Vortragsbuch

Nr. 64

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Inhaltsverzeichnis zum Vortragsbuch Nr. 64

Komplexe Analysis, 26. 8. - 1. 9. 1984

(Fortsetzung der Berichte)

Quasihomogene Singularitäten

Unter den Singularitäten komplexer Räume nehmen die quasihomogenen eine besondere Stellung ein, die sich unter anderem aus dem Zusammenhang mit der Theorie der automorphen Formen ergibt. Für die Topologie einer quasihomogenen Untervarietät X von $\mathbb{C}^{m+1}$ ist der zugehörige Umgebungsrand Σ maßgeblich. Eine wesentliche Vereinfachung ergibt sich durch die Voraussetzung, daß X ein n-dimensionaler vollständiger Durchschnitt mit isolierter Singularität ist. Bei der Untersuchung der ganzzahligen Homologie von Σ kommt es dann allein auf $H_{n-1}(\Sigma; \mathbb{Z})$ an. Es lassen sich explizite Formeln für den Rang dieser Gruppe und - falls n ungerade ist - ihre Torsionsuntergruppe angeben. Im Hyperflächenfall ($m=n$) läßt sich diese Torsionsgruppe für beliebiges n berechnen.

Helmut A. Hamm (Münster)

Relative groups of automorphisms of Kähler morphisms.

If $\varphi: X \to S$ is a surjective holomorphic map with X Kähler smooth, and whose generic fiber is irreducible projective then this generic fiber is prehomogeneous in case X and S have the same algebraic dimension. In particular φ is the composition $\tau\circ\alpha$ $\alpha: X \to T$, $\tau: T \to S$ of two meromorphic maps such that:
. the general fiber of τ is a torus and .. for generic $s \in S$, $\alpha_s: X_s \to T_s$ is a fiber bundle (in fact, flat) with fiber a

unirational prehomogeneous manifold. This last result has been proved by Fujiki in case the generic fibers of $\mathcal{C}$ are not assumed to be projective, but only not possessing non-trivial subspaces of algebraic dimension zero. (property $*$)
Problem: is also the conclusion of the first statement true (prehomogeneity) when the projectivity assumption is replaced by property $*$?

F. Campana (Nancy - France.)

Das lokale Modulproblem für 1-konvexe Räume

Ein <u>graduierter</u> <u>komplexer</u> <u>Raum</u> ist ein $\mathbb{C}$-geringter Raum $X=(X,\mathcal{O}_X)$, dessen Strukturgarbe eine $\mathbb{N}$-Graduierung $\mathcal{O}_X = \coprod_{i\in\mathbb{N}} (\mathcal{O}_X)_i$ trägt, so daß gilt: (1) $(X,(\mathcal{O}_X)_0)$ ist ein komplexer Raum, (2) $(\mathcal{O}_X)_i$ ist ein kohärenter $(\mathcal{O}_X)_0$-Modul für jedes i aus $\mathbb{N}$. (3) $\mathcal{O}_X$ ist eine $(\mathcal{O}_X)_0$-Algebra von endlicher Darstellung. Für jedes $a\in\mathbb{N}$ ist dann die Abschneidung $X_{(\leq a)} := (X, \coprod_{i\leq c}(\mathcal{O}_X)_i)$ wieder ein solcher Raum. – Seien nun $f: X \longrightarrow Y$ eine Abbildung graduierter komplexer Räume, $K \subseteq X$ und $L \subseteq Y$ abgeschlossene Teilmengen mit $f(K)\subseteq L$ und $(f,K,L): (X,K) \longrightarrow (Y,L)$ der durch diese Daten definierte Abbildungskeim sowie $c \in \mathbb{N}$. Weiter sei $S=(S,0)$ ein komplexer Raumkeim. Unter einer <u>Deformation</u> <u>von</u> $(f,K,L)^{(\geq c)}$ <u>über</u> S verstehen wir ein kommutatives Diagramm

$$\begin{tikzcd} \mathcal{X} \arrow[rr, "f"] \arrow[dr] & & \mathcal{Y} \arrow[dl] \\ & S & \end{tikzcd}$$

graduierter komplexer Räume, wobei $\mathcal{X}$ und $\mathcal{Y}$ flach über S seien, mit $\mathcal{X}(0) = (X,K)$, $\mathcal{Y}(0) = (Y,L)$, $f(0) = (f,K,L)$, zusammen mit einem Isomorphismus $f_{(\leq c-1)} \Rightarrow (f,K,L)_{(\leq c-1)} S$. Letzteres bedeutet, daß $(f,K,L)_{(\leq c-1)}$ trivial deformiert wird.

