MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20|1974

Ringe, Moduln und homologische Methoden

12.5. bis 18.5.1974

Die Tagung wurde von A. Rosenberg (Ithaca, USA) und F. Kasch (München) geleitet.

Sie war wiederum von Mathematikern aus zahlreichen Ländern besucht, die über aktuelle Forschungsergebnisse berichteten.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Teilnehmer

G.Azumaya, Bloomington, USA

R.Baer, Zürich, Schweiz

I.Beck, Bergen, Norwegen

J. Brewer, Lawrence, USA

H.Brungs, Gießen

V.Dlab, Ottawa, Canada

G.Ehrlich, College Park, Maryland, USA

D.Eisenbud, Waltham, USA

E.G.Evans, Urbana, USA

J.L.Fisher, Edmonton,Canada

E.Formanek, Princeton, USA

A.W.Goldie, Leeds, England

G.Hauger, München

J.Herzog, Regensburg

A.Hirschelmann, Bonn

C.U.Jensen, Kopenhagen,Dänem.

A.Jategaonkar, Ithaca, USA

T.Józefiak, Torun, Polen

S.Jøndrup,Kopenhagen,Dänemark

F. Kasch, München

M.Knebusch, Regensburg

G.Krause, Winnipeg, Canada

L.van Leeuwen, Groningen, Holland

T.H.Lenagan, Edinburgh, Schottland

H.Lenzing, Paderborn

B.J.Müller, Hamilton, Canada

H. Müller, Bielefeld

H.-J.Nastold, Münster

U.Oberst, Innsbruck, Österreich

B.Pareigis, München

D.Popescu, Bukarest, Rumänien

R.Raphael, Montreal, Canada

I.Reiten, Trondheim, Norwegen

A.Rosenberg, Ithaca, USA

K.Sugano, Sapporo, Japan

F.Szasz, Budapest, Ungarn

W.Scharlau, Münster

H.-J.Schneider, München

E.J.Taft, Berkeley, USA

Vortragsauszüge

Goro Azumaya: Semi-perfect and Perfect Modules

Let P be a projective left R-module. E. A. Mares called P semi-perfect if every homomorphic image of P has a projective cover, while P is called perfect if the direct sum of any number of copies of P is semi-perfect. The following theorems are to be proved: THEOREM 1. The following conditions are equivalent: (1) P is semi-perfect, (2) every proper submodule of P is contained in a maximal submodule of P and every simple homomorphic image of P has a projective cover, (3) P is a direct sum of local R-modules and the radical J(P) is small in P (where a local R-module means a projective cover of a simple R-module). THEOREM 2. P is perfect if and only if every proper submodule of P is contained in a maximal submodule of P and every semi-simple R-module which is a direct sum of simple homomorphic images of P has a projective cover.

István Beck: Projective and free modules

We consider some submodules of a free module F which have a surjection onto F. Let for instance J denote the Jacobson radical in the ring R, F a free R-module and F' a submodule of F such that F = F' + JF. Then F' has a surjection onto F.

James Brewer: Prime ideals and localization in commutative group rings

Let K be a field, G an abelian group of finite torsion-free rank and denote by K[G] the group ring of G over K.

Theorem 1. K [G] satisfies the first chain condition for prime ideals. Moreover, if K is of characteristic zero, then each localization of K [G] at a prime ideal is a regular local ring. These results are used in the proof of Theorem 2. If $n \geq 2$ is a positive integer, then there exists a non-Noetherian quasi-local UFD of characteristic zero and Krull dimension n . In addition, corresponding results to those of Theorem 1 are valid for fields of characteristic $p \neq 0$ and for the ring of integers.

H.H.Brungs: Rings with a distributive lattice of right ideals

Let R be a ring with unit element and no zero divisors. R is called right distributive if the lattice of right ideals is distributive and a chainring if the right ideals are linearely ordered.

Thm. R is a right distributive ring if and only if $S = R \setminus N$ is a right Ore-system for every maximal right ideal N in R and R_N is a chain ring.

Thm. For a right noetherian ring R the following conditions are equivalent:

1. R is right distributive
2. For any two right ideals $A \subseteq B$ in R there exists a right ideal C in R with A = BC.

