in A.
Let $\Lambda_0 = \Lambda$, and $\Lambda_{i+1} = \mathcal{O}_\ell(J(\Lambda_i))$ be the left order of the Jacobson radical of Λ_i. Then

$$\Lambda_0 \subsetneq \Lambda_1 \subsetneq \Lambda_2 \subsetneq \cdots \subsetneq \Lambda_s = \Lambda_{s+1} = \cdots = \Lambda_\infty .$$

It is shown that $s = d - (e-1)$, where

$\mathcal{D}_{K/k} = \mathfrak{P}^d$, $\mathfrak{P}$ the maximal ideal of $\mathcal{O}$, $\mathcal{D}_{K/k}$ the different, $e = e(K/k)$ the ramification index. Also, the type numbers of the hereditary order Λ_∞ are
$\underbrace{(f, f, f, \dots, f)}_{e/m \text{ times}}$ where $f = f(K/k)$ is the inertial degree, and m is the Schur index of A.

Gerald Cliff
Edmonton

Class groups of orders in algebras over function fields

Theorem: Let k be a field, $R = k[t]$, $K = k(t)$, Λ a heriditary R-order in a central simple K-algebra of prime index ℓ. Then $C\ell(\Lambda)$ is finite if
(a) k global $ch(k) \neq \ell$ or
(b) k f.g. over $\mathbb{Q}$ and there is a maximal left Λ-ideal $M \subset \Lambda$ such that $\ell \nmid \dim_k \Lambda/M$

To prove the theorem we choose a Galois extension k' of k such that $K' := k' \otimes_k K$ splits A and such that $\Lambda' := k' \otimes_k \Lambda$ only ramifies at k'-rational primes of $R' := k' \otimes_k R$.
We then construct a group homomorphism $\Phi : C\ell(\Lambda) \longrightarrow H^1(k'/k, K_1(\Lambda'))$. The proof of theorem then consists of two parts. First we prove that $\operatorname{Im} \Phi$ is finite using class field theory in (a) and the Mordell-Weil-Neron theorem in (b). Then we consider $\operatorname{Ker} \Phi$ and using a theorem of Merkurjev-Suslin we are able to relate this group to $H^3_{et}(k, \mu_\ell^{\otimes 2})$.

Per Salberger
Göteborg

Units of Crossed Product Orders

Let K be either a totally real number field, normal over $\mathbb{Q}$, or a totally imaginary quadratic extension of such a field. Then K admits complex conjugation $x \to \bar{x}$. Let G be a finite group and J the involution on KG defined by $J(\sum_g \alpha_g g) = \sum \bar{\alpha}_g g^{-1}$. R_K = algebraic integers in K.

<u>Theorem</u> Let Λ be any R_K-order in KG which contains $R_K G$. Then $U_J(\Lambda) = \{\lambda \in \Lambda : \lambda J(\lambda) = 1\}$ is a finite group containing G. If $G \subseteq H \subset (KG)^\times$ with H finite, then $H \subseteq U_J(\Lambda)$ for some order Λ.

Consider the case G = Frobenius group of order $p(p-1)$, p an odd prime. One can explicitly determine the orders in $\mathbb{Q}G$ containing $\mathbb{Z}G$. They can be indexed as $\Gamma_1, \cdots, \Gamma_{p-1}$ and for these orders $U_J(\Gamma_i)$ can be determined. In 4 cases this group is $\langle -1 \rangle \times \mathrm{Sym}(p)$. If $p+1$ is divisible by 4, 6, 8, or 12, then some groups are $\langle -1 \rangle \times PGL_2(p)$ (at most 8). The remaining groups are $\langle -1 \rangle \times G$.

Gerald J Janusz 8.6.84
Urbana, IL USA

Zeta-Functions of two-sided Ideals in Arithmetic Orders

We define Z- and L-series of two-sided ideals in arithmetic orders. We obtain explicit formulas for the zeta functions for some particular classes of orders, and give some examples. We also study, in the simple case, the behavior of the zeta functions at their largest pole. The discussion ends with some possible generalizations of the prime ideal theorem to two-sided ideals of arithmetic orders in simple algebras.

Gerardo Roggi 8,6,84
Urbana, IL. U.S.A.

Galoismodules and embedding problems.

We combine the embedding problem with the problem of Galois module structure of rings of integers. We derive a necessary condition for the solvability of the "embedding problem with prescribed free Galois module structure". This is analoguous to the classical condition of Hasse-Wolf for the embedding problem. Our approach leads to a number of explicit results, for example: no cyclic extension of the rationals of odd prime power order has a normal integral basis over any proper sub intermediate field. A basic tool is a map from a Hochschild-Serre sequence to a Fröhlich-Wall sequence. One intriguing feature of this diagram is that two of its vertical maps are not in general a homomorphism, but only have a weak multiplicative property.

Jan Brinkhuis, 8-6-84
Rotterdam.

Permutation modules and group cohomology

The following theorem provides a means of computing the p-part of cohomology from p-local subgroups.

<u>Theorem</u> Let the finite group G act simplicially on the simplicial complex Δ such that for each simplex $\sigma\in\Delta$ the isotropy group G_σ fixes σ pointwise. Suppose that for each subgroup $H\leq G$ with $H/O_p(H)$ cyclic the fixed-point complex Δ^H has Euler characteristic $\chi(\Delta^H)=1$. Then

(a) $\mathbb{Z}_{(p)} \cong \sum_{\sigma\in\Delta/G} (-1)^{\dim\sigma}\, \mathbb{Z}_{(p)}\uparrow^G_{G_\sigma}$ (mod projectives) in the

Green ring $A(\mathbb{Z}_{(p)},G)$

(b) For any $\mathbb{Z}G$-module M and integer n

$$H^n(G,M)_p = \sum_{\sigma\in\Delta/G} (-1)^{\dim\sigma} H^n(G_\sigma,M)_p \quad \text{in } K_0(\text{finite abelian groups}, 0)$$

The hypotheses of the theorem are satisfied when Δ is either the simplicial complex of elementary abelian p-subgroups of G, or of all p-subgroups of G, or Δ is a Tits building of a finite Chevalley group in defining characteristic p.

Peter Webb (Manchester)

An affine plane with desarguesian and non-desarguesian (3,1)-nets. (Conference on Wil Geometry, June 10 - June 15, 1984)
A (3,1)-net $\mathfrak{N} = (P, \mathfrak{G}_1, \mathfrak{G}_2, \mathfrak{G}_3)$ is called desarguesian, if there is a desarguesian affine plane $(P, \mathfrak{G})$, such that $\mathfrak{N}$ is a substructure of $(P, \mathfrak{G})$, otherwise non-desarguesian. If $\mathfrak{N}$ is a desarguesian (3,1)-net then the corresponding double loop $(K, +, \cdot)$ is a division ring. The (non-desarguesian) plane given by Tschetweruchin in 1927 (J. d. DMV <u>36</u>, 134-136) is an example of a non-desarguesian plane containing desarguesian (3,1)-nets as Salzmann recognized in 1971. A second example of such an affine plane is constructed over the field $K = K_1((t))$ of the formal Laurent series, where K_1 is a field of characteristic $\neq 2$. This plane occurs as affine derivation of a non-ovoidal Möbius plane of Hering class III_2.

Hans-Joachim Kroll (TU München)

WEB GEOMETRY
JUNE 10 THROUGH JUNE 15, 1984

Sophus Lie's Fundamental Theorems for local analytical loops.

A local analytical loop is a vector space L together with an analytical multiplication $\circ : B\times B \longrightarrow L$ defined on some open 0-neighborhood B such that $0\circ x = x\circ 0 = x$ for $x\in B$ and that, as a consequence, $\circ$ has an expansion $x\circ y = x+y+q(x,y)$ $+ s(x,x,y) + t(x,y,y) + f_4(x,y) + \dots$ with a bilinear map q and two trilinear maps s and t, and with homogeneous polynomials f_n of degree n. We define the commutator $[x,y]$ by $\lim_{t\to 0} t^2((tx\circ ty)/(ty\circ tx))$ with the loop quotient "/" defined locally by the implicit function theorem. Similarly we define the associator $\langle x,y,z\rangle = \lim_{t\to 0} t^3(((tx\circ ty)\circ tz)/(tx\circ(ty\circ tz)))$. We find $[x,y] = q(x,y)-q(y,x)$ and $\langle x,y,z\rangle = q(q(x,y),z)-q(x,q(y,z))+s(x,y,z)+s(y,x,z)-t(x,y,z)-t(x,z,y)$. The commutator and associator are linked by the relation

(A) $\sum_{\sigma\in S_3} \mathrm{sgn}(\sigma)\langle x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)}\rangle = J(x_1,x_2,x_3)\ (:= \sum_{\sigma\in A_3}[[x_{\sigma(1)}, x_{\sigma(2)}], x_{\sigma(3)}])$

Any algebra $(A,[\cdot,\cdot],\langle\cdot,\cdot,\cdot\rangle)$ with a skew-bilinear $[\cdot,\cdot]$ and a trilinear $\langle\cdot,\cdot,\cdot\rangle$ satisfying (A) is called an Akivis algebra. (For $\langle x,y,z\rangle\equiv 0$ one obtain a Lie algebra!). Lie's first theorem: Starting from the local analytical loop $(L,\circ)$ we obtain an Akivis algebra $(L,[\cdot,\cdot],\langle\cdot,\cdot,\cdot\rangle)$. Lie's third theorem (K. Strambach, K.H.H). For each Akivis algebra $(L,[\cdot,\cdot],\langle\cdot,\cdot,\cdot\rangle)$ there are an $n\binom{n}{3}$-dimensional affine variety full of trilinear maps (s,t) (where $n=\dim L$) such that $x\circ y = x+y+\frac{1}{2}[x,y]+r(x,x,y)$ $+ s(x,y,y)$ defines a local analytical loop whose associated Akivis algebra is the given one.

Karl H. Hofmann, TH DARMSTADT

Moufang 3-webs and homogeneous spaces

The canonical connection of a 3-web defined by M. A. Akivis induces a linear connection on a horisontal leaf satisfying the structure equations
$d\omega^i = -\omega^i_j \wedge \omega^j + a^i_{jk}\,\omega^j \wedge \omega^k; \quad d\omega^i_j + \omega^i_k \wedge \omega^k_j = 0.$
In the case of a Moufang web the connection $\overset{*}{\omega}{}^i_j = \omega^i_j + \frac{2}{3} a^i_{jk}\omega^k$ determines on this leaf a local reductive homogeneous space with canonical connection. If we denote $[X,Y] = a^i_{jk} X^j Y^k e_i$, the torsion and curvature tensors of $\overset{*}{\omega}{}^i_j$ can be expressed as

$$(*)\quad T(X,Y) = \frac{2}{3}[X,Y], \quad R(X,Y)Z = \frac{4}{9}\{[[Z,X],Y] + [[Y,Z],X] - [[X,Y],Z]\}.$$

A horisontal leaf of a 3-web can be identified with the coordinate loop of the web. We have investigated the question: how can be the coordinate loop multiplication reconstructed from the reductive homogeneous space induced by a Moufang 3-web.

Theorem: Let L be a Moufang loop, $\mathfrak{m}$ is the corresponding tangent Malcev algebra with bilinear multiplication $[X,Y]$. We consider the reductive homogeneous space G/H defined by the enveloping Lie algebra $\mathfrak{g} = \mathfrak{m} \oplus \mathfrak{h}$ of $\mathfrak{m}$ and the canonical connection $\overset{*}{\omega}{}^i_j$ with torsion and curvature tensors $(*)$ satisfying $\overset{*}{\nabla} T = \overset{*}{\nabla} R = 0$. Then the geodesic loop multiplication $x \circ y := \exp_x \circ \tau_{e,x} \circ \exp^{-1} y$ corresponding to the invariant connection $\omega^i_j = \overset{*}{\omega}{}^i_j - T^i_{jk}\omega^k$ $(T(X,Y) = T^i_{jk} X^j Y^k e_i)$ is equal to the original loop multiplication of L in a neighbourhood of the identity $e \in L$.

Péter T. Nagy, Bolyai Institute,
Szeged University, Hungary

Canonical imbeddings of homogeneous systems into the enveloping groups

A homogeneous system (G, η) is a set G with a ternary operation $\eta: G\times G\times G \to G$ satisfying the conditions (1) $\eta(x,x,y)=y$, (2) $\eta(x,y,x)=y$, (3) $\eta(x,y,\eta(y,x,z))=z$ and (4) $\eta(x,y,\eta(u,v,w))=\eta(\eta(x,y,u),\eta(x,y,v),\eta(x,y,w))$. In this lecture, we considered a connected and simply connected analytic manifold G with an analytic homogeneous system η. We constructed the enveloping group $A=G\times K$ of (G,η) at a point e of G, where K denotes the closure of the left inner mapping group of the binary multiplication $xy=\eta(e,x,y)$ in the Lie group of all analytic automorphisms of (G,η). Then, we presented the following result: The homogeneous system (G,η) is decomposed into a direct product of a K-semisimple symmetric homogeneous system (G_1,η_1) and a homogeneous system (G_2,η_2) of a semisimple Lie group G_2, if and only if the following conditions are satisfied: (i) A is semisimple, (ii) the canonical decomposition $\mathfrak{a}=\mathfrak{g}\oplus\mathfrak{k}$ of the Lie algebra $\mathfrak{a}$ of A satisfies $\varphi(\mathfrak{g},\mathfrak{k})=0$, when φ is the Killing form of $\mathfrak{a}$, and (iii) G is imbedded as a totally geodesic submanifold of A under the canonical imbedding of G into A.

Michihiko Kikkawa
SHIMANE UNIVERSITY

Algebraization Problem for Hypersurfaces

An important class of examples of webs is the class of algebraic webs, constructed as follows. Given an algebraic subvariety V of dimension n and degree d in $\mathbb{P}^{n+k}$, there is a d-web of codimension n in $Gr(k,n+k)$ (the Grassmannian of k-planes in $\mathbb{P}^{n+k}$), where the d leaves through a $\Lambda\in Gr(k,n+k)$ are the Schubert cycles $\sigma(x_1),\cdots,\sigma(x_d)$, where $\Lambda\cap V=\{x_1,\cdots,x_d\}$ and $\sigma(x)=\{\Lambda'\in Gr(k,n+k)\mid x\in\Lambda'\}$.

An important problem, then, is to characterize algebraic webs. This breaks down into two stages. (1) The Grassmannization problem: to determine when the leaves of a web are equivalent to Schubert cycles $\sigma(x)$, as above. (2) Algebraization problem: (after dualizing) to determine when d local pieces of submanifold are contained in an algebraic subvariety of degree d.

We solve the algebraization problem for hypersurfaces. By a projection argument, Akivis and Little have then reduced the general codimension case to this codimension 1 case. To state the precise result, assume $x_0, \dots, x_n$ are affine coordinates on $\mathbb{P}^{n+1}$ and line coordinates $(m,b) = (m_1, \dots, m_n, b_1, \dots, b_n)$ on $Gr(1,n)$ s.t. $\ell(m,b) = \begin{cases} x_1 = m_1 x_0 + b_1 \\ x_n = m_n x_0 + b_n \end{cases}$.

If $\gamma_1, \dots, \gamma_d$ are d local pieces of hypersurface, intersected transversely by a line $\ell_0 = \ell(0,c)$, let $X_i(m,b) = 0^{th}$ coord of $\gamma_i \cap \ell(m,b)$. Then we have:

Thm There exists an algebraic hypersurface γ of degree d with $\gamma_i \subset \gamma \iff$

$$\sum_{i=1}^{d} \frac{\partial^2 X_i}{\partial b_j \partial b_k} = 0 \quad , \text{for all } j,k = 1,2,\dots,n, \quad \text{all } (m,b) \text{ near } (0,c).$$

There are several proofs of this result. We gave one which stems from some old work of Lie and Scheffers, which generalizes to the above setting.

Jay A. Wood, Univ of Chicago.

On Lie's Approach to the Study of Translation manifolds

The classical theorem of Lie and Wirtinger states that any hypersurface of double translation $S \subset \mathbb{C}^{n+1}$, given by

$$x_i = \sum_{j=1}^{n} \alpha_{ji}(t_j) = \sum_{j=n+1}^{2n} \alpha_{ji}(t_j)$$

(such that $\frac{\partial t_j}{\partial t_1} \neq 0$ for all $n+1 \le j \le 2n$) is part of the theta-divisor of an algebraic curve of genus $n+1$ (or a singular "limit" of such curves). The key step in the proof is to show that the "curves of tangents" $\dot{\alpha}_j$ $1 \le j \le 2n$ in $\mathbb{P}^n$ lie on an algebraic curve of degree $2n$.

Lie studied the cases $n=2$ and $n=3$ and tried to deduce this by studying the integrability conditions for the system of PDE whose solutions are the equations of the hypersurfaces S. We showed how these integrability conditions can be interpreted for general n. The $\dot{\alpha}_j$ are either (part of) the intersection of $\binom{n-1}{2}$ quadric hypersurfaces, in which case the result follows almost immediately, or the $\dot{\alpha}_j$ lie on $\binom{n-1}{2}$ quadrics which intersect in a surface of minimal degree in $\mathbb{P}^n$. In this last case (which, a posteriori, corresponds to curves of genus $n+1$ with a g^1_3 or a g^2_5 by the Enriques-Petri theorem), we indicated an analog of the Reiss relation, which characterizes the algebraic curves on the surfaces of minimal degree, and which should also be a consequence of the integrability conditions mentioned above.

John Little
Holy Cross College

Abelian equations of webs

I. Significance of web geometry

α) As a generalization of the geometry of projective varieties, because a projective variety defines a Grassmann web. But webs are more general, as there exist non-linearizable webs.

β) To describe the polyhedral behavior of the boundary of a domain. Cf. recent work of J. Baumann.

γ) To study the relation between the orbits of a space under the action of an intransitive group. Cf. Gelfand-McPherson-Damiano

II Abelian equations and rank of a web.

Upper bounds by Chern, Damiano, Chern-Griffiths.

Fundamental Problem (Unsolved) To determine all webs of max rank.

III An application of web geometry to the theorem of Lie-Wirtinger on manifolds of double translation

Proof of linearization by web geometry.

S. S. Chern
Berkeley, California

Rank Problems for Webs $W(d,2,r)$

r-rank and 1-rank problems (the upper bound for rank and a description of webs of maximum rank) were discussed for d-webs $W(d,2,r)$ of codimension r on (2r)-dimensional differentiable manifold. Almost Grassmannizable and almost algebraizable webs $W(d,2,r)$ play an important role in these problems. Webs $W(4,2,2)$ of maximum 2-rank are exceptional in the sense that they are not necessarily algebraizable. Some quadratic and cubic exterior forms are associated with such exceptional webs $W(4,2,2)$ of maximum 2-rank and their properties were given.

Vladislav Goldberg
New Jersey Institute of Technology, U.S.A.

Surfaces with two families of conics

Starting with the problem to determine all surfaces with a hexagonal 3-web of conics in projective 3-space there are considered socalled Beutel surfaces (generated by a one-parameter family of conics) such that the conjugate family of curves are conics, too. The very complex system of non-linear differential equations of third order, which is overdetermined, can be completely solved. From this arises the geometric result, that the planes carrying the conics form a pencil for both families.

The remaining differential equations can be completely integrated such that an explicit algebraic representation is established. In the case of intersecting axes the surfaces are double translation surfaces. In the general case a classification can be derived from a projection out of a projective 5-space. Besides of quadrics and some few exceptional cases the surfaces turn out to be ~~of~~ third or forth order and to be the complex-projective transforms of Dupin's cyclides.

W. Degen
Stuttgart

Complex Structures, Exterior Algebras, and Commutative Moufang Loops.

Commutative Moufang loops were introduced as non-associative generalisations of abelian groups that tend to appear in certain cases where abelian groups might be expected. Two examples were given: Manin's related algebraic structures on equivalence classes of rational points on cubic hypersurfaces, and Kikkawa's homogeneous systems (without the 4th axiom) under the assumption that there is a unique ternary operation with the required properties.
Exterior algebras underly the two main known constructions (Bruck's and Malbos') of non-associative commutative Moufang loops with large nilpotence class. Verification that these constructions do give Moufang loops is by tedious calculation, and it would be very desirable to give more conceptual proofs.
Complex structures can be used to give a streamlined version of Malbos' construction. The problem of giving a natural explanation for the form of this construction was raised.

Jonathan D.H. Smith

Darmstadt/Philadelphia/Ames

Wednesday, 13 June 1984. A new record for return from St. Roman to the Institute: $52\frac{1}{2}$ minutes (walking).

Jay A Wood Leo Weddes

Neuere Resultate über Sechseckgewebe

Auf einer negativ gekrümmten Fläche $F \in C^4$ des E^3 wird das drei-Gewebe betrachtet, das aus den beiden Scharen der Krümmungslinien und der einen Schar der Asymptotenlinien besteht. Für dieses Gewebe wird eine Integralformel bewiesen. Ferner werden folgende Fragen untersucht: 1) Wann ist dieses Gewebe ein Sechseckgewebe? (dann sagt man: Die Fläche F hat die Sechseckgewebeeigenschaft). 2) Man bestimme alle Flächen, die die Sechseckgewebeeigenschaft besitzen. 3) Welche Flächen konstanter mittlerer Krümmung haben die Sechseckgewebeeigenschaft? 4) Existiert eine infinitesimale Verbiegung einer Minimalfläche derart, daß die Sechseckgewebeeigenschaft erhalten bleibt?

N. K. Stephanidis
Thessaloniki, Griechenland

Webs on Complex-Analytic Polyhedra

Complexanalytic polyhedra arise in classic Complex Analysis as one extreme class of domains of holomorphy compared to the strictly pseudoconvex domains. As there the geometry of the boundary is very important for the mapping theory of these domains. The analytic fibrations of the boundary of complexanalytic polyhedra (which actually are defined as $\{z \in \mathbb{C}^n : |f_j(z)| < 1,\ j = 1 \ldots\}$) continue in the interior and form a web of complex analytic surfaces, that are preserved under proper holomorphic mappings. A meromorphic version of the classical Blaschke-Bol calculus for the $*$-transformation bases the construction of invariant for proper holomorphic mappings between such domains

John Baumann, Syd

On a maximally mobile G-structure

1. Let M be a n-dimensional manifold and $P(M)$ be a G-structure on M of a finite type. Then the group H of all automorphisms of $P(M)$ is a Lie group and there exists a positive integer $N = N(n, G)$ such that $\dim H \le N$ for any G-structure on M. A G-structure is called <u>maximally mobile</u> if $\dim H = N$.

2. Let $W(3,2,r)$ be a 3-web of r-dimensional foliations on a $2r$-dimensional manifold M. This web defines a G-structure on M with $G = GL(r)$. This structure is of a finite type and for it $N = r^2 + 2r$.

<u>Theorem (Gvozdovich - 1981)</u>. A web $W(3,2,r)$ is maximally mobile if and only if the web is parallelizable.

<u>Theorem (Gvozdovich - 1981)</u>. Let m be the dimension of the full group of automorphisms of a web $W(3,2,r)$. If $m \ge r^2 - r$, then m must be one of the following numbers: r^2+2r; r^2+r, r^2+r-1; r^2+2, r^2+1, r^2; r^2-r+6; r^2-r+4, r^2-r+3, r^2-r+2, r^2-r+1, r^2-r.

3. Let $T_1, \ldots, T_s$ be pairwise orthogonal distributions on a differentiable manifold M, such that for any $x \in M$, $T_1(x) \oplus \cdots \oplus T_s(x) = M_x$. This is a G-structure with $G = O(n_1) \times \cdots \times O(n_s)$, where $n_i = \dim T_i$, $i = 1, \ldots, s$. This G-structure is of a finite type and for it $N = \sum_{i=1}^{s} \frac{n_i(n_i+1)}{2}$.

In our talk we present a complete global classification of maximally mobile G-structures with $G = O(n_1) \times \cdots \times O(n_s)$ both for the simply-connected case (Gauchman - 1978, Cattani and Mann - 1979), and for the non simply-connected case (Gauchman - 1978)

Hillel Gauchman
Ben Gurion Univ. of the Negev
Beer Sheva, Israel

NETS AND BIMAGIC SQUARES

H. Lenz

[5-Minuten-Vortrag]

Let $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}$ be the vectors in $GF(3)^4$
2101, 0112, 1021, 2120, respectively. They form a basis.
The 9 vectors $\mathfrak{o}, \mathfrak{a}, 2\mathfrak{a}, \mathfrak{b}, \mathfrak{a}+\mathfrak{b}, 2\mathfrak{a}+\mathfrak{b}, 2\mathfrak{b}, \mathfrak{a}+2\mathfrak{b}, 2\mathfrak{a}+2\mathfrak{b}$,
written as columns,

$$\begin{array}{ccccccccc}
0 & 2 & 1 & 0 & 2 & 1 & 0 & 2 & 1 \\
0 & 1 & 2 & 1 & 2 & 0 & 2 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 \\
0 & 1 & 2 & 2 & 0 & 1 & 1 & 2 & 0
\end{array}$$

form an orthogonal array (OA).
Hence they determine a 4-net. The same holds for the
9 vectors $\mathfrak{o}, \mathfrak{c}, 2\mathfrak{c}, \mathfrak{d}, \mathfrak{c}+\mathfrak{d}, 2\mathfrak{c}+\mathfrak{d}, 2\mathfrak{d}, \mathfrak{c}+2\mathfrak{d}, 2\mathfrak{c}+2\mathfrak{d}$.

In the addition table

$\mathfrak{o}$ $\quad \mathfrak{a}$ $\quad 2\mathfrak{a}$ $\quad \mathfrak{b}$ $\quad \mathfrak{a}+\mathfrak{b}$ $\quad 2\mathfrak{a}+\mathfrak{b}$ $\quad 2\mathfrak{b}$ $\quad \mathfrak{a}+2\mathfrak{b}$ $\quad 2\mathfrak{a}+2\mathfrak{b}$
$\ldots\ldots\ldots\ldots$

$\mathfrak{c}$ $\quad \mathfrak{a}+\mathfrak{c}$ $\quad 2\mathfrak{a}+\mathfrak{c}$ $\quad \mathfrak{b}+\mathfrak{c}$ $\ldots\ldots\ldots$

$2\mathfrak{c}$ $\ldots\ldots\ldots$

$\mathfrak{d}$ $\quad \mathfrak{a}+\mathfrak{d}$ $\ldots\ldots\ldots$

$\mathfrak{c}+\mathfrak{d}$ $\ldots\ldots\ldots$

$2\mathfrak{c}+\mathfrak{d}$ $\ldots\ldots\ldots$

$2\mathfrak{d}$ $\quad \mathfrak{a}+2\mathfrak{d}$ $\ldots\ldots\ldots$

$\mathfrak{c}+2\mathfrak{d}$ $\ldots\ldots\ldots$

$2\mathfrak{c}+2\mathfrak{d}$ $\ldots\ldots\ldots$

the 9 vectors in each row or column (and diagonal) form an OA. Now consider each vector (x_1, x_2, x_3, x_4) as a triadic number $x_4 + 3x_3 + 9x_2 + 27x_1$. Then the above addition table becomes a bimagic square ($\sum_i a_{ik} = \sum_k a_{ik} = \text{const}$, $\sum_i a_{ik}^2 = \sum_k a_{ik}^2 = \text{const}'$):

0	64	47	14	75	31	25	62	42
34	17	78	36	19	56	50	3	67
59	39	22	70	53	6	72	28	11
69	52	8	74	27	10	58	41	21
13	77	30	24	61	44	2	63	46
38	18	55	49	5	66	33	16	80
48	4	68	35	15	79	37	20	54
73	29	9	57	40	23	71	51	7
26	60	43	1	65	45	12	76	32

Permuting the coordinates of the vectors yields other bimagic squares. See CAZALAS, Carrés magiques; Paris 1934. This construction, as well as many others, such as bimagic squares of order 8, is due to G. TARRY

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Quasidomains and Generalized Isotopies

A quasidomain $(D,+,\cdot)$ is defined as an algebraic structure with 2 binary operations s.t. both, the additive and the multiplicative systems are quasigroups. Distributive law is not required.

A type (QQ, QL, GQ etc.) is described by the type of the additive and the multipl. system: Q-quasigroup, L-loop, G-group, A-Abelian group.

I. Finite quasidomains of type AQ. (i.e. $(D,+)$-an Abelian group $(D-0,\cdot)$ - a quasigroup.)

Albert's theorem for quasigroups: A finite non-assoc. quasidomain of type AQ (and char $\neq 2,3,5$) cannot be distributive if Q is an isotope of a power-associative quasigroup which is not a group.

Thus being power-ass. or an isotope of a power-associative (but not associative) quasigroup is a sufficient condition for Q to be distributivity-intolerant. But this cond. is not necessary.

Question: find a nec. and sufficient condition for Q to be distributivity-intolerant.

II. Generalized isotopies of quasidomains of type GQ.

A quadruple $(\alpha,\beta,\gamma,\delta)$ of bijections on D is called generalized isotopism $(D,+,\cdot)\to(D,\oplus,\circ)$ if α is an isomorphism $(D,+)\to(D,\oplus)$ and (β,γ,δ) is an isotopism $(D-0,\cdot)\to(D-0,\circ)$.

A sufficient (but not nec.) condition for $(\alpha,\beta,\gamma,\delta)$ to preserve distributive property of $(D,+,\cdot)$ is that α is an automorphism of $(D,+)$. A necessary (but not suff.) is that $\beta^{-1}\delta$ and $\gamma^{-1}\delta$ are automorphisms of $(D,+)$.

Question: find a necessary and sufficient condition.

Hala Pflugfelder
Temple University
Philadelphia, PA, U.S.A.

Differential Geometry of Webs: the School of M.A. Akivis

The study of webs $W(d,n,r)$ of foliations of codimension r, $r \geq 1$, on nr-dimensional differentiable manifold was initiated by G. Bol (webs $W(3,2,2)$) and S.S. Chern (webs $W(3,2,r)$). This study was systematically developed by M.A. Akivis and his students for webs $W(3,2,r)$ and by V.V. Goldberg for webs $W(n+1,n,r)$.

In the talk the following topics were covered:

I. The main equations of $W(n+1,n,r)$.
II. Webs and almost Grassmann structures.
III. Webs and local differentiable quasigroups.
IV. Geometry of $W(3,2,r)$.
V. Other geometrical structures connected with webs.
VI. Webs formed by surfaces of different dimension.

Vladislav Goldberg
New Jersey Institute of Technology, U.S.A.

Geometry of 3-nets.

Similarities occurring in the study of foundations of plane projective geometry and of 3-nets geometry were pointed out by W. Blaschke (cf. "Projektive Geometrie" p. 192). In the talk we showed that these similarities extend in a very natural way also to results obtained studying projective planes and 3-nets from von Staudt's point of view.

Were considered: general properties of the group of projectivities of a line in a geometric structure, constructions of free 3-nets, the group of projectivities of a free 3-net and Staudt's theorems for loops

Adriano Barlotti

Projective webs from (A,B)-regular spreads.

In case of (A,B)-regular spreads there are incidence-propositions caracterizing different classes of translation planes. For better understanding one uses projective incidence loops, see Geom. Ded. 6, 471-484 (1977).
Moreover, up to isomorphism, the (A,B)-regular spreads are in one-one-correspondance with those three-webs on a $S_{n,n}$ (appropriate n), where the horizontals and verticals are the two sheares of n-spaces, wheras the transversals are quadratic Veronese surfaces V_n^2 on the $S_{n,n}$.

A. Herzer, Mainz.

A Charakterisation of the Hall Planes.

A generalised homology α of a (finite) projective plane π is a non-planar collineation which fixes an antiflag (C, ℓ) and at least three points on ℓ. The point C is called the centre of α, the line ℓ its axis. Every homology is a generalized homology. If π is (C,ℓ)-transitive, then α is the product of a planar collineation with a (C,ℓ)-homology, where the planar collineation fixes both C and ℓ. Non-Desarguesian planes admitting gen. homologies are, for instance, the André planes. It is shown that the order $o(\alpha)$ of a gen. homology α which moves ts points on its axis ℓ such that all non-trivial cycles on ℓ have length s is $o(\alpha) = rs$, $r \leq ts-2$. (The restriction that all non-trivial cycles on ℓ have the same length is not as stringent as it looks, since this can always be achieved by raising α to an appropriate power). For $t=1$ we have $o(\alpha) \leq s(s-2)$ and, if equality holds, the plane contains lots of projective subplanes of order $s-1$. Using this we show that the Hall planes are precisely those translation planes which admit a generalized homology of order $(n+1)(n-1)$ which moves exactly $n+1=s$ points on its axis such that the moved points on ℓ form a cycle of length s. This holds for all Hall planes of order n^2, $n \neq 2$.
The talk is based on joint work with D. Jungnickel.

Klaus Vedder (Giessen)

Webs and characteristic classes.

I. Application of web geometry to the geometry of projective varieties. Bounds on geometric genus

II. Formula of Gelfand – McPherson on generalised dilogarithm (cf. Adv. Math. (1982))

S. S. Chern

Foliations transverse to a foliation

We treat the following problems. All the foliations are of class C^∞, codimension one and transversally orientable.

Problem A. When a foliation $\mathcal{F}$ admits transverse foliations $\mathcal{G}$?

Problem B. If $\mathcal{F}$ does so, then classify the foliation $\mathcal{G}$.

Tamura and Sato resolved Problem B for the Reeb component $\mathcal{F}_R$ of $S^1 \times D^2$ and, as to Problem A, showed that the foliation $\mathcal{F}_k \cup \mathcal{F}_R$ of $S^3 = C(k) \cup T(k)$ canonically constructed from a fibered knot $k \subset S^3$ does not admit transverse foliations.

We generalize their result on Problem B to the 'generalized' Reeb component of $S^1 \times S^2(h)$ where $S^2(h)$ means the h-punctured 2-sphere, and give a criterion to Problem A for the foliations obtained by attaching a finite number of 'generalized' Reeb components.

Furthermore we show that the 'generalized' Reeb component of the one-punctured torus bundle E over S^1 admits transverse foliations if and only if $\operatorname{Trace} \Phi_E \geq 2$ where Φ_E is the monodromy of E.

Toshiyuki Nishimori (Sapporo)
Hokkaido University, 〃, JAPAN.

Geometrie der Loops und Doppelloops.

Es wurde versucht aufzuzeigen, daß die Theorie der Gewebe Ordnungsprinzipien für nichtassoziative algebraische Strukturen bereitstellt und daß mit ihrer Hilfe rein geometrische Beweise algebraischer Sätze geführt werden können. (So läßt sich etwa eine Lenz-Barlotti-Klassifikation für Loops einführen und die Theorie freier Loops vollständig „geometrisieren.")

Karl Strambach (Erlangen)

On projectivities in free Benz planes

M. Funk proved that the (maximal) free Benz planes are 6-regular with respect to a certain group of projectivities. Karzel and Kroll proposed a larger group of projectivities. This brings up a new problem of regularity. Let F be the free planar extension of a Benz plane M with a "transcendental" point x on one of its blocks. We propose a definition of the group G of projectivities of M which makes G a homomorph of G(F, M), the Galois group of F over M. (An isomorph if M is an open nondeg. plane). For a subgroup G_0 of G, where no perspectivities are parallel ~~constr~~ projections, we establish 6-regularity when M is a (minimal) free Moebius or Laguerre plane. The proof is by chasing obstructions to Barlotti's proof for the projective case.

O. Iden, Bergen.

Reference.

Fortragsbuch nr 16. Oberwolfach, p 189.

Arranging lines in finite projective spaces.

There are various questions (having their roots in combinatorics) concerning lines in finite projective spaces which are also related to topics in classical algebraic geometry. We discussed some of these questions in the lecture.

Some specific topics discussed included

(a) parallelisms ~~of~~ of lines in finite projective spaces (= packings)

(b) arranging the invariant lines of a symplectic polarity into spreads

(c) A combinatorial analogue of (a) and the work of Baranyai

(d) A question of Denniston concerning the embedding of collections of spreads in a packing

(e) Partial spreads and a problem of ~~Kantor~~ Cameron and Liebler.

A. A. Bruen
U. of W. Ontario, London,
Ontario, Canada.

On a Characterisation Problem for Nets

Let X be a hyperplane and x a point on X in $PG(n+2, F)$, $n \geq 0$. Let G be the group of elations, centre x, axis X. Then $|G| = |F|$. Given a subgroup H of G, it is well known that the incidence structure of the point and hyperplane orbits of length $\neq 1$ of H has a parallelism and so does its dual. This gives an example of what Drake and Jungnickel call an (s, μ) symmetric net with $s = q/h$, $\mu = hq^n$, $|H| = h$ in case $F = GF(q)$.

These symmetric nets have been characterised for the case $h=1$, $s \neq 2$ by the property that the intersection of all blocks containing two non-parallel points has order s. For $h=1$, $s=2$, Jungnickel has characterised them by the property that the intersection of all blocks containing 3 non-parallel points has order 4. The first case was proved by Mavron and Leemans by different techniques. Analoguous characterisations for $h \neq 1$ are not known.

Let $(F, +, \cdot)$ be a cartesian group with commutative addition. For $n \geq 2$, define an incidence structure on the elements of $F^n = \{\underline{x} = (x_1, x_2, \ldots, x_n) \mid x_i \in F\}$ as points where for each $\underline{a} \in F^n$ a block is defined thus: $\{\underline{x} \in F^n \mid 0 = x_1 + a_1 x_2 + \cdots + a_{n-1} x_{n-1} + a_n\}$. In fact, more generally, one can use different cartesian groups with same + loop for $n-1$ coordinate positions. Then we have, in case $|F| = q$, a (q, q^{n-2}) symmetric net.

For $n \geq 3$, this class of nets can be characterised by the property that they can be extended by adjoining new blocks to a complete (q, q^{n-2}) net having $n-1$ parallel classes $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_{n-1}$ of new blocks satisfying:

(i) $\exists\ B_i \in \mathcal{B}_i$ such that $\phi \neq \bigcap_{i \neq t} B_i \neq \bigcap_{i=1}^{n-1} B_i$ for any t, $1 \leq t \leq n-1$.

and (ii) if $B \in \mathcal{B}_i$, any i, and C is any block of the complete net then $B \cap C$ is contained in q blocks.

VCMavron (UCW Aberystwyth)

Geometric orders and direct differential geometry
(June 10 – June 16, 1984)

Scheitelsätze und Spitzen von Kaustiken

J. W. Bruce, P. J. Giblin, and C. G. Gibson (Caustics through the looking glass, The Mathematical Intelligencer 6, no. 1 (1984), 47–58, dort weitere Literatur) betrachten ein ebenes Ovals S als Spiegel und untersuchen, welche generischen Formen die (reale oder virtuelle) Kaustik K haben kann. Das reflektierte Strahlenbündel besitzt eine Orthogonaltrajektorie (Wellenfront) W_0, welche der Lichtquelle L entspricht. Liegt L innerhalb S, so ist W_0 sternförmig bzgl. L. W_0 hat 4 Scheitel, K hat daher 4 Spitzen (falls K nicht entartet). Liegt L außerhalb S, so hat W_0 einen Doppelpunkt und K braucht nur 2 Spitzen zu haben. Schneidet ein Kreis um L den Spiegel S in $2n$ Punkten, dann schneidet ein größerer Kreis W_0 auch in $2n$ Punkten, und der $2n$-Scheitelsatz liefert $2n$ Spitzen der Kaustik K.

Erhard Heil, Darmstadt

Singularities of curves in the real projective plane

Let Γ be a directly differentiable curve in P^2. A point p of Γ is ordinary if Γ is locally convex at p (char (1,1)), otherwise p is singular. We assume that the singular points of Γ are $n_1(\Gamma)$ inflections (char (1,2)), $n_2(\Gamma)$ cusps of the first kind (thorns, char (2,1)), and $n_3(\Gamma)$ cusps of the second kind (beaks, char (2,2)). Let $n(\Gamma) = n_1(\Gamma) + n_2(\Gamma) + n_3(\Gamma)$ and $\tilde{n}(\Gamma) = n_1(\Gamma) + 2n_2(\Gamma) + n_3(\Gamma)$. Under certain conditions (most notably that every line in P^2 meets Γ with a positive even multiplicity), we determine minimum values for $n(\Gamma)$ and $\tilde{n}(\Gamma)$.

J. Bisztriczky

Convex space curves in real projective 3-space

Let Γ be a directly differentiable elementary curve in affine 3-space A^3. Γ is convex if it is the set of extreme points of its convex hull. Again a point $p \in \Gamma$ is ordinary, if Γ has a neighbourhood of p which meets every plane in at most 3 points (then p is regular and has char $(1,1,1)$; $\kappa \neq 0$ and $\tau \neq 0$); otherwise p is singular. First we show that Γ has at least 2 singular points. Since Γ has a supporting plane at each point, singular points with char. $(1,2,2)$ and $(2,1,2)$ cannot occur. If the only singular points are of char $(1,1,2)$ (inflection points, where $\kappa \neq 0$ and τ changes sign) then Γ is called inflectional and has at least 4 such points.

Tibor Bisztriczky & Jonathan Schaer

An n-vertex theorem for convex space curves

If a directly differentiable elementary convex inflectional space curve in real affine space intersects a plane in n points in a certain way, then it has at least n inflection points.

T. Bisztriczky

A convexity property of arcs of order n in n-space.

Let Γ denote a differentiable arc of order n in real affine n-space; $\Gamma_{n-1}(t)$ = osculating $(n-1)$-flat of Γ at the point $\Gamma(t)$; $H(\Gamma)$ = convex hull of Γ. Then

$$\Gamma_{n-1}(t_1) \cap \Gamma_{n-1}(t_2) \cap \operatorname{Int} H(\Gamma) = \emptyset$$

if the points $\Gamma(t_1)$ and $\Gamma(t_2)$ are distinct.

P. Scherk
and T. Bisztriczky.

Examples of direct differential geometry.

I gave five lectures outlining five well-known classical examples of direct differential geometry as follows:

(1) Direct linear differentiability.

(2) Direct conformal differentiability.

(3) Direct conical differentiability.

(4) Direct polynomial differentiability.

(5) Direct parabolic differentiability.

In each of these examples a characteristic of a differentiable point of an arc is defined. The characteristic determines the geometric order of the arc at that point.

N. D. Lane

The foundations of direct differential geometry (three lectures).

(1) Quasigraphs.

The book Geometrische Ordnungen by O. Haupt and H. Künneth starts with a certain set of axioms. These axioms require certain adaptations for work in direct differential geometry.
In this first lecture we present the notion of characteristic quasigraph, which is meant to replace the Haupt-Künneth notion of characteristic curve in our context. We also discuss the isotopic families of quasigraphs, the notions of mutual support and of mutual intersection of quasigraphs, and the local decomposition of the plane by a finite subset of an isotopic family of quasigraphs.

(2) Ordered geometry and matroids.

The dimension of a family of quasigraphs is defined by means of matroid theory.

(3) Partial results.

Under certain conditions, we obtain a general proof of the basic lemma that makes possible the definition of the numerical characteristic of a point on the basic arc. A second result, under the same conditions, shows that certain of these characteristics are never realized. These conditions and properties are verified in all the classical cases.

N. D. Lane, Peter Scherk and Jean M. Turgeon

NONLINEAR FUNCTIONAL ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS

JUNE 17 – 23, 1984

New Results on the attractors for the Navier Stokes equations

It is known that as time goes to infinity, the solutions of the two-dimensional Navier-Stokes equations associated to time independant forces, converge to a functional invariant set (f.i.s). There is even an attractor bounded in the H^1 norm, compact in L^2, to which converges any solution; it is called the universal attractor (Foias – T., J.M.P.A, 1979). It was known also that the universal attractors (which contains all the f.i.s.) has a finite Hausdorff and fractal dimensions.

The object of the lecture is to give an estimate on this dimension in term of the data. Using refined estimates on the Lyapunov numbers, one can show that the dimension is bounded by cG, where c is an absolute constant and G is the so-called Grashoff number $= \frac{\|f\| |\Omega|}{\nu^2}$, $\|f\|$ the L^2-norm of the driving forces, ν the kinematic viscosity, $|\Omega|$ = the (two dimensional) measure of the domain Ω. The details will appear in a memoir of AMS (Constantin – Foias – T., for the general results and the study of

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3-D flows in connection with turbulence) and in the lecture of the author in the proceedings of the 1983 - Berkeley Summer Research Institute, AMS Symposium series, to appear.

Roger Temam (Paris-Orsay)

Periodic solutions of prescribed energy of Hamiltonian systems

A sketch was made of a proof of the following recent result of V. Benci and the speaker:

Theorem: Let $H = H(p,q) \in C^1(\mathbb{R}^n \times \mathbb{R}^n, \mathbb{R})$ satisfy $H^{-1}(1)$ is the boundary of a compact neighborhood of 0 and is a manifold (i.e. $H' \neq 0$ on $H^{-1}(1)$) + $p \cdot H_p > 0$ if $p \neq 0$. Then the corresponding Hamiltonian system $\dot{p} = -H_q$, $\dot{q} = H_p$ possesses a periodic solution on $H^{-1}(1)$.

The proof involves the use of a minimax argument to find a critical point of $A(z) \equiv \int_0^{2\pi} p \cdot \dot{q}\, dt$ on the set $\mathcal{H}(z) \equiv \frac{1}{2\pi}\int_0^{2\pi} H(z(t))\, dt = 1$ where $z = (p,q)$ is 2π periodic. This critical point then ~~gives~~ is a periodic solution of the Hamiltonian system.

Paul H. Rabinowitz
Madison, Wisconsin

An index for periodic orbits

We define an index for isolated sets of periodic orbits of a semiflow in the following situation: X is an ANR, $\Phi = (\varphi_t)_{t \geq 0}$ is a semiflow on X possessing a compact attractor, and there is a $T \geq 0$ such that φ_T is locally compact. Let $\Pi(\Phi) := \{(x,t) \in X \times [0,\infty) \mid \varphi_t x = x\}$, let Ω be open in $X \times [0,\infty)$ such that $\Pi(\Phi) \cap \partial\Omega = \emptyset$ and such that $pr_2(\Omega \cap \Pi(\Phi))$ is contained in a compact subset of (T_0, ∞). Let $P \subset \Omega$ be an isolated subset of $\Pi(\Phi)$. We then define an index of fixed point type, $i(\Phi, P)$ for P. Our definition generalizes the approach by Fuller (Amer. J. Math. 89 (1967) 133-148) in that we do not assume smoothness and we do not need to assume that Φ is generated by an autonomous (functional) differential equation.

Christian Fenske (Gießen)

Some free boundary problems for reaction-diffusion systems.

The solutions of some nonlinear systems with nonlinearities which are not locally Lipschitz can be zero on some subdomains. We give sufficient and/or necessary conditions for the existence of this "dead core" (its boundary is called the "free boundary"), and also some information about its size and location. The results are proved by using comparison arguments involving local supersolutions. Some applications are given to systems arising in combustion theory and Lotka-Volterra systems with nonlinear diffusion.

Jesús Hernández

(Madrid)

A free boundary problem for minimal surfaces

For manifolds $S \subset \mathbb{R}^3$ diffeomorphic to the standard sphere in $\mathbb{R}^3$ the existence of non-constant minimal surfaces bounded by S and intersecting S orthogonally along their (free) boundaries is deduced.
The proof uses an approximation argument introduced by Sacks and Uhlenbeck for the study of harmonic mappings and a minimax characterization of critical values.

Michael Struwe

Variational Inequalities in Orlicz-Sobolev Spaces

We work in the complementary system of Orlicz-Sobolev spaces $(W_0^1L_M(\Omega), W_0^1E_M(\Omega); W^{-1}L_{\bar M}(\Omega), W^{-1}E_{\bar M}(\Omega))$ and consider a mapping $T: D(T) \subset W_0^1L_M(\Omega) \to W^{-1}L_{\bar M}(\Omega)$ corresponding to a second order nonlinear differential expression in divergence form. Assumptions of the Leray-Lions type are made on the coefficients of this differential expression which guarantee that T is finitely continuous, pseudo-monotone and satisfies, in a weak sense, a boundedness and a coercivity condition. Let $K \subset W_0^1L_M(\Omega)$ be convex, $\sigma(W_0^1L_M(\Omega), W^{-1}E_{\bar M}(\Omega))$ closed and such that: (*) $K \cap W_0^1E_M(\Omega)$ is $\sigma(W_0^1L_M(\Omega), W^{-1}L_{\bar M}(\Omega))$ dense in K. Let f be given in $W^{-1}E_{\bar M}(\Omega)$. Then there exists $u \in K \cap D(T)$ solution of the variational inequality $\langle u-v, Tu\rangle \leq \langle u-v, f\rangle$ for all $v \in K$. Our purpose in this talk is to discuss condition (*) on K. This is a problem of approximation within a convex set. We show that (*) holds for the obstacle problem when the obstacle function satisfies some mild regularity condition. (Joint work with V. Mustonen).

Jean-Pierre Gossez (Bruxelles).

Some multiplicity results for semilinear equations crossing higher eigenvalues

The purpose of this talk was to survey recent results ~~of the~~ on the Dirichlet problem $\Delta u + f(u) = h(x)$ under the assumptions that the interval $(f'(-\infty), f'(+\infty))$ contains eigenvalues of the Laplacian with Dirichlet B.C.s Four classes of results were surveyed, (a) if $\{\lambda_1 \ldots \lambda_n\} \in (f'(-\infty), f'(+\infty))$ (b) if $\{\lambda_k \ldots \lambda_{k+n}\} \in (f'(-\infty), f'(+\infty))$ (c) the corresponding more detailed results for the O.D.E. and d) results for operators without compact ~~[illegible]~~ inverses. Several natural conjectures were mentioned

Joe McKenna

Periodic solutions to equations of magnetohydrodynamics of incompressible and compressible fluids.

The lecture is based on results of the following two papers by O. Vejvoda and M. Štědrý:

1) Small time-periodic solutions of equations of magnetohydrodynamics as a singularly perturbed problem, Aplikace matematiky 28 (1983), 344–356,

2) A note on equations of magnetohydrodynamics

of compressible fluid, to appear in Aplikace matematiky.

In the first paper one proves the existence of a periodic solution to MHD equations of incompressible fluid and its convergence to a similar solution of the shortened system of MHD equations as considered e.g. by O.A. Ladyženskaja and V.A. Solonnikov.

In the second one, using the idea of A. Valli, initial-boundary and periodic problems for MHD equation are treated.

O. Vejvoda (Praha)

On infinitely many solutions to some nonlinear homogeneous equation.

In $L = L_1(\Omega) + L_\infty(\Omega)$, Ω being a measure space with σ-finite positive measure, the equation of the form $Su + Tu = F(u)$ is investigated, where S is a linear operator in L, T a bounded linear perturbation of S and F is a superposition operator. The existence of infinitely many large solutions, if F is superlinear, and of infinitely many small solutions, if F is sublinear is established under the main assumptions that S and T are symmetric (in a certain sense) and F is an odd function.

V. Lovicar (Praha)

We give a short discussion of the equations of motions of strings. There are two cases the extensible and the inextensible string. The equations are of indefinite type and there is no rigorous result on the time evolution of these objects. In the stationary cases (including uniformly rotating configurations) the problem of indefiniteness of the space part of the differential operator can be completely resolved. The equations can be equivalently represented by a family of elliptic problems parametrized by the measurable subsets of the parameter interval of the string. Each of the elliptic problems can be treated by current methods of non-linear functional analysis.

M. Reeken
(Wuppertal)

The range of some semilinear elliptic operators

Clarke - Ekeland's dual least action is used to prove the existence of solutions for the abstract equation in $V \subset L^2(\Omega, \mathbb{R}^N)$

(1) $$Lu = \nabla_u F(x,u)$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain, $L : D(L) \subset V \to V$ is self-adjoint with closed range, nontrivial kernel and spectrum $\sigma(L) = \{ \dots < \lambda_{-1} < 0 < \lambda_1 < \dots \}$, with λ_i eigenvalues of finite multiplicity, $F(x,\cdot)$ is convex, $\nabla_u F(x,\cdot)$ exist and $\nabla_u F(\cdot, u(\cdot)) \in V$ for $u \in D(L)$. We assume moreover that $F(x,u) \geq (l(x),u) - \beta(x)$ for some $l \in L^2(\Omega, \mathbb{R}^N)$ and $\beta \in L^2(\Omega, \mathbb{R}_+)$.

The result is the following: Assume that

i) $\exists\, \alpha \in L^\infty(\Omega)$, $\operatorname{ess\,inf}_\Omega \alpha > 0$ such that $\overline{\lim}_{|u|\to\infty} \frac{F(x,u)}{|u|^2} \le \alpha(x)$ unif. in x.

ii) $\alpha(x) \le \lambda_1$ and $\int_\Omega [\lambda_1 - \alpha(x)](v^1(x))^2 dx > 0$ for all $v^1 \in \ker(L - \lambda_1 I) \setminus \{0\}$

iii) $\int_\Omega F(x, \bar{u}(x))\,dx \to +\infty$ if $\|\bar{u}\| \to \infty$, $\bar{u} \in \ker L$.

Then problem (1) has a solution minimizing on $R(L)$ the dual action $X : v \mapsto \int_\Omega [-(Kv, v) + F^*(x,v)]dx$

with K the right inverse of L and $F^*(x,.)$ the Fenchel transform of $F(x,.)$.

Applications are given to periodic solutions of Hamiltonian systems, hyperbolic equations and to elliptic problems of the form

$$-\Delta u = f(x,u) \quad \text{in } \Omega$$
$$u = 0 \text{ or } \frac{\partial u}{\partial n} = 0 \quad \text{in } \partial\Omega$$

with $f(x,.)$ non decreasing.

This is joint work with Ward - Willem and Willem.

J. Mawhin
(Louvain-la-Neuve)

On the asymptotic behaviour of solutions of nonlinear evolution equations.

The following ergodic theorem for semigroups of nonexpansive mappings holds;

Thm: Let $C \subset X$ be a closed subset of a Banach space X and $f : C \to X$ be uniformly continuous and bounded on bounded subsets of C. Let $S(t)$, $t \ge 0$ be a semigroup of contractions on C. If for some $x \in C$ the trajectory $\gamma(x) = \{S(t)x : t \ge 0\}$ is precompact then $\omega(x)$ is a nonempty commutative group and

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T f(S(t)x)\,dt = \int_{\omega(x)} f(\xi)\,d\xi$$

where $d\xi$ is the unique normalized Haar measure on $\omega(x)$.

A. Pazy
(Jerusalem)

The Morse Smale property for some semilinear parabolic equations.

The parabolic initial value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(x,u)$ $x \in [0,\pi], t>0$ with Dirichlet bdy conditions generates a semiflow T_t on $X = H_0^1(0,\pi)$. If there is an $M>0$ such that $u \cdot f(x,u) \le 0$ for $|u| \ge M$, then $A_f = \{u \in X : u$ lies on a complete bdd. orbit of $T_t\}$ is compact ($\{T_t : t \ge 0\}$ is the semiflow). An abstract result of Hale and others asserts that A_f is structurally stable (w.r.t. perturbations of $f(x,u)$) if the semiflow has the Morse Smale property. We prove the following

Thm If all fixed points of $\{T_t : t \ge 0\}$ are hyperbolic, then $\{T_t : t \ge 0\}$ has the M.S. property.

The proof uses the "lap-number" introduced by Hiroshi Matano.

Sigurd Angenent (Leiden).

New degree theories and their applications

Two theories of degree of mapping are considered which generalize the Leray-Schauder theory and have significant applications to problems in nonlinear partial differential equations, the first to the study of periodic solutions of nonlinear wave equations, the second to fully nonlinear elliptic equations of arbitrary even order.

1) In studying periodic solutions of nonlinear wave equations of the form $u_{tt} - u_{xx} + g(u) = h(t,x)$, $0 \le x, t \le \pi$, Brézis and Nirenberg have considered operators in a Hilbert space H of the form $L + N$ with L a closed, densely defined linear operator having $\operatorname{Im}(L) = \operatorname{Ker}(L)^{\perp}$; P the orthogonal projection of H on $\operatorname{Ker}(L)$ and L having a compact inverse on $\operatorname{Im}(L)$. Suppose that the nonlinear operator N is continuous and bounded, and

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satisfies P-$(S)_+$: if $Pu_j \to Pu$, $(I-P)u_j \to (I-P)u$, $\overline{\lim}\,(Nu_j, u_j - u) \leq 0 \Longrightarrow u_j \to u$.

The existence and uniqueness of a classical degree function established for the class of maps $L+N$, using a generalized Galerkin procedure introduced by Mawhin in 1982.

(2) The existence and uniqueness of a classical degree theory is established for nonlinear Fredholm operators of index zero, under the hypothesis that their differentials df_x fall in a convex class of Fredholm linear operators of index zero closed under the addition of compact linear operators. Strongly elliptic differential operators of any given order $2m$ under Dirichlet boundary conditions generate such a class. One must assume only that for the target point y_0, $f^{-1}(y_0)$ is compact. This theory forms a part of a broader theory published by the writer in 1976 for a broader class of mappings given by topological conditions and closed under addition of arbitrary nonlinear compact maps. Emphasis must be placed on verifying the compactness (rather than precompactness) of the inverse image during a degree-preserving homotopy. For fully nonlinear partial differential equations with possible non-elliptic solutions, such verification involves not only a-priori bounds but an a-priori verification of uniform ellipticity at all the elliptic solutions considered in the deformation.

Felix Browder (University of Chicago).

The Parity of Curves of Fredholm Operators, and Global Bifurcation for NonLinear Elliptic Problems.

We outline some recent results of the speaker and J. Pejsachowicz

First we show that a fully nonlinear elliptic boundary value problem whose formal linearization gives rise to a properly elliptic problem covered by a normal family of boundary conditions verifying the Lopatinsky - Shapiro conditions may be written as the zeros of a map F whose functional setting is as follows:

X, Y and Z are Banach spaces, $Z \subset\subset X$, $F: X \to Y$

$$F(x) = L_x(x) + C(x)$$

where: $C: X \to Y$ is compact

$L_z \in \Phi_0(X,Y)$, for $z \in Z$

$z \mapsto L_z$ is continuous from Z to $\mathcal{L}(X,Y)$.