<u>Satz.</u> Seien $f: X \longrightarrow Y$ eine Abbildung graduierter komplexer Räume, $c \in \mathbb{N}$, $E := \mathrm{Supp}(\coprod_{i\geq c}(\mathcal{O}_X)_i)$, $L \subseteq Y$ eine endliche Teilmenge und $K := f^{-1}(L)\cap E$. Liegt K eigentlich über Y und $\mathrm{Supp}(\mathcal{T}^1_{gr}(f,\mathcal{O}_Y)_{(\geq c)})$ in L, so besitzt $(f,K,L)^{(\geq c)}$ eine semiuniverselle Deformation.

Aus diesem Satz, dessen Beweis gemeinsam mit Herrn Kosarew ausgearbeitet wurde, folgen fast alle bekannten Existenzsätze der analytischen Deformationstheorie.

Eine weitere Anwendung ist die Lösung des Modulproblems für 1-konvexe Räume.

J. Bingener (Regensburg)

Reelle algebraische Geometrie (2.9.84 – 8.9.84)

Relations among analytic functions. I. Local invariants (Joint work with P. D. Milman).

We discuss the variation of local invariants (like the Hilbert-Samuel function) associated with formal relations among real or complex analytic functions. Zariski semicontinuity of these invariants, which we conjecture in general and have proved in many cases (e.g., algebraic), has striking applications to the geometry of semialgebraic and subanalytic sets, as well as to the main problems of differential analysis. The key idea is the relationship among several invariants: the Hilbert-Samuel function, an estimate of Chevalley, and a "diagram of initial exponents" which we can use to stratify a subanalytic set with explicit special generators of its ideal sheaf along each stratum.

Edward Bierstone (Toronto)

Relations among analytic functions. II Applications to C^∞ geometry (Joint work with E. Bierstone).

The composition, division and extension conjectures for ~~analytic~~ differentiable functions are all consequences of our fundamental assertion in analytic geometry: semicontinuity of local invariants. Our explicit techniques provide a better understanding even in the classical coherent case: we give simple new proofs of the semicontinuity of the Hilbert-Samuel function, a uniform version of the Artin-Rees theorem and, as a consequence of formal division by the special generators, Malgrange's C^∞ division theorem. In this talk we sketch the proof of our main theorem on C^∞ functions.

Pierre Milman (Toronto).

Quantitative geometry of semialgebraic sets and mappings and some applications in real analysis.

It is well known, that the Betti numbers of any fiber $P^{-1}(\xi)$ of a polynomial mapping $P: R^n \to R$, are bounded by some constants, depending only on n and the degree of P. ~~[illegible]~~ Now let f be a k times differentiable function on a bounded domain in R^n, with all the derivatives of order k, bounded by a constant M_k. We can think M_k as the measure of a deviation of f from a polynomial of degree $k-1$; as far as the deviation in a C^j-norm is concerned, $j \le k-1$, the Taylor formula gives the precise expression for it.

The important general phenomenon is that also in much more delicate questions, concerning the topology and the geometry of the mapping f, its deviation from the "polynomial behavior" can be bounded in terms of M_k.

We give some theorem, illustrating this phenomenon, in particular, for the property of polynomials, mentiond in the beginning.

Yosef Yomdin (Beer-Sheva, Bonn)

Topological properties of inclusions of real algebraic sets.

Problem: Given K a complex, $L \subset K$ a subcomplex, does there exist real algebraic sets $W \subset V$ and an homeomorphism $\Phi: K \xrightarrow{\sim} V$ such that $\Phi(K) = W$? Here are some necessary conditions.

I) Sullivan's local Euler characteristic theorem may be formulated as follows: if V is a real algebraic set, $a \in V$, then $\chi(V) \equiv \chi(V-a) + \chi(a)$ (mod 2) and it can be easily generalised to $\chi(V) \equiv \chi(V-F) + \chi(F)$ (mod 2) where F is any algebraic subset of V. I have no actual application of this formula.