3. Every right ideal in R is a product of prime ideals
4. Every right ideal $A \neq R, \neq 0$ in R can be written as a product $A = P_1 \dots P_n$ of prime ideals $P_i \neq R$ in R such that $P_i \supsetneq P_j$ implies $i > j$. Such a factorization is unique up to the order of commuting factors.

V. Dlab: Coxeter functors and representation theory

An oriented moduled graph $(\Gamma, \mathfrak{M}, \Omega)$ is a finite set of skew fields F_i together with (finite-dimensional) F_i-F_j-bimodules ${}_iM_j$ such that (i) ${}_jM_i \approx \mathrm{Hom}_{F_i}({}_iM_j, F_i) \approx \mathrm{Hom}_{F_j}({}_iM_j, F_j)$, and (ii) for every ${}_iM_j \neq 0$, the pair $\{i,j\}$ is ordered. Denote by $\mathcal{L}$ the (abelian) category of all representations of $(\Gamma, \mathfrak{M}, \Omega)$. For a (+)-admissible ordering $i_1, i_2, \dots i_n$ of all vertices of Γ, define the Coxeter functor $C^+ : \mathcal{L} \to \mathcal{L}$ by $C^+ = S^+_{i_n} \dots S^+_{i_2} S^+_{i_1}$; here, $S^+_i : \mathcal{L} \to \mathcal{L}(\Gamma, \mathfrak{M}, s_i\Omega)$ is defined as follows: $(S^+_i V)_j = V_j$ for $j \neq i$, $0 \to (S^+_i V)_i \to \oplus V_j \otimes {}_jM_i \xrightarrow{(i \varphi j)} V_i$ and $S^+_i\, {}_j\gamma_i : (S^+_i V)_i \otimes {}_iM_j \to V_j \otimes {}_jM_i \otimes {}_iM_j \to V \otimes F_j \to (S^+_i V)_j$. Dually, we define $C^- : \mathcal{L} \to \mathcal{L}$. The relationship between the Coxeter functors and the Coxeter transformations in the Weyl group of Γ is explored to show that (i) $\mathcal{L}$ is of finite type iff Γ is a Dynkin diagram and $V \to (\dim V_i)$ induces a bijection between the indecomposable objects of $\mathcal{L}$ and the positive roots of Γ.
(ii) If Γ is an "extended" Dynkin diagram, $\mathcal{L}$ has two types of indecomposable objects: discrete and homogeneous. Again there is a bijection between discrete indecomposable objects of $\mathcal{L}$ and positive roots of Γ given by $V \to (\dim V_i)$.

Gertrude Ehrlich: Units and one-sided units in regular rings

A ring R with identity is unit-regular if, for each $a \in R$, there is a unit $x \in R$ such that $axa = a$. Semi-simple Artinian rings, strongly regular rings, von Neumann continuous rings, and regular group rings are unit-regular, while the(regular) ring of all linear operators on an infinite dimensional vector space is not. We give two new characterizations of unit-regularity:
1. A regular ring R with identity is unit-regular if and only if, for each $a \in R$, there is a unit $u \in R$ such that $aR \oplus u\,(Ra)^r = R$, where $(Ra)^r$ is the right annihilator of Ra in R. 2. A regular ring R with identity is unit-regular if and only if it is von Neumann finite (i.e., contains no strictly one-sided units). One-sided unit-regularity, defined in analogous fashion, can also be characterized in ideal-theoretic terms. The ring of all linear transformations on an arbitrary vector space is one-sided unit-regular. We pose the problem whether every regular ring with identity is one-sided unit-regular.

David Eisenbud: Ideals of small height

A survey, including some of my recent work with David Buchsbaum, but concentrating on the very remarkable developements due to Barth and to Hartshorne-Ogus.