(Here $\Phi_0(X,Y)$ are Fredholm maps of index 0, $\mathcal{K}(X,Y)$ will denote the compact maps)

A mapping as above we call quasi-linear Fredholm (q.l.F.).

When $\mathcal{O} \subseteq X$ is open and bounded, $F: X \to Y$ is q.l.F., and $0 \notin F(\partial \mathcal{O})$ we define a topological degree

$$\operatorname{Deg}(F, \mathcal{O}, 0).$$

By using well-known results on compact families of linear Fredholm operators we rewrite $F(x)=0$ as the fixed point of a compact map and then the definition proceeds via the Leray-Schauder degree.

The degree is independent of the representation of F; it has the existence and additivity properties, the degree at $x=0$ can be computed by linearization when the linearization is invertible and one has a weak homotopy property.

These properties allow us to keep track of orientation and so prove multiplicity and bifurcation results.

Def: Let $\alpha : [0,1] \to \Phi_0(X,Y)$ be continuous with $\alpha(0)$ and $\alpha(1)$ invertible. Choose $\beta : [0,1] \to \mathcal{L}(X,Y)$ continuous, with $\beta(0)=0$ and such that $\alpha(t)+\beta(t)$ is invertible for $0 \le t \le 1$. We define the parity of α, $p(\alpha)$, by

$$p(\alpha) = \deg_{L.S.}\left((\alpha(1)+\beta(1))^{-1}\alpha(1), B(0,1), 0 \right).$$

The parity is well-defined and we have the

Theorem: Let $F_\lambda : \mathbb{R}\times X \to Y$ be a continuous family of q.l.F. maps. Assume $F_\lambda(0) \equiv 0$. Let F be differentiable in x and let

$$\alpha(t) = \frac{\partial F_t}{\partial x}\Big|_{x=0}, \quad 0 \le t \le 1.$$

Assume $\alpha(0)$ and $\alpha(1)$ are invertible and $p(\alpha) \neq 1$. Then there is global bifurcation from $\{ (t,0) \in \mathbb{R}\times X \mid 0 \le t \le 1 \}$ of nontrivial solutions of

$$F_\lambda(x) = 0.$$

Patrick M. Fitzpatrick (University of Maryland)

On some extension of convex analysis and application to systems of PDEs.

Although ordinary convexity is appropriate for the study of scalar variational problems whose Euler equations are just one single pde, it is too strong a condition for vectorial problems whose Euler equations are systems of pdes. The appropriate notion is the quasiconvexity condition introduced by Morrey. In this talk we present some extensions of well known results of convex analysis (in particular Carathéodory's Theorem) to quasiconvex functions.

B. Dacorogna (Ecole Polyt. / Lausanne)

On the uniqueness and the singular set of weak solutions of the Navier-Stokes equations.

We consider the Navier-Stokes equations

$$u' - \nu\Delta u + u\cdot\nabla u + \nabla\pi = f,$$
$$\nabla\cdot u = 0,$$
$$u|\partial\Omega = 0,$$
$$u(0) = \varphi$$

over $(0,T)\times\Omega \subset \mathbb{R}^{n+1}$. We improve on Serrin's conditions for uniqueness and regularity for weak solutions. We prove the following theorems:

Thm. 1: Let u^1, u^2 be two weak solutions, having the same data, let u^1 fulfill the energy inequality in $t=0$, let $u^2 \in L^s([0,T), L^r(\Omega))$ for some s, r with $\frac{2}{s}+\frac{n}{r} = 1$, $r>n$, then $u^1 = u^2$ on $[0,T]$. If $u^2 \in C^0([0,T], L^n(\Omega))$ then also $u^1 = u^2$ on $[0,T]$.

The first part of Thm. 1 was proved by Serrin for $n = 2,3,4$. We also show that $L^\infty([0,T), L^n(\Omega))$ is a uniqueness class for weak solutions.

Thm. 2: Let $u \in L^\infty([0,T), L^n(\Omega))$. Then u has at most countably many singular points in t, i.e. in these t $u(t)$ is no C^2-function. If $u \in L^s([0,T), L^r(\Omega))$ with s, r as above then u is regular for $n = 3,4,5,6$.

This work was done together with H. Sohr

(Paderborn).

Wolf von Wahl

A connection between the generalized Hardy inequality and some nonlinear boundary value problem

Conditions on the weight functions $a_0, a_1, \dots, a_N$ are derived which guarantee the validity of the estimate

$$(*) \quad \left(\int_\Omega |u(x)|^q a_0(x)\, dx \right)^{1/q} \leq c \left(\sum_{i=1}^{N} \int_\Omega \left| \frac{\partial u}{\partial x_i} \right|^p a_i(x)\, dx \right)^{1/p}$$

for a rather wide class $V \,(\subset W^{1,1}(\Omega))$ of functions $u = u(x)$. Here Ω is an open set in $\mathbb{R}^N$, p, q are real numbers, $1 \leq q \leq p < \infty$. Inequality $(*)$ can be considered as an generalized N-dimensional Hardy inequality.

The cases $N = 1$, $q > p$, and $N > 1$, $q = p$, are treated and it is shown, that the conditions on a_i $(i = 0, 1, \dots, N)$ can be expressed in terms of solvability of a certain boundary value problem on $\Omega \cup \partial\Omega$, for an equation, in which the weight functions appear as coefficients. E.g. for $N > 1$, $p = q$ this equation has the form

$$\sum_{i=1}^{N} \frac{\partial}{\partial x_i} \left(a_i \left| \frac{\partial w}{\partial x_i} \right|^{p-1} \operatorname{sgn} \frac{\partial w}{\partial x_i} \right) + a_0 |w|^{p-1} \operatorname{sgn} w = 0 .$$

Examples are given.

Alois Kufner
Math. Inst. Acad. Sci
Prague, Czechoslovakia

Solitary waves in stratified fluids

We report on recent work with C. Amick. The problem analysed is that of two-dimensional wave motion in a heterogeneous, inviscid fluid confined between two rigid, horizontal planes and subject to gravity g. It is assumed that a fluid of constant density ρ_+ lies above a fluid of constant density $\rho_- > \rho_+ > 0$ and that the system is nondiffusive. Progressing solitary waves can be described in a moving coordinate system by $\lambda = g/c^2$, c the wave speed, and w where $w(x,\eta)+\eta$ is the height at a horizontal position x of the streamline which has height η at $x = \pm\infty$. It is shown that among the nontrivial solutions of a quasilinear elliptic eigenvalue problem for (λ, w) is an unbounded, connected set in $\mathbb{R} \times (H_0^1 \cap C^{0,1})$. Various properties of the solutions are shown.

Robert E. Turner
Madison, Wisconsin

Pinched hypersurfaces and Hamiltonian systems.

Let $S \subset \mathbb{R}^{2n}$ be a compact hypersurface, bounding a convex set, and $J \in \mathcal{L}(\mathbb{R}^{2n})$ the matrix $\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$. We consider the Hamiltonian flow on S, given by:

(E) $\qquad \dot{x} = J\,n(x) \quad , \; x \in S$

where $n(x)$ is the normal to S at x, and we are interested in its closed trajectories.

We say that S is δ-pinched, for some $\delta \in (0,1)$, if (a) the second fundamental form always lies between δI and I, and (b) the surface lies between two concentric balls of radii 1 and $1/\delta$.

We then prove the following:

(i) if S is δ-pinched, all closed trajectories of (E) satisfy $\oint \frac{1}{2}(Jx, dx) \geq \pi$ (this estimate is originally due to Croke and Weinstein)

(ii) if S is δ-pinched, there is at least one closed trajectory with $\oint \frac{1}{2}(Jx, dx) \leq \pi\delta^{-2}$

(iii) if S is $(\sqrt{2})^{-1}$-pinched, or δ-pinched with $\delta \in [\frac{1}{\sqrt{2}}, 1]$, then (E) has at least one closed trajectory which is linearly stable.

Ivar Ekeland
CEREMADE
Université Paris-Dauphine

On the Semigroup Approach to Boundary Value Problems

Let X be a Banach space and A be the infinitesimal generator of a linear C_0-semigroup $S(\cdot)$ on X.
We consider the Cauchy problem

(1) $\frac{d}{dt} x(t) = A(I + B(t))\, x(t), \quad t \geq t_0$

$x(t_0) = x_0$

where $B(\cdot)$ is a family of (nonlinear) operators in X.

We give conditions that ensure that (1) is well-posed and apply the results to delay equations and age-dependent population dynamics.
Finally, we investigate the qualitative behavior of the solutions in terms of the spectrum of A.

D. Schappacher (Graz).

Multiplicity results for some nonlinear second order ODE's

We prove a multiplicity result of Lazer and McKenna, which relates the number of solutions of a two point boundary value problem with the number of eigenvalues crossed by the nonlinearity, by constructing bifurcation branches of an appropriate bifurcation problem. The same technique can also be applied to some superlinear problems, establishing a relation between the number of solutions and the number of eigenvalues which are

<u>not</u> crossed by the nonlinearity

Bernhard Ruf (Trieste)

<u>Elliptic nonlinear problems involving the critical Sobolev exponent</u>

Let Ω be a bounded domain in $\mathbb{R}^n$, $n \geq 3$, $\lambda \in \mathbb{R}$, $2^* = \frac{2n}{n-2}$. Consider the problem

$$(1) \quad -\Delta u - \lambda u - u|u|^{2^*-2} = 0 \qquad u \in H_0^1(\Omega)$$

Since the embedding $H_0^1(\Omega) \hookrightarrow L^{2^*}(\Omega)$ is not compact, the ~~fe~~ energy functional associated to (1) does not satisfies, in general, the Palais-Smale condition. In a joint paper with Cerami-Struwe, the existence of non trivial solutions of (1) near arbitrary eigenvalue λ_k of $-\Delta$ ~~[illegible]~~ is proved. Moreover it can be proved that if ~~[illegible]~~ $n \geq 7$, (1) has nontrivial solutions for any $\lambda > 0$ and if $n = 5, 6$ (1) has non trivial solutions for $\lambda > 0$, ~~and~~ $\lambda \notin \sigma(-\Delta)$ ($\sigma(-\Delta)$ denotes the spectrum of $-\Delta$).

Donato Fortunato (Bari)

Application of A-proper mapping theory to the solvability of ODE' and PDE's

Let $y \in C([0,T])$ and $C^2([0,T])$ be the Banach space of twice continuously differentiable function with the norm $|x|_2 = \max\{|x|_0, |x'|_0, |x''|\}$. We establish the (constructive) solvability of ODEquation of the form

$$(A) \quad x'' = f(t, x, x', x'') - y(t) \qquad 0 \leq t \leq T,$$

subject to one of the boundary conditions (I) $u(0)=u(T)=0$; (II) $u'(0)=u'(T)=0$; (III) $u(0)=u(T)$, $u'(0)=u'(T)$; (IV) $-\alpha x(0)+\beta x'(0)=0$, $a x(T)+b x'(T)=0$ for suitable $\alpha, \beta, a, b \geq 0$. The main novelty of the result is that f depends on the highest order derivative x'' and has a very general growth of the form

$$|f(t,x,r,q)| \leq A(t,x)r^2 + B|q| + C(t,x), \quad 0 \leq B < 1,$$

A and C are cont. fts on $[0,T] \times R$, bounded on compact subset of $[0,T] \times R$. The function $f(t,x,0,q)$ is assumed to satisfy conditions of the form $x \geq M \Rightarrow f(t,x,0,q) > a$ and $x \leq -M \Rightarrow f(t,x,0,q) < b$ for $t \in [0,T]$ and $q \in R$, and $b \leq y(t) \leq a$. When $a=b=0$ and f is independent of x'' we obtain the result of Granas, Guenther and Lee. The Neumann problem is also considered.

The second result deals with the solvability with the PDE

$$(B) \quad \sum_{|\alpha| \leq m} (-1)^{\alpha} D^{\alpha} A_{\alpha}(x, D^{\gamma}u) - \lambda \sum_{|\beta| \leq m-1} (-1)^{\beta} D^{\beta} B_{\beta}(x, D^{\gamma}u) = f \quad |\gamma| \leq m$$

It is shown that if A_α and B_β are close to odd and λ-hom. asymptotes given by A_α^0 and B_β^0, then (B) has a weak solution in V, $\mathring{W}_p^m \subset V \subset W_p^m(Q)$, $Q \subset R^n$ bounded domain provided A_α and A_α^0 generate A-proper maps from $V \to V^*$ and B_β and B_β^0 generate compl. cont. maps from $V \to V^*$ and $\lambda \neq 0$ is not a generalized eigenvalue of

$$(C) \quad \sum_{|\gamma| \leq m} (-1)^{\alpha} D^{\alpha} A_\alpha^0(x, D^{\gamma}x) - \lambda \sum_{|\beta| \leq m-1} (-1)^{\beta} B_\beta^0(x, D^{\gamma}x) = 0.$$

W. V. Petryshyn.

Critical point theory and nonlinear elliptic partial differential equations in $\mathbb{R}^N$

Henri Berestycki (Univ. Paris 13)

This talk reports on a joint work with Clifford TAUBES. We consider the problem (for example):

$$(1)\quad \begin{cases} -\Delta u + u = a(x)\,|u|^{p-1}u & \text{in } \mathbb{R}^N \\ u \in H^1(\mathbb{R}^N) \end{cases}$$

Theorem: Assume $a \in C^1(\mathbb{R}^N, \mathbb{R})$, $0 < a_\infty = \lim_{|x|\to\infty} a(x)$ and $a(x) \geq a_\infty + e^{-\nu|x|}$ for large $|x|$, with $\nu < 1$. Then, (1) admits infinitely many solutions.

Similar results are obtained in more general settings, for problems like

$$\begin{cases} -\partial_i\,(a_{ij}(x)\,\partial_j u) + c(x)\,u = f(x,u) & \text{in } \Omega \\ u \in H^1_0(\Omega) \end{cases}$$

under suitable assumptions on a_{ij}, c, f, with Ω "looking like an angle" outside some ball.

These results are obtained using some new methods in critical point theory which rely on work by C. Taubes about Yang-Mills-Higgs equations.

Lastly, we further indicate many examples of non-existence, of existence of only finitely many or existence of non-spherically symmetric solutions ~~to (1)~~, (when $a(x) = a(|x|)$) for equation (1).

H.B.

"On some quasilinear elliptic problems at resonance."

Sergio Solimini
S.I.S.S.A. – Trieste.

The existence of solutions of some elliptic problems at resonance with respect to a simple eigenvalue is studied. The ~~same~~ proof works for a class of nonlinearitis which includes as particular cases of main interest the periodic functions and the functions wich tend to zero at infinity with their primitive. The proof of the main result consits in establishing a suitable "saddle point theorem" for a functional which does not satisfy completely the Palais-Smale compactness condition.

S. Solimini.

" Solutions of prescribed minimal period for convex Hamiltonian systems"

Helmut Hofer, Bath UK.

Since the seminal work of Rabinowitz in 77-78 much work has been in studying the existence of periodic solutions of Hamiltonian systems $-J\dot{x} = H'(x)$ having a prescribed period T. However a question already raised by Rabinowitz and then frequently repeated by other authors if among the periodic solutions found at least one has minimal period T remained unanswered. However if the Hamiltonian is convex a rather satisfactory answer can be given. Always if the mountain pass theorem is applicable to the dual functional Ψ or a local minimum is found the corresponding periodic solution have minimal period. (The result is a joint work with J. Thelen? Paris)

"Some remarks on the continuation method and the method of monotone iterations." Philippe Clément, Delft, The Netherlands

Let (E, P) denote an ordered Banach space with nonempty interior $\mathring{P}$, and let $a, b \in E$ such that $b - a \in \mathring{P}$ (not: $a \ll b$), $\lambda_1, \lambda_2 \in \mathbb{R}$ with $\lambda_1 < \lambda_2$. Consider $K : [\lambda_1, \lambda_2] \times [a, b] \to [a, b]$ continuous with relatively compact range in $[a, b]$. Let $S := \{(\lambda, u) \in [\lambda_1, \lambda_2] \times [a, b] \mid u = K(\lambda, u)\}$. Let us further assume that K satisfies the following assumptions:

(i) For each $\lambda \in (\lambda_1, \lambda_2)$, $K(\lambda, \cdot)$ is strictly increasing. (i.e. $u < v \Rightarrow K(\lambda, u) \ll K(\lambda, v)$).

(ii) For each $(\lambda, u) \in S$ and $\mu_1 < \lambda < \mu_2$, $K(\mu_1, u) < K(\lambda, u) < K(\mu_2, u)$

(iii) $K(\lambda_1, u) = u \Rightarrow u = a$ and $K(\lambda_2, u) = u \Rightarrow u = b$

Then (i) C the component of (λ_1, a) in S meets (λ_2, b) (continuation)

(ii) for each $\lambda \in (\lambda_1, \lambda_2)$ there exists a minimal solution $(\lambda, \check{u}(\lambda))$ and a maximal solution $(\lambda, \hat{u}(\lambda))$.

(iii) Any closed connected set D in S which meets $(\lambda_1, a), (\lambda_2, b)$ contains all minimal $(\lambda, \check{u}(\lambda))$ and maximal $(\lambda, \hat{u}(\lambda))$ fixed ~~the~~ points of $K(\lambda, \cdot)$, $\lambda \in (\lambda_1, \lambda_2)$.

As an example we consider

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega \text{ bdd, open domain of } \mathbb{R}^N. \\ u = 0 & \text{on the boundary.} \end{cases}$$

where $f \in C^1(\mathbb{R})$ $(f(0) > 0)$, $f(u) > 0$, $u \in [0, \alpha)$, $f(\alpha) = 0$. ~~and satisfy~~ Then every ~~[illegible]~~ ~~[illegible]~~ maximal (resp. minimal) solution $(\lambda, \hat{u}(\lambda), \lambda, \check{u}(\lambda))$ such that $\hat{u}(\lambda)$ (resp. $\check{u}(\lambda)$) $\in [0, \alpha]$ belongs ~~to~~ to the component of S in $\mathbb{R} \times C^1$ which contains $(0, 0)$.

"A maximum principle for an elliptic system and applications to semilinear problems". Djairo G. de Figueiredo (U. of Brasilia)

The Dirichlet problem for the semilinear elliptic systems

(*) $\quad -\Delta u = f(x,u) - v, \quad -\Delta v = \delta u - \gamma v \quad$ in Ω,

where Ω is a bounded smooth domain in R^N, is studied. Here δ and γ denote positive constants. The solutions (u,v) of (*) represent steady state solutions of reaction diffusion systems of relevance in Biology. The authors ~~question~~ (this is joint work with Enzo Mitidieri) consider general classes of nonlinearities f, which are modelled in examples that often appear in the applications. Namely (i) f behaving like $\lambda u - u^3$ where $\lambda > 0$ is some real parameter, and (ii) $f(u) = u(u-a)(1-u)$ where $0 < a < 1/2$ is some given real number. In order to ascertain if solutions of (*) are positive one needs maximum principle for systems like

$$-\Delta u = +\lambda u - v + f(x) \qquad -\Delta v = \delta u - \gamma v$$

It is proved that if $-\gamma + 2\sqrt{\delta} \le \lambda < \lambda_1 + \delta/(\gamma + \lambda_1)$ then $f \ge 0$ in Ω implies $u, v > 0$ in Ω.

D. G. de Figueiredo.

On periodic-parabolic Eigenvalue Problems

We consider the eigenvalue problem

$$(*)\begin{cases} Lu = \lambda m u & \text{in } \Omega \times \mathbb{R} \\ Bu = 0 & \text{on } \partial\Omega \times \mathbb{R} \\ u(\cdot, 0) = u(\cdot, T) & \text{on } \overline{\Omega} \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a suff. smooth ~~bounded~~ domain, $T > 0$ a prescribed period, B denotes either a Dirichlet boundary operator or else a Neumann boundary operator and $Lu := \frac{\partial}{\partial t} u - a_{jk}(x,t)\partial_j\partial_k u + a_j(x,t)\partial_j u + c_0(x,t) u$, where $c_0 \ge 0$ and

a_{ijk} is uniformly pos. def. Finally a_{jk}, a_j, a and m (which has not to be positive) belong to $\{u \in C^{\alpha,\alpha/2}(\bar{\Omega} \times \mathbb{R}) : u$ is T-periodic in $t\}$. Then we have the

<u>Thm</u>: (*) admits a positive eigenvalue $\lambda_1(m)$ having a positive eigenfunction u iff $P(m) > 0$, where
$$P(m) := \int_0^T \sup_{x\in\Omega} m(x,t)\,dt .$$

A. Beltramo

Closed trajectories for hamiltonian flows on starshaped manifolds.

Let S be a C^1 manifold in $\mathbb{R}^{2N}$, radially diffeomorphic to the unit sphere. We consider the problem of finding periodic solutions for the hamiltonian system
$$\dot z = J\nu(z) \qquad z \in S, \quad \nu(z) \text{ denoting the}$$
external normal at z.

We show that if S is suitably nested between two ellipsoids, the associated hamiltonian flow has at least N closed orbits.

Mancini

Constructive Methods for the Practical Treatment of Integral Equations
25.–30. Juni 1984

Stability results for discrete Volterra equations: Numerical experiments

In this lecture we propose a local stability criterion for linear multistep discretizations of first- and second-kind Volterra integral equations with finitely decomposable kernels. In a large number of numerical experiments this criterion is tested. We did not find examples that behaved unstable while the stability criterion predicted stability. Conversely, we found many examples which behaved stable whereas the stability criterion predicted instability. A possible explanation might be the fact that the decomposition of the kernel does not enter into the stability criterion, that is, the criterion holds for the most ill-conditioned decomposition and consequently it will be rather conservative.

P.J. van der Houwen

"Evolutionary problems of Volterra type":—

Amongst possible competitors as numerical methods for Volterra integral equations of the second kind are (i) quadrature methods, (ii) extended Runge-Kutta methods and (iii) mixed quadrature-Runge-Kutta methods. The error analysis for such methods, and stability analysis for the first two, already exist. Although the mixed quadrature Runge-Kutta methods have certain attractions computationally, they appear to have fallen out of favour. We argue that such methods should not be too readily discarded: we consider the stability analysis of mixed quadrature-Runge-Kutta methods using $\{\rho,\sigma\}$–reducible quadrature rules and establish the existence of A_0-, A- and $A(\alpha)$–stable methods. Extensions of the stability analysis to convolution equations and integro-differential equations can be made. C.T.H. Baker, U. Manchester.

Die Monotonie der Templeschen Quotienten

In einem reellen Hilbert-Raum $(H; \langle\cdot,\cdot\rangle)$ sei eine Eigenwertaufgabe
$M\varphi = \lambda N\varphi$, $\varphi \in D(M)$ mit Operatoren
$M: D(M) \to H$ symmetrisch, positiv definit
$N: D(N) \to H$ symmetrisch, positiv definit $\Big\}$ $D(M) \subset D(N) \subset H$
gegeben. Beim „Verfahren der schrittweisen Näherungen" (s. L. Collatz, Eigenwertaufgaben mit technischen Anwendungen) hat man zunächst eine Folge $\{F_p\}$ gemäß $F_0 \in D(N)$, $MF_p = NF_{p-1}$, $F_p \in D(M)$ für $p \in \mathbb{N}$ zu berechnen (es seien F_0, F_1 linear unabhängig), dann die Folge $\{a_{p+q}\}$ gemäß $a_{p+q} = \langle F_p, NF_q\rangle$ für $p+q \in \mathbb{N}_0$ sowie die Folge $\{\mu_p\}$ gemäß $\mu_p = a_{p-1}/a_p$ für $p \in \mathbb{N}_1$ (Schwarz'sche Quotienten) und schließlich die Folge $\{\tau_p\}$ der Temple'schen Quotienten gemäß $\tau_p = (La_{p-1} - a_{p-2})/(La_p - a_{p-1})$ für $p \in \mathbb{N}_2$ (vorausgesetzt wird dabei $\mu_1 < L < \lambda_2$). Neben $\lambda_1 \le \dots < \mu_3 < \mu_2 < \mu_1$ (L. Collatz, EWA) gilt dann $\tau_2 < \tau_3 < \tau_4 < \dots \le \lambda_1$ (F. Goerisch).
Arbeitet man nicht nur mit einer Folge $\{F_p\}$, sondern mit n Folgen $\{F_p^{[\nu]}\}$ $(\nu = 1, \dots, n)$, so lassen sich – unter der Voraussetzung $\mu_1^{[s]} < L < \lambda_{s+1}$, wobei $s \le n$ – Intervallschachtelungen $(\sigma = 1, \dots, s)$
$\tau_2^{[\sigma]} < \tau_3^{[\sigma]} < \tau_4^{[\sigma]} < \dots \le \lambda_\sigma \le \dots < \mu_3^{[\sigma]} < \mu_2^{[\sigma]} < \mu_1^{[\sigma]}$ beweisen (F. Goerisch).

Julius Albrecht, Clausthal

Optimal control of a Volterra process involving hysteresis

We consider the problem of optimal control

Minimize $L_T(x(T), y(T))$
subject to $\dot{x} = f(t, x, y, u)$, $x(0) = x_0$
$y = W[Sx, y_0]$, $u(t) \in \Omega$

where W is a hysteron of 1st respectively 2nd kind in the sense of Krasnoselski. For some special situations we prove theorems on existence of optimal controls and on necessary optimality conditions

Martin Brokate, Hamburg

Integral treatment of the O.D.E $y'' = f(x, y, y')$ with Splines

A method for approximating the solution of the initial value problem: $y'' = f(x,y,y')$ with Spline Functions is presented. Here, the Spline Function approximating the solution is not necessarily polynomial spline. It is a one-step method and it has been shown that if the function f is r-times differentiable and if $f \in \mathrm{Lip}_{M_\circ} \alpha_\circ$, $0 < \alpha_\circ \leq 1$, then the method is $O(h^{r+2+\alpha_\circ})$ in $y^{(i)}(x)$ $\forall\, i = 0, 1, \ldots, r+2$.

Tharwat Fawzy
Suez-Canal Univ. Ismailia, Egypt.

Spline-Galerkin method for solving some quantum mechanic integral equations

An investigation is made of the Galerkin technique with cubic B-splines approximants to solve some quantum mechanic integral equations. The problem is to find the numerical solution of a linear integral equation of the second kind. This equation has a singular kernel, and a non-smooth (cusp) behaviour in the solution function. The discontinuity in the solution function is built into the spline approximation by combining knots (multiple knot) at the point of discontinuity. A one-dimensional example is used to test the performance of both the Galerkin and iterated Galerkin methods

David Eyre
NRIMS, CSIR, South Africa

The Solution of Nonlinear Integral Equations in Acoustic Scattering Theory

We consider the problem of determining the shape of an obstacle from a knowledge of the far field pattern of the scattered acoustic wave. This problem is complicated by the fact that it is nonlinear and improperly posed. Two methods of solution are presented, both of which require the minimization of a nonlinear functional subject to constraints. The second approach, which has yet to be numerically implemented, has the advantage of a simple Frechét derivative and avoids the need to solve an integral equation at each step of the iterative procedure for obtaining the solution.

David Colton
University of Delaware

Arbitrarily Slow Convergence, Uniform Convergence and Superconvergence of Galerkin-like Methods

Let $M: X \to (X_n)$ be a method for the approximate solution of $x - Tx = y$, where T is a continuous linear operator in a Banach space X, which assigns to each $x \in X$, an $x_n^M \in X_n$, $\dim X_n = n$, $X_n \subset X$. The method M is said to be converging, if for all $x = (I-T)^{-1}y$ hold $\lim_{n\to\infty} x - x_n^M = 0$. A converging method is said to be arbitrarily slow converging, if for each monotone decreasing sequence (w_n) there is an $x \in X$ such that $\lim w_n^{-1} \|x - x_n^M\| = \infty$. Otherwise the method is said to be uniformly converging. It is shown that the Galerkin method converges arbitrarily slow, the iterated Galerkin method and the Kantorovich method in Hilbert spaces are equivalent and converge uniformly. In the space of continuous functions the Kantorovich method converges uniformly but the iterated Galerkin method converges arbitrarily slow, if it uses interpolating projections.

Eberhard Schock
Kaiserslautern

Wiener-Hopf integral equations and their finite section approximation

This talk is concerned with Wiener-Hopf integral equations of the form

$$x(s) - \frac{1}{\lambda}\int_0^\infty \kappa(s-t)\,x(t)\,dt = y(s), \qquad s \in \mathbb{R}^+$$

and their finite-section approximation

$$x_\beta(s) - \frac{1}{\lambda}\int_0^\beta \kappa(s-t)\,x_\beta(t)\,dt = y(s), \qquad s \in \mathbb{R}^+,$$

where $\beta \in \mathbb{R}^+$. It is assumed that $\kappa \in L_1(\mathbb{R}^*)$, and that y, x and x_β belong to X^+, the space of bounded continuous functions on $\mathbb{R}^+$ with the uniform norm. With the finite-section equation written as $(I - \frac{1}{\lambda}K_\beta)x_\beta = y$, recent joint work with P. M. Anselone has established that $(I - \frac{1}{\lambda}K_\beta)^{-1}$ exists and is uniformly bounded as an operator on X^+ for all β sufficiently large. The key is a 'sliding' variant of the Arzelà-Ascoli theorem. It follows from earlier results of K. E. Atkinson that $x_\beta(s) \to x(s)$ as $\beta \to \infty$, uniformly for s in finite intervals.

Ian Sloan (Univ. of New South Wales)

"On the numerical solution of Volterra integral and integro-differential equations with weakly singular kernels".

It is well known that Volterra integral equations of the second kind, or Volterra integro-differential equations, with weakly singular kernels of the form $(t-s)^{-\alpha}\cdot K(t,s)$ $(0 < \alpha < 1)$ possess solutions which exhibit a nonsmooth behavior near the leftendpoint of the interval of integration $[0,T]$. As a consequence, if such equations are solved by collocation in polynomial spline spaces with respect to quasi-uniform mesh sequences $\{\Pi_N\}$, the resulting order of convergence is given by $O(N^{-(1-\alpha)})$, regardless of the degree of the approximating spline. The optimal order can be recovered if one employs suitably graded meshes of the form $t_n := \left(\frac{n}{N}\right)^r \cdot T$ $(n = 0, \ldots, N)$, with grading exponent r given, respectively, by $r = (m+1)/(1-\alpha)$ and $r = (m+1)/(2-\alpha)$, where m is the degree of the approximating spline. Proofs of these convergence results will be given, and we discuss the application, as well as the limitations, of spline collocation on graded meshes.

Hermann Brunner
(Université de Fribourg, Switzerland)

"Numerical solution of a first kind Fredholm integral equation arising in atomic physics"

In a study of dispersion relations for electron-atomic scattering, the following first kind Fredholm integral equation arises ([1]):

$$\Delta(x) = \pi^{-1} \int_{x_0}^{\infty} \frac{\rho(y)\,dy}{x+y} .$$

Here, the function $\Delta(x)$ is given in 23 points of the interval $[0,500]$ with an accuracy of about 3%. Moreover, $\Delta(x)$ may be assumed to vanish for $x > 500$. The unknown function $\rho(y)$ is known to tend to zero, as $y \to \infty$, at least as fast as the function $y^{-1/2}$.
Experiments with the regularization method of Phillips and Tikhonov will be reported. The results obtained are acceptable to the physicist, at least in a qualitative sense.

[1] R. Wagenaar, Small angle elastic scattering of electrons by noble gas atoms, Doctor's Thesis, Amsterdam, 1984.

Herman J. J. te Riele
(Centre for Mathematics and Computer Science, Amsterdam)

"Stability results for Abel equation"

If we pose:

$$I^{\alpha} u(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} u(t)\,dt \qquad 0 < x < 1$$

where $0 < \alpha < 1$, we obtain for $0 < \theta < 1$

$$\|u\|_p \leq C(\alpha,\theta) \left\{ |u|_{\theta,p}^{\frac{\alpha}{\theta+\alpha}} + \|I_\alpha u\|_p^{\frac{\alpha}{\theta+\alpha}} \right\} \|I_\alpha u\|_p^{\frac{\theta}{\theta+\alpha}}$$

for $1 \leq p < +\infty$. Where:

$$|u|_{\theta,p} = \left(\int_0^1\int_0^1 \frac{|u(x)-u(t)|^p}{|x-t|^{1+p\theta}}\,dx\,dt \right)^{1/p}.$$

and:

$$\|u\|_p \leq C(\alpha) \left\{ \|u'\|_p^{\frac{\alpha}{1+\alpha}} + \|I_\alpha u\|_p^{\frac{\alpha}{1+\alpha}} \right\} \|I_\alpha u\|_p^{\frac{1}{1+\alpha}}$$

for $1 \leq p \leq +\infty$.

Sergio Vessella

Constrained approximation methods for integral equations

Classical techniques for solving integral equations use an approximation to the function, and obtain the unknown parameters of this approximation by interpolation. Thus collocation interpolates on a given set of points, and the Galerkin method finds a 'good' set of interpolation points. We suggest that the complete continuous approximation problem is solved, which invariably will give a better set of parameters for the approximation than an interpolation scheme. If this approach is made then constraints on the function and/or the parameters are easily incorporated into the method. Also the theory developed in the proposed method is extendable to problems with more than one variable.

Mike Brannigan. (Univ. of Georgia)

On the condition of boundary integral equations in scattering theory

The question of non-uniqueness in boundary integral equation formulations of exterior boundary value formulations in time-harmonic acoustic and electromagnetic scattering can be resolved by seeking the solutions in the form of a combined single- and double-layer potential in acoustics or a combined electric- and magnetic-dipole field in electromagnetics. We present an analysis of the appropriate choice of the coupling parameters which is optimal in the sense of minimizing the condition number of the boundary integral operators.

Rainer Kress (Göttingen)

Inclusions of regular and singular solutions of certain types of integral equations

Bei der Funktionalgleichung $u = Tu$ für eine Funktion $u(x) = u(x_1, \dots, x_n)$ sei der gegebene (lineare oder nichtlineare) Operator T „monoton zerlegbar" im Sinne von J. Schröder, und vollstetig. Man habe, ausgehend von 2 Funktionen v_0, w_0 mit Hilfe eines Iterationsverfahrens v_1, w_1 berechnet mit $v_0 \leq v_1 \leq w_1 \leq w_0$. Dann existiert (Schauder's Fixpunktsatz) mindestens eine Lösung u im Intervall $[v_1, w_1]$. In vielen Fällen ist dies die einzige praktisch brauchbare Möglichkeit einer Einschliessung von u. Diese Methode erfasst Hammersteinsche Integralgleichungen

Fortsetzung S. 76 Mitte

Collocation Methods for Integral Equations on the half-Line.

Convergence results are proved for projection methods for integral equations of the form $y(t) = f(t) + \int^{\infty} k(t,s)y(s)ds$. The conditions on $k(t,s)$ are such that the Weiner-Hopf integral equations are included in our analysis. The convergence results indicate that the iterated collocation solution may exhibit superconvergence. The case of collocation using piecewise-constant basis functions applied to an integral equation with kernel $e^{-|t-s|}$ is discussed in detail and numerical results are given.

Alastair Spence.
(Bath University, England)

(Fortsetzung von S. 75), aber auch weitere Typen nichtlinearer Integralgleichungen. In Fällen, in denen die Lösungen Singularitäten haben, hat man zu unterscheiden, ob man die Lage der Singularitäten kennt oder nicht (versteckte Singularitäten). In neuerer Zeit sind insbesondere dreidimensionale Singularitäten zu Bedeutung gelangt. Es wird über numerische Erfahrungen mit räumlich verteilten Singularitäten im R_3 berichtet.

L. Collatz, Hamburg

Spline collocation for singular integral equations and integrodifferential equations

Spline collocation is the most frequently used approximation of boundary integral equations in the boundary element method. Here the singular integral equations with Cauchy kernel form a special subclass which at the same time is the most important model problem. If the plane closed non intersecting curves forming the boundary are all given by 1-periodic parameter representations then the equations and unknowns can be considered as to be 1-periodic systems of equations and vector valued functions on $\mathbb{R}$. For the approximation we use

$\mathcal{S}_d(\Delta)$, the 1-periodic splines of degree d in C^{d-1} subordinate to the partition $\Delta = \{0 = t_0 < t_1 < \dots < t_N = 1\}$ and we apply the naive collocation method: Find $u_\Delta \in (\mathcal{S}_d(\Delta))^p$ and $\omega \in \mathbb{R}^q$ satisfying

$$Au_\Delta(\tau_k) + B\omega_\Delta(\tau_k) = f(\tau_k), \quad k = 1,\dots, N, \quad \int_0^1 u_\Delta \, dt = \beta \in \mathbb{R}^q$$

where A denotes an integrodifferential or pseudodifferential operator or, in the simplest case, a Cauchy singular integral operator such as

$$Au(\tau) = a(\tau)u(\tau) + \frac{b(\tau)}{i\pi} \oint_0^1 \frac{u(t)\,d\zeta}{\zeta - e^{2\pi i \tau}} + \int_0^1 L(\tau, t)\, u(t)\, dt, \quad \zeta = e^{2\pi i t}$$

and

$$\tau_k = \begin{cases} t_k & \text{for } d \text{ odd}, \\ t_k + \frac{1}{2}(t_{k+1} - t_k) & \text{for } d \text{ even}. \end{cases}$$

In the Sobolev spaces $H^\sigma(\Gamma)$ we obtain optimal order asymptotic error estimates

$$|\omega - \omega_\Delta| + \|u - u_\Delta\|_\sigma \le c\, h^{\tau - \sigma} \|u\|_\tau$$

with $0 \le \sigma \le \frac{d+1}{2} \le \tau \le d+1$ for d odd and arbitrary families of meshes Δ of meshwidth $h = \max(t_{k+1} - t_k) \to 0$ and with $0 \le \sigma \le \tau \le d+1$ and $\frac{1}{2} < \tau, \sigma < d + \frac{1}{2}$ for uniformly grades families of meshes, d odd or even — if (and only if) A is strongly elliptic, i.e. in the above case

$$\det\left(a(\tau) + \eta\, b(\tau)\right) \ne 0 \quad \text{for all } \eta \in [-1, 1] \text{ and all } \tau.$$

These results can be extended to the integrodifferential and pseudodifferential equations.

This all is a short survey on joint work with D. N. Arnold, Univ. of Maryland USA, and J. Saranen, Univ. Oulu Finland. (Arnold + Wendland in Math. Comp. <u>41</u> (1983) and a paper in preparation; Saranen + Wendland in Math. Comp. in press).

Wolfgang Wendland
Technische Hochschule Darmstadt

On the numerical solution of ill-posed problems with weakly singular integral operators

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Regularization methods for the numerical solution of ill-posed problems based on the finite element method are very sensitive to the computation of the corresponding matrix elements. Several ideas are presented to overcome these difficulties for the case of Fredholm integral equations with weakly singular kernels.

Jürg T. Marti
(ETH Zürich)

Discrepancy principles for the choice of the regularization parameter for solving integral equations of the 1st kind by Tikhonov-regularization

Since the solution of an integral equation of the first kind is in general an ill-posed problem, one has to use regularization methods, e.g. Tikhonov regularization. There the problem of a choice of the regularization parameter that leads to the optimal convergence rate arises. We are interested in such parameter choices that are "a-posteriori" choices in the sense that the parameter is calculated from quantities that appear during the calculations. A class of such methods are the "so-called" "discrepancy principles" due to Morosov and Arcangeli. It is known that these methods do not yield the optimal convergence rate.
We propose a variant of the discrepancy method that yields optimal convergence rates. This variant is applicable to different versions of Tikhonov regularization (classical, with

differential operators, in Hilbert scales). Finally we present some preliminary results on constrained Tikhonov regularization and numerical results.
(Joint work with A. Neubauer).

Heinz Engl (Linz)

Identification of ODEs

The identification problem of estimating certain functions in a system of linear ordinary differential equations from measured data of its state is considered. The approach consists in an imbedding of the problem into a family of parameter-dependent problems which can be solved at least numerically. The corresponding solutions are proved to converge to the unknown functions as the parameters tend to infinity. Stability results with respect to disturbances in the measurements and the initial data are developed as well. The method is applied to determine mass exchange rates in compartmental systems of pharmaco kinetic models.
These results are joint work with Jürgen Sprekels.

K.-H. Hoffmann, Augsburg

Numerische Verfahren für singuläre Integralgleichungen

Bei Umströmung eines dünnen Profils mit zusätzlicher Ausblaswirkung an der Hinterkante des Profils entsteht für die Wirbelbelegung $\gamma_p(x)$, $0<x<1$, auf der Profillinie und $\gamma_S(x)$, $1<x$, auf der Strahllinie die singuläre Integralgleichung

(1) $$W\gamma \equiv \int_0^\infty \frac{\gamma(y)}{y-x}\,dy - \int_0^\infty \gamma(y)\,\frac{y-x}{(y-x)^2+b^2}\,dy = \mathring{r}(x)$$

~~Integro-differential inequalities~~

~~A general method of obtaining inequalities~~

$$\cancel{\|A^{*m}f\| \le C_{n,m}\,\|f\|^{\frac{n-m}{n}}\,\|A^{*n}f\|}$$

mit einer rechten Seite $\mathring{r}(x)$, die außer im Punkt $x=1$ stetig ist und für $x=1$ einen endlichen Sprung aufweist. Es ist

$$\gamma(y)=\begin{cases}\gamma_p(x), & 0<x<1,\\ \gamma_S(x), & 1<x.\end{cases}$$

Aus physikalischen Erwägungen gilt $\gamma(y)\to\infty$ für $y\to 0$ und $y\to 1$, außerdem $\gamma(y)\to 0$ für $y\to\infty$.

Die Lösung der Gl. (1) wird in der Form $\gamma(y)=F(y)+S(y)$ gesucht, wobei die Funktion $S(y)$ den Sprung der rechten Seite $\mathring{r}(x)$ aufnimmt. $F(y)$ ist Lösung der Gleichung $W_0F=r$ mit stetiger rechter Seite.

Diese Gleichung wird durch Reduktion des Integrationsgebiet $(0,\infty)$ auf ein endliches Gebiet $(0, G)$ mit hinreichend großem $G > 0$ ersetzt:

$$(2) \quad \int_0^G g(y)\sqrt{\frac{G-y}{y}}\,\frac{dy}{y-x} - \int_0^\infty g(y)\sqrt{\frac{G-y}{y}}\,\frac{y-x}{(y-x)^2+b^2}\,dy = r(x), \quad 0<x<G.$$

Diese Gleichung wird näherungsweise durch Gauß-Quadraturformeln mit Jacobischen Gewicht gelöst. Durch Koordinatentransformation wird eine direkte Anwendung der Gauß-Quadraturformeln auf Gleichungen der Art (1) mit stetiger rechter Seite möglich.

Frieder Kuhnert
TH Karl-Marx-Stadt

Product integration for two dimensional weakly singular integral equations

Convergence results are proved for product integration methods for multi-dimensional integral equations with weakly singular kernel. Furthermore, an integral equation is considered which arises in electrical engineering: when an alternating current flows in a conducting bar then the current is displaced towards the surface of the conductor – that is the so-called skin effect. The unknown current distribution satisfies an integral equation with a logarithmic kernel. For a rectangular region (the cross-section of the conductor) and a tensor product mesh a slightly better order of convergence result is proved (for a mid-point product integration) than for the general case. Numerical results are given.
These results are joint work with Ivan G. Graham (University of Melbourne)

Claus Schneider, Mainz.

Beyond superconvergence of collocation methods for Volterra integral equations of the first kind.

We discuss superconvergence aspects of the numerical solution of Volterra integral equations of the first kind (VIE1) by collocation methods with piecewise polynomials of degree $\leq p$ on a uniform mesh. It is well-known that (i) under "normal" conditions convergence of order $O(h^{p+1})$ is achieved {De Hoog & Weiss (1973) (ii) under slightly more special conditions, $O(h^{p+2})$ convergence is achieved at special points inside each subinterval (Brunner (1978), E. (1982)). (iii) higher order convergence is impossible (Brunner (1978)). We show here that it is possible to do some postprocessing on the "superconvergence" collocation solution to obtain $O(h^{p+3})$ convergence at yet another set of special points. This possibility is based on the oscillating behavior of the error in the collocation solution inside each subinterval. For the "pure" differentiation case of VIE1 this is easily shown (applying Lagrange interpolation). For the general case it follows from this special case and from the closeness of the projectors associated with the collocation method under compact perturbations. Some numerical illustrations are presented. There appears to be a connection between the above and the convergence analysis of certain Runge-Kutta methods for VIE1 (Keech, 1978) but the details are not yet clear.

Paul Eggermont,
University of Delaware

An adaptive stepsize control for Volterra integral equations

Stepsize control strategies normally rely on an error expansion of the local and/or global error. For Volterra integral equations of the second kind such an error expansion exists e.g. in the case of extended Runge Kutta methods (Hairer, Lubich, Nørsett). We develop an adaptive stepsize control which differs from the strategies known in ordinary differential equations. The main point is that we always look whether integrations that have to be performed at a later time will be carried out according to the prescribed precision.

Herbert Arndt (Bonn)

The design of acoustic torpedos

The homing mechanism of an acoustic torpedo requires that its nose should contain a circular disc. The design problem is to connect this vertical flat disc with the main body which is a right circular cylinder.

The pressure distribution of the flow around the torpedo needs to be kept as high as possible to prevent separation and so, by Bernoulli's equation, this means that the maximum speed of the flow should be kept as low as possible.

A Fredholm integral of the second kind with a logarithmic singularity can be derived which is satisfied by the speed of flow. This was obtained first by F. Vandrey but published only as an internal report for the British Admiralty; the derivation depended on hydrodynamical considerations. An alternative approach based on Green's third identity was outlined here.

In order to find a numerical solution of the equation a new type of Gaussian quadrature was developed, and used to calculate the flow for a variety of shapes. It was found that within the restricted class considered the best shape was when in the generating curve of the torpedo the flat nose and flat back was connected by the quadrant of an ellipse chosen so that the whole curve was continuous and had a continuous derivative

D. Kershaw (Lancaster, England)

A Unified Analysis of Discretization Methods for Volterra type Equations

This talk presents some of the main results of my D. Phil. thesis at Oxford University (submitted June 1984). The following abstract is taken from that thesis.

Numerical functional analysis is used to present a unified analysis of discretization methods for Volterra type equations. The concept of analytic and discrete fundamental forms is introduced. Prolongation and restriction operators reduce the problem of comparing the exact solution with the numerical solution to that of considering the effect of perturbations in the fundamental forms; new Gronwall inequalities are then employed to obtain error estimates. A concept of optimal consistency permits two-sided error bounds to be presented.

The analysis is illustrated by considering the convergence of a general class of quadrature methods for first kind Volterra integral equations and, in particular, reducible quadrature provides an illustrative example.

Jennifer Scott
University of Oxford.

Die Fehlernorm spezieller Gauß-Quadraturformeln

Man approximiere das Integral

$$I(f) = \int_{-1}^{1} w(x) f(x)\,dx, \quad w \geq 0, \quad \|w\|_1 > 0,$$

durch die Gauß-Quadraturformel

$$Q_n(f) = \sum_{i=1}^{n} w_i f(x_i).$$

In gewissen Räumen holomorpher Funktionen ist das Fehlerfunktional $R_n := I - Q_n$ stetig. Für $w(x) = \frac{1}{k - x^2}(1-x)^{\alpha}(1+x)^{\beta}$, $\alpha, \beta = \pm 1/2$, $k > 1$, wird die Norm von R_n angegeben.

G. Akrivis
Universität München

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Integro-differential inequalities

A general method for obtaining inequalities

$$\|A^{*m} f\| \le C_{n,m} \|f\|^{\frac{n-m}{n}} \|A^{*n} f\|^{\frac{m}{n}}$$

where A is an arbitrary symmetric operator in Hilbert spaces, is worked out, together with a method of calculating best constants $C_{n,m}(A)$. Applications are given to differential operators in $L^2(0,\infty)$ and $L^2(0,1)$ to get new integro-differential inequalities. A problem of how to apply this method to integral operators of Carleman's type is posed.

Vu Quoc Phong.

Solving Integral Equations on Surfaces in Space

A method is described for solving integral equations defined on piecewise smooth surfaces in three dimensions. The surface is triangulated and approximated with quadratic isoparametric elements. A collocation method is described and analyzed for the solution of integral equations of the second kind. We then discuss several questions that are important for the practical implementation of the method, especially when applying the results to equations from potential theory. First, how is the surface to be described and how is the triangulation to be carried out, including how to store the triangular elements. Next, how should we numerically evaluate the weakly singular collocation integrals that arise in applications to potential theory. Finally, we consider iterative methods of solution for the resulting large linear systems.

Kendall E. Atkinson,
University of Iowa

Tikhonov Regularization of the Radon Transform

When the Tikhonov regularization is applied to an ill-posed problem two questions arise. First the regularization norm has to be selected which can be done with the help of available information on the solution. The difficult task is then the selection of the optimal regularization parameter. For the Radon transform this problem is treated via an explicit representation of the regularized solution with the help of the complete singular system of the Radon transform. Its effect on the solution is studied. Also the limited angle problem is discussed in this framework.

Alfred Louis, Kaiserslautern

Integrable Hamiltonian systems and algebraic geometry
(July 1 – July 7)

Loop groups and integrable systems

One of the properties of 'integrable' evolution equations such as the KdV, or modified KdV, equations is that they have a fairly large class of globally meromorphic solutions which can be expressed in terms of certain entire functions, known as τ-functions: in special cases these are essentially theta functions (or degenerations of these). Several equivalent descriptions of these solutions are known: the 'method of algebraic curves' (Krichever), the 'Grassmannian method' (M. and Y. Sato, Date, Jimbo, Kashiwara, Miwa, ...) and 'dressing the vaccuum' (Zakharov, Shabat). The last of these has the advantage that it applies immediately to the equations associated by Drinfeld and Sokolov to any affine Kac-Moody algebra. The τ-functions arise in a completely natural manner if one repeats the 'dressing' construction working with the universal central extension of a loop group, rather than with the rather trivial loop group itself.

George Wilson (IHES).

A class of solvable dynamical systems

This class obtains assuming $\psi(x,t)$ to be a polynomial in x of degree n satisfying an appropriate second order linear PDE and setting $\psi[x_j(t),t] = f_j(t)$, $j=1,2,\dots,n$. There thus obtains a system of n second-order coupled ODEs for the $2n$ quantities $x_j(t)$ and $f_j(t)$, that can be written in fairly compact, and quite explicit, form; it is of course linear in the n f_j's and nonlinear in the n x_j's. Solvable dynamical systems obtain ~~by~~ then positing n additional relations, f. i. $f_j(t) = F_j[\underline{x}(t),t]$, $j=1,2,\dots,n$, with the $F_j[\underline{x},t]$ n arbitrarily chosen functions. Several explicit examples of systems of n coupled second-order nonlinear ODEs for the n $x_j(t)$'s, obtained in this manner, are exhibited; they are explicitly integrable, namely their solution is reduced to solving an explicit algebraic (or transcendental) equation. Other examples of systems of n coupled second-order nonlinear ODEs for the n $x_j(t)$'s are also exhibited, that can be reduced by appropriate nonlinear mappings to n decoupled non autonomous nonlinear first-order ODEs.

2.7.1984

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Kac-Moody Lie algebras and $A_1^{(1)}$

It is shown, starting with the AKNS system (a 2×2 system version of mKdV and NLS), that the complete integrability of this system can be derived from the loop algebra of $sl(2, \mathbb{C})$. In addition, by introducing certain fluxes, it is shown that the polynomial eigenvalue problem for AKNS also results in completely integrable soliton equations. A method of deriving the standard differential Lie algebraic approach of the Russian school is given. This enables one to obtain e.g. the derivative nonlinear Schrödinger equation. Finally it is proved that the Hamiltonian structure for the stationary equations is a restriction to a Poisson submanifold of the Hamiltonian structure of the usual AKNS equations.

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Fredholm determinant formalism for KdV and other soliton equations

Another method of describing solutions of soliton equations in terms of τ functions (cf. G. Wilson's talk) is presented: Fredholm determinants of suitably chosen integral operators.

For the case of KdV:

Let f be a solution of the "linearized KdV" $f_t + 8 f_{xxx} = 0$ decaying sufficiently fast for $x \to -\infty$. Define F by

$$F\varphi(s) := \int_{-\infty}^{0} f(2x+s+\sigma, t) \quad .$$

Then

$$p := \det(1+F)$$

is a τ function for the KdV equation, in particular

$$u = -2 \frac{d^2}{dx^2} \log p$$

solves KdV. (This process is formally equivalent to solving the well-known Gel'fand-Levitan-Marčenko integral equations of inverse spectral theory; however, the assumptions on f are quite different.)

For the proof, first introduce a linear functional on some suitable space of integral operators: for K defined by $K\varphi(s) := \int_{-\infty}^{0} k(s,\sigma)\,\varphi(\sigma)\,d\sigma$ let $[K] := k(0,0)$. Some computation rules for $[\cdot]$ can be introduced that are derived from (and resemble very much) the product rule of ordinary calculus. Using these rules, the proof is a straightforward calculation.

Moreover, the same technique allows to prove more statements of soliton theory: From p and $\tilde{p} := \det(1-F)$, solutions of the modified KdV equation can be constructed; a simple constant-coefficient o.d.e. among solutions of the linearized KdV can be shown to induce a Bäcklund transformation among the corresponding KdV solutions; eigenfunctions of the "scattering operator" for KdV $-\frac{d^2}{dx^2}+u$ can be expressed in terms of the eigenfunctions of the unperturbed operator $-\frac{d^2}{dx^2}$; finally, solutions of all the ~~members~~ higher order members of the KdV hierarchy can be constructed the same way from solutions of their respective linearizations $f_t + \text{const.}\ \partial_x^{2n+1} f = 0$.

Question: How does this relate to the algebraic approaches to τ functions?

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The Intersection of 4 quadrics in $\mathbb{P}^6$ and geodesic flow in SO(4).

It was shown that geodesic flow with a 4th quadratic invariant occurred precisely when the span of the 4 quadrics contained a curve of rank 3 quadrics. Then the affine part of the intersection of the 4 quadrics was the affine part of an abelian variety given as ~~the~~ a Prym variety of a natural 2-fold cover of the curve of rank 3 quadrics. The flow linearizes on this variety. The curve can either be an elliptic curve (or some limit) or a rational curve. These 2 cases correspond respectively to the cases of Clebsch and Lyapunov-Steklov of the flow of a solid rigid body in a perfect fluid

Mark Adler
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Complexified Fermi Curves

We study the family of complexified Fermi curves of a two dimensional perfect crystal. A representation of the monodromy is constructed from the density of states. The ramification points and monodromy determine the imbedding of the family in $\mathbb{C}^* \times \mathbb{C}^* \times \mathbb{C}$. This in turn determines the potential q in ~~XXXXX~~ the discrete periodic Schrödinger operator $(-\Delta + q)$. (Joint work with D. Gieseker, UCLA)

E. Trubowitz
ETH Zürich

Quasi-periodic solutions of some soliton equations.

We considered the problem of constructing quasi-periodic solutions of soliton equations. Most of soliton equations are expressed as the compatibility condition of linear equations. Our result is based on a characterization of solutions of these linear equations (bilinear identity).

Using these identities and the method of Krichever, we can construct quasi-periodic solutions. i) For the KP hierarchy ~~the~~ corresponding τ-functions are "essentially" expressed in terms of theta functions on Jacobian varieties. ii) For the BKP hierarchy theta functions on Prym varieties associated with involutions with two fixed points appear. ~~iii)~~ For the Landau-Lifshitz equation, theta functions on some Prym varieties associated with fixed point free involutions appear. (Joint work with M. Jimbo, M. Kashiwara and T. Miwa)

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JAPAN

Hamiltonian actions of compact groups.

In this lecture we discussed some properties of a hamiltonian action of a compact group K on a symplectic manifold (M, σ). Let J denote the corresp. moment map. More specifically: convexity questions for the image $J(M)$, and formulas for $J_*(dm)$ where dm is the Liouville measure on M. For a semisimple Lie algebra $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ (Cartan dec.) with rank$(\mathfrak{g})$ = rank$(\mathfrak{k})$ the pushforward $J_*(dm)$ in the case of a regular elliptic orbit M, viewed as a hamiltonian K-space, is computed explicitly. As an application it ~~was~~ briefly ~~was~~ sketched how this enables one to prove Blattner's formula for the K-types of the discrete series of the Lie group G.

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_ An overview of the Schottky problem _

the problem consists in giving an analytical characterization of the jacobian locus J_g inside the Siegel generalized half plane H_g.

We first describe the Schottky-Jung approach and the Schottky locus $S_g \subset H_g$. The basic result in that direction are the one of Igusa who proves that $S_4 = J_4$ and the one of Van Geemen who shows that J_g is an irreducible component of S_g.

We then describe the Andreotti-Mayer Approach and their locus $N_{g-4} \subset \mathcal{H}_g$. We present their proof of the fact that J_g is an irreducible component of N_{g-4}. In that direction we mention the work of A. Beauville.

We finally describe the approach via the Kadomcev Petviashvili equations as signified by the work of Novikov, Krichever, Dubrovin, Fay, Mumford and we present the recent work of Gunning and Welters. We then proceed to describe our joint work with Corrado de Concini in which we give a complete analytical characterization of the jacobian locus J_g, by showing that irreducible Riemann matrices are characterized by the fact that the corresponding theta function satisfies a certain number of equations of the K.P. hierarchy.

4.7.1984 Enrico Arbarello
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Recent results on the inverse spectral problem

We survey recent results on the inverse spectral problem for elliptic differential operators on compact manifolds. For simplicity we just consider the following three special cases: 1) the Laplace operator on a compact manifold with Riemann metric (work of Vignéras

Wolpert and Gordon-Wilson); 2) domains in $\mathbb{R}^2$ with Dirichlet or Neumann boundary conditions (work of Marvizi-Melrose, Urakawa and others) and 3) the inverse problem for the Schroedinger operator on compact boundaryless manifolds (e.g. work of Eskin, Trubowitz, Ralston)

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The algebraic geometry of some classical completely integrable systems.

