MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 22/1974

Finite Geometries

27.5. bis 31.5.1974

This year's conference on 'Finite Geometries' was held under the supervision of Professor D.R.Hughes (London) and Professor H.Lüneburg (Kaiserslautern).
The topics covered included Projective Geometry, Graph and Coding Theory and was notable in emphasising the close interaction between them.

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J. ANDRÉ: Finite nearaffine spaces

A nearaffine space is a structure of the type $\mathfrak{F} = (X, \mathcal{L}, \cup, \parallel)$ such that $X \neq \phi$ (points), $\mathcal{L} \subseteq \mathfrak{P}(X)$ (lines = Linien), $\cup : X^2 \setminus \Delta_X \twoheadrightarrow \mathcal{L}$ (not necessarily commutative), $\parallel \subseteq \mathcal{L}^2$ is an equivalence relation (parallelity); moreover some further axioms hold such that becomes an affine space if additionally $\cup$ is commutative. The only known proper finite nearaffine spaces are of the form $X = F^d$ such that $(F, +_i, \cdot)(i \in \{1,\ldots,d\})$ are finite nearfields and $\cup$ is defined by $x \cup y := F(y-x) + x$ where multiplication is given componentwise and the addition by $(\xi_i) + (\eta_i) = (\xi_i +_i \eta_i)$ $(\xi_i, \eta_i \in F)$. A general theory of finite nearaffine spaces will be developed. Some number-properties will be proved, e.g. two different points are incident with either exactly one or exactly n lines where n is the number of points on a line (the order of the space).

E.F. ASSMUS, Jr.: Some conjectures on the ranks of projective planes.

Let π be a projective plane of order n and p a prime dividing n. Let $Rk_p(\pi)$ denote the mod p rank of the incidence matrix of π. We have the following

Theorem: $Rk_p(\pi) \leq \frac{n^2+n+2}{2}$ and, moreover, equality whenever p^2 does not divide n.

It is well-known that $Rk_p(\mathbb{P}_2(GF(p^a))) = \binom{p+1}{2}^a + 1$. Thus,

$Rk_3(\mathbb{P}_2(GF(g))) = 37$. The other three planes of order 9 that are known each have rank 41. We have the following

<u>Conjecture 1</u> : If π is a plane of order $p^a = n$, then $Rk_p(\pi) \geqq \binom{p+1}{2}^a + 1$ with equality if and only if $\pi \approx \mathbb{P}_2(GF(p^a))$

or, more strongly,

<u>Conjecture 2</u> : If k is the smallest of the ranks of all projective planes of order n (for some fixed p dividing n), then any two planes of order n and rank k are isomorphic.

Observe that either conjecture would imply that there is a unique plane of order p (a well-known conjecture). Moreover Conjecture 2 would imply that there is at most one plane of order 10. If one could prove that any cyclic plane of order p^a had rank $\binom{p+1}{a}^a + 1$, then the conjectures would imply that cyclic planes of order p^a are Desarguesian, another well-known conjecture.

P.J. CAMERON: <u>Parallelisms and Multiply Transitive Groups</u>

Parallelisms of the set of all k-subsets of an n-set are considered. First I discuss those with a certain "parallelogram property", and exploit connections with Steiner systems and perfect codes to obtain a complete determination: the only non-trivial examples are the usual parallelism of lines in affine d-space over GF(2), with $k = 2$, $n = 2^d$, and a unique example with $k = 4$, $n = 24$. Next I consider parallelisms whose automorphism group has the highest possible degree of transitivity (namely k+1): such a parallelism satisfies either the parallelogram property or the condition $k = 2$, $n = 6$. The final theorem asserts that a (k+1)-transitive group of degree n which acts imprimitively on k-subsets is an automorphism group of a parallelism.

Ph. DELSARTE: <u>On the nonexistence of perfect codes in triangular type association schemes</u>

For integers k, v, with $2 \leq k \leq [v/2]$, let X be the set formed by all k-subsets of a given v-set. Define an "adjacency relation" R on X as follows : $R = \{(x,y) \in X^2; \quad |x \cap y| = k - 1\}$. Then (X,R) is a <u>metrically regular graph</u> : it generates the triangular type association scheme T(k,v).

