MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 25/1974

Potentialtheorie

16.6. bis 22.6.1974

Die Tagung über Potentialtheorie am Mathematischen Forschungsinstitut Oberwolfach in der Woche vom 16. bis 22. Juni 1974 stand unter der Leitung von Herrn Prof. Dr. H. Bauer. Dabei war es nach etwa 10 Jahren (Tagung des C.N.R.S. in Orsay) zum erstenmal wieder gelungen, führende Vertreter aller Teilgebiete der Potentialtheorie, wie der klassischen, der axiomatischen, der wahrscheinlichkeits- und prozeßtheoretischen, zu versammeln.
Die Fülle von 38 Vorträgen, die sämtlich über neue Forschungsergebnisse berichteten, läßt erkennen, daß sich einerseits das Interesse an der Potentialtheorie in letzter Zeit wieder verstärkt hat und daß es andererseits wünschenswert wäre, die Zeitabstände zwischen derartigen Tagungen über Potentialtheorie nicht zu groß werden zu lassen.

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Vortragsauszüge

J. DÉNY: Dirichlet Forms and Operators "carré du champs".

(Survey on three recent papers on Dirichlet forms and related topics by G. ALLAIN (Orsay 1973); L.E. ANDERSSON (Mittag-Leffler Inst. 1974); J.P. ROTH (C.R. avril 1974).)

There is an integral representation theorem for Dirichlet forms:

Theorem 1: For X a locally compact space let Q be a Dirichlet form, whose domain V is dense in $\mathcal{K}(X)$. Then there exists:

(i) a positive Radon measure μ on X

(ii) a positive Radon measure σ on $X^2 \setminus \Delta$

(iii) a local Dirichlet form N on V

such that $Q(f) = \int |f|^2 \, d\mu + \frac{1}{2} \iint |f(x) - f(y)|^2 d\sigma(x,y) + N(f)$ for all $f \in V$, and such a decomposition is unique.

Concerning the local form in theorem 1 the following theorem holds:

Theorem 2: If X is an euclidian domain in $\mathbb{R}^n$ and if $\mathcal{K}(X) \cap \mathcal{C}^1(X) \subset V \subset \mathcal{K}(X)$, then there exists a positive Radon measure v on X and n^2 borel functions $a_{ij} \in L_{loc}(v)$ satisfying $a_{ij} = a_{ji}$ and $\sum_{ij} a_{ij}(.)f_i f_j \geq 0$ such that for all $f \in \mathcal{K}(X) \cap \mathcal{C}^1(X)$

$$N(f) = \sum_{ij} \int a_{ij} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} dv(x) .$$

These results give new and simple proofs for LÉVI-KHINTCHIN representation formulas of operators satisfying the positive maximum principle.

M. FUKUSHIMA: On Closed Extensions of Markov Symmetric Forms.

A symmetric form on a real L^2-space is said to be a Dirichlet form if it is closed and the associated L^2-semigroup is markovian. Given a closable symmetric form $\mathcal{E}$ introduce the following semiorder in the family DE ($\mathcal{E}$) of all Dirichlet extensions of $\mathcal{E}$:

$$\mathcal{E}^{(1)} \succ \mathcal{E}^{(2)} \text{ iff} \begin{cases} D(\mathcal{E}^{(1)}) \supset D(\mathcal{E}^{(2)}) \\ \mathcal{E}^{1}(u,u) \leq \mathcal{E}^{(2)}(u,u) \quad \text{for } u \in D(\mathcal{E}^{(2)}), \end{cases}$$

and look for the maximum elements of various subfamilies of DE($\mathcal{E}$). We give a concrete expression of the KREIN-EXTENSION and present an example where the Krein extension generates a semigroup which is not even positive.

When $\mathcal{E}$ is given by the usual Dirichlet integral D on an euclidian domain $\Omega \subset \mathbb{R}^n$ with $D(\mathcal{E}) = \mathcal{C}_0^\infty(\Omega)$, then the maximum element is known to be the celebrated SOBOLEV space $(\mathcal{E}, H^1(\Omega))$. We establish analogous theorems for the more general integro-differential expression introduced by BEURLING and DENY.

CHR. BERG: Fourier Transformation of Positive Definite Measures.

Many symmetric kernels in potentialtheory are positive definite measures, and it turns out to be of interest to Fourier-transform such measures. If the underlying space is $\mathbb{R}^n$ this can be done, since every positive definite measure on $\mathbb{R}^n$ is a tempered distribution.

Using the theorem of BOCHNER it is possible to prove the following result:

Theorem 1: Let G be a locally compact abelian group with Haar measure dx and dual group $\hat{G}$. Then for every positive definite measure μ on G there exists a uniquely determined measure σ on $\hat{G}$ such that for all $f \in \mathcal{K}(G)$ and $x \in G$

(i) $\int |\hat{f}(\gamma)|^2 \, d\sigma(\gamma) < \infty$

(ii) $\mu * f * \tilde{f}(x) = \int (x,\gamma) |\hat{f}(\gamma)|^2 \, d\sigma(\gamma)$

Theorem 2: This Fourier transformation is an idempotent homeomorphism between the cones of positive definite measures of G and $\hat{G}$ respectively endowed with the vague topologies.

G. RITTER: On a Construction of Resolvents and Semigroups by Means of Absorbing Sets.

Let V be a complex kernel on a locally compact space X with a countable base. It is supposed that V has "sufficiently" many absorbing sets ($A \subset X$ is absorbing if $\bar{A} = A$ and $\mathrm{supp}(V(x,.)) \subset A$ for $x \in A$). Then V^n can be defined by composition and it is shown that there exists a resolvent family $(V_p)_{p \in \mathbb{C}}$ of kernels such that $V_0 = V$.

The condition on the existence of "many" absorbing sets is satisfied for hyperbolic and parabolic potential kernels.

It is shown how SEBASTIÃO e SILVA's spectral theory in algebras with a bounded structure can be applied to such resolvents. This yields a sufficient condition for the existence of a semigroup whose Laplace transform is the resolvent.

J.L. DOOB: Probabilistic Versions of Potential Theoretic Ideas.