The algebraic geometry of the Kowalewski's top and the Clebsch's case of Kirchhoff's equations describing the motion of a solid body in a perfect fluid was discussed. These 2 systems were integrated at the end of last century in terms of genus 2 hyperelliptic functions by Kowalewski (1889) and Kötter (1892) as a result of some "mysterious" computations. It was shown that the concept of algebraic complete integrability throws a new light on these two systems. In both cases the affine varieties in $\mathbb{C}^6$ obtained by intersecting the 4 polynomial invariants of the flow are affine parts of Prym varieties of genus 3 curves which are double covering of elliptic curves with 4 branchpoints. Such Prym varieties are not principally polarized and so they are not isomorphic but only isogenous to Jacobi varieties of genus 2 hyperelliptic curves.

Luc Haine
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The Geometry of the Benjamin-Feir Instability

We consider here the stability of quasi-periodic solutions of the sine-Gordon equation from two points of view. The first, modulation theory, supposes that a real solution, u, of the sine-Gordon equation, $u_{tt} - u_{xx} + \sin u = 0$, can be approximated by an N-phase solution

$$u \simeq u_N(x, t; E_1(X,T), \ldots, E_{2N}(X,T))$$

in which the parameters E_i depend slowly on space and time $X = \epsilon x$, $T = \epsilon t$. Accepting Whitham's theory we postulate that the E_i are governed by the averages of the first 2N conservation laws of the sine-Gordon hierarchy. By geometric techniques we find an invariant representation of the modulation equations in which the E_i, regarded as branch points of a hyperelliptic Riemann surface, are the Riemann invariants of these equations. We determine that unless all branch points are initially real, u_N is modulationally unstable.

The second point of view, perturbation theory, constructs a linearized stability theory, in the L^2-completion of $C_L^\infty \times C_L^\infty$ (L=spatial period), for periodic sine-Gordon solutions. Loosely summarized our results are (i) if u is generic (i.e. all degrees of freedom are excited) then u is linearly stable; (ii) if u has a degree of freedom which is not excited, an exponential stability may occur; (iii) we characterize the exponentially unstable modes and derive explicit formulae for the rates of instability in terms of Riemann surface differentials. These instability results may be viewed as a generalization of the classical Benjamin-Feir instability. (Joint work with G. Forest (Ohio State) and D. McLaughlin (Univ. of Arizona, Tucson)

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Characterization of Jacobian varieties in terms of soliton equations

We show that the following conditions for a principally polarized abelian variety $X = \mathbb{C}^g/\underbrace{(\mathbb{Z}^g + \Omega\mathbb{Z}^g)}_{=\Gamma}$, $\Omega \in \mathfrak{H}_g$, are all equivalent:

(A) There exist 3 vectors $a_1, a_2, a_3 \in \mathbb{C}^g$ and a quadratic form $Q(t) = \sum_{i,j=1}^{3} Q_{ij} t_i t_j$ such that

i) for all $\zeta \in \mathbb{C}^g$ the function

$$\tau(t) = \tau(t, \zeta) = \exp(Q(t))\, \vartheta\left(\sum_1^3 t_i a_i + \zeta\right)$$

satisfies the KP equation

$$(D_1^4 + 3D_2^2 - 4D_1D_3)\tau \cdot \tau = 0 \quad ;$$

ii) There ~~exists~~ is no abelian subvariety $Y \subset X$ such that $Y + \overset{\exists}{C} \subset \Theta \subset X$ and $\mathbb{C}a_1 \bmod \Gamma \subset Y$.

(Here and in what follows $\vartheta(z) = \vartheta(z, \Omega)$ is Riemann's theta function for the abelian variety X)

(B) $X \cong \mathrm{Jac}(C)$ for some smooth curve C over $\mathbb{C}$.

(C) There exist a $g \times \infty$-matrix $\bar{A} = (a_1, a_2, \cdots)$, $a_j \in \mathbb{C}^g$, and a quadratic form $\bar{Q}(t) = \sum_{i,j=1}^{\infty} Q_{ij} t_i t_j$ such that

(*) ——— rank $\bar{A} = g$ (i.e. of maximal rank)

and the function

$$\bar{\tau}(t) = e^{\bar{Q}(t)}\, \vartheta(\bar{A}t)$$

is a τ-function for the KP hierarchy.

(D) i) Same as (C) except that the condition (*) is replaced by the weaker condition

$$a_1 \neq 0 \quad (a_1 : \text{the first column of } \bar{A}) ;$$

ii) Either the condition ii) in (A) or the assumption that the variety X is irreducibly principally polarized.

In (C) and (D) the condition may be weakened so that one has only to consider the first $2g+1$ time evolutions in the KP hierarchy, since for a g-dimensional solution to the KP hierarchy it is automatic to extend the solution from the $(t_1, \cdots, t_{2g+1})$-space to the whole time parameter space. In (A) the condition ii) is very likely to be ~~weakened~~ replaced by the irreducibility assumption on X as in (D).

We prove the equivalence as:

$(\underline{A}) \Longrightarrow (\underline{D}) \Longrightarrow (\underline{C}) \Longrightarrow (\underline{B}) \Longrightarrow (\underline{A})$

- $(\underline{A}) \Rightarrow (\underline{D})$: quasiperiodic extension procedure for the KP hierarchy
- $(\underline{D}) \Rightarrow (\underline{C})$: a) algebraic argument based on the fact that the $\otimes$-action of $Pic^0(C)$ on $\overline{Pic^{g-1}(C)}$ is algebraic; b) Krichever's result
- $(\underline{C}) \Rightarrow (\underline{B})$: Burchnall-Chaundy-Krichever theory
- $(\underline{B}) \Rightarrow (\underline{A})$: Krichever's construction of a quasiperiodic solutions to the KP ~~hierarchy~~ equation

Presumably, the same method works to characterize the principally polarized Prym varieties by the BKP equation(s).

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We discuss ordinary differential equations

$$x' = A(t)x, \quad A = \begin{pmatrix} a & b \\ c & -a^t \end{pmatrix},$$

ie Hamiltonian systems of ODEs. We introduce a Floquet exponent for such equations, in particular a rotation number based on the Bott-Arnold idea of ~~rotation~~ intersection number in a space of Lagrange subspaces. Applications of the

Floquet exponent are discussed, to the spectral properties of ordinary differential operators (Results of Pastur, Molchanov, Pastur-Sobin; Johnson-Moser, and others).

Russell Johnson

Systèmes sous-holonomes d'équations différentielles linéaires sur les variétés abéliennes.

Soient X une variété algébrique complexe, complète lisse et connexe, et $D \subset X$ un diviseur. Notons A la composante neutre de la variété de Picard de X. Lorsque D est suffisamment ample, on peut associer à ces données un système d'équations différentielles linéaires sur A. On sait décrire la variété caractéristique de ce système et associer à chaque point de X-D une solution de ce système. L'exemple fondamental est le cas où X est une courbe et D un point. Les "constantes de structures" de ce système sont alors des solutions des équations K.P. . Le cas où X est une courbe munie d'une involution et où D est un ensemble de deux points permutés par cette involution, mène à des systèmes d'équations différentielles linéaires liés à l'équation $-\Delta+Q$ où Δ est le laplacien à deux variables et Q un potentiel. Ce cas a été étudié par Kričever et Novikov. On donne un exemple où X est une surface, D une courbe hyperelliptique de genre 3 et où le système construit ci-dessus est lié au laplacien à trois variables.

Jean-Louis Verdier
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The self-duality equation as an integrable system

With the exception of gravity all known forces in nature are described by connections in principal bundles over space-time. The self-duality eq. $F = *F$ for the curvature (= field strength) of such a G-connection arises in various physical contexts, in particular over $M = S^4$ (instantons) and $M = R^3 \times S^1$ (magnetic monopoles, calorons). This eq. shares the 'integrability' properties of the KP hierarchy etc., which here have nice geometrical interpretations.

Consider the twistor bundle of local complex structures over M (with fibre $\mathbb{C}P^1$). The patching fct. f of this bundle satisfies the Lax pair eqs.

$$(\partial_u - \lambda \partial_{\bar v}) f = (\partial_v + \lambda \partial_{\bar u}) f = 0, \quad u = x^0 + ix^1,\ v = x^2 + ix^3,\ \lambda \in \mathbb{C}P^1.$$

On this patching fct. one may act by 'dressing' transformations given by maps $S^1 \to G$. The connection can be recovered from f. Using this construction, R. Ward showed that analytic solutions on a domain in $\mathbb{C}^4 \setminus S$, S a non-characteristic hypersurface, are meromorphic on the whole domain.

For solutions on $R^3 \times S^1$ which are translationally invariant w.r.t. S^1, one writes the connection as $A = \sum_1^3 A_i dx^i + \varphi\, dx^0$ and interprets φ as Higgs field, i.e. a scalar field taking non-vanishing values φ_∞ in the physical vacuum. The self-duality eq. yields $F = *D\varphi$ on R^3. Solutions with $\mathrm{tr} \int F \wedge *F < \infty$ yield magnetic monopoles. The Higgs field looks like $\varphi = \tilde\varphi + O(e^{-cr})$, where $\tilde\varphi$ is an algebraic H-connection, with H = centralizer of φ_∞ in G. The locus of singularities of $\tilde\varphi$ is the real section of an algebraic curve in $\mathbb{C}^3$.

Writing the Lax pair as $(\varphi + iz + i\sum u_k D^k)\chi = 0$, $\sum_k y_k D^k \chi = 0$, $u \in R^3$, $u^2 = 1$, $y \in \mathbb{C}^3$, $u \times y = iy$, z a real parameter, and specializing to square integrable solutions of the first eq. along lines in R^3, one obtains an algebraic curve in $T\mathbb{C}P^1$ (N. Hitchin) with a linear flow on its Jacobian given by translation in z. The envelope of this family of lines in R^3 is the locus of singularities of $\tilde\varphi$.

Considering square integrable orthonormal solutions of the Weyl eq. $q_\mu^+ D^\mu \psi = 0$, q_μ quaternions, and defining a new potential in a dual space $T = \sum_1^3 T_i(z)\, dp^i + T_0(z)\, dz$ by $T_i = -i\int \psi^+ x^i \psi\, d^3x$, $T_0 = \int \psi^+ \frac{\partial}{\partial z} \psi\, d^3x$, one obtains a commutative diagram

$$\begin{tikzcd}
\{A \mid F_A = *F_A\} \arrow[d] & \{T \mid F_T = *F_T\} \arrow[d] \\
\{\psi \mid q_\mu^+ D_A^\mu \psi = 0\} \arrow[ur] & \{v \mid q_\mu^+ D_T^\mu v = 0\} \arrow[ul]
\end{tikzcd}$$

which reduces the explicit determination of the solutions of the P.D.E. to the solution of the completely integrable O.D.E. $F_T = *F_T$, which has Hitchin's spectral curve as invariant and yields the flow on its Jacobian considered above.

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Hyperelliptic generalized Jacobians and the non-linear Schrödinger equation.

By Krichever's method, hyperelliptic ϑ-functions are used to construct quasi-periodic solutions to the non-linear Schrödinger equation (NLS): the higher NLS flows linearize on a generalized Jacobian G. A parametrization of an affine open subset of G is then provided, in terms of algebraic functions on the curve (cf. the Jacobi–Mumford construction for KdV). This allows one to convert the transcendental expression for solutions into an algebraic one and has applications to the determination of: reality conditions; existence of solutions for all time; isospectral manifolds; embedding of the flows into a (finite-dimensional) completely integrable system; construction of the solitons as limits of quasi-periodic solutions.

Emma Previato
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Kowalewski's asymptotic method and Abelian surfaces

When is a system algebraically completely integrable? A system is algebraically comp. integrable (a.c.i.) when it is completely integrable in the usual sense with polynomial invariants and moreover when the real Arnold-Liouville tori can be extended to abelian varieties. Implicit in the work of Kowalewski is the following theorem: if a system Hamiltonian is algebraically completely integrable, then the differential equations admit Laurent solution at infinity with enough degrees of freedom. Conversely in order to prove that a given system is a.c.i. the expansion can be used very effectively to actually build the abelian varieties on which the flow linearize. The nature of those tori can then be completely investigated, including its period matrix, some divisors on it, etc.... Applied to geodesic flow on $SO(4)$, it leads to three classes of metrics for which geodesic flow on $SO(4)$ is a.c.i.. The last case has some quartic integrals.

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ger

Multiplets of Indecomposable Highest Weight Modules of Infinite Dimensional Lie Algebras

We review some developments in the representation theory of semisimple Lie groups (SSLG) and algebras. We work in the context of the elementary (=generalised principal series) representations which exhaust <u>all irreducible representations</u> of any SSLG. We show some relevant examples of our work in that direction, in particular the multiplet classification of the indecomposable elementary representations. We demonstrate how to extend these results to Kac-Moody Lie Algebras and we give <u>all</u> indecomposable highest weight modules of $A_1^{(1)}$ and $A_2^{(1)}$. Work on $A_\ell^{(1)}$, $\ell > 2$, is in progress. We make a <u>conjecture</u> how to obtain the KdV hierarchy of integrable nonlinear equations (bypassing the algebraic-geometric machinery). Namely each such equation shall correspond to a highest weight vector of some indecomposable module in the multiplet of the basic module of $A_1^{(1)}$. Presumably indecomposable modules of the other multiplets and other KM algebras shall give other hierarchies of integrable nonlinear equations (and possibly some new equations).

V.K. Dobrev, TU Clausthal and INRNE Sofia

<u>Monopoles, Toda equations and their integrability</u>

Self dual monopoles in four dimensional spontaneously broken gauge theories constitute analogues of solitons in two dimensional theories and it would be interesting to understand their quantum theory. The radial shape of a spherically symmetric monopole is governed by the Toda equations

$$\frac{d^2}{dr^2}\phi_a = e^{\sum K_{ab}\phi_b}$$

where K is the Cartan matrix for the original gauge algebra.

The solution for which the monopole is regular at the origin has been constructed.

Analogous equations exist in two dimensions and/or involving Cartan matrices for Kac-Moody algebras, together with zero curvature potentials A_μ. In the one dimensional case the quantities $\mathrm{Tr}\,A^N$ are conserved, and constitute candidate Hamiltonians. In two dimensions a condition was established whereby the space component of the zero curvature potential, A_x, could be gauge transformed into abelian form when it forms a series of conserved densities which are candidate Hamiltonian densities.

Using the fundamental Poisson bracket relation between matrix elements of A_x, A_t is constructed for any of the candidate Hamiltonians which then have vanishing mutual Poisson brackets.

The treatment is representation independent and may be suited to a quantum generalisation

David Olive, Imperial College, London.

Chaos in Perturbations of Integrable Hamiltonian Systems

We reviewed the Poincaré-Melnikov method for proving the existence of Poincaré-Birkhoff-Smale horseshoes. This states that if $\dot{x} = X(x)$, $x \in \mathbb{R}^2$ is Hamiltonian with a homoclinic orbit $\bar{x}(t)$ and Hamiltonian H_0, and is perturbed by a Hamiltonian vector field with Hamiltonian $\varepsilon H_1(x,t)$, T periodic in t, ε small, the perturbed separatrices split by $\varepsilon M(t_0) + O(\varepsilon^2)$ where

$$M(t_0) = \int_{-\infty}^{\infty} \{H_0, H_1\}_{(\bar{x}(t-t_0),\, t)}\, dt\,;$$

one assumes M has simple zeros. Generalizations to 2 degree of freedom Hamiltonian systems, systems with symmetry and PDE's were discussed. The examples presented were: a forced pendulum, the Leggett equations for superfluid helium (a result of R. Montgomery), and the equations of a forced and damped beam. Finally, recent work of Holmes, Marsden and Scheurle on the equation

$$\ddot{\varphi} + \sin\varphi = \delta\varepsilon\cos(t/\varepsilon)$$

were discussed. Here one develops an expansion in δ

$$\text{splitting} = M^{(1)}(t_0) + M^{(2)}(t_0) + M^{(3)}(t_0) + \cdots$$

Both $M^{(1)}$ and $M^{(2)}$ are exponentially small, of orders $\varepsilon e^{-\pi/2\varepsilon}$ and $\varepsilon^2 e^{-\pi/2\varepsilon}$ respectively. However due to certain resonance phenomena, $M^{(3)}(t_0)$ appears to contain a term of order ε^3. Such results would be applicable to situations arizing in unfolding theory and KAM theory.

Jerry Marsden, UC Berkeley.

Graphentheorie, 84:07:08-14

Richard K. Guy scripsit

Outerthickness and outercoarseness of graphs.

A graph is planar if it can be imbedded in the plane (or sphere).

A graph is outerplanar if such an imbedding can be found with all vertices on the boundary of one of the cells which are the complement of the graph in the surface.

Halin [3] has shown that a graph is outerplanar just if it does not contain a subgraph homeomorphic to the complete graph, K_4, , or to the complete bipartite graph, $K_{2,3}$, .

The thickness of a graph is the least number of parts in an edge-partition of the graph, where each part is planar.

The coarseness of a graph is the greatest number of parts in a partition of the edges of the graph into parts which are each non-planar.

The outerthickness, θ_o, and outercoarseness, ξ_o, are defined similarly, but with planar (resp nonplanar) replaced by outerplanar (resp nonouterplanar).

The values of the outercoarseness of the complete graph, K_n, and of the complete bipartite graph, $K_{m,n}$, were given by Beineke [1]:

$$\xi_o(K_n) = \lfloor n(n-1)/12 \rfloor \quad , \quad \xi_o(K_{m,n}) = \lfloor mn/6 \rfloor ,$$

except that $\xi_o(K_{1,n}) = 0$ for all n in the second case.

We give corresponding results for the outerthickness,

$$\theta_o(K_n) = \lfloor n/4 \rfloor + 1 \quad , \quad \lceil mn/(2m+n-2) \rceil \le \theta_o(K_{m,n}) \le m ,$$

where, in the second case, we assume $m \le n$.

Richard K. Guy, 84:07:09.

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2. Richard K. Guy, Outerthickness & outercoarseness of graphs, in Mavron & McDonough, Combinatorics (British Combin. Conf, Aberystwyth, 1973), Cambridge Univ. Press, 1974, 57-60.

3. R. Halin, Über einen graphentheoretischen Basisbegriff und seine Anwendung auf Färbensprobleme, Doctoral thesis, Köln, 1962.

Intersection Graphs of Balls in $\mathbb{R}^n$

T.D. Parsons, California State University, Chico

(The following is joint work with S. Krantz and P. Erdös).

Let $\mathcal{G}_n$ be the family of all finite graphs G representable as intersection graphs $G = G(\mathcal{F})$, for $\mathcal{F}$ a set of balls in $\mathbb{R}^n$. If $0 < \varepsilon < 1$, let $\mathcal{G}_{n,\varepsilon}$ be the subfamily of $\mathcal{G}_n$ such that each graph $G \in \mathcal{G}_{n,\varepsilon}$ has a representation $G = G(\mathcal{F})$ where no ball in $\mathcal{F}$ contains more than $1-\varepsilon$ of the volume of another ball in $\mathcal{F}$.

<u>Theorem 1</u>: There exists an integer $c = c(n,\varepsilon)$ such that $\chi(G) \leq c$ for all $G \in \mathcal{G}_{n,\varepsilon}$ [$\chi(G)$ = chromatic no.]

<u>Theorem 2</u>: There exists an integer $k = k(n)$ such that each $G \in \mathcal{G}_n$ has some vertex x for which the neighbor set $N(x)$ contains no independent set of size $> k$.

The results hold for balls defined by any norm on $\mathbb{R}^n$.

T.D. Parsons 10 July 1984

<u>Homomorphisms into Odd Cycles</u>

Paul A. Catlin, Wayne State U., Detroit MI 48202

A homomorphism $\theta: G \to H$ is a function from a graph G into a graph H such that $x \sim y$ implies $\theta(x) \sim \theta(y)$, where $\sim$ denotes adjacency. We consider homomorphisms as a generalization of the concept of vertex coloring (the case H complete).

There is a constructive characterization of the edge-minimal series-parallel graphs with no homomorphism into C_5.

For the case $H = C_{2k+1}$, there are various results, including a sufficient condition for a homomorphism $G \to C_{2k+1}$ to be unique up to isomorphism. This last result is work of Lai Hongjian.

GRAPH POLYNOMIALS WHOSE ZEROS ARE REAL

Let G be a graph on n vertices: $v_1, v_2, \dots, v_n$.

The characteristic polynomial of G is $\phi(G,x) = \det(x\underset{\sim}{I} - \underset{\sim}{A})$, where $\underset{\sim}{I}$ is the unit matrix, $\underset{\sim}{A}$ is the adjacency matrix of G. The matching polynomial of G is $\alpha(G,x) = \sum_{k=0}^{n/2} (-1)^k p(G,k)\, x^{n-2k}$, where $p(G,k)$ is the number of k-matchings in G and $p(G,0)=1$.

Let $A_1, A_2, \dots, A_n$ be non-negative real numbers.

Proposition 1 The zeros of $\sum_{i=1}^{n} A_i\, \phi(G-v_i, x)$ are real. The zeros of $\sum_{i=1}^{n} A_i\, \alpha(G-v_i, x)$ are real.

Proposition 2 The zeros of $\phi(G,x) + \sum_{i=1}^{n} A_i\, \phi(G-v_i,x)$ are real. The zeros of $\phi(G,x) - \sum_{i=1}^{n} A_i\, \phi(G-v_i,x)$ are real. The zeros of $\alpha(G,x) + \sum_{i=1}^{n} A_i\, \alpha(G-v_i,x)$ are real. The zeros of $\alpha(G,x) - \sum_{i=1}^{n} A_i\, \alpha(G-v_i,x)$ are real.

Statements about the location of the zeros of the above polynomials as well as some further graph polynomials with real zeros are given.

Ivan Gutman, Kragujevac, Yugoslavia

Matchings in infinite graphs.
Ron Aharoni, Haifa.

A necessary and sufficient condition is given for a graph of arbitrary size (= possibly infinite) to possess a perfect matching. Using it a generalization of Tutte's 1-factor theorem is proven, as follows:

For any subset S of V construct a bipartite graph $\Pi(G,S)$ whose one side is S and the other consists of the factor critical components in $G-S$. Connect a component P to a vertex s in S

iff there exists some line edge from P to S in G.

Theorem: G has a perfect matching if and only if for every $S \subseteq V$ there exists a matching from the factor critical components of $G-S$ into S in $\Pi(G,S)$.

<u>The Mobility of a Graph</u> (jointly with J. Rooney)

In this talk I discussed kinematic structures constructed from links and joints. I showed how certain types of structure can be represented by direct or interchange graphs, and indicated how such graphs can be used to enumerate kinematic structures. After introducing the mobility of a structure (mobility = #(degrees of freedom) − # (constraints)) I showed how to define the mobility of a graph, and looked in detail at the two cases: (a) mobility 0 and binary links, (b) mobility 1 and binary joints. Finally I related this material to the bracing of rectangular frameworks, using the result that such a framework is rigid if and only if an associated graph is connected.

Robin J. Wilson, The Open University, England.

Two Theorems on Graph Substitution

Suppose we want to represent the maximum clique weight $\omega_G(x)$ of G as a function of the node weights $x = (x_v \mid v \in V)$ in a decomposed way as

$$\omega_G(x) = f\,[\,f_1(x_v \mid v \in V_1), \ldots, f_m(x_v \mid v \in V_m)\,],$$

where $V_1, \ldots, V_m$ form a partition of V and $f, f_1, \ldots, f_m$ are real-valued functions. Under certain conditions (no f_i may map a subset homeomorphic to $\mathbb{R}^2$ bijectively into $\mathbb{R}^1$) we show that G must then decompose according to graph substitution (or X-join) as

$$G = G'\,[\,G/V_1, \ldots, G/V_m\,],$$

where G/V_i is the subgraph of G induced by V_i and G' is the quotient graph of G with respect to the partition $\{V_1, \ldots, V_m\}$.

Applying results on the asymptotic relative frequency of (substitution-) indecomposable partial orders, we then show that almost all comparability graphs are uniquely partially orderable (UPO), i.e.

$$\lim_{n\to\infty} \frac{\#\text{ UPO comp. graphs with } n \text{ points}}{\#\text{ comp. graphs with } n \text{ points}} = 1 .$$

As a consequence, the number of comparability graphs is asymptotically equal to half the number of partial orders.

Rolf H. Möhring, Aachen

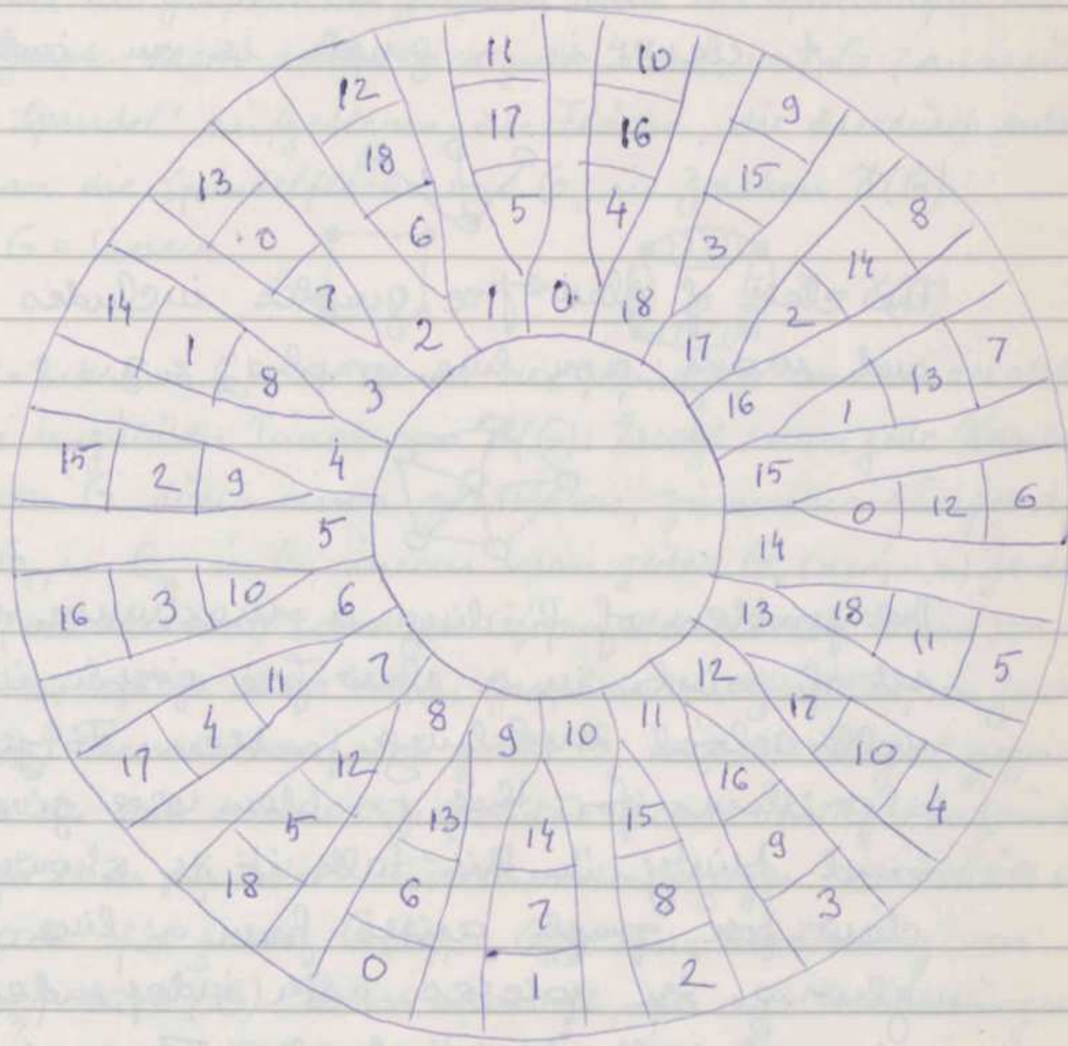
Zur Struktur primer 1-optimaler Graphen

Ein 1-einbettbarer Graph mit α_0 Ecken und α_1 Kanten heißt prim 1-optimal, wenn $\alpha_1 = 4\alpha_0 - 8$ und die Zusammenhangszahl $z(G) = 6$ ist. Ist Γ^* die Menge aller primen 1-optimalen Graphen, so ist die Bestimmung der chromatischen Zahl $\cancel{\chi(G)}$ $\chi(\Gamma^*)$ gleichbedeutend mit der Lösung des von G. Ringel angegebenen 6-Farbenproblems für Landkarten. Auf Γ^* läßt sich eine Ordnungsrelation $>_R$ definieren, deren Minimalbasis $M(>_R, \Gamma^*)$ die Menge der Graphen $2 * \hat{C}_{2m}$, $m \in \mathbb{N} \setminus \{1,2\}$ ist, wobei $\hat{C}_{2m}$ ein Kreis der Länge $2m$ ist, in dem zusätzlich je zwei Ecken mit dem Abstand 2 durch eine Kante verbunden sind. Für Graphen $G \in \Gamma^* \setminus M(>_R, \Gamma^*)$ gilt:

1. Ist k eine Kante von G, so ist k bei jeder 1-Einbettung von G kreuzungsfrei oder k ~~ist~~ wird bei jeder 1-Einbettung von G von einer anderen Kante von G gekreuzt.
2. Löscht man in G alle sich kreuzenden Kanten, so spannen die Ecken mit minimalem Eckengrad in dem so erhaltenen Graphen $V(G)$ einen Wald auf, dessen Komponenten Wege oder Dreiersterne sind.

Heinz Schumacher, Kiel

Beweis der Heawood'schen Imperium
Vermutung in der Ebene



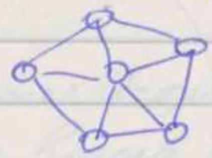
Diese Abbildung zeigt eine Landkarte mit 19 paarweise benachbarte Staaten, wobei jeder Staat aus 4 getrennten Ländern besteht. Wenn man auf Symmetrie verzichtet, so gibt es sogar 24. Allgemein wird gezeigt, es gibt in der Ebene $6m$ paarweise benachbarte m-teilige Staaten ($m \geq 2$). Bisjetzt war dies nur bewiesen für $m = 2$ von Heawood 1890 und für $m = 3$ oder 4 von Taylor 1980. Daß es nicht mehr als $6m$ paarweise benachbarte m-teilige Staaten geben kann, hat schon Heawood bemerkt.

Gerhard Ringel Santa Cruz
California

Independent sets in claw-free graphs

A claw in a graph is an induced $K_{1,3}$:

The class of claw-free graphs includes all line-graphs, and many non-line graphs, e.g.



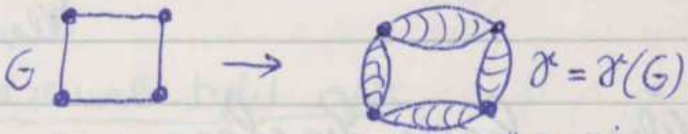
The problem of finding a maximum independent set of points in a claw-free graph includes the well-solved matching problem. Polynomial time algorithms for this problem were given by Sbihi and Minty. In this talk it is shown that every claw-free graph arises from a line-graph by gluing on pieces with independence number ≤ 2 in a well-described way. This gives rise to an algorithm which reduces the independence number problem for claw-free graphs to a matching problem.

László Lovász
(Budapest and Bonn)

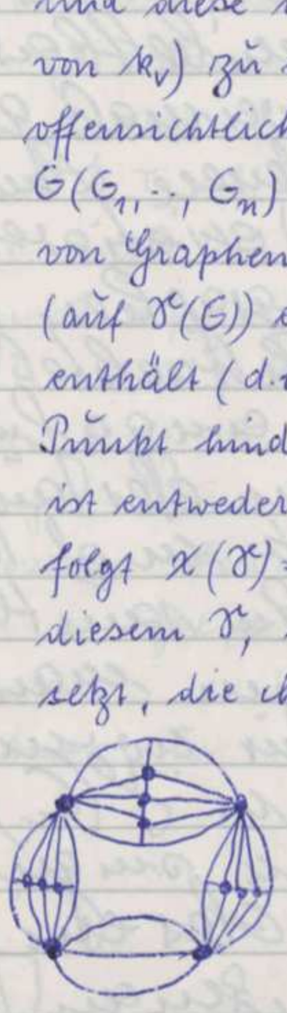
Graphen von Graphen und Spindelflächen

Die Ebene (oder was das Gleiche bedeutet, die Kugel) hat die chromatische Zahl $\chi = 4$, die projektive Ebene hat die chromat. Zahl $\chi = 6$, der Torus $\chi = 7$ u.s.w. Es geht darum, (neue) Flächen $\mathcal{F}$ zu finden mit der chromat. Zahl $\chi(\mathcal{F}) = 5$. Es ergibt sich, zu jeder natürlichen Zahl N gibt es eine solche

Fläche $\mathcal{F}$ mit $\chi(\mathcal{F}) = 5$ und $\chi(G) \leq 4$ für alle in $\mathcal{F}$ einbettbaren Graphen G mit der Eckenzahl $|E(G)| \leq N$. Der „Schlüssel" zu diesem Ergebnis sind die Graphen von Graphen und die Spindelflächen.
Sei G ein Graph. Man denke sich jede Kante von G (anschaulich) zu einer „Spindel" aufgeblasen. Die Fläche, die hierdurch entsteht, nenne man die Spindelfläche von G, in Zeichen $\mathcal{F}(G)$.
Beispiel: G = Viereck



Die Punkte (= Ecken von G), in denen die „Spindeln" aneinanderstoßen, heißen die singulären Punkte von $\mathcal{F}(G)$. Ersetzt man jede Kante $k_1, \ldots, k_n$ von G durch einen plättbaren, zusammenhängenden Graphen $G_1, \ldots, G_n$ in G, indem man jedes G_ν $(\nu = 1, \ldots, n)$ jeweils mit zwei (verschiedenen) Ecken von G_ν in den beiden Ecken von k_ν anheftet, so erhält man einen Graphen von Graphen, in Zeichen $G(G_1, \ldots, G_n)$. Denkt man sich jedes G_ν auf der Kugel gezeichnet und diese in den beiden betreffenden Ecken von G_ν (Endpunkte von k_ν) zu einer „Spindel" auseinander gezogen, so erhält man offensichtlich für jedes $G(G_1, \ldots, G_n)$ eine Einbettung von $G(G_1, \ldots, G_n)$ in $\mathcal{F}(G)$. Es ist sinnvoll, für alle Einbettungen von Graphen in die Fläche $\mathcal{F}(G)$ zu verlangen*), daß keine Kante (auf $\mathcal{F}(G)$) einen singulären Punkt von $\mathcal{F}(G)$ in ihrem Innern enthält (d.h. anschaulich, daß keine Kante durch einen singul. Punkt hindurchläuft). Dann gilt der Satz: Für jedes $\mathcal{F} = \mathcal{F}(G)$ ist entweder $\chi(\mathcal{F}) = 4$ oder $= 5$ oder $= \chi(G)$. Im obigen Beispiel folgt $\chi(\mathcal{F}) = 5$. Man beachte, daß der folgende Graph auf diesem $\mathcal{F}$, der sich aus drei $K_5 - k$ und einem K_2 zusammensetzt, die chromat. Zahl 5 hat. Ist speziell (auch noch der



„äußere" Graph) G plättbar, so folgt aus dem Satz unmittelbar $\chi(\mathcal{F}) = 4$ oder $= 5$ und weiter:
$\chi(\mathcal{F}(G)) = 5 \iff G$ enthält einen Kreis.
Hieraus folgt das oben angegebene Resultat.
Zum Schluß wird noch die Spindelfläche $\mathcal{F}_1$

*) da man andernfalls Länder bekommt, auf deren Grenze „halbe" Kanten liegen würden.

betrachtet, die genau einen singul. Punkt enthält.

γ_1

Es geht um den Kuratowskischen Satz für γ_1, d.h. um alle (minimalen) Graphen, die nicht in γ_1 einbettbar und die aber nach Löschen oder Kontraktion jeder (beliebigen) Kante derselben stets in γ_1 einbettbar sind. Der K_6 und der Petersensche Graph sind zwei solche (minimalen) G.

Klaus Wagner, Köln.

Über ebene Graphen

Ein endlicher, ungerichteter, schlichter und in die Kugel F_0 einbettbarer Graph $G=(E,K)$ ist dadurch ausgezeichnet, daß er höchstens einmal geplättet werden kann und daß der Grenzgraph $G(L)$ für jedes Land L der ebenen Landkarte (G, F_0-G) ein Husimi-Baum. Ist nun $F \neq F_0$ eine beliebige orientierbare oder nichtorientierbare Fläche, so lassen sich stets in F einbettbare Graphen G finden, die mindestens zweimal geplättet werden können oder bei denen Länder L in der Landkarte $(G, F-G)$ existieren, bei denen die zugehörigen Grenzgraphen $G(L)$ keine Husimibäume sind. Betrachtet man nun die von K. Wagner (s.o.) eingeführten Spindelflächen $\gamma(C_n)$, bei denen der äußere Graph G ein Kreis C_n ($n \geq 3$) ist und die inneren Graphen $G_1, G_2, \dots, G_n$ sämtlich gleich dem K_2 sind, so kann man zeigen, daß diese Spindelflächen für unendlich viele Graphen, die sich überdies in $\gamma(C_n)$ so einbetten lassen, daß die n singulären Punkte von $\gamma(C_n)$ als Ecken des eingebetteten Graphen vorkommen, genau eine die Kugel F_0 die Sättigungseigenschaft und die Landkarteneigenschaft erfüllen. Im Gegensatz zur Kugel mit $\chi(F_0) = 4$ ist die chromatische Zahl $\chi(\mathcal{F}(C_n))$ für jedes $n \geq 3$ $\chi(\mathcal{F}(C_n)) = 5$.

Rainer Bodendiek, Kiel

Matchings and maximal tight sets

A tight set $T \subseteq V$ of a graph (V, E) is a matchable set such that $s(F) = T$ for all matchings F of T. ($s(F) = \bigcup\{\{x,y\} \mid \{x,y\} \in F\}$). Every graph has a maximal tight set and every maximal matchable set is a maximal tight set. But if $G = (V, E)$ is countable, then every maximal tight set is also a maximal matchable set of vertices. Therefore every countable set of vertices has a maximal matchable set of vertices. Define by $cl(T) = T \cup \{x \in V \setminus T \mid N(x) \subseteq T\}$ the closure of T and by $\ker G = \bigcup\{cl(T) \mid T \subseteq V \text{ tight}\}$ the kernel of G. Then for every maximal tight sets S, T we have $cl(T) = cl(S)$, so the kernel of G is equal to the closure of any maximal tight set. One can now prove a decomposition theorem for the kernel of a graph. If G is finite, then this theorem is the Edmonds-Gallai decomposition theorem for matchings.

K. Steffens, Hannover

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Edge disjoint complete subgraphs

The problem is to determine the maximum number of edge disjoint K_k's (complete subgraphs of k vertices) contained by any graph of n vertices with m edges. The conjecture is that any graph of n vertices with $t_{k-1,n} + l$ edges ($t_{k-1,n}$ is the edge number of the k-1 partite Turán graph of n vertices) contains $\left(\frac{2}{k} + o(1)\right) l$ edge disjoint K_k's. If $l = o(n^2)$ then $(1+o(1))l$ is proved. For k=3 we have $\left(\frac{5}{9} + o(1)\right) l$ edge disjoint triangles.

At the same time it is proved that any graph of n vertices can be decomposed into $2 \cdot t_{k-1,n}$ edge disjoint K_k's and K_2's, which generalizes a theorem of Bollobás. $(k > 3)$.

Ervin Győri
Budapest

Asymptotic Enumeration of Latin Rectangles (joint work with B.D. McKay, Canberra)

Let $L(k,n)$ denote the number of $k \times n$ Latin rectangles We have established the following asymptotic formula for $L(k,n)$, valid for $k = o(n^{6/7})$:

$$L(k,n) \approx \frac{(n!)^{n+k}}{n^{nk}\,((n-k)!)^n} \exp\left(k(k-1)\,l(k,n)\right)$$

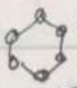
where

$$l(k,n) = \frac{1}{4n} + \frac{k-1}{6n^2} + \frac{k^2-k-1}{8n^3} + \frac{12k^3-13k^2-13k-6}{120n^4}$$
$$+ \frac{15k^4-18k^3-18k^2-28k+47}{180n^5}$$

The strongest previous result was valid only for $k = o(n^{1/2})$. As with all earlier work on this problem, our formula was derived by estimating the number of ways in which a $k \times n$ rectangle R can be extended to a $(k+1) \times n$ rectangle by adding an extra row. The important new feature in our approach was the use of some of the recently developed theory concerning the matchings and rook polynomials.

From this we have that the number of extensions of R can be expressed as

$$\int_0^\infty e^{-x} r(x)\, dx$$

where $r(x)$ is a polynomial associated with R. This leads to an estimate for the number of extensions in terms of n, k and the number of certain small subgraphs ('s, 's, 's) in a k-regular bipartite graph naturally associated with R. The average value, over all $k \times n$ Latin rectangles R, is determined by another method. This finally yields the formula stated above.

Chris Godsil

Burnaby

Packing and Covering of the complete graph by Trees.

Let $P(n, G)$ be the maximal number of pairwise edge disjoint graphs G in the complete graph K_n, and $C(n, G)$ the minimum number of graphs G whose union is K_n.
It is shown that:

$$P(n, T_k) = \left[\frac{n(n-1)}{2(k-1)}\right] \text{ and } C(n, T_k) = \left\{\frac{n(n-1)}{2(k-1)}\right\}, \quad n \geq 9$$

where T_k is every tree of order $k \leq 6$.
([x] denotes the largest integer not exceeding x and {x} the least integer less than x)

It was asked the following questions:

I) Is it true that for each tree T_k of order $k>6$:

$$P(n, T_k) = \left[\frac{n(n-1)}{2(k-1)}\right] \text{ and } C(n, T_k) = \left\{\frac{n(n-1)}{2(k-1)}\right\}, \quad n \geq n_0 \;?$$

(n_0 some constant to be defined).

II) Is it true that for each integer $k>6$, $\exists\, m,n$ integers such that if $(k-1)\mid mn$ then $K_{m,n}$ is decomposed into isomorphic copies of each tree, T_k, of order k?

Ex: for $k=6$. Then $K_{4,5}$ is decomposable into each tree of order 6.

Y. Roditty
Tel-Aviv.

Critical families.

Let $F = (F(i) \mid i \in I)$ be a family of sets. F is called critical if F has an injective choicefunction and $f[I] = \bigcup\{F(i) \mid i \in I\}$ for every injective choicefunction. Critical families can be defined as follows:

Let α be an ordinal:

If $\alpha = \bigcup\alpha$, then $F \in \mathcal{C}_\alpha$ iff there is a chain $(I_\beta)_{\beta<\alpha}$ such that $I = \bigcup\{I_\beta \mid \beta < \alpha\}$ and $(F(i) \mid i \in I_\beta) \in \mathcal{C}_\beta$ for every $\beta < \alpha$.

If $\alpha = \beta+1$, then $F \in \mathcal{C}_\alpha$ iff $F \in \mathcal{C}_\beta$ or there is an $i \in I$ and an $a \in F(i)$ s.t. $F(i) \subseteq \{a\} \cup \bigcup\{F(j) \mid j \in I \setminus \{i\}\}$ and $(F(i) \setminus \{a\} \mid j \in I \setminus \{i\}) \in \mathcal{C}_\beta$

Then F is critical iff there is an α such that $F \in \mathcal{C}_\alpha$.

Klaus P. Podewski
Hannover

t-perfect graphs

The following concept is due to V. Chvátal: An undirected graph $G=(V,E)$ is t-perfect if the coclique polytope (= the convex hull of the characteristic vectors in $\mathbb{R}^V$ of cocliques (stable sets)) is determined by the inequalities:

$$\begin{cases} x_v \geq 0 & (v \in V), \\ x_v + x_w \leq 1 & (vw \in E), \\ \sum_{v \in C} x_v \leq \frac{1}{2}|C| - \frac{1}{2} & (C \text{ odd circuit in } G). \end{cases}$$

If G is t-perfect, we can find a maximum weighted coclique by linear programming methods. It is easy to see that K_4 is not t-perfect. In fact, K_4 seems to be an important forbidden minor for t-perfection. We show that G is t-perfect if G ~~is~~ does not contain a homeomorph of K_4 as subgraph in which all triangles of K_4 have become odd circuits. This generalizes work of Boulala, Fonlupt and Uhry.

A. Schrijver

Eulerian Subgraphs, Cuts and Certain Binary Matroids

We study the convex hull of the incidence vectors of the cycles of a binary matroid. We prove that a description of the facets of this polytope can be obtained from a description of the facets that contain any vertex. A complete and nonredundant description of this polytope by linear equations and inequalities is given for those binary matroids with no F_7^*, R_{10} or $M(K_5)^*$ minor. This implies a convenient characterization of the convex hull of the incidence vectors of Eulerian subgraphs of a graph and of the convex hull of the incidence vectors of the cuts in a graph not contractible to K_5.

Martin Grötschel, Augsburg

EDGE-DISJOINT PATHS IN PLANAR GRAPHS

The following theorem is presented.

THEOREM In a planar graph $G=(V,E)$ k pairs of terminals on the boundary are specified. Every node not in the boundary has even degree. Then there exist k edge-disjoint paths in G between the corresponding terminals iff

$$\sum \text{surplus}(C_i) \geq \frac{q}{2}$$

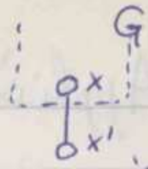
for every family $\{C_1, C_2, \cdots C_\ell\}$ of $\ell \leq |V|$ cuts, where q denotes the number of components in $G-C_1-C_2-\cdots-C_\ell$ which are odd.

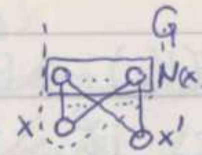
(A set $X \subseteq V$ is called odd if the number of edges leaving X and the number of terminals in X have different parity. The surplus of a cut C is the difference between the number of edges in C and the number of terminals separated by C.)

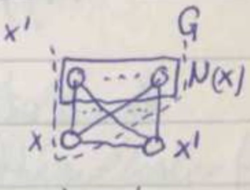
This theorem is a common generalization of earlier results of Okamura-Seymour and of the author.

András Frank, Budapest

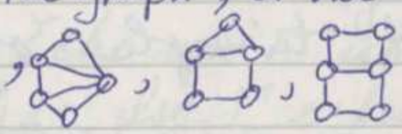
DISTANCE-HEREDITARY GRAPHS (joint work with H.J. Bandelt)

A distance-hereditary graph is a connected graph in which all induced subgraphs are isometric (i.e. all induced paths have the same length, and so are shortest paths). Examples of such graphs are trees, complete multi-partite graphs and ~~even circ~~ ptolemaic graphs. Every finite distance-hereditary graph on ≥ 2 vertices can be obtained from K_2 by a sequence of applications of the following two operations : (i) adding a pendant vertex : 

(ii) splitting a vertex : 

non-adjacent splitting , 

adjacent splitting

Using this result we deduce characterizations of (infinite) distance-hereditary graphs : in terms of the distance function d (involving some four-point-condition); in terms of the interval function I of the graph ; or via forbidden isometric subgraps i.e. C_n $(n \geq 5)$,  . Related results on ptolemaic graphs and parity graphs are given.

Henry Martyn Mulder, Amsterdam

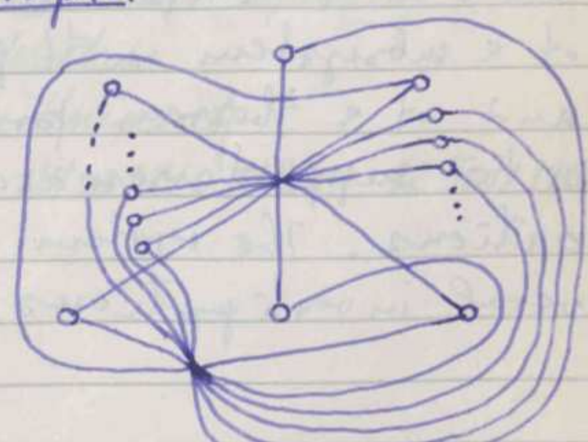
Toroidal graphs admit a 5 flow.

Theorem (M. Möller, Bielefeld): Tutte's 5-flow conjecture is valid for graphs of genus 1

Walter Deuber (Bielefeld)

Multiple crossings in drawings of graphs.

Realizations of a graph in the plane are considered where two lines have at most one point in common, either an endpoint or a crossing. For graphs with 2m vertices at most m-fold crossings are possible. It is proved, that the maximum number of m-fold crossings is 2 for m = 3 and m = 4, and at least 2 in general, as follows from the above figure.



Heiko Harborth (Braunschweig)

Application of small Turán numbers.
(joint work with Y. CARO)

1. Let $f: E(K_n) \to E(K_n)$, $f(e) \neq e$, a K_4 say A is called free if $f(e) \notin E(A)$ for $e \in E(A)$.

Theorem. The smallest n which inforces a free K_4 for every f is 10.

2. Let $f: E(K_n) \to E(K_n)$.

Theorem. The smallest n which inforces either an edgewise fixed triangle or a free triangle is 13.

J. Schönheim, Tel-Aviv

Programming System "Graph" – an Expert System for Graph Theory

Interactive programming system "Graph" for the classification and extension of knowledge in graph theory has been implemented at University of Belgrade, Faculty of Electrical Engineering, in the last four years (1980.–1984.). The system "Graph" consists of a computerized bibliography of graph theory, of a subsystem with graph theoretical algorithms and of a theorem prover. The purpose of the system is to support research in graph theory and applications. The system has already been used in several investigations.

D. Cvetković, Belgrade

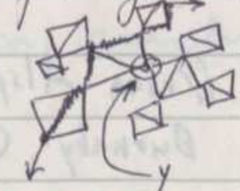
Infinite Paths Containing only Shortest Paths and How to avoid them

Hier werden nur zusammenhängende unendliche lokalfinite Graphen Γ betrachtet. Ein 2-seitig (bezw., 1-seitig) unendlicher Weg A in Γ heißt Achse (bezw. Halbachse), wenn ein kürzester Weg, der je zwei Ecken von A verbindet, in A selbst enthalten ist.

Nach einem bekannten Satz von D. König enthält Γ einen 1-seitig unendlichen Weg. Wir beweisen das stärkere Lemma: Jede Ecke von Γ ist Anfangspunkt einer Halbachse. Satz 1: Ist Γ eckentransitiv, dann liegt jede Ecke auf einer Achse. Satz 2. Ist A eine Achse in einem eckentransitiven Graphen Γ, dann hat $\Gamma \setminus A$ keine unendliche Komponente. Unmittelbar folgt Korollar: Seien A Achse in Γ und y Ecke von $\Gamma \setminus A$. Dann liegt y auf einer mit A disjunkter Halbachse.

Es wird untersucht, in welchen eckentransitiven Graphen Γ man in obigen Korollar "Halbachse" durch "Achse" ersetzen kann. Falls Γ nicht 2-fach zusammenhängend ist wird diese Frage vollständig beantwortet.⊛ Wir vermuten, daß in 2-fach zusammenhängenden Graphen Γ, die unendlich viele Enden haben, eine durch y gehende mit A disjunkte Achse (für jede A und y in $\Gamma \setminus A$) immer existiert.

⊛ Beispiel!



Mark Watkins
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12 Juli 1984

Some new results on the Oberwolfach problem (jointly with R. Häggkvist)

Let $F(\ell_1, \ell_2, \ldots, \ell_r)$ denote a 2-regular graph whose components are cycles of lengths $\ell_1, \ell_2, \ldots, \ell_r$. Write $F(\ell_1, \ell_2, \ldots, \ell_r) | G$ if the edge-set of G can be decomposed into copies of $F(\ell_1, \ell_2, \ldots, \ell_r)$. The following problem is called the Oberwolfach problem:

i) If $\ell_i \geqq 3$ and $\sum_i \ell_i = n$, does $F(\ell_1, \ell_2, \ldots, \ell_r) | K_n$ when n is odd or does $F(\ell_1, \ell_2, \ldots, \ell_r) | K_n - I$, where I denotes a 1-factor, when n is even?

ii) If $\ell_i \geqq 4$, ℓ_i even and $\sum_i \ell_i = n$, does $F(\ell_1, \ell_2, \ldots, \ell_r) | K_{n,n}$ when n is even or $F(\ell_1, \ell_2, \ldots, \ell_r) | K_{n,n} - I$ when n is odd?

When all the ℓ_i's are the same, say ℓ, we simply write $F(\ell)$.

Theorem 1. $F(2m) | K_{2rm} - I$ for all $r \geqq 1$ and all $m \geqq 2$.

Theorem 2. If ℓ_i is even and $\ell_i \geqq 4$ for all i, then $F(\ell_1, \ell_2, \ldots, \ell_r) | K_{4m+2, 4m+2}$ when $\sum_i \ell_i = 4m+2$ and $F(\ell_1, \ell_2, \ldots, \ell_r) | K_{4m,4m}$ when $\sum_i \ell_i = 4m$.

(Note: I learned about the first part of Theorem 2 during this week at Oberwolfach when T. Andreae informed me that his colleague W. Piotrowski has proved it.)

Theorem 3. If $F(\ell) | K_n$, n odd, then $F(d\ell) | K_{dn}$, d odd.

Corollary 4. $F(\ell) | K_n$ when ℓ is odd, $\ell \equiv 0 \pmod 3$ and n is an odd multiple of ℓ.

Brian Alspach
Burnaby, Canada

A Game of Cops and Robbers played on finite connected Graphs

There are two players $\underline{c}$ = cop player and $\underline{r}$ = robber player. First $\underline{c}$ places s cops at some of the vertices of a given graph G ("board of the game"). $\underline{r}$ then places a robber at some vertex. Thereafter the players move alternately. A move of $\underline{c}$ consists of moving some of the cops along edges to adjacent vertices. Similarly, a move of $\underline{r}$ is defined. $\underline{c}$ wins if he catches the robber, $\underline{r}$ wins if he avoids this forever. Let $c(G)$ be the minimal number of cops that are sufficient to catch the robber ("cop-number of G"). Aigner and Fromme proved that $c(G) \leq 3$ if G is planar, and that, in general, $c(G)$ can be arbitrarily high. Here it is shown that, for each finite graph H, there is a minimal $\alpha(H) \in \mathbb{N}$ such that $c(G) \leq \alpha(H)$ if H is not a subcontraction of G. Further, $\alpha(K_n) \leq (n-1)(n-3)$ for $n \geq 4$, $\alpha(K_5) = 3$, $\alpha(K_{3,3}) = 3$, $\alpha(K_5^-) = 2$, $\alpha(W_n) \leq \lceil n/3 \rceil + 1$ (W_n = wheel with n rim vertices, K_n (K_n^-) = complete graph with n vertices (minus an edge)). Other results: 1. Quilliot showed $c(G) \leq 2n+3$ if G has genus n, 2. an algorithmic characterization of the graphs with $c(G) = 1$ is due to Quilliot and (independently) Nowakowski/Winkler, 3. $c(G) \leq 2$ if $|V(G)| \leq 10$ with one exception, namely $c(P) = 3$ for the Petersengraph P.

Thomas Andreae
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West Germany

On universal graphs

A class $\mathcal{G}$ of infinite graphs is said to have a universal element $G_0 \in \mathcal{G}$ if $G \leq G_0$ holds for every $G \in \mathcal{G}$.

Theorem 1. The class of countable planar graphs does not have a universal element.

Theorem 2. (P. Komjáth & J. P.)
Assume GCH. Let $1 \leq \alpha \leq \beta \leq \gamma$ be cardinals, $\alpha < \omega \leq \gamma$. Then there exists a universal element in the class $\mathcal{G}_\gamma(K_{\alpha,\beta})$ of all $K_{\alpha,\beta}$-free graphs on γ vertices if and only if

(i) $\gamma > \omega$

or

(ii) $\gamma = \omega$, $\alpha = 1$ and $\beta = 3$.

The special cases $(\alpha = \gamma > \omega, \beta = 1)$ and $(\alpha = \beta = 2, \gamma = \omega)$ were proved by Shelah, 1973, and Hajnal-Pach, 1981.

János Pach

Eine Abschätzung für die Ramsey-Zahl $r(K_5-x)$

Unter der Ramsey-Zahl $r(G)$ eines Graphen G versteht man die kleinste natürliche Zahl p, so daß bei jeder 2-Färbung der Kanten des K_p ein einfarbiger Teilgraph G vorkommt. Bislang ist $r(K_n)$ nur für $n \leq 4$ exakt bekannt, für $r(K_5)$ weiß man nur $42 \leq r(K_5) \leq 55$. Als eine Art Annäherung an $r(K_5)$ ist $r(K_5-x)$ von Interesse (K_5-x entsteht aus K_5 durch Entfernen einer Kante). Die beste bisher bekannte obere Schranke für $r(K_5-x)$, 24, wird zu 23 verbessert. Die beste bisher bekannte untere Schranke ist 21.

Ingrid Mengersen
Techn. Univ. Braunschweig

A SURVEY OF n-LINKED GRAPHS

A graph is called n-linked if there for distinct vertices $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n$ exist n disjoint paths, the first from x_1 to y_1, the second from x_2 to y_2, etc. The study of n-linked graphs was initiated by independently M. Watkins and R. Halin in 1967. Halin asked if a graph is n-linked under the assumption that it has a sufficiently high connectivity $h(n)$. The problem has four different versions (undirected or directed graphs; vertexdisjoint or edgedisjoint paths).

The talk gave a survey of some of the problems and results on Halin's problem and the more general related <u>subgraph homeomorphism problem</u>, among them: 1) In the undirected vertexdisjoint case the existence of h(n) proved by H.A. Jung and D.G. Larman & P. Mani, based on a result of W. Mader, 2) In the undirected edgedisjoint case a generalized version of Menger's theorem by K.E. Strange and B. Toft, 3) In the directed edgedisjoint case the complete solution of Halin's problem by Edmonds' branching theorem.
In the directed vertexdisjoint case Halin's problem is still open.