II) Codimension 1 case: let $\Phi: K \to V$ be a semi-algebraic triangulation of a real algebraic set of dimension d, σ a $d-1$ simplex of K. Let $g(\sigma) = \#\{\tau \mid \tau \text{ } d\text{-simplex of } K, \sigma \text{ a face of } \tau\}$. Then

i) (well known) $g(\sigma)$ is even.

ii) if σ and σ' are $d-1$ simplexes such that $\Phi(\sigma)$ and $\Phi(\sigma')$ are contained in the same irreducible algebraic subset W of V of codimension 1,

$$g(\sigma) \equiv g(\sigma') \pmod 4$$

These facts may be proved by translating them in the language of real spectrum, and then using the classical relationship between orders and valuation rings in the field of rational functions $K(V)$ (for V irreducible): i) and ii) are then deduced from the formula

$$g(\alpha) = \sum_{B \in \mathcal{B}(\operatorname{supp}\alpha)} 2 \cdot n(B,\alpha)$$

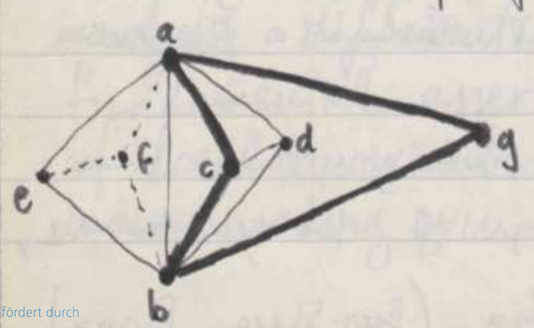
where $\alpha = (\operatorname{supp}\alpha, \leq_\alpha) \in \operatorname{Spec}_R(\mathcal{P}(V))$, $\operatorname{supp}\alpha$ of height 1,

$g(\alpha) = \#\{\beta \text{ orders on } K(V), \beta \text{ generisation of } \alpha\}$,

$\mathcal{B}(\operatorname{supp}\alpha) = \{B \text{ valuation rings in } K(V), B \text{ dominates } \mathcal{O}(V)_{\operatorname{supp}\alpha}\}$,

$n(B,\alpha) = \#\{\text{orders on } B/m_B \text{ which extend } \leq_\alpha \text{ on the residue field of } \mathcal{O}(V)_{\operatorname{supp}\alpha}\}$.

Here is an exemple of application of ii: K is the 2-dimensional complex $[abc] \cup [abd] \cup [acd] \cup [bcd] \cup [abe] \cup [abf] \cup [aef] \cup [bef] \cup [ag] \cup [bg]$. L is the 1-dimensional subcomplex $[ac] \cup [bc] \cup [ag] \cup [bg]$. Both K and L are homeomorphic to real algebraic sets,



but there is no homeomorphism $\Phi : K \xrightarrow{\sim} V$ on a real algebraic set such that $\Phi(L)$ is algebraic.

Michel Coste, Rennes.

On real algebraic mappings into S^m.

Let $M \subset \mathbb{R}^q$ and $N \subset \mathbb{R}^p$ be real algebraic sets. Let

$$\mathcal{P}(M) = \{\varphi : \varphi = f|M \quad \text{for some } f \in \mathbb{R}[X_1, \dots, X_q]\}$$

be the ring of polynomial functions on M and let

$$\mathcal{R}(M) = \{\varphi/\psi : \varphi, \psi \in \mathcal{P}(M), \psi^{-1}(0) = \phi\}$$

be the ring of entire rational functions on M. A map $f : M \to N$ is said to be polynomial (resp. entire rational) if there exist $\varphi_1, \dots, \varphi_p \in \mathcal{P}(M)$ (resp. $\mathcal{R}(M)$) such that $f(x) = (\varphi_1(x), \dots, \varphi_p(x))$ for all $x \in M$. We denote by $\mathcal{P}(M, N)$ (resp. $\mathcal{R}(M, N)$) the set of polynomial (resp. entire rational] mappings $M \to N$. Very little seems to be known about the structure of $\mathcal{P}(M, N)$ or $\mathcal{R}(M, N)$, the classification of their elements, the relationship with other classes of functions, etc. We got (jointly with W. Kucharz from Albuquerque, U.S.A) some results in this direction, mostly for $N = S^m = \{x \in \mathbb{R}^{m+1} : \sum_{i=1}^{m+1} x_i^2 = 1\}$. It appears that the behaviour of polynomials and entire rational mappings are often quite different.

A sample of results. Assume that M is a compact real algebraic manifold, i.e. a nonsingular compact real algebraic set. Let $\mathcal{E}(M, N)$ be the set of C^∞ mappings $M \to N$, equipped with the C^∞ topology.