A nonempty subset $Y \subset X$ is called a <u>perfect e-code</u>, for a given integer e = 1,2,...,k-1, if each vertex $x \in X$ is at distance $\leq$ e from exactly one vertex $y \in Y$ in the graph (X,R). By use of the <u>Lloyd theorem</u> for perfect codes in metrically regular graphs, a strong necessary condition is obtained. A typical result, for e = 1, is the following : Any perfect 1-code yields a t-design $S_\lambda(t,k,v)$, with t = solution of $t = (k-t-1)(v-k-t-1)$ and $\lambda = \binom{v-t}{k-t}(t+1)(v-t)$. Conjecture : for $t \geq 2$, there are no t-designs with such parameters, hence no perfect 1-codes.

R.H.F. DENNISTON: The fifteen schoolgirls walk for thirteen weeks

A t-design $(k,\lambda;v)$ can sometimes be resolved into t'-designs $(k,\lambda';v)$: this simply means that, considered as a set of blocks, it has been partitioned. Thus, the schoolgirl problem is to find a 2-design $(3,1;v)$ resolvable into 1-designs $(3,1;v)$. A further problem is to resolve the complete 3-design $(3,1;v)$ into 2-designs $(3,1;v)$ which are again resolvable into 1-designs $(3,1;v)$; and for this two methods are known, both classical in the case $v = 9$. The one involves addition and multiplication in a ring of order $v - 2$, and has been used where $v = 33$ (Schreiber), 51, 75, 105 - also where $v = 15$, though in that case multiplication is unhelpful. The other method is to draw a pattern across three copies of a 2-design $(3,1;\frac{1}{3}v)$: this has been used where $v = 3^n$, 3^n5^2, 45, and 135. For $v = 24$ and 48, the complete 3-design $(3,1;v)$ has been resolved into 2-designs $(3,2;v)$ resolvable into 1-design $(3,1;v)$; and a 3-design $(4,1;32)$ exists which has been resolved into 2-designs $(4,3;32)$ resolvable into 1-designs $(4,1;32)$.

VINCENZO DICUONZO : On pseudo Möbius planes over fields of characteristic > 2.

It is the object of this paper to construct and study a class of the so called Minkowski geometries. To this aim, given a projective plane π over a field K of characteristic > 2 and order q, let I and J be two distinct points of π and denote by r the line through I and J. The points of π, not belonging to r, and the lines of π through I or J are assumed as points of an incidence structure ψ; certain conics of π through I and J are assumed as blocks of ψ and called circles of ψ. By construction an involutorial automorphism of ψ is associated to every circle. Such an automorphism, called inversion, gives rise to the definition of orthogonality between two circles. On the planes ψ, constructed in this way, the quadruplets of mutually orthogonal circles may be of four types and two of them occur when $\frac{1}{2}(q+1)$ is odd, while the other ones when $\frac{1}{2}(q+1)$ is even. It follows that, with respect to the types of such quadruplets of circles, the planes ψ may be divided into two classes, according a $\frac{1}{2}(q+1)$ is odd or even. Successively the group G, generated by the inversions of ψ, is studied and it is demonstrated that every element of G may be represented as product of at most five generators.

MANFRED DUGAS: <u>Charakterisierungen endlicher desarguesscher uniformer Hjelmslev-Ebenen</u>.

Eine Kollineations-gruppe G einer endlichen projektiven Hjelmslev-Ebene $\mathfrak{H}$ heisst Rang3-Gruppe, falls für jeden Punkt P von $\mathfrak{H}$ der Stabilisator G_P genau 3 Punktbahnen hat. Das kanonische epimorphe Bild von $\mathfrak{H}$ sei $\mathfrak{H}$.

<u>Satz 1</u> : Ist $\mathfrak{H}$ eine endliche projektive Hjelmslev-Ebene und G eine Rang3-Gruppe von $\mathfrak{H}$, so ist $\mathfrak{H}$ uniform; ist die

Ordnung von $\bar{\mathcal{H}}$ ungerade, so ist $\mathcal{H}$ desarguessch.