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Edge-colouring of graphs. (jointly with A.G. Chetwynd)

Let $\chi'(G)$ and $\Delta(G)$ denote the edge-chromatic number and the maximum degree respectively of a simple graph. By Vizing's theorem $\chi'(G) = \Delta(G)$ (in which case it is called Class 1) or $\chi'(G) = \Delta(G)+1$ (in which case it is called Class 2). If $|E(G)| > \Delta(G) \cdot \left\lfloor \frac{|V(G)|}{2} \right\rfloor$, then call G an <u>overfull</u> graph.

It is well-known that an overfull graph is Class 2.

<u>Theorem 1</u>. Let G have r vertices of maximum degree, let $|V(G)| = 2n$ or $2n+1$ and let $\Delta(G) \geq n + \frac{7}{2}r - 3$. If G is Class 2, then

<u>either</u> G is overfull

<u>or</u> G has an edge-cut S with $|S| < r-2$

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such that $G \setminus S = G_1 \cup G_2$, where $G_1 \cap G_2 = \phi$, $\Delta(G_1) = \Delta(G)$ and G_1 is overfull.

Theorem 2. Let $|V(G)|$ be even and let G be a regular graph of degree $d(G)$. Let $d(G) \geq \frac{6}{7}|V(G)|$. Then $\chi'(G) = \Delta(G)$.

A. J. W. Hilton, Department of Mathematics,
University of Reading, Whiteknights, Reading, U.K.

ISOTROPIC SYSTEMS . RECOGNIZING CIRCLE GRAPHS

Soit un ensemble fini V. On considère l'espace vectoriel K de dimension 2 sur le corps à 2 éléments muni de la forme bilinéaire alternée non nulle $(x,y) \mapsto xy$ et la structure symplectique induite sur K^V par la forme bilinéaire alternée $(\alpha, \beta) \longmapsto \sum_{v \in V} \alpha(v)\,\beta(v)$. Un <u>système isotrope</u> $(\mathcal{L}, V)$ est défini par un sous-espace totalement isotrope $\mathcal{L} \subseteq K^V$ de dimension égale à $|V|$.

Des systèmes isotropes peuvent être associés aux graphes 4-réguliers – les <u>systèmes graphiques</u> – et aux paires de matroïdes binaires duaux. De façon générale les systèmes isotropes unifient certaines propriétés de ces structures. Tel est le cas par exemple du polynôme de Tutte symétrique (obtenu par identification des 2 variables) d'un matroïde binaire et du polynôme de Martin (fonction génératrice des décompositions eulériennes d'un graphe 4-régulier). L'unification de ces deux polynômes dans le cadre des systèmes isotropes permet de prouver une conjecture de Las Vergnas "$T(3,3) = s\,|T(-1,-1)|$ avec s impair pour le polynôme de Tutte d'un matroïde binaire".

Une autre application provient du fait qu'un système isotrope est naturellement associé à un graphe simple considéré aux complémentations locales près (remplacement d'un sous-graphe induit sur le voisinage d'un sommet par son complémentaire). Alors les graphes de cordes (obtenus en associant des sommets aux cordes d'un cercle et en reliant deux sommets si les cordes correspondantes intersectent) correspondent aux systèmes graphiques. Une théorie de la 3-connexité des systèmes isotropes similaire à celle de Tutte pour les matroïdes permet de donner un algorithme polynomial de reconnaissance des graphes de cordes.

A. Bouchet

Parity Theorems for Paths and Cycles in Graphs.
(a report of joint work with Fay Halberstam)

A graph is <u>even</u> (resp. <u>odd</u>) if every vertex has even (resp. odd) degree. Multiple edges are permitted. Define:

$p_i(u)$: number of paths of length i with initial vertex u;
p_i : number of paths of length i;
p : number of paths; v: number of vertices.

Using the 'lollipop' technique of A. G. Thomason, we obtain the following results:

1/ $p_i(u)$ is even if G is even and $i \geq 1$ or G is odd and $i \geq 2$;
2/ p_i is even if G is even + bipartite and $i \geq 1$ or G is odd + bipartite and $i \geq 3$;
3/ $p \equiv v \pmod 2$ if G is even, $p \equiv \frac{1}{2} v \pmod 2$ if G is odd.

A <u>generalized friendship graph</u> is one in which any two distinct vertices are connected by a unique path of length k (k fixed). A. Kotzig has conjectured that no such graph exists for $k \geq 3$. It is easily seen that, for $k \geq 2$, every GFG is even. Applying result 1 we have $p_k(u)$ even. Since $p_k(u) = v - 1$ (from the definition), v is odd. Thus every GFG is of odd order.

Let $c(e)$ denote the number of cycles containing the edge e. The following result of Toida can be proved using the lollipop technique.

4/ $c(e)$ is odd if G is even.

B. Richter and H. Shank have pointed out two interesting corollaries.

5/ If G is even, the number of paths connecting any two vertices is even;

6/ If G is even, the number of decompositions into cycles is odd.

Adrian Bondy
University of Waterloo

Density theorems for hypergraphs
(jointly with Peter Frankl)

Let $G_i^r(n_i, e_i)$, $i = 1, 2, \dots$ $n_i \to \infty$ be a sequence of r-uniform hypergraphs of n_i vertices and e_i edges. The density of this sequence is α, $0 \le \alpha \le 1$, if α is the largest real number for which there is a subgraph $G_i^r(m_i, e_i')$ of $G_i^r(n_i, e_i)$ for which $m_i \to \infty$ and $\lim e_i' / \binom{m_i}{r} = \alpha$. The theorem of Erdős, Stone and Simonovits states that for $r = 2$ the only possible values of the densities are 1 and $1 - 1/t$ where $t = 1, 2, \dots$.

It was conjectured by P. Erdős that for $r \ge 3$ the set D_r of possible densities for r-uniform hypergraphs forms a well ordered set. We disprove this ~~theorem~~ conjecture and show the following

<u>Theorem</u>: For every $\varepsilon > 0$, $r \ge 3$ there exists $l(r)$ such that for any $l > l(r)$

$$1 - \frac{1}{l^{r-1}} < d < 1 - \frac{1}{l^{r-1}} + \varepsilon$$

for some $d \in D_r$.

Vojtěch Rödl
Prague

Partitions of digraphs into elementary paths or directed cycles (with W. Bienia)

Let $G=(V,E)$ a directed graph; let $|\omega^+(A)|$ $A\subset V$ the cardinality of the set of directed edges going from A to $V\setminus A$; let $\lambda = \sup_{A\subset V} |\omega^+(A)|$

It is proved that for pseudosymmetric graphs (i.e. $d^+_G(x)=d^-_G(x)\ \forall x$) it is possible to partition the directed edges into λ elementary directed cycles.
For bipartite graphs it is possible to partition the edges into λ paths or directed cycles of length at most two. It is a general result by Maamoun (to appear in JCTB) that for any directed graph it is possible to partition the edges into λ paths or directed cycles.
We give two conjectures

C_1 If G is pseudosymmetric, it is possible to partition the edges into $\leq |V|$ directed cycles.

C_2 It is possible to partition the edges into $\leq \lambda + 2|V|$ paths (a consequence of C_1)

Henry Meyniel (CNRS Paris)

On 3-partitions of a set
(jointly with P. Erdös)

We give two results relevant to the Ramsey functions $R_3(n,n)$ and $R_3(n,n,n)$.

It is well known that $2^{c_0 n^2} < R_3(n,n) < 2^{2^{c_1 n}}$ for some $c_0, c_1 > 0$. The next theorem says that the lower bound can not be improved using „justly distributed" partitions

Th 1. $\exists \alpha > \frac{1}{2}\ \exists c_\alpha > 0\ \forall m\ \forall (K_0,K_1)([m]^3 = K_0 \cup K_1 \Rightarrow \exists H \subset m\ \exists i<2$

$$|H| \geq c_\alpha \sqrt{\log m} \wedge |[H]^3 \cap K_i| \geq \alpha \binom{|H|}{3})$$

Th. 2 $R_3(n,n,n) \geq 2^{\frac{n^2 \log n}{\log\log n}}$ for $n > n_0$.

We conjecture that Th 1. extends to every $\alpha < 1$.

For history and earlier results see "Combinatorial Set Theory" by Erdős Hajnal Máté and Rado

András Hajnal

On minimally n-connected digraphs

Für einen minimal n-fach zusammenhängende, gerichteten Graphen D sei D_0 der von den Kanten (x,y) mit $\gamma^+(x;D) > n$ und $\gamma^-(y;D) > n$ erzeugte Teilgraph, wobei $\gamma^+(x;D)$ bzw. $\gamma^-(x;D)$ den Außengrad bzw. Innengrad von x in D bedeute. Es wird gezeigt, daß D_0 keinen alternierenden Zyklus enthält. Dies ist äquivalent dazu, daß ein D_0 zugeordneter paarer Graph keinen Kreis enthält. Hieraus ergeben sich ähnliche Resultate wie im ungerichteten Fall. (1) Für jeden endlichen, minimal n-fach zusammenhängenden gerichteten Graphen D gelten $|\{x \in E(D) : \gamma^+(x;D) = n\}| \geq n$ und $|\{x \in E(D) : \gamma^+(x;D) = n\}| + |\{x \in E(D) : \gamma^-(x;D) = n\}| \geq \frac{n-1}{2n-1}\, 2|D| + \frac{2}{2n-1}$. (2) Für jeden endlichen, minimal n-fach zusammenhängenden, gerichteten Graphen D gilt für die Kantenzahl $\|D\| \leq 2n|D| - n(n+1)$ und im Falle $|D| \geq 4n+5$ sogar $\|D\| \leq 2n(|D|-n)$, wobei die D charakterisiert werden, für welche das Gleichheitszeichen gilt.

W. Mader (Hannover)

Path Partitions and Packs of Digraphs

Let G be a digraph. A path partition $\mathcal{P} = \{P_1, \ldots, P_m\}$ is a partition of $V(G)$ into directed paths. Denote by $|\mathcal{P}|_k = \sum_{P_i \in \mathcal{P}} \min\{|P_i|, k\}$. A path partition is k-optimal if $|P|_k$ is as small as possible.

A partial k-colouring is a collection $\mathcal{C}^k = \{C_1, \dots, C_k\}$ of k disjoint independent sets. A path partition P and a partial k-colouring $\mathcal{C}^k$ are orthogonal if ~~for~~ each $P_i \in P$ meets exactly $\min\{|P_i|, k\}$ different colour classes of $\mathcal{C}^k$. Berge made the following conjecture: For every k-optimal path partition P of G there exists a partial k-colouring $\mathcal{C}^k$ orthogonal to P. We report on recent progress on the problem, ~~on~~ and on ~~the~~ ~~problem~~ a dual problem.

Ruth Ben-Arogo Hartman

Konvexe Körper

15 – 21 Juli 1984.

Affine Surface Area

Let K be a convex body in n-space such that, for some $\tau_0 > 0$, the spherical image of that part of the boundary of K outside the half-space $\langle x, u\rangle \le H(u) - \tau$ is any open hemisphere whenever $0 < \tau < \tau_0$ and u is any direction; here H denotes the support function of K. Then for each $\delta > 0$ and sufficiently small, there is a unique convex body $K(\delta)$, each of whose support hyperplanes cuts off from K a set of volume δ. Set $\Psi_\delta(K) = (V(K) - V(K(\delta), K, \dots, K))/\delta^{2/(n+1)}$, where V signifies volume or mixed volume. $\Psi_\delta(K)$ is a unimodular affine invariant. Symmetrization arguments show: (*) $\Psi_\delta(K) \le \Psi_\delta(\rho B)$, where ρB is a ball with $V(\rho B) = V(K)$. Inequality (*) implies that $\limsup\limits_{\delta \to 0+} \Psi_\delta(K) = \Psi(K)$ exists and also that $\Psi^{n+1}(K) \le \beta_n V^{n-1}(K)$ with β_n a constant for each n. $\Psi(K)$ is a unimodular affine invariant which vanishes for polytopes and equals the classical affine surface area of Blaschke and Santaló when K is smooth enough. See also the abstract of K. Leichtweiss in this Tagung for a different, independent development leading to similar results

Wm. J. Firey
Corvallis, Oregon, USA.

Polyhedra with transitivity properties

This talk was concerned with finite (bounded) polyhedra in $\mathbb{E}^3$ whose faces have no mutual or self-intersections. For such a polyhedron P let $S(P)$ be the group of symmetries (isometries that map P onto itself) and say that P is

V-transitive if $S(P)$ is transitive on the vertices of P,

E-transitive if $S(P)$ is transitive on the edges of P,

F-transitive if $S(P)$ is transitive on the faces of P.

A number of theorems were stated showing the connections between these concepts and transitivity on the various sorts of flags. In particular:

1) An E-transitive polyhedron is convex;

2) An F-transitive polyhedron is star-shaped and its faces are star-shaped;

3) The vertices of a V-transitive polyhedron lie on a sphere, and all its faces are convex. There exist such polyhedra of genera 0, 1, 3, 5, 7, 11 and 19.

The talk concluded with some remarks on enumerating the "types" (in a well-defined sense) of F-transitive and V-transitive polyhedra. In particular, the construction of the V-transitive polyhedra of genus greater than 0 was described and illustrated by models.

G. C. Shephard
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The minimal number of circuits in a finite set in R^d

A subset C of R^d is a <u>circuit</u> if C is affinely dependent, and every proper subset of C is affinely independent.

Denote $C(S)$ = the number of circuits included in S, and $m(d,s) = \min\{C(S) : S \subset R^d, \text{card } S = s\}$. The problem of finding $m(d,s)$ has been posed by Eckhoff in 1978. A partial solution ($d=2$ and $s \leq 3(d+1)/2$) was published in 1980 by J-P. Doignon. This work settles the case of any <u>odd</u> dimension d, proving Doignon's conjecture

M. Kallay
Jerusalem, Israel

Linear and non-linear inequalities for mixed volumes

Let a_n be a finite or infinite sequence. Then

(1) a_n log convex $\Rightarrow$ a_n convex.

(2) a_n concave $\Rightarrow$ a_n log concave.

Example 1: Let r be the equator radius of a body of revolution in E^d. Then Hadwiger showed that $a_n = r^n W_n$ (W_n quermassintegral) is concave. By (2) this implies (and improves) the Fenchel-Alexandrov inequalities for bodies of revolution.

Example 2: Let two convex bodies be given which have the same projection. Then Bonnesen showed that the mixed volumes V_0, V_1, V_2 or V_0, V_1, V_d form concave sequences. (2) shows that this an improvement of Minkowski's inequalities.

Example 3: Let K_0 be a summand of K_1. The the mixed volumes V_n form a convex sequence.

Example 4: Let R be the circumradius of a convex body. Together with J. Bokowski it was shown that $a_n = (n+1)R^n W_n$ is convex. The sequences of examples 3 and 4 are not log convex, but log concave. Are there similar situations where one can use (1)?

Erhard Heil, Darmstadt

On compact packing of circles

Let O and O_i be the centres of the circles c and c_i on the unit sphere. A packing of circles (= spherical caps of radius less than $\pi/2$) is said to be compact if each circle c of the packing satisfies the following three conditions:

(i) c has a finite number of neighbours (= circles touching c);

(ii) if c has n neighbours, $c_1, \dots, c_n$, they can be numbered so that c_1 touches $c_2, \dots, c_n$ touches c_1;

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(iii) C is covered by the union of the triangles $OQ_1Q_2, \dots, OQ_nQ_1$.
In this talk a lower bound for the density of a compact packing of a finite number of circles with radii $\leq R \leq \pi/3$ is given. This bound is attained if and only if the circles are the in-circles of a regular tiling $(p,3)$, for $p = 2, 3, 4, 5$.
A similar result is proved for compact packings of circles in the hyperbolic plane. The above definition of a compact packing is due to L. Fejes Tóth who considered compact packings in the Euclidean plane.

A. Florian, Salzburg

Toric varieties and lattice polytopes

Since about 14 years a connection between algebraic geometry and convex body theory has developed. Part of the development is centered around the concept of a toric variety:
Let $\{\sigma_\alpha\}$ be a system of cones in $\mathbb{R}^n$ generated positively by lattice vectors each, and none of them containing a linear subspace of positive dimension. If σ_β is a face of σ_α then it is to belong to the system; each intersection of different cones again lies in the system (cell complex properties). To any σ_α we assign its dual cone $\check{\sigma}_\alpha$ and consider the affine variety $U_\alpha :=$ spec $\mathbb{C}[\check{\sigma}_\alpha \cap \mathbb{Z}^n]$. We glue together U_α and U_β in spec $\mathbb{C}[\check{\sigma}_\alpha \cap \check{\sigma}_\beta \cap \mathbb{Z}^n]$, obtaining what is called a toric variety (or DeMazure-variety) $X_{\{\sigma_\alpha\}}$.
For $n > 2$ little is known ~~by~~ about the structure of toric varieties. It is possible, for example, to resolve singularities by applying special types of morphisms that are induced by stellar subdivisions of the complex of cones. We call them generally σ-processes.
Thm. Given a compact projective toric variety there exist stellar subdivisions ~~or~~ inverse stellar subdivisions

$\sigma_1, \dots, \sigma_k$ such that $\sigma_k \cdots \sigma_1 X_{\{\sigma_\alpha\}} = \mathbb{P}^n$ where n is the dimension of $X_{\{\sigma_\alpha\}}$.
Since compactness means that $\{\sigma_\alpha\}$ covers $\mathbb{R}^n$, and since projectiveness of $X_{\{\sigma_\alpha\}}$ is equivalent to $\{\sigma_\alpha\}$ being generated by the facets of a lattice polytope by projecting them from an interior point of the polytope the proof of the theorem reduces to a result of G.C. Shephard and the author from 1973.
For $n=3$ the following is shown for arbitrary toric varieties $X_{\{\sigma_\alpha\}}$ that are compact:
Thm. There exist stellar subdivisions $\sigma_1, \dots, \sigma_s, \tau_1, \dots, \tau_t$ such that $\sigma_s \cdots \sigma_1 X_{\{\sigma_\alpha\}} = \tau_t \cdots \tau_1 \mathbb{P}^3$.
The proof is achieved by proving the following fact (which solves for $d=2$ an old problem of p.l.-topology):
Thm. Let Σ, Σ' be simplicial complexes of dimension $d=2$, and let $|\Sigma| = |\Sigma'|$. Then there exist stellar subdivisions $\sigma_1, \dots, \sigma_s, \tau_1, \dots, \tau_t$ such that $\sigma_s \cdots \sigma_1 \Sigma = \tau_t \cdots \tau_1 \Sigma'$.

Günter Ewald

Weakly neighborly polyhedral maps (Common work with U. Brehm)

A weakly neighborly polyhedral map (w.n.p. maps) is a 2-dim. topological cell-complex which decomposes a closed 2-manifold without boundary, such that every two vertices belong to a common facet. There are infinitely many w.n.p. maps on the 2-sphere (all the pyramids and the triangular prism), but only finitely many on any other 2-manifold. So far, we know all the orientable w.n.p. maps of genus 0 (∞), 1 (5) and 2 (0), and all the non-orientable w.n.p. maps of Euler characteristics 1, 0, −1, −2. In the talk we discuss the 5 w.n.p. maps on the torus, and we show that precisely 2 of them are not geometrically realizable.

Amos Altshuler
Ben-Gurion University of the Negev
Beer-Sheva, Israel

Finite Packing and Covering

We consider the following two problems in euclidean E^d: Determine, for a given convex body $K \subset E^d$ and a given $k \in \mathbb{N}$

1) The maximal volume of all convex bodies, into which k translates of K can be packed,
2) The maximal volume of all convex bodies, which can be covered by k translates of K.

The convexity is essential and the relation to convexity is much closer than in the classical packing and covering problems. One can replace the volume by the surface area or other intrinsic volumes. The case if $K = B^d$ is the unit ball is of particular interest.

The geometric behaviour depends strongly on d ($d=2$ classical results by L. Fejes Toth, Bambah, Rogers, Woods, Zassenhaus, Groemer). For $d=3$ and 4 one gets sausage catastrophes and for $d \geq 5$ sausages and bones.

The talk gives a survey on the known results.

Jörg M. Wills (Siegen)

Sausages, densities and the lattice-point enumerator

Let C_n denote the convex hull of the centres of n non-overlapping translates of the unit ball B^d in E^d and let S_n be a line-segment of length $2(n-1)$. In 1975 L. Fejes Tóth conjectured that for $d \geq 5$

$$V(S_n + B^d) \leq V(C_n + B^d)$$

("sausage-conjecture"). This inequality is at least "almost" true, since for $d \geq 2$

$$V(S_n + B^d) < \left(2 + \sqrt{2} + \frac{2}{\sqrt{d-1}}\right) \cdot V(C_n + B^d).$$

In particular, in terms of finite densities $\delta_{S\!S}^{d}$ this yields

$$\sqrt{1-\frac{1}{d+1}} < \sqrt{\frac{2}{\pi}}\sqrt{d}\ \delta_{S\!S}^{d}(B^d) < 2+\sqrt{2}+\frac{2}{\sqrt{d-1}}.$$

Furthermore, there is a close connection between the sausage-conjecture, or more generally finite packings, and some lattice-point problems. Thus, applying results on the density of finite packings, we obtain certain inequalities for the lattice-point enumerator.

Peter Gritzmann, Siegen

Tarski's circle-squaring problem.

Two sets A, B in $\mathbb{R}^n$ are equidecomposable if $A = \bigcup_{i=1}^{n} A_i$, $B = \bigcup_{i=1}^{n} B_i$, $A_i \cap A_j = \emptyset = B_i \cap B_j$ for $i \neq j$, and $B_i = \tau_i A_i$ for isometries τ_i. Tarski asked in 1925 whether a circle (with interior) in $\mathbb{R}^2$ and a square are equidecomposable. Two convex bodies A, B in $\mathbb{R}^n$ are convex equidecomposable if each A_i and B_i in the definition above is also a convex body and $\operatorname{int}(A_i) \cap \operatorname{int}(A_j) = \emptyset = \operatorname{int}(B_i) \cap \operatorname{int}(B_j)$. It follows from Banach and Tarski's work that two polygons in $\mathbb{R}^2$ are equidecomposable if and only if they are convex equidecomposable. We gave examples to show that this does not hold for convex bodies, answering a question of Sallee in 1969. We also stated the result that two convex bodies in $\mathbb{R}^2$ are Q-equidecomposable if and only if they are Q-convex-equidecomposable. Here Q is the group of rational translations, and Q-(convex)-equidecomposable means that τ_i above belongs to Q. It follows that the circle and the square are not Q-equidecomposable.

Richard Gardner (Dhahran, Saudi Arabia)

TWO GEOMETRIC CURIOSITIES

Applications of a result of Jacobi on determinants lead to two geometric curiosities. First, let L be a rational linear r-space in $\mathbb{E}^n$, let $\mathbb{Z}^n$ be the ordinary lattice of integer points of $\mathbb{E}^n$, and let d denote determinant of sublattices. Then $d(\mathbb{Z}^n \cap L) = d(\mathbb{Z}^n \cap L^{\perp})$, where $L^{\perp}$ is the orthogonal complement of L.

Similarly, $d(\mathbb{Z}^n | L) = d(\mathbb{Z}^n | L^\perp)$, where $|L$ denotes orthogonal projection on to L. Possibly these properties characterize $\mathbb{Z}^n$.
Second, let C be the unit n-cube in $\mathbb{E}^n$, and let V_r denote r-dimensional volume. Then for each r-space L, $V_r(C|L) = V_{n-r}(C|L^\perp)$. However, centrally symmetric polytopes satisfy the same property (for all r and L) if and only if they belong to the class of orthogonal direct products of members of $\mathcal{S}_x$, which consists of unit line segments and polygons P of unit area, for which $P-P$ has 4-fold symmetry. Certain non-centrally symmetric members of this class also share the projection property, but the characterization is as yet incomplete.

Peter McMullen, London.

Shellings and stellar equivalences

Let $\mathcal{M}$ be a triangulation of a closed p.l. n-manifold and $\mathrm{link}(A;\mathcal{M}) = \mathcal{B}(B)$ for a simplex $B \notin \mathcal{M}$. Then $\chi_{(A,B)}\mathcal{M} := (\mathcal{M} \setminus A \cdot \mathcal{B}(B)) \cup \mathcal{B}(A) \cdot B$ is called a bistellar operation. The following holds:

Theorem 1

$\mathrm{set}\, \mathcal{M} \overset{p.l.}{\cong} \mathrm{set}\, \mathcal{M}' \iff \mathcal{M} \overset{bst}{\sim} \mathcal{M}'$

Similar to shellings bistellar equivalences are closely related to problems concerning f-vektors of simplicial spheres.
Using Theorem 1 it can be shown:

Theorem 2

Every triangulation of a p.l. n-sphere is the boundary complex of a shellable simplicial $(n+1)$-ball.
This property is a necessary condition for the shellability of triangulations of p.l. spheres, which is an unsolved problem for $n \geq 3$.

Uwe Pachner (Bochum)

Tomography of convex bodies.

P.C. Hammer asked in 1961 the following question: Suppose we have a convex hole K in a homogeneous solid and assume that an X-ray picture gives the length of the chord along any ray. How many X-ray pictures are needed to reconstruct K, if a) the X-rays issue from a finite source; b) the X-rays are assumed to be parallel?

The following theorem holds:

Theorem

If P_1, P_2 and P_3 are three non collinear points in the projective plane, then X-ray pictures taken from them determine uniquely any convex body K containing the points.

This result, together with the previous results of Giering, Gardner, McMullen and Falconer, solves Hammer's X-ray problem in the planar case, at least from the point of view of uniqueness theorems.

A reconstruction algorithm is presented (joint work with Kölzow and Kuba), based on an algorithm valid for directionally convex binary patterns.

Aljoša Volčič
Trieste

Centrally Symmetric Convex Bodies

The talk is concerned with an investigation of various classes of centrally symmetric convex sets. These classes range from the zonoids at one extreme to the class of all centrally symmetric bodies at the other. For $1 \le j \le d-1$ and $1 \le k \le d-j$, $P(j,k)$ comprises all K with $\dim K \ge j+1$ and for which

$$V(K, j; L, k; B, d-j-k) \le V(K, j; M, k; B, d-j-k)$$

whenever

$$V(L, k; B, d-j-k : E) \le V(M, k; B, d-j-k : E)$$

for all E in the Grassmannian $G(d, d-j)$. Thus mixed volumes involving these bodies reflect properties of certain intrinsic volumes in lower dimensional subspaces. We find various characterizations of these classes, some involving measures on Grassmann manifolds. Finally, we mention some connections with a related problem in stochastic geometry.

Paul Goodey
(Norman)

Inequalities for random flats meeting a convex body.

We choose a uniform random point in a given convex body K in n-dimensional Euclidean space and through that point the secant of K with random direction chosen independently and isotropically. Given the volume of K, the expectation of the length of the resulting random secant of K was conjectured by Enns and Ehlers (1978) to be maximal if K is a ball.

Combining some methods of integral geometry with results from convex body theory, we prove this, and we also treat moments of higher order and higher-dimensional sections defined in an analogous way. By similar methods, we also show that certain geometric probabilities, connected with a finite number of independent isotropic uniform random flats meeting K, become maximal when K is a ball.

Rolf Schneider (Freiburg i. Br.)

Sets of constant width

In a joint work with E. Schulte we proved following statements.

Theorem 1. Let L be a body of constant width 2. Assume that a ball of radius $r > 0$ slides freely in L and let $2-r < c < 2$ and C be a compact, convex subset of L with diameter c. Then there exists a body K of constant width c such that

(a) $C \subset K \subset L$ and $K \cap \operatorname{bd} L = C \cap \operatorname{bd} L$,

(b) Each point in $(C \cap \operatorname{bd} K) \setminus \operatorname{bd} L$ is at the end of a chord of C,

(c) Each common symmetry of C and L is a symmetry of K.

(d) Each singular boundary point x of K is a singular boundary point of C (relative to aff C) and, if $x \in K \setminus \operatorname{bd} L$, then $S(x,K) = \operatorname{Sing}(x,C)$.

Theorem 2. Let L be a set of constant width 2 and $1 \leq c < 2$. Each subset C of diameter c is embeddable into a body K of constant width c contained in L (and with $K \cap \operatorname{bd} L = C \cap \operatorname{bd} L$), if and only if a ball of radius $r = 2-c$ slides freely in L (with exactly one point of contact with $\operatorname{bd} L$).

The following characterization of the ball is an immediate consequence of theorem 2.

Corollary. The unit ball is the only set L of diameter 2, for which each subset of diameter c, $c \geq 1$, is embeddable into a body K of constant width c contained in L.

Siniša Vrećica, Belgrade

Extremal properties of regular 3-zonotopes

We call a 3-zonotope regular, if its faces are congruent rhombs and its vertex figures are regular polygons. There are exactly three regular 3-zonotopes: the cube, the rhombic dodecahedron and the rhombic triacontahedron, generated by 3, 4 resp. 6 unit line segments.
Theorem: The regular 3-zonotopes have maximal inradius and maximal surface area among all 3-zonotopes generated by the same number of unit line segments.
Conjecture: There is an analogous theorem concerning the minimal circumradius and the maximal volume.

Johann Linhart, Salzburg

On boxes and large angles

A box in d-space is a set of the form $\{x \in R^d : a_i \leq x_i \leq b_i, i = 1, \dots, d\}$. The box, $\text{box}(p,q)$ of two points $p, q \in R^d$ is defined as the smallest box containing p and q. The following theorem answers a question of A. Gyárfás and J. Lehel and is proved in a joint work with J. Lehel:
Thm 1 Each compact set $V \subset R^d$ contains a subset $S \subset V$, $|S| \leq f(d)$

such that $V \subset \bigcup_{p,q \in S} \text{box}(p,q)$ where $f(d)$ is a constant depending on d only.

Similar results are true if one defines the "neighbourhood" of two points differently. For instance, let $\text{hull}_\varepsilon(p,q)$ be the set of points $x \in R^d$ for which $\sphericalangle(p x q) \geq \pi - \varepsilon$.

<u>Thm 2.</u> Each compact set $V \subset R^d$ contains a subset $S \subset V$, $|S| \leq f(d,\varepsilon)$ such that $V \subset \bigcup_{p,q \in S} \text{hull}_\varepsilon(p,q)$ where $f(d,\varepsilon)$ is a constant depending on d and ε only.

These results are related to some classical theorems due to Erdős and Szekeres.

Imre Bárány
Budapest

Construction of neighborly 3-spheres

Given a 2-neighborly 3-sphere S on n vertices, $x \in \text{vert}\, S$, and H a Hamiltonian circuit on $\text{link}(x,S)$ separating $\text{link}(x,S)$ into the triangulated disks K^+, K^-, one obtains a 2-neighborly 3-sphere S' on $n+1$ vertices by replacing x and the cells incident to it by two new vertices x^+, x^- and

$$\{[x^+ x^-, F] : F \in H\} \cup \{[x^+, F] : F \in K^+\} \cup \{[x^-, F] : F \in K^-\}.$$

Each S' on $n+1$ vertices having a universal edge (edge of valence $n-1$) arises from an S on n vertices in this way. As an application of this two results of I. Shemer on universal edges of neighborly polytopes are also proved for non-polytopal spheres (in case of 4 dimensions), and a Steinitz criterion for 2-neighborly 3-spheres in terms of universal edges is given.

Christoph Schulz
Hagen (z.Zt. Siegen)

Über Polytope ohne dreieckige 2-Seiten

Im $\mathbb{R}^d$ sei $\mathcal{P}_\Delta$ die Klasse der Polytope ohne dreieckige 2-Seiten, d.h. der Polytope, bei denen keine Seite eine Pyramide ist; $\mathcal{P}_\Delta^s$ sei die Klasse der einfachen Polytope aus $\mathcal{P}_\Delta$. $f_j(P)$ sei die Anzahl der j-Seiten eines Polytops P. Im $\mathbb{R}^d$ besteht für $P \in \mathcal{P}_\Delta$ und für jedes $j \in \{0, \ldots, d-1\}$ die

Vermutung: $f_j(P) \geq 2^{d-j}\binom{d}{j}$, und das Gleichheitszeichen gilt genau dann, wenn P komb. äquivalent zum d-Kubus ist. (by Kupitz 1982)

Diese Vermutung ist richtig speziell für $d \leq 4$, sie ist richtig bei beliebigem d und beliebigem j für $P \in \mathcal{P}_\Delta^s$, sie ist richtig bei beliebigem d und für alle $P \in \mathcal{P}_\Delta$ für $j = d-1, d-2, d-3$.

Roswitha Blind, Stuttgart

A new approach to mixed volume

We start by giving a dissection of the Minkowski-sum $P_1 + P_2$ for d-dimensional polytopes P_i. Then we use this dissection to obtain a formula for the mixed volume $V(P_1, i, P_2, d-i)$ which is independent of a coordinate system. Finally we give a localisation formula for some cases which is a generalisation of the familiar area-measures.

Ulrich Betke, Siegen

Covering three dimensional convex bodies with smaller homothetical copies

Let A be a convex body of Euclidean space E^n. Let $L(A)$ denote the minimum number of smaller positive homothetical copies of A whose union covers A. Hadwiger formulated the conjecture that $L(A) \leq 2^n$. This known problem remains unsolved for $n \geqslant 3$. Boltjanskiĭ showed that $L(A)$ is equal to the smallest number of directions which illuminate $\mathrm{bd}\, A$. Remember that $p \in \mathrm{bd}\, A$ is said to be illuminated by a direction δ if the half-line with the vertex p and of direction δ has non-empty intersection with $\mathrm{int}\, A$.

THEOREM. Let $A \subset E^3$ be a convex body. If A is centrally symmetric, then $\mathrm{bd}\, A$ can be illuminated by some 4 pairs of opposite directions. If A is of constant width, $\mathrm{bd}\, A$ can be illuminated by some 3 perpendicular pairs of opposite directions.

COROLLARY 1. $L(A) \leq 8$ for any centrally symmetric convex body.

COROLLARY 2. $L(A) \leq 6$ for any convex body $A \subset E^3$ of constant width.

COROLLARY 3. Any centrally symmetric set of diameter 1 of three dimensional normed space can be partitioned into 8 subsets of diameters smaller than 1.

PROPOSITION. $L(A) \leq 20$ for any convex body $A \subset E^3$.

Marek Lassak

An Upper Bound Theorem for Polytope Pairs

Let P be a simple d-dimensional convex polytope and F be a simple k-dimensional polytope that is a face of P. Then (P, F) is a polytope pair of type (d, v, k, r) if P has v facets and F has r facets. Define $P \sim F$ to be the simple unbounded d-polyhedron obtained from P by applying a projective transformation that sends a supporting hyperplane defining F onto the hyperplane at infinity. In 1981 upper bounds were determined for the possible numbers of faces of all dimensions of P and of $P \sim F$, but were not yet proven to be tight in all cases. Joint work with Barnette and Kleinschmidt has now remedied this situation, yielding an upper bound theorem for polytope pairs and unbounded simple polyhedra.

Carl W. Lee
Lexington, Kentucky
& Bochum

Neighborly Polytopes

This is a survey talk on even-dimensional neighborly polytopes, mainly on work of Dr. Ido Shemer. (See refs. below)

1.) Denote by $\mathcal{N}^{2m}(v)$ the class of neighborly $2m$-polytopes with v vertices. If $P \in \mathcal{N}^{2m}(v)$ then P is simplicial and has $\frac{v}{m+1}\binom{v-m-2}{m}$ missing faces, all of size $m+1$. (If $v \le 2m+3$ then P is cyclic.)

2.) The face structure of a neighborly $2m$-polytope determines the face structure of all its subpolytopes.

3.) If $P \in \mathcal{N}^{2m}(v)$, $v \geq 2m+5$ and P is not cyclic, then it has at most $2m$ cyclic subpolytopes with $v-1$ vertices. [2]

4.) If $P \in \mathcal{N}^{2m}(v)$ and Φ_{2j} is a face of P with $2j$ vertices $(0 \leq j \leq m)$ then Φ_{2j} is <u>universal</u> if either $j = m$, or $j < m$ and the quotient polytope P/Φ_{2j} is in $\mathcal{N}^{2(m-j)}(v-2j)$.

The universal edges of P form a graph $G(P)$. If $v \geq 2m+3$ then $G(P)$ is either a hamiltonian circuit (if P is cyclic), or a disjoint union of simple paths.

A non-cyclic $P \in \mathcal{N}^{2m}(v)$ has at most $v-2$ universal edges. Those P's with $v-2$ universal edges have been described in [3].

5.) The <u>sewing</u> construction associates with a polytope $P \in \mathcal{N}^{2m}(v)$ and a <u>tower</u> $\Phi_2 \subset \Phi_4 \subset \cdots \subset \Phi_{2m}$ of universal faces of P a point z such that $\operatorname{conv}(P \cup \{z\}) \in \mathcal{N}^{2m}(v+1)$.

Using the sewing construction Shemer has shown that the number $g(2m+\beta, 2m)$ of combinatorial types of polytopes in $\mathcal{N}^{2m}(2m+\beta)$ grows superexponentially as $\beta \to \infty$ (with $m \geq 2$ fixed), and also as $m \to \infty$ (with $\beta \geq 4$ fixed)

6.) $g(10, 6) = 37$ (due to Bokowski & Shemer, [4].)

References

[1] I. Shemer, Neighborly polytopes, Israel J. Math., 43 (1982), 291–314

[2] —, How many cyclic subpolytopes can a non-cyclic polytope have? To appear in Israel J. Math.

[3] —, Almost cyclic polytopes, to appear in JCT.

[4] J. Bokowski & I. Shemer, Neighborly 6-polytopes with 10 vertices, to appear.

Micha A. Perles
Hebrew University
Jerusalem, Israel

Billiards

(1) Most (convex) billiard tables have the property that no trajectory ends up in the boundary $(d \geq 2)$

(2) For most billiard tables there does not exist a caustic $(d \geq 2)$

(3) For most billiard tables for any $\varepsilon > 0$ there is a trajectory with the following property: For any point p and any direction v there is a point q on the trajectory at which the trajectory has direction w such that $|p-q| < \varepsilon$, $|v-w| \leq \varepsilon$ $(d=2)$

Peter M. Gruber
(Techn. Univ. Wien)

Remarks to W.J. FIREY's lecture on the affine surface-area for convex bodies.

A convex body K is called "ε-smooth" if a ball of fixed radius ε rolls freely in the interior of K, and the class of all such K is denoted by S_ε. Then the following generalization of a theorem of BLASCHKE in the analytic case 1923 holds: If K belongs to S_ε then there exists

$$\lim_{\delta \to 0} c_n \cdot \frac{V(K) - V(K_{[\delta]}, K, \dots, K)}{\delta^{\frac{2}{n+1}}}$$

where n is the dimension of the space, c_n a dimension depending constant and $K_{[\delta]}$ the difference of K and the union of all plane sections of K with the volume $\delta > 0$. This equiaffinely invariant value may be also used as a definition of an affine surface-area $A_{aff}(K)$ for K. In the case of an arbitrary K $\inf_{\tau > 0} A_{aff}(K + \tau E)$, being independent from the choice of the ellipsoid E, plays the same role. – The proofs are based on theorems of A.D. ALEKSANDROV concerning the twice differentiability of a convex function almost everywhere and the notion of the area function of K on the unit sphere.

K. Leichtweiß (Univ. Stuttgart)

A sausage-skin conjecture for coverings

The proof of the following theorem has been outlined: If in the euclidean plane the boundary of a convex domain C is covered by k unit circles then C has perimeter less than or equal to $4(\sqrt{k^2-1} + \arcsin\frac{1}{k})$. Equality occurs only in the case when the centres of the circles are equally spaced at distance $2\sqrt{1-\frac{1}{k^2}}$ on a straight line l and C is the intersection of the union of the circles with the parallel region of l at distance $\frac{1}{k}$. Thus the convex domain with maximal perimeter the boundary of which can be covered by k unit circles has the form of a „sausage". It is conjectured that analogous statments hold for convex bodies in E^d, if we maximize the j-th intrinsic volume of bodieds ($j < d$) the j-skeleton of which can be covered by a given number of unit balls.

Gábor Fejes Tóth (Budapest)

Random polytopes on the torus

A subset of a compact metric manifold is called convex if for any two points of the subset all geodesic segments (i.e. all curves of minimal length on the manifold joining the two points) are completely contained in the subset.
In order to answer the question of determining the expected volume of the convex hull of a fixed number of independent and uniform random points, the convex sets on the manifold have to be classified and the probability that the convex hull belongs to a certain class has to be determined.
In the case that the manifold is the sphere, the problem was solved by Wendel, Cover and Efron using a result due to Steiner and Schläfli. We (joint paper with Tichy) investigate the case of the torus.

Christian Buchta (Wien)

Polytopes and number-theoretical partitions

A new Steinitz-type theorem for convex polytopes is presented.
Let M denote a finite set, $\mu = \{M_1, \dots, M_{r-1}\}$ a partition of M into $r-1$ subsets. A subset X of M is a transversal of μ, if X intersects every M_i in exactly one element. Let $T(\mu)$ denote the set of all transversals of μ and let $\mathcal{X}$ be a subset of $T(\mu)$. Then $\mathcal{X}$ is nested, if it has the following property:

For every $X, Y \in \mathcal{X}$, $X \neq Y$ there exists at least one $U \in \mathcal{X}$ with
$U \subset X \cup Y$ and $\#(U \cup X) = r$ or $\#(U \cup Y) = r$.

Now suppose that $\mathcal{X}$ is nested and ~~that~~ let $H(\mathcal{X})$ denote the set of all r-subsets of M containing exactly one transversal of $\mathcal{X}$ as a subset. Then $H(\mathcal{X})$ describes the combinatorial structure of a convex polytope P as follows: There exists a bijection α of the facets of P onto M and a bijection β of the vertices of P onto $H(\mathcal{X})$ such that the vertex e is on the facet F if and only if $\alpha(F)$ is not an element of $\beta(e)$.

This result characterises a class of convex polytopes combinatorially and therefore is a Steinitz-type theorem. The so-obtained polytopes are simple $(m-r)$-polytopes with m facets. The class includes the class of simple $(m-3)$-polytopes with at most m facets, first characterised by M. Perles.

An equivalent formulation leads to a connection to number-theoretical partitions: Let F denote the Ferrer-diagram of ~~to~~ an $(r-2)$-dimensional partition of ~~to~~ a certain natural number ξ. I.e. F is a function on $\mathbb{N}^{r-1}$ having the following properties:

(i) $F(v) \in \{0, 1\}$ for every $v \in \mathbb{N}^{r-1}$

(ii) $F(v) = 1$, $u \leq v$ implies $F(u) = 1$ for every $u, v \in \mathbb{N}^{r-1}$

(iii) $\sum_{v \in \mathbb{N}^{r-1}} F(v) = \xi$

Now fix a $w \in \mathbb{N}^{r-1}$ having the property $F(v) = 1$ implies $v \leq w$. Denote by $H(F, w)$ the set of all pairs (u, v), $u, v \in \mathbb{N}^{r-1}$ such that:

(i) $v \leq w$, (ii) $F(v) = 0$, (iii) $F(u) = 1$

(iv) There exists exactly one $i \in \{1, \dots, r-1\}$ such that $u_i \neq v_i$.

Then $H(F, w)$ describes the combinatorial structure of a convex polytope analogously as $H(\mathcal{X})$ does. Here m corresponds to

$w_1 + \dots + w_{n-1}$, $\mathcal{H}$ corresponds to $\{v \in N^{n-1} : F(v) = 1\}$ and $H(\mathcal{H})$ corresponds to $H(F, w)$

The proof is based on Gale-diagram techniques

Franz Heinz (Dortmund)

LA JONCTION METRIQUE DANS UN ESPACE DE MINKOWSKI

Si M est un espace de Minkowski dont la distance est d, définissons le joint métrique de a et b par

$$ab = \{x \in M : d(a,x) + d(x,b) = d(a,b)\}$$

Si A est une partie de M, posons $Ab = \bigcup_{a \in A} ab$ et convenons que A est métriquement convexe si $xy \subset A$ chaque fois que $x \in A$ et $y \in A$.

Si $\dim M = 2$,

1°) l'application $(x, y) \mapsto xy$ est continue,

2°) les joints métriques sont métriquement convexes,

3°) $\forall x, y, z \in M$, $(xy)z = x(yz)$,

mais ces propriétés ne sont pas vraies en général si $\dim M > 2$.

Si la boule unité B de M est un polytope, la propriété (2°) est équivalente à : pour toute face à $n-2$ dimensions de B, le sous-espace vectoriel engendré par cette face rencontre la frontière de B suivant des faces à $n-2$ dimensions de B. Si l'on suppose de plus $\dim M = 3$, la condition (3°) est équivalente à : la boule unité B est une bipyramide centrée en O

Guy Valette (Bruxelles)

Inequalities of the form $\sum c_i R^i W_i \geq 0$

Let K be a convex body in $\mathbb{E}^d$,
W_i its i-th quermass integral and
R the radius of its outer sphere.
The following theorem holds (J. B. / Erhard Heil):

$$c_{ijk} R^i W_i + c_{jki} R^j W_j + c_{kij} R^k W_k \geq 0 \qquad 0 \leq i < j < k \leq d$$

The coefficients $c_{ijk} := (k-j)(i+1)$ are best possible if we require that equality holds for balls.
Equality holds if and only if K is a ball or $W_k = 0$.

Jürgen Bokowski (Darmstadt)

Geometric Inequalities and Total Mean Curvature of Polyhedral Surfaces

There are formulas involving the ideas of integral geometry which are analogous to the Leibnitz formula for differentiating an integral. For example,

$$\frac{d\alpha}{dt} = c_n \int_{p \in L} \frac{d\alpha(L)}{dt}\, d\mu(L)$$

where α is the measure of a varying angle in $\mathbb{E}^n$, $\alpha(L)$ is the measure of the orthogonal projection of the angle onto the plane L, and μ is the invariant measure on the planes L through some fixed point p. The applications include Schläfli's differential formula, Connelly's flexible polyhedral surfaces, and geometric inequalities.

Ralph Alexander (Urbana)

Hirsch polytopes and polytope pairs

Properties are discussed which guarantee the existence of "short-paths" in the edge-graphs of polyhedra. One such property is "vertex-decomposability", a special shelling process first introduced by Billera and Provan. We give a counterexample to the conjecture that the boundary-complex of every simplicial polytope is vertex-decomposable.
We show that a class of neighborly polytopes (due to Barnette) is "weakly-vertex-decomposable" and conclude that this class allows paths between any pair of vertices which are at most twice as long as the bound given by the Hirsch-conjecture.
These polytopes also are the solution of an Upper-bound-conjecture for "polytope-pairs", i.e. polytopes with a preassigned local structure.
It is shown how Balinski's proof of the Hirsch-conjecture for dual transportation polyhedra can be used to find efficient algorithms for primal transportation problems.

Peter Kleinschmidt (Bochum)

Translative Poincaré formulae

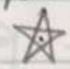
With every k-rectifiable subset of $\mathbb{R}^d$ we associate two zonoids by means of projection maps, and if the generating measures of these zonoids are not concentrated on a great sphere, we obtain two other convex bodies from Minkowski's theorem on the existence of a convex body with given area function. This enables us to formulate translative Poincaré formulae for rectifiable sets by means of mixed volumes of these auxiliary convex bodies, and to derive some inequalities where the sets of constant brightness are characterized by the equality case.

J. A. Wieacker (Freiburg)

Slope-critical Configurations in the Plane

In 1970 P. R. Scott asked if n noncollinear points in the plane always determine at least $n-1$ slopes, with equality only for odd n. This was proved by P. Ungar (JCT, 1982) using the combinatorial reformulation of Goodman and Pollack. The classification of the critical configurations – which attain the minimum – remains open. There are 4 infinite families and 102 sporadic examples known. Four of these are not centrally symmetric:


$n=7$


$n=11$

(for example) Nonetheless, one can show there is always a combinatorial "center" of symmetry – i.e., a pt z so that each line L thru z contains equal numbers of pts of the configuration on each side of z. Let d be the number of connecting lines thru z and $(\nu_1, \dots, \nu_d)$ be the nr's. of pts on d consecutive half lines thru z. It can be shown that if d is even, then the configuration must be centrally symmetric. Other limitations include the following:

1) if $d > 2$, and $\nu_i > 1$, $\nu_{i+1} > 1$, then $\nu_i + \nu_{i+1} \le 8$

2) if $d > 2$, and $\nu_i > 1$, $\nu_{i+1} = 1$, $\nu_{i+2} > 1$, then $\nu_i \le 3$ and $\nu_{i+2} \le 3$.

It is impossible to restrict d in general since there are two infinite families each with d arbitrarily large. The largest sporadic value of d is 16 and it is conjectured this is best possible.

Robert Jamison (Clemson, S.C. USA)

Edge effects in particle counting

Let $\mathcal{K}^d$ be the set of all convex bodies in $\mathbb{R}^d$ and let X be a stationary point process on $\mathcal{K}^d$. If $K_0 \in \mathcal{K}^d$ is a 'sampling window' and if $N(X, K_0)$ denotes the number of particles of X intersecting K_0, the particle density N_V of X is defined by $N_V = \lim_{r \to \infty} \mathbb{E}\, N(X, rK_0)/V_d(rK_0)$ ($\mathbb{E}$ expect., V_d volume). Let φ_0 be the Gaussian curvature measure. It is shown that $\sum_{K \in X} \varphi_0(K, \operatorname{int} K_0)$ for $V_d(K_0) = 1$ is an unbiased estimator of N_V which corrects the edge effect of particles $K \in X$ intersecting the boundary of K_0.

Wolfgang Weil (Karlsruhe)

On the Chessboard Conjecture

Theorem 1. (Chessboard Conj.) Let Λ_C be a lattice of translates of a convex body C in $\mathbb{R}^n$ with the property that the removal of all members of Λ_C splits up the space into bounded pieces. Then $\operatorname{Vol} C \geq \frac{1}{2} \operatorname{Vol} E$, where E is an elementary cell of the lattice, and equality holds iff Λ_C is a 'chessboard-like' configuration.

The proof is based on the following result:

Theorem 2. Let P and P' be two n-dimensional polytopes whose facets are denoted by

F_i $(1 \le i \le m)$ and F_j' $(1 \le j \le m')$, respectively. Assume that P' is convex and

$$\sum \{ \mathrm{Vol}_{n-1} F_i : v \text{ is an outward normal of } F_i \}$$
$$\le \sum \{ \mathrm{Vol}_{n-1} F_j' : v \text{ is an outward normal of } F_j' \}$$

holds for every $v \in \mathbb{R}^n$. Then we have

$$\mathrm{Vol}\, P \le \mathrm{Vol}\, P'$$

with equality iff $P = P'$.

The above results and some other isoperimetric inequalities for convex polytopes will appear in a forthcoming paper of I. Bárány, K. Böröczky, E. Makai, Jr. and J. Pach.

János Pach (Budapest)

Mixed Projection Inequalities

If $K_1, \dots, K_{n-1}$ are convex bodies in $\mathbb{R}^n$, and for a given direction $u \in S^{n-1}$ we use $v(K_1^u, \dots, K_{n-1}^u)$ to denote the $(n-1)$-dimensional mixed volume of the images of the projections of $K_1, \dots, K_{n-1}$ onto the hyperplane orthogonal to u, then the mixed projection body $\Pi(K_1, \dots, K_{n-1})$ can be defined as the body whose support function is given by

$$h_{\Pi(K_1, \dots, K_{n-1})}(u) = v(K_1^u, \dots, K_{n-1}^u).$$

Some (sharp) inequalities involving mixed projection bodies are obtained. One of the inequalities is the following generalization of the Petty projection inequality:

$$V(K_1)\cdots V(K_{n-1})\,V(\Pi^*(K_1,\dots,K_{n-1}))\le(\omega_n/\omega_{n-1})^n,$$

with equality if and only if the K_i are homothetic ellipsoids. In this inequality $\Pi^*(K_1,\dots,K_{n-1})$ denotes the polar body of $\Pi(K_1,\dots,K_{n-1})$ and ω_n denotes the volume of the unit ball in $\mathbb{R}^n$.

Erwin Lutwak (New York)

Random Steiner Symmetrizations

If, starting with a convex body $K\subset\mathbb{R}^d$, we iterate at random the Steiner symmetrization, then we obtain almost always some Euclidean ball as limit figure

Peter Mani (Bern)

A Traffic Flow Problem.

We have m starting points $1,\dots,m$ for cars and q destination points $1,\dots,q$. Let t_{ij} be the number of cars starting at i and finishing at j. Let $T=(t_{ij})$. We do not know T but we do know (e.g. by traffic counters) the number of cars O_i which start from i and the number of cars D_j which arrive at j. We also have a probability matrix (p_{ij}), produced from previous data in similar situations, with p_{ij} being the probability that a car starting at i will finish at j. The problem is to determine the most likely matrix T to occur. The conjectured solution was the unique matrix of the form $t_{ij}=x_i p_{ij} y_j$, $x_i>0$, $y_j>0$. We shall show that this is not so in a particular example by showing that this would lead to the conclusion that $\log 2$ was rational.

David Larman (London)

Simplices

The description of the closed (and open) finite-dimensional Choquet simplices lead me years ago to the conjecture that most algebraically closed Choquet simplices are of the following type

$$S = \{x : f_i(x) \le \alpha_i ,\ i \in \mathcal{I}\}$$

where the f_i's are affinely independent (or linearly independent with $\alpha_i = 0$ for each $i \in \mathcal{I}$, if S is a cone with apex 0).

We showed that this conjecture is true in a lot of cases. Moreover, we gave an example of a free family $\mathcal{F} = \{f_i : i \in \mathcal{I}\}$ such that

$$S = \{x : f_i(x) \le 0,\ i \in \mathcal{I}\}$$

is not a Choquet simplex. Similar results for algebraically open Choquet simplices were also given.

René Fourneau (Liège).

Inner Parallel Bodies and a Steiner-type formula

Let K and E be convex bodies in E^n where r is the inradius of K relative to E and $K_{-\lambda}$, $0 \le \lambda \le r$, its inner parallel bodies. Then Matheron conjectured for $n>2$, and proved for $n=2$,

$$V(K_{-\lambda}) \ge \sum_{i=0}^{n} \binom{n}{i} (-\lambda)^i\, V(K, \dots, K, \underbrace{E, \dots, E}_{i}).$$

The inequality is false for $n>2$. In the case n is odd it is false for the $n-1$ tangent bodies and for n even, $n>2$, it is false for the $n-2$ tangent bodies.

For $n=3$ and $E=B$, the unit ball, if equality ~~for~~ holds for a body K, where $K \ne K_{-r} + rB$, then the inequality above is valid when K is replaced by its inner parallel bodies

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Potentialtheorie

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Weak Duality and Potential Theory.

Weak duality is a setting in which a good potential theory may developed based on a pair of processes X and $\hat{X}$. One constructs a σ-finite measure on paths defined on a random time interval $]\alpha, \beta[$ (Z_t). In one direction Z looks like X and in the other like $\hat{X}$. For example, if $\tau_K = \inf\{t : Z_t \in K\}$, $\lambda_K = \sup\{t : Z_t \in K\}$ where K is transient, then $P[\tau_K \in dt, Z_{\tau_K} \in dx] = dt\, \hat{\pi}_K(dx)$ and $P[\lambda_K \in dt, Z_{\lambda_K -} \in dx] = dt\, \pi_K(dx)$ where π_K and $\hat{\pi}_K$ are the capacitary and co-capacitary measures of K. In particular, $P[\tau_K \in dt] = C(K)\,dt = P(\lambda_K \in dt)$ where $C(K)$ is the capacity of K.

Ronald K. Getoor

Dirichlet and Neumann Problems for Schrödinger's equation

For bounded domain D in R^d, and $q \in K_d$ (Kato class), the Gauge for (D, q) is defined to be

$$u(x) = E^x\left\{e^{\int_0^{\tau_D} q(X_s)\,ds}\right\},$$ where $\{X_s, s \geq 0\}$ is the Brownian motion and τ_D the first exit time from D. If $u \not\equiv \infty$ in D, then u is bounded in $\bar{D}$ (Chung-Rao-Zhao). In this case the unique solution of $\left(\frac{\Delta}{2} + q\right)\varphi = 0$ in D, $\varphi = f$ (continuous on ∂D) is given by

$$\varphi(x) = E^x\left\{e^{\int_0^{\tau_D} q(X_s)\,ds} f(X_{\tau_D})\right\}.$$

Several equivalent conditions for $u \not\equiv \infty$ are given. There is a similar theorem for the Neumann problem, (thesis of P. Hsu).

Kai Lai Chung

This is a report on the thesis of Ming Liao done at Stanford under Prof. K. L. Chung. K. L. Chung introduced an important condition on the potential density guaranteeing the validity of some well-known results of classical potential theory. Ming Liao in his dissertation weakens the conditions of Chung. At the same time he proves that the process admits a strong Markov dual albeit some branch points exist.

Murali Rao.

Ninomyia operators for the generalized Dirichlet problem. Fine strict maxima.

Let X denote a harmonic space and U be a relatively compact open subset of X. We shall denote by $\mathcal{P}_U$ the cone of continuous potentials on X and harmonic on U. The operator A sending continuous functions on ∂U into real-valued functions on U is said to be a Ninomyia operator, provided A is linear and positive, $A(p_{|\partial U}) = p_{|U}$ for every $p \in \mathcal{P}_U$ and there exists a strict potential q such that $A(q_{|\partial U})$ is subharmonic. Main result: The following conditions are equivalent: (1) There is exactly one N. operator on U; (2) The set of irregular points of U is negligible. The relation to the Keldyš type theorem is discussed.

Let $f: \mathbb{R}^m \to \mathbb{R}$ be an arbitrary function. Define $M(f) = \{x;\ \{y; f(y) \geq f(x)\}$ is thin at $x\}$. ($M(f)$ is called the set of strict fine maxima.)

Main result (jointly with J. Král): If f is a Borel function, then $M(f)$ is a polar set. The case of an arbitrary function is also treated. Relations to a series of results from real analysis are also mentioned.