Theorem 1. For each $m \in \mathbb{N}$ and each $k = 1, 2, 4$, the set $\mathcal{R}(S^m, S^k)$ is dense in $\mathcal{E}(S^m, S^k)$.

This theorem contrasts with a result of ~~[illegible]~~ Wood (Invent. Math. 1969) saying that $\mathcal{P}(S^m, S^k)$ contains only constant polynomials if $m = 2^p$, $k < m$. It is not known whether Theorem 1 is ~~[illegible]~~ valid for $k \neq 1, 2, 4$.

Theorem 2. Let M be a compact algebraic manifold. Then $\mathcal{R}(M, S^2)$ is dense in $\mathcal{E}(M, S^2)$ in each of the following cases:

(i) M is a connected nonorientable surface of genus odd.

(ii) M is a connected nonorientable surface with $H_1^{alg}(M, \mathbb{Z}_2) = H_1(M, \mathbb{Z}_2)$.
(iii) $H^2(M, \mathbb{Z}) = 0$.

Theorem 3. Let $q_1, \dots, q_k$ be a sequence of $k \geq 2$ positive integers and let $n = \sum_{i=1}^{k} q_i$. Then the following conditions are equivalent:
(i) Each entire rational map $S^{q_1} \times \dots \times S^{q_k} \to S^n$ is null homotopic.
(ii) n is even and at least one q_i is odd.

Theorem 4. Let M be a compact oriented algebraic manifold of odd dimension m. Then for each even number $p \in \mathbb{Z}$ there exists $f \in R(M, S^m)$ of topological degree p.

J. BOCHNAK (AMSTERDAM)

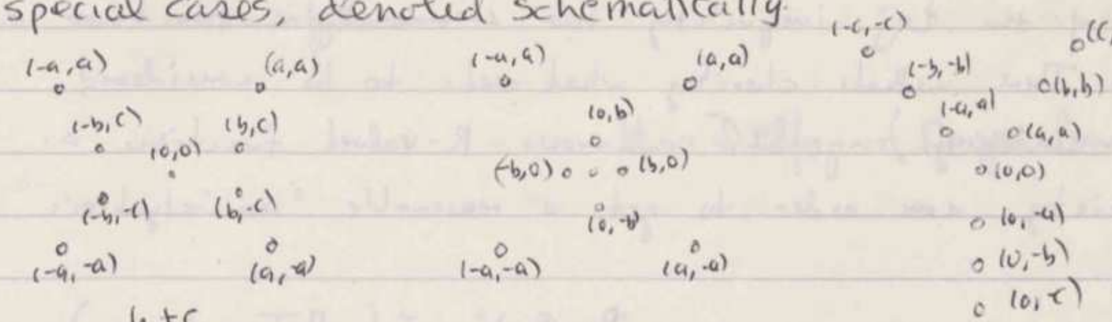
Generalizations of the Arithmetic-Geometric Inequality

The arithmetic-geometric inequality [(1) below] is one of the central inequalities in mathematics. One way of deriving it is to solve for the coefficients of a polynomial in n variables vanishing to order 2 at $(1, \dots, 1)$. This preliminary report considered certain special cases of vanishing to higher even order.

For $\underline{u} = (u_1, \dots, u_n) \in \mathbb{R}^n$ and $\underline{x} \geq 0$ let $\underline{x}^{\underline{u}} = x_1^{u_1} \cdots x_n^{u_n}$. Suppose $\underline{u}_{k+1} = \sum_{i=1}^{k} \lambda_i \underline{u}_i$, $\lambda > 0$, $\sum \lambda_i = 1$, Then the arithmetic geometric inequality is

(1) $$\lambda_1 \underline{x}^{\underline{u}_1} + \dots + \lambda_k \underline{x}^{\underline{u}_k} \geq \underline{x}^{\underline{u}_{k+1}} \quad \text{for } \underline{x} \geq 0 \text{ with equality at } (1, \dots, 1) = \underline{1}$$