E.G. Evans: Remarks on Bourbaki's Theorem on Torsion Free Modules over Normal Domains

Let R be a commutative noetherian ring and M a finitely generated module. If R is Dedekind and M is torsion free then $M \cong R^n \oplus I$

where I is an ideal and the isomorphism class of I depends only on M. This generalizes to Serre's theorem that projectives of big enough rank have free summands, to Bass's theorem that if P is projective of big enough rank and $P \oplus R \cong Q \oplus R$ then $P \cong Q$, and to Eisenbud and Evans's theorem that if M needs n generators at every localization at a prime ideal then $\exists\, m \in M$ which is a minimal generator of M_Q for all primes Q of height $< n$. A different direction of generalization is Bourbaki's theorem which states that if R is a normal domain and M is torsion free then $\exists\, F \subset M$ with F free and M/F isomorphic to an ideal. The remarks are the following: 1) Bourbaki follows quickly from Eisenbud-Evans, 2) The isomorphism type of I is never uniquely determined by M if dim $R > 1$, 3) If one can get an I which contains an R sequence of length 3 then $M \cong R^n \oplus I$ and any other J one gets which contains an R sequence of length 3 is equal to it, and 4) In case M is projective one can describe certain "generic" properties of the ideals that are obtainable.

James L. Fisher: The category of epic R-fields

Let R be an algebra with 1 over a skew field k. The category of epic R-fields is shown to be isomorphic to a category of R'-Ore domains with morphisms being homomorphisms, where R' is a certain R-ring. This shows that a local subring corresponding to a morphism of epic R-fields is equivalent to a local subring obtained from a torsion theory of R'.

Edward Formanek: Polynomial Identity Rings

A survey talk on recent results in polynomial identity rings.

Günther Hauger: Aufsteigende Kettenbedingung für Moduln und perfekte Endomorphismenringe

Für einen endlich erzeugten selbst/projektiven R-Modul M mit Endomorphismenring S sind folgende Aussagen äquivalent:

a) S ist rechts perfekt

b) Der Radikalfaktormodul M/RaM ist halbeinfach und jeder M-erzeugte Modul hat kleines Radikal

c) Für jeden R-Modul X wird jede aufsteigende Folge von M-zyklischen Untermoduln stationär

d) Zu jedem M-erzeugten Modul X existiert ein wesentlicher Epimorphismus $P \longrightarrow X$, so daß P M-erzeugt und $\mathcal{E}M$-projektiv ist

e) Jeder M-erzeugte Modul ist komplementiert.

Dabei heißt ein Modul M-zyklisch (M-erzeugt), falls er epimorphes Bild von M (einer direkten Summe von Kopien von M) ist. Die Menge der M-erzeugten Moduln werde mit $\mathcal{E}M$ bezeichnet.

Jürgen Herzog: Rings of characteristik p and Frobenius functors

Let R be a commutative noetherian ring of characteristic p, where $p > 0$ is a prime number. The endomorphisms $f^\nu : R \longrightarrow R$, $f^\nu(r) := r^{p^\nu}$ are called Frobeniushomomorphisms. R considered as R-module via f^ν will be denoted by f^ν_R. We give criteria for a finitely generated module to be of finite projective dimension resp. of finite injective dimension.

<u>Theorem 1:</u> If M is a finitely generated R-module, then the following conditions are equivalent:

a) p.d. M $< \infty$

b) $\mathrm{Tor}_i^R(M^{f^\nu}, R) = 0$ for all $i > 0$ and all $\nu > 0$

c) There exists an unbounded sequence of integers (ν_k), such that $\mathrm{Tor}_i^R(M^{f^{\nu_k}}, R) = 0$ for all $i > 0$ and all $k = 1,2,\ldots$

<u>Theorem 2:</u> If M is a finitely generated R-module and if the Frobeniushomomorphism is finite, then the following conditions are equivalent:

a) i.d. M $< \infty$

b) $\mathrm{Ext}_R^i(f_R^\nu, M) = 0$ for all $i > 0$ and all $\nu > 0$

c) There exists an unbounded sequence of integers (ν_k), such that $\mathrm{Ext}_R^i(f_R^{\nu_k}, M) = 0$ for all $i > 0$ and all $k=1,2,\ldots$

The implications a) $\Rightarrow$ b) in 1) and 2) are due to L. Szpiro and C. Peskine (Dimension projective finie et cohomologie local, F.H.E. S.42(1973)) We use Theorem 1 and 2 to characterize regular local rings (compare E. Kunz, Characterizations of regular local rings of characteristic p, AmJ.Math.91(1969)) and to characterize Gorenstein rings.

A. V. Jategaonkar: <u>Principal ideal theorem for noetherian rings</u>

The following form of the principal ideal theorem was proved.

