MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 19/1974

General Group Theory

5.5. bis 11.5.1974

Die diesjährige Tagung stand wieder unter der Leitung der Herren Professoren W.Gaschütz (Kiel) und K.W.Gruenberg (London). Es haben 44 Mathematiker teilgenommen. 31 Vorträge aus der allgemeinen Gruppentheorie wurden gehalten. Behandelt wurden viele Fragen sowohl über endliche als auch über unendliche Gruppen, so daß sich ausreichend Stoff für vertiefende und anregende Diskussionen ergab.

<u>Teilnehmer</u>

R.Baer, Zürich
G.Baumslag, New York
B.Beisiegel, Mainz
H.Bender, Kiel
D.Blessenohl, Kiel
A.Brandis, Heidelberg
D.E.Cohen, London
D.J.Collins, London
R.S.Dark, Galway
K.Doerk, Mainz
E.Formanek
W.Gaschütz, Kiel
D.Gildenhuys, Montreal
F.Groß, Kiel
W.Gruenberg, London
T.O.Hawkes, Coventry
H.Heineken, Würzburg
Chr. Hering, Tübingen
J.A.Hulse, Edinburgh
B.Huppert, Mainz
Z.Janko, Heidelberg
W.Knapp, Tübingen
O.U.Kramer, Kiel
C.R.Leedham-Green, London
J.C.Lennox, Cardiff
F.Levin, Bochum
F.P.Lockett, Coventry
R.Maier, Tübingen
S.Moran, Canterbury
M.L.Newell, Galway
A.Rae, Uxbridge
D.J.S.Robinson, Zürich
K.W.Roggenkamp, Bielefeld
R.Sandling, Manchester
K.U.Schaller, Kiel
P.Schmid, Tübingen
R.Schmidt, Kiel
M.Selinka, Tübingen

U.Stammbach, Zürich
V.Stingl, Mainz
S.E.Stonehewer, Coventry
R.Strebel, Zürich
H.Wielandt, Tübingen
J.S.Williams, Bielefeld

Vortragsauszüge

R.BAER: Endliche Gruppen, deren Hauptfaktoren und darin induzierte Automorphismengruppen teilerfremde Ordnung haben.

Diese Gruppenklasse werde mit $\underline{\underline{H}}$ bezeichnet. Zunächst wird eine präzise Beschreibung der endlichen Gruppen G mit folgenden Eigenschaften gegeben: G ist auflösbar, gehört nicht zu $\underline{\underline{H}}$, während jede echte Untergruppe von G eine $\underline{\underline{H}}$-Gruppe ist. Diese Charakterisierung der minimalen nicht-$\underline{\underline{H}}$-Gruppen ermöglicht u.a. den Beweis des folgenden Satzes.

Die folgenden Eigenschaften der endlichen Gruppe G sind äquivalent:
(1) $G \in \underline{\underline{H}}$.
(2) Jede von zwei Elementen aus G erzeugte Untergruppe von G ist eine $\underline{\underline{H}}$-Gruppe.
(3) G ist auflösbar und jeder Faktor von G ist p-normal für jede Primzahl p.
(4) Ist $U \leq G$ und o(U) durch höchstens n verschiedene Primzahlen teilbar, so ist die Fittinglänge von U höchstens n.
(5) Einreihige Faktoren von G sind Sylowturmgruppen.
(6) G ist auflösbar; ist U eine Untergruppe von G und S eine Sylowuntergruppe von U, sind a und b Elemente aus U, so daß a und a^b in S liegen, so gibt es ein Element c im Normalisator von S in U mit $a^b = a^c$.

B.BEISIEGEL: Simple groups with Sylow-2-subgroups of order at most 2^{10}.

Problem (P) Suppose G is a group satisfying the following conditions
(a) G is finite, simple and nonabelian.
(b) The order of a Sylow-2-subgroup of G is at

most 2^{10}.

(c) The centralizers of involutions in G are 2-constrained.

Determine the structure of the group G!