On the other hand, with the same notation, if $H(\underline{x}) = \sum_{i=1}^{k+1} a_i \underline{x}^{\underline{u}_i}$ is asked to satisfy $H(\underline{1}) = \frac{\partial H}{\partial x_j}(\underline{1}) = 0$ then, up to a multiple, $H(\underline{x}) = \sum \lambda_i \underline{x}^{\underline{u}_i} - \underline{x}^{\underline{u}_{k+1}}$. I discussed some easier cases of the following. Let $\underline{u}_1, \dots, \underline{u}_m$ be given and suppose $H(\underline{x}) = \sum a_i \underline{x}^{\underline{u}_i}$, satisfying $\frac{\partial^r H}{\partial x_1^{r_1} \cdots \partial x_n^{r_n}}(\underline{1}) = 0$ for all $\sum r_i = r \leq 2s-1$, is determined uniquely up to a multiple. Under what conditions on m, n, s and the points u_i is $H(x) \geq 0$ for all $x \geq 0$. If $n=1$ and $m=2s+1$ no additional condition is required by Descartes' Rule of Signs. If $n=2$ and $2s=4$, we have the following special cases, denoted schematically.



$b \neq c$
H not ≥ 0 $\qquad$ $H \geq 0$ $\qquad$ $H \geq 0$

Bruce Reznick (Urbana)

Geometric Rings and Convergent Power Series

If X is a real algebraic set in $\mathbb{R}^n$, there is a dictionary relating semi-algebraic subsets of X and the sheaf of s.a. functions to the constructible subsets in $\operatorname{Sper} A$ (A = coordinate ring of X) and the sheaf of abstract semi-algebraic functions. In this talk we show how statements about the behavior of s.a. functions on ~~$\mathbb{R}^n$~~ real alg. sets can be transfered to yield results about abstract s.a. functions on ~~the~~ real spectrum of any ring. An example is an abstract Łojasiewicz inequality which reads:

> Let $X \subseteq \operatorname{Sper} A$ be constructible, f, g be two abstract s.a. functions on X such that $f=0 \Rightarrow g=0$. Then $\exists\, m \in \mathbb{N}$ and an algebraic positive function p on $\operatorname{Sper} A$ such that $|g|^m \leq p\,|f|$ on X.

We then define a class of rings, geometric rings, which allow us to interpret statements about abstract s.a. functions on $\operatorname{Sper} A$ as statements about sets and functions on the $\mathbb{R}$-rat'l points of A defined by finitely many inequalities. More generally, we define relative and local geometric rings. An example of the latter is the ring of germs of convergent power series over a complete valued real-closed field. Here abstract s.a. functions correspond to semi-analytic function germs. This gives a purely algebraic proof of the Łoj. inequality for semi-analytic function germs. These methods clarify what needs to be considered when considering rings of continuous R-valued functions on a variety in order to get a reasonable "semialgebraic" theory.

R. Robson (Regensburg)

The proper base change theorem in semialgebraic geometry

We consider a cartesian square

$$\begin{array}{ccc} X & \xleftarrow{g'} & X\times_Y Y' \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle f'} \\ Y & \xleftarrow[g]{} & Y' \end{array}$$

of morphisms between abstract semialgebraic spaces. Suppose a sheaf $\mathcal{F}$ is given on X.

Theorem 1. If f is proper and of finite type, then the canonical base change homomorphism

$$g^* R^q f_* \mathcal{F} \to R^q f'_* (g'^* \mathcal{F}) \quad (q \geq 0),$$

is an isomorphism.

Among other things the proof uses the following important result.

Theorem 2. Let M be a semialgebraic space over a real closed field R, $\mathcal{F}$ be a sheaf on M

and $R \subset S$ be a real closed field extension. Let $M(S)$ and $\mathcal{F}(S)$ be the base extensions of M and $\mathcal{F}$. Then the canonical homomorphism

$$H^q(M,\mathcal{F}) \longrightarrow H^q(M(S),\mathcal{F}(S))$$

is an isomorphism.

H. Delfs (Regensburg)

The ξ-"topology" on an η_ξ-class

In On Numbers and Games J. H. Conway defined a field No that is a proper class and is a real-closed field. Given subsets of numbers L and R for which $x^L \in L$ and $x^R \in R \Rightarrow x^L < x^R$, we will write $L < R$. Conway then constructs a new number $\{L \mid R\} \equiv x \in No$. N.b., $L < \{x\} < R$.

Let ξ be an ordinal number for which $\xi > 0$ and for which $\aleph_\xi$ (i.e., ω_ξ) is regular. One can modify Conway's construction so that it is required also that $|L| + |R| < \aleph_\xi$. When one does this one defines a real-closed subfield ξNo that is an η_ξ-set. [Ref. Alling, Trans. Amer. Math. Soc. '84?]