<u>Theorem:</u> In a right Noetherian ring, a prime ideal minimal over a normalizing element has rank at most one. The proof uses a modification of the notion of Krull dimension.

S. Jøndrup: Flat and Projective Modules

We consider the projectivity of finitely generated flat modules on a given ring. In particular we consider the problem: M finitely generated flat left module $^{M}/J(R)M$ $^{R}/J(R)$ -projective. Is M projective? It is known to be true if R is commutative, we prove the result for R a P.I. ring. Problem of the type: Under which conditions on R does it hold that finitely generated flat left modules are projective iff finitely generated right modules are projective.

Tadeusz Józefiak: Homology of graded commutative algebras

For a graded connected commutative algebra A over a field K we define homology $H_{**}(A)$ to be $Tor^{A}_{**}(K,K)$. It is known that $H_{**}(A)$ admits the structure of a bigraded Γ-algebra over K.

Theorem 1. $H_{**}(A)$ is generated as a bigraded Γ-algebra over K by $H_{1*}(A)$ and $H_{2*}(A)$ iff A is a graded complete intersection.

Theorem 2. Let A be a finitely generated K-algebra. The homology $H_{**}(A)$ is a finitely generated bigraded Γ-algebra over K iff A is a graded complete intersection.

If K is a field of characteristic zero Theorem 2 settles the long standing conjecture that $H_{**}(A)$ is finitely generated as a bigraded algebra over K iff A is a graded complete intersection.

L.C.A. van Leeuwen: The hereditariness of the upper radical

All rings are associative, but neither commutativity nor the existence of a unity is presupposed.

A class K of rings is said to be hereditary, if every ideal of a K-ring is again a K-ring. If M is a hereditary class, then UM= $\{R$: R cannot be mapped onto a non-zero ring of M homomorphically$\}$ is a radical class, the upper radical class of M.

Theorem. Let M be a class of simple prime rings of characteristic zero. Then UM is hereditary if and only if every ring in M has a unity. Theorem. Let M be a class of simple prime rings of characteristic p (p a prime). Assume that for any ring $I \in M$ the centroid C(I) is transcendental over Z_p. Then UM is hereditary if and only if every ring in M has a unity.

T. H. Lenagan: Krull dimension and invertible ideals

Let R be a right Noetherian ring and I an invertible ideal of R. Denote the Krull dimension (eg. R-Gordon & J.C.Robson, Mem.Amer.Math.Soc. 133(1973)) by $|R|$. If I is contained in the Jacobson radical of R then $|R| = |R/I| + 1$. If $S = \{(1-i) \mid i \in I\}$ and $T = \bigcup_{n=1} I^{-n}$, then $R_S = S^{-1}R$ and $T_S = S^{-1}T$ exist and I_S is an invertible ideal contained in the Jacobson radical of R_S. We find $|R| = \sup(|R_S|, |T|)$ and, consequently, $|R| = |R/I| + 1$ or $|T|$. Examples are given to illustrate the two possibilities.

H. Müller: Ringe mit regulärer, flacher, epimorpher Ringerweiterung

R sei ein Ring mit 1, es gilt dann:

R besitzt genau dann eine reguläre, rechtsflache, epimorphe Ringerweiterung, wenn gilt:

(1) R ist links nichtsingulär

(2) Zu jedem endlich erzeugten Linksideal A von R gibt es ein endlich erzeugtes Linksideal B von R mit $A \cap B = 0$ und $A + B \hookrightarrow R$ ist wesentlich.

(3) Jedes endlich erzeugte Linksideal von R ist wesentlich endlich präsentierbar.

U. Oberst: <u>Zur Theorie der affinen algebraischen Gruppen in Primcharakteristik</u>

Der Vortrag behandelte affine algebraische Gruppen über im allgemeinen perfekten Körpern k der Charakteristik $p > 0$. In der ersten Hälfte des Vortrages wurden einige grundlegende Definitionen, Sätze und Beispiele aus der Theorie dargestellt. (Literatur: Borel, Linear algebraic groups, Benjamin 1969; Demazure-Gabriel, Groupes algebriques, Masson 1970; Demazure-Grothendieck, SGA3, Springer L.N. 151-153; Seligman, Modular Lie-algebras, Springer 1967). Insbesondere wurde für das Studium endlicher affiner Gruppen geworben. Im zweiten Teil des Vortrages wurden Probleme über endliche affine Gruppen diskutiert, mit denen sich einige Innsbrucker Mathematiker (Meyer, Oberst, Sauer) zur Zeit befassen.