The PERRON-WIENER-BRELOT method of solving the Dirichlet problem for classical harmonic functions leads to a new criterion for the uniform integrability of a martingale:

A martingale $(\mathcal{U}_n)$ is uniformly integrable iff for any $\varepsilon > 0$ there is a supermartingale (X'_n) and a submartingale (X''_n) such that $X'_n \leq \mathcal{U}_n \leq X''_n$ and $E[X''_1 - X'_1] < \varepsilon$.

The probabilistic version of the existence of LEBESGUE-POINTS of a function leads to a new convergence theorem for conditional expectation:

Given an increasing sequence $(\mathcal{F}_n)$ of σ-fields on Ω, X measurable with respect to $\bigvee_n \mathcal{F}_n$. Put for $w' \in \Omega$ $Y(w) := X(w) - X(w')$. Then the proof of the Lebesgue point theorem leads to the proof of

$$\lim_{n\to\infty}(E^{\mathcal{F}_n}[|Y|])(w') = 0 \qquad \text{a.e. on } \Omega .$$

P.A. MEYER: Duality up to a Polar Set.

Assume E to be a Borel set in some compact metric space and let (X_t) be a Markov process on $E \cup \{\Delta\}$ $(\Delta \notin E)$, (P_t) and $(\mathcal{U}_p)$ its semigroup and resolvent respectively. Let (X_t) furthermore satisfy the "hypotheses from the right" (i.e. (X_t) right continuous, without branching points, p-excessive functions nearly borel and a.s. right continuous along paths), $\mathcal{U}$ be a proper kernel on E and suppose all the measures $\mathcal{U}(x,dy)$ to be absolutely continuous to some fixed measure m.

Then it is possible to remove a polar set from E and to make a time-change (more precisely an accelaration) of the process such that $(\mathcal{U}_p)$ has co-resolvent $(\hat{\mathcal{U}}_p)$ satisfying the hypotheses for a RAY-COMPACTIFICATION.

So without any hypotheses on the existence of a dual all the theory of martin boundary can be carried out.

M. NAGASAWA: A Class of Excessive Measures of Branching Markov Processes.

Let S be a compact metric space and define convolution on the generalized sequence space $\pi_{\ell_1} \mathcal{C}(S^n) =: \ell^1(S)$ as in ℓ_1. Then a convolution preserving semigroup (T_t) of bounded linear operators on $\ell^1(S)$ induces a dual semigroup (S_t) on the (generalized) $\ell^\infty(S)$ by $\langle T_t f,\mu\rangle = \langle f,S_t\mu\rangle$ with the following property. If for a bounded measure m on S $\hat{m}$ is defined on $\bigcup_{n=0}^{\infty} S^n$ by $\hat{m}|_{S^n} := m^n$ the formula $S_t\hat{m} = \widehat{(S_t\hat{m})_1}$ holds (KAC's propagation of chaos).

Now the non linear semigroup $H_t m := (S_t\hat{m})_1$ provides the solution of Boltzmann equation. Furthermore if (S_t) preserves absolute continuity with respect to $\hat{m}$, we can define a semigroup

(P_t) on $\mathcal{B}(\cup S^n)$ by $S_t(\hat{f}\ \hat{m}) = (P_t\hat{f})\hat{m}$ where $\widehat{fm} =: \hat{f}\ \hat{m}\ (f \in \mathcal{B}(S)_1)$, which satisfies branching property $P_t\hat{f} = \widehat{(P_t\hat{f})_1}$.

Now if $\hat{m}$ is excessive for (P_t), we have duality between (T_t) and (P_t). Concerning the existence of such excessive measures we have for Galton-Watson the reference [(*)] and for the general case we have the following

<u>Theorem:</u> If there are non-negative solutions $\eta_1 \le \eta_2$ (at most two) of $\sum q_k\ u^k = u$, then $\hat{m}$ is an excessive measure when

$$\eta_1 \le 2q_o \le \eta_2 \quad \text{and} \quad {}^1/\eta_2 \le m \le {}^1/2q_o$$

when $q_o = o$ (e.g. dual of collision process), $\eta_1 = o$. If there is another solution $\eta_2 > o$, then we can find $m \ge {}^1/\eta_2$.

H. FÖLLMER: <u>Relative Densities of Semimartingales.</u>

A semimartingale $X = (X_t)_{t\ge o}$ over a "nice" system $(\Omega,\mathcal{F},\mathcal{F}_t,P)$ can be represented as a finite signed measure P^X on the σ-field $\mathcal{P}$ of predictable sets in $\Omega \times (o,\infty]$. Introduce the relative density D^YX of X with respect to a semimartingal Y as the Radon-Nikodym density of P^X with respect to P^Y.

It turns out that there are various identifications of D^YX, which are motivated by classical analysis on $\mathbb{R}^1$ (Lebesgue's theorem on the differentiability of functions of bounded variation, l'Hôpital rules) respectively by potential theory (Dynkin's formula, Mokobodzki's theorem on "differentiability" of excessive functions, Fatou theorems).

Fatou theorems of the form $D^YX(w,t) = \lim_{s\uparrow t} \frac{X_s}{Y_s}(w)$ P^Y a.s. for example hold on the parts of $\Omega \times (o,\infty]$ where the life time $\zeta(w,t) = t$ is predictable.

(*) Proc. Jap. Acad. vol 49 (1973)

H. KUNITA: The Controllability Problem and the Maximum Principle.

Let us consider the control system defined on $\mathbb{R}^d$:

$$(*) \qquad \frac{dx}{dt} = Y(x) + \sum_{j=1}^{r} X_j(x)\, u_j(t)$$

where Y, X_j are $\mathcal{C}^\infty$-vectorfields on $\mathbb{R}^d$ and $u(t) = (u_1(t), \dots, u_r(t)$ are vector functions called the control. The controllability-region of $x \in \mathbb{R}^d$ is defined as the set of all y, which can be reached in positive time from x by trajectories of (*) with a suitable choice of control. We discuss the relation between the controllability region and the maximum domain admitting the maximum principle of the elliptic operator

$$L = \sum_{j=1}^{r} X_j^2 + Y, \quad \text{where } X_j = \sum_{i=1}^{d} \sigma^{ij} \frac{\partial}{\partial x^i}, \quad Y = \sum_{i=1}^{d} \ell^i \frac{\partial}{\partial x^i}$$

The way of combining these two problems is via supports of diffusion process corresponding to L, which was studied by STROOCK-VARADHAN .