Die projektive Hjelmslev-Ebene heisst Moufang-Hjelmslev-Ebene, falls fur jede Gerade g von $\mathcal{H}$ die Menge aller Ttanslationen von $\mathcal{H}$ mit Achse g auf den nicht zu g benachbarten Punkten von $\mathcal{H}$ transitiv operiert. Es gilt der

<u>Satz 2</u> : Jede eindliche uniforme Moufang-Hjelmslev-Ebene ist desarguessch.

B. GANTER: <u>Partial pairwise balanced designs</u>

Let K be a nonempty set of integers ≥ 2. Any finite partial PBD with block sizes from K can be embedded into a finite PBD with block sizes from K.

A. GARDINER: <u>Distance-transitive graphs</u>

Let Γ be a finite graph with distance function ∂ and vertex set $V\Gamma$. Γ is G-<u>distance-transitive</u> if $G(\leq \mathrm{Aut}\Gamma)$ acts transitively on each set $\Gamma_i := \{(u,v) \in V\Gamma \times V\Gamma : \partial(u,v) = i\}$. The theory of such graphs is still unable to use the "group" assumption effectively. Only when Γ has valency p+1, p a prime, has the hypothesised group been effectively used:

Γ is (G,s)-transitive if $G(\leq \mathrm{Aut}\Gamma)$ acts transitively on s-arcs but not on (s+1)-arcs of Γ.

<u>Theorem 1</u>. If Γ is (G,s)-transitive of valency p+1, p a prime, then (i) $p = 2$, $s \leq 5$, or (ii) p odd, $s \leq 4$, or (iii) $p = 3$, $s = 7$.

Theorem 2. If Γ is G-distance-transitive of valency p+1, p a prime, then

(a) $p \leq 3$ and Γ is known, or (b) $\Gamma \cong (2 \cdot K_{p+2})_p$, or

(c) Γ contains triangles, or (d) Γ is (G,s)-transitive, $2 \leq s \leq 4$, $5 \leq p$.

For small values of $p \geq 5$, one can obtain partial classifications.

J.M. GOETHALS: The regular two-graph on 276 vertices

Let V denote a finite set of n elements, and $V^{(i)}$ denote the set of all i-subsets of V. An ordinary graph consists of a vertex set V and an edge set $E \subset V^{(2)}$. A two-graph (V,Δ) is a pair of a vertex set V and a triple set $\Delta \subset V^{(3)}$ having the property that each 4-subset of V contains an even number of triples of Δ. A two-graph (V,Δ) is regular if each 2-subset of V is contained in a constant number of triples of Δ. The switching class of graphs belonging to the two-graph (V,Δ) is the set of all graphs with vertex set V, which have Δ as the set of triples of vertices carrying an odd number of edges. With respect to any labelling of V, any graph (V,E) is described by its (-1,1)-adjacency matrix $A = (a_{i,j})$ as follows : $a_{i,i} = 0$ for all $i \in V$, and $a_{i,j} = -1$ or 1, according as $\{i,j\}$ belongs to E or not. The adjacency matrix of any graph in the switching class of a regular two-graph has only two distinct eigenvalues.

The aim of the present lecture is to prove that, up to taking complements, there exists a unique non-trivial regular two-graph on 276 vertices. The eigenvalues of its switching class of graphs are 55 and -5.

HEINZ-RICHARD HALDER: On i-regular (k,n)-arcs in finite planes

A k-set K in a finite projective plane of order q is said to be a (k,n)-arc, if there are n but no n+1 collinear points in K ($n \leq q$). For a fixed number i with $0 < i < n$ a (k,n)-arc K is said to be i-regular if $|K \cap G| = i$ holds for all lines G with $0 < |K \cap G| < n$.

Theorem 1. Let $q \equiv 1 (\mathrm{mod}\ n)$, $i = 1$ or $i = n-1$, $k = (n-1)q+1$ and K be an i-regular (k,n)-arc in PG(2,q). Then the lines G with $|G \cap K| = i$ (tangent lines) form an i-regular (k,n)-arc in the dual plane.