D. M. Popescu: <u>A strong approximation theorem over discrete valuation rings</u>

We consider the following: (P) <u>Problem</u>: Let R be a noetherian ring and $\underline{m}$ an ideal of R. Let $F_i \in R[y_1,\dots,y_n]$, $i = 1,\dots,m$ a m-system of polynomials. Does there exist a function $\nu: \mathbb{N} \longrightarrow \mathbb{N}$ with the following: <u>Property</u> For any positive integer $c \geq 1$ and for any $\hat{y} \in R^n$ so that $F_i(\hat{y}) \equiv 0 \bmod \underline{m}^{\nu(c)}$, there exists a solution $y \in \underline{\tilde{R}_m}^n$ of (F_i) with $\equiv \hat{y} \bmod \underline{m}^c$ [$\underline{\tilde{R}_m}$ is the henselization of R with respect to $\underline{m}$]. M. Greenberg answered affirmatively (P) when R is an excellent discrete valuation ring. M. Artin has shown that (P) is true when

$R = k[x_1,\ldots,x_n]$, where k is a field and $\underline{m} = (x)$. In his report to International Math. Congress 1970 in Nice, M. Artin stated that (P) is true when $R=S[x_1,\ldots,x_n]$, $\underline{m}=(x)$ where S is an excellent discrete valuation ring. Our main result of this work is the solution of the Artin's conjecture.

R. Raphael: An order-relation on commutative semiprime rings

The talk will summarize joint work with W. Burgess which has proceeded from the fact that a reduced ring (one with no non-zero nilpotent elements) becomes a partially ordered set under the definition $a \leq b \Leftrightarrow ab = a^2$. The work has attempted to relate the algebraic properties with the order-theoretic ones. A subset S of a ring is called orthogonal if $ab = 0$ whenever $a,b \in S$ and $a \neq b$. It is called boundable if $ab(a-b) = 0$, whenever $a,b \in S$. The natural motivation for these definitions is given and completeness is defined for them:- R is orthogonally complete if every orthogonal subset of R has a supremum, and complete if every boundable subset of R has a supremum. A class of rings called I-dense (idempotent dense) is introduced and it is shown that for these rings one has both completions and they coincide. This is in accord with the Boolean case. Examples of other rings are studied. A representation technique due to Banaschewaki is used to study the completions of I-dense rings.

Idun Reiten: Stable equivalence (of selfinjective algebras)

Let Λ be an artin algebra, $\underline{r}$ its radical, and denote by mod Λ the category of finitely generated left Λ-modules. Denote by mod Λ/P the category whose objects are the same as those in mod Λ , denoted by $\underline{M}$, and where the morphisms are given by Hom $(\underline{M},\underline{N}) = \mathrm{Hom}_\Lambda(M,N)/P(M,N)$. Here $P(M,N)$ denotes the subgroup of Hom (M,N) consisting of the maps $f:M \to N$ which factor through a projective module.

Λ and Λ' are said to be stably equivalent if mod Λ/P and mod Λ'/P' are equivalent categories. Theorem: An artin algebra Λ is stably equivalent to a selfinjective algebra if and only if each indecomposable direct factor algebra of Λ is either selfinjective or generalized uniserial with $\underline{r}^2 = 0$

Corollary: If an indecomposable artin algebra Λ with $\underline{r}^2 \neq 0$ is (the direct factor of an algebra) stably equivalent to a selfinjective algebra, then Λ is selfinjective.

W. Scharlau: Involutions on algebras and orders

A survey report on the following problems

1) The existence and classification problem for involutions $*$ on algebras and orders, in particular over semisimple algebras, maximal orders, hereditary orders, etc.

2) The canonical involution $R[G] \to R[G]$, $g \mapsto g^{-1}$ on a group algebra

   Special cases: $R = \mathbb{R}$ (Frobenius, Schur)

   G p-group (Roquette)

3) The existence of involution invariant orders

   Theorem: A separable F-algebra, F quotientfield of Dedekind domain R, Ω R-order. If $\Omega = \bar{\Omega}'$ then there exists a hereditary order $\Gamma \supset \Omega$ such that $\Gamma^* = \Gamma$.