$U \subset \xi No$ is called ξ-open if there exists $\beta < \omega_\xi$, $(a_\alpha)_{\alpha<\beta}$ and $(b_\alpha)_{\alpha<\beta}$ points in ξNo for which $U = \bigcup_{\alpha<\beta}(a_\alpha, b_\alpha)$. Then $X - U$ is ξ-closed. One can define the relative ξ-"topology", ξ-connected subspaces, as well as ξ-compact subspaces. Then S is ξ-connected $\Leftrightarrow$ S is an interval. For intervals S of ξNo conditions can be given on the upper and lower characters of S such that S is ξ-compact. [Ref. Alling, Math. Reports, Royal Soc. Canada, June, '84]

No has additional structure called its "birth-order" structure, which has the following properties: $\exists\, b: No \to Ord$, the class of all ordinal numbers such that for all $L, R \subset No$, with $L < R$, $\exists!\, x \in No$ such that $L < \{x\} < R$, with $b(x)$ minimal. Further, $\forall x \in No$ $\exists$ such L & R with $b(L) \cup b(R) < b(x)$.

Using this structure Conway constructs e.g., a copy of $\mathbb{R}$ in No; a map $x \in No \longmapsto \omega^x \in No^+$ with very nice properties etc. $\forall y \in No$, with $|y| > 0$, $\exists!\, x \in No$ such that y & ω^{-x} are in the same Archimedean class. Let $V(y) \equiv x$; then this defines a valuation on No. Using the birth-order structure on No one can show that every pseudo-convergent sequence in No (resp. $\S No \S$ of length $< \omega_\xi$) has a unique pseudo-limit x with $b(x)$ minimal. Let $x \equiv \operatorname{Lim}_{\alpha<\lambda} a_\alpha$.

Let $(a_n)_{0 \le n < \omega}$ be in No (resp. $\S No$), & let $z \in No$ (resp. $z \in \S No$). Let $S_m(z) \equiv \sum_{n=0}^{m} a_n z^n$. $\exists\, \varepsilon > 0$ such that $\forall\, |z| < \varepsilon$ implies that $(S_m(z))_{0 \le m < \omega}$ is pseudo-convergent. For such z let $\sum_{n=0}^{\infty} a_n z^n \equiv \operatorname{Lim}_{0 \le m < \omega} S_m(z)$ exists.

It is hoped that some of these ideas will allow one to do analysis over surreal & surcomplex number fields.

N.L. Alling (Rochester, NY)

Rational equivalence and homology of cycles of small dimension

Let X be a smooth projective $\mathbb{R}$-variety and z an r-dimensional cycle whose homologyclass in $H_r(X(\mathbb{R}), \mathbb{Z}/2)$ is zero. For $r \leq 2$ it is shown that then z is rationally equivalent to a zycle z' with $|z'|_{reg}(\mathbb{R}) = \emptyset$.

F. Ischebeck (D-4400 Münster)

Real closed spaces

Real closed spaces are locally ringed spaces that generalize the semi-algebraic spaces of Delfs & Knebusch much in the same way as schemes generalize algebraic varieties (classical). For real closed spaces a theory can be developed which is reminiscent of the theory of schemes. This theory can then be used to gain results (in particular, affineness results) for semi-algebraic spaces.

Niels Schwartz (München)

Connected components and H^1 of real algebraic surfaces

Let $X(\mathbb{C})$ be a smooth surface defined over $\mathbb{R}$ and $X(\mathbb{R})$ it's real part. Using methods developed by Kharlamov and Rokhlin plus a few other ingredients one can prove the following theorems:

Th. 1: If $H^*(X(\mathbb{C}), \mathbb{Z})$ is torsion free then:

$$h^1(X(\mathbb{R})) \leq B_1 + B_2 - h^{0,2} \quad \text{if } h^{0,2} \equiv 0 \pmod{8}$$
$$h^1(X(\mathbb{R})) \leq B_1 + B_2 - h^{0,2} - 1 \quad \text{if } h^{0,2} \equiv \pm 1 \pmod{8}$$
$$h^1(X(\mathbb{R})) \leq B_1 + B_2 - h^{0,2} - 2 \quad \text{if } h^{0,2} \equiv \pm 2 \text{ or } 4 \pmod{8}$$
$$h^1(X(\mathbb{R})) \leq B_1 + B_2 - h^{0,2} - 3 \quad \text{if } h^{0,2} \equiv \pm 3 \pmod{8}$$

where $h^1(X(\mathbb{R})) = \dim_{\mathbb{Z}/2} H^1(X(\mathbb{R}), \mathbb{Z}/2)$, $B_i = \dim H^i(X(\mathbb{C}), \mathbb{Q})$, $h^{0,2} = \dim_{\mathbb{C}} H^{0,2}$