Theorem 2. In case $q \not\equiv 0 (\mathrm{mod}\ n)$ there are no 1-regular $((n-1)q+1,n)$-arcs for $(n-1)^2 > q$. In a desarguesian plane there are no 1-regular $((n-1)q+1,n)$-arcs for $q = (n-1)^2+n$ and $q \not\equiv 0 (\mathrm{mod}\ n)$. For $(n-1)^2 = q$ the 1-regular $((n-1)q+1,n)$-arcs are the embeddable unitals.

Theorem 3. $(n-1)q+1$ is the least upper bound for k for the existence of i-regular (k,n)-arcs in a plane of order $q \equiv 1 (\mathrm{mod}\ n)$.

WERNER HEISE: Minkowski (pseudo euclidean) planes of even order

Every finite Minkowski (pseudo-euclidean) plane of even order is representable as the geometry of plane sections with an hyperboloid in a 3-dimensional projective space and hence unique. This is analogous to the result of P. Dembowski, that every Möbius plane of even order is ovoidal. As a consequence each sharply triply transitive set Γ of permutations of odd degree with id $\in \Gamma$ is isomorphic to the linear fraction group over a galois-field of characteristic two.

J.W.P. HIRSCHFELD: Ovals in desarguesian planes of even order

In PG(2,q), for q even, an oval is a set of q + 2 points no three of which are collinear. Any such oval can be written as

$$D(F) = \{(1,t,F(t)) \mid t \in GF(q)\} \cup \{(0,1,0),\ (0,0,1)\}$$

where $F(0) = 0$, $F(1) = 1$ and F is a permutation polynomial.

Necessary and sufficient conditions for D(F) to be an oval are given. Also, two particular cases are investigated:

(a) $F(t) = t^k$ for some positive integer k;

(b) $F(x+y) = F(x) + F(y)$ for all x, y in GF(q).

In the latter case, ovals are completely classified.

XAVIER HUBAUT: Locally polar spaces

This is a joint work of F. Buekenhout and X. Hubaut.

A locally polar space is a set of points E together with a collection of subsets of cardinality at least 4 called circles. Two points lying on the same circle are called adjacent. The main requirement is the following : given a circle γ and a point p which doesnot belong to γ p is adjacent to 0,2 or all points of γ. Moreover we require that the space is connected and that there exist at least two points not on a circle. Then one can prove that E_p, the set of points adjacent to p, together with the "lines" (i.e. the circles through p $-\{p\}$) is a polar space. If the associated group is strongly regular, and if the automorphism graph is rank 3 then one can determine all the locally polar spaces : the automorphism graphs are

$O^{\pm}_{2n+1}(2)$, $O^{\pm}_{2n+1}(2) \cdot E_{2^{2n+2}}$, M(22), McL, $PSU_5(4)$,

$PSL_4(2) \cong$ Alt(8), $PO_6^-(3)$ and Alt(10).

MICHAEL J. KALLAHER: Affine planes in which the stabilizer of every line is doubly transitive

Heinz Lüneburg has shown that affine planes described in the title are translation planes. (This has also been proven independently by Norman L. Johnson and myself). Such planes are a natural generalization of rank 3 affine planes. Investigating the structure of such planes, we are able to show that in the case of non-square order these planes either

(1) are semi-field planes (and hence in this case are rank 3 planes),

(2) are generalized André planes, or

(3) possess a desarguesian decomposition.

In case (3) it can be proven in many cases that the planes either are rank 3 planes or possess a doubly transitive collineation group.

C. LEFEVRE: Tallini sets and quadrics

Segre's famous characterization of conics in planes of odd order lead to a similar question for quadrics in higher dimension. This problem was attacked by Tallini (1956).

Let $P = P(n,q)$ be a finite projective space of order q and dimension n. Then a Tallini set in P is a set of points Q such that every line of P intersecting Q in more than two points is contained in Q.

Tallini characterized two of the three infinite classes of quadrics (at the exclusion of the elliptic quadrics), by a classification of all Tallini sets of cardinality at least $q^n + q^{n-1} + \cdots + 1$. We classify all Tallini sets of cardinality at least $q^n + q^{n-2} + \cdots + 1$ and so we characterize all (non degenerant) quadrics.