H.-J. Schneider: Finite algebraic groups

Let R be a comm. ring, e. g. $R = \mathbb{Z}$ or R=ring of integers of an alg. number field, or R a field. A finite algebraic group G is a functor from R-algebras to groups, which is represented by an algebra A such that A is finitely gen. and projective over R. (A is a comm. finite Hopfalgebra)

Examples: 1) $G = \mu_n$, $\mu_n(S) = \{s \in S \mid s^n = 1\}$, S R-algebra, $A = R[\mathbb{Z}/(n)]$ (group algebra). 2) D an abstract finite group, $G = D_R$, $A = R^D$ which is dual to the groupalgebra $R[D]$.

Finite alg. groups occur in the study of p-torsionpoints of abelian varieties as ascending chains (G_n, i_n), G_n finite of rank $p^{n \cdot h}$, p prim, h fixed ,("p-divisible groups").

Tate and Serre asked whether there are nontrivial p-div. gr. over $\mathbb{Z}$ (which are not products of groups as in ex. 1), 2)).

In this talk a construction of extensions

$$1 \rightarrow \mu_n \rightarrow G^{\alpha,\beta} \rightarrow D_R \rightarrow 1$$

is given: $\alpha : D \times D \rightarrow R^*$ is a factorset, $\beta : D \rightarrow R^*$, $\alpha^n = \delta\beta$, $G^{\alpha,\beta}$ is a subgroup of the group of units of a twisted groupalgebra. Using this construction all finite $\mathbb{Z}$-groups of rank 4 are classified: There are exactly 8 iso. classes. One of these groups is the first component of a nontrivial 2-divisible group over $\mathbb{Z}$ (this answers the question of Tate-Serre for p = 2). Using the notion of integral (Larson-Sweedler) it is shown that there are only finitely many iso. classes of alg. groups of fixed finite rank over R=ring of integers of an alg. numberfield. Tate-Oort's classification of $\mathbb{Z}$-groups G of rank p, p prime ($G \cong \mu_p$ or $\mathbb{Z}/(p)$) is generalized using a new construction of integrals: Let G be a $\mathbb{Z}$-group of rank $p^n, n \geq 1$.

$G \cong \mu_p^n \Leftrightarrow \mathbb{Z}/(p) \otimes G$ is of height ≤ 1. $G \cong D_{\mathbb{Z}}$, D an abstract group $\Leftrightarrow \mathbb{Z}/(p) \otimes G$ is separabel.

Kozo Sugano: Endomorphisms of separable extensions

Let Λ be a ring Γ a subring of Λ which has the common identity element. We denote the center of Λ by C and $V_\Lambda(\Gamma) = \{ x \in \Lambda \mid xr = rx$ for all $r \in \Gamma$ by $\Delta \}$. First we have

Theorem For Λ and Γ the following conditions are equivalent.

(1) ${}_\Lambda(\Lambda \otimes_\Gamma \Lambda)_\Lambda \; < \oplus \; {}_\Lambda(\Lambda \oplus \cdots \oplus \Lambda)_\Lambda$

(2) Δ is C-finitely generated projective, and the following map is a Λ-Λ- isomorphism

$\eta : \Lambda \otimes_\Gamma \Lambda \longrightarrow \mathrm{Hom}({}_C\Delta, {}_C\Lambda)$ $\quad (\eta(x \otimes y)(d) = xdy \,,\; x, y \in \Lambda \,,\; d \in \Delta \;)$

(3) The following map g is a C-isomorphism

$g : \Delta \otimes_C (\Lambda \otimes_\Gamma \Lambda)^\Lambda \longrightarrow (\Lambda \otimes_C \Lambda)^\Gamma$ $\quad (g(d \otimes \alpha) = d\alpha$ for $\alpha \in (\Lambda \otimes_C \Lambda)^\Lambda, d \in \Delta)$

(4) For any two sided Λ-module M, the following map g is a C-isomorphism

$g : \Delta \otimes_C M^\Lambda \longrightarrow M^\Gamma$ $\quad (g(d \otimes m) = dm \,,\; m \in M^\Lambda \;\; d \in \Delta \;)$

We shall call that Λ is an H-separable extension of Γ if Λ and Γ satisfy the above conditions.