C. DELLACHERIE: Ensembles analytiques et temps d'arrêt.

Soient $\Omega := \mathbb{N}^{\mathbb{N}}$, X_n les applications coordonnées, $\mathcal{F}_n := \sigma\{X_r, r \leq n\}$ et $\mathcal{T}$ l'ensemble des temps d'arrêt sur Ω muni de la topologie de la convergence simple sur Ω. $\mathcal{T}$ est un espace compact mêtrisable. Une application $x \mapsto T_x$ de E dans $\mathcal{T}$ est dit s.c.s. si $x \mapsto T_x(w)$ est sêmi-continue supêrieurement pour tout $w \in \Omega$.

Proposition: Il existe une bijection canonique entre les schêmas de Souslin sur les fermês de E et les applications s.c.s. de E dans $\mathcal{T}$. Le noyau de schêma de Souslin est alors l'image rêciproque par $x \mapsto T_x$ de l'ensemble $\mathcal{P}$ des temps d'arrêt non finis.

Théorie de l'indice (LUSIN-SIERPINSKI) $i: \mathcal{T} \to I \cup \{\pi\}$

(Où π est le premier ordinal non dénombrable)

Théorème 1: L'ensemble $\{(S,T) \in \mathcal{T} \times \mathcal{T} \, ; \, i(S) \le i(T)\}$ est analytique.

Théorème 2: Soient S et T deux temps d'arrêt.

$i(S) \le i(T) \Longleftrightarrow$ Il existe un pliage f telque $S \le T \circ f$.

[Séminaire de Probabilités de Strasbourg, vol. IX].

NGUYEN-XUAN LOC: A Martin Compactification of a Borel Finely Open Set in the Kunita-Watanabe Hypotheses.

Let X be a Hunt process with the resolvent in duality of a coresolvent $(\hat{W}_\alpha)$ in the sense of Kunita-Watanabe. The part of $X = (X_t)$ on a Borel, finely open subset $\mathcal{U}$ of the state space E is defined as follows:

$$X_{\mathcal{U}}(t,w) := \begin{cases} X(t,w) & t < \tilde{\zeta}(w) \\ \Delta & t \ge \tilde{\zeta}(w) \end{cases}$$

where $\zeta(w) := \inf\{t \ge 0; \, X_t(w) \notin \mathcal{U}\}$.

We try to construct the exit space of $X_{\mathcal{U}}$. The method consists of three steps:

(a) Construct a "nice" compactification $\mathcal{E}$ of $\mathcal{U}$. (b) Prove that for every excessive function h, which is integrable with respect to the standard measure, we have $\lim_{t \uparrow \zeta(w)} \tilde{X}_{\mathcal{U}}(t,w)$ exists P_x^h - a.s. in $\mathcal{E}$ $(x \in \mathcal{U})$. (c) Use the random variable $X_{\zeta-}(w)$ to represent h and to characterize the exit space of $\mathcal{U}$ for $X_{\mathcal{U}}$.

An application of this result is the Martin representation of non-negative, finely hyperharmonic function on a finely open set of a Brelot axiomatic with axiom (D).

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C. CONSTANTINESCU: A Non-Linear Dirichlet Problem.

There is one method to solve the Dirichlet problem in axiomatic potentialtheory which uses instead of the linearity of the sheaf of harmonic functions the resulting property that sums of hyperharmonic functions are hyperharmonic. Taking advantage of this situation a weaker non-linear axiomatic is constructed in which the proof of resolutivity still holds.

This axiomatic may be used in order to solve the Dirichlet problem for some quasi-linear systems of partial differential equations. More precisely: given an open set $\mathcal{U}$ in euclidean space and finitely many differential equations such that each of it defines a harmonic space on $\mathcal{U}$. Then adding a perturbation in order to mix up the unknown functions and to get non-linear differential equations, the resulting sheaf satisfies this non-linear axiomatic.

R.M. HERVÉ: Quelques propriétés des fonctions surharmoniques associées à un opérateur elliptique dégénéré.

Problème de Dirichlet pour une tel opérateur: Soit

$Lu = \sum_{k=1}^{r} X_k^2 u + Yu + cu$ défini sur un ouvert $\Omega \subset \mathbb{R}^n$, X_k et Y des champs de vecteurs $\in \mathcal{C}^\infty(\Omega)$, $c \in \mathcal{C}^\infty(\Omega)$. Soit $\mathcal{L}(X_1,\dots,X_r)$ l'algèbre de Lie engendrée par $X_1,\dots,X_r$. Si le rang de $\mathcal{L}(X_1,\dots,X_r)$ est n en tout point y de Ω on montre:

1) les fonctions surharmoniques sont localement intégrables et caractérisées par $Lu \leq 0$.

2) les potentiels à support ponctuel donné sont proportionnels.

Par suit, étant donnée une axiomatique de M. Brelot possédant "suffisantement" de fonctions harmoniques $\in \mathcal{C}^\infty$, l'unicité de p_y est verifiée pour $y \in \mathcal{U}$, $\mathcal{U}$ un ouvert dense dans Ω .

D'autre part, on montre que la solution du problème de Dirichlet dans un ouvert $w \subset \bar{w} \subset \Omega$ (à frontière $\mathcal{C}^\infty$ et très régulier au sens de Bony) au sens variationnel, construité par DERRIDJ coïncide avec la solution classique de PERRON-WIENER-BRELOT.

Ü. KURAN: <u>On Sets where a Harmonic Function Vanishes or not.</u>

It is wellknown that if P is a polynomial (on $\mathbb{R}^n$) then the number $\beta_o([P = o])$ of the connected components of the set where P vanishes is finite and so is $\beta_o([P \neq o])$.

In the opposite direction there are the following results:

(I) If h is harmonic in $\mathbb{R}^2$ and $\beta_o([h \neq o]) = k < \infty$ $(k > o)$, then h is a polynomial of degree n with $\frac{1}{2} k \leq n \leq k$.