J.H. VAN LINT: Nearaffine planes

At last year's meeting J. André introduced the concept of nearaffine planes. The essential thing was that the lines $x \cup y$ and $y \cup x$ were not necessarily the same. However if this is always the case then the plane is an affine plane. General construction methods were not known. We describe a construction due to H. Wilbrink and J. van de Schoot which works for all odd n. For $1 \leq i \leq \frac{n-1}{2}$ define $f_i(x)$ for $1 \leq x \leq \frac{n-1}{2}$ by $[f_i(x)] = \text{circ}\left(1, \frac{n-1}{2}, \frac{n-3}{2}, \cdots, 2\right)$. Furthermore let $f_i(x) := -f_i(-x)$ and $f_{n-i}(x) - f_i(x)$. Finally let $f_i(0) = 0$, $1 \leq i \leq n-1$.

These n-1 functions have the property that for every $x \neq 0$ there is a y such that $f_i(x+y) \neq f_i(x) + f_i(y)$. Let $M: = \,_n$, $V = M^2$ and define the line $x \cup y$, where $x = (\xi,\xi')$, $y = (\eta,\eta')$ by $\{(\xi,\alpha') \mid \alpha' \in M\}$ if $\xi = \xi'$, $\{(\alpha,\eta) \mid \alpha \in M\}$ if $\eta = \eta'$ and $\{(\xi+\alpha,\ \xi' + f_i(\alpha) \mid \alpha \in M\}$ otherwise, where we take the unique i s.t. $f_i(-\xi+\eta) = -\xi' + \eta'$. Let ∞, 0, i respectively be the direction of these lines. Two lines are parallel iff they have the same direction. It is now simple to show that all of André's axioms hold.

RUDOLF METZ: $(3,6,\frac{1}{3}(4^n+2))$

Starting from the well known Steiner system of type (3,6,22), for any n a Steiner system of type $(3,6,\frac{1}{3}(4^n+2))$ can easily be constructed. Define on any plane of PG(n-1,4) a S(3,6,22) and identify all the improper points.

T.G. OSTROM: Finite translation planes of dimension 3

A non-solvable minimal non f.p.f. subgroup of a translation plane of odd order and odd dimension over its kernel, either has A_7 as an homomorphic image or is ismorphic to SL(2,u) for some u. For dimension 3 over the kernel $u \leq 13$ or is a power of the characteristic of the plane.

When u is a power of the characteristic p, the various possibilities for the nature of p-elements in terms of their minimal polynomials. Some cases can be eliminated; for the others the question remains open as to whether planes exist admitting SL(2,p) with p-elements of the type in question.

STANLEY E. PAYNE: <u>Coordinates for Generalized Quadrangles</u>

Let $P_4 = (P,B,I)$ be a generalized quadrangle of order s (alternately, P_4 is a partial geometry (1+s, 1+s, 1) in the sense of Bose). A collineation θ of P_4 is a symmetry about a line $L \in B$ provided that θ fixes all lines meeting L. If the group $S(L)$ of symmetries about L is complete, i.e. has order s, then L is called a centre of symmetry. If L is a centre of symmetry, a projective plane ¶ based at L may be constructed and used in conjunction with $S(L)$ to provide a system of coordinates for P_4. Each known generalized quadrangle of order s has some point x_∞ such that each line through x_∞ is a centre of symmetry. A partial converse is proved : If P_4 has three centres of symmetry concurrent at x_∞, then each line through x_∞ is a centre of symmetry and P_4 is a translation generalized quadrangle in the sense of Thas. If s=4 and P_4 has even two concurrent lines that are centres of symmetry, it must be the unique translation quadrangle of order 4.

VERA PLESS: Classification and Enumeration of Self-Dual Codes

This is a joint work of Vera Pless and N.J.A. Sloane.