Ivan Netuka, Charles University, Prague, Czechoslovakia

An example of instability ~~of~~ for the potential theoretic structure of quasi-isometric Riemmanian manifolds and reversible markov chains.

Two metrics $g, \tilde{g}$ on a manifold M are quasi-isometrically equivalent if

$g_x(u,u)/\tilde{g}_x(u,u) \in [\frac{1}{c}, c] \subset \mathbb{R}_+$ for every $u \in TM_x$ and $x \in M$. ~~Two~~

A countable set X and a symmetric function $\underline{a}: X \times X \to \mathbb{R}_+$ determine a reversible Markov chain on X providing $\pi_x = \sum_{y \in X} a_{xy}$ is strictly positive and finite for each x by putting $p_{xy} = a_{xy}/\pi_x$ and using p as a transition kernel. Two reversible Markov chains on X are quasi equivalent if $a_{xy}/b_{xy} \in [\frac{1}{c}, c] \subset \mathbb{R}_+$ for all $x, y \in X$.

We consider the following question: If (M, g) admits no nonconstant ~~posit~~ bounded harmonic functions then can $(\tilde{M}, \tilde{g})$; and also the analogous question for Markov chains. We give the following negative answer:

There is a ~~complete~~ Riemann surface M and a second one $\tilde{M}$ quasi-conformally (or isometrically) equivalent to the first such that M has trivial Martin boundary and $\tilde{M}$ admits a two point boundary - and each extremal positive harmonic function is bounded. Analysis of a rather complicated counter example to the analogous Markov chain problem plays a vital role.

Terry Lyons
Imperial College, London

Biharmonic Spaces

We consider a biharmonic structure on a locally compact space X with a countable base in the same way as E. P. Smyrnelis but without the hypothesis of the compatibility of the pairs of biharmonic functions. We give the following characterisation: $(X, \mathcal{H})$ is a biharmonic space if and only if there exist a unique harmonic space $(X, \mathcal{H}_1)$ and $(X, \mathcal{H}_2)$ having a common base $\mathcal{U}$ of regular open sets, and a unique positive section of continuous and reals potentials represented by a family $(p_U)_{U \in \mathcal{U}}$ of potentials on U such that for all $U \in \mathcal{U}$:
$\mathcal{H}(U) = \{(h_1, h_2) \in \mathcal{C}(U) \times \mathcal{C}(U) \,/\,$ for all $V \in \mathcal{U}$ with $\bar{V} \subset U$ we have $h_1 = H_V^1 h_1 + K_V h_1,\ h_2 = H_V^2 h_2\}$, H_V^1 and H_V^2 are the harmonic measures corresponding to V in $(X, \mathcal{H}_1)$ and $(X, \mathcal{H}_2)$ respectively and K_V is the potential kernel associated to p_V.
This characterisation of biharmonic space simplifies many proofs of results of E. P. Smyrnelis and shows an intimate connection between biharmonic and harmonic structures.

Abderrahman BOUKRICHA, Faculté des sciences de Tunis, Tunisia.

One-sided growth conditions for the coefficients of stochastic differential equations

We consider the stochastic differential equation

$(*) \quad d\xi_t = \sigma(\xi_t)\, d\beta_t + b(\xi_t)\, dt$

in a separable Hilbert space H. Here $(\beta_t)_{t \geq 0}$ is Brownian motion with covariance Q (positive, nuclear operator on H) and mean 0, σ is a "locally" Lipschitz function $H \to L(H, H)$ (bdd. linear operator on $\mathbb{H}$), and b is "locally" Lipschitz $\mathbb{H} \to \mathbb{H}$.

Th: a) If i) $\|\sigma(y)\| = O(\|y\|)$ and

ii) $(y, b(y)) \leq K(1+\|y\|^2)$

then for each $x \in \mathbb{H}$ there exists exactly one solution $(\xi_{x,t})$ to (*) with infinite lifetime.

b) $P_t f(x) := E[f \circ \xi_{x,t}]$ is a semigroup whose generator is an extension of the differential operator

$$\mathcal{L} f(x) = \frac{1}{2} \operatorname{tr} f''(x)\, \sigma(x) Q\, \sigma^*(x) + f'(x)\, b(x)$$

(f sufficiently regular).

Under the stronger conditions iii) $\|\sigma(y)\| \leq K$, iv) $|(y, b(y))| \leq K(1+\|y\|^2)$, $(P_t)_{t \geq 0}$ induces a Feller semigroup, i.e., a strongly continuous semigroup on the functions that are small outside bdd sets and uniformly continuous on bdd. sets.

If $\sigma(y)$ = identity the case of discontinuous drift terms b as arising in Euclidean quantum field theory was also considered.

Gunter Ritter, Universität Passau

Potentialtheoretische Methoden bei der Approximation von Lösungen elliptischer Gleichungen höherer Ordnung

Es sei L ein linearer, eigentlich elliptischer Differentialoperator im R^n mit reellen, beliebig oft differenzierbaren Koeffizienten. Der adjungierte Operator L^* besitze die eindeutige Fortsetzungs-Eigenschaft. $\Omega \subset R^n$ sei ein beschränktes Gebiet mit dem glatten Rand $\partial\Omega$ und auf $\partial\Omega$ sei ein System $B_1, \dots, B_m$ von Randoperatoren der Ordnung $\leq 2m-1$ mit glatten Koeffizienten gegeben, welches normal ist und den Operator L auf $\partial\Omega$ überdeckt. $\Gamma \subset \Omega$ sei eine glatte, k-dimensionale $(1 \leq k \leq n-1)$ Fläche, die Ω nicht zerlegt, $V \subset \partial\Omega$ sei eine vorgegebene offene Teilmenge des Randes. Wir betrachten

$L_v(\Omega) = \{u \in C^\infty(\bar{\Omega}) : Lu = 0 \text{ in } \Omega,\ B_j u|_{\partial\Omega \setminus V} = 0\ (j = 1, \dots, m)\}$
sowie $L_v(\Gamma) = L_v(\Omega)|_\Gamma$. Man kann dann zeigen, daß $L_v(\Gamma)$ in den Sobolev-Slobodeckij-Räumen $W_p^s(\Gamma)$ ($1 < p < \infty$, $s \geq 0$ reell) dicht liegt. Ist Γ eine hinreichend reguläre, geschlossene Fläche, die innerhalb Ω ein Teilgebiet Ω_1 begrenzt, für welches das Dirichlet-Problem der homogenen Gleichung $Lu = 0$ eindeutig lösbar ist, so läßt sich ferner die Dichtheit von $L_v(\Gamma)$ im Raum $W^{m-1}(\Gamma)$ der Whitney-Taylor-Felder der Ordnung $m-1$ zeigen, falls man noch zusätzlich die Existenz einer globalen Fundamentallösung voraussetzt. Ein Teil der Resultate wurde gemeinsam mit U. Hamann erzielt.

Günther Wildenhain (Rostock)

Reversible measures for diffusion processes in Hilbert spaces.

One of the important problems in the study of evolution processes is to describe the equilibrium states.
We deal with a real separable Hilbert space and diffusion processes in this space arising as solutions to stochastic differential equations. Here equilibrium behaviour is given by means of probability measures on H, which are invariant or reversible (i.e. symmetric with respect to time reversal) for the corresponding semigroup.
We characterise reversibility by the socalled (and in finite dimensions well known) "principle of detailed balance" for a suitable differential operator $\mathcal{L}$. Furthermore we give sufficient conditions on the (generally unbounded and nonlinear) drift part of the stochastic differential equation [the diffusion part is identity] in order to prove existence of reversible measures. The theory covers examples of "continuous spin models" of statistical mechanics which occur in the "lattice approximation" ~~of~~ to Euclidean quantum field theory.

G. Lehe, Erlangen.

The asymptotics of solutions of boundary value problems in non-smooth domains or for jumping boundary conditions

If Ω is a domain with a boundary being C^∞ except some conical point or a wedge or when the boundary conditions have a smooth jump (as for the classical Zaremba problem) the standard C^∞ regularity up to the boundary of solutions of elliptic boundary problems has to be replaced by another notion. It turns out that we have to expect an asymptotic expansion of the form

$$\sum_{j=0}^{\infty} \sum_{k=0}^{m_j(y)} \xi_{jk}(y,\varphi)\, t^{p_j(y)} (\log t)^k \quad \text{as } t \to 0 \ (*)$$

when, for instance, we have jumping conditions, where the jump is over a submanifold Y of $\partial\Omega$ of codimension 1, $y \in Y$, (φ, t) polar coordinates in a plane orthogonal to Y, such that (y, φ, t) are local coordinates in Ω near Y. The exponents $p_j(y)$ are in $\mathbb{C}$, $j \in \mathbb{Z}_+$, and $m_j(y) \in \mathbb{Z}_+$. The expansion $(*)$ is a generalization of the Taylor expansion, that holds in the case of smoothness up to $\partial\Omega$. For varying $y \in Y$ we have to expect a very general branching behaviour of the exponents that generate clouds of points $p_j(y) \in \mathbb{C}$. This leads to an extension of the concept of C^∞ spaces with asymptotics. On $\mathbb{R}_+$ for $t \to 0$ it has the form

$$u(t) \sim \sum_{j=0}^{\infty} \langle \zeta_j(w), t^{-w} \rangle ,$$

where $\zeta_j \in \mathcal{A}'(\Lambda_j)$ is a sequence of analytic functionals carried by compact sets Λ_j in $\mathbb{C}$. The Mellin image of $\langle \xi_j(w), t^{-w} \rangle$ is the potential of ξ_j with respect to a fundamental solution of the Cauchy-Riemann operator. The parametrix construction in spaces with asymptotics may be carried out in terms of operators with a complete symbolic level, namely pseudo-differential, Mellin, Green operators, strongly degenerate pseudo-differential operators and trace and potential operators. The results are contained in a number of papers jointly with S. Rempel.

Bert-Wolfgang Schulze, Berlin
AdW der DDR

Fine potential theory Consider a finely open subset of a $\mathfrak{P}$-harmonic space X with a countable base (or, more generally, of a standard balayage space). We define $\mathcal{W}(U)$ as the set of all $f \geq 0$ with the following property:

If $W \subset U$ is finely regular (i.e. $eW = beW$), then there is a sequence $\{q_n\}$ of elements of $\mathcal{W}\ (= \mathcal{H}_+^*(X))$ such that $q_n - \hat{R}_{q_n}^{eW} \nearrow f$ on W.

A function g is said to be U-quasi-τ-l.s.c. on $M \subset U$ if

$$\bigwedge \{ u \in \mathcal{W}(U) : g \text{ is } {}^{\tau}\text{l.s.c. on } [u<1] \cap M \} = 0$$

The U-natural topology is defined as the coarsest topology on U such that it is finer than the initial one and $\hat{R}_p^{eU}$ is continuous in it for every continuous $p \in \mathcal{P}$.

A function $f \geq 0$ on U belong to $\mathcal{W}(U)$ if and only if for every finely open set $V \subset \bar{V} \subset U$ and $x \in V$ we have

$$\int_U^* f \, d\varepsilon_x^{eV} \leq f(x)$$

and f is finely l.s.c. (J. Malý & W. Hansen).

Let $f \geq 0$ on U. Suppose that f is finely l.s.c. on U and for every open set $G \subset X$ and $x \in G \cap U$ there is $A \subset \bar{A} \subset G \cap U$ such that

$x \notin b\complement A$ and

$$\int_U^* f \, d\varepsilon_x^{\complement A} \le f(x).$$

Then the following assertions are equivalent,

(i) $f \in W(U)$

(ii) f is U-quasi l.s.c. on every compact subset of U

(iii) f is l.s.c. in the U-natural topology

(iv) f is U-quasi U-naturally l.s.c. on (every compact subset of) U

The proof uses the Bauer minimum principle on the Choquet boundary and the following observation: If $f \ge 0$ satisfies (iv), then for every compact subset K of U we have

$$\bigwedge \{ v \in W(U) : f + v \text{ is l.s.c on } K \} = 0 .$$

Jan Malý, Praha

Removable singularities of solutions of the heat equation

A subset E of real numbers is called negligible for caloric functions in the class C^1, if each continuously differentiable function f on an open set $U \subset R^n$ which satisfies the heat equation on an open subset $U_0 \subset U$ such that $f(U \setminus U_0) \subset E$ must necessarily satisfy the heat equation on the whole of U. Necessary and sufficient condition for E to be negligible for caloric functions in the class C^1 reads as follows: each compact $Q \subset E$ must be at most countable.

Similar results of Radó's type for sets E to be negligible for caloric functions in the classes of functions whose gradient satisfies the Hölder condition can be expressed in terms of the corresponding Hausdorff measures. Related conjectures for subcaloric functions and for solutions (or subsolutions) of more general differential equations were discussed.

Josef Král, Praha

Comparison of Greens functions for parabolic operators.

Let G^L_Ω be the Greens function of $L - \frac{\partial}{\partial t}$ for some second order linear elliptic partial differential operator on a domain $\Omega \subset \mathbb{R} \times \mathbb{R}^n$.

" $\exists\, c>0, \gamma>0 : \frac{1}{c} G^{1/\gamma \Delta}_\Omega \le G^L_\Omega \le c\, G^{\gamma\Delta}_\Omega$ on $\Omega \times \Omega$ " (*)

was proved by Aronson 1968 for general L and $\Omega =]0,T[\times \mathbb{R}^n$ and ~~B~~ by Maagli in his Tunis thesis 1984 for $L = a(t)b(x)\frac{\partial^2}{\partial x^2} + a(t)c(x)\frac{\partial}{\partial x} + d(t,x)$ and $\Omega =]0,T[\times]0,a[$ (and is conjectured to hold far more generally). As a consequence of (*) $\gamma\Delta - \frac{\partial}{\partial t}$ and $L - \frac{\partial}{\partial t}$ have the same Martin boundary of Ω. This is shown in the framework of axiomatic potential theory in the Thesis of H Missi (Tunis 1984). In fact a certain regularity (Poisson regularity) of boundary points is inherited by the structure in the middle of (*) from the structures at the left and at the right. Poisson-regularity permits to identify the ~~topological boundar~~ topological-parabolic boundary with the Martin-boundary.

M. Sieveking

The potential theory of the Laplace Kohn operator

I'll consider the Laplace Kohn operator Δ_K on $\mathbb{R}^3 = \{(x,y,t)\}$, which is given by $\Delta_K = X^2 + Y^2$, where

$$X = \frac{\partial}{\partial x} + 2y\frac{\partial}{\partial t} \; ; \quad Y = \frac{\partial}{\partial y} - 2x\frac{\partial}{\partial t} .$$

This operator is invariant under the group structure on $\mathbb{R}^3$ which is given by $(x,y,t)(x',y',t') = (x+x', y+y', t+t'+2(x'y-xy'))$. $\mathbb{R}^3$ with this group structure is the first Heisenberg group.

The harmonic space associated to Δ_K is a Brelot space. There are many analogies to classical potential theory, and I'll give a short introduction into results of Koranyi/Vagi, Folland, Gaveau, Greiner, and Jerison. My own contributions to this subject are mainly the following:

Similar to the classical situation the potential theory of Δ_K on $H_+ = \{(x,y,t) \mid t>0\}$ is equivalent to the potential theory on a certain "ball" – the Koranyi ball. The Poisson space of this ball is homeomorphic to the topological boundary.

W. Hueber, Bielefeld

Measure representations in harmonic spaces

In classical potential theory the sheaf of harmonic functions is defined by the Laplacian operator Δ : h harmonic $\Longleftrightarrow \Delta h = 0$.

The abstract theory of harmonic ~~functions~~ spaces starts with a sheaf of functions without the intervention of an defining operator. As a substitute F.-Y. Maeda introduced the notion of a measure representation and developed his theory of Dirichlet integrals for those harmonic spaces admitting a measure representation σ.

By definition σ is a homomorphism of the sheaf R of local differences of continuous superharmonic functions into the sheaf M of signed Radon measures such that : f superharmonic $\to \sigma(f) \geq 0$.

In this lecture the following result is presented:

Every harmonic space with a countable base admits a measure representation satisfying additional continuity properties.

Monika Schirmeier (Eichstätt)

Natural localization and natural sheaf property in standard H-cones of functions.

The aim of this paper is to develop a theory of localization in a standard H-cone of functions S on a set X.

If G is a fine open subset of X we denote by $S(G)$ the set of all positive functions f on G which are finite on a fine dense subset of G and such that there exists a sequence $(s_n)_n$ in S, $s_n < \infty$ for which the sequence $(s_n - B^{X \setminus G} s_n)_n$ increases to f. We show that $S(G)$ is a standard H-cone of functions on G.

If G is an open subset of X and if f is a positive lower semicontinuous function on G which is finite on a dense subset of G, then $f \in S(G)$ iff for any $x \in G$ there exists a fundamental system $\mathcal{V}_x$ of open neighbourhoods of x such that

$$\varepsilon_x^{X \setminus D}|_G(f) \leq f(x) \quad (\forall) \ D \in \mathcal{V}_x, \ \overline{D} \subset G,$$

where ε_x^A is the unique measure on X defined by

$$\varepsilon_x^A(s) = B^A s(x) \quad (\forall) \ s \in S.$$

Many charaterisations are given for each of the properties:

$$G \longrightarrow S(G) \text{ is a fine sheaf}$$

$$G \longrightarrow S(G) \text{ is a natural sheaf.}$$

It is shown, for instance, that the fine sheaf property is equivalent with axiom D and also with the union between the natural sheaf property and the axiom of nearly continuity.

N. Boboc. (Bucuresti)

Keldyš type operators and continuability of the Riesz α-harmonic functions ($0 < \alpha < 2$).

A bounded Borel measurable function h on $\mathbb{R}^m$ is said to be α-harmonic on an open set G [$\equiv h \in \mathcal{H}_i^\alpha(G)$]

provided it is continuous on G and for every ball $B = B(x,r) = \{y;\ |x-y|<r\}$, $\bar{B} \subset G$ we have $\varepsilon_x^{CB}(h) = h(x)$.
Define $H^\alpha(G) := \mathcal{H}_b^\alpha(G) \cap \mathcal{C}(R^m)$ and consider it with the sup-norm.

If there is an $h \in H^\alpha(G)$ for which $h \notin H^\alpha(G')$ for any $G' \supset G$, $G' \neq G$, then the set of such functions is a dense $\mathcal{G}_\delta$-subset of $H^\alpha(G)$. (Some other equivalent conditions involving the density of the set of regular points $\partial_r G$ in ∂G were given).

Denote $\tilde{H}^\alpha(G) = H^\alpha(G) \cap \{f;\ \lim_{x\to\infty} f(x) \in R\}$. An operator $A: \mathcal{C}_b(CG) \to \mathcal{H}^\alpha(G)$ is said to be the K-operator on G if it is monotone and for every $h \in \tilde{H}^\alpha(G)$ we have $A(h/_{CG}) = h$.

If $H^G f$ is the Perron type solution of the Dirichlet problem for f and G, then $Af(x) = H^G f(x)$ for every $f \in \mathcal{C}_b(CG)$ and every $x \in R^m \setminus \partial_i G$, where $\partial_i G$ is the set of irregular points of G.

Jiří Veselý, Charles University, Prague

A Measure-Theoretic Boundary Limit Theorem

The main result of this joint work with Jürgen Bliedtner is a purely measure-theoretic limit theorem for which the proof is quite simple. From this result, the fine limit theorem of Fatou-Naïm-Doob and its extensions to general potential theories follow immediately by using the order isomorphism between finite measures and positive harmonic functions and also employing some simple facts about reduced functions.

Peter Loeb, University of Illinois,
Urbana-Champaign, Ill. USA

Boundary value problems with respect to a non-linearly perturbed structure of a harmonic space

Let $(X, \mathcal{H})$ be a connected self-adjoint P-harmonic space such that $1 \in \mathcal{H}(X)$, G be a symmetric Green function on X, σ the canonical measure representation associated to G, $D[f, g]$ the mutual Dirichlet integral of f and g defined in terms of σ and $D[f] = D[f, f]$. Let X^* be a resolutive compactification of X and ω be the harmonic measure on $\partial^* X = X^* \setminus X$. Consider a linear subspace Φ of $\Phi_{BD} = \{\varphi \in L^\infty(\omega) \mid D[H_\varphi] < \infty\}$ which is closed w.r.t. the BD-topology and on which the unit contraction operates; and a linear subspace of Φ_{BD} containing $\Phi \cup \{1\}$. Let $\mathcal{M}_F$ be the space of finite signed measures ν on X such that $G|\mu|$ is bounded continuous and let $R(\mathcal{D}) = \{H_\varphi + G\nu \mid \varphi \in \mathcal{D},\ \nu \in \mathcal{M}_F\}$. Given mappings (non-linear !) $F : \mathcal{R}(\mathcal{D}) \to \mathcal{M}_F$ and $\beta : \mathcal{D} \to L^1(\omega)$, and given $\tau \in \mathcal{D}$, we discuss the following boundary value problem which generalizes some semi-linear boundary value problems in P.D.E.: Find $u = H_\varphi + G\nu \in \mathcal{R}_F(\mathcal{D})$ satisfying (E) $\sigma(u) + F(u) = 0$ on X, (B-1) $\varphi - \tau \in \Phi$, (B-2) $D[u, H_\psi] - \int_X H_\psi \, d\sigma(u) + \int_\Gamma \psi \beta(\varphi) \, d\omega = 0$ for all $\psi \in \Phi$.

F.-Y. Maeda (Hiroshima Univ.)

A Martin type compactification of harmonic spaces and its applications

We consider a compactification of harmonic spaces on which the notion of minimal thinness is defined and is connected closely with the topology.
For a harmonic morphism of harmonic spaces we construct a fine cluster set at a minimal boundary point. Then we develope theorems analogous to those of Riesz-Frostman-Nevanlinna and Fatou-Plessner and the caracterization of morphisms of type Bl which are obtained by Constantinescu-Cornea for analytic mappings of Riemann surfaces.

Teruo Ikegami (Osaka City Univ.)

Some constructions of Markov processes.

The simplest exposed is: consider two transition semigroups (Q_t^1), (Q_t^2), the first having as state space $E^1 \oplus \Delta^1$, Δ^1 being a „cemetery" set, the second having E^2 ~~and~~ as state space, consider, on a prob. space, a (Q_t^1)-process (x_t^1) (with right cont sample paths until the first visit ζ^1 in Δ^1) and a (Q_t^2)-process (x_t^2), constituting altogether a Markov family, such that the cond. distrib of x_0^2 with respect to x_∞^1, on $(\zeta^1 < \infty)$, is a given one T^{12}, and construct $x_t^{12} = x_t^1$ for $t < \zeta^1$, $x_t^{12} = x_{t-\zeta^1}^2$ for $t \geq \zeta^1$; it is a (Q_t^{12})-process for a well determined (Q_t^{12}) on $E^1 \oplus E^2$. Then the case of a sequence of semigroups (Q_t^n) and transition probabilities $T^{n,n+1}$ is considered, and a cemetery set equal to the product of their cemetery sets is organized for the resulting semigroup. If all (Q_t^n) are the same, as well as all $T^{n,n+1}$, we can „group" the states in the resulting semigroup and obtain an analogue of the „minimal transition semigroups". Also the case when the second process has time set $(0,\infty)$, T^{12} is a mapping from Δ^1 to the set of entrance laws for (Q_t^2), etc, is considered, case in which the hypothesis „the condit. distrib of ζ^1 with respect to x_0^1, x_∞^1 charges no singleton $\{t\}$" must be assumed.

As applications: recollement, processes with denumerable state space and without fictitious states, processes with denumerable state space in which there is no totally ordered descending sequence of jumps „longer" than a sequence, Itô's construction (in Berkeley 1972) of a process having an instantaneous state and given excursions (the corresp Markov process and infinite mass entrance law), generalised to the case of more instantaneous states, random evolutions.

Ioan Cuculescu (Bucharest Univ, Romania)

A quasi-linear potential theory

Inspired by familiar phenomena in the theory of qr-mappings Martio, Granlund and Lindqvist proved that weak solutions of the non-linear DE $\nabla \cdot \nabla_h F(x, \nabla u) = 0$, $F: G \times \mathbb{R}^n \to \mathbb{R}$, G domain $\subseteq \mathbb{R}^n$, a fairly general variational kernel, satisfy many familiar properties from the potential theory. Based on these observations, we propose axioms which would cover such situations as well as the usual linear harmonic spaces. These contain axioms of quasi-linearity, resolutivity, quasi-linear positivity, convergence and completeness similar, but mostly somewhat modified, as to the usual linear theory of harmonic spaces.

Ilpo Laine (Joensuu, Finland)

The fine Dirichlet problem

The concept of the solution of the Dirichlet problem for the fine topology in the framework of abstract harmonic spaces was discussed. The central role is played by the notion of finely hyperharmonic functions defined by the hypermean value property. Omitting the domination axiom D in fine potential theory, the proof of the fine minimum principle is done using fine topology methods, in particular, by help of the Lusin - Menchoff property. We introduce also an alternative approach to the generalized (fine) Dirichlet problem weaking the classical concept. For this purpose quasi-topological methods are used. The role of small sets is played by the elements of evanescent families.

Jaroslav LUKEŠ, Charles University, Prague

A Dirichlet problem for distributions and application to the prediction problem for Gaussian generalized random fields

We explicitly construct consistent conditional distributions for a large class of Gaussian measures defined on the space of (tempered) distributions on a domain D in $\mathbb{R}^d$. The conditional distributions are with respect to an (uncountable) family of σ-fields associated with the complements of the (relatively compact) open subsets of D. The construction involves solving a Dirichlet problem whose "boundary data" is given by a distribution. Furthermore, the associated set of Gibbs states is studied. We characterize the extreme Gibbs states, prove that they have the global Markov property and, using the Dirichlet solution for distributions, we can represent any Gibbs state in terms of extreme Gibbs states.

Michael Röckner, Universität Bielefeld

Resolvents on topological sums

Let X be a right process on a state space $(E,\mathcal{E})$. Let $E = \bigcup_{i\in I} E_i$ be a disjoint union of finely open nearly optional sets, where $(I,\mathcal{I})$ is a $\mathcal{U}$-space such that the canonical projection $\pi : E \to I$ is measurable. Then $T := \inf\{t : \pi\circ X_t \neq \pi\circ X_0\}$ is a perfect, exact and terminal stopping time. If (U^α) is the resolvent of X, then the resolvent (V^α) of the process Y obtained by killing X at time T is exactly subordinated to (U^α) and $U^\alpha = V^\alpha + P_T^\alpha U^\alpha$ for the α-order hitting kernel P_T^α and each E_i is absorbing for ~~t.~~ the right process Y. Similar results can be obtained by purely analytic methods for a Ray resolvent (U^α) on a compact space E which is the direct topological sum of compact spaces. Using results on perturbations of Ray resolvents by BEN SAAD, one can prove that U is ~~[illegible]~~ a weak coupling of Ray resolvents on the components. Details can be found in a joint papers with BEN SAAD.

Klaus Janssen (Tunis / DÜSSELDORF)

Fine and Parabolic Limits.

Consider $Lu = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(a_{ij}(x,t)\frac{\partial u}{\partial x_i}\right) - \frac{\partial u}{\partial t}$ whose coefficients are measurable and satisfy certain weak conditions on $X = \mathbb{R}^n \times (0,T)$. These conditions include the classical case of uniformly parabolic L with bounded, Hölder-continuous coefficients. It is shown that the weak solutions of $Lu = 0$ form a $\mathfrak{P}$-harmonic space on X and the notion of semi-thinness at points of $B = \mathbb{R}^n \times \{0\}$ is introduced. The relationships between fine, semi-fine and parabolic convergence at points of B are examined and the fine limit theorem is used to deduce that every positive weak solution has finite parabolic limits Lebesgue a.e. on B.

Bernard Mair (Kingston, Jamaica)

On a distance invariant under Möbius transformations on $\mathbb{R}^n$

Let $(X,\mathcal{H})$ be a Brelot space. We study the metric

$$\rho(x,y) = \log \sup\left\{\frac{h(x)}{h(y)} : h \in \mathcal{H}^+(X)\right\} - \log \inf\left\{\frac{h(x)}{h(y)} : h \in \mathcal{H}^+(X)\right\},$$

assuming hereby that the set $\mathcal{H}^+(X)$ of pos. harmonic functions separates the points of X. The metric ρ has the following properties: (1) It is complete, (2) on the unit ball B_n of $\mathbb{R}^n$ $(n \geq 2)$ it agrees (up to a constant factor) with the Poincaré distance, and (3) it is invariant under Möbius transformations of $\mathbb{R}^n$. In case (X,g) is a Riemannian manifold and $\mathcal{H}(X)$ denotes the solutions of the Laplace-Beltrami operator, we also study the corresponding differential metric.

Heinz Leutwiler (Erlangen)

Three applications of the analytic set theory to potential theory

① Let V a bounded (positive) measurable kernel from a Suslin measurable space $(E, \mathcal{E})$ into another $(F, \mathcal{F})$. Then V is basic (i.e. there exists a probability λ on $(F, \mathcal{F})$ such that $\varepsilon_x V$ is absolutely continuous w.r.t. λ for every $x \in E$) iff for every universally measurable positive function f on F, the function Vf is measurable on E.

② Let E a metrizable compact space and $E^{\#}$ the space of subprobability measures on E endowed with the vague topology. Let H a Borel or more generally an analytic part of $E \times E^{\#}$ and set, for every positive Borel function f on E

$$Nf^x = \sup_{\mu \in H(x)} \mu(f)$$

where $H(x)$ is the section of H at x (if $H(x) = \phi$, set $Nf^x = 0$). For $\mu, \lambda \in E^{\#}$ define

$$\mu \le \lambda N \quad \text{iff} \quad \forall f \in \mathcal{B}^+ \quad \mu(f) \le \lambda(Nf)$$

where $\mathcal{B}^+$ is the set of Borel positive functions. Since Nf is universally measurable (actually, it is an "analytic" function), it makes sense. Now we have the following extension of a well known Strassen's theorem: we have $\mu \le \lambda N$ iff there exists a sequence (P_n) of Borel kernels s.t. $P_n \le N$ for every n (i.e. $\varepsilon_x P_n \le \varepsilon_x N \quad \forall x \in E$) and $\mu = \lim_n \lambda P_n$ where the limit is in norm.

③ Let $E, E^{\#}$ as before and H a Borel subset of $E \times E^{\#}$ with countable sections $H(x)$ (equivalent to: H is a

countable union of graphs of Borel kernels)
Define N as before and say that $f \in \mathcal{B}^+$ is a pure excessive function w.r.t. N if $Nf \leq f$ and for any $g \in \mathcal{B}^+$ ($g \leq f$ and $Ng = g \Rightarrow g = 0$). Define the potential operator G_N associated to N by: $G_N f$, for $f \in \mathcal{B}^+$, is the least solution of the Poisson equation $u = f + Nu$. Then we have: an excessive (finite) function u is pure ~~exc.~~ iff ① For every kernel P with graph included in H, u is a pure excessive function w.r.t. P (or u is a potential w.r.t. P)

② The set $\{u > 0\}$ is proper for G_N, i.e. there exists a Borel function φ strictly positive on $\{u > 0\}$ s.t. $G_N \varphi$ is finite everywhere.

Dellacherie (Université de Rouen)

Opérateurs de subordination des résolvantes

Soit X un espace compact métrisable, $(V_\lambda)_{\lambda \geq 0}$ une résolvante de Ray sur X. On étudie et on caractérise les propriétés de noyaux positifs P sur X, portés par l'ensemble X_0 des points de non-branchement et vérifiant les conditions suivantes:

1) $U_0 = V_0 - PV_0$ vérifie le principe complet du maximum et la résolvante $(U_\lambda)_{\lambda \geq 0}$ d'opérateur terminal U_0, est subordonnée à (V_λ), c'est à dire que $U_\lambda \leq V_\lambda$ pour tout $\lambda \geq 0$.

2) si $\mathcal{S}$ désigne le cône des fonctions excessives bornées par rapport à (V_λ) et si $(V'_\lambda)_{\lambda \geq 0}$ est une famille résolvante admettant le même cône $\mathcal{S}$ de fonctions excessives, alors $U'_0 = V'_0 - PV'_0$ vérifie le principe complet du maximum ~~de~~ et la résolvante (U'_λ) associée est subordonnée à (V'_λ); de plus (U_λ) et (U'_λ) engendrent le même cône de

fonctions excessives –

Une condition nécessaire et suffisante pour que 1) et 2) soient vérifiées est que P vérifie la condition de subordination:

(S) : pour tous $u, v \in \mathcal{S}$, $s_1, s_2 \in \mathcal{S}$ avec $s_1 \geq s_2$,

$$\left[\inf\left(u, Pu + v - Pv + P(s_1 - s_2)\right)\right]_{X_0} \in \mathcal{S}_{|X_0}$$

(Sous la condition générale évidente ~~et que~~ $Pu \leq u$ $\forall u \in \mathcal{S}$ et $Pu \in \mathcal{S}$ (ou encore Pu fortement surmédiane $\forall u \in \mathcal{S}$)

G. MOKOBODZKI (PARIS)

On the bisubharmonic functions in $\mathbb{R}^n$

A locally summable function ω in an open set $\Omega \subset \mathbb{R}^3$ is said to be bisubharmonic if $\Delta^2 \omega \geq 0$ in the sense of distribution. Identify such a function with the pair (ω, s) where s is subharmonic and $\Delta\omega = s$; note that ω is the difference of two subharmonic functions a.e.

Then using the Brelot Kernel $B_n(x,y) = \begin{cases} -\dfrac{1}{|x-y|} & \text{if } |y| < 1 \\ -\dfrac{1}{|x-y|} + \dfrac{1}{|y|} + \sum\limits_{1}^{n-1} \dfrac{H_m |x|^m}{|y|^{m+1}} & \text{if } |y| \geq 1 \end{cases}$

where $H_m = P_m(\cos\theta)$, P_m the Legendre polynomial and θ the angle between Ox and Oy, one obtains the result:

If (ω, s) is a bisubharmonic pair in $\mathbb{R}^3$, then $\omega(x) = \frac{1}{4\pi}\int B_n(x,y)\, s(y)\, dy$ iff $\int_1^\infty \frac{|s(y)|}{|y|^{n+1}}\, dy < \infty$.

This result has a variant in the form of a generalized Riesz decomposition theorem. Also there is a natural generalization of this result to the case of a polysubharmonic function in $\mathbb{R}^n$, $n \geq 2$.

Victor ANANDAM (Rabat)

The Martin boundary of subdomains (of a harmonic space) satisfying corkscrew conditions

Let $(X, \mathcal{H})$ be a Brelot harmonic space endowed with a metric d such that $z, y \in B(x,r) \subset B(x,2r)$, $h \in \mathcal{H}_+(B(x,2r))$

implies $h(y) \leq C\, h(z)$,

where $C > 0$ is a constant independent of x, r, h. On such a harmonic space we consider NTA (= non-tangential accessible) domains which were introduced into classical potential theory by D.S. Jerison and C.E. Kenig. For these domains we can show that the Martin boundary coincides with the topological boundary. This result is a wide generalisation of a result of R. Hunt, R. Wheeden (1970). Further applications to Fatou type theorems are indicated.

R. Wittmann

The convolution kernels of logarithmic type and the closure of Hunt convolution kernels

Let X be a locally compact, σ-compact abelian group. We denote by the set of all Hunt convolution kernels on X and by $B(X)$ the set of all convolution kernels satisfying the balayage principle. J. Deny proposed the following problem

Does the equality $\overline{H(X)} = B(X)$ hold?

Here $\overline{H(X)}$ is the closure of $H(X)$ in the weak* topology.

By discussing the potential-theoritic properties of convolution kernels of logarithmic type, we show that if $X \not\cong R^n \times Z^m \times F$ with $n+m \leqq 2$ and with a compact abelian group, $\overline{H(X)} \neq B(X)$.

M. Ito

(Totally) partially harmonic functions

Contrary to the usual theory of polyharmonic functions we consider (totally) partially harmonic functions in $\mathbb{R}^n$, i.e. if $\mathbb{R}^n = \mathbb{R}^{n_1} \times \dots \times \mathbb{R}^{n_k}$, $k>1$ we consider Dirichlet problems of the type

$$(*) \quad L(D)u := \frac{1}{2^k} \Delta_1 \cdot \Delta_2 \cdots \Delta_k u = f \quad \text{in } G$$

$$D^\alpha u/_{\partial G} = g_\alpha \quad |\alpha| \le k-1,$$

where the Δ_i's are the Laplacians in $\mathbb{R}^{n_i}$, $1 \le i \le k$ and the g_α are given functions on the boundary ∂G of the bounded domain $G \subset \mathbb{R}^n$. The differential operator $L(D)$ is not hypoelliptic. In the homogeneous case ($f \equiv 0$) a solution of $(*)$ is harmonic in each subspace $\mathbb{R}^{n_i}$. One can show that this Dirichlet problem has for all $f \in L^2(G)$ a unique (weak) solution.

If $L^\sim$ denotes the closure of the differential operator $L(D)$ in $L^2(G)$ and G is the cartesian product of open bounded sets $G_j \subset \mathbb{R}^{n_j}$

$$D(L^\sim) \text{ is dense in } L^2(G)$$

and there exists a positive constant c, such that the estimate

$$\|(\lambda I - L^\sim)^{-1}\| \le \frac{c}{1+|\lambda|}$$

holds for all $\lambda \in \mathbb{C}$ with $\operatorname{Re}\lambda \le 0$. Thus it is possible to solve a Cauchy problem for the first order evolution equation

$$u'(t) + L^\sim u(t) = f(t) \qquad (t>0)$$

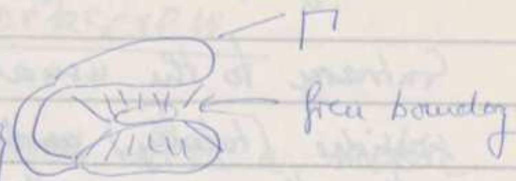
with the initial condition $u(0) = u_0$.

K. Doppel, Berlin

VARIATIONSRECHNUNG, 29.7 – 3.8.84

An Approach to Soap Films That Touch Themselves

Let Γ be a C^r – Jordanarc in $\mathbb{R}^3$, B be the unit disc in $\mathbb{R}^2$ and set $M_\Gamma = \{ v \in H^{1,2}(B, \mathbb{R}^3) \mid v \text{ parametr. } \Gamma \text{ on } \partial B \}$.



With respect to $v \in H^{1,2}(B, \mathbb{R}^3)$, we define $Ad(v) = \{ \text{open subsets } \Theta \text{ of } B \mid v \circ \varphi = v \text{ in } \Theta, \text{ for some conformal map } \varphi : \Theta \to \varphi(\Theta) \subset B \setminus \Theta \}$. We consider the problem

$$\frac{1}{2} \int_{B \setminus \Theta} |\nabla v|^2 \, dx \longrightarrow \min \quad \left(v \in M_\Gamma,\ \Theta \in Ad(v) \right).$$

Imposing additional constraints on v and Θ, one can solve the problem. Thus, the solution of the considered problem is reduced to a priori estimates, in particular for the length of the free boundary $v(\partial\Theta)$. For points away from Γ, these estimates are given. The boundary estimates, however, are yet in preparation.

P. Tolksdorf, Bonn

Rearrangements of functions

We prove a conjecture of Jeff Rauch: Let $\Omega = \Omega' \times [0, a] \subset \mathbb{R}^n$ and let u be the second eigenfunction to the Laplacian under Neumann boundary conditions. Then u attains its maximum and minimum on the boundary. u represents the fundamental mode of an acoustical standing wave.

The proof uses rearrangement techniques and is based on a discussion of the equality sign in $\int_\Omega |\nabla u|^p dx \geq \int_\Omega |\nabla u^*|^p dx$, $p > 1$, where u^* denotes a rearrangement of u. For other rearrangements there are opposing conjectures (by Lieb, Pólya and Szegö) on the question whether equality of the integrals implies $u = u^*$. It is shown that they are both right, since this conclusion is false for one dense subset of $W_0^{1,p}(\Omega)$ and correct for another dense subset.

This has applications e.g. to the symmetry and multiplicity of solutions of some semilinear elliptic boundary value problems on annuli.

We then present starshaped rearrangements and combine a result of Bandle and Marcus with new own results. As an application one can prove Lipschitz continuity of free boundary problems, e.g. the dam problem or the minima of $\int_{\mathbb{R}^n} |\nabla u|^2 + \chi_{\{u>0\}}\,dx$ over $\{u \in H^{1,2}_{loc} \mid u \equiv 1 \text{ in } \Omega\}$.

Bernd Kawohl, Erlangen.

Optimal existence results for large surfaces of prescribed constant mean curvature

A direct variational approach to the Plateau problem for surfaces of prescribed constant mean curvature (H-surfaces) is presented which permits the following improvement of recent results by Steffen and this author, resp. Brezis and Coron:

Suppose there exists an H-surface which is a "strict" local minimum for the energy functional associated to the Plateau problem, then there exists a second solution to the Plateau problem which is geometrically distinct from the first, independently of any geometric condition on the supporting curve.

Michael Struwe, z. Zt. Zürich

Non coercive functionals in elasticity

I consider the problem of minimizing functionals of the type

$$E(u) = \int_\Omega \| P_K \varepsilon(u) \|^2 - \int_\Omega f u \, dx - \int_{\partial\Omega} F u \, d\mathcal{H}^{n-1}$$

where Ω is an open set in $\mathbb{R}^n$, $u: \Omega \to \mathbb{R}^n$; $f: \Omega \to \mathbb{R}^n$, $F: \partial\Omega \to \mathbb{R}^n$ are given, $\varepsilon(u)$ is the strain tensor of u, K is a closed convex cone in the space V of the $n \times n$ symmetric matrices, $P_K: V \to K$ is the projection on K with respect to a given scalar product $\langle \, , \rangle$ on V, whose associated norm is $\| . \|$. A special feature of these functionals is that they are not coercive in general when the forces are zero, while they become coercive under suitable assumptions on f, F. One application of the theory is to the static equilibrium for elastic materials which do not resist to traction. These materials are a first mathematical model for masonry or concrete. Functions of bounded deformation, measure theory and the direct method of Calculus of Variations are the basic tools used.

Gabriele Anzellotti,
Trento (Italy)

ON THE TRANSSONIC FLOW PROBLEM

An irrotational, steady, adiabatic, non-viscous, compressible fluid is considered in bounded domain governed by the equation $-\operatorname{div}(\rho(|\nabla u|^2)\nabla u) = 0$, where the density $\rho(s) = \rho_0 \left(1 - \frac{\varkappa - 1}{2a_0^2} s\right)^{\frac{1}{\varkappa - 1}}$. The key point of considerations is the proof of compactness to the set $\Delta u \leq M$, $|\nabla u| \leq c < \infty$, functions, satisfying simplified entropy condition.
On this set can be minimised the energy functional $\frac{1}{2}\int_\Omega \left(\int_0^{|\nabla u|^2} \rho(t)\,dt\right)dx - \int_{\partial\Omega} g u\, ds$ or the alternating functional, which gives a possibility of the construction to the approximate sequence to the physical solution. This generalises the second modulus method for variational inequalities.

Jindřich Nečas

Praha, ČSSR

The second variation of minimal surfaces in $\mathbb{R}^p$ with polygonal boundary – A uniqueness theorem

We assume that $\Gamma \subset \mathbb{R}^p$ $(p \geq 2)$ is a polygon with $N+3$ corners $(N \geq 1)$. Then the set of minimal surfaces is contained in the set of critical points of a certain function θ in N variables. This function was originally introduced by I. Marx and M. Shiffman. By E. Heinz was proved, that θ is analytic, and for the family of quasiminimal surfaces singular expansions in the corners were given.

Now we connect the 2^{nd} variation of the area functional of a minimal surface bounded by Γ to the Hessian

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of θ. We prove a theorem for the pos-semi-def. and pos.-def. character of the Hessian of θ.
As an application we receive a uniqueness theorem for minimal surfaces.

Fritz Sauvigny
Clausthal-Zellerfeld

On embedded minimal disks in convex bodies

In a joint paper with J. Jost we prove the following result.
Suppose $A \subset \mathbb{R}^3$ is open, bounded, strictly convex of class C^4. Then there exists an embedded minimal disk in A meeting ∂A orthogonally. Related results have been established by Simon-Smith, Sacks-Uhlenbeck, Struwe. We work in the framework of geometric measure theory and use the important concept of almost minimizing varifolds. This was developed by Pitts to show the existence of compact minimal submanifolds of codim 1 in arbitrary compact Riemannian manifolds of dim ≤ 7. For the regularity we use ideas of Almgren-Simon, Meeks-Simon-Yau and Schoen-Simon. The regularity at the free boundary follows from a general regularity result for varifolds with mean curvature in L^p ($p > n$ = dim of the varifold) which was proved by the authors in a second paper and which generalizes fundamental results of Allard. The control of the topological type of the solution follows along the lines of the same proof in Simon-Smith. Our result generalizes to the case where the ambient space is a Riemannian manifold of class C^5.

Michael Grüter, Düsseldorf

The notion of graded harmonic map.

(Collaboration with J.H. Rawnsley)

Let $\mathcal{V} \to M$ and $\mathcal{W} \to N$ be graded Riemannian manifolds (sense of Kostant); and Φ a map between them. We define the Hilbert–Schmidt norm^2 $\|d\Phi\|^2$ of the differential of Φ, using the supertrace. With that as Lagrangian, say that Φ is a <u>graded harmonic map</u> if it is an extremal.

<u>Example.</u> Let A be the exterior algebra $\Lambda\mathbb{R}^\infty$, which is $\mathbb{Z}_2$-graded by parity: $A = A_0 + A_1$. $S\mathbb{R} \to \mathbb{R}$ denotes the spinor bundle of $\mathbb{R}$; its fibres $\mathbb{R}+\mathbb{R}$ are $\mathbb{Z}_2$-graded algebras multiplicatively generated by an element e with $e^2+1=0$. Take $\mathcal{V}$ to be the sheaf of sections of $A \otimes S\mathbb{R} \to \mathbb{R}$; and N a Riemannian manifold with $\mathcal{W} \to N$ the sheaf of smooth functions. In that context a map Φ consists of a pair (φ, ψ), where $\varphi : \mathbb{R} \to N$ is a path and $\psi = (\psi_1 + e\psi_2)/\sqrt{2}$ with $\psi_1\psi_2 + \psi_2\psi_1 = 0$, is a section of $A_1 \otimes S\mathbb{R} \otimes \varphi^{-1}TN \to \mathbb{R}$. Then

$$\|d\Phi\|^2 = \frac{1}{2}\left|\frac{d\varphi}{dt}\right|^2 + \frac{1}{2}\left\langle \psi, \frac{D\psi}{dt}\right\rangle + \frac{1}{12}\left\langle R^N(\psi_1,\psi_1)\psi_2, \psi_2\right\rangle$$

is the Lagrangian of the supersymmetric bosonic chiral model [E. Witten, Phys. Rev. 16 (1977), 2991–2994; Alvarez-Gaumé and D.Z. Freedman, Comm. Math. Phys. 80 (1981), 443–451].

James Eells, Warwick.

Remarks on Hedberg's Theorem.

Hedberg's theorem reads as follows: Given $u \in W^{m,p} \equiv W^{m,p}(\mathbb{R}^N)$ $(m \in \mathbb{N}, 1 < p < \infty)$. Then, there exists a sequence $u_n \in W^{m,p} \cap L^\infty$ such that $\|u - u_n\|_{m,p} \to 0$ and $|u_n(x)| \leq |u(x)|$, $\operatorname{sgn} u_n(x) = \operatorname{sgn} u(x)$ a.e. The original proof by Hedberg (1972 and 1978) as well as re-proofs by Webb (1980) and Brezis-Browder (1982) demands a considerable amount of non-trivial potential theory. In this lecture, a new proof of this theorem is presented. In case $p = 2$, the proof is completely elementary, if $p \neq 2$ some L^p-estimates for elliptic differential equations with constant coefficients are needed.

C. G. Simader (Bayreuth)

Liouville-theorems for harmonic maps on riemannian manifolds with partially negative Ricci-curvature

If M is a smooth, non compact, connected, complete riemannian manifold with Ricci-curvature bounded from below (globally) and nonnegative outside a bounded set $\Omega \subseteq M$, and if N is a smooth, simply connected, complete riemannian manifold, we prove:

a) Every harmonic map $u \in C^2(M,N)$ satisfying the growth-condition $\limsup_{a \to \infty} \frac{1}{\sqrt{a}}\, \delta(a) = 0$ is a constant map, provided the sectional curvature of N is nonpositive. $\delta(a) := \sup\{\operatorname{dist}_N(u(x), z_0) \mid x \in \overline{B}_a(x_0)\}$, $z_0 \in N$ fixed.

b) Every harmonic map $u \in C^2(M,N)$ with "small image" $u(M) \subseteq B_R(z_0)$, $R < \frac{\pi}{2\sqrt{K}}$, is a constant map, provided the sectional curvature of N is bounded from above by the pos. constant K and the geodesic ball $B_R(z_0)$ in N does not meet the cut locus of its center z_0.

Jürgen Kampmann
(BOCHUM)

Nonlinear Stochastic Homogenization

Let $\mathcal{F}$ be the class of all integral functionals

$$F(u,A) = \int_A f(x, Du(x))\,dx$$

with $f(x,p)$ measurable in x, convex in p and such that $c_1|p|^\alpha \le f(x,p) \le c_2(1+|p|^\alpha)$ for $c_2 \ge c_1 > 0$ and $\alpha > 1$ fixed constants. Our aim is to study measures on $\mathcal{F}$ and their convergence, in such a way that variational problems with randomness can be treated, as for instance the thermal behavior of a two-fases material with random chessboard structure. First, we define a metric on $\mathcal{F}$ such that $\mathcal{F}$ becomes a compact metric space and the function

$$F \in \mathcal{F} \longrightarrow \min_u \{F(u,A) + \int_A \varphi u\,dx : u = u_0 \text{ on } \partial A\}$$

is continuous for any given A, φ, u_0. Then, we take a random integral functional, that is a measurable map from a probabilistic space Ω into $\mathcal{F}$ and we consider the homogenization process

$$F_\varepsilon(\omega)(u,A) = \int_A f(\omega, \tfrac{x}{\varepsilon}, Du(x))\,dx \qquad (\varepsilon > 0,\ \omega \in \Omega).$$

Our main result assures that, if F is stochastically periodic, then (F_ε) almost everywhere converges to a random integral functional F_0, whose integrand may be calculated by taking the limit of the minima of Dirichlet's problems with linear boundary values.

Luciano Modica (Pisa)

Double periodic minimal surfaces

We discuss an index theorem for minimal surfaces of genus 1. We start with the Tromba model for the boundary curves of the Plateau problem and develops a functiontheoretical method (elliptic functions)

Karlheinz Schüffler (Düsdorf)

Area maximizing hypersurfaces having an isolated singularity

Radially symmetric solutions of the maximal surface equation in Minkowski space

$$\operatorname{div}\left(\frac{Du}{\sqrt{1-|Du|^2}}\right) = 0 \qquad |Du| < 1$$

are given by

$$w^{\pm}(r) = \pm \int_0^r \frac{K}{\sqrt{t^{2(n-1)} + K^2}}\, dt \quad , \quad K > 0$$

They have a light-cone-like singularity at 0, i.e. they are asymptotic to the upper (resp. lower) lightcone at this point.
We prove that this type of isolated singularity is the only one that can occur for area maximizing hypersurfaces (which include maximal hypersurfaces).
Furthermore we show that the only entire maximal hypersurfaces having an isolated singularity are (up to Lorentz-transformations) radially symmetric maximal surfaces.

Klaus Ecker (Heidelberg)

On the obstacle problem for nonlinear elliptic equations

Let $h_1, h_2 \in H^1$ be such that $h_1 \leq h_2$ in Ω and $h_1 \leq 0 \leq h_2$ in $\partial\Omega$, L an elliptic op. with smooth coefficients and $f: \Omega \times \mathbb{R} \to \mathbb{R}$ satisfying the Caratheodory conditions. Let $K = \{v \in H^1_0 \mid h_1 \leq v \leq h_2\}$ and let a be the Dirichlet form associated to L. We are looking for $u \in K$ such that

$$(*) \qquad a(u, v-u) + \langle f(x,u), v-u \rangle \geq 0 \qquad (v \in K)$$

under no monotonicity assumptions on $f(x, \cdot)$. There is only one paper treating such a question without monotonicity on $f(x,\cdot)$, namely a paper of Chang in Comm. Pure Appl. Math. 1980. However, Chang requires $h_1, h_2 \in C^2$ and $f \in C^\gamma$. We show that if f satisfies the Carathéodory conditions and

$$|f(x,u)| \le a(x) \qquad (h_1(x) \le u \le h_2(x))$$

with $a \in L^2$ (and $h_1, h_2 \in H^2$) then a solution u to $(*)$ can be obtained as a solution of a sequence $(u_n)_n$ of solutions to

$$Lu + g_n(x,u) = 0$$

where g_n are bounded modification of f. It follows that under Chang's hypotheses we have better regularity on the solution u. Since the set of solution is not necessarily convex under ~~these~~ our hypotheses, the question of uniqueness and multiplicity of solutions arises in a natural way. After having indicated some results on these lines, the talk ends with two open problems related to the eigenvalue problem

$$a(u, v-u) \ge \lambda \langle f(x,u), v-u \rangle \qquad (v \in A)$$

related to a paper by Benci (JMAA, 1977), A being a suitable closed convex subset of H^1.

G. Vidossich (Trieste).

Minimal Surfaces of higher topological type

In this talk, we first outline a proof of the existence of minimal surfaces of higher genus and/or connectivity, provided a so-called Douglas condition is satisfied i.e. the infimum over the area of surfaces of the given topological type is strictly less than the area ~~of~~ of surfaces of lower topological type. The proof uses in an essential way the global existence of conformal parameters of competing surfaces. A global variational method providing these conformal representations is outlined as well.

In the second part of the talk, we turn to the existence of embedded minimal surfaces of prescribed topological type. Again we can show existence under a Douglas type condition, where in this case, however, we have to assume that the boundary curve lies on a surface with nonnegative mean curvature.

Finally, we look at a mixed boundary problem and find an embedded minimal disk with holes, having a given fixed boundary curve lying again on a suitable barrier and its free boundaries on some solid which the surface is not allowed to penetrate.

Jürgen Jost

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Deterministic Ergodic Control

Consider

$$\frac{dy}{d\tau} = g(y(\tau), v(\tau)) \qquad y(0) = y$$

$$J_\alpha(v(\cdot)) = \int_0^\infty e^{-\alpha\tau} \psi(y(\tau), v(\tau)) d\tau$$

$$\chi_\alpha(y) = \operatorname*{Inf}_{v(\cdot)} J_\alpha(v(\cdot))$$

The problem of ergodic control is to study the behaviour of $\chi_\alpha(y)$ as $\alpha \to 0$. Several techniques are possible, namely through the definition by control theoretic arguments or by PDE techniques through the Hamilton Jacobi equation

$$\alpha \chi_\alpha(y) = \operatorname*{Inf}_{v} \left[\psi(y, v) + D\chi_\alpha \cdot g(y, v) \right].$$

The results that can be expected are of the following type: there exists a sequence ρ_α (scalar) and a function $\Lambda(y)$ such that

$$\chi_\alpha(y) - \frac{\rho_\alpha}{\alpha} - \Lambda(y) \to 0 \qquad \text{pointwise}.$$

$$\rho_\alpha \to \rho \quad \text{as } \alpha \to 0$$

When control theory can be applied, ρ and Λ have a control interpretation. There may happen that this interpretation is not available, while convergence can still be proven by P.D.E. techniques

Alain BENSOUSSAN (Paris)

Continuous and discontinuous disappearance of capillary surfaces

In a cylindrical tube closed at one end by a base Ω, we seek a capillary surface covering Ω and making a prescribed contact angle γ with the cylinder walls. To each Ω there exists $\gamma_0(\Omega)$, $0 \leq \gamma_0 \leq \pi/2$, such that a surface exists if $\gamma_0 < \gamma \leq \pi/2$, and no surface exists if $0 \leq \gamma < \gamma_0$. We ask what happens when $\gamma = \gamma_0$ and consider first the case $0 < \gamma_0 < \pi/2$. It is shown that if $\Sigma = \partial\Omega$ is smooth then there is no surface at γ_0; however if Σ has one or more corners it can happen that a surface will exist. In both cases, the qualitative behavior as $\gamma \searrow \gamma_0$ can be characterized.

It is shown by examples that varying kinds of behavior can occur when $\gamma_0 = 0$.

R Finn

Generalized Wiener Conditions, U. Mosco

The classical Wiener condition for regularity of boundary points of Dirichlet problem can be generalized to a class of variational problems involving functionals of type:

$$E(u) = \int_\Omega |Du|^2\, dx + \int_\Omega u^2\, d\mu \quad , \Omega \in \mathbb{R}^n ,$$

where μ is a non-negative Borel measure in $\mathbb{R}^n$, such that $\mu(A) = 0$ if $\mathrm{cap}(A) = 0$.

One obtains estimates for any local minimizing function u ($u \in H^1(\Omega) \cap L^2(\Omega,\mu)$ s.t. $E(u) \le E(v)$ for every $v \in H^1_0(\Omega) \cap L^2(\Omega,\mu)$), of the type

$$V(r) \le k\, V(R)\, \omega(r,R)^{\beta} \qquad (0 < r \le \tfrac{1}{10} R),$$

where $V(r) = \sup\limits_{B_r(x_0)} u^2 + \int\limits_{B_r(x_0)} |Du|^2 |x-x_0|^{2-n} dx + \int\limits_{B_r(x_0)} u^2 |x-x_0|^{2-n} d\mu$,
$k = k(n)$, $\beta = \beta(n) > 0$,
x_0 being a given fixed point in Ω, and $\omega(r,R)$ is the Wiener modulus of μ at x_0, defined to be the function

$$\omega(r,R) = \exp\left(- \int_r^R \left[\mathrm{cap}_\mu (B_\rho(x_0), B_{2\rho}(x_0)) / \mathrm{cap}(B_\rho(x_0), B_{2\rho}(x_0)) \right] \frac{d\rho}{\rho} \right)$$

Here cap is the usual Newtonian capacity ($n \ge 3$), while cap_μ is a capacity associated with the measure μ:

$$\mathrm{cap}_\mu^n (B_\rho, B_{2\rho}) = \inf \left\{ \int_{B_{2\rho}} |Du|^2 dx + \int_{B_\rho} u^2 d\mu \;\middle|\; u \in H^1(B_{2\rho}),\ 1-v \in H^1_0(B_{2\rho}) \right\}$$

Similar estimates can be given for elliptic and parabolic obstacle problems and hold for general 2nd order elliptic p.d.o. with discontinuous coefficients. The above result has been obtained jointly with G. Dal Maso.