(II) Suppose that f and g are harmonic in $\mathbb{R}^2$, positive in a domain D and vanish on its finite boundary ∂D. Then there exists a strictly positive function φ on $\mathbb{R}^2$ such that $f = \varphi g$. More precisely $\varphi = {}^{a+b}/_{|\psi|^2}$ where a,b are non-negative constants, $a + b > o$ and $b > o$ if and only if there exists an entire function ψ, $|\psi| > o$ and $\operatorname{Im} \psi = g$.

H. AIRAULT: <u>Formes harmoniques et processus de diffusion.</u>

M est une variété riemannienne de dimension n. On indique deux methodes pour étudier d'un point de vue probabiliste les formes harmoniques sur M, c'est à dire les formes differentielles ϕ qui satisfont à l'équation $\Box \phi = o$ où $\Box$ est l'opérateur de De Rham-Hodge sur M.

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(1) La méthode employée par P. MALLIAVIN utilisant le fibré de repères orthonormés O(M) de la variété M (à paraitre Jour. of Funct. Analysis).

(2) La méthode qui consiste à considérer les formes harmoniques comme des fonctions sur le fibré tangent, linéaires sur les fibres dans le cas des formes de degré 1, p-linéaires alternés dans le cas des formes de degré p.

On construit un processus dans les deux cas.

J. FARAUT: Semi-groups de Feller invariants sur les espaces homogènes non moyennables.

Let G be a locally compact group and $(\mu_t)_{t>0}$ a convolution semigroup of probability measures on G with $t \to \mu_t$ is vaguely continuous. μ_0 being idempotent, it follows that μ_0 is the normalized Haar measure of a compact subgroup K of G. The associated convolution operators $(P_t)_{t>0}$ constitute a Feller semigroup on the quotient space $X := {}_K\backslash^G$. Having defined the notion "type α of the semigroup (μ_t)" it turns out, that $(P_t)_{t>0}$ is integrable if $\alpha < 0$ (i.e.: for each f continuous at x and with compact support $x \to \int_0^\infty P_t f(x)\, dt$ is an element of $\mathcal{C}_0(X)$).

The type α of (μ_t) is strictly negative in case of a non amenable group G, $\mu_t \neq \mu_0$, and K being maximal among the closed subgroups of G.

The hypotheses on G and K are satisfied if X is a riemannian symmetric space of non compact type which is irreducible.

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M. BRELOT: Fine Limits for a Family of Functions.

Let X be a Hausdorff topological space. Consider a cone C of lower semi continuous functions, with $\infty \in C$ and every $\sum_{n=0}^{\infty} u_n$ is contained in C. There is a corresponding fine topology and a notion of thinness and strong thinness.

Suppose thinness implies strong thinness.

Now given a family F of real functions, a sequence f_p with fine limits at x_0, a sequence φ_q of positive functions with a fine limit $\lambda_q \to 0 (q \to \infty)$. If for any $f \in F$ and any fixed q there is a p such that $f_p - \varphi_q \leq f \leq f_p + \varphi_q$, then there exists a set α thin at x_0 such that any f tends to a limit at x_0 off α.

One of the applications is as follows: Let w be a domain in a strong harmonic space in the sense of Bauer; $\mathcal{U}', \mathcal{U}''$ harmonic in w with finite fine limits at x_0 (polar, $\complement$ w thin at x_0), then there exists a common thin set for the family of all harmonic u with $\mathcal{U}' \leq u \leq \mathcal{U}''$.

B. FUGLEDE: An Optimal Boundary Minimum Principle.

Let Ω denote a strong harmonic space with axiom (D) and an adjoint sheaf in the sense of M^me^ HERVÉ. Let G denote the Green kernel on $\Omega \times \Omega$ and let there be given a potential $p = G\mu$ on Ω. For a finely hyperharmonic and $\geq -p$ in a finely open set $\mathcal{U} \subset \Omega$ consider the sets:

$$E := \left\{ y \in \partial_f \mathcal{U} \; ; \; \underset{x \to y,\, x \in \mathcal{U}}{\text{fine-lim inf}}\; u(x) < 0 \right\}$$

$$e := \left\{ y \in \partial_f \mathcal{U} \; ; \; \underset{x \to y,\, x \in \mathcal{U}}{\text{fine-lim inf}}\; u(x) = -\infty \right\}, \text{ and}$$

$$E_r := E \cap b(\mathcal{U}).$$

Theorem: E_r and $\mathcal{e}$ are Borel sets. If E_r is a nullset for

$$\varepsilon_x^{\complement\,\mathcal{U}}(x \in \mathcal{U}) \text{ and if } \mu(\mathcal{e}\setminus b^*(\complement\,\mathcal{U})) = 0 \quad \text{then } u \geq 0.$$

This theorem is best possible in the sense that neither of the smallness conditions on E_r and $\mathcal{e}$ can be weakened. In case of a bounded $\mathcal{U}$ and a lower bounded u it reduces to a result due to BRELOT (1950/51), whereby the latter condition drops out.

K. GOWRISANKARAN: Holomorphic Functions on the Polydisc.

Let f be a holomorphic function on the polydisc $\mathcal{U}^n$ in the Nevanlinna class (i.e.: $\log^+ |f|$ has a n-harmonic majorant). ZYGMUND (1949) proved that the iterated non-tangential limits of f exist almost everywhere on the n-Torus T^n and he conjectured that these iterated limits are independent of the order of iteration.

One can show a more general result: If f is defined only on a proper open subset of $\mathcal{U}^n$ and has iterated limits (on a suitable subset of T^n) then these limits are independent of the order of iteration, except for a set of measure zero. Similar results are true for positive n-harmonic functions and in this case even the existence of the limits can be proved.

E.P. SMYRNELIS: Axiomatique des fonctions biharmoniques.

Les axiomatiques harmoniques sont inspirées des équations aux dérivées partielles linéaires du second ordre et ne s'appliquent pas à des équations simples d'ordre plus élevé comme, par exemple, l'équation biharmonique $\Delta^2 h = \Delta(\Delta h) = 0$. Pour traiter cette équation, on peut la remplacer par le système $\Delta h_1 = -h_2, \Delta h_2 = 0$. En utilisant un faisceau de couples (h_1,h_2) compatibles, nous avons développé une axiomatique biharmonique locale applicable aussi à des équations du type $L_2(L_1 h) = 0$ où $L_j (j=1,2)$ est un opérateur linéaire du second ordre elliptique ou parabolique. Ainsi, nous obtenons, parmi d'autres résultats, la solution généralisée du problème de RIQUIER.