Suppose the group G^* satisfies (a), (b), (c) and also the following conditions

(d) The sectional 2-rank $r(G^*)$ of G^* is greater than 4.

(e) G^* contains a nonsolvable 2-local subgroup.

It is sufficient to determine the structure of the group G^* in order to get a solution for (P).

A preliminary result. Suppose the group G^* satisfies

(f) G^* contains a 2-local subgroup M such that the group $M/O_2(M)$ has a subgroup isomorphic to PSL(2,7) or to PSL(2,8). Then G^* is isomorphic to one of the following groups: GL(5,2), M_{24}, PSP(6,2), PSL(3,8).

A conjecture. Suppose G^* satisfies (not f). Then we have $G \cong$ PSP(4,4).

H.BENDER: <u>The generalized p -core of a finite group.</u>

Definition: H is a p^*-group $\Longleftrightarrow H = NO^p(C_H(N_p))$ whenever $N \lhd H$ and $N_p \in Syl_p(N)$.

$O_{p^*}(G)$ = the unique largest normal p^*-subgroup of G.

$O_{p^*}(G)$ behaves in a similar way as $O_{p'}(G)$ does for solvable G.

Some important facts:

(1) $O_{p^*}(C_G(P)) \leq O_{p^*}(G)$, for any p-subgroup P of G.

(2) If $P \in Syl_p(O_{p^*,p}(G))$, then $C_G(P) \leq O_{p^*,p}(G)$.

(3) If $C_{G_p}(A) \leq A \lhd G_p \in Syl_p(G)$, then every A-invariant p^*-subgroup lies in $O_{p^*}(G)$.

(4) Let P be a p-group acting on a p^*-group K.

(i) $K = [P,K]O_{p^*}(C_K(P))$;

(ii) $[P,K]$ is a p^*-group equal to $[P,[P,K]]$;

(iii) if K has a normal subgroup K_o such that $C_K(K_o) \leq K_o{}^P$ and $O_{p^*}(K_o) \leq C_K(P)$, then $C_K(P) = K$.

(5) $O_{p^*}(G) = O_{p',E}(G)O_{p'}(C_G(P))$ with $P \in Syl_p(O_{p',F^*}(G))$, provided any simple factor group X of $E(G/O_{p'}(G))$ satisfies: $C_{Aut(X)}(X_p)$ is p-solvable for $X_p \in Syl_p(X)$. By a theorem of Glauberman, this is true if p = 2.

A.BRANDIS: Untergruppen freier Gruppen.

Sei G ein Graph, $\mathfrak{W}$ seine Wegegruppe. Ein Erzeugendensystem B in $\mathfrak{W}$ heißt Basis, wenn für alle u, v, w $\in$ B, die paarweise verschieden sind, gilt:

$$l(uvw^{\varepsilon}) > l(u) + l(w) - l(v), \quad \varepsilon = \pm 1,$$

wobei l(u) die Länge des Weges u bedeutet.
Es gilt der Satz:
Jede Basis ist frei. Es gibt einen vollständigen Baum in G, so daß B in bekannter Weise ein zu diesem Baum gehöriges Erzeugendensystem ist.
Dieser Satz ist eine Verallgemeinerung eines Satzes, den der Vortragende zusammen mit K.Reidemeister bewiesen hat.
Der Satz wurde für den Fall interpretiert, wo der Graph der Restklassenkomplex einer freien Gruppe nach einer Untergruppe ist. (Beweis des Freiheitssatzes von Schreier und Nielsen).

D.J.COLLINS: Magnus subgroups of one relator groups.

B.Neuman has proved that, as abelian subgroups, a one-relator group may have only locally cyclic groups or free abelian groups of rank at most 2.
We report on the result of G.Bagherzadeh, which states that if the direct product H $\times$ K is embeddable in a one-relator group then H and K are free and one is cyclic.
The proof uses the following lemma:
Let M be a Magnus subgroup of a one-relator group G (i.e. M is a free subgroup generated by a subset of the generators of G) and let g $\in$ G $\setminus$ M. Then $g^{-1}Mg \cap M$ is cyclic.