Theorem If ${}_\Lambda(\Lambda \otimes_\Gamma \Lambda)_\Lambda < \oplus \; {}_\Lambda(\Lambda \oplus \cdots \oplus \Lambda)_\Lambda$, Λ is a separable extension of Γ, that is, the map $\Lambda \otimes_\Gamma \Lambda \longrightarrow \Lambda$ $(x \otimes y \mapsto xy)$ splits as Λ-Λ-map.

Azumaya algebra Λ is an H-separable extension of its center C. And we can see that many properties of Azumaya algebra hold in H-separable extension. For example, we have

Proposition Let Λ be an H-separable extension of Γ. Then the following three maps are ring isomorphisms.

$\eta_l : \Delta \otimes_C \Lambda^\circ \longrightarrow \mathrm{Hom}({}_\Gamma\Lambda, {}_\Gamma\Lambda)$ $\quad \eta(d \otimes x^\circ)(y) = dyx$

$\eta_r : \Lambda \otimes_C \Delta^\circ \longrightarrow \mathrm{Hom}(\Lambda_\Gamma, \Lambda_\Gamma)$ $\quad \eta(x \otimes d^\circ)(y) = xyd$

$\eta_t : \Delta \otimes_C \Delta^\circ \longrightarrow \mathrm{Hom}({}_\Gamma\Lambda_\Gamma, {}_\Gamma\Lambda_\Gamma)$ $\quad (d \otimes e^\circ)(y) = dye$

for $x, y \in \Lambda$ and $d, e \in \Delta$.

Let $\mathcal{L}_e = \{B \mid$ intermediate subring of Λ and Γ, $B \otimes_\Gamma \Lambda \longrightarrow \Lambda$ $(b \otimes x \mapsto bx)$ splits as B-Λ-map, ${}_B B_\Gamma <\oplus\, {}_B \Lambda_\Gamma \}$ and $\mathcal{U} = \{D \mid$ C-subalgebra of Δ, $D \otimes_C \Delta \longrightarrow \Delta$ $(d \otimes x \mapsto dx)$ splits as D-Δ-map, ${}_D D <\oplus\, {}_D \Lambda \}$. And let $\mathcal{L} = \{B \mid$ subring of Λ which is a separable extension of Γ, ${}_B B_B <\oplus\, {}_B \Lambda_B \}$, and $\mathcal{U} = \{D \mid$ separable C-subalgebra of $\Delta \}$. Clearly we see $\mathcal{L} \subset \mathcal{L}_e$.

Theorem. If Λ is an H-separable extension of Γ, then $\mathcal{U} < \mathcal{U}_e$, and the correspondence $V : A \to V_\Lambda(A)$ yields one to one correspondece between $\mathcal{L}_e$ and $\mathcal{U}_e$, and between $\mathcal{L}$ and $\mathcal{U}$, with V^2 = identity.

Finally we have

Theorem If Λ is an H-separable extension of Γ with ${}_\Gamma \Gamma <\oplus\, {}_\Gamma \Lambda$ (or $\Gamma_\Gamma <\oplus\, \Lambda_\Gamma$), and if the center of Γ is C-f·g projective, then every ring endomorphism of Λ which fixes all elements of Γ is an automorphism.

Earl J. Taft: <u>Hopf algebras with non-semisimple antipodes</u>

Let H be a finite-dimensional pointed Hopf algebra over a field F of characteristic p. Let S be the antipode of H. We had previously shown that $S^{2ep^m} = I$, where e is the exponent of the group G(H) of group-line elements of H, and $p^m \geq n \geq p^{m-1}$ where $H=H_n$ in the coradical filtration of H. Here we construct an example for $p > 2$ where dimension H is p^3, H is pointed irreducible (so e = 1) and S has order 2p. This is the first known example for $p > 2$ for which H is not semisimple. The algebra structure of H involves some interesting ring-theoretic and Lie algebra-theoretic ideas.

F. Kasch (München)

Gefördert durch
DFG Deutsche Forschungsgemeinschaft