U. Mosco

The index theorem for k-fold connected minimal surfaces

Using the global formulation of Plateau's problem due to A.J. Tromba (AMS Memoirs 1977) I show the manifold structure of the set M of k-fold connected minimal surfaces bounded by given curves. I show that the map Π which sends a minimal surface to its boundary curve is Fredholm and calculate its index depending on the number and order of branch points. The index calculation gives the same number as R. Böhme and A.J. Tromba had in their index paper in the disc case (Annals of Math. 113, 1981), this means, that the number k of connectivity does not come into the index formula.

In consequence of that Smale's infinite dimensional version of Sard's theorem gives generic isolateness and stability and, under some further condition in the case $k=2$, also a generic finiteness result. For a certain class of extreme pairs of Jordan curves (if $k=2$) I can also derive the existence of an embedded solution of Plateau's problem.

Ulla Thiel (Heidelberg)

Some Remarks on Degenerate Parabolic Systems

We consider the degenerate parabolic system

$$\frac{\partial u^i}{\partial t} - \mathrm{div}\left(|\nabla u|^{p-2}\nabla u^i\right) = 0, \quad 1\le i\le m \quad \text{on } \Omega\times(0,T),\ \Omega\subset\mathbb{R}^N$$

with some $p>2$ and assume a weak solution

$$u^i \in L_\infty((0,T), L_2(\Omega)) \cap L_p((0,T), W_p^1(\Omega)) \text{ be given.}$$

Based on results of Di-Benedetto, Friedman (J. Reine & Angew. Math. 349, 83-128 (1984)) we prove, that $\nabla u \in C_{\alpha, loc}$ for some α, depending only on N and p.

M. Wiegner
Bayreuth

A Counter Example in R^3 to a Conjecture of H. Hopf

<u>Counter Example Theorem</u>: There exist closed immersed surfaces of genus one in R^3 with constant mean curvature. (In fact, we exhibit a countably infinite number of isometrically distinct surfaces)

We exhibit the surface by producing a conformal map $\bar{x}: R^2 \to R^3$ of constant mean curvature which is doubly periodic with respect to a rectangle in R^2. To produce such a mapping one looks at the 1^{st} and 2^{nd} fundamental forms.

$$I = dx\cdot dx = E(du^2+dv^2) = e^{2\omega}(du^2+dv^2)$$

$$II = -dx\cdot d\xi = L\,du^2 + 2M\,du\,dv + N\,dv^2$$

Let $\omega(u,v)$ be a solution to the D.E. $\Delta\omega + \sinh\omega\cosh\omega = 0$ which is positive on a rectangle $\Omega_{AB} = (0,A)\times(0,B)$ vanishing on the boundary. By odd reflections ω extends to a doubly periodic solution on R^2. Set $L = e^{\omega}\sinh\omega$, $M=0$, $N = e^{\omega}\cosh\omega$ so that $k_1 = e^{-\omega}\sinh\omega$, $k_2 = e^{-\omega}\cosh\omega$ and $H = 1/2$. One verifies that the Gauss and Codazzi equations are satisfied and so one produces a map $\bar{x}: \mathbb{R}^2 \to \mathbb{R}^3$ of mean curvature $H = 1/2$ with the given 1st and 2nd fundamental forms. The map has nice symmetries. In particular

a) $\bar{x}(u+2A, v) = \bar{x}(u,v) + \bar{a}$ for some vector $\bar{a}$

b) $\bar{x}(u, v+2B) = \Theta \cdot \bar{x}(u,v)$

where Θ is a rotation about a line l whose direction is given by $\bar{a}$. The surface will close up if $\bar{a} = \bar{0}$ and $\Theta/2\pi$ is rational. We show that there exist rectangles Ω_{AB} for which this is the case.

An important step is to look for large solutions (positive) to the D.E. $\Delta W + \lambda(e^W - e^{-W}) = 0$ on a rectangle Ω, $W|\partial\Omega = 0$ as $\lambda \to 0$. An integral equation method used by V. Weston for the D.E $\Delta W + \lambda e^W = 0$ and extended by R.L. Moseley to include more general D.E.'s can be made to work in our case

Henry C. Wente
(Toledo, Ohio, U.S.A.)

Harnack's inequality and quasilinear diagonal systems

In this talk, we present a new approach to establish Liouville theorems, Phragmén-Lindelöf type theorems, and local as well as global continuity estimates for bounded weak solutions of quasilinear elliptic systems in diagonal form. By employing the Harnack inequality for weak supersolutions of elliptic equations, one obtains considerably simplified proofs ~~of~~ for various results that have previously been known. The method also yields a number of generalizations concerning certain critical cases.
In particular, an extension of a Liouville theorem for harmonic maps due to Hildebrandt-Jost-Widman is proved.
Moreover, we describe a procedure to derive a priori estimates for the

Hölder norm of weak solutions which can easily be adapted to the case of variational inequalities for vector-valued functions and to quasilinear parabolic systems of diagonal form.
Finally, a removable singularity theorem for harmonic maps into manifolds with nonpositive sectional curvature is presented, extending the corresponding result by Carleson for Laplace's equation. The proof rests upon appropriate mean value inequalities concerning distribution solutions to differential inequalities.

Michael Meier (Berkeley/Bonn)

Let Γ be a contour inside a solid $2p$ torus N_p with ∂N_{2p} having non-negative inward mean curvature. Suppose there exists a basis of $\pi_1(N_{2p})$ (free on $2p$ generators) such that $[\Gamma] \in \pi_1(N_{2p})$ is the commutator of this basis. Then Γ bounds a minimal surface of genus p.

Moreover, let Γ be a contour inside a solid torus T such ∂T has non-negative inward mean curvature. Suppose $[\Gamma] \in 2\pi_1(T)$, $[\Gamma] \neq 0$. Then Γ bounds a Möbius minimal surface.

A. J. Tromba (Bonn - Santa Cruz)

The Regularity of Minimal Surfaces with Free Boundary

Let S be a compact surface embedded in $\mathbb{R}^3$ and Γ a homotopically non trivial Jordan curve in the complement of S. We consider the problem of minimizing Dirichlets integral in the class of all mappings from the closed unit disc into $\mathbb{R}^3$ whose boundary values lie on S and form a homotopically non trivial loop in $\mathbb{R}^3 \setminus \Gamma$. It is proven that for analytic S minimizing maps are immersed up to the boundary. The proof uses methods for the corresponding situation arising in Plateau's problem.

F. Tomi (Heidelberg)

Plateau's problem in Minkowski space

We prove that the boundary Γ of a spacelike $C^{3,\alpha}$ immersion $f: B \to M^n$ of the unit ball $B \subset \mathbb{R}^{n-1}$ into M^n, the n-dim. Minkowski space, also bounds a spacelike maximal hypersurface, that means an extremum of the volume induced from M^n. In M^3, we show by examples that these surfaces are not geometrically unique for fixed boundary Γ, but we derive a sufficient geometric condition for Γ which asserts uniqueness.

N. Quien (Heidelberg)

Algebraische Zahlentheorie

5. – 11. August 1984

Remarks on the quadratic reciprocity law in totally real algebraic number fields

Let S denote the sum and N the product over numbers and/or variables attached to the conjugate fields K_i of a given field K. For instance $S(\alpha) = \sum \alpha_i$ the trace of a number, but $S(\alpha\tau) = \sum \alpha_i \tau_i$ with complex variables τ_i. Let $\mathfrak{d}$ be the different and $\mathfrak{g}$ an integral ideal equivalent with $\mathfrak{d}$, and $(g) = \mathfrak{g}\mathfrak{d}^{-1}$. We consider odd numbers $a \in K$ which are congruent units ε_a mod 4 : $a \equiv \varepsilon_a \bmod 4$, $b \equiv \varepsilon_b \bmod 4$ and so on. The elementary theta function is defined as

$$\vartheta_g(\tau) = \sum_\omega e^{2\pi i S(g\omega^2\tau)}, \quad \omega \in \mathfrak{o} = \text{all integers of } K.$$

For

$$\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} \in Sl(2, \mathfrak{o}) \text{ with } \gamma \equiv 0 \bmod 4\mathfrak{g},\ \delta \equiv \varepsilon_\delta \bmod 4$$

we have

$$(1) \qquad \vartheta_g(\tau) = \vartheta_g\left(\frac{\alpha\tau+\beta}{\gamma\tau+\delta}\right) j\left(\begin{matrix}\alpha & \beta\\ \gamma & \delta\end{matrix}\Big|\tau\right)$$

where

$$j\left(\begin{matrix}\alpha & \beta\\ \gamma & \delta\end{matrix}\Big|\tau\right) = \left(\frac{\gamma}{\delta}\right)\left(\frac{\delta\varepsilon_\delta^{-1}}{\mathfrak{g}}\right) e^{\frac{\pi i}{4} S(1-\operatorname{sign}\varepsilon_\delta)} \left|N(\gamma\tau+\delta)\right|^{-1/2},$$

and for all i: $g_i > 0$, $\gamma_i > 0$, $-\frac{\pi}{2} < \arg(\gamma_i\tau_i + \delta_i) \leq \frac{\pi}{2}$.

(1) plays an important rôle in the theory of modular forms of half integral weight, especially if $K = \mathbb{Q}$. In this case it has been proved by Hecke.

The theta function is related to the Gaussian sums by

$$(1a) \qquad \lim_{\lambda \to 0} N(\lambda)^{1/2} \vartheta_g\left(\frac{a}{b} + \frac{i}{2}\lambda\right) = G_g(a,b)\,|N(b)|^{-1} |N(\mathfrak{g})|^{-1/2};$$

and the Gaussian sums are connected with the Legendre symbol by

$$(1b) \qquad G_g(a,b) = \left(\frac{a}{b}\right) G_g(1,b),$$

the latter being

$$(1c) \qquad G_g(1,b) = \left(\frac{b\varepsilon_b^{-1}}{\mathfrak{g}}\right) e^{\frac{\pi i}{4} S(\operatorname{sign} g b - \operatorname{sign} g \varepsilon_b)}.$$

Inserting (1a) - (1c) into (1) we get

(2) $$\left(\frac{a}{b}\right) = \left(\frac{A}{B}\right)\left(\frac{\gamma}{\delta}\right)(-1)^{W(b,B)}$$

with
$$A = \alpha a + \beta b \quad \text{and} \quad W(b,B) = \tfrac{1}{4} S[(\operatorname{sign} g b + 1)\operatorname{sign}(g B - 1)]$$
$$B = \gamma a + \delta b \qquad\qquad - \tfrac{1}{4} S[(\operatorname{sign} g \varepsilon_b + 1)(\operatorname{sign} g \varepsilon_B - 1)]$$

According to (2) the Legendre symbol behaves similarly like a modular form. (2) can be derived from the quadratic reciprocity law

(3) $$\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) = (-1)^{w(a,b)}, \quad w(a,b) = \tfrac{1}{4} S[(\operatorname{sign} a - 1)\operatorname{sign}(b-1)]$$

for $a \equiv \varepsilon_a \bmod 4$, $b \equiv \varepsilon_b \bmod 4$ $$- \tfrac{1}{4} S[(\operatorname{sign} \varepsilon_a - 1)(\operatorname{sign} \varepsilon_b - 1)].$$

(3) differs from the formerly known reciprocity law (Hasse, Hecke ($n=2$), Siegel)

(3') $$\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) = (-1)^{w'(a,b)}, \quad w'(a,b) = \tfrac{1}{4} S[(a-1)(b-1)]$$

for $a \equiv b \equiv 1 \bmod 2$ $$- \tfrac{1}{4} S[(\operatorname{sign} a - 1)(\operatorname{sign} b - 1)]$$

(3) and (3') are actually 2 different versions of the reciprocity law, covering different cases. In the case of a real qudratic field the comparison leads to the following congruence: Let $\varepsilon_a, \varepsilon_b$ be two units, both $\equiv 1 \bmod 2$. Then

$$S[(\varepsilon_a - 1)(\varepsilon_b - 1)] \equiv S[(\operatorname{sign} \varepsilon_a - 1)(\operatorname{sign} \varepsilon_b - 1)] \bmod 8,$$

which can easily be checked.

Equation (3) can be proved by using the theta function and its behaviour under $\tau \to -\tau^{-1}$. The rational case had been treated by Kronecker, and the generalization is straightforward.

M. Eichler (Basel)

Iwasawa invariants of abelian number fields

For a prime p, the minus-part of the Iwasawa invariants of an imaginary abelian field K can be decomposed as $\mu_p^- = \sum_\chi \mu_\chi$, $\lambda_p^- = \sum_\chi \lambda_\chi$, where χ

runs through all odd characters $\neq \omega^{-1}$ of K (ω the Teichmüller character) and where μ_χ and λ_χ are the Iwasawa invariants of the power series representing the p-adic L-function attached to $\chi\omega$. This holds true provided p^2 (or 8, if $p=2$) does not divide the conductor of K, which in fact may be assumed without loss of generality. I present a simple proof that $\mu_\chi = 0$ and $\lambda_\chi < p^{r(p-1)/2}$ if $p>2$ and p does not divide f_χ, the conductor of χ; here r is such an integer that $p^r > 2f_\chi^2$. An analogous result is proved for $p=2$ and $p=3$ without the restriction $p \nmid f_\chi$. This extends my previous work where similar results were proved under the assumption that $B^1(\chi)$ is prime to p. It is interesting to compare the bound of λ_χ with recent results of D. Barsky.

Tauno Metsänkylä

Turku

"Höhere Reziprozitätsgesetze und Modulfunktionen vom Gewicht Eins"

Für einen imaginär-quadratischen Zahlkörper $\Sigma = \mathbb{Q}(\sqrt{-d_\Sigma})$ der Diskriminante $-d_\Sigma < 0$ existiere ein $f \in \mathbb{N} - \{0\}$, so daß der Ringklassenkörper modulo f über Σ, der mit N_f bezeichnet werde, eine zyklische Erweiterung vom Grad 4 über Σ ist. N_f enthält außer Σ genau zwei weitere quadratische Zahlkörper $K_1 = \mathbb{Q}(\sqrt{-d_1})$ und $K_2 = \mathbb{Q}(\sqrt{d_2})$, wobei $-d_1 < 0$ und $d_2 > 0$ die quadratfreien Kerne der entsprechenden Diskriminanten sind. Sei R_f die Ordnung zum Führer f in Σ und $\Phi(x)$ das Ringklassenpolynom zur Ordnung R_f. Bekanntlich hat $\Phi(x)$ ganze rationale Koeffizienten und ist über Σ irreduzibel. N_f ist der Zerfällungskörper von $\Phi(x)$ über Σ. Es gilt folgender

Satz Sei p eine Primzahl, die das Produkt $d_2 f$ nicht teilt und $\mathbb{F}_p$ der endliche Körper mit p Elementen. Dann gilt:

$$\#\{x \in \mathbb{F}_p \mid \Phi(x) = 0\} = 1 + \left(\frac{d_2}{p}\right) + a(p).$$

Dabei ist $a(p)$ der p-te Koeffizient der Fourierentwicklung einer Spitzenform die mit hilfe Theta-Reihen definiert ist.

Aus dem Satz ergibt sich der höhere Reziprozitätsgesetz für die Erweiterung $N_f/\mathbb{Q}$, nämlich:

$$Spl(\Phi) = \{p \in \mathbb{P} \mid p \nmid d_2 f, \left(\frac{d_2}{p}\right) = \left(\frac{-d_1}{p}\right) = 1 \text{ und } a(p) = 2\}.$$

Jannis A. Antoniadis

Ring Class Fields and the 168-Tesselation

For the principal quadratic form of discriminant $d\,(<0)$

$$F_d(x,y) = \begin{cases} x^2 - (d/4)\,y^2 \\ x^2 + xy - ((d-1)/4)\,y^2 \end{cases} \qquad d \equiv \begin{cases} 0 \\ 1 \end{cases} \bmod 4$$

with $d = d_0 f^2$ (belonging to $k = \mathbb{Q}(\sqrt{d_0})$), Weber's Theorem states for a prime p

$$(p \nmid 2d),\ p = F_d(x,y) \iff p \text{ splits completely in } K = k \,/\, j\left(\frac{d+\sqrt{d}}{2}\right)),$$

(K is the ring class field). We consider the special case $f = b^t$, ($t = 0, 1, 2, \ldots$), so we have to know how to find $j(z)$, $j(bz)$, $j(b^2 z)$, ... iteratively.

The modular equation for $j(bz)$ is of degree $M = b \prod (1 + 1/r)$ (r = prime divisors of b) and it gives $j(z/b)$ as a conjugate so an iterative process $j(z) \to j(bz)$ might be repeated to produce $j(z)$

(e.g., for $b<11$)

(instead of $j(b^2 z)$). For a modular equation of genus 0, we have

$$\begin{cases} j(z) = \psi(\tau) & \text{(degree } M \text{ in } \tau) \\ j(bz) = \psi(\tau^*), & \tau^* = 1/\tau . \end{cases}$$

To prevent "reversibility (and guarantee iteration) we use $\tau \to \tau'$, (a conjugate of $\tau = \psi^{-1}(j(z))$), and set $\tau^* = 1/\tau'$ instead. Then for cases where both the modular equation and its closure are of genus 0, (i.e., $b = 2, 3, 4, 5$) we can set up the following (typical) system (for b odd prime).

$$b = 3,5 \quad \begin{cases} j(z) = \psi(\tau), & \tau: (M=) b+1 \text{ valued for } j(z), \\ \tau = \phi(\omega), & \omega: (b-1)/2 \text{ valued for } \tau, \\ \omega = \zeta^b, & \zeta: \; b \text{ valued for } \omega, \end{cases}$$

This can be done, (as in Fricke-Klein Elliptischen Modulfunktionen, see author's paper in Math. Ann. 255 (1981), also), so that the Galois group of the modular equation $PSL(2,b)$, (order $b(b^2-1)/2$ for b odd prime), has an action on ζ through rotations of ~~a~~ the

3-dihedral group for $b=2$
tetrahedral group for $b=3$
octahedral group for $b=4$
icosahedral group for $b=5$.

In the present paper we take $b=7$, where the modular equation is of genus 0 but its closure field is of genus 3. The system becomes

$$b = 7 \quad \begin{cases} j(z) = (\tau^2+3\tau+9)(\tau^2+235\tau+1201)^3/(\tau-5)^7 \\ \tau = (1-3\omega^2-\omega^3)/(\omega+\omega^2) \\ \zeta^7 = -\omega^3/(1+\omega) \qquad \text{(genus 3)}. \end{cases}$$

Thus finding a conjugate for τ requires rotating the (ω, ζ) Riemann-surface according to Klein's 168-Tesselation pattern.

By setting up the above system as an iteration process for $j(7^t z)$, we have the following numerical result:

$4p = x^2 - d_0 b^{2t} y^2 \Longleftrightarrow$ the splitting occurs through t iterations; (this means $-(\omega^3/(1+\omega))^{1/7}$ exists for t levels of iteration modulo p). We incidentally must restrict $p \equiv 1 \mod 7$ in order to take seventh roots.

Harvey Cohn

Kummer's system of congruences

Let l be an odd prime. We call by Kummer's system of congruences the following system

$$(K)\quad \varphi_{l-2j}(t)B_{2j} \equiv 0 \pmod{l} \quad \left(1 \le j \le \frac{l-3}{2}\right),$$

where B_{2j} means the Bernoulli number and $\varphi_i(t) = \sum (-1)^{v-1} v^{i-1} t^{v}$ $(1 \le v \le l-1)$ are the Mirimanoff polynomials for $2 \le i \le l-1$. The system (K) is ~~[illegible]~~ connected with the first case of Fermat last theorem. ~~as follows~~

In this lecture there are introduced systems of congruences (S) and (T) depending on the Stickelberger ideal $\mathcal{I}^-$ and (T) depends also on the set T of all $l-1$-tuples $(\gamma_0, \gamma_1, \ldots, \gamma_{l-2})$ of integers.

The main result is the following theorem: "Let τ be an integer, $\tau \not\equiv -1 \pmod{l}$. Then the following statements are equivalent:

(a) τ is a solution of (K),

(b) $-\tau$ is a solution of (S),

(c) $-\tau$ is a solution of (T)."

By suitable choice of elements from the Stickelberger ideal $\mathcal{I}^-$ and the set T we get the following corollaries:

1) If there exists a solution τ of the system (T) such that $\varphi_{l-1}(-\tau) \equiv 0 \pmod{l}$ and $\tau \not\equiv 0 \pmod{l}$, then

$$q(2) = \frac{2^{l-1}-1}{l} \equiv 0 \pmod{l}.$$

2) If there exist solutions τ_1, τ_2 of the system (T) such that $\varphi_{l-1}(-\tau) \equiv \varphi_{l-1}(-\tau) \equiv 0 \pmod{l}$ and

$\tau_1 \cdot \tau_2 \not\equiv 0 \pmod{l}$, $\tau_1 \not\equiv \tau_2 \pmod{l}$, then

$$q(3) = \frac{3^{l-1}-1}{l} \equiv 0 \pmod{l}.$$

3) If there exists a solution τ of the system (T) such that $\varphi_{l-1}(-\tau) \equiv 0 \pmod{l}$ and $\tau \not\equiv 0, -1 \pmod{l}$, $\tau^2 \not\equiv -1 \pmod{l}$, then

$$q(5) = \frac{5^{l-1}-1}{l} \equiv 0 \pmod{l}.$$

If we assume that the first case of Fermat last theorem fails, then we get $q(2) \equiv 0 \pmod{l}$ (Wieferich 1909), $q(3) \equiv 0 \pmod{l}$ (Mirimanoff 1911) and $q(5) \equiv 0 \pmod{l}$ (Vandiver 1914).

Ladislav Skula (Brno)

<u>The status of certain old problems from elementary and analytic theory of algebraic numbers.</u>

A report was given on the situation in 12 old problems, which included: Lehmer's & Schinzel-Zassenhaus conjectures, Robinson problems concerning the distribution of complete sets of conjugated integers on the plane, various questions concerning factorizations in ~~and~~ algebraic number fields, ~~and~~ (i.e. the problem of characterizing fields with given class-groups in terms of factorizations) and divisibility questions for class-numbers.

W. Narkiewicz (Wrocław)

Endliche Galoismoduln und automorphe Formen

Mit Hilfe von endlichen Moduln über der absoluten Galoisgruppe $G_k = \mathrm{Gal}(\bar{k}/k)$ eines Zahlkörpers k werden sogenannte komplexe Weil-Darstellungen von G_k konstruiert und, basierend auf neueren Ergebnissen von Arthur, Flicker und Kazhdan, automorphen Darstellungen zugeordnet. Dabei ergibt sich die Holomorphie der Artinschen L-Reihen (primitiver) dreidimensionaler Weil-Darstellungen. Schließlich wird anhand eines Beispiels auf einen möglichen Zusammenhang zwischen der L-Funktion einer elliptischen Kurve und den Artinschen L-Reihen der zu den Torsionspunkten dieser elliptischen Kurve gehörigen Weil-Darstellungen hingewiesen.

Hans Opolka, Münster

Realisierung endlicher Gruppen als Galoisgruppen

Satz 1: Alle sporadischen einfachen Gruppen mit höchstens einer Ausnahme J_4 sind als Galoisgruppen über $\mathbb{Q}^{ab}(t)$ und $\mathbb{Q}^{ab}$ realisierbar.

Zusatz: Mindestens 17 der 26 sporadischen einfachen Gruppen sind als Galoisgruppen über $\mathbb{Q}(t)$ und $\mathbb{Q}$ realisierbar.

Def: Eine endliche Gruppe G mit $Z(G)=1$ besitzt eine GAR-Darstellung über $k(t)$ $\Leftrightarrow$

(G) $G \cong \mathrm{Gal}(N/k(t))$ mit $N/k(t)$ regulär,

(A) $\mathrm{Aut}(G) \leq \mathrm{Aut}(N/k)$ mit $N^{\mathrm{Inn}(G)} = k(t)$,

(R) $K/N^{\mathrm{Aut}(G)}$ regulär mit $\bar{k}K = \bar{k}(t) \rightsquigarrow K/k$ rational.

Satz 2: Ist G eine endliche Gruppe, deren Kompositionsfaktoren GAR-Darstellungen über $k(t)$ mit einem Hilbertkörper $k \geq \mathbb{Q}$ besitzen, so ist G als Galoisgruppe über k realisierbar.

Zusatz: Ist G eine endliche Gruppe, deren Kompositionsfaktoren zyklisch sind oder GAR-Darstellungen über $k(t)$ mit $(k:\mathbb{Q}^{ab})<\infty$ besitzen, so ist G als Galoisgruppe über k realisierbar.

Kostproben von Beweisen zu Satz 1 mit Zusatz wurden vorgeführt sowie Beispiele von Gruppen mit GAR-Darstellungen über $\mathbb{Q}(t)$ und $\mathbb{Q}^{ab}(t)$ angegeben.

B. H. Matzat (Karlsruhe)

On the absolute Galois group of a $\wp$-adic field

The absolute Galois group $G_k = Gal(\bar{k}/k)$ has a completely explicit description in the case that k is a local field over $\mathbb{Q}_p$ and $p \neq 2$ or $k(\sqrt{-1})/k$ is unramified, (e.g. $\sqrt{-1} \in k$). For $p \neq 2$ this was done by Jannsen and Wingberg in Inventiones 70 (1982). If k contains a primitive p-th root of unity ζ_p we have the following result (for $p=2$ see Crelle 350 (1984)):

Theorem: Let $n = [k : \mathbb{Q}_p]$, $\zeta_p \in k$, $q = \#(\mu_{p^\infty} \cap k^{nr})$, $q \neq 2$, f = residue class field degree of $k/\mathbb{Q}_p$. Then G_k is generated by $n+3$ elements $\sigma, \tau, x_0, \dots, x_n$ which satisfy the following defining conditions:

1) the normal subgroup generated by $x_0, \dots, x_n$ is a pro-p-group,
2) "tame relation": $\sigma\tau\sigma^{-1} = \tau^{p^f}$,
3) $\sigma x_0 \sigma^{-1} = (x_0\tau)^{\pi g} x_1^q [x_1, x_2][x_3, x_4] \dots [x_{n-1}, x_n]$, where $\pi \in \hat{\mathbb{Z}}$ such that $\pi \hat{\mathbb{Z}} = \mathbb{Z}_p$, $g \in \mathbb{Z}_p$ such that $\sigma(\zeta_q) = \zeta_q^g$ for a q-th root of unity ζ_q and a Frobenius automorphism σ.

The structure of $G_{\mathbb{Q}_2}$ is still unknown. There is a hope that the theorem above remains true. The condition 3) then simply means: $\sigma x_0 \sigma^{-1} = (x_0\tau)^{\pi} x_1^q$.

Volker Diekert, (Hamburg).

Einverstanden

Universal Γ-norms for abelian varieties

Let $K/\mathbb{Q}_p$ be a finite extension, K_∞/K be a ramified $\mathbb{Z}_p$-extension, and $A_{/K}$ be an abelian variety with good reduction. We are interested in the study of the following subgroup of $A(K)$:

$NA(K)$:= subgroup of universal norms in $A(K)$ w.r.t. K_∞/K.

The basic invariant of A which is needed is

r := p-rank of the reduction of A (i.e. $p^r = \# A(\overline{\mathbb{F}}_p)_p$).

Our result then is the

Theorem: $\operatorname{rank}_{\mathbb{Z}_p} A(K)/NA(K) = (\dim A - r)\cdot[K:\mathbb{Q}_p]$.

Using the theory of the logarithm of a formal Lie group, and important result of S. Sen about the Galois cohomology of the ring of integers in a local field, and the techniques of flat cohomology (in particular the local flat duality theorem) the above assertion is reduced to a statement about p-divisible groups over the residue class field k of K.

Let R, resp. R_∞, be the ring of integers in K, resp. K_∞, and put $\Gamma := \operatorname{Gal}(K_\infty/K)$.

Proposition: For any local-local p-divisible group $G_{/k}$ we have

$$\operatorname{corank} H^1(\Gamma, G(R_\infty \otimes_R k)) = \operatorname{height}(G)\cdot[K:\mathbb{Q}_p].$$

Via the theory of Dieudonné modules this is reduced to a problem about the Γ-structure of the formal Lie group $\hat{CW}_{/k}$ of Witt covectors. Although this group is infinite-dimensional it is easier to handle since it has a filtration $\hat{W}_1 \subseteq \hat{W}_2 \subseteq \hat{W}_3 \subseteq \ldots \subseteq \hat{CW}_{/k}$ where the factors a isomorphic to the formal additive group. This enables us to solve that problem.

Peter Schneider
(Heidelberg)

On Local Galois Characters

From the observation that any finite Galois extension M/K shows up as the composite field of the fixed fields belonging to the kernels of all irreducible complex characters χ of the absolute Galois group G_K of K which are trivial on G_M, one is led to study the following two questions: (1) What is the structure of $G_\chi = G_K/\ker\chi$? (2) Can G_χ be described in terms of parameters that depend on K alone? When K is a finite extension of the p-adic number field $\mathbb{Q}_p$ Local Class Field Theory provides an answer to (2) as long as χ is abelian; here the answer to (1) is simply "G_χ is cyclic". The Local Langlands Conjecture seems to indicate that for non-abelian χ the groups G_χ are somehow related to the multiplicative group $D^\times$ with D being a central division algebra over K of an index s that is a multiple of the degree of χ. In order to be able to investigate such a relationship one first has to study the group G_χ itself, and, in fact, not only its group theoretical structure but also its arithmetical properties: so the ramification subgroups, the conductor of χ, $\det\chi$, and finally the Artin L-function belonging to χ have to be determined. Particularly the discussion of the Artin L-function requires an explicit Brauer formula "χ = sum of monomial characters" for the given χ. In the first case where non-monomial χ occur, namely when the degree is p, a complete answer is given.

Jürgen Ritter (Augsburg)

Ein Analogon zur Fundamentalgruppe einer Riemann'schen Fläche im Zahlkörperfall

Sei K ein algebraischer Zahlkörper vom CM-Typ mit maximal total reellem Teilkörper K^+; der Körper K enthalte die Gruppe μ_p der p-ten Einheitswurzeln ($p \neq 2$), also $K = K^+(\mu_p)$. Weiter sei K_∞, $K(p)$ bzw. K_S die zyklotomische, die maximale p-Erweiterung bzw. die maximale p-Erweiterung von K, die außerhalb einer Primstellenmenge $S \supseteq S_p \cup S_\infty$ unverzweigt ist ($S_p \cup S_\infty = \{\mathfrak{g} \mid p\infty\}$).
Wird nun mit $\tilde{K}$ die maximale bei p positiv-zerlegte p-Erweiterung von K bezeichnet (die durch die Bedingung definiert ist: $\tilde{K}_{\mathfrak{g}} \subset K^+(p)(\mu_p)_{\mathfrak{g}}$ für alle Komplettierungen bezüglich $\mathfrak{g} \mid p$), so gilt für endliche Erweiterungen $E|K_\infty$ mit $E \subset \tilde{K}$ ein zum Funktionenkörperfall völlig analoger Satz, wenn E_S durch die Erweiterung $\tilde{E}_S = E_S \cap \tilde{K}$ ersetzt wird:

<u>Satz</u>: Sei die Iwasawa-μ-Invariante von K null und die Komplettierungen $K^+_{\mathfrak{g}}$ für $\mathfrak{g} \mid p$ enthalten nicht die Gruppe μ_p. Dann gilt:

(i) Die Galoisgruppe $Gal(\tilde{E}_{S_p} | E)$ ist trivial oder eine Demuškingruppe vom Rang $2g_E$, wobei g_E durch die Riemann-Hurwitz Formel gegeben ist

$$2 - 2g_E = (2 - 2\lambda_1(K^+))[E:K_\infty] - \sum_{\mathfrak{g} \notin S_p(E)} (e_{\mathfrak{g}} - 1)$$

($e_{\mathfrak{g}}$ die Verzweigungsindizes der Erweiterung $E|K_\infty$, $\lambda_1(K^+)$ der $\mathbb{Z}_p$-Rang von $Gal(K^+_{S_p}|K_\infty)^{ab}$).

(ii) Die Galoisgruppe $Gal(\tilde{E}_S | E)$ für endliches $S \supsetneq S_p \cup S_\infty$ ist eine freie pro-p-Gruppe vom Rang

$$\operatorname{rang} Gal(\tilde{E}_S|E) = 2g_E + \#S \setminus (S_p \cup S_\infty) - 1 .$$

Für $S \supseteq S_p \cup S_\infty$ besitzt $Gal(\tilde{E}_S | E)$ Erzeugende x_i, y_i, $1 \le i \le g_E$,

$u_{\mathfrak{g}}$, $\mathfrak{g} \in S \setminus S_p \cup S_\infty$ mit der einzigen definierenden Relation

$$\prod_{i=1}^{g_E} [x_i, y_i] \prod_{\mathfrak{g} \in S \setminus S_p \cup S_\infty} u_{\mathfrak{g}} = 1$$

und es ist

$$\langle u_{\mathfrak{g}} \rangle = T_{\mathfrak{P}}(E(p)|E)$$

für geeignetes $\mathfrak{P}|\mathfrak{g}$ ($T_{\mathfrak{P}}$ die Trägheitsgruppe bezüglich $\mathfrak{P}$).

Kay Wingberg
(Regensburg)

Embedding problems in $\mathbb{Z}_p$-extensions

For a numberfield k, and a fixed prime p, let $\tilde{k}$ be the compositum of all $\mathbb{Z}_p$-extensions of k. We give an explicit answer to the following question:

If N/k is a given abelian p-extension, is N contained in $\tilde{k}$?

The problem is solved by introducing a Log-map in the following way:

Let $S = \{\mathfrak{p}/p \text{ in } k\}$ and let $C = \prod_{\mathfrak{p} \in S} k_{\mathfrak{p}}$, $U = \prod_{\mathfrak{p} \in S} U_{\mathfrak{p}}$ be the product of completions of k on S, and the product of corresponding local principal units; let $\log : U \to C$ be the usual $p$adic logarithm, and put $\mathrm{Log} : U \to C/V$, where V is the $\mathbb{Q}_p$-subspace of C generated by the log of (global) units of k; we extend Log to I: if $\mathfrak{a} \in I$, and $\mathfrak{a}^n = (a)$, $a \in k^\times \cap U$, then we put $\mathrm{Log}\, \mathfrak{a} = \frac{1}{n} \mathrm{Log}\, a$ in C/V.

In the following diagram,

$$\begin{tikzcd}
\tilde{k} \arrow[r, dash, "T"] \arrow[d] \arrow[dd, dash, bend right=60, "\tilde{G}"'] & \hat{k} \arrow[ddl, dash, bend left=40, "G"] \\
\tilde{N} \arrow[r, dash] \arrow[d, dash] & N \arrow[u, dash] \\
k &
\end{tikzcd}$$

$\hat{k}$ is the maximal abelian S-ramified p-extension of k, its Galois group G is $\overline{\alpha I}$, where $\alpha : I \to G$ is the Artin map, and $T = \mathrm{Gal}(\hat{k}/\tilde{k})$ is the torsion part of G.

As Log is 0 on $\mathrm{Ker}\, \alpha$, there exists a Log-function on a dense subgroup of G ($\simeq I/\mathrm{Ker}\, \alpha$), and then on G. We have $\mathrm{Ker}\, \mathrm{Log} = T$ and then we obtain a canonical isomorphism:

$$\mathrm{Log} : \tilde{G} \xrightarrow{\simeq} \overline{\mathrm{Log}\, I}$$

($\overline{\mathrm{Log}\, I}$ is numerically known, as soon as classes and units of k are known).

...

The main corollary is that if A is the Artin group of N, then the Artin group of $\tilde{N} = N \cap \tilde{k}$ is $\tilde{A} = \{ \mathfrak{a} \in I, \log \mathfrak{a} \in \overline{\mathrm{Log} A} \}$ (then $\tilde{N} = N \Leftrightarrow \tilde{A} = A$, and this condition can be tested numerically, from the knowledge of A).

The case of kummerian extensions (i.e. $\mu_p \subset k^\times$) is also solved: if N is the field fixed by G^p, then $N = k(\sqrt[p]{R})$, $\tilde{N} = k(\sqrt[p]{\tilde{R}})$ ($\tilde{R} \subset R \subset k^\times / k^{\times p}$), and $\tilde{R}$ is numerically computable from R, via the use of the p-power residue symbol $\left(\frac{a}{\mathfrak{a}}\right)$, for $a \in R$, $\mathfrak{a} \in \tilde{A}$.

Numerical examples where given for the computation of $\tilde{R}$, and also some related questions concerning Tate's theory of K_2 and p-ramification where examined.

Georges Gras
(Besançon)

Galois modules and elliptic functions.

The main theme of the talk was the use of Kummer extensions of group laws to describe rings of integers in local and global situations:

(1) Local case: Let K denote a finite non-ramified extension of $\mathbb{Q}_p$ of degree n. Let F denote a Lubin-Tate formal group for K. We fix $m \geq 1$, we let π (resp ϖ) denote a primitive p^{2m} (resp. p^m) division point on F. We set $N = K(\pi)$, $M = K(\varpi)$. We then described the associated order $\mathfrak{A}$ of the ring of integers $\mathfrak{O}$ of N (wrt the extension N/M), and in particular we showed that $\mathfrak{O}$ is $\mathfrak{A}$-free on $\varpi\pi^{-1}$.

(2) Global case: We then described some corresponding global results for abelian extensions of $\mathbb{Q}$ (resp. quadratic imaginary number field K) by imitating the local construction and using the multiplicative (resp. an elliptic) group law.

Martin Taylor
Trinity College, Cambridge.

On Serre's formula for the trace form.

(Eine fröhliche Einleitung zu orthogonalen Darstellungen von Galoisgruppen). Serre's formula gives an expression of the Hasse-Witt invariant of the trace form, and this is generalised in the context of orthogonal representations of Galois groups. Applications to the embedding problem are given. Throughout the only hypothesis on the underlying field is that its characteristic is not 2.

A. Fröhlich
(Cambridge - London)

The absolute Galois group of a pseudo p-adically closed fields I.

Definition: A field K is said to be pseudo p-adically closed if for every nonempty absolutely irreducible variety V defined over K ~~there~~ has a K-rational point provided $V_{sim}(\bar{K}) \neq \phi$ for every p-adic closure $\bar{K}$ of K.

Definition: A diagram

$$\begin{tikzcd} & G \arrow[d, "\varphi"] \\ B \arrow[r, "\alpha"'] & A \end{tikzcd}$$

of profinite groups with α surjective is said to be a p-adic embedding problem for G if for every closed subgroup H of G which is isomorphic to $G(\mathbb{Q}_p)$ (the absolute Galois group of $\mathbb{Q}_p$) there exists a homomorphism $\gamma: H \to B$ such that $\alpha \circ \gamma = \varphi$ on H.

Definition: A profinite group G is said to be p-adically projective if

a) every finite p-adic embedding problem for G is solvable (i.e., there exists a homomorphism $\gamma: G \to B$ such that $\alpha \circ \gamma = \varphi$); and

b) the collection of all closed subgroups H of G which are isomorphic to $G(\mathbb{Q}_p)$ is closed.

<u>Theorem:</u> The absolute Galois group of every pseudo p-adically closed field K is p-adically projective.

A sketch for the proof of a) was given. ~~As for b)~~ ~~and~~ ~~From~~ Krasner's Lemma implies the the collection of all subgroups $G(\bar{K})$ where $\bar{K}$ is a p-adic closure of K is closed. It turns out that this implies that if $H \leq G(K)$ and $H \cong G(\mathbb{Q}_p)$, then $H = G(\bar{K})$ ~~with R~~ for some p-adic closure $\bar{K}$ of K. This leads us to the following

<u>Problem:</u> Let E be a field of characteristic 0 such that $G(E) \cong G(\mathbb{Q}_p)$. Is E p-adically closed?

The case where E is an algebraic extension of K has been affirmatively answered by Neukirch

Moshe Jarden (Tel-Aviv)

The absolute Galois group of a PpC field II

<u>Theorem:</u> For every p-adically ~~closed~~ projective group G there exists a pseudo p-adically closed field K such that G(K) (= the absolute Galois group of K) = G.

A Sketch of the proof has been given. One has to pass to a more suitable category, which requires the following

<u>Definition:</u> Let $\bar{\Phi} = \varprojlim_m \mathbb{Q}_p^\times/(\mathbb{Q}_p^\times)^m$ and $f: \mathbb{Q}_p^\times \to \bar{\Phi}$ the completion map. A pair (π, φ) consisting of a place $\pi: K \longrightarrow \mathbb{Q}_p \cup \{\infty\}$ and a homomorphism $\varphi: K^\times \to \bar{\Phi}$ is called a <u>$\mathbb{Q}_p,\bar{\Phi}$-site</u> if for all $\alpha \in K$: $\pi(\alpha) \in \mathbb{Q}_p^\times \Rightarrow \varphi(\alpha) = f \circ \pi(\alpha)$.

Let $\tilde{\bar{\Phi}} = \tilde{\mathbb{Q}}_p \times \bar{\Phi}/\mathbb{Q}_p$ and $\tilde{f}: \tilde{\mathbb{Q}}_p^\times \to \tilde{\bar{\Phi}}$ the map induce by f. Then one can similarly define $\tilde{\mathbb{Q}}_p, \tilde{\bar{\Phi}}$-sites.

Let L/K be a Galois extension. We denote $X(L/K) = \{(\pi, \varphi) \mid (\pi, \varphi)$ is a $\tilde{\mathbb{Q}}_p, \tilde{\bar{\Phi}}$-site of L and $\pi(K) \subseteq \mathbb{Q}_p, \varphi(K^\times) \subseteq \bar{\Phi}\}$

This is a Boolean space, and one can define for every $(\pi, \varphi) \in X(L/K)$ a homomorphism $d(\pi, \varphi): G(\mathbb{Q}_p) \to G(L/K)$, (essentially induced by $\pi: L \to \tilde{\mathbb{Q}}_p$). We call

$$\underline{G}(L|K) = \langle \mathcal{G}(L|K),\ X(L|K),\ d: X(L|K) \to \mathrm{Hom}(G(\mathbb{Q}_p), \mathcal{G}(L|K)) \rangle$$

a $G(\mathbb{Q}_p)$-<u>structure</u> <u>associated</u> <u>with</u> $L|K$.

Using a suitable definition of (abstract) $G(\mathbb{Q}_p)$-structures we show that for every projective $G(\mathbb{Q}_p)$-structure $\underline{G}$ there is a pseudo p-adically closed field K with $\underline{G}(K) = \underline{G}$. From this one deduces the Theorem.

Dan Haran

Tel Aviv, z. Z. Erlangen

On the $\mathbb{Z}_p$-torsion of certain Galois modules

Leopoldt's well-known formula gives an expression for the residue of $\zeta_p(s)$ at $s=1$, where $\zeta_p(s) = \zeta_p(s,K)$ is the p-adic zeta function attached to an abelian totally real algebraic number field. In his Durham talk (1975), John Coates gave an analogous formula for the residue at $s=1$ of an Iwasawa function $Z_p(s,K)$ attached to the cyclotomic $\mathbb{Z}_p$-extension of a totally real field K verifying Leopoldt's conjecture. In fact, Coates' proof shows that there should be an intimate connection between the behaviour of the function Z_p (the "main conjecture" states that Z_p and ζ_p are essentially the same) and the structure of the $\mathbb{Z}_p$-torsion module $\mathcal{T}_K$ of the Galois group of the maximal abelian p-ramified p-extension of K. We clarify this by showing that for <u>any</u> algebraic number field K verifying Leopoldt's conjecture, there exists an exact sequence linking $\mathcal{T}_K$ to roots of unity and the dual of a quotient of a certain Iwasawa module. This enables us to generalize Coates' formula by using the p-adic logarithm introduced by G. Gras (see his talk). Other applications are related to K-theory and to the embedding problem into $\mathbb{Z}_p$-extensions

NGUYEN-QUANG-DO

(Paris VII)

p-adische L-Funktionen zu Rankin-Produkten von Modulformen

Der Ansatz von Mazur und Swinnerton-Dyer, gewissen Spitzenformen durch p-adische Interpolation spezieller Werte der zugehörigen Mellin-Transformierten eine p-adische L-Funktion zuzuordnen, läßt sich nach Resultaten von Shimura, Sturm, Amand und Pančiškin auch für das Rankin-Produkt der Mellin-Transformierten mit sich selbst durchführen. Dies liefert insbesondere für Weil-Kurven $E/\mathbb{Q}$ eine analytisch definierte p-adische L-Funktion zum Motif $H^1(E) \otimes H^1(E)$. Andererseits haben wir eine strukturell definierte p-adische L-Funktion zu gewissen Twists von Selmer-Gruppen der Kurve via Iwasawa-Theorie. Die 'Hauptvermutung für symmetrische Quadrate', daß die beiden L-Funktionen im wesentlichen übereinstimmen, läßt sich für Kurven mit komplexer Multiplikation als eine Folgerung der sogenannten 2-Variablen-Haupt-vermutung ~~nachweisen~~ über entsprechend definierte p-adische L-Funktionen zum Motif $H^1(E)$ nachweisen.

Claus Schmidt
(Bures-sur-Yvette)

Neubegründung der Klassenkörpertheorie

Durch die Beobachtung, daß sich jeder Automorphismus einer Körpererweiterung durch eine einfache Manipulation in einen Frobenius-Automorphismus verwandeln läßt, ergibt sich die Möglichkeit, die Klassenkörpertheorie in elementarer, rein gruppentheoretischer Weise abzuhandeln. Die Manipulation sei am Beispiel einer galoisschen Erweiterung $L|K$ lokaler Körper erläutert. Sei $\tilde{L}$ bzw. $\tilde{K}$ die maximale unverzweigte Erweiterung von L bzw. K. Ist $\sigma \in Gal(L|K)$, so wähle man eine Fortsetzung $\tilde{\sigma}$ von σ auf $\tilde{L}$, derart daß $\tilde{\sigma}|_{\tilde{K}} = \varphi_K^n$, $n \in \mathbb{N}$, wobei $\varphi_K \in Gal(\tilde{K}|K)$ der Frobenius-automorphismus ist. Die Überraschung: Ist Σ der Fixkörper von $\tilde{\sigma}$, so ist $\tilde{L}|\Sigma$ die maximale unverzweigte Erweiterung von Σ und $\tilde{\sigma}$ ist der Frobenius-Automorphismus φ_Σ von $\tilde{L}|\Sigma$. Durch diese Feststellung ist man gezwungen die Reziprozitätsabbildung

$$r_{L|K} : G(L|K) \longrightarrow K^*/N_{L|K}L^*$$

in direkter Weise durch

$$r_{L|K}(\sigma) = N_{\Sigma|K}(\pi_\Sigma) \bmod N_{L|K}L^*$$

zu definieren, wobei π_Σ ein Primelement von Σ ist. Nachdem man das „Klassenkörperaxiom" $(K^* : N_{L|K}L^*) = [L:K]$ für zyklische Erweiterungen bewiesen hat läßt sich sofort das Reziprozitätsgesetz verifizieren:

$$G(L|K)^{ab} \cong K^*/N_{L|K}L^*.$$

Jürgen Neukirch

(Regensburg)

Eine Klasse von Polynomen mit gewissen Frobeniusgruppen als Galoisgruppen

(Auszug einer gemeinsamen Arbeit mit N. Yui)

p sei eine Primzahl > 3 und F_{pl} die Frobeniusgruppe vom Grade p und der Ordnung pl, $l \mid p-1$. Es werden Polynome $p$ten Grades in $\mathbb{Q}[X]$ mit F_{pl} als Galoisgruppe konstruiert. Falls $p \equiv 3 \bmod 4$ und $l = \frac{p-1}{2}$ ist, können derartige Polynome mit Hilfe von Tschebyschef-„polynome" explizit angegeben werden

C. U. Jensen (Kopenhagen)

Reducibility of polynomials in several variables

A rational function $\varphi \in K(x_1, \dots, x_n)$ is called reducible over K if the numerator in its reduced form is reducible over K. For $\varphi \in K(x)$ let $\operatorname{ord} \varphi$ be the order of the pole of φ at ∞ and $c(\varphi)$ the constant term in the Laurent expansion of φ at ∞. The following theorem has been presented

<u>Theorem</u> Let $n \geq 3$, $f_i \in K(x_i) \setminus K$ $(i=1,2,\dots,n)$. $\sum_{i=1}^{n} f_i$ is reducible over K if and only if at least one of the following three conditions is satisfied

(i) there exists an additive polynomial $L \in K[t]$ and $g_i \in K(x_i)$ such that

$$f_i - c(f_i) = L(g_i(x)) \quad (i = 1, 2, \dots, n)$$

and $L(t) + \sum_{i=1}^{n} c(f_i)$ is reducible over K.

(ii) char $K = 2$ and there exist $c, d, g_0 \in K$ and $g_i \in K(x_i)$ such that $\sqrt{d} \notin K$

$$f_i - c(f_i) = \frac{c}{g_i^2 + d} \quad (i = 1, 2, \dots, n) \quad \text{and} \quad \sum_{i=1}^{n} c(f_i) = \frac{c}{g_0^2 + d} \text{ or } 0$$

(iii) char $K = 2$, $\operatorname{ord} f_i \leq 0$ $(i = 1, 2, \dots, n)$, the condition (i) is satisfied with K replaced by $\tilde{K}$, a quadratic inseparable extension of K and $L(t) + \sum_{i=1}^{n} c(f_i)$ is not a constant multiple of the square of a polynomial irreducible over $\tilde{K}$

(A polynomial $L \in K[t]$ is additive if $L(x+y) = L(x) + L(y)$).

<u>Corollary</u>. If char $K = 0$, $n \geq 3$, $f_i \in K(x_i) \setminus K$ $(i = 1, \dots, n)$ then $\sum_{i=1}^{n} f_i$ is irreducible over K.

This corollary generalizes an old result of Ehrenfeucht and Pełczyński and answers a recent question of M. Jarden.

Andrzej Schinzel (Warszawa)

Trace Forms of Algebraic Number Fields

To an arbitrary finite, separable field extension F/K we can associate the trace form $q_{F/K}(v) = \mathrm{trace}_{F/K}(v^2)$ $(v \in F)$. This makes F into a quadratic space over K. There are 2 fundamental questions:

[I] given F/K, what does $q_{F/K}$ look like?

[II] given a form f over K, when is f equivalent to $q_{F/K}$ for some finite sep. ext. F?

We can study the ~~following~~ relation of the trace form $q_{F/K}$ up to
a) Witt equivalence in $W(K)$ b) Rational Equivalence
c) Equivariant Integral Equivalence (when F/K is normal)

a). Def. A Witt class $X \in W(K)$ is algebraic if X is represented by a trace form $q_{F/K}$ for some finite sep. ext. F/K.

Examples: In $W(\mathbb{C}) = \{0,1\}$, The class 1 is algebraic, 0 is not.
In $W(\mathbb{Q}_p)$ p odd, There are 12 algebraic classes and 4 non-algebraic classes.

Using a Theorem of Olga-Taussky, PE Conner + The lecturer have proved
Th$^{\underline{m}}$ 1: $X \in W(\mathbb{Q}) \Leftrightarrow \mathrm{sgn}(X) \geq 0$.

One step in the proof is:

Th$^{\underline{m}}$ 2: If k = alg. nr. field, and if $\alpha \in k^*$, Then The 1-dimensional form αx^2 is Witt equivalent to a trace form $q_{F/k}$ if and only if α is totally positive in k.

If " k is totally complex, Then every $\alpha \in k^*$ is totally positive, so every 1-dimensional form αx^2 is algebraic in our sense. Using This one proves more:

<u>Th^m 3</u>: If k is a totally complex alg. number field, then every $X \in W(k)$ is algebraic.

Theorems 1 and 3 strongly suggest:

<u>Conjecture</u>: If K = alg. nr. field, Then $X \in W(k)$ is algebraic if and only if for of every real embedding $\sigma : K \to \mathbb{R}$ we have

$$\mathrm{sgn}_\sigma (X) \geq 0.$$

b). There are several Theorems concerning the classification of trace forms $q_{F/Q}$ (not: ground field = Q). Only 1 is mentioned here:

If F/Q has degree n, and if the normal closure of F/Q has <u>odd</u> degree over Q, Then

$$q_{F/Q} \underset{Q}{\sim} x_1^2 + \cdots + x_n^2.$$

It follows That all <u>normal</u> cubics have "The same" (i.e. Q-equivalent) trace forms.

Rational Invariants of trace forms:

Disc = field discriminant d_F (mod Q^{*2})

Rank = $[F:Q]$

Signature = number of real embeddings $r_1(F)$ (by a Th. of Olga Taussky)

Hasse symbols = $h_p(q_{F/Q})$

$p = 2, 3, 5, \ldots, \infty$.

The Hasse symbols are given by <u>diagonalizing</u>

$$q_{F/Q} \sim a_1 x_1^2 + \cdots + a_n x_n^2,$$

+ forming the product

$$h_p(q_{F/Q}) \overset{\text{def}}{=} \prod_{i<j} (a_i, a_j)_p,$$

where $(a_i, a_j)_p$ = Hilbert symbol. Hasse proved that this def. is independent of the way $q_{F/\mathbb{Q}}$ is diagonalized.

Serre's formula: $h_p(q_{F/\mathbb{Q}}) = w_2(\rho_p)\cdot(2, d_F)_p$

where $w_2(\rho_p)$ = "p-part" of the second Stiefel-Whitney class $w_2(\rho) \in H^2(G_{\mathbb{Q}}, \pm 1)$ of the representation

$$\rho : G_{\mathbb{Q}} \longrightarrow O_n(\bar{\mathbb{Q}})$$

given by the action of $G_{\mathbb{Q}}$ on the n cosets of G_F.

$$\begin{array}{l} \bar{\mathbb{Q}} \\ \big| \\ F \\ \big|\, n \\ \mathbb{Q} \end{array}$$

Using results of Deligne + Tate's Durham Conference lecture (1975), (which removes Deligne's assumption that his representations are virtual of degree 0) we can say

Thm 4: $$h_p(q_{F/\mathbb{Q}}) = \frac{W(\rho_p)}{W(\det \rho_p)}\cdot(2, d_F)_p$$

where $W(\rho_p)$ = Artin root number of the orthogonal rep. ρ described above, restricted to $G_{\mathbb{Q}_p}$,

and $W(\det\rho_p)$ = ~~Artin~~ root number of

$$G_{\mathbb{Q}} \xrightarrow{\rho} O_n(\bar{\mathbb{Q}}) \xrightarrow{\det} O_1(\bar{\mathbb{Q}})$$

again restricted to $G_{\mathbb{Q}_p}$.

c). Equivariant Integral Equivalence.
Assume $F/\mathbb{Q}$ is normal, with Galois group $G = G(F/\mathbb{Q})$. Let $\mathcal{O}_F$ = ring of integers.

Then G acts as a group of isometries of $(\mathcal{O}_F, q_{F/\mathbb{Q}})$ and we can ask to

classify the $\mathbb{Z}[G]$ module $\mathcal{O}_F$ up to equivariant integral isometry.

Th^m 5: Let $E/\mathbb{Q}$ and $F/\mathbb{Q}$ be cyclic extensions of the same prime degree p. Identify $G(E/\mathbb{Q}) \cong G(F/\mathbb{Q}) = \mathbb{Z}/p\mathbb{Z}$.

Then the $\mathbb{Z}[\mathbb{Z}/p]$ - modules $\mathcal{O}_E$ and $\mathcal{O}_F$ are isometric (i.e., equivariantly isometric) with respect to the trace forms $q_{E/\mathbb{Q}}$, $q_{F/\mathbb{Q}}$ if and only if the discriminants agree

$$d_E = d_F \quad (\text{in } \mathbb{Z}).$$

The proof uses deep results of Leopoldt on the Galois-module structure of the rings $\mathcal{O}_E$ and $\mathcal{O}_F$.

Robert Perlis (Baton Rouge)

On certain quantitative diophantine results in algebraic number theory

Let R be a finitely generated integral domain over $\mathbb{Z}$ with quotient field K, let R^* be the unit group of R, let G be a finite extension field of K, and let $0 \neq \beta \in R$, $\gamma \in R$. Suppose that R is integrally closed in K. The polynomials $f, f^* \in R[X]$ will be called R-equivalent if $f^*(X) = f(X+a)$ with some $a \in R$. In this case they have the same discriminant.

THEOREM. There are only finitely many pairwise R-inequivalent monic polynomials $f \in R[X]$ with roots in G and discriminant β.

Let L be a finite extension field of K, and S an integral extension ring of R in L having L as its quotient field. The elements $\alpha, \alpha^* \in S$ will be called R-equivalent if $\alpha^* - \alpha \in R$ and weakly R-equivalent if $\alpha^* - u\alpha \in R$ with some $u \in R^*$. If α, α^* are R-equivalent then for their discriminants $D_{L/K}(\alpha^*) = D_{L/K}(\alpha)$ holds.

COROLLARY 1. There are only finitely many pairwise R-inequivalent $\alpha \in S$ with $D_{L/K}(\alpha) = \beta$.

COROLLARY 2. There are only finitely many $\alpha \in S$ with $D_{L/K}(\alpha) = \beta$ and $N_{L/K}(\alpha) = \gamma$.

COROLLARY 3. There are only finitely many $\alpha \in S^*$ with $D_{L/K}(\alpha) = \beta$.

If $S = R[\alpha]$ with some $\alpha \in S$ then $S = R[\alpha^*]$ for every $\alpha^* \in S$ which is weakly R-equivalent to α.