R.S.DARK: A complete group of odd order.

Let G be the semilinear group of the field K of order 7^3, i.e. G = ACU with A = Aut K $\cong C_3$, C = $K^{\times} \cong C_{342}$, U = $K^{+} \cong E_{7^3}$.
Then G is complete (Rose) but has even order.
Define P = $\langle x, y, z: P^7 = P'' = P_5 = 1 \rangle$, Γ = Aut P, Δ = $C_{\Gamma}(P/P')$. Then Γ/Δ = Aut U, Δ is a 7 group. Thus AC $\leq \Gamma/\Delta$; & AC $\leq \Gamma$ (Schur-Zassenhaus). ACP is still complete. Now

$|P_2/P_3| = 7^3$, $|P_3/P_4| = 7^8$, $|P_4| = 7^{15}$.
Also $P_3/P_4 = V \times X$, $P_4 = W \times Y$ as AC-modules, where $|V| = |W| = 7^3$ and, writing $B = C'^8 \cong C_{19}$, there is an AB-isomorphism $\vartheta : V \to W$. Regard V,X as subgroups of P' (Maschke), and define $R = \langle Y, vv^{\vartheta} : v \in V \rangle$.
Then $R \triangleleft ABP$; in fact $N_C(R) = B$.
If S/R is the centre of ABP/R, then $|S/R| = 7^2$ and ABP/S is a complete group of order $3.19.7^{12}$.

E.FORMANEK: The Automorphism Group of a Free Group is Complete.

Any group G acts on itself by conjugation, and this induces a homomorphism $G \to A(G)$, where A(G) denotes the automorphism group of G. G is said to be complete if $G \to A(G)$ is an isomorphism. An outline of the proof of the following result, which affirmed a conjecture of G.Baumslag, was given:
Theorem (J.L.Ryer - E.Formanek) Let F be a free group of finite rank $n \geq 2$. Then A(F) is complete.

W.GASCHÜTZ: Eine historische Bemerkung zum Satz von Schreier über die Untergruppen freier Gruppen.

Hinweis auf eine Arbeit in Crelles Journal aus dem Jahre 1902 von "Herrn Hoyer", in der die Erzeugendenzahlformel $n(\nu - 1) + 1$ für die genannten Untergruppen schon zu finden ist.

W.GASCHÜTZ: Über normale Fittingklassen.

Die Gruppenkonstruktion von Lausch zu normalen auflösbaren Fittingklassen wird auf etwas anderem Wege als bei Lausch durchgeführt und in ihrer Universalität diskutiert.

D.GILDENHUYS: The cohomology of groups acting on trees.

Suppose a group G acts on a tree X. Let us write V(X) for the set of vertices of X, E(X) for the set of edges, G_P (resp. G_x) for the stabilizer of a vertex P (resp. edge x), cd G for the cohomological dimension of G and

$$n_V = \sup \{ cd\ G_P \mid P \in V(X) \} \leqslant \infty ,$$
$$n_E = \sup \{ cd\ G_x \mid x \in E(X) \} \leqslant \infty .$$

We give a spectral sequence for the cohomology of G and prove that

$$n_V \leqslant cd\ G \leqslant \sup (2, n_V) \quad \text{if } n_E < n_V$$
$$n_V \leqslant cd\ G \leqslant n_V + 1 \quad \text{if } n_E = n_V .$$

If the graph of orbits $G \setminus X$ is also a tree, G is atree product, and its cohomology is described by a generalized Mayer-Vietoris sequence.
We derive Lyndon's theorem on the cohomological dimension of a one-relator group F/(r) from the above inequalities. We then generalize his theorem to a situation where F is replaced by a free product of finitely generated torsion-free abelian groups.

K.W.GRUENBERG: Generation of Frobenius groups.

Let G be a finite soluble group and take aminimal free presentation: $1 \to R \to F \to G \to 1$.
Then the relation module R/R' is indecomposable if,and only if, G is not a 2-Frobenius group or a Frobenius group with cyclic complement. This reports on joint work with K.W.Roggenkamp.