M. ARSOVE (with H. Leutwiler): Harmonic and Potential Bands.

A superharmonic semigroup is a regularly partially ordered abelian semigroup $(\mathcal{U},+,\leq)$ with 0 such that the ordering "$\leq$" is finer than the specific order "$\preccurlyeq$". Two further axioms are imposed on $\mathcal{U}$. Examples of superharmonic semigroups are the nonnegative superharmonic functions in classical case, in the axiomatics of BAUER, BRELOT and CONSTANTINESCU-CORNEA, and the excessive functions of MOKOBODZKI's Stresa notes. The following results are obtained:

- $(\mathcal{U},\preccurlyeq)$ is a Dedekind, σ-complete, Archimedean lattice.
- There exists a mixed lattice structure on $\mathcal{U}$ (denoted by $\curlywedge$ and $\curlyvee$ which are not commutative) such that for any subset A of $\mathcal{U}$ right resp. left orthogonal complements exist, denoted by $A^\perp$ resp. ${}^\perp A$.
- (Pre-)harmonic and potential semigroups are defined. It turns out that there exists a greatest harmonic subsemigroup $\mathcal{H}^*$ of $\mathcal{U}$.
- Introducing the notion of $\mathcal{H}$-potentials and $\mathcal{P}$-preharmonic elements (where $\mathcal{H}$ is a preharmonic, $\mathcal{P}$ a potential semigroup) it is proven that the $\mathcal{H}$-potentials form the potential semigroup $\mathcal{H}^\perp$, the $\mathcal{P}$-preharmonic elements form the preharmonic semigroup ${}^\perp\mathcal{P}$.

- (Pre-)harmonic and potential bands are defined. One obtains various results about the relation of these notions and the "mixed" orthogonal complements.
- Furthermore one obtains a Riesz-decomposition and, using the notion of Φ-(quasi-)boundedness and Φ-singularity (Φ a non-empty subset of $\mathcal{U}$) one gets a decomposition of $\mathcal{U}$, which is related to the PARREAU- decomposition in classical case.

H. LEUTWILER (with M. Arsove): <u>Superharmonic Semigroups and their Spectral Theorem.</u>

$\mathcal{U}$ is a superharmonic semigroup on which multiplication by scalars $\in \mathbb{R}_+$ is defined and for which a stronger form of one of the two axioms holds.
Fixing an element $e \in \mathcal{U}$ a one-parameter family of operators $S_\lambda (\lambda \geq 0)$ is considered, defined by $S_\lambda u = \min\{w \in \mathcal{U}: u \leq w+\lambda e\}$.
It is shown that $u \in \mathcal{U}$ is e-quasibounded iff $Su = 0$ and e-singular iff $Su = u$, where $Su = \inf\limits_{\lambda} S_\lambda u$.

The operators S_λ form a semigroup, i.e. $S_\lambda S_\mu = S_{\lambda+\mu}$.
A generalized infitesimal operator A is defined by $Au = \sup\limits_{\delta>0} \frac{u-S_\delta u}{\delta}$.
This operator is idempotent.

It is shown that the set $\mathcal{E}$ of invariant elements of A consists precisely of the set of extremal elements of the convex set $B_e = \{u \leq e\}$. In classical case, if $\mathcal{U}$ is the set of nonnegative superharmonic functions and $e \equiv 1$, the harmonic functions in $\mathcal{E}$ are the generalized harmonic measures in the sense of M. HEINS (i.e. defined by $h \wedge (1-h) = 0$) and the potentials in $\mathcal{E}$

are the fine capacitary potentials. The following generalization of the spectral theorem of H. FREUDENTHAL is obtained:

For every e-quasibounded element u $\in \mathcal{U}$ the representation

$$u = \int_0^\infty e_\lambda \, d\lambda \quad \text{holds, where} \quad e_\lambda = AS_\lambda u.$$

A. ANCONA: Fonction de Green dans les espaces fonctionnels.

Soit (H,a) un espace fonctionnel à forme coercive construit sur (X,ξ), où X est localement compact, et ξ une mesure de Radon positive sur X. On suppose que la contraction module opère sur (H,a). On introduit une notion de classe de fonction a-excessive, et on en donne quelques propriétés. On caractérise ensuite le cas où la bimesure associée à (H,a) est une densité par rapport à $\xi \otimes \xi$. On peut alors introduire un noyau potentiel de base ξ, associé à un cône de fonctions excessives, et construire une fonction de Green ayant de bonnes propriétés.

Enfin, on illustre les considérations précédentes en étudiant le cas où X est homogène et (H,a) est invariant par translation.

N. BOBOC: H-cones.

The "H-cone" is intended to construct potential theory which provides a natural framework for the duality in harmonic spaces and which has as principal models the cones of superharmonic, finely superharmonic and excessive functions.

A convex subcone $\mathcal{S}$ of positive elements of an ordered vector space is called "H-cone" if any subset of $\mathcal{S}$ has an infimum (resp. a supremum if it is upper directed and dominated) belonging to $\mathcal{S}$ and if the Riesz-decomposition property holds in $\mathcal{S}$. The dual $\mathcal{S}^*$

of $\mathcal{S}$ is the ordered convex cone of all maps $\mu : \mathcal{S} \to \mathbb{R}_+ \cup \{+\infty\}$ which are additive, increasing, continuous in order from below, and finite on a subset of $\mathcal{S}$ which is dense in order from below.

Many constructions and results from the ordinary potential theory (balayage, réduite, specific order) are given and their relations with the dual are studied.

GH. BUCUR: Standard H-cones.

An element s of an H-cone $\mathcal{S}$ is called continuous if any increasing family converging in order to s is uniformly convergent with respect to any weak order-unit. An H-cone is called standard if it possesses a countable dense - in order from below - subset of continuous elements. If C is a standard H-cone, then the following results are obtained:

a) the dual C* is also standard

b) an integral representation theorem holds

c) C is isomorphic with the cone of excessive functions with respect to a resolvent family of continuous kernels on a compact metrisable space

d) If C is represented as a cone of functions then the fine topology, thin sets, balayages in C and in C* are investigated.