Th. 2

$$^{\#}X(\mathbb{R}) \leq \frac{B_1 + B_2 - h^{0,2} + 1}{2} \quad \text{if } h^{1,1} - h^{0,2} - 1 \equiv 0 \pmod{8}$$

$$^{\#}X(\mathbb{R}) \leq \frac{B_1 + B_2 - h^{0,2}}{2} \quad \text{if } h^{1,1} - h^{0,2} - 1 \equiv \pm 1 \pmod{8}$$

$$^{\#}X(\mathbb{R}) \leq \frac{B_1 + B_2 - h^{0,2} - 1}{2} \quad \text{if } h^{1,1} - h^{0,2} - 1 \equiv \pm 2 \text{ or } 4 \pmod{8}$$

$$^{\#}X(\mathbb{R}) \leq \frac{B_1 + B_2 - h^{0,2} - 2}{2} \quad \text{if } h^{1,1} - h^{0,2} - 1 \equiv \pm 3 \pmod{8}$$

where $^{\#}X(\mathbb{R})$ is the number of connected components of $X(\mathbb{R})$.

For small values of $h^{0,2}$ ($h^{0,2} \leq 5$) one can find many examples ~~showing that~~ where the bounds given in Th. 1 are reached. For Th. 2 examples are more difficult to construct but one can also give quite a few examples where the bounds are reached.

Th. 2 leads to the following corollary:

Corollary: If X is a surface of degree m in $\mathbb{P}^3$ then:

$$^{\#}X(\mathbb{R}) \leq \frac{5m^3 - 18m^2 + 25m}{12} \quad \text{if } m \equiv 0 \pmod{16} \text{ or } m \equiv 1 \pmod{4}$$

$$^{\#}X(\mathbb{R}) \leq \frac{5m^3 - 18m^2 + 25m - 6}{12} \quad \text{if } m \equiv \pm 2 \bmod 16$$

$$^{\#}X(\mathbb{R}) \leq \frac{5m^3 - 18m^2 + 25m - 12}{12} \quad \text{if } m \equiv \pm 4 \text{ or } 8 \pmod{16} \text{ or } m \equiv 3 \pmod{4}$$

$$^{\#}X(\mathbb{R}) \leq \frac{5m^3 - 18m^2 + 25m - 18}{12} \quad \text{if } m \equiv \pm 6 \pmod{16}$$

The bound given here is known to be sharp for $m = 4$ is it still the case for $m \geq 5$?

R. SILHOL (ANGERS)

Real Differential Algebraic Stellensätze

Some of the stellensätze of real semialgebraic algebra and geometry have differential algebraic counterparts. These are obtained using a differential version of the real spectrum of a differential ring (A, D) containing the rationals, an Artin-Schreier theory for such rings, and the model theory of ordered differential fields. The real differential spectrum X of (A, D) is the (typically very small) subset of the real spectrum (typically very big) of A consisting of orderings the center of which is a differential ideal. X is nonempty if and only if -1 is not a sum of squares in A. Abstract stellensätze are the following. Let Σ be the semiring generated by the squares and a multiplicatively closed set S in A. Let I be a differential ideal. Then

i) (nichtnegativstellensatz) $\ell \geq 0$ on $\{I=0\} \cap \{S>0\} \Leftrightarrow \ell(s\ell^{2m}+\sigma_1) = \sigma_2 \pmod{I}$ for some $s \in S$, $\sigma_i \in \Sigma$.

ii) (nullstellensatz) $\ell = 0$ on $\{I=0\} \cap \{S>0\} \Leftrightarrow s\ell^{2m}+\sigma_1 = \sigma_2 \pmod{I}$ for some $s \in S$, $\sigma_i \in \Sigma$

iii) (nichtnullstellensatz or abstract differential Hörmander Łojasiewicz relation)

$\ell \neq 0$ on $\{I=0\} \cap \{S>0\} \Leftrightarrow \exists N$ such that $(s+\sigma_1)\sum_{k=0}^{N} (D^k\ell)^2 = s+\sigma_2 \pmod{I}$

Corresponding results about solutions of systems of algebraic differential equations in some closed ordered differential field are obtained using work of Michael Singer which shows that the theory of such systems admits elimination of quantifiers.