T.O.HAWKES: Fitting Length and other invariants.

A proof of the following result, a conjecture of Kovacs, was discussed:
Let G be a finite soluble group whose Sylow subgroups all have d generators. Then the Fitting length of G is at most $2d^2 + 4d + 1$.
(It is likely that a bound of 2d will suffice.)

H.HEINEKEN: Inheritance of properties by soluble groups from their Frattini quotients.

An example of a nilpotent-class-two by abelian, finitely

Gefördert durch DFG Deutsche Forschungsgemeinschaft

generated group is given which is not finitely related but its Frattini quotient is. This shows that being finitely presented is not a property which can be read off the Frattini quotient of a finitely generated soluble group.

J.A.HULSE: Local Systems in Groups.

A local system for a set S is a subset $\mathcal{L} \subseteq \mathfrak{P}(S) = \{P \mid P \subseteq S\}$ such that for all finite $F \subseteq S$, $\exists\, L \in \mathcal{L}$ s.t. $F \subseteq L \in \mathcal{L}$. $\mathcal{L}$ is upper-directed if $\forall\, L_1, L_2 \in \mathcal{L}\ \exists\, L_3 \in \mathcal{L}$ s.t. $L_1 \cup L_2 \subseteq L_3$. $\mathcal{L}$ is weakly upper-directed if $\forall\, L \in \mathcal{L}$, $\forall\, x \in S\ \exists\, L' \in \mathcal{L}$ such that $L \cup \{x\} \subseteq L'$.
Let $\mathbf{L}$, $\mathbf{L}_u$ and $\mathbf{L}_w$ be the corresponding operations on classes of groups, so that for instance $G \in \mathbf{L}_w \underline{X}$ if and only if G has a weakly upper-directed local system of $\underline{X}$-subgroups.
If $\mathbf{A}$ is an operation on classes of groups, i.e. $\underline{X} \subseteq \mathbf{A}\underline{X} \subseteq \mathbf{A}\underline{Y}$ for all $\underline{X} \subseteq \underline{Y}$, then $\mathbf{A} \leq \mathbf{B} \Longleftrightarrow \mathbf{A}\underline{X} \subseteq \mathbf{B}\underline{X}$ for all $\underline{X}$; $\mathbf{A}^{\alpha}\underline{X} = \mathbf{A}(\bigcup\{\mathbf{A}^{\beta}\underline{X} \mid \beta < \alpha\})$ for all ordinals α and $\mathbf{A}^{*}\underline{X} = \bigcup\{\mathbf{A}^{\alpha}\underline{X} \mid \alpha \text{ an ordinal}\}$. If $\mathfrak{Q} \subseteq \mathfrak{P}(K)$ then $\mathbf{L}\mathfrak{Q} = \{S \mid S \subseteq K$ and S has a local system $\mathcal{L} \subseteq \mathfrak{Q}\}$. $\mathbf{L}_u\mathfrak{Q}$ and $\mathbf{L}_w\mathfrak{Q}$ are defined similarly.
The following translation theorem was discussed:
Theorem: Let K be any countably infinite set. Then there exist groups A_Q for each $Q \subseteq K$ such that $A_\emptyset = 1$ and the map

$$\nu : \mathfrak{Q} \longmapsto \underline{Q} = \{G \mid G \cong A_Q \text{ for some } Q \in \mathfrak{Q}\}$$

is a bijection from $\{\mathfrak{Q} \mid \emptyset \in \mathfrak{Q} \subseteq \mathfrak{P} = \mathfrak{P}(K)\}$ onto the set of all classes of groups contained in $\underline{P} = \mathfrak{P}\nu$. Further if $\mathbf{A}$ is any one of $\mathbf{L}_u^{\alpha}$, $\mathbf{L}_w^{\alpha}$, $\mathbf{L}_u^{*}$, $\mathbf{L}_w^{*}$ or $\mathbf{L}$ then

$$\nu : \mathbf{A}\mathfrak{Q} \longmapsto \mathbf{A}\underline{Q}$$

for all $\mathfrak{Q}$.