A. CORNEA: Duality in H-cones and Applications to Harmonic Spaces.

A balayage B on a standard H-cone C is called regular if the image of any continuous element is also a continuous element. The balayage is called coregular if it is a continuous map with respect to the weak topology $\sigma(C, C_0^*)$, where C_0^* is the set of all continuous elements of C*. The regular and coregular balayages are in duality.

The relations between regular (resp. coregular) balayages and regular (resp. completely determining) domains on a harmonic space are investigated. For any standard H-cone of functions a presheaf with respect to the fine topology is constructed. Conditions under which the sheaf property for C and C* holds are investigated.

B. WALSH: Characterization of the Space of Affine Functions on a Simplex.

Let E be a Banach-space ordered by a closed, normal, generating "positive" cone K. We show that the following conditions are equivalent:

(1) E' is lattice ordered by K'

(2) If T: E → E is an operator of finite rank and $T \geq o$ $(T[K] \subseteq K)$, then $tr(T) \geq o$.

(3) The identity operator on E is the pointwise (simple) limit of operators $x \to \Sigma \langle x, y_i' \rangle y_i$ $(y_i \in K, y_i' \in K')$.

If E has an order unit (so that it is the space of affine functions on a base B of K') then (1) is equivalent to B being a simplex. In this case "finite rank" in (2) may be replaced by "nuclear", "pointwise" in (3) by "uniformly on compacta" and there is a fourth equivalent metric condition:

(4) If T : E → F, F a Banach space, is cone-absolutely-summing then it is integral, with $\| T \|_K = \| T \|_1$.

J. BLIEDTNER, W. HANSEN: Simplexes and Harmonic Spaces.

Let $(Y,\mathcal{H}^*)$ be a $\mathcal{P}$-harmonic space with countable base, $\mathcal{P}$ its cone of real continuous potentials and let $X \subset Y$ be compact. Let $\mathcal{T}$ be a set of $\mathcal{P}$-dilations on X, i.e. kernels T on Y such that

1) $\forall p \in \mathcal{P}$: Tp is hyerharmonic and $Tp \leq p$

2) $\forall x \in X$: supp $T(x,\cdot) \subset X$ and $\forall y \in X$: $T(y,\cdot) = \varepsilon_y$.

Theorem: $S_{\mathcal{T}} = \{S \in \mathcal{C}(X): TS \leq S \quad \forall T \in \mathcal{T}\}$ is a simplicial cone (i.e. for any $x \in X$ there exists a unique representing measure supported by the Choquet boundary $CH_{S_{\mathcal{T}}}X$). The maximal measures μ_x are obtained by balayage of ε_x on $CH_{S_{\mathcal{T}}} X \cup \complement X$.

For any subset A of Y, the essential base $\beta(A)$ of A is defined as the maximal subset of the fine closure $\bar{A}^f$ of A which is not thin at any of its point. $\beta(A)$ is the smallest subset of $\bar{A}^f$ which is finely closed and differs from $\bar{A}^f$ by a semi-polar set.

Theorem: $Ch_{S_{\mathcal{T}}} X = X \cap \beta(A_{\mathcal{T}})$, where $A_{\mathcal{T}}$ is the (finely closed) set of points $y \in Y$ with $T(y,\cdot) = \varepsilon_y \ \forall T \in \mathcal{T}$.

It follows that every real continuous function on X which is "finely harmonic" on the fine interior of X is the uniform limit of a sequence of functions which are harmonic on a neighborhood of X. Moreover, this theorem yields a characterization of "axiom P" (semi-polar = polar) and "axiom T" (semi-polar = totally thin).

P.A. LOEB: Applications of Nonstandard Analysis to Ideal Boundary Theory of Harmonic Spaces.

After an introduction to nonstandard analysis, one considers a harmonic space $(W,\mathcal{H})$, an enlargement of a structure S containing $(W,\mathcal{H})$ and the extension $(^*W,^*\mathcal{H})$ in $^*S$. Assume 1 is $\mathcal{H}$-superharmonic.

<u>Theorem 1:</u> If Ω is a region, $h \in {}^*\mathcal{H}_{*\Omega}$, $h \geq -n$, and $h(x_o) \leq n$ for some $n \in \mathbb{N}$ and $x_o \in \Omega$, then ${}^o h \in \mathcal{H}_\Omega$.

<u>Theorem 2:</u> There is an internal, Dirichlet-regular, relatively compact region $\Omega \subset \overline{\Omega} \subset {}^*W$ so that if K is compact in W, then ${}^*K \subset \Omega$. We say that $\Omega \subset \mu(\infty)$.

Assume $\overline{W}$ is a compactification of W, and let $\Delta = \overline{W} \setminus W$. Given $f \in \mathcal{C}(\Delta)$, let $\tilde{f} \in \mathcal{C}(\overline{W})$ extend f into W, and let $h_f^\Omega = {}^o H({}^*\tilde{f} \mid \partial\Omega, \Omega)$.

<u>Theorem 3:</u> The mapping $f \to h_f^\Omega \in \mathcal{H}_W$ is well-defined and linear, and $\underline{H}(f,W) \leq h_f^\Omega \leq \overline{H}(f,W)$.

<u>Theorem 4:</u> (CORNEA-LOEB): Assume W is 2nd countable, $\overline{W}$ is the Wiener-compactification of W, and Γ is the harmonic part of Δ. Let $x \in \Delta$. Then $x \in \Gamma$ iff for each internal, regular region Ω with $\Omega \subset \mu(\infty)$, $\mu(x) \cap \partial\Omega \neq \emptyset$.

A nonstandard approach to Martin boundary theory is also discussed.