COROLLARY 4. There are only finitely many pairwise weakly R-inequivalent $\alpha \in S$ with $S = R[\alpha]$.

The condition that R is integrally closed can be weakened. In the corollaries 1 to 3 it is enough for example to assume that the index $[(S \cap K)^+ : R^+]$ is finite.

The above results were earlier proved by me in 1982 in ineffective forms, and in 1984 in effective forms. Recently, we derived jointly with J. H. EVERTSE good explicit upper bounds for the numbers of elements under consideration which depend only on a few parameters. In my talk these bounds were presented. In the special case when $R = \mathbb{Z}$, $K = \mathbb{Q}$ and L is an algebraic number field of degree $n \geq 2$ with ring of integers S, our quantitative version of Corollary 4 gives e.g. the upper bound $7^{4(n+1)!}$ for the $\mathbb{Z}$-equivalence classes of $\alpha \in S$ for which $S = \mathbb{Z}[\alpha]$.

Kálmán Győry (Debrecen, Hungary)
(K. Győry)

On the polynomials $X^4 + uX^2 + 1$

See Vortragsbuch No 61, page 136

A. Prestel (Konstanz)

On an irreducibility theorem of Ostrowski

Theorem: If $f \in \mathbb{Z}[X_1, \dots, X_n]$ is absolutely irreducible, then $f \bmod p$ is absolutely irreducible for all $p \geq p_0$. This theorem was proved by Ostrowski 1919, then by E. Noether 1922, then by many other people (including Kneser - Roquette). Analyzing Noether's proof, W. Schmidt gave 1976 („Equations over Finite Fields") a first explicit bound for p_0, which was horribly large. W. Ruppert (in a thesis in Erlangen under construction) has obtained the following results which were given (partly) with proof: (1) p_0 does not depend on n (the crucial case is $n=2$), but on $d = \deg f$ and H = height of f. (2) $p_0 = C_1(d) \cdot H^{d^2-1}$ is a valid bound with explicit $C_1(d)$. (3) $p_0 = C_2(d) \cdot H^{3(d-1)}$ is a valid bound with explicit $C_2(d)$, at least for all f, where $f = 0$ is a smooth projective curve. (4) There are examples (modulo the conjecture of Bunjakowski from ≈ 1850 about prime values of irreducible polynomials) showing that the exponent in (3) is best possible.

W.D. Geyer (Erlangen)

Units in $\mathbb{Z}[D_{2m}]$

The structure of the unit group $U = \mathbb{Z}[D_{2m}]^{\times}$ (D_{2m} = m-th dihedral group) is studied by decomposing $\mathbb{Q}[D_{2m}] \cong A_1 \times \ldots \times A_q$ into a direct product of simple algebras A_i, looking for the image of $\mathbb{Z}[D_{2m}]$ in the product and interpreting the occurring „coupling relations" between the components group-theoretically. This is carried out for m = odd squarefree and $m = p^n$ = odd prime power.

Ernst Kleinert (Köln)

Zerlegungsgesetze und binäre quadratische Formen

Sei K ein biquadratisch-bizyklischer Zahlkörper, in dem 2 nur einen Primteiler hat, $t \geq 0$, K_t der Strahlklassenkörper mod 2^t über K und K_t' die maximale 2-Erweiterung innerhalb K_t. Es sind drei Fälle zu unterscheiden: I) $K = \mathbb{Q}(\sqrt{d}, \sqrt{q})$, II) $K = \mathbb{Q}(\sqrt{\Delta}, \sqrt{q})$, III) $K = \mathbb{Q}(\sqrt{\Delta}, \sqrt{d})$ mit $\Delta, d, q \in \mathbb{Z} \setminus \{0,1\}$ quadratfrei, $\Delta \equiv 2 \bmod 4$, $d \equiv 3 \bmod 4$, $q \equiv 5 \bmod 8$. Dann gelten folgende Zerlegungsgesetze:

I) Sei $t \geq 2$ und enthalte K eine Einheit $\eta \equiv \sqrt{d} \bmod 2$. Dann gilt für eine Primzahl p: Genau dann ist p vollzerlegt in K_t, wenn $p = x^2 - 4^t d y^2 = u^2 - 4^t q^* v^2 = a^2 - 4^t q b^2$ ($dq = m^2 q^*$, q^* quadratfrei) mit $x \equiv 1 \bmod 2^t$, $a \equiv 1 + 2^t b \bmod 2^{t+1}$, $y \equiv v \bmod 2$. Enthält K spezielle Einheiten, so kann auf eine der drei quadratischen Formen verzichtet werden.

II) Sei $t = 1$. Genau dann ist p vollzerlegt in K_1, wenn $p = x^2 - 4\Delta y^2 = u^2 - 4\Delta^* v^2$ ($\Delta q = \Delta^* m^2$, Δ^* quadratfrei) und $y \equiv v \bmod 2$.

III) Sei $\Delta > 0$, $N\varepsilon_\Delta = -1$ und zerfalle p voll in K_1. Genau dann ist p vollzerlegt in K_t, wenn $p = x^2 - 4^t d y^2 =$

$= u^2 - 4^t \Delta^* v^2$ ($\Delta d = u^2 \Delta^*$, Δ^* quadratfrei) und $x \equiv 1 \bmod 2^{t+1}$!
Durch Anwendung dieser Kriterien in Spezialfällen erhält man Potenzrestkriterien, welche insbesondere die Vermutungen von E. Lehmer (Crelle 268/269) und Leonard & Williams (Rocky Mountain J. of Math. 9), aber auch viele andere in der Literatur bewiesenen Kriterien umfassen.

F. Halter-Koch (Graz)

Torsion subgroups of elliptic curves

~~Theorem~~ Let K be a number field such that there exists finitely many groups $G_1, \dots, G_r$ with the property that for any elliptic E defined over K, $E(K)_{tor} \cong G_i$ $(1 \le i \le r)$.

Theorem Let $L = K(\sqrt{d})$ be a quadratic extension of K and E as above. Then up to a factor $\mathbb{Z}/2\mathbb{Z}$,

$$E(L)_{tor} \cong \text{a subgroup of } G_i \times G_j,$$

$1 \le i, j \le r$.

Jasbir S. Chahal (Provo, UTAH)

Units in Finite Abelian Group Rings ~~& Galois Module Theory~~

Suppose Γ is a finite, abelian group. Let $\| \ \|$ be a Euclidean norm on $\mathbb{C}\Gamma$. For each $x \in \mathbb{C}\Gamma$ let x' denote central projection $x' = x\|x\|^{-1}$. The idea is to study the flow of the x' as x runs over $\mathbb{Z}\Gamma^\times$. Let U be any open subset of $C_1 = \{x \in \mathbb{C}\Gamma \mid \|x\| = 1\}$. Define

$$\Lambda_U(s) = \sum_{x \in \mathbb{Z}\Gamma^\times} f_U(x')(\log\|x\|)^{-s}$$

where f_U is the characteristic function of U. The surprise is that the convergence and behaviour of $\Lambda_U(s)$ do not depend upon the volume of U, rather, upon how many points from a certain finite set are contained in U.

One can use Delange-Ikehara to obtain counting function theorems.

There is another application to Galois-module theory. Using $\Lambda_U(s)$ and some Diophantine Approximation theory one can describe the divisibility of normal integral bases in tame, abelian extensions.

G. Everest (U.E.A, Norwich)

Division algebras and equivalence of number fields

Let K be a field, G a finite group. G is called K-admissible iff there exists a division algebra finite dimensional and central over K which is a crossed product for G.

Conjecture: $\forall G$, G $\mathbb{Q}$-admissible $\Leftrightarrow$ every Sylow subgroup of G is metacyclic. ($\Rightarrow$ is true)

Theorem. The conjecture is true for all solvable G.

Theorem. Let K, L be number fields (finite over $\mathbb{Q}$). Suppose for all G, G K-admissible $\Longleftrightarrow$ G L-admissible. Then K and L have the same normal closure over $\mathbb{Q}$.

Jack Sonn (Haifa)

Topologie

12. – 18. August 1984

Equations over groups and singular surfaces in 3-manifolds

A long-standing problem in group theory asks whether a system of n equations in n unknowns over a group G, whose exponent-sum matrix is nonsingular, has a solution in some overgroup of G. The conjectured answer is yes, and some partial results are known, but the general problem remains unsolved.

In a paper published last summer (Math. Z. 184 (1983) pp 1-17), J. Stallings related this problem to a problem about immersed surfaces in 3-manifolds, and solved the corresponding problem for embedded surfaces. Explicitly, let $M^3 \hookrightarrow N^3$ be a tame embedding of orientable 3-manifolds, and let S^2 be a compact orientable surface immersed in N^3, such that $\partial S \subset M^3$. Assume also $H_2(N,M)=0$, then ∂S also bounds a compact orientable surface T^2 immersed in M^3.

Conjecture T^2 can be chosen with genus $T^2 \leq$ genus S^2.

Proposition This conjecture implies the above group theoretic conjecture.

Theorem If S^2 is embedded, then T^2 can be chosen to be embedded, with genus $T^2 \leq$ genus S^2.

J. Howie (Glasgow)

Contact forms on (n-1)-connected (2n+1)-manifolds

An odd dimensional manifold M^{2n+1} admits a contact form if there exists a 1-form ω such that $\omega \wedge (d\omega)^n \neq 0$ at all points of M. Contact forms are an odd dimensional analogue to the more familiar symplectic forms (closed 2-forms Ω of maximal rank), and the condition for an almost contact structure is that the structural group of the tangent bundle of the (oriented) manifold M^{2n+1} admit reductions from $SO(2n+1)$ to $U(n) \oplus (1)$. Classically contact forms arise in the study of Hamiltonian systems, and it is of considerable interest to know the conditions under which an almost structure is integrable. For closed manifolds we have the following results.

Theorem 1 Every orientable M^3 is contact.

A quick proof of this uses an "open book" decomposition of M^3. With more care one can show that there is a (1-1) correspondence between homotopy classes of almost structures & suitably defined equivalence classes of contact forms (R. Lutz / J. Martinet).

Theorem 2 If M^5 is 1-connected, $w_2(M) = 0$ and $H_2(M, \mathbb{Z})$ contains no element of order 3, then M^5 is contact.

The proof uses the decomposition of M^5 into the connected sum of prime manifolds (Smale) of the form $S^2 \times S^3$ (classically contact) or M_{p^t} ($H_2 \cong \mathbb{Z}/p^t \times \mathbb{Z}/p^t$). The latter class can be described by the complex polynomial $z_0^{p^t} + z_1^3 + z_2^3 + z_3^3$, and the real form $\frac{i}{2}\left(\frac{1}{p^t}(z_0 d\bar{z}_0 - \bar{z}_0 dz_0) + \frac{1}{3}(z_1 d\bar{z}_1 - \bar{z}_1 dz_1) + \dots\right)$ restricts to a contact form. Finally we use a theorem of C. Meckert on the compatibility of contact structure with connected sums to complete the proof.

Theorem 2 is a special case of a more general theorem for highly connected manifolds, which uses the classification by C.T.C. Wall (Topology 1967), and which suggests that, at least under connectivity assumptions, some almost contact structure is always integrable.

Problems: study symplectic structures on 1-connected 6-manifolds.

C. B. Thomas (Cambridge).

Teichmuller Spaces of Punctured Surfaces

Let F_g^s = genus g surface with s punctures, s, $2g-2+s > 0$, and let $\mathcal{T}_g^s$ = Teichmuller space of F_g^s. We choose a puncture P of F_g^s once and for all and define a <u>geodesic</u> in F_g^s to be a bi-∞ geodesic (in the usual sense) tending to P in both directions. If h is a horosphere about P and c is a geodesic, we define $d_h c$ = hyperbolic length along c from h to h.

We uniformize by taking $\Gamma \subset SO(2,1)$ so that $H^+/\Gamma \cong F_g^s$, where H^+ = upper sheet of the unit hyperboloid in Minkowskii space $\mathbb{M}$, and choose, once and for all, a point $x_0 \in$ light cone corresponding to P. If c is a geodesic and $\gamma(c) \in \Gamma$ the corresponding covering translation, we define the λ-<u>length</u> of c to be $\lambda(c) = \sqrt{-\langle x_0, \gamma(c)x_0\rangle}$, where $\langle \cdot,\cdot\rangle$ is the inner product in $\mathbb{M}$.

<u>Lemma 1</u>: If c_1 and c_2 are geodesics, then

$$\lim_{h\to P} e^{d_h c_1 - d_h c_2} = \left[\frac{\lambda(c_1)}{\lambda(c_2)}\right]^2$$

A <u>triangulation</u> Δ of F_g^s is a disjointly embedded collection of simple geodesics so that components of $F_g^s - \Delta$ are either tri-gons or punctured mono-gons. A <u>cell decomposition</u> Δ' of F_g^s is a subset of a triangulation so that components of $F_g^s - \Delta'$ are either simply connected or boundary-parallel.

<u>Theorem 2</u>: If Δ is a triangulation of F_g^s, then λ-lengths of geodesics in Δ give $\mathbb{R}$-analytic projective coords on $\mathcal{T}_g^s$.

Let MC_g^s = mapping class group of F_g^s fixing P.

<u>Theorem 3</u>: MC_g^s acts on λ-length coords for $\mathcal{T}_g^s$ by rat'l map (homogeneous of degree 1).

<u>Cor 4</u> (to Lemma 1): Γx_0 doesn't accumulate in the light cone.

Let $H(\Gamma)$ = Euclidean convex hull of Γx_0 and let $\Delta(\Gamma)$ be the collection of geodesics corresponding to extremal edges of $H(\Gamma)$.

<u>Theorem 5</u>: $\Delta(\Gamma)$ is a cell decomposition of F_g^s.

Fix a cell decomposition Δ of F_g^s, and let $C(\Delta) = \{\Gamma \in \mathcal{T}_g^s : \Delta(\Gamma) = \Delta\}$. The overall goal is to prove that $\mathcal{C} = \{C(\Delta) : \Delta$ is a cell decomposition of $F_g^s\}$ is a MC_g^s-invariant cell decomposition of $\mathcal{T}_g^s$. The difficult part is showing that each $C(\Delta)$ is contractible, and the theorem at present is

Theorem 6: $\mathcal{C}$ forms a MC_0^s-invariant cell decomp. of $\mathcal{T}_0^s$ so that $C(\Delta')$ is a face of $C(\Delta)$ iff $\Delta' \subset \Delta$.

Remarks 1) The restriction $g=0$ is believed to be unnecessary in Thm 6.

2) The techniques above in fact suggest a cell decomp of a certain compactification of $\mathcal{T}_g^s$. This has yet to be made precise

3) Each cell of $\mathcal{C}$ has a natural cpx structure, and it is hoped that these cpx structures are coherent and consistent with the cpx structure of $\mathcal{T}_g^s$.

R.C. Penner (Princeton)

Perturbation theory and small models for the chains of certain spaces

Given a fibre square
$$\begin{array}{ccc} E_f & \to & E \\ \downarrow & & \downarrow \pi \\ X & \xrightarrow{f} & B \end{array}$$
, under suitable strong hypotheses which, in particular, imply that H_*X, H_*B, H_*E are coalgebras, a model for the singular chains of E_f can be constructed which only involves H_*X, H_*B, H_*E, $H_*\Omega B$, H_*f, $H_*\pi$ and, as an additional ingredient, a "twisting cochain" $\tau : H_*B \to H_*\Omega B$ which is essentially the transgression in the homology Serre spectral sequence of the path fibration of B. Away from the prime 2, the obvious diagonal map on this model yields a correct diagonal map in the sense that in homology and cohomology the correct map is induced. An application of this is the determination of the cohomology rings of almost all homogeneous spaces of compact Lie groups away from the prime 2. Even without the

invertibility hypothesis of the prime 2, a complete description of the cohomology ring of $GL_n(\mathbb{F}_q)$ can be given away from the characteristic p, where $q = p^s$ (say); the description is in terms of a model of the kind mentioned before, with a diagonal which arises from the obvious one by a suitable "perturbation".

The proofs of these results rely on a perturbation theory which cuts the chain model coming from the Eilenberg-Moore theorem to seize.

Johannes Huebschmann (Heidelberg)

Ends of hyperbolic 3-manifolds.

We consider hyperbolic 3-manifolds with finitely generated fundamental group, and more precisely want to study the geometric behaviour of their ends. The simplest of these manifolds are the so-called geometrically finite ones, which are quotient of the hyperbolic 3-space $\mathbb{H}^3$ by a discrete group of isometries admitting a finite polyhedron as fundamental domain. To study limits of these (geometrically finite) manifolds, Thurston introduced the notion of "geometrically tame hyperbolic 3-manifold", proved that such manifolds enjoy many interesting properties, and conjectured that every hyperbolic 3-manifold with finitely generated fundamental group is geometrically tame. We prove this conjecture under the hypothesis that the fundamental group does not split as a free product. As a corollary, this proves the so-called "Ahlfors conjecture" on measures of limit sets for indecomposable Kleinian groups, and provides a different approach to the proof of Thurston's hyperbolisation theorem.

Francis BONAHON (Orsay).

The total curvature of a knotted torus in $\mathbb{R}^3$.

The total (absolute) curvature of a closed embedded submanifold $M \subset \mathbb{R}^N$ can be defined in terms of linear functions $z : (\mathbb{R}^N, 0) \longrightarrow (\mathbb{R}, 0)$, $\|z\| = 1$, by

$$\tau = \mathcal{E}_z \, \mu_z$$

Here μ_z is the number of critical points of the, in general nondegenerate, function $z|M$ and τ is the expectation value $\mathcal{E}$ (mean) with respect to the invariant measure on the unit sphere $\{z : \|z\| = 1\}$.

For a closed curve γ one has $\tau(\gamma) = \int |\rho \, d\sigma| / \pi$, and for a closed surface M_g of genus g $\tau(M_g) = \int |K d\sigma| / 2\pi$ in usual notations.

An embedded torus $T \subset \mathbb{R}^3$ divides $S^3 = \mathbb{R}^3 \cup \infty$ into two parts one of which by Alexander is standard. Let γ be a core curve, $\infty \notin \gamma$, for this standard solid torus part.

<u>Theorem</u> If T is knotted, then the infimum of $\tau(T')$ for T' isotopic to T is $4B(\gamma)$, where $B(\gamma)$ is the bridgenumber of γ (a knot). Moreover the infimum is never attained

$$\tau(T) = \int |K d\sigma| / 2\pi > 4 \; B(\gamma).$$

This is analogous to a theorem of Fenchel-Fary-Fox-Milnor concerning closed curves

$$\tau(\gamma) \geq 2B(\gamma)$$

and $\quad \tau(\gamma) > 2B(\gamma)$ for γ knotted.

For surfaces of genus $g \geq 3$ minima for knotted embeddings do occur.

This lecture concerns joint work with W.H. Meeks III

Nicolaas H. Kuiper.
(Bures sur Yvette
France)

Concordance of G-actions on spheres.

Let G be a finite group of order q, and let $\varphi_1, \varphi_0 : G\times M \to M$ be two smooth actions. φ_1 and φ_2 are called concordant if there exists $\Phi : G\times M\times[0,1] \to M\times[0,1]$ such that $\Phi(M\times\{i\}) = \varphi_i$, $i=0,1$. Concordance is an equivalence relation and is defined for various classes of G-manifolds. Concordance classes of semifree G-actions on S^n with tangential representation ρ at some $x\in(S^n)^G$ form an abelian group $C_n(\rho)$ under equivariant connected sum. If $\varphi: G\times S^n \to S^n$ is semifree such that $(S^n, (S^n)^G)$ is diffeomorphic to (S^n, S^k), then φ is called <u>almost linear</u>. The almost linear concordance classes of such actions form an abelian group $C_n^{AL}(\rho)$ and we have a natural homomorphism $C_n^{AL}(\rho) \xrightarrow{i} C_n(\rho)$. Define <u>the Swan invariant</u> of X, $H_*(X;\mathbb{Z}/q) \cong H_*(S^k;\mathbb{Z}/q)$, $\dim X < \infty$ via $\theta(X) = \sum_{i\neq 0,k} (-1)^i \sigma_G(|H_i(X)|)$.

where σ_G is the Swan homorphism, $\theta(X)\in \tilde{K}_0(\mathbb{Z}G)$. Mod q h-cobordism classes of mod q homology spheres of dimension k with vanishing Swan invariant and ρ-structure on normal bundle in S^n form an abelian group $\Theta_k(0)$. <u>Theorem</u>. Let $\dim\rho - \dim\rho^G > 2$. Then there exists a homomorphism $\Delta : \Theta_k(0) \to C_{n-1}^{AL}(\rho)$ such that the following sequence is exact: $\mathcal{F}$ = taking fixed set

$$\cdots\cdots \to C_n^{AL}(\rho) \xrightarrow{i} C_n(\rho) \xrightarrow{\mathcal{F}} \Theta_k(0) \xrightarrow{\Delta} C_{n-1}^{AL}(\rho) \to \cdots\cdots$$

<u>Corollary</u>: If two smooth actions φ_1, φ_2 on S^n have diffeomorphic fixed point sets (respecting ρ-structure on normal bundle), then they are G-homeomorphic. In fact, there is an almost linear action σ such that $\varphi_1 \# \sigma$ is G-diffeomorphic to φ_2.

<u>Corollary</u>. smooth $\varphi : G\times S^n \to S^n$ is G-homeomorphic to $\partial\psi : G\times S^n \to S^n$ where $\psi : G\times D^{n+1} \to D^{n+1}$ (smooth also) if and only if $\mathrm{Fix}(\varphi)$ bounds a mod q homology disk with zero Swan invariant (extending ρ-structure as well.)

Amir H. Assadi (Charlottesville)

Strong shape and Steenrod-Sitnikov homology

The following definition of a strong shape category SSh is the result of joint work with Ju. T. Lisica. We first define a coherent prohomotopy category of spaces CPHTop. Its objects are inverse systems of spaces $\underline{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$. A coherent map of systems $\underline{X} \to \underline{Y} = (Y_\mu, q_{\mu\mu'}, M)$ is given by an increasing function $\varphi : M \to \Lambda$ and by maps $f_{\mu_0 \dots \mu_n} : \Delta^n \times X_{\varphi(\mu_n)} \to Y_{\mu_0}$, $\mu_0 \le \dots \le \mu_n$, which satisfy certain compatibility conditions with respect to the boundary and the degeneracy operators ∂^n_j, σ^n_j. Morphisms of CPHTop are classes of coherently homotopic coherent maps. A morphism $p : X \to \underline{X}$ of pro-Top of a space X into an inverse system of ANR's $\underline{X}$ is called a resolution of X provided any map $f : X \to P \in ANR$ factors approximately through $\underline{X}$ and if two approximation are sufficiently good, the factorizations are arbitrarily close. A morphism $X \to Y$ of SSh is given by ANR-resolutions $p : X \to \underline{X}$, $q : Y \to \underline{Y}$ and by a morphism of CPHTop $\underline{X} \to \underline{Y}$. The Steenrod homology H^s of a space X is defined as the homology $H(\underline{X})$ of any ANR-resolution of X, where $H(\underline{X})$ is the homology of a chain complex $C(\underline{X})$ associated with $\underline{X}$. Its p-chains x consist of $(p+n)$-singular chains $x_{\lambda_0 \dots \lambda_n}$ of X_{λ_0} and the boundary operator d is given by the formula

$$(-1)^n (dx)_{\lambda_0 \dots \lambda_n} = \partial(x_{\lambda_0 \dots \lambda_n}) - p_{\lambda_0\lambda_1} x_{\lambda_1 \dots \lambda_n} - \sum_{j=1}^{n} (-1)^j x_{\lambda_0 \dots \hat{\lambda}_j \dots \lambda_n}.$$

H^s is a functor on SSh. Extending the theory to pairs, one obtains a homology theory for paracompact spaces and their closed subsets, which satisfies all the

Eilenberg - Steenrod axioms and on metric compacta coincides with the classical Steenrod - Sitnikov homology.

Sibe Mardešić (Zagreb, Yugoslavia)

Link homotopy of 2-spheres in 4-space.

Let $\Sigma^2 \cup \Gamma^2$ be two 2-spheres in $\mathbb{R}^4$ (possibly singular). We ask whether it is possible to undo this link by a link homotopy. That is a homotopy in which components are allowed to pass through themselves but in which different components remain disjoint. We show (joint work with D. Rolfsen) that if Σ^2 is a spun knot then this is possible for any Γ^2. More technical conditions on Σ^2 allow more complicated results to be obtained. The question is asked whether these link can always be undone if Σ^2 is non singular. As a counterpoint an example is given where Σ^2 and Γ^2 have an isolated double point each and which cannot be undone by a link homotopy. The invariant used to prove this can be defined as follows: let d be a double point in Σ^2 and let z be a closed path starting and finishing at d on different branches. Then $\rho \equiv \sum_d lk(z, \Gamma^2) \bmod 2$ is the required invariant. There is another invariant due to Haefliger which is defined as follows: let $f_1, f_2 : S^2 \to R^4$ define Σ^2, Γ^2 respectively and let $\varphi : S^2 \times S^2 \to S^3$ be defined by $\varphi(x,y) = f_1(x) - f_2(y) / \| f_1(x) - f_2(y) \|$. Then the homotopy class $[\varphi] \in \mathbb{Z}_2$. We ask the question is $[\varphi] = \rho$?

Roger Fenn (Sussex)

Dehn surgery and 3-manifolds with cyclic fundamental group

Let M be a compact, irreducible, orientable 3-manifold which is not Seifert-fibered and whose boundary is a single torus. A simple closed curve μ in ∂M is called a weak meridian (of order $n>0$) if the group $|\pi_1(M):[\mu]|$ is finite cyclic (of order n). The following result is joint work with Marc Culler and Cameron Gordon.

Theorem. If μ and μ' are weak meridians then the algebraic intersection number $|\mu\cdot\mu'|$ is ≤ 5. If μ, μ' are ~~both~~ of order $\neq 2$ then $|\mu\cdot\mu'| \leq 4$.

As a consequence one sees that there are at most 5 classical knots with a given complement. It is conjectured that the bound in this theorem can be reduced to 1. Examples due to Fintushel-Stern and Przytycki show that this would be best possible.

Using Thurston's uniformization theorem, the proof reduces to the case where $\mathring{M}$ is hyperbolic. In this case one finds a curve C in the complex affine variety of characters of representations of $\pi_1(M)$ in $SL_2(\mathbb{C})$ and seeks to rule out most curves $\mu \subset \partial M$ as weak meridians by finding representations of $\pi_1(M)$, with characters in C, which have non-abelian images and map μ to $\pm I$. This involves understanding the degrees of certain evaluation functions on C. This approach breaks down when these functions take certain distinguished values at ideal points in the projective completion of C rather than in C itself. However, these ideal points then define actions of $\pi_1(M)$ on trees, which determine systems of surfaces in M; the geometric information so obtained is enough to complete the proof of the theorem.

Peter B. Shalen (Rice)

On the torus rank of certain spaces
(joint work with C. Allday)

Using a "cochain complex" version of the localization theorem for singular equivariant cohomology and Sullivan's theory of minimal models it is shown that the torus rank (i.e. the maximal dimension of those tori that act almost freely on a given space) of a "reasonable" space X with $\pi_{even}(X) \otimes \mathbb{Q} = 0$ is bounded from above by the dimension of the centre of the rational homotopy Lie algebra $\pi_*(X) \otimes \mathbb{Q}$. This generalizes a result of S. Halperin's (namely the case when the centre is 0).
The $\mathbb{Z}/p\mathbb{Z}$-version of the localization theorem can be applied to give a simple and unified proof of results of G. Carlsson and W. Browder concerning (free) p-torus actions on product of spheres.

V. Puppe (Konstanz)

Local Surgery and Space Forms
(joint work with I. Madsen)

Let $1 \to \mathbb{Z}/m \to \pi \to \sigma \to 1$ be a 2-hyperelementary group with m odd and $\sigma \cong \mathbb{Z}/2^k$ with twisting $t: \sigma \to (\mathbb{Z}/m)^\times$. This is a group with periodic integral ~~th~~ cohomology of period 2^{l+1} where $\mathrm{Im}\, t \cong \mathbb{Z}/2^l$. If $\ker t \neq 1$ then π has a free linear representation V of complex dimension 2^l obtained by inducing to π a faithful character $\chi: \mathbb{Z}/m.2^{k-l} \to \mathbb{C}$. Let $N = S(V)/\pi$ be the corresponding linear space form and $g(N) \in H^{2^{l+1}}(\pi;\mathbb{Z}) \cong \mathbb{Z}/|\pi|$ its k-invariant (Chern class). Any other space form in this dimension has homotopy type described by a k-invariant $r.g(N)$ where $(r,|\pi|) = 1$. Our problem is

(i) for which r does there exist a free (topological) action of π on S^{q-1} with k-invariant $r \cdot g(N)$ for S^{q-1}/π? Here $q \equiv 0 \pmod{2^{l+1}}$.

(ii) for which r does there exist a semi free (topological) action of π on $\mathbb{R}^q$ fixing only 0?

By fixing orientation, assume that $r \equiv 1 \pmod 4$.

Thm 1 Let $q = 2^{l+1} \cdot s \geq 5$ and $r \equiv 1 \pmod 4$. Assume $|\ker t| = 2$ and $-1 \in \operatorname{Im} t$. Then π acts freely on S^{q-1} with k-invariant $r.(g(N))^s$ if and only if $r \in (\mathbb{Z}/m)^{\times 2^l}$.

(By contrast the linear homotopy types have $r \in (\mathbb{Z}/|\pi|)^{\times 2^l}$ so many non-linear types actually occur).

Thm 2 Let $q = 2^{l+1} \cdot s \geq 4$ and $\ker t \neq 1$, $r \equiv 1 \pmod 4$. Then π acts semi-freely on $\mathbb{R}^q$ fixing only 0 if and only if $r \in (\mathbb{Z}/d)^{\times 2}$ for all divisors $d \mid m$ such that the subgroup $\mathbb{Z}/d \rtimes \sigma \subseteq \pi$ has $-1 \in \operatorname{Im} t$.

Corollary 3 Let $k=2$, $l=1$ so $\pi = Q(4m)$ is a quaternionic group. Then π acts semi-freely on $(\mathbb{R}^{4s}, 0) \Leftrightarrow$ the homotopy type is linear.

(This result is a necessary step in the existence problem (i) for type II groups $1 \to \mathbb{Z}/ab \to Q(8a,b) \to Q8 \to 1$).

We introduce a new classifying space for degree r normal maps $p: M \to X$ (covered by bundle map $\hat{p}: \nu_M \to \xi$) and note that there exists such a normal map $N \to X$ where X is the homotopy type specified by k-invariant $r.g(N)$. This normal map has a surgery obstruction in $L'_{q-1}(\mathbb{Z}[\frac{1}{r}]\pi)$ and we prove the above results by relating this obstruction to that of any degree 1 normal map $M \xrightarrow{f} X$.

I. Hambleton (Hamilton)

Doubly sliced knots
(joint work with Neal W. Stoltzfus)

A knot $K^n \subset S^{n+2}$ is said to be doubly sliced if there exists a trivial $(n+1)$-knot $\Sigma^{n+1} \subset S^{n+3}$ such that

$\Sigma^{n+1} \cap S^{n+2} = K^n$. This notion is due to D. Summers. We shall say that a knot K^n is stably doubly sliced if there exist doubly sliced knots K_1^n and K_2^n such that $K^n \# K_1^n$ is isotopic to K_2^n. Let K^{2q-1} be a simple knot, $q \geq 2$.

Theorem 1: Assume that the knot module of K^{2q-1}, $q \geq 2$ is annihilated by a square-free polynomial $\lambda \in \mathbb{Z}[t, t^{-1}]$, and that K^{2q-1} is stably doubly sliced. Then K^{2q-1} is doubly sliced.

D. Summers and C. Kearton have proved that a simple knot K^{2q-1}, $q \geq 2$ is doubly sliced if and only if the associated Blanchfield form is hyperbolic. Therefore theorem 1 is a consequence of the following

Theorem 2 A stably hyperbolic Blanchfield form on a module which is annihilated by a square free polynomial $\lambda \in \mathbb{Z}[t, t^{-1}]$ is hyperbolic.

Eva Bayer-Fluckiger (Genève)

Integral monodromy of some plane curve singularities.
(Counterexamples to Orlik's conjecture).
(Joint work with Françoise Michel).

Let $f: \mathbb{C}^{n+1} \longrightarrow \mathbb{C}$ be a polynomial map, $f(0)=0$, 0 being an isolated singularity. Let Σ be the Milnor fiber and let h be the monodromy. Then h_* is an automorphism of $H_n(\Sigma; \mathbb{Z})$, giving this last group the structure of a $\mathbb{Z}[t,t^{-1}]$-module. Call it $M(f)$.
P. Orlik's conjecture states that $M(f)$ is the direct sum of cyclic modules, at least if h_* is of finite order.

Theorem: This conjecture is false. Counter-examples:
Take the singularity $f \equiv (x^a - y^b)(x^c - y^d) = 0$ with:
$\gcd(a,b) = 1$ $\gcd(c,d) = 1$ $\frac{c}{d} < \frac{a}{b}$.
Moreover, let b and c be two distinct prime such that:
$a+c = b^k$ $b+d = b^{k'}$ $k < k'$.
Then $M(f)$ is not $\cong \oplus$ cycles
The simplest example is $(x^1 - y^2)(x^3 - y^{14}) = 0$.

Sketch of proof: A presentation matrix for $M(f)$ is

$$\begin{pmatrix} (t-1)\,\alpha(t) & \alpha(t) \\ 0 & \beta(t) \end{pmatrix} \quad \text{where } \alpha(t) = \frac{t^{b(a+c)}-1}{t^{a+c}-1},\ \beta(t) = \frac{t^{c(b+d)}-1}{t^{b+d}-1}$$

Reduction mod b and mod c give conditions on the minimal polynomial of t_* which are impossible to be met by any $\oplus$ cycle of $M(f)$.

Claude Weber (Genève).

Jones' invariant of oriented links
(Expository talk)

Let $K = \mathbb{C}(\sqrt{t})$ be the rational function field on the indeterminate $\sqrt{t}$. Let A be the K-algebra with generators $e_1, e_2, \dots, e_n, \dots$ and relations

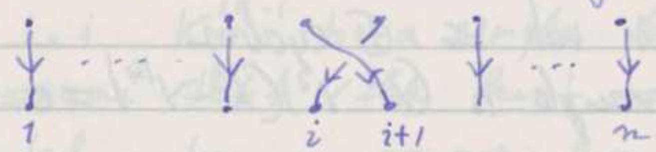
$$e_i^2 = e_i \qquad i = 1, 2, \dots$$
$$e_i e_j = e_j e_i \qquad \text{for } |i-j| \geq 2,$$
$$e_i e_{i+1} e_i = \tau e_i \qquad i = 1, 2, \dots$$
$$e_i e_{i-1} e_i = \tau e_i \qquad i = 2, \dots$$

where $\tau = t/(1+t)^2 \in K$.

V. Jones proves (Inventiones 72 (1983), p. 1) that A possesses a trace $\mathrm{Tr}: A \to K$ satisfying

1) $Tr(1) = 1$ and 2) $Tr(a e_m) = \tau\, Tr(a)$ if a belongs to the subalgebra generated by $e_1, \dots, e_{m-1}$.
Now, the braid groups B_n have natural maps $\pi : B_n \to A$ given by $\pi(\sigma_i) = \sqrt{t} \cdot (t e_i - (1 - e_i))$, where σ_i $i = 1, .., n-1$ are the standard generators of B_n.

σ_i :



$i = 1, .., n-1$.

If $\alpha \in B_n$, the Jones invariant of α is defined to be

$$V_\alpha(t) = \mu^{n-1} \cdot Tr(\pi(\alpha))$$

where $\mu = -(1+t)/\sqrt{t}$. (The notation V is believed to stand for von Jones.)
It turns out (as a consequence of Markov's theorem) that V_α depends only on the closed braid $\hat{\alpha}$ associated with α and thus yields a link invariant.
If c is the number of components of L, $V_L(t) \in \mathbb{Z}[t, t^{-1}]$ for c odd and $V_L(t) \in \sqrt{t} \cdot \mathbb{Z}[t, t^{-1}]$ for c even.
Proof If $\alpha \in B_n$ is such that $\hat{\alpha} = L$, let $\alpha = \sigma_{i_1}^{\varepsilon_1} \cdots \sigma_{i_r}^{\varepsilon_r}$. Then, the exponent of $\sqrt{t}$ in $V_\alpha(t)$ is $1 - n + e$, where $e = \sum_i \varepsilon_i$. Now, let $p(\alpha)$ be the permutation associated with α. We have $\text{sign}\{p(\alpha)\} = (-1)^e$ and writing $p(\alpha)$ as a product $\gamma_1 \cdots \gamma_c$ of disjoint cycles, $\text{sign}\{p(\alpha)\} = (-1)^{\sum_{i=1}^{c}(\ell_i + 1)}$, with $\ell_i = \ell(\gamma_i)$. Since $\sum \ell_i = n$, one concludes that $e \equiv n + c \mod 2$ as desired.
The fact that $V_\alpha(t)$, resp. $\sqrt{t} \cdot V_\alpha(t)$ is a Laurent polynomial in $\mathbb{Z}[t, t^{-1}]$ follows by an easy estimate on the exponents of $1+t$ occuring in the denominators of $Tr(\pi(\alpha))$.
$V_\alpha(t)$ has nice properties:
1) $V_{L^{-1}}(t) = V_L(t^{-1})$, where $L^{-1} = \widehat{\alpha^{-1}}$ if $L = \hat{\alpha}$.
2) $V_{L \amalg L'}(t) = \mu \cdot V_L(t) \cdot V_{L'}(t)$
3) $V_{L \# L'}(t) = V_L(t) \cdot V_{L'}(t)$.

Examples are: For $\beta = (\sigma_1^{-1}\sigma_2)^3$, $\hat{\beta}$ = the Borromean rings, one gets

$$V_\beta(t) = 1 - (1+t+t^{-1})(5 - 4(t+t^{-1}) + t^2+t^{-2})$$

For $\omega = \sigma_1\sigma_2^{-2}\sigma_1\sigma_2^{-1}$, $\hat{\omega}$ = the Whitehead link, one gets

$$\sqrt{t}\cdot V_\omega(t) = t^{-1} + (1+t+t^{-1})(2-t-3t^{-1}+t^{-2})$$

Generally, if $\sqrt{t}$ is specialized to $e^{i\pi/3}$, the value of $V_\alpha(t)$ is $(-1)^{c+1}$, where c is the number of components of $\hat{\alpha}$.

V. Jones reputedly claims that if $K = \hat{\alpha}$ is a knot, then $V_K(i)$ is the Robertello invariant of K. (Comm. Pure & Applied Math., 1965.)

Michel Kervaire (Genève)

Compact nilmanifolds and stable homotopy

Let G be a connected Lie group of dimension m and Γ a discrete subgroup such that G/Γ is compact. The tangent bundle of G/Γ admits a left-invariant trivialization, and thus we get an element $[G/\Gamma] \in \pi_m^S$ by the Thom - Pontrjagin construction.
We concentrate on the case where G is nilpotent and simply connected. In this case the discrete and cocompact subgroups are exactly the arithmetic subgroups; they were investigated by Malcev.
Using the Atiyah - Singer index theorem, it is shown that if $m \equiv 1$ or $2 \pmod 8$ and $m > 2$ then $d\,[G/\Gamma] = 0$. With other words, compact nilmanifolds of dimension ≥ 3 bound as spin manifolds.
The proof requires the determination of the dimension of harmonic spinors on G/Γ, i.e. the dimension of the kernel of the Dirac operator

Then the Adams e-invariant is computed in special cases, using the theorem of Atiyah - Patodi - Singer: Let $H(n)$ be the Heisenberg group (of dimension $2n+1$) and $\Gamma(n)$ its standard arithmetic subgroup. Then $e[H(n)/\Gamma(n)]$ is essentially given by the value of the Riemann zeta-function at the place $-n$ (for n odd). The proof requires a complete spectral analysis of the Laplace operator $H(n)/\Gamma(n)$; it turns out that the square of the Dirac operator is closely related to the Laplace operator. This computation of the e-invariant is joint work with Ch. Deninger

Wilhelm Singhof (Köln)

Andras Szücs: Multiple points and singular points.

Using normal forms of singularities one can generalize the Pontryagin Thom construction to cobordisms of (source) singular maps.

The classifying spaces for cobordisms of singular maps provide a model for the loop space of Thom space. This model can be applied to the following question:

Fix a set $\{\alpha_1, \alpha_2, \dots, \alpha_r\}$ of Boardman symbols of singularity types. Can a map of an n-manifold into R^{n+k} have a single point $P \in f(M)$ such that $f^{-1}(P)$ consists of r points which are of types $\alpha_1, \alpha_2, \dots, \alpha_r$?

Examples 1, $\alpha_1 = \alpha_2 = \dots = \alpha_r = \Sigma^0$ (nonsingular points). This case was solved by Eccles in codimension $k=1$ for immersions. ~~So~~ Particularly he showed that no such a map when n is even. We can extend this result to maps having singular points of multiplicity $\leq n-1$.

2, $\alpha = \{\Sigma^1, \Sigma^0\}$ No such a map i.e a map $f: M^{2k+1} \to R^{3k+1}$ can not have 1 double singular point

3, $\alpha = \{\Sigma^1, \Sigma^0, \Sigma^0\}$ No such a map i.e a map $f: M^{3k+1} \to R^{4k+1}$ can not have 1 singular triple point.

Some more applications of the model mentioned above were formulated

1, Th: Denote $\mathrm{Imm}(n,k)$ the cobordism group of immersions of n-manifolds into R^{n+k}. Suppose $n < 2k-1$.

Then $\mathrm{Imm}(n,k) \underset{\mathrm{mod}\,2}{\approx} \begin{cases} \Omega_n & \text{if } k \text{ is odd} \\ \Omega_n \oplus \Omega_{n+k} & \text{if } k \text{ is even} \end{cases}$

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~~Th~~ 2) How many elements of $\Omega_i(M^n)$ (the i-th bordism group of an n-manifold M^n) can be realised by immersions?

Th: Let $\mathcal{D}$ be a class of groups (in the sense of Serre) containing all finite 2 groups.

Suppose that $H^i(M^n) \in \mathcal{D}$ if $i \equiv 1 \bmod 4$

Then the factor group $\Omega_i(M^n)/\{\text{realizable elements}\}$ belongs to $\mathcal{D}$.

Where {realizable elements} denotes the ~~subgroup~~ set of elements, which are realizable by immersion.

András Szűcs (Budapest)

Larry SIEBENMANN: Exotic quasi-3-spheres in S^4 arising from Gromov's horizon of certain Coxeter-Davis groups. (with R. Ancel)

Let W^4 be a compact contractible 4-manifold with non-simply connected boundary M^3. One can so triangulate M^3 that M^3 becomes a full simplicial 3-complex, in which every quadrilateral in the 1-skeleton (a cycle of 4 simplexes) has at least one diagonal present as a 1-simplex of M^3. The Coxeter group Γ with one generator of order 2 for each vertex v, say $x(v)$, $x(v)^2=1$, and one relation for each edge $e=[v,v']$, namely $(x(v)x(v'))^2=1$, is combinatorially hyperbolic in the sense of Gromov (ICM Warsaw 1983). Following M. Davis (Annals early '80's) we make the Poincaré dual 3-cells into mirrors of reflection for an action of Γ on an open contractible 4-manifold X^4, with fundamental region $W^4 \subset X^4$. Davis observed that X^4 is not homeomorphic to $\mathbb{R}^4$. We show that the double $(\hat{X}^4 \cup \hat{X}^4)/\{Gr(\Gamma)=Gr(\Gamma)\}$ is homeomorphic to S^4, where $\hat{X}=X\cup Gr(\Gamma)$ is Gromov's compactification of X by the horizon $Gr(\Gamma)$ of the combinatorially hyperbolic group Γ. Further, the resulting pair $(S^4, Gr(\Gamma))$ is topologically homogeneous, i.e. given x and y in $Gr(\Gamma)$ there exists a homeomorphism h of S^4 respecting $Gr(\Gamma)$ and sending x to y. $Gr(\Gamma)$ can be identified as a homogeneous infinite connected sum of copies of M^3 of a sort constructed (initially for $M^3=S^3$) by W. Jakobsche about 1977 (see Fundamenta Math 1981). The same holds in higher dimensions as soon as the special triangulation can be found.

L. Sieb (Orsay)

Geometric representation theory of finite group

1. Point of view. Geometric representation theory is concerned with:
i) group actions on spheres, disks, Euclidean spaces (up to homotopy)
ii) systematic results for large classes of groups.
iii) using methods, results, and ideas from ordinary (algebraic) representation theory.
iv) The study of the geometry of unit spheres SV of orthogonal representations V.
v) the analysis of the role of SV's for general actions on spheres.

2. Homotopy representations and representation forms.
A homotopy representation of G is a finite-dimensional G-complex X such that i) For each subgroup $H \subset G$ the fixed point set X^H is homotopy-equivalent to a sphere $S^{n(H)-1}$; ii) $\dim X^H = S^{n(H)-1}$. iii) $H \in \mathrm{Iso}(X)$, $K \supsetneq H \Rightarrow n(L) < n(H)$. iv) $\mathrm{Iso}(X)$ is closed under intersections.

A representation form is an action with similar properties where all X^H are actual spheres.

3. Problem: Classify homotopy representations up to G-homotopy type.
Two basic invariants are suitable for this classification.
i) The Dimension function $\mathrm{Dim}\, X : H \mapsto n(H)$.
ii) Degree functions. Suppose $\mathrm{Dim}\, X = \mathrm{Dim}\, Y$. Let $f: X \to Y$ be a G-map. Degree function is $d_f : (H) \mapsto$ degree of f^H.

4. Stable classification Let $V^\infty(G)$ be the Grothendieck group of X with join as composition law. Then $\mathrm{Dim}: V^\infty(G) \to C(G)$, $X \mapsto \mathrm{Dim}\, X$ is a homomorphism into the group $C(G)$ of integer valued functions on conjugacy classes. The kernel $v^\infty(G)$ of Dim is isomorphic to the Picard group of the Burnside ring.

5. Dimension functions.
Let $D \subset C(G)$ be a function. Then D is the dimension function of a homotopy representation if and only if the following

conditions are satisfied

(I) $D(H) \geq 0$; $H \subset K \Rightarrow D(H) \geq D(K)$

(II) The set $\operatorname{Iso} D = \{H \mid D(H) > 0,\ H \subsetneq K \Rightarrow D(K) < D(H)\}$ is closed under intersections.

(III) Let $H \subset G$, $WH = NH/H$, $WH(p)$ p-Sylow subgroups of WH. Define a function $D_{WH(p)} \in C(WH(p))$ by $D_{WH(p)}(U/H) = D(U)$, $U/H \subset WH(p)$. Then assume that there exists a finitely dominated $\mathbb{Z}_{(p)}$-homology sphere $Y(H,p)$ for $WH(p)$ such that $D_{WH(p)} = \operatorname{Dim} Y(H,p)$

(IV) Write $W = WH$, $W(p) = WH(p)$, $Y = Y(H,p)$. For $w \in W$ let $K = W(p) \cap wW(p)w^{-1}$. Have two K-spaces Y, namely $K \times Y \to Y$, $(k,y) \mapsto ky$ and $K \times Y_w \to Y_w = Y$, $(k,y) \mapsto w^{-1}kwy$. Assume: There are given a K-equivalences $\lambda_w : Y_w \to Y$ such that i) $\lambda_w \simeq l_w$ (left w-translation) for $w \in WH(p)$, ii) $\lambda_{w_1} \lambda_{w_1} \simeq \lambda_{w_1 w_2}$

(V) For each p-group $P = U/H_\ast \subset WH$ and $w \in WH$ with $wPw^{-1} = P$ we have degree $\lambda_w^P = (-1)^{D(U) - D(w,U)}$ where (w,U) is the pre-image in NH of the group generated by w and U.

Tammo tom Dieck (Göttingen).

Algebraic K-theory of Spaces
19.8.84 – 25.8.84

A non-connective delooping of the algebraic K-theory of spaces

Let Y be an A_∞ ring space which is ringlike ($\pi_0 Y$ is a ring). Its K-theory KY is $K_0(\pi_0 Y) \times (BgLY)^+$, where $(BgLY)^+$ is the plus-construction on the classifying space of the telescope of the invertible components $gl_d Y$ in $m_d Y \simeq Y^{d^2}$. In particular if X is a based space and $Y = \Omega^\infty\Sigma^\infty((\Omega X)_+)$ [$(\ )_+$ denotes the addition of a base-point], then KY is a possible definition for the algebraic K-theory of X.

Imitating Wagoner, one can deloop KY non-connectively as follows, perhaps more informatively than the usual way. There is a sequence $Y, sY, s^2Y, \ldots$ of ringlike A_∞ ring spaces such that $KY \simeq \Omega KsY$, $KsY \simeq \Omega Ks^2Y, \ldots$. Here sY is such that $\pi_q sY \cong S\pi_q Y$ (locally finite matrices over $\pi_q Y$ modulo finite ones), etc. It is got from a bar construction, using the general principle that a construction on semirings extends to A_∞ ring spaces provided one never adds two equal terms.

Richard Steiner (Glasgow)

Surgery theory, automorphisms of manifolds, and higher algebraic K-theory
(joint work with Bill Dwyer)

For a spectrum A with $\mathbb{Z}/2$ action we construct a "Tate cohomology" fibration, $H_\cdot(\mathbb{Z}/2, A) \xrightarrow{N} H^\cdot(\mathbb{Z}/2, A) \to \hat{H}(A)$, e.g. A = Waldhausen's $A(X)$ with Vogell's involution. If $A^{(n)} = A$ twisted by n copies of the flip representation, then $\Omega^n \hat{H}(A) \simeq \hat{H}(A^{(n)})$.

For M^n a topological manifold, let $H(M)$ = (simple) homotopy automorphisms of M and $TOP(M)$ = homeomorphisms of M

Conjecture: There exists a commutative diagram of natural transformations

$$\begin{array}{ccccc} M_+ \wedge L(\mathbb{Z}) & \longrightarrow & L(\mathbb{Z}\pi) & \longrightarrow & S(M) \\ \downarrow & & \downarrow & & \downarrow \theta \\ \hat{H}(M_+ \wedge A(*)) & \longrightarrow & \hat{H}(A(M)) & \longrightarrow & \hat{H}(Wh(M)) \end{array} \qquad (*)$$

such that $\frac{H(M)}{TOP(M)}$ is the homotopy fiber of the map

$$\Omega^{n+1}\mathcal{S}(X) \xrightarrow{\Omega^{n+1}\theta} \Omega^{n+1}\hat{H}(Wh(X)) \xrightarrow{\simeq} \Omega\hat{H}(Wh(X)^{(n)}) \xrightarrow{\partial} H_\cdot(\mathbb{Z}_2, Wh(X)^{(n)}).$$

Thus (*) would be the "glueing data" between surgery theory and the algebraic K-theory of spaces.

Evidence for the conjecture comes from the work of Hatcher, Hsiang-Sharpe, and Burghelea-Lashof.

Bruce Williams (Notre Dame)

An algebraic description of the transfer induced by a fibration on K_0 and K_1

For certain fibrations $F \to E \xrightarrow{p} B$ there is a geometrically defined homomorphism $p^! \colon K_i(\mathbb{Z}[\pi_1(B)]) \to K_i(\mathbb{Z}[\pi_1(E)])$ using the pull-back construction. Using chain-complexes with a ~~so called~~ twist one can define pairings $K_i(\mathbb{Z}[\pi_1(B)]) \otimes K_0(\mathbb{Z}[\Delta]-\pi_1(E)) \xrightarrow{\otimes_t} K_i(\mathbb{Z}[\pi_1(E)])$ where $K_0(\mathbb{Z}[\Delta]-\pi_1(E))$ is the Grothendieck-group of $\mathbb{Z}[\Delta]$-chain complexes with a $\pi_1(E)$-twist. A $\pi_1(E)$-twist is ~~give~~ a homotopy extension of the Δ-action to an $\pi_1(E)$-action. Δ denotes the kernel of $p_*\colon \pi_1(E) \to \pi_1(B)$. It defines p an element $L(p) \in K_0(\mathbb{Z}[\Delta]-\pi_1(E))$ and $p^!$ is just $? \otimes_t L(p)$. If $\pi_1(E)$ acts trivial (up to homotopy) on the pointed fibre $p_* \circ p^!$ and $p^! \circ p_*$ vanish. If Δ is contained in center $(\pi_1(E))$ and is free as abelian group and $G_1(F) = \pi_1(F)$, then $p^!$ is trivial.

Wolfgang Lück

Lower K-theory and parametrized spaces with bounded control

(jt. work with D. R. Anderson) Let (Z, ρ) be a metric space. The category $\underline{Top}^c/Z$ has as objects all maps $p: X \to Z$, X any space; a morphism $f: (X,p) \to (Y,q)$ is a map $f: X \to Y$ with $\rho(p(x), qf(x))$ bounded.

We develop an "algebraic topology" for $\underline{Top}^c/Z$, including chain-, homology-, and homotopy "groups" that take values in a certain abelian category $\underline{\underline{A}}(X,p)$ [for $Z = pt$, $\underline{\underline{A}}(X,p)$ becomes $\mathbb{Z}[\pi_1 X]$-mod].

We prove a Hurewicz-, and a Whitehead-theorem.

The results are applied to study "simple homotopy theory with bounded control". There results obstructions in a group $Wh(\underline{\underline{A}}(X,p))$ which is constructed from the category of "boundedly, finitely generated" projectives in $\underline{\underline{A}}(X,p)$ in a standard way.

For $Z = \mathbb{R}^k$, if $\pi_1(X)$ is "uniformly boundedly defined" one has $Wh(\underline{\underline{A}}(X,p)) \cong \tilde{K}_{1-k}(\mathbb{Z}\pi_1(X))$.

H. J. Munkholm (Odense)

The equivariant Top_G/PL.

(joint work with M. Rothenberg). Let G be a finite group of odd order. If V is an $\mathbb{R}G$-module, write $Top_G(V)$ resp. $PL_G(V)$ for the groups of equivariant homeomorphisms (resp. PL-isomorphisms) of V. Let $Top_G = \varinjlim_V Top_G(V)$, $PL_G = \varinjlim_V PL_G(V)$

THEOREM. $\pi_k(Top_G/PL_G) = \Sigma^{\oplus} L^{\langle -\infty \rangle}_{k+1}(\mathbb{Z}[NH/H]) / L^{s}_{k+1}(\mathbb{Z}[NH/H]) \quad k \neq 3$

$= \Sigma^{\oplus} L^{\langle -\infty \rangle}_{k+1}(\mathbb{Z}[NH/H]) / L^{s}_{k+1}(\mathbb{Z}[NH/H]) \oplus A(G) \otimes \mathbb{Z}/2, \; k = 3.$

Here $A(G)$ is the Burnside ring, L^s_{k+1} is the simple surgery groups of Wall and $L^{\langle -\infty \rangle}_{k+1}(\mathbb{Z}\Gamma) = L^s_{k+1+j}(\mathbb{Z}[\Gamma \times \mathbb{Z}^j])^{INV}$, j large.

The groups $L_k^{\langle -\infty \rangle}(\mathbb{Z}G)/L_k^s(\mathbb{Z}G)$ are easy to tabulate. If k is odd then $L_k^{\langle -\infty \rangle}(\mathbb{Z}G)/L_k^s(\mathbb{Z}G) = \hat{H}^1(\mathbb{Z}/2; K_{-1}(\mathbb{Z}G))$ where $\tilde{K}_0(\mathbb{Q}G) \oplus \tilde{K}_0(\hat{\mathbb{Z}}_{|G|}G) \to \tilde{K}_0(\hat{\mathbb{Q}}_{|G|}G) \to K_{-1}(\mathbb{Z}G) \to 0$ is exact. If k is odd, $L_k^{\langle -\infty \rangle}(\mathbb{Z}G)/L_k^s(\mathbb{Z}G) \subset \hat{H}^1(\mathbb{Z}/2; K_1(\mathbb{R}G))$ (Its rank is $rk R_{\mathbb{C}}G - rk R_{\mathbb{Q}}G$). The fibration sequence $F_G/PL_G \to F_G/Top_G \to B(Top_G/PL_G)$ gives on homotopy groups: $0 \to L_k^s(\mathbb{Z}G)_{(2)} \to L_k^{\langle -\infty \rangle}(\mathbb{Z}G)_{(2)} \to L_k^{\langle -\infty \rangle}(\mathbb{Z}G)/L_k^s(\mathbb{Z}G) \to 0$ when $k \neq 3$ (and G is abelian)

Ib Madsen (Århus)

DICKSON – HUỲNH MÙI'S INVARIANTS AND THE HOMOLOGY COALGEBRAS OF LOOP SPACES $\Omega^q S^q X$.

This talk announces some current researchs of our seminar in Hanoi, particularly of Huỳnh Mùi and the author, on applications of modular invariants to Algebraic Topology.

We introduce the mod p Dickson characteristic classes for finite coverings over paracompact spaces derived from the Dickson – Huỳnh Mùi's invariants of $GL(n, \mathbb{Z}/p)$. These Dickson classes are closely related to the classical Stiefel – Whitney or Chern classes.

Cohomology algebras of the (universal) loop spaces $\Omega^q S^q$ are determined using the isomorphisms $H^*(\Omega_0^q S^q; \mathbb{Z}/p) \cong H^*(F(\mathbb{R}^q, \infty)/\mathfrak{S}_\infty; \mathbb{Z}/p)$ and the Dickson classes for the $\mathfrak{S}_\infty$ – principal covering over $F(\mathbb{R}^q, \infty)/\mathfrak{S}_\infty$. The actions of Steenrod operations on $H^*(\Omega_0^q S^q; \mathbb{Z}/p)$ are computed by reducing them to those on the $GL(n, \mathbb{Z}/p)$ – invariants.