B.HUPPERT: Absolutely indecomposable modules.

Absolute indecomposability of a KG-module M is best defined by $\mathrm{Hom}_{KG}(M,M)/J(\mathrm{Hom}_{KG}(M,M)) = K$. Discussion of some properties of this concept.

Z.JANKO: On thin simple groups - 2-constrained case

The following result will be proved:

Theorem: Let G be a smallest unknown simple "thin" group. (i.e. if H is any 2-local subgroub of G, then every odd order Sylow subgroup of H is cyclic) Let M be a maximal 2-local subgroup of G and let $1 \neq E$ be a normal elementary abelian subgroup of M. Then there exists a maximal 2-local subgroup N of G such that $N \neq M$ and $N \supseteq E$.

W.KNAPP: Finite uniprimitive permutation groups

Theorem: Let (G,Ω) be a finite primitive permutation group with a subconstituent $G_\alpha^{\Delta(\alpha)}$ of degree $d > 1$, p a prime number, $\beta \in \Delta(\alpha)$. Then:

(a) $O_p(G_{\alpha\beta}) \trianglelefteq G_\alpha \Rightarrow O_p(G_{\alpha\beta}) = 1$.

(b) $O_p(G_{\alpha\beta}^{\Delta(\alpha)}) = 1 \Rightarrow O_p(G_{\alpha\beta}) = 1$.

(c) $F(G_{\alpha\beta}^{\Delta(\alpha)}) = 1 \Rightarrow F(G_{\alpha\beta}) = 1$ and $|G_\alpha| \,|\, d![(d-1)!]^d$.

Several corollaries can be derived from this result, e.g.

(1) $G_\alpha^{\Delta(\alpha)}$ regular $\Rightarrow F(G_{\alpha\beta}) = 1$ and $|G_\alpha| \,|\, d[(d-1)!]^d$.

(2) $G_\alpha^{\Delta(\alpha)}$ regular and solvable $\Rightarrow G_{\alpha\beta} = 1$, i.e. $G_\alpha^{\Delta(\alpha)}$ is a faithful subconstituent.

(3) $G_\alpha^{\Delta(\alpha)}$ abelian or Hamiltonian $\Rightarrow G_{\alpha\beta} = 1$ and G is a primitive Frobeniusgroup, $G_\alpha^{\Delta(\alpha)}$ is therefore cyclic or cyclic x quaternion.

O.-U.KRAMER: Finite groups with subgroups with relatively prime indices

Let $\mathfrak{X}$ be a class of groups. $\mathfrak{X}$ has the property (Σ_n) if:
$(G \in \mathfrak{X} \Leftrightarrow$ ex. U_i (i 1,..,n); $U_i \in \mathfrak{X}$; $(|G:U_i|, |G:U_j|) = 1.$)
In a similar way: $\mathfrak{X}$ has the property (M_n), (N_n) resp. if the subgroups U_i are modular or normal resp.

We prove:

Theorem 1: Let $\mathfrak{X}$ be a saturated formation, $\mathfrak{X} \subseteq \mathfrak{N}^2$. Then $\mathfrak{X}$ has the properties (Σ_4), (M_3) and (N_2).

For arbitrary formations we get:

Theorem 2: Let $\mathfrak{X}$ be locally defined by $\mathfrak{x}(p)$.

(a) If the $\mathfrak{x}(p)$ have the property (N_n), (Σ_n) resp., then $\mathfrak{X}$ has the property (N_n), (Σ_{n+2}) resp.

(b) Let the $\mathfrak{S}_p\, \mathfrak{x}(p)$ be saturated. If the $\mathfrak{x}(p)$ have the property (M_n), then so has $\mathfrak{X}$.