A. DE LA PRADELLE (avec D. Feyel): <u>Faisceaux d'espaces de Sobolev et Principe du minimum.</u>

Dans $\mathbb{R}^m (m \geq 1)$, on considère l'opérateur $Lf = \operatorname{div}(Af' + f X) - (Y,f') - cf$ où $f' = \operatorname{grad} f$, A est une matrice carrée; X, Y des champs de vecteurs et c une fonction; $A,X,Y,c \in \mathbb{L}^\infty_{loc}(\mathbb{R}^m)$, L uniformement localement elliptique. Pour tout ouvert w $W^2(w)$ désigne l'espace des $f \in \mathbb{L}^2(w)$ telle que $f' \in \mathbb{L}^2(w)$; W^2_{loc} désigne le faisceau des $W^2_{loc}(w)$. Pour tout ouvert $\mathcal{U}$ $f \in W^2_{loc}(\mathcal{U})$ est une sursolution locale faible et on a

$$B_u(f,\varphi) = \int_{\mathcal{U}} (Af',\varphi') + f(X,\varphi') + \varphi(Y,f') + \alpha f \, d\tau \geq 0$$

$\forall \varphi \in \mathcal{D}(\mathcal{U})$, $\varphi \geq 0$ ($d\tau$ = mesure de Lebesgue). Les sursolutions dans $\mathcal{U}$ forment un cône convexe $\mathcal{F}(\mathcal{U})$ - les $\mathcal{F}(\mathcal{U})$ forment un faisceau $\mathcal{F} \subset W^2_{loc}$. On montre:

- $\mathcal{F}$ satisfait un principe du minimum relativement à la base de tous les ouverts bornés coercitifs w à frontière ∂w Lipschitzienne.
- $\mathcal{F}$ est maximal dans W^2_{loc} pour ce principe du minimum.

En utilisant un resultat de hypoellipticité de STAMPACCHIA on associe à $\mathcal{F}$ un faisceau $\widehat{\mathcal{F}}$ de représentants s.c.i. $> -\infty$ tel que $\mathcal{F}$ soit isomorphe à $\widehat{\mathcal{F}}$. De plus $\widehat{\mathcal{F}}$ vérifie le principe du minimum usuel relativement à la même base.

D. FEYEL (avec A. de la Pradelle): Faisceaux d'espaces de Sobolev et principe du minimum.

On poursuit l'étude des sursolutions faibles de l'opérateur L en liaison avec le principe du minimum. On montre qu' il n'existe qui un seul préfaisceau de cônes convexes de fonctions s.c.i. $> -\infty$ maximal pour le principe du minimum, déterminé par le faisceau des sursolutions locales faibles. On en déduit que ce faisceau maximal est identique au faisceau des hyperharmoniques associées à L par Mme.HERVÉ. Dans une deuxieme partie on étudie les propriétés de continuité de la réduite variationelle de STAMPACCHIA. Cela permet de retrouver assez simplement les propriétés de régularité des fonctions hyperharmoniques.

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Pour conclure on sait que l'étude de l'opérateur L peut se ramener essentiellement à l'étude des espaces de Dirichlet, du principe du minimum et de l'hypoellipticité.

I. LAINE: <u>Harmonic Bl-mappings between General Harmonic Spaces.</u>

There are announced recent results devoted to covering properties of harmonic Bl-mappings between harmonic spaces in the general axiomatics of CONSTANTINESCU-CORNEA. A continuous mapping is harmonic if it preserves hyperharmonic functions ($f' \circ \varphi$ is hyperharmonic on $\varphi^{-1}(\mathcal{U}')$ whenever $\mathcal{U}' \subset X'$ is open, $\varphi^{-1}(\mathcal{U}') \neq \emptyset$ and f' is hyperharmonic on $\mathcal{U}'$). A Bl-mapping, by definition, preserves locally bounded potentials. Let $\mathcal{U}' \subseteq X'$ be a $\mathcal{P}$-domain with $\varphi^{-1}(\mathcal{U}') \neq \emptyset$.

The following results are considered:

1) If $\varphi(V)$ is open then $\overline{\varphi(V)}$ is an absorbent set in $\mathcal{U}'$.

2) If the image D'_φ of the branch set of φ is nowhere dense and its polar points form a polar set and if $n(\varphi,x',V)$ as a function of x' is l.s.c. then
$F' = \{x' \in \mathcal{U}' \mid n(\varphi,x',V) < \sup n(\varphi,\mathcal{U}',V)\}$ is polar in $\mathcal{U}'$ or $\operatorname{int}(F') \neq \emptyset$. The same result holds if φ is open and D'_φ polar.

3) Some special cases with stronger conclusions are treated.

4) Some open problems are shortly discussed.

K. JANSSEN: A Co-fine Domination Principle for Harmonic Spaces.

Let $(X,\mathcal{H}^*)$ be a $\mathcal{P}$-harmonic space in the sense of CONSTANTINESCU-CORNEA. Then every $u \in {}_+\mathcal{S}_X$ has an integral representation by a unique measure μ_u on the Martin space M of $(X,\mathcal{H}^*)$. For every extremal function s in ${}_+\mathcal{S}_X$ the filter of "co-fine neighborhoods of $\tilde{s} \in M$" is given by

$T_{\tilde{s}} := \{E \subset X : R_s^{\complement E} \neq s\}$.

Theorem: Let $v \in {}_+\mathcal{H}^*_X$. If $\limsup_{T_{\tilde{s}}} (\frac{v}{s}) \geq 1$ for μ_u-almost every $\tilde{s} \in M$, then $v \geq u$.

This implies a "co-fine" minimum principle. As an application we obtain that every finite potential is a sum of continuous potentials (i.e. the strong domination principle) if $(X,\mathcal{H}^*)$ has a symmetric Green function.

G. FORST: Convolution Semigroups of Local Type.

A vaguely continuous convolution semigroup $(\mu_t)_{t>0}$ of positive bounded measures of total mass 1 on a locally compact abelian group G induces - by convolution - contraction semigroups $(P_t)_{t>0}$ on various Banach spaces of functions on G, and the convolution semigroups $(\mu_t)_{t>0}$ of local type (i.e. for which the infinitesimal generator for $(P_t)_{t>0}$ is a local operator) are studied.

First a measure μ on $G\setminus\{0\}$, the Lévy-measure for $(\mu_t)_{t>0}$, is constructed and several characterizations of μ are given. Then it is shown that $(\mu_t)_{t>0}$ is of local type iff μ vanishes. Using this result, the general "form" of a convolution semigroup of local type is given. Finally, in the case of a transient convolution semigroup $(\mu_t)_{t>o}$, the property of locallity is studied in terms of the potential kernel of $(\mu_t)_{t>o}$.