Generalizing these results Huỳnh Mùi describes the coalgebra structures of $H_*(\Omega^q S^q X)$ by introducing the homology operations derived from the modular invariants, which are certain linear combinations of iterated Dyer – Lashof operations, on the loop spaces $\Omega^q S^q X$. The invariants lead us to overcome the Adem phenomenon occurring in the Dyer – Lashof approach.

Nguyễn H. V. Hưng
(Hanoi)

CALCULATION of the RATIONAL K-THEORY of spaces via CYCLIC HOMOLOGY.

In this lecture $HH_*(X)$ and $HC_*(X)$ denote the Hochschild resp cyclic homology with rational coefficients of X.

Proposition: Given two spaces X and Y one has the following exact sequence

$$0 \leftarrow HC_*(X) \underset{HC_*(*)}{\square} HC_*(Y) \leftarrow HC_*(X \times Y) \leftarrow \Sigma\big(\underset{HC_*(*)}{\mathrm{Cotor}}\, HC_*(X),\, HC_*(Y)\big) \leftarrow 0$$

and if $HC_*(Y)$ is a quasifree $HC_*(*)$-comodule of the form $HC_*(Y) = HC_*(pt) \underset{\mathbb{Q}}{\otimes} W_* + V_*$ (with $HC_*(pt) \underset{\mathbb{Q}}{\otimes} W_*$ the free part and V_* the trivial part) then $HC_*(X \times Y) = HC_*(X) \underset{\mathbb{Q}}{\otimes} W_* + HH_*(X) \underset{\mathbb{Q}}{\otimes} V_*$. If Y is a suspension or $K(Z,n)$ then $HC_*(Y)$ is quasifree and explicit formulas for both $HH_*(Y)$ and $HC_*(Y)$ are given.

If X has $(\Lambda[x_\alpha], d)$ as a Sullivan minimal model ($\deg x_\alpha \geq 2$) and $\Lambda[x_\alpha, \bar{x}_\alpha, u], \mathcal{D}$ denotes the commutative differential graded algebra with $\deg \bar{x}_\alpha = \deg x_\alpha - 1$, $\deg u = 2$ and $\mathcal{D} x_\alpha = d x_\alpha + \bar{x}_\alpha u$, $\mathcal{D} u = 0$ and $\mathcal{D} \bar{x}_\alpha = \beta(d x_\alpha)$ ($\beta : \Lambda[x_\alpha] \to \Lambda[x_\alpha, \bar{x}_\alpha]$ the unique derivation with $\beta(x_\alpha) = \bar{x}_\alpha$) then:

THEOREM (joint work with M. Vigué Poirrier): $HC^*(X) = H^*(\Lambda[x_\alpha, \bar{x}_\alpha, u], \mathcal{D})$ with $HC^n(X) = \mathrm{Hom}(HC_n(X), \mathbb{Q})$

Corollary: If X is CP^n or QP^n (quaternionic projective space) $HC_*(X)$ is quasifree and explicit calculations are provided for $HC_*(X)$ and $HH_*(X)$ (Similarly for complex grassmanians)

Combined with the known relationship between $\underset{\cdot}{A}(X) \otimes \mathbb{Q}$ and $HC(X)$ these results recover all known computations of $\underset{\cdot}{A}(X) \otimes \mathbb{Q}$ and permit few other.

D. Burghelea
Columbus OHIO

Transfer in Whitehead Theory and G-actions

Let $\tilde{X} \longrightarrow X$ be a G-covering, where G is finite and $|G| = q$, and X is finitely dominated and $\tilde{X}$ is homotopy equivalent to a finite complex Y (fixed). The obstructions to choose a finite complex X' homotopy equivalent to X so that $\tilde{X}'$ be homotopy equivalent to Y via a π-simple homotopy equivalence lie in an abelian group $Wh_1^T(\pi_1 Y \to \pi_1 X)$. In general, for an extension $1 \to \pi \to \Gamma \to G \to 1$, $|G| = q < \infty$, one has a long exact sequence

$$Wh_1(\Gamma) \xrightarrow{Tr} Wh_1(\pi) \xrightarrow{\beta} Wh_1^T(\pi \to \Gamma) \xrightarrow{\alpha} Wh_0(\Gamma) \to Wh_0(\pi)$$

and a commutative diagram, when $\Gamma = \pi \times G$

$$\begin{tikzcd} Wh_1(\pi) \arrow[rr, "\beta"] \arrow[dr] & & Wh_1^T(\pi \to \Gamma) \\ & Wh_1(\pi; \mathbb{Z}/q) \arrow[ur, "\gamma"'] & \end{tikzcd}$$

This exact sequence can be shown to be the lower portion of an exact sequence of homotopy associated to the fibration obtained by delooping a geometric transfer between $\mathbb{W}h(B\Gamma) \xrightarrow{\tau} \mathbb{W}h(B\pi)$ where $\mathbb{W}h = \Omega^{-1} Wh$, Wh = Hatcher's Whitehead theory and $\tau = \Omega^{-1} T$, where T is the transfer constructed using Burghelea-Lashof type arguments on their geometric transfer between concordance spaces.

There are geometric applications for Wh_1^T in transformation groups, which show that Wh_1^T is the analogue of K_0 functor in the case of G-actions on non-simply-connected spaces.

Amir H. Assadi
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Cyclic Homology, Monads and Group Actions

Connes' ~~action~~ notion of a cyclic set is analyzed. It is shown that for a cyclic set $X_*: \Lambda^{op} \to Sets$, the Connes-Gysin sequence relating cyclic homology to simplicial homology can be obtained from a fibration of the form $|X_*| \simeq \mathrm{hocolim}_{\Delta^{op}} X_* \longrightarrow \mathrm{hocolim}_{\Lambda^{op}} X_* \longrightarrow B\Lambda^{op} \simeq \mathbb{C}P^\infty$. It is then shown that there is a natural S^1 action on the geometric realization of a cyclic set and that the usual adjunction between simplicial sets and topological spaces extends to give a combinatorial description of S^1 actions. This combinatorial description is then generalized to describe actions by a certain limited class of Lie groups. For these groups G_* one can define a similar category $\Lambda[G_*]$ and for combinatorial G_* actions on simplicial sets described by functors $X_*: \Lambda[G_*]^{op} \longrightarrow Sets$ one has a similar fibration sequence $\mathrm{hocolim}_{\Delta^{op}} X_* \to \mathrm{hocolim}_{\Lambda[G_*]^{op}} X_* \to B\Lambda[G_*]^{op}$ and that this fibration sequence can be naturally identified up to homotopy with $|X_*| \longrightarrow |X_*| \times_{|G_*|} E|G_*| \longrightarrow B|G_*|$. This result can be used to give a conceptual proof of the isomorphism $HC_*(k[\Omega_* X]) \cong H_*(X^{S^1} \times_{S^1} ES^1)$.

Z. Fiedorowicz
Columbus, OH

Two Questions in Integral Algebraic K-theory

We discuss two conjectures in K-theory which are integral analogues of rational constructions. The first involves a configuration-space model for K-theory. The space $CGL(R) = \coprod_{n\geq 0} C(n;\mathbb{R}^\infty) \times_{\Sigma_n} BGL_n(R)/\sim$ is analyzed, in analogy to $C(X) = \coprod_{n\geq 0} C(n;\mathbb{R}^\infty) \times_{\Sigma_n} X^n/\sim \;\cong\; Q(X)$. We show that there is a map $BGL(R)^+ \to CGL(R)$ which is a rational homotopy equivalence. The motivation for the construction of $CGL(R)$ is that there is a map $CGL(R) \to$

$NGL(R) = \coprod_{n\geq 0} * \times_{\Sigma_n} BGL_n(R)/\sim \ \simeq \prod_{n\geq 1} K(\pi_n, n)$ which is a rat.

eq., together with a map $Sp_\infty(\Sigma_\infty \backslash B(R)) \to NGL(R)$ which induces a map $HC_*(D^0_*(R)) \to \pi_*(\Sigma_\infty \backslash BGL(R)) = \pi_*(NGL(R))$ $\to K_*(R)$ <u>rationally</u>; $D^0_*(R)$ a certain cyclic subcomplex of $C_*(R) =$ Connes complex. It is conjectured that the ∞-loop space $CGL(R)$ is either algebraic K-Theory, or algebraic K-Theory "away" from $Q(S^0)$. In particular, one can construct $C\hat{GL}(R)$ for the ring up to homotopy $R = Q(\Omega X_+)$, and it is conjectured that $C\hat{GL}(Q(S^0))$ is either $A(*)$ or $Wh^{Diff}(*)$ <u>integrally</u> (This is true rationally).

The second question involves the integral K-Theory of a ~~K-Theory~~ square-zero ideal I. We construct maps $I^{\otimes n}/\sim \xrightarrow{L} K_*(I) \xrightarrow{\Phi} I^{\otimes n}/\sim$ (where $\sim$ is the cyclic relation in cyclic homology) whose composition is mult. by n. By Staffeldt, the map $I^{\otimes n}/\sim \to K_*(I)$ is a rational isomorphism, but it is shown not to be an integral one. However, we conjecture that $\mathrm{im}(L)$ generates $K_*(I)$ as an ideal in the graded ring $K_*(\mathbb{Z}\oplus I)$ integrally.

Crichton Ogle
Colombus, OH

<u>Comparison of</u> <u>Involutions on $A(X)$</u> (joint with W-C. Hsiang)
W. Vogell constructs involutions τ_ξ on $A(X)$, corresponding (up to sign) to the involution on pseudoisotopy theory for manifolds X with tangent sphere fibration $\simeq \xi$.
On the other hand R. Steiner has proved that the operation $A \mapsto \bar{A}^t$ on matrices over $Q(G_+)$ also gives rise to an involution on $A(X)$, defined as $\mathbb{Z}\times B\hat{GL}(Q(G_+))^+$ (Here $G = \Omega X$, and conjugation induced by $g \mapsto g^{-1}$ on G).

Theorem 1 This involution corresponds to Vogell's τ_e, where e is the trivial spherical fibration.

The proof of this uses a geometric (~"manifold") version of $\hat{GL}(Q(G_+))$.

For computations one would also like to define the more general τ_ξ on $B\hat{GL}(Q(G_+))^+$. In view of Vogell's work, it suffices to identify the maps $\xi\cdot : A(X) \to A(X)$ using the $\hat{GL}$-definition.

Given ξ, there is a homomorphism $\alpha: G \to \Omega^d S^d$ — the loops on the classifying map. If $f: S^n \to S^n(G_+)$ represents an element of $Q(G_+)$, let $f^\alpha: S^d \wedge S^n \to S^d \wedge S^n(G_+)$ be defined by $f^\alpha(u,x) = (\alpha(g)u, y, g)$, where $f(x) = (y, g)$.

Theorem 2 $f \mapsto f^\alpha : QG \to QG$ induces a map $B\hat{GL}(Q(G_+))^+ \to B\hat{GL}(Q(G_+))^+$, which corresponds to Vogell's $\xi\cdot: A(X) \to A(X)$.

Björn Jahren,
Oslo (Norway).

Delooping K-theory by parametrized modules (joint with EK Pedersen)

Given an additive category $\mathcal{A}$ and a metric space X, one can construct a category $\mathcal{C}_X(\mathcal{A})$ of $\mathcal{A}$-objects parametrized by X (in a locally finite way), the morphisms being given by "bounded matrices". The point of the lecture was that the K-theory spaces of $\mathcal{A} = \mathcal{C}_0$, $\mathcal{C}_{\mathbb{R}}(\mathcal{A})$, $\mathcal{C}_{\mathbb{R}^2}(\mathcal{A})$, ... form a nonconnective infinite loop spectrum, at least when all short exact sequences split in $\mathcal{A}$. This allows us to define the negative K-theory of $\mathcal{A}$. In fact, we recover the definition given by Karoubi in SLN#136 (1968), naturally in a different form. The hope is that this machinery works in case short exact sequences in $\mathcal{A}$ do not split, but the problem at present is defining an exact structure on the category $\mathcal{C}_X(\mathcal{A})$.

Assuming all s.e.s. split in $\mathcal{A}$, we can define the K-theory of $\mathcal{C}_X(\mathcal{A})$ by allowing only split monomorphisms. For convenience, assume $\mathcal{A}$ idempotent complete. Then we prove: $K(\mathcal{C}_{X\times[0,\infty)}) \simeq *$

$K(\mathcal{C}_Y) \to K(\mathcal{C}_X) \to K(\mathcal{C}_X/\mathcal{C}_Y)$ is a fibration

$$\begin{array}{ccc} K(\hat{\mathcal{C}}_X) & \longrightarrow & K(\mathcal{C}_{X\times[0,\infty)}) \simeq * \\ \downarrow & & \downarrow \\ * \simeq K(\mathcal{C}_{X\times(-\infty,0]}) & \longrightarrow & K(\mathcal{C}_{X\times\mathbb{R}}) \end{array}$$

is homotopy cartesian

From this it follows that $\Omega^n K(\mathcal{C}_{\mathbb{R}^n}) \simeq K(\mathcal{A})$, and that $K(\mathcal{C}_{\mathbb{R}^n})$ is a covering space of $\Omega K(\mathcal{C}_{\mathbb{R}^{n+1}})$.

Chuck Weibel
New Brunswick NJ (USA)

A commutativity formula for Nil-groups

Let A and B be two rings and ${}_A S_B$ and ${}_B T_A$ be two bimodules. Consider the category of objects (P, Q, p, q) where $P \in \mathcal{P}_A$ and $Q \in \mathcal{P}_B$ are finitely generated projective right modules and $p : P \to Q \otimes_B T$, $q : Q \to P \otimes_A T$ are maps. The category $\mathcal{N}il(A, B; S, T)$ of such objects which are nilpotent in an obvious sense is an exact category and we have a K-theoretical spectrum $K\mathcal{N}il(A, B; S, T)$ which split: $K\mathcal{N}il(A, B; S, T) \simeq K(A) \times K(B) \times K\widetilde{\mathcal{N}il}(A, B; S, T)$.

Theorem If S and T are free on each side, the rule $(P, Q, p, q) \to (P, q \circ p)$ gives a homotopy equivalence of spectra:

$$K\widetilde{\mathcal{N}il}(A, B; S, T) \xrightarrow{\sim} K\widetilde{\mathcal{N}il}(A, S \otimes_B T)$$

Corollary In the same conditions we have a homotopy equivalence:

$$K\widetilde{\mathcal{N}il}(A, S \otimes_B T) \simeq K\widetilde{\mathcal{N}il}(B, T \otimes_A S)$$

As a consequence of these result we see that the Nil functor defined by Waldhausen in the computation of Mayer Vietoris exact sequence in algebraic K-theory associated to push out of groups, are of the form $K\widetilde{\mathcal{N}il}(A, S)$ and are contractible in many cases. For example we have the following:

Theorem Let $\begin{array}{ccc} H & \hookrightarrow & G \\ \downarrow & & \downarrow \\ G' & \to & \pi \end{array}$ be a push-out of groups. Suppose that

$H \cap \alpha H \alpha^{-1} \cap \beta H \beta^{-1}$ is regular coherent for every $\alpha \in G - H$ and $\beta \in G' - H$

Then we have a cartesian square of spectra:

$$\begin{array}{ccc} Wh(H) & \longrightarrow & Wh(G) \\ \downarrow & & \downarrow \\ Wh(G') & \longrightarrow & Wh(\pi) \end{array}$$

Pierre Vogel
Nantes (France)

Hochschild homology outside algebraic K-theory

The fact that the Hochschild homology of the algebra $\mathcal{D}_n = \mathbb{C}\langle x_1, .., x_n, \frac{\partial}{\partial x_1}, .., \frac{\partial}{\partial x_n}\rangle$ of differential operators on the affine space of dimension n is given by

$$H_i(\mathcal{D}_n, \mathcal{D}_n) = \begin{cases} \mathbb{C} & \text{if } i = 2n \\ 0 & \text{otherwise} \end{cases}$$

is explained in connection with Feigin-Tsigan's work on the cohomology of certain Lie algebras of matrices and with the notion of semi-tensor product invented in the early sixties by Massey-Peterson for topological purposes. This semi-tensor product allows to construct numerous non-commutative algebras used in various fields (algebra, analysis, topology, Kac-Moody Lie algebras) and to compute their Hochschild homology by means of a spectral sequence.

Christian Kassel (Strasbourg).

Hochschild homology and stable K-theory.

Let R be a ring and M a bimodule over it. Supposing that R is an algebra over a ground ring k one defines the Hochschild homology $H_k(R,M)$ as the simplicial object

$$[n] \longmapsto \begin{array}{ccc} & \otimes_k R \otimes_k \cdots & \\ R^{\otimes_k \cdots} & & \cdots \otimes_k R \\ \otimes_k & M & \otimes_k \end{array}$$

(n factors R – the circular display is to indicate that the j-th face map is given by the collapsing of the j-th tensor product sign). It turns out that the construction can be extended to a framework of "rings up to homotopy" (one uses monads, and algebras over monads, to carry this out technically). The interest of the extended construction is in its use to compute the stable K-theory $K^S(R,M)$. The assertion (whose proof is fairly difficult) is that the natural map $K^S(R,M) \longrightarrow H_k(R,M)$ is a homotopy equivalence provided that for the ground ring k one takes the "universal" ring up to homotopy, QS^0, whose homotopy groups are the stable homotopy groups of spheres. (Note, if $M = R = k$ then $H_k(R,M) \simeq k$, and if $M = R = QS^0$ then $K^S(R,M) = A^S(*)$, so this generalizes the assertion that $A^S(*) \to QS^0$ is a homotopy equivalence.)

Friedhelm Waldhausen (Bielefeld)

K-theory and stable K-theory

Using étale homotopy theory one constructs a commutative diagram of 2-complete spaces

$$\begin{tikzcd} K(\mathbb{Z})^{\wedge}_{(2)} \arrow[r] \arrow[d] & (\mathbb{Z}\times BO)^{\wedge}_{2} \arrow[d] \\ K(\mathbb{F}_3) \arrow[r] & (\mathbb{Z}\times BU)^{\wedge}_{(2)} \end{tikzcd}$$

This defines a map of $K(\mathbb{Z})^{\wedge}_{(2)}$ to the pullback of the other spaces in the diagram. After taking connected cover of the pullback this maps the subgroups of $\pi_* K(\mathbb{Z})^{\wedge}_{(2)}$ generated by étale K-theory & the Borel classes (i.e. all known homotopy in $K(\mathbb{Z})^{\wedge}_{(2)}$) isomorphically to the homotopy groups $\pi_*(JK(\mathbb{Z}))$.

Theorem 1: $K(\mathbb{Z}) \to JK(\mathbb{Z})$ is not a 7-connected map

The proof uses the Hochschild homology $H_{QS^0}(\mathbb{Z},\mathbb{Z})$ (see the preceding lecture).

Theorem 2: $H_{QS^0}(\mathbb{Z}) = \mathbb{Z} \times \prod_r \mathbb{Z}/r\,[2r-1]$

where $\mathbb{Z}/r\,[i]$ denotes the i^{th}-dimensional Eilenberg-MacLane-space of dimension i.

Assume Theorem 1 false. Then, the space $JK(\mathbb{Z})$ gives a concrete model for $K(\mathbb{Z})$ (at least in a dimension range). Direct computation using this model gives two conflicting results about the maps

$$\begin{cases} H^7(K(\mathbb{Z});\mathbb{Z}/4) \to H^7(H(\mathbb{Z});\mathbb{Z}/4) \\ H^7(K(\mathbb{Z})_1;\mathbb{Z}/4) \to H^7(H_{QS^0}(\mathbb{Z})_0;\mathbb{Z}/4) \end{cases}$$

Marcel Bökstedt

Some generalities on continuous functors, monads and rings up to homotopy.

Functors $F: TOP_* \to TOP_*$ (which commute with directed colimits, are continuous and has $F(pt) \sim pt$) are models for abelian groups up to homotopy. $F(-)$ codifies structures on $F(S^0)$. If F is such a functor then $F^S(-) = \underrightarrow{\mathrm{colim}}\,\Omega^n(F(S^n \wedge -))$

is a reduced homology theory. Composition of functors gives a monoidal structure. This leads to the notion of A_∞-monads and a theory for homotopy invarians of such structures. Multiplicative structures are preserved under stabilization. In the stable case A_∞-monads can be changed to monads. K-theory is defined for monads as in classical theory (ring:=monad), in particular constructions used in the analysis of the algebraic K-theory of spaces can be performed using monads (as demonstrated by F. Waldhausen)

Thomas Gunnarsson
(Luleå)

Involutions on A(X).

Various models of the algebraic K-theory of spaces functor A(X) were described & it was shown how to put natural involutions on these. There is a notion of equivariant Spanier-Whitehead duality underlying the construction of these involutions. It turns out that the correct notion of equivariant duality to use is a generalization of Ranicki duality where the group acting on the spaces under consideration is no longer discrete but is allowed to be any simplicial group. The involutions constructed depend on a chosen spherical fibration ξ over X. If X is a manifold these involutions on A(X) are shown to correspond to the natural involution on the stable concordance space $\mathcal{C}(X)$, where ξ is the (fibrewise one-point-compactification of the) tangent bundle of X.

Wolrad Vogell
(Bielefeld)

Komplexe Analysis, 26.8. – 1.9. 1984

Invariants of arithmetic varieties

Let $\mathcal{D}$ be a hermitian bounded symmetric domain and $\Gamma \subset Aut(\mathcal{D})$ an arithmetic subgroup. For simplicity we assume that Γ is neat. Then $X = \mathcal{D}/\Gamma$ is a manifold, which is in general not compact.

There is a minimal compactification X^* of X, called Baily-Borel compactification and an ample invertible sheaf $\mathcal{L}^*$ on X^* extending the canonical sheaf $\Omega_X = \Omega_X^n$. The space of automorphic forms of weight k is then defined as

$$A_k(\Gamma) = H^0(X^*, \mathcal{L}^k).$$

If one considers a desingularization $\tilde{X}$ of X^* along a boundary then we again have a natural identification

$$A_k(\Gamma) = H^0(\tilde{X}, (\Omega^n_{\tilde{X}})^k(kE)) \quad \text{where } E = \tilde{X} - X.$$

The space of cusp forms of weight k is defined similarly as

$$S_k(\Gamma) = H^0(\tilde{X}, (\Omega^n_{\tilde{X}})^k((k-1)E)).$$

One of the main problem is to calculate the dimension of these spaces.

The new methods of toroidal compactification due to Mumford allows us to get the first approximation of this dimension formula thanks to the generalized Hirzebruch's proportionality. Let $\check{\mathcal{D}} \supset \mathcal{D}$ the compact dual of $\mathcal{D}$ and $P(k) = \dim H^0(\check{\mathcal{D}}, (\Omega^n_{\check{\mathcal{D}}})^k)$. Moreover let $n' = \dim(X^* - X)$. Then there is a polynomial $P'(k)$ of degree $\leq n'$ s.t. for $k \geq 2$

$$\dim S_k(\Gamma) = c\,P(k-1) + P'(k)$$

$$\text{where} \quad c = vol(X)/vol(\check{\mathcal{D}})$$

In the case of $n' = 0$, $P'(k)$ is just a number and it has two important signification, i.e. on the one hand it is a special value of a certain L-series and on the other hand is a kind of 'index defect'. (Hirzebruch's conjecture in the case of Hilbert-modular group, now proved by Atiyah-Donelly-Singer).

In general we have a conjecture saying that $P'(k)$ has a similar decomposition into sum of multiples of a certain univ. constant, a special value of L-series, and a universal polynomial of degree ν, when the latter two depends on the boundary components of dim ν.

Such decomposition is known for the Siegel upperhalf plane and principal congruence subgroups of $Sp(2p, \mathbb{Z})$ due to Shintani and Hashimoto, which gives a non-trivial support for this conjecture.

Namikawa
浪川幸彦
(Nagoya Univ. z.Z. Bonn)

Projectivity criteria for threefolds

A well-known theorem of Moišezon states that every Moišezon manifold[1] which is Kähler is already projective. For many purposes however the Kähler assumption is too strong. In the case dim $X = 3$ the assumption can be weakened in the following form:

Any Moišezon threefold without effective curves homologous to 0 is projective.

By an effective curve we mean a finite linear combination $\sum n_i C_i$, where all $n_i \geq 0$ and $C_i \subset X$ are irreducible curves.

The theory of positive closed currents is essentially used for the proof. Denoting by $P_a^1(X)$ the cone of positive currents of the form $\lim_{j\to\infty} \sum_{i=1}^{n_j} \lambda_{ij} T_{C_{ij}}$ (in the weak topology),

[1] i.e. a compact complex manifold with dim X algebraically independent meromorphic functions.

where $d_{ij} \in \mathbb{R}$ and $C_{ij} \subset X$ irreducible curves one proves under the assumption: $P_a^1(X) \cap d\mathcal{D}^3(X) = \{0\}$ ($\mathcal{D}^3(X)$ = space of 3-currents; X Moišezon) the existence of a real (1,1)-form ω and a real 2-form φ such that:

i) ω is positive definite; ii) $d(\omega - \varphi) = 0$, iii) $\int_C \varphi = 0$ for all curves C.

Since any Moišezon manifold carrying such ω, φ is already projective (generalizing Moišezon's theorem), a sufficient and necessary condition for X to be projective is $P_a^1(X) \cap d\mathcal{D}^3(X) = \{0\}$.

In the case of $\dim X = 3$ it can be shown that this is already true if we only assume the non-existence of effective curves ~ 0.

As application it is shown that any Moišezon manifold which is topologically isomorphic to $\mathbb{P}_3$ is biholomorphic equivalent to $\mathbb{P}_3$.

Thomas Peternell
(Münster)

Hypersurfaces of the flagmanifold: Classification and deformation.

A homogeneous-rational manifold is a compact, projective-algebraic, homogeneous manifold, which is bimeromorphic to a projective space. By results of Goto, Borel and Remmert these are exactly the manifolds of the form $Z = G/H$ with a complex, semisimple and simply-connected Liegroup G and a complex parabolic subgroup H.

Every linebundle on Z is homogeneous in the sense of Bott, hence uniquely determined by the weight λ of the corresponding representation of Lie H on $\mathbb{C}$. We define a natural number p, the defect of positivity of L, which can be red off from the explicit form of λ, and prove the following Lefschetz-theorem:

Let X be the smooth zero-variety of a section $s \in H^0(Z, L)$. Then the natural mappings $H_j(X) \to H_j(Z)$ are bijective for $j \le n-2-p$ and surjective for $j = n-1-p$, $n = \dim Z$.

In the case $p = 0$ one obtains the well-known theorem for positive linebundles.

The proof makes use of an invariant metric of L, whose curvature-form can be expressed by the representation. Then it goes along the lines of the old proof of Bott using Morse-theory.

A simple example of a homogeneous-rational manifold of rank 2 is the flag-manifold $\mathbb{F}$ of point and line in $\mathbb{P}_2$; we have $\mathrm{Pic}(\mathbb{F}) = \mathbb{Z}^{\oplus 2}$. We classify the hypersurfaces $X = \mathrm{Var}(s)$, $s \in H^0(\mathbb{F}, L)$ and $L \in \mathrm{Pic}(\mathbb{F})$, according to their bidegree $(d_1, d_2) \in \mathbb{N} \times \mathbb{N}$, give the connection with the Kodaira-dimension of the two-dimensional surface X and determine those cases, where the natural family of hypersurfaces in $\mathbb{F}$, parametrized by $H^0(\mathbb{F}, L)$ is a complete resp. not complete deformation.

In particular we obtain a family of K3-surfaces X with Picard-number $\rho \geq 2$ and Kodaira-Spencer map $T_0 S \to H^1(X, \Theta_X)$ of codimension 2.

Joachim Wehler (München)

Über Hodge-Strukturen auf abelschen komplexen Lie-Gruppen

Hodge-Strukturen auf den Cohomologiegruppen kompakter Kählermannigfaltigkeiten oder algebraischer Varietäten stellen ein wichtiges Hilfsmittel für die Behandlung verschiedener Probleme der algebraischen und analytischen Geometrie dar; aber auch in anderen Kategorien scheinen Hodge-Strukturen interessante Anwendungen zuzulassen.

Wir betrachten hier als Objekte exakte Sequenzen $0 \to Z \to X \xrightarrow{\pi} Y \to 0$ von zusammenhängenden abelschen komplexen Lie-Gruppen, wobei Y kompakt und Z Steinsch (also $Z \cong \mathbb{C}^n \times (\mathbb{C}^*)^n$) ist. Die kompakten komplexen Lie-Gruppen sowie die kommutativen algebraischen Gruppen können in natürlicher Weise als Teilkategorien der so definierten Kategorie aufgefaßt werden.

Hier tragen die Cohomologie-Gruppen $H^r(Y, \mathbb{Z})$

und $H^r(Z, \mathbb{Z})$ natürliche reine Hodge-Strukturen vom Gewicht r bzw. $2r$ (die Hodge-Struktur auf $H^r(Z, \mathbb{Z})$ ist vom Typ (r,r)).
Die exakte Sequenz der Cohomologiegruppen

$$0 \to H^r(Y, \mathbb{Z}) \to H^r(X, \mathbb{Z}) \to H^r(Z, \mathbb{Z}) \to 0$$

gestattet es, auf $H^r(X, \mathbb{Z})$ eine gemischte Hodge-Struktur $(W_\cdot, F^\cdot)$ ($W_\cdot$ Gewichtsfiltrierung, $F^\cdot$ Hodge-Filtrierung) zu definieren, die man als (separierte) Erweiterung der Hodge-Struktur auf Z durch die Struktur auf Y gewinnt. Die so definierten Erweiterungen von Hodge-Strukturen verhalten sich funktoriell bei Morphismen von exakten Sequenzen und stimmen im kompakten sowie im algebraischen Fall mit den üblichen Strukturen überein.

Anwendungen:

Wir fixieren auf der abelschen komplexen Lie-Gruppe X eine "pseudo-algebraische" Struktur

$$0 \to Z \to X \to Y \to 0.$$

1) Benutzung der Hodge-Struktur auf $H^2(X, \mathbb{Z})$ und eine einfache Analyse der exakten Exponentialsequenz ergibt die folgende Beschreibung der Néron-Severi-Gruppe

$$NS(X) = H^2(X, \mathbb{Z}) \cap (H^{1,1} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{2,2})$$
$$= H^2(X, \mathbb{Z}) \cap (F^1 \cap \overline{F^1})$$

2) Die Hodge-Struktur auf $H^1(X, \mathbb{Z})$ definiert in natürlicher Weise ein Motiv $u: L \to \mathcal{J}^1$, wobei L das Gitter $H^{1,1} \cap W_2/W_1$ und $\mathcal{J}^1$ die Jacobi-Varietät $H^1(X, \mathbb{C})/F^1 + H^1(X, \mathbb{Z})$ ist.
In unserem Fall ist $\mathcal{J}^1$ isomorph zum

dualen Torus $\hat{Y}$ (insbesondere ist Y durch die Hodge-Struktur auf $H^1(X,\mathbb{Z})$ bestimmt) und der Cokern des Motivs $\bar{u}$ ist isomorph zur Gruppe $Pic^0_\Theta(X) \subset Pic^0(X)$ der topologisch trivialen Theta-Bündel.

Probleme:

1.) Torelli-Problem: Inwieweit ist die Struktur von X (d.h. die komplexe Struktur und die durch die exakte Sequenz $0 \to Z \to X \to Y \to 0$ gegebene "pseudo-algebraische" Struktur) bestimmt durch die Erweiterung von Hodge-Strukturen?

2.) Welche Erweiterungen von Hodge-Strukturen kommen im Falle von abelschen komplexen Lie-Gruppen vor?

Erich Selder (Osnabrück)

The canonical hermitian form for an isolated singularity of hypersurface

For $f: X \to D$ a Milnor representative of a germ of holomorphic function with a isolated singularity on $\mathbb{C}^{n+1}$ we study the module of asymptotic expansions $\mathcal{M}$ for the integrals $\int_{f=s} \varphi$ (where $\varphi \in C_c^\infty(X)$ is a (n,n) form) at $s=0$.

By considering the following sesquilinear (on $\mathbb{C}\{s\}$) hermitian quasi horizontal form on the relative De Rham cohomology $\Omega^n_X / df \wedge \Omega^{n-1} + d\Omega^{n-1} = \underline{F}$ endowed with the Gauss-Manin connexion ∇

($\nabla \omega = \frac{d\omega}{df}$)

$$H(\omega,\omega') = \underset{\text{at } s=0}{\text{expansion}} \left[i^n \int_{f=s} \rho\, \omega \wedge \bar{\omega}' \right] \bmod \mathbb{C}[[s,\bar{s}]]$$

which is canonical ($\rho \in C_c^\infty(X)$ $\rho = 1$ near 0)
we have the following theorems

Th1

If $\omega_1 \ldots \omega_\mu \in \Omega^n_X$ generates $\underset{\sim}{F}$ on $\mathbb{C}\{s\}$ then M is generated on $\mathbb{C}[[s,\bar{s}]]$ by 1 and the expansions of the integrals $i^{n^2} \int_{f=s} \rho\, \omega_i \wedge \bar{\omega}_j$ $i,j \in [1,\mu]$

Then we define on $E = H^n(X(s_0), \mathbb{C})$ a hermitian form h invariant by the monodromy T, associated to H called the canonical hermitian form

Th2

If $1 \notin \mathrm{Spec}(T)$ h correspond to the hermitian intersection form on E (if $1 \notin \mathrm{Spec}\, T$ the natural map $H^n_c(X(s_0), \mathbb{C}) \to H^n(X(s_0), \mathbb{C})$ is an isomorphism)

Th3

The canonical hermitian form h is non degenerated

Daniel BARLET (NANCY)

Let M be a Cauchy-Riemann submanifold of $\mathbb{C}^n$ (generically embedded). Assume that there exists a smooth distribution $L : M \to T(M)$ and a smooth CR function $A : M \to GL(n, \mathbb{C})$ such that the following inequalities holds

$$|\mathrm{Im}\, v| < |\mathrm{Re}\, v| \quad \text{for} \quad v \in A(p) L_p .$$

Then there exists a neighbourhood Ω of M in $\mathbb{C}^n$ with the following property:
for any smooth CR function $f : M \to \mathbb{C}$ and any

strongly pseudoconvex domain $D \Subset \Omega$ there exist a sequence of holomorphic functions F_ν on $\bar{D}$ which uniformly approximates ~~f~~ on $\bar{D} \cap M$ the CR function ~~f~~ f.

In the proof is used a generalized local approximation integral formula of Baouendi - Treves and the first Cousin problem with bounds.
It will be interesting to replace the assumptions on the distribution L and the function A by another assumptions.

Roman Dwilewicz
(Warsaw)

Zum Kürzungproblem für kompakte komplexe Räume

Es sei $\mathcal{C}$ die Kategorie der reduzierten irreduziblen kompakten komplexen Räume. Für $d \in \mathbb{N}$ bezeichne $\mathcal{T}_d$ die Klasse der $X \in \mathcal{C}$ mit folgender Eigenschaft:

Es gibt einen Torus $T = \mathbb{C}^d/\Gamma$, $k \in \mathbb{N}+1$, $X_0 \in \mathcal{C}$ und $\chi \in \mathrm{Hom}(\Gamma/k\Gamma, \mathrm{Aut}\, X_0)$ derart, daß $X = T \times X_0 / \mathrm{graph}\, \chi$.

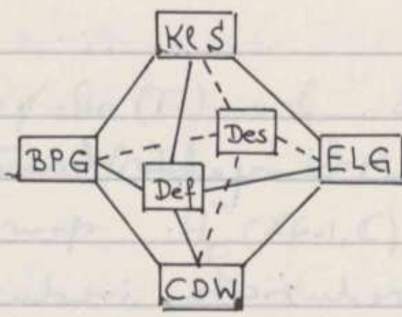
Weiter sei $\mathcal{T} := \bigcup_{d>0} \mathcal{T}_d$.

Kürzungssatz: Sei $X \times Z \cong Y \times Z$ in $\mathcal{C}$.
Ist $\{X, Y, Z\} \not\subset \mathcal{T}_d$ für alle $d > 0$, so ist $X \cong Y$.
Insbesondere ist jedes $Z \in \mathcal{C} \setminus \mathcal{T}$ kürzbar.

Camilla Horst (München)

Deformation Kleinscher Singularitäten und klassische Invariantentheorie

Die Kleinschen Singularitäten (KlS) stellen vermöge ihrer Desingularisations- und Deformationstheorie eine Korrespondenz her zwischen den binären Polyedergruppen (BPG) und den einfachen komplexen Liegruppen mit homogenen Wurzelsystemen (ELG) (unter Einbeziehung kombinatorischer Daten (CDW)). Es wurden zunächst einige der bekannten Relationen an Hand der folgenden Figur erläutert:



Aus früheren Arbeiten von Schlessinger und Pinkham wurde folgendes Ergebnis abgeleitet (gemeinsame Arbeit mit K. Behnke und H. Knörrer), bei dem der klassische Clebsch-Gordan-Isomorphismus eine wesentliche Rolle spielt:

<u>Satz 1</u>. Sei G eine (kleine) endliche Untergruppe von $GL_2 = GL_2(\mathbb{C})$, $X = \mathbb{C}^2/G$ die zugehörige Quotientensingularität und T^1_X der Vektorraum der infinitesimalen Deformationen von X; ferner bezeichne S_{χ^k} die mit der k-ten Potenz des Charakters $\chi = \det$ tensorierte kanonische Darstellung von GL_2 auf $S = \mathbb{C}[u, v]$. Dann gilt

$$(T^1_X)^* \cong \operatorname{coker}\Big(\bigoplus_{j=1}^n S^G_\chi \longrightarrow S^G_{\chi^2} \oplus S^G_{\chi,\geq 1}\Big),$$
$$(\omega_1, \dots, \omega_n) \mapsto \Big(\sum_{j=1}^n J(\omega_j, P_j), \sum (\deg P_j) P_j \omega_j\Big).$$

(Hierbei bezeichnet $P_1, \ldots, P_n$ ein Fundamentalsystem homogener Invarianten von $\mathcal{S}^G$, $\mathcal{S}_{\geq 1}$ bezeichnet die Polynome vom Grad ≥ 1, und $J(\omega, P)$ ist die Jacobische von ω und P).

<u>Korollar</u>. Für $G \subset SL_2$ gilt $(T^1_X)^* \cong \mathcal{S}^G / \langle J(P,Q) : P, Q \in \mathcal{S}^G \rangle$

Mit Hilfe von Satz 1 wurde von Constantin Kahn gezeigt:

<u>Satz 2</u>. $\dim T^1_X = \dim \widetilde{T}^1_X + (e-4)$,
falls die Einbettungsdimension e von $X = \mathbb{C}^2/G$ größer oder gleich 4 ist (d.h. $G \not\subset SL_2$; $\dim \widetilde{T}^1_X$ ist die Dimension der "Artin"-Komponente, die leicht aus den Selbstschnittzahlen b_ρ der Komponenten der exzeptionellen Menge in der minimalen Auflösung von X berechnet werden kann als $\sum_\rho (b_\rho - 1)$).

Oswald Riemenschneider (Hamburg)

Levi-Metriken und $\bar{\partial}$-Neumann-Problem
(nach Lieb/Range)

Es sei $\mathcal{G} \subset\subset X$ ein streng pseudokonvexes glatt berandetes Gebiet in einer n-dimensionalen komplexen Mannigfaltigkeit X, auf der eine hermitische Metrik ds^2 gewählt wird. Zu $\bar{\partial} : L^2_{0q} \to L^2_{0q+1}$ wird der Hilbertraum-adjungierte Operator $\bar{\partial}^* : L^2_{0q+1} \to L^2_{0q}$ definiert; der Definitionsbereich von $\bar{\partial}^*$ ist durch die – nicht-elliptische – komplexe Neumann-Bedingung abgegrenzt. 1963 bewies KOHN die „basic estimate"

$$(1) \qquad \|f\|_{H^{1/2}} \leq \mathrm{const}\left(\|f\|_{H^0} + \|\bar{\partial} f\|_{H^0} + \|\bar{\partial}^* f\|_{H^0}\right)$$

für $f \in \operatorname{Dom} \bar{\partial} \cap \operatorname{Dom} \bar{\partial}^* \subset L^2_{0q}$, $q \geq 1$. Die H^α sind die L^2-Sobolev-Räume. Die Metrik darf beliebig sein.

Für feinere Abschätzungen benötigt man eine dem Gebiet angepaßte Metrik: es sei $G = \{r < 0\}$, r streng plurisubharmonisch, und $d\sigma^2 = \sum (\partial^2 r/\partial z_i \partial \bar{z}_j)\, dz_i\, d\bar{z}_j$ in lokalen Koordinaten. $d\sigma^2$ ist eine Kähler-Metrik in einer Umgebung des Randes von G. Jede hermitische – i.a. nichtkählersche – Metrik ds^2 auf X, die in einer Umgebung von bG die Gestalt $ds^2 = f(x)\, d\sigma^2$, $f(x) > 0$, für eine geeignete Randfunktion r hat, heißt eine Levi-Metrik für G. Levi-Metriken sind auf dem Bündel der holomorphen Tangentialräume an bG bis auf konforme Äquivalenz eindeutig bestimmt. Wir zeigen folgende „basic estimate" in C^k- und Höldernormen:

$$(2) \qquad \|f\|_{C^{k+1/2}} \le \text{const} \left(\|f\|_{L^2} + \|\bar\partial f\|_{C^k} + \|\bar\partial^* f\|_{C^k} \right)$$

für $f \in \text{Dom}\, \bar\partial \cap \text{Dom}\, \bar\partial^* \subset L^2_{0q}$, $q \geq 1$, falls ds^2 eine Levi-Metrik ist. Für $q = 0$ lautet die Abschätzung

$$(3) \qquad \|f - Pf\|_{C^{k+1/2}} \le \text{const}\, \|\bar\partial f\|_{C^k},$$

wobei P die Bergman-Projektion $L^2_{00} \to L^2_{00} \cap \mathcal{O}(G)$ ist; die Metrik darf beliebig sein. (2) ist neu, (3) war vorher bekannt gewesen (PHONG/STEIN; GREINER/STEIN; LIGOCKA; AHERN/SCHNEIDER). (2) und (3) enthalten einen großen Teil der Theorie der Cauchy-Riemannschen Differentialgleichungen auf G.

Der – für (2) und (3) einheitliche – Beweis beruht auf einer neuen Integraldarstellung, die durch Kombination potentialtheoretischer Methoden mit der Theorie der Cauchy-Fantappiè-Kerne gewonnen wird, und auf Symmetrieeigenschaften der gewonnenen Kerne, die eine „Auslöschung von Singularitäten" bewirken.

Ingo Lieb

Harmonische Abbildungen und schwache 1-Vollständigkeit

Für ein relativ-kompaktes Gebiet Ω in einer ~~St~~ komplexen Mannigfaltigkeit M werden folgende Begriffe betrachtet:

<u>Def.</u> a) Ω heißt pseudokonvex (pk), wenn es zu jedem $q \in \partial\Omega$ eine Umgebung V und auf $V \cap \Omega$ eine streng plurisubharmonische Funktion φ gibt mit $\{\varphi < c\} \cap \partial\Omega = \emptyset$ für alle $c \in \mathbb{R}$

b) Ω heißt schwach 1-vollständig, wenn es auf Ω eine C^∞ plurisubharmonische Funktion φ gibt mit $\{\varphi < c\} \subset\subset \Omega \ \forall c \in \mathbb{R}$

Es gilt immer b) $\Rightarrow$ a). Andererseits impliziert b) wichtige Verschwindungs- und Endlichkeitssätze (Nakano, T. Ohsawa u.a.), während a) leichter nachprüfbar und lokal ist. Es stellt sich deshalb die <u>Frage</u>: Gilt auch a) $\Rightarrow$ b)?
Daß die Antwort darauf i.a. NEIN ist, folgt aus:

<u>Theorem 1.</u> Es gibt ein lokal triviales holomorphes Kreisbündel $\Omega \to H$ über einer Hopf-Fläche H, so daß Ω nicht durch relativ kompakte pk. Gebiete ausschöpfbar ist. (Diederich/Fornæss 1983)
(Man bemerke, daß Ω in dem zugehörigen $\mathbb{P}^1$-Bündel $\hat{\Omega}$ pk. mit glattem C^ω-Rand ist)

Über kompakten Kähler-Mannigfaltigkeiten sind solche Gegenbeispiele jedoch nicht möglich:

Theorem 2 (Diederich/Ohsawa 1984) Sei $\tilde{M}$ eine komp. Kähler-Mannigfaltigkeit und $\Omega \to \tilde{M}$ ein lokal-triviales hol. Kreisbündel. Dann ist Ω schwach 1-vollständig

Der Beweis geschieht zunächst für den Fall, daß das zugehörige $\mathbb{P}^1$-Bündel $\hat{\Omega} \to \tilde{M}$ keinen flachen Schnitt zuläßt. Dann läßt sich nämlich mit Hilfe der Techniken von Eells, Sampson, Hamilton ein harmonischer Schnitt $s: \tilde{M} \to \hat{\Omega}$ konstruieren. Die faserweise genommene Poincaré-Distanz zum Schnitt (im Quadrat) erweist sich (mit Sätzen von Siu) als plurisubh. Ausschöpfungsfunktion. –

Klas Diederich

Die Dimension glättender Komponenten

(Bericht über eine gemeinsame Arbeit mit E. Looijenga)

Sei (X,x) komplexer Raumkeim mit isolierter Singularität und $F: (Y,y) \to (S,s)$ die semiuniverselle Deformation von (X,x). Eine irreduzible Komponente (S',s) von (S,s) heißt glättende Komponente, falls die generische Faser über diese Komponente glatt ist. Sei Δ die komplexe Einheitskreisscheibe und $j: (\Delta,0) \to (S',s)$ eine holomorphe Abbildung, so daß die generische Faser X_t von $f = F^*(j): (\mathcal{X},x) \to (\Delta,0)$ glatt ist. Dann gilt:

Satz: $\dim (S',s) = \dim_{\mathbb{C}} \operatorname{coker} (\Theta_{\mathcal{X}/\Delta,x} \to \Theta_{X,x})$

wobei $\Theta_{X,x}$ bzw. $\Theta_{\mathcal{X}/\Delta,x}$ die Vektorfelder auf (X,x) bzw. die relativen Vektorfelder bezeichnet.

Der Satz ist eine Folgerung aus der folgenden allgemeineren Aussage:

Sei f wie oben das pull back von j aber X_t sein nicht notwendigerweise glatt. Dann gilt:

$$\dim (S',s) = \dim_{\mathbb{C}} \operatorname{coker}(\Theta_{X/D,x} \to \Theta_{X,x}) + \sum_{x \in \operatorname{Sing}(X_t)} \dim S(X_t,x)$$

wobei $S(X_t,x)$ die Basis der semiuniversellen Deformation von (X_t,x) ist.

Der obige Satz war von J. Wahl (Topology 1980) vermutet und in einigen Spezialfällen bewiesen worden. Außerdem hat er dort gezeigt, daß die rechte Seite der Gleichung des Satzes im Fall normaler Flächensingularitäten (X,x) explizit durch Invarianten der minimalen Auflösung und der topologischen Eulercharakteristik von X_t ausgedrückt werden kann. Ähnliche Formeln wie oben gelten entsprechend ~~für~~ in ganz allgemeinem Zusammenhang in komplexen gefaserten Gruppoiden über komplexen Raumkeimen für Objekte mit semiuniverseller Deformation, also z.B. für Deformationen kompakter komplexer Räume, Deformationen von holomorphen Vektorbündeln auf kompakten komplexen Räumen, Deformationen kohärenter Garben mit isolierten Singularitäten etc.

Gert-Martin Greuel

Deformationen des $\mathbb{P}^3 = \mathbb{P}^3(\mathbb{C})$.

Es wird eine Beweisskizze der folgenden Aussage gegeben:

$\{X_t \mid t \in Y = \{|t| < 1\}\}$ sei eine holomorphe Familie kompakter, komplexer 3-Mannigfaltigkeiten. Für $t \in Y$, $t \neq 0$, sei X_t analytisch isomorph zum $\mathbb{P}^3$. Dann ist X_0 analytisch isomorph zum $\mathbb{P}^3$.

N. Kuhlmann (Essen)

The Babylonian tower theorem in the local case

Let $\mathcal{E}$ be the germ of a coherent module on $(\mathbb{C}^n, 0)$ with isolated singularity, i.e. $\mathcal{E}$ is defined in some neighbourhood V of 0 and locally free except at 0. $\mathcal{E}$ is said to be extendable to $(\mathbb{C}^{n+k}, 0)$ if there is a coherent module $\mathcal{F}$ on $(\mathbb{C}^{n+k}, 0)$ with isolated singularity such that $\mathcal{F}|\mathbb{C}^n \times \{0\}$ and $\mathcal{E}$ are isomorphic on some punctured neighbourhood $U = V \setminus \{0\}$ of 0 in $\mathbb{C}^n$. Horrocks has posed the problem if there is always a k (depending on $\mathcal{E}$) such that $\mathcal{E}$ cannot be extended to $(\mathbb{C}^{n+k}, 0)$. One of the main results shown in this talk is that this is indeed true. The corresponding question in the projective case was well known before (Barth – Van de Ven 1974, Sato, Tyurin 1976, 1977), and in the local case it has been solved if in addition $\mathrm{Ext}^i(\mathcal{E}, \mathcal{O}_{\mathbb{C}^n}) = 0$ for some i in the range $1 \le i \le n-2$ (Horrocks 1966 if $i = 1$, Evans – Griffiths 1983/84). The main ~~result~~ idea used in our proof is to apply formal deformation theory. A similar result can be obtained for locally complete intersections instead of modules.

Hubert Flenner

Flächen im $\mathbb{P}^4$

Die Klassifikation der glatten Flächen im $\mathbb{P}^4$ vom Grad ≤ 8 wird angegeben. Die folgende Liste enthält alle diese Flächen außer den vollständigen Durchschnitten (d = Grad, π = Geschlecht eines glatten Hyperebenenschnittes)

d	π	Fläche	d	π	Fläche
			$\cancel{8}$ 7	4	$\tilde{\mathbb{P}}^2(x_1, \dots, x_{11})$
3	0	$\mathbb{F}_1$	7	5	$\tilde{Y}_0(y_0)$, $Y_0 = S_{(2,2,2)}$
4	0	$\mathbb{P}^2$ (Veronese)	7	6	$g_{\lvert K \rvert}: Y \to \mathbb{P}^1$ ell.
5	1	$\mathbb{P}_C(E)$	8	5	$\tilde{\mathbb{P}}^2(x_1, \dots, x_{11})$
5	2	$\tilde{\mathbb{P}}^2(x_0, x_1, \dots, x_7)$ (Castelnuovo)	8	6	$\tilde{\mathbb{P}}^2(x_1, \dots, x_{16})$
6	3	$\tilde{\mathbb{P}}^2(x_1, \dots, x_{10})$ (Bordiga)	8	6	$\tilde{Y}_0(y_0)$, Y_0 K3
			8	7	$g_{\lvert K \rvert}: Y \to \mathbb{P}^1$ ell.

Ch. Okonek

Finiteness results for algebraic K3 surfaces

The 2^{nd} cohomology lattice of an algebraic K3 surface contains the Néron-Severi group NS as a primitive nondegenerate sublattice of hyperbolic signature. The set $\{ x \in NS \otimes \mathbb{R} \mid x^2 > 0 \}$ consists of 2 disjoint connected cones only one of which contains ample classes: $\mathcal{C}$. We construct a rational polyhedral fundamental domain for the action of a certain arithmetic subgroup of the orthogonal group of $NS \otimes \mathbb{R}$ on $\mathcal{C}_+ :=$ convex hull of $(\bar{\mathcal{C}} \cap NS \otimes \mathbb{Q})$ which satisfies a finiteness-property: Siegel-property. This subgroup is related to the automorphism group of the K3 surface. This relation and the above mentioned finiteness property enable us to prove that the automorphism group has only a finite number of orbits in the set of complete linear systems which contain an irreducible curve of fixed selfintersection.

Hans Sterk
(Nijmegen, Niederlande)

Moduli polarisierter, kompakter Kähler-Mannigfaltigkeiten

Unter einer polarisierten Kähler-Mannigfaltigkeit versteht man eine solche Mannigfaltigkeit zusammen mit einer Kählerklasse.

<u>Satz:</u> Sei $\mathcal{R}$ eine Klasse polarisierter, kompakter Kähler-Mannigfaltigkeiten. Dann gibt es einen groben Modulraum für $\mathcal{R}$, falls gilt:

(i) $\mathcal{R}$ ist stabil unter kleinen Deformationen

(ii) Sind $(X \to S, \lambda_{X/S})$ und $(Y \to S, \lambda_{Y/S})$ Familien von polarisierten Mannigfaltigkeiten aus $\mathcal{R}$ so ist $\mathrm{Isom}_S^{\lambda}(X,Y) \to S$ eigentlich.

<u>Satz:</u> Die Klasse aller kompakten, polarisierten Kählermfr., welche keine Regelmannigfaltigkeiten sind und keine Deformationen in solche zulassen, besitzt einen groben Modulraum.

Es wurden auch Aussagen über die Modulräume $M_{c_1 \le 0}$ und $M_{c_1 = 0}$ gemacht.

Georg Schumacher (Münster)

Quotients by $\mathbb{C}^* \times \mathbb{C}^*$

(joint work with A. Bialynicki-Birula)
Univ. of Warsaw

Let $T \approx (\mathbb{C}^*)^k$. Let $T \times X \to X$ be a meromorphic action of T on a normal compact complex space X. We are interested in the following questions.

Question a) Classify all Zariski open $U \subseteq X$ with $U \to U/T$ a compact geometric or semi-geometric quotient.
b) study the geometry of such quotients.

The answer to the above when U/T is projective is partially given by Mumford's theory. It should be noted that even if X is a 4 dimensional quadric and $k=1$, there are U with complete non-projective schemes as quotients.

For $k=1$ a very complete answer to a) and a start on b) has been given. Let $F_1, \dots, F_r$ be the connected components of the fixed point locus of T. F_i is said to be directly less than F_j if there is an x such that $\lim_{t \to 0} tx \in F_i$, $\lim_{t \to \infty} tx \in F_j$, x is not a fixed point. F_i is said to be less than F_j, written $F_i < F_j$ if there is a sequence $F_i = F_{i_1}$ directly less than F_{i_2} .... directly less than $F_{i_n} = F_j$. A cross section of $\{F_1, \dots, F_r\}$ is a partition (A^-, A^+) of $\{F_1, \dots, F_r\}$ into two sets where $F_i < F_j \in A^- \Rightarrow F_i \in A^-$.

Theorem I $U \subseteq X$ with compact geometric quotient $U/\mathbb{C}^*$ are in 1-1 correspondence with ~~cr~~ cross sections. Precisely given (A^-, A^+)

$$U = \bigcup_{\substack{F_i \in A^- \\ F_j \in A^+}} \{x \in X \mid x \text{ not fixed pt}, \lim_{t \to \infty} tx \in F_j, \lim_{t \to 0} tx \in F_i\}$$

Here a ~~we~~ weak condition (satisfied if X is projective or Kähler, or a scheme) is used; this condition called local linearity says that each $x \in X$ has a Stein nbhd. equivariantly embedded in $\mathbb{C}^N$ with a linear action [all known examples have this property].

The ~~ab~~ above result was shown by A. Bialynicki-Birula + Sommese.
The generalization to char p + semi geometric quotients ~~they~~ was shown by
A. Bialynicki-Birula + J. Swiecicka
The generalization to semi-geometric quotients in the analytic category
by D. Gross (Notre Dame Thesis)

Using the above the solution of a conjecture of Mumford on quotients of open sets of $(\mathbb{P}^1)^n$ by $SL(2,\mathbb{C})$ is given (see Trans. A.M.S. (1983))

Also the betti numbers of $U/_T$ can be computed by using the Weil conjectures.

<u>Thm II</u> (B-B + So). Let X be an algebraic manifold with $T = \mathbb{C}^*$ as above. Let U have a compact geometric quotient $U/_T$ where U is associated to the cross section (A^-, A^+). Let $P(Z)$ denote the Poincaré polynomial of a space Z. Then:

$$P(U/_T) = \sum_{F_i \in A^-} \left(\frac{t^{2d_i^+} - t^{2d_i^-}}{t^2 - 1} \right) P(F_i)$$

Here d_i^+ = rank of plus part of Normal bundle of F_i
d_i^- = " " minus " " " " "

Recently using the moment map and an associated complex called the moment complex Bialynicki-Birula + ~~say~~ I have made progress for $k>1$. In particular for $k=2$, X a compact Kähler manifold, and geometric quotients, a complete answer analogous to THM. I has been given (to appear in TRans. Amer. Math Soc.)

Andrew J. Sommese

Quadric bundles associated to conic bundles and compactification of $M_{\mathbb{P}_3}(0,2)$

The space $M_{\mathbb{P}_3}(0,2)$ is the moduli scheme of stable rank 2 vectorbundles over $\mathbb{P}_3$ with Chern classes $c_1=0$, $c_2=2$. A compactification of it is described by a quadric bundle over the scheme of conics in the Graßmannian $G_{2,4} \subset \mathbb{P}_5$. More generally to any conic bundle one can associate a quadric bundle whose fibres consist of conics satisfying the Poncelet property with respect to a given one. This Poncelet property for regular conics $\gamma \subset \mathbb{P}_2$, $\gamma' \subset \mathbb{P}_2^*$ is: there exist triangles in $\mathbb{P}_2$ with vertices on γ such that the dual triangle in $\mathbb{P}_2^*$ has vertices on γ'. The set of such pairs including degenerate conics is a hypersurface $Q \subset \mathbb{P}_5 \times \mathbb{P}_5^*$ defined by a form of bidegree 2,2. The functor description allows us to associate quadric bundles to conic bundles. The singularities of such quadric bundles are well understood. In the case of the above compactification there is an interpretation for the corresponding classes of torsion free sheaves. It turned out that the sheaves corresponding to singular points of the compactified moduli scheme are exactly the semi-stable ones.

Günther Trautmann
(Kaiserslautern)

(Fortsetzung der Berichte „ Komplexe Analysis" in folgendem Buch)

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