From these therems you can derive estimations for the number of prime divisors of so-called $\mathfrak{X}$-critical groups. Finally we prove :

Theorem 3: Let $\mathfrak{H}$ be a saturated formamation, $\mathfrak{H} \subseteq \mathfrak{N}^2$.

$$G \in \mathfrak{H} \Leftrightarrow \text{ex.} U_1, U_2, U_3 \leq G, U_1 \in \mathfrak{N} \; ; \; U_2, U_3 \in \mathfrak{H} \text{ and } (|G:U_i|, |G:U_j|) = 1.$$

C.R.LEEDHAM-GREEN: On-p-groups of maximal class

If P is a group of order p^n and class n-1 then P is of maximal class .Using ideas of Blackburn we prove that ,if n is big enough , P has a subgroup P_1 of index p and class at most 3. Moreover the order of $\gamma_3(P_1)$ is bounded in terms of p alone. It follows that ,modulo a subgroup of order bounded in terms of p alone , every p-group of maximal class is a split extension of a group of class at most 2 by a p-cycle.

Using simple ideas from number theory and the theory of difference equations we construct all such p - groups. This is joint work with Susan Mc Kay.

J.C.LENNOX: Some Frattini properties of finitely generated soluble by finite groups.

P is a Frattini property of a class X of groups if whenever the Frattini factor group $G/\Phi(G)$ of an X-group G has P then G has P . It is well known (P.Hall) that nilpotency is a Frattini property of the class Y of all finitely generated soluble by finite groups.

It is proved that finiteness , supersolvability and polycyclicity are also Frattini properties of Y-groups.

F.P.LOCKETT: The Fitting class F^*

Given an arbitrary Fitting class F of finite soluble groups, we exploit the fact that $(G \times G)_F = G_F \times G_F$ is not in general true for all G by defining F^* to be the class $\{G : (G \times G)_F = G_F \times G_F \langle (g^{-1}, g) : g \in G \rangle\}$.

F^* is a Fitting class containing F , close in content and properties to F . Whenever F=sF , F =QF or F = R_0F then $F=F^*$, whereas F is normal if and only if F^* is the class of all finite soluble groups.The only known examples of classes not closed under the $*$ operation are intersections of the type $F^* \cap$(a normal Fitting class). We conjecture that all such Fitting classes occur in this way, equivalently: for each Fitting class F , $F = F^* \cap$(the normal Fitting class generated by F).

S.MORAN : <u>Presentations of groups</u>

A group is said to have p^λ - smooth presentation on a set X of generators if and only if $G \cong F(X)/R(X)$ with $R(X) \subseteq F(X)^{p^\lambda} F(X)'$.

Theorem: Suppose $\langle x_1,\dots,x_d ; x_1^{p^\lambda \beta_1} = \dots = x_d^{p^\lambda \beta_d} = u_1 = \dots = u_{r(k)} = 1 \rangle$ is p^λ -smooth presentation of a finite group, where every u_i belongs to $F(X)_{k,p}$. Then $r(k) > \text{Max}\{(d/2)^k - d(2/d)^{p^\lambda - k}, (d/k)^k (k-1)^{k-1} - d\}$ for $p^\lambda \geq k \geqslant 2$.

M.L.NEWELL: <u>Supplements in abelian-by-nilpotent groups</u>

Let A be an abelian normal subgroup of a group G . If G/A belongs to the class of groups $\mathfrak{X}$, then

1) A has a self - normalizing $\mathfrak{X}$- supplement in G and
2) all such complements are conjugate,
   whenever A is finite or torsionsfree divisible of finite rank and $\mathfrak{X}$ is the class of nilpotent, hypercentral or locally-nilpotent groups.

If A satisfies the minimum condition on subgroups , then 1) and 2) hold for the class of hypercentral and locally- -nilpotent groups.

If A satisfies the minimum condition on subgroups and G/A is nilpotent , then

1^*) A has a nilpotent supplement S in G where S has finite index in its normalizer and

2^*) if S and T are any such supplements , then there is a finite normal subgroup B of G contained in A such that BS and BT are conjugate.

A.RAE : <u>p-groups acting on finite p-soluble groups</u>

Let the group A act on the p-soluble group G , where A has order p^n and G has p-length l_p. Then if l_p is greater than some linear function f(n) , it is conjectured that A should have fixed points on some p'-section of G and that , if A acts non trvially on G/O^p ,some non trivial quotient of A should have free modules on A invariant p- and p'-sections of G. The relations between these statements are discussed and some answers given for A elementary abelian of order p^2 , or cyclic.The cases p=2 and p=3 both have to be considered individually.