A complete classification is given of all [22,11] and [24,12] self-dual codes. There are 26 distinct indecomposable self-dual codes od length 24 over GF(2), including unique codes of minimum weights 8 and 6, whose groups are respectively the Mathieu group M_{24} and the maximal subgroup of index 1771 in M_{24}. For each code we give the order of its group, the number of codes equivalent to it, and its weight distribution. Several theorems on the enumeration of self-orthogonal codes are used, including formulas for the number of such codes with minimum distance ≥ 4, and for the sum of the weight enumeration of all self-dual codes.

DIJEN K. RAY-CHAUDHURI: Characterization of projective incidence structures and projective graphs

Alan Spragne and Dijen K. Ray-Chaudhuri proved the following two theorems. Let $\pi = (P,L,I)$ be a finite incidence structure. $G(\pi)$ will denote a simple graph whose vertex set is P and two points are adjacent iff there is a line joining them in π. Let $d(p,p')$ denote the distance between p and p' in $G(\pi)$. Define $d(p,\ell) = \min_{p'\in\ell} d(p,p')$, $d(\ell,m) = \min_{p\in\ell} d(p,m)$. Let $3 \leq s \leq d-2$ and q (a prime power) be integers. Let V be a d-dimensional vector space over GF(q). Let W_s be the class of s-dimensional subspaces.

Consider the incidence structure $\pi = (W_{s-1}, W_s, \subseteq)$ whose points and lines are respectively elements of W_{s-1} and W_s and incidence is inclusion. Then π satisfies

(1) $G(\pi)$ is connected.

(2) Any two points are joined by at most one line.

(3) If $d(p,p') = 2$, then there are $q+1$ lines ℓ which pass through p' such that $d(p,\ell) = 1$.

(4) If $d(p,\ell) = 1$, then are $q+1$ lines in passing through p and intersecting ℓ.

Theorem 1. Let $q \geq 2$ and π' be an incidence structure satisfying the axioms (1), (2), (3) and (4). Let s and d be defined by $\bar{k} = \frac{q^s-1}{q-1}$, $\bar{r} = \frac{q^{d+1-s}-1}{q-1}$ where $\bar{k}$ = average number of points on a line, $\bar{r}$ = average number of lines on a point. If $3 \leq s \leq d - 2$, then $\pi' \cong \pi$ and q is a prime power. Let G be a graph whose vertices are elements of W_s, two vertices being adjacent iff they intersect in an $s-1$ dimension subspace.

Theorem 2. The graph G can be reconstructed from 4 parameters and an inequality $d \geq \psi(s,q)$.

JOACHIM RÖHMEL: Smooth incidence structures

Let $\mathfrak{J} = (\mathfrak{P}, \mathfrak{B}, I)$ be a finite incidence structure, (for notations see Dembowski, Finite Geometries) with

(i) $x,y,z \in \mathfrak{P}$, $y \neq z$, $x \notin ((y,z)) \Rightarrow [x,y,z] = f = \text{const.}$

(ii) $B,D \in \mathfrak{B}$, $B \neq D$, $[B,D] \neq 0 \Rightarrow [B,D] = \mu = \text{const.}$

(iii) $[x] \le b-1$, $[B] \le v-1$ for all $x \in \mathcal{P}$, $B \in \mathcal{B}$. There is a point $x \in \mathcal{P}$ with $[x] \ge 2$.

Then $\mathfrak{J}$ is a smooth design. The lines and blocks through any fixed point p form a symm. design $\mathfrak{J}_p^*$.

Let $\bar{\mathfrak{J}}$ be a symm. design. Call $\mathfrak{J}$ an extension of $\bar{\mathfrak{J}}$ if (i) $\mathfrak{J}$ is a smooth design, and (ii) there is a point p such that $\bar{\mathfrak{J}} \cong \mathfrak{J}_p^*$. If $\mathfrak{J}$ is an extension of a 2-(4n-1, 2n-1, n-1) Hadamard-design, then either any line contains exactly t = 2 points and $\mathfrak{J}$ is the affine 3-(4n,2n,n-1) design, constructed by Hughes and Dembowski, or any line contains t = 3 points and $\mathfrak{J} \cong \mathcal{P}_{d-1}(d,2)$.