E. CABALLERO: Fonctions indifférentes en théorie axiomatique du potentiel.

En se plaçant dans le cadre axiomatique de M. BRELOT, on étudie les fonctions indifférentes (introduites en théorie classique par M. BRELOT), qui sont définies comme suit:
On considère la famille W des ouverts de la form $\omega = \{x \in \Omega : p(x) > \lambda\}$, où p est un potentiel à support compact et $\lambda \in \mathbb{R}^+$. Une fonction harmonique dans Ω sera dite indifférente si elle coïncide avec sa solution pour le problème de Dirichlet dans ω pour tout $\omega \in W$. On montre pour u harmonique > 0 dans Ω :

$$u \text{ est indifférente dans } \Omega \iff \inf_{\substack{V\uparrow\Omega \\ V\in\mathcal{V}}} (R_u^{\omega\cap\complement V})_\omega = 0 \quad \forall \omega \in W$$

$$\iff \inf_{\substack{V\uparrow\Omega \\ V\in\mathcal{V}}} H_{u_V}^{V\cap\omega} = 0 \text{ sur } \omega \quad \forall \omega \in W$$

où $\mathcal{V}$ désigne la famille des tous les ouverts rel.compacts de Ω, et

$$u_V = \begin{cases} 0 & \text{sur } \partial V' \cap \partial \omega \\ u & \text{sur } \partial V' \cap \omega \end{cases} \qquad V' = V \cap \omega.$$

Ceci nous permet d'étudier un problème de Dirichlet appellé problème de Dirichlet asymptotique (que correspond au problème de Dirichlet radial de M. BRELOT, en théorie classique).

J. GUILLERME: Balayage des fonctions surharmoniques dans un espace sans potentiel > 0.

On considère un espace harmonique de BRELOT Ω à base dénombrable, telle que les constantes sont harmoniques et il n'existe pas de potentiel > 0. Si u est une fonction surharmonique dans Ω et A une partie de Ω, on cherche à obtenir une minorante à peu près surharmonique de u, égale à u sur A et harmonique dans $\Omega \setminus \bar{A}$. Si A est relativement compact, ceci suppose que u admette une minorante harmonique hors d'un compact; u est dite dans ce cas admissible. Si u et v sont deux fonctions surharmoniques admissibles et u_H, v_H leurs plus grandes minorantes harmoniques hors du compact non localement polaire H, u et v sont dites équivalentes à l'infini ($u \sim v$ à l' ∞), si $|u_H - v_H|$ est bornée à l'infini (indépendent de H). Soit pour u admissible et $A \subset \Omega$:

$$\mathcal{B}(u,A) = \{v \mid v \text{ surharm.}, v \geq u \text{ sur } A \cup H_v (H_v \text{ compact})\}$$

$$\mathcal{F}(u,A) = \{v \mid v \text{ surharm.}, v \geq u \text{ sur } A, v \sim u \text{ à l' } \infty\}$$

$$B_u^A = \inf \{v \in \mathcal{B}(u,A)\}, \quad F_u^A = \inf \{v \in \mathcal{F}(u,A)\}.$$

On montre que B_u^A et F_u^A sont égales a u sur A, harmoniques dans $\Omega \setminus \bar{A}$, et équivalents à u à l' ∞ (A non loc. polaire, u admissible). Pour obtenir l'égalité entre B_u^A et F_u^A; on montre des propriétés analogues à celles de la réduite. A partir de ceci, on peut facilement construire des capacités telles qu' un ensemble soit loc. polaire si et seulement si il est de capacité égale à $-\infty$.

J.M. REAY: Multiply Superharmonic Functions.

If X,Y are harmonic spaces with countable base (in the sense, say, of CONSTANTINESCU-CORNEA) we can define the cone of positive doubly superharmonic functions as follows:

u: $X \times Y \to \bar{\mathbb{R}}_+$ is doubly superharmonic if it is multiply superharmonic and for all $f \in \mathcal{C}_+(\bar{X})$, $g \in \mathcal{C}_+(\bar{Y})$ [$\bar{X},\bar{Y}$ Alexandroff-compactifications of X,Y] the functions

$$(x,y) \to g.\, u^x(y)\ , \qquad (x,y) \to f.u^y(x)$$
$$(x,y) \to f.(g.u^\cdot(y))(x), \ (x,y) \to g.(f.u^\cdot(x))(y)$$

are multiply superharmonic [modulo "small" sets].

If $\mathcal{S}_+^{\otimes}(X,Y)$ denotes the cone of doubly superharmonic functions, we can define the generated vector space $[\mathcal{S}^{\otimes}](X,Y) = \mathcal{S}_+^{\otimes}(X,Y) - \mathcal{S}_+^{\otimes}(X,Y)$. We give it a Hausdorff locally convex linear topology, analogous to the T-topology.

<u>Theorem:</u> For X,Y connected Brelot-spaces

$\mathcal{S}_+^{\otimes}(X,Y) \cong$ {the closure of $\mathcal{S}_+(X) \otimes \mathcal{S}_+(Y)$ in the projective topology on $[\mathcal{S}](X) \hat{\otimes} [\mathcal{S}](Y)$}

and $\mathcal{S}_+^{\otimes}(X,Y)$ is a lattice cone in its own order with a compact base.

<u>Corollary</u> (GOWRISANKARAN): If X,Y are connected Brelot spaces, $\forall v \in \mathcal{S}_+^{\otimes}(X,Y)$ $\exists$ a finite Radon measure μ_v on the cartesian product of the extreme points of $\mathcal{S}_+(X)$ with the extreme points of $\mathcal{S}_+(Y)$ so that

$$v(x,y) = \int u(x)\, v(y)\, d\mu(u,v) \qquad \forall x \in X,\ \forall y \in Y.$$

An extension of this is the

<u>Theorem:</u> If X is a connected Brelot space, Y an arbitrary harmonic space then the cone $\mathcal{S}_+^{\otimes}(X,Y)$ is a lattice cone.

V. Dembinski, G. Leha (Erlangen)