Theorem : Let A be elementary of order p^2 and G have trivial p-radical.Then if $l_p \geqslant 5$ $(p \neq 2)$ or 7 (p=2) A has a fixed point on some p'-section of G.Moreover this section is acted on fixed point freely by some A invariant p-subgroup of G.
(The final sentence shows that the section is , for Sylow theory , non trivial.)

D.J.S.ROBINSON:Finitely generated $\mathfrak{S}_o$-groups are minimax

It is proved that a finitely generated $\mathfrak{S}_o$-group - or soluble group with finite abelian section rank - is a minimax group, thus answering a question raised some years ago.This theorem is deduced from a result on the cohomology of soluble minimax groups with coefficients in a module which,as an abelian, is a direct sum of p - groups satisfying Min.

K.W.ROGGENKAMP: Decomposition of the augmentation ideal of finite groups

Theorem 1: Let G be a finite group ,U a π- Hall subgroup which is a T.I. set , and if $S=N_G(U) \neq U$,then S is a Frobeniusgroup with kernel U.Then the integral augmentation ideal of G decomposes.

Theorem 2: If G is a finite soluble group,there is the following equivalence:
(i) The augmentation ideal of G decomposes.
(ii) $\pi(G)=\pi_1 \dot{\cup} \pi_2$, $\pi_i \neq \emptyset$ and G has no mixed elements.
(iii) $\mathbb{Z}$ is not a Heller module
(iv) G is a Frobenius or 2-Frobenius group.

(Joint work with K.W.Gruenberg)

U.STAMMBACH: <u>On groups with P-local homology</u>

Let P be a set of primes (possibly empty), P' the complementary set of primes . The following two classes of groups are considered:

$G \in UP'R$ $\Leftrightarrow$ to every $x \in G$ and every P' number n there exists a unique $y \in G$ with $y^n = x$ (unique P'-roots)

$G \in HPL$ $\Leftrightarrow$ H_nG $(=H_n(G,\mathbb{Z}))$ is P-local for $n \geq 1$. (the homology is P- local)

Proposition 1: $G \in UP'R \Rightarrow \mathfrak{z}_{i+1}G/\mathfrak{z}_iG$ is P-local , $G/\mathfrak{z}_iG$ is UP'R ; $i \geq 0$.

Proposition 2: $G \in HPL \Rightarrow G_i/G_{i+1}$ is P- local , G/G_i is HPL , i

Proposition 3: (i) G abelian. G is HPL $\Leftrightarrow$ G is UP'R $\Leftrightarrow$ G is P-local.

(ii) G finite . G is HPL $\Leftrightarrow$ G is UP'R

(iii) G nilpotent . G is HPL $\Leftrightarrow$ G is UP'R

(iv) HPL $\not\equiv$ UP'R in general .

Proposition 4: Let G be nilpotent. Let $l: G \to m_PG$ be the Malcevcompletion of G with respect to P . Then $l: H_nG \to H_n(m_PG)$, $n \geq 1$ is the P-localizing map. In particular , m_PG is HPL.

Proposition 5 : G nilpotent . Among nilpotent groups K and homomorphisms $f: G \to K$ the Malcevcompletion $l: G \to m_PG$ is characterized by the fact that $f_*: H_nG \to H_nK, n \geq 1$ is the P-localizing map.

V. STINGL : <u>A characterization of J_2.</u>

The following theorem is proved:

Theorem: Let G be a finite simple group and t an involution in such that $C_G(t)/O(C_G(t))$ is isomorphic to the direct product of a fourgroup and PSL(2,q), where q is congruent 3 or 5 mod 8. Then G is isomorphic to J_2.

(Remark: J_2 is the uniquely determined simple group of order 604800)

Sketch of proof : 1) Determine the possible structure of $N_G(A)/C_G(A)$, where A is a Sylow -2-subgroup of $C_G(t)$.