If $\mathfrak{J}$ is an extension of a projective plane of order n then either any line contains t = n+1 points and $\mathfrak{J} \cong \mathcal{P}_2(3,n)$ or t = n and $\mathfrak{J} \cong \mathcal{A}_2(3,n)$ or t = n or $n = t(t^2+1)$.

N.J.A. SLOANE: Upper bounds for modular forms, lattices and codes (Joint work with C.L. Mallows and A.M. Odlyzko)

Let $W(z) = 1 + A_d e^{2\pi i dz} + A_{d+1} e^{2\pi i(d+1)z} + \cdots$ be a modular form of weight n for the full modular group. Then for every constant b there exists an $n_o = n_o(b)$ such that if $d \ge \frac{n}{6} - b$ and $n \ge n_o$, then one of A_d, A_{d+1}, $\cdots$, has a negative real part. This implies that there is no even unimodular lattice in E^n, for $n \ge n_o$, having minimum nonzero squared length $\ge \frac{n}{12} - b$. A similar argument shows that there is no binary self-dual code of length $n \ge n_o$ having all weights divisible by 4 and minimum nonzero weight $\ge \frac{n}{6} - b$. A corresponding result holds for ternary codes.

N.J.A. SLOANE: The orchard problem (Joint work with S.A. Burr and B. Grünbaum.)

A (p,t)-arrangement consists of p points and t (straight) lines in the Euclidean plane chosen so that every line has exactly three points on it. The orchard problem is to find an arrangement with the greatest t, t(p) say, for each value of p. The exact value of t(p) is given for $1 \leq p \leq 12$ and $p = 16$, and upper and lower bounds are obtained. These imply that $t(p) \sim p^2/6$ as $p \to \infty$.

J.A. THAS: 4-gonal configuarations with parameters $r = q^2 + 1$ and $k = q + 1$.

Main results:

(a) Up to an isomorphism there is only one 4-gonal configuaration with parameters $r = 5$ and $k = 3$. An immediate corollary is the well-known theorem that up to an isomorphism there is only one strongly regular graph with parameters $v = 27$, $n_1 = 10$, $p^1_{11} = 1$, $p^2_{11} = 5$.

(b) If the 4-gonal configuaration S with parameters $r = q^2 + 1$ and $k = q + 1$, where q is even and $q > 2$, possesses a regular point, then S is isomorphic to a 4-gonal configuration T(O) arising from an ovoid O of PG(3,q).

(c) If the 4-gonal configuration S with parameters $r = q^2 + 1$ and $k = q + 1$, with q odd, possesses a regular point x and if the corresponding inversive plane $\pi(x)$ is egglike, then

S is isomorphic to the 4-gonal configuration Q(5,q) arising from a non-singular hyperquadric of index 2 of PG(5,q).

(d) If each point of the 4-gonal configuration S with parameters $r = q^2 + 1$ and $k = q + 1$ $(q > 2)$ is regular, then S is isomorphic to Q(5,q).

(e) Suppose that the 4-gonal configuration S = (P,B,I), with parameters $r = q^2 + 1$ and $k = q + 1$ $(q > 1)$, has a 4-gonal subconfiguration s' = (P',B',I'), with parameters k' = r' = k, for which the following condition is satisfied : if $x,y,z \in P'$, with $x \not\sim y$, $y \not\sim z$, $z \not\sim x$, then the triple (x,y,z) is regular and moreover $sp(x,y,z) \subset P'$. Then we have

(i) S has an involutorial automorphism θ which fixes P' pointwise,

(ii) S' is isomorphic to the 4-gonal configuration Q(4,q) arising from a non-singular hyperquadric of PG(4,q).

H.C.A. VAN TILBORG: <u>t-designs, generated by uniformly packed codes</u>

An e-error correcting code of length n is called uniformly packed if there exists an r such that

(1) any word at distance e from a codeword is at distance e+1 from r-1 other codewords.

(2) Any word at distance >e from the code is at distance (e+1) from r codewords.

These codes imply the existance of (e+1) - $(n+1,w,\alpha(w))$ designs, for weights w occurring in the extended code. $\alpha(w)$ can be evaluated by a recursive formula $\alpha(2e+2) = r-1$.