2) Determine the possible structure of a Sylow 2-subgroup E of N(A).

3) Determine the possible structure of a Sylow-2-subgroup T of G.

4) By using "Transfer Lemmas" one gets that T must be isomorphic to a Sylow-2-subgroup of J_2.

5) Apply Gorenstein - Harada - characterization of J_2.

S.E.STONEHEWER: <u>Nilpotent residuals of subnormal subgroups</u>

We prove the following generalizations of results of Wielandt (1957) about groups with a composition series:

Theorem 1: Let G be a minimax group generated by subnormal subgroups H,K and suppose that $H/H^{\mathfrak{N}}$, $K/K^{\mathfrak{N}}$ are nilpotent. (Here $H^{\mathfrak{N}}$ denotes the intersection of all normal subgroups of H modulo which H is nilpotent.) Then $G^{\mathfrak{N}} = \langle H^{\mathfrak{N}}, K^{\mathfrak{N}} \rangle$ and $G/G^{\mathfrak{N}}$ is nilpotent.

Theorem 2: Let $G = \langle H,K \rangle$, H,K, subnormal in G and suppose that $G^{\mathfrak{N}} = \langle H^{\mathfrak{N}}, K^{\mathfrak{N}} \rangle$. Then $H/H^{\mathfrak{N}}$ nilpotent $\Rightarrow$ $G^{\mathfrak{N}} = H^{\mathfrak{N}} K^{\mathfrak{N}}$.

R.STREBEL: <u>Some finitely generated metabelian groups which are infinitely related although their multiplicators are finitely generated</u>.

Theorem: Let $G = \langle a,t : t^r a^m t^{-r} = a^n \rangle$, $mnr \neq 0$. Then:

(i) If either (a) g.c.d. $(m,n) > 1$, or
(b) $|r| > 1$ and $mn=1$, then
$H_2(G/G'')$ is not f.g. ,and thus G/G'' is not finitely related.

(ii) If either (a) $|r| = 1$, and $|m|$ or $|n| = 1$, or
(b) $|r| > 1$, and $|m|$ or $|n| = 1$, and $mn \neq 1$,
then G/G'' is finitely presented.

(iii) If g.c.d.$(m,n) = 1$ and $|m| \neq 1 \neq |n|$, then G/G'' is not finitely related, although $H_2(G/G'')$ is finitely generated.

Finally, even though $H_2(G/G'')$ is f.g. in (III), none of the relation modules of G/G'' is f.g.

Remark: In case $m=2, n=3, r=1$, the group G/G'' is the split extension of a locally cyclic group by an infinite cyclic group. It has trivial multiplicators but is infinitely related.

H.WIELANDT: <u>Über die Existenz von Normalteilern in dreifach transitiven Permutationsgruppen</u>

Sei G eine Gruppe und $1 < N \trianglelefteq H \trianglelefteq G$. Wann ist N der Durchschnitt der normalen Hülle N^G mit H? Für endliche Gruppen ist diese

Frage oft untersucht worden , mit Hilfe von Verlagerung, Charakteren und Graphen . Hier wird auf neuem Wege ein kürzlich von Hale und Shult entdeckter Satz (Math.Z.135) von allen Endlichkeitsvoraussetzungen befreit. Wir kenn= zeichnen Die Punktstabilisatoren in den Normalteilern einer beliebigen 3-transitiven Gruppe G auf einem Raum X.

Satz: Sei $H = G_o$ der Stabilisator eines Punktes $o \in X$ in G. Sei $s \in G - H$, und (✱) sei s zu einem Element von N in G konjugiert. Genau dann ist $N^G \cap H = N$ (und daher $G/N^G \cong H/N$) , wenn aus $h \in H$ und $s^{-}hs \in H$ stets $h^{-}s^{-}hs \in N$ folgt . Die Zusatzforderung (✱) ist entbehrlich,wenn H/N abelsch ist.

O.-U. Kramer (Kiel)
K.-U. Schaller (Kiel)