The 2-error correcting BCH-codes of length $n = 2^{2k+1} - 1$ form the first known infinite sequence of uniformly packed codes. In this case $r = \frac{n-1}{6}$.

HEINRICH WERNER: Automorphism groups of Steiner Triple Systems (STS)

A subsystem S of a STS T of order v is said to be "large in T" if $|S| = \frac{1}{2}(v-1)$ and this is the case iff every subsystem of T has a nonempty intersection with S.

Proposition: 1. If a finite STS has a large subsystem it is simple.

2. If a finite STS has a point which together with every pair of noncollinear points generates a PG(2,2) then it is simple.

Theorem: Every finite STS which is not simple is a large subsystem of some simple STS having the same Automorphism Group.

Proposition: If all morphisms $S \to T$ and $T \to S$ between the STSs S and T are constant $\mathrm{Aut}(S\times T) \cong \mathrm{Aut}\, S\ \mathrm{Aut}\, T$ holds.

Corollary: If G is isomorphic to the automorphism group of some finite STS then so is $G \times G$.

RUDOLF WILLE: Partial designs with respect to conclusion sentences

For an axiomatic class $\mathfrak{A}$ of designs and a conclusion sentence we define $\mathfrak{A}_\sigma$ to be the class of all A in $\mathfrak{A}$ satisfying σ . A partial $\mathfrak{A}_\sigma$-design is defined as a relative substructure of a design in $\mathfrak{A}_\sigma$; $\mathcal{P}\mathfrak{A}_\sigma$ is the class of all partial $\mathfrak{A}_\sigma$-designs.

A general theorem is stated which roughly says that almost all $\mathcal{P}\mathfrak{A}_\sigma$ are not finitely axiomatizable with respect to $\mathfrak{A}$; for instance, $\mathcal{P}\mathfrak{A}_\sigma$ is not finitely axiomatizable with respect to $\mathfrak{A}$ if $\mathfrak{A}$ is the class of all projective planes and σ the Fano-Axiom, Desargues' theorem or Pappus' theorem or if $\mathfrak{A}$ is the class of all inversive planes and σ the bundle theorem or Miguel's theorem.

KARL ERICH WOLFF: Konstruktion von Partialblockplänen

1. Problem: Kann man 24 Teilnehmer einer Versammlung 8 Tage lang so zum Mittagessen an Tische mit je 4 personen verteilen, dass je 2 personen wenigstens einmal zusammensitzen. (ungelöst)

2. Verallgemeinerung: Man untersuche taktische Konfigurationen mit $[x,y] \geq 1$, $r(k-1) = v$. Gibt es Parallelismen in diesen Inzidenzstrukturen (genannt Fast-Blockpläne)?

3. Zur Konstruktion von Fast-Blockpläne werden zwei Verfahren verfeinert und angewendet:

(a) Verallgemeinerung der "method of differences" fur beleibige (insbes. auch nichtkommutative) Gruppen; Konstruktion zweier Fast-Blockpläne mit $k = 3$, $r = 12$.

(b) Präzisierung der "method of extension" durch f-Summen von Inzidenstrukturen : $\mathfrak{J}_1 +_f \mathfrak{J}_2$.

Die Anwendung liefert Serien von Fast-Blockplänen mit $k = 3$. Dabei werden Parallelismen in den Fast-Blockplänen mit $k = 2$ benutzt, die aus Parallelismen des vollständigen Graphen $K_v (v \equiv 0 \ (2))$ konstruierbar sind. Es wird ein neuer Parallelismus von K_v angegeben.

4. Aus Fast-Blockplänen mit $k = 2$ oder $k = r = 4$ lassen sich 3-Assoziationsschemata konstruieren.

5. Ist $\mathfrak{J}$ eine (r,k,t)-Partialgeometrie, $t < r$, mit Parallelismus oder ein Blockplan mit $\lambda = 1$ und Parallelismus, so ist fur $k < s = v/k$ $\mathfrak{J}$ dual ein 3-Partialblockplan.

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