Gilbert Stengle (Lehigh)

<u>On orderings and completions</u>

Let $(A, \mathfrak{m}, \kappa)$ be a local excellent domain, A^h its henselization and $\hat{A}$ its completion. Let $\mathfrak{p}_1, \dots, \mathfrak{p}_r$ be the zero-divisors of A^h and $\hat{\mathfrak{p}}_1, \dots, \hat{\mathfrak{p}}_r$ the ones of $\hat{A}$; we call the quotients $A^h_i := A^h/\mathfrak{p}_i$ henselian branches of A and the quotients $\hat{A}_i := \hat{A}/\hat{\mathfrak{p}}_i$ formal branches. Let α be a total ordering in A.

<u>THM 1</u>.- The following conditions are equivalent: (1) $\mathfrak{m}$ is α-convex; (2) α extends to some A^h_i (resp. $\hat{A}_i$). In that case i is unique and α extends uniquely to A^h_i.

This thm. raises the question of computing the number $e(\alpha)$, eventually $+\infty$, of extensions of α to $\hat{A}_i$. To be precise let us consider an ordering α_0 in κ and its real closure k, and denote by Ω the space of all α generalizing α_0 (Harrison topology). Then:

THM 2.- The set of all $\alpha \in {}_L\Omega_L$ such that $e(\alpha)=1$ is dense in ${}_L\Omega_L$.

In order to go further in this question we need a new concept:

DEF.- The ring A has Zariski-dense selection (= ZDS) if for any α_0 and any $f_1, \dots, f_m \in A$ positive in some $\alpha \in {}_L\Omega_L$ there is a residually trivial local embedding $\sigma: A \hookrightarrow k[[t]]$ such that $\sigma(f_1)>0, \dots, \sigma(f_m)>0$.

Now we have:

THM 3.- Let us assume A has ZDS: (1) If $\dim A = 2$, the set of all $\alpha \in {}_L\Omega_L$ with $e(\alpha)=2$ is dense. (2) If $\dim A \geq 3$, the set of all $\alpha \in {}_L\Omega_L$ with $e(\alpha)=+\infty$ is dense.

Of course an essential problem is to characterize the A's with ZDS. We have:

THM 4.- (1) If A dominates A', the extension $A \supset A'$ is essentially finite and A' has ZDS, then A has ZDS. (2) If $A = B_{\mathfrak{p}}$ for a local excellent henselian domain B and some non-maximal prime ideal $\mathfrak{p} \subset B$, then A has ZDS.

Using 4 we prove that localizations of finitely generated algebras over a field, Nash local algebras over a field, or analytic local algebras over a complete non-discrete valued field have ZDS. So thm. 3 applies to all these rings. Also we obtain more precise information for such an A. For instance: (1) if $\dim A = 2$, $e(\alpha)=1$ or 2 for all $\alpha \in {}_L\Omega_L$, (2) if $\dim A = 3$, $e(\alpha)=1, 2$ or $+\infty$ for $\alpha \in {}_L\Omega_L$. All these results lead us to propose two questions:

(I) Does there exist a bound $b = b(\dim A)$ such that $e(\alpha) \leq b$ or $e(\alpha)=+\infty$ for all $\alpha \in {}_L\Omega_L$?

(II) If the set of all $\alpha \in {}_L\Omega_L$ such that $e(\alpha)=e$ is non-empty, is it necessarily dense?

JESÚS RUIZ (COMPLUTENSE. MADRID)

Diffeomorphisms of Quotientsingularities.

Let X be a smooth manifold, G a Liegroup acting properly-discontinuous on X. X/G ist the ~~O~~ orbitspace with the smooth structure given by the G-invariant smooth functions. The singularities of X are called quotientsingularities (QS).

Special case: $X = \mathbb{R}^n$, $G \subset O(n)$ finite subgroup.
Then the quotient is given by G-invariant polynomials $(Q_1, \dots, Q_r): \mathbb{R}^n \to \mathbb{R}^n/G \subset \mathbb{R}^r$, so $\mathbb{R}^n/G$ is semi-algebraic.

Well known: All QS's are of this type.

Theorem: Let $G, \tilde{G} \subset O(n)$ be finite subgroups. $f: \mathbb{R}^n/G \to \mathbb{R}^n/\tilde{G}$ a smooth diffeomorphism. Then f has a lifting, that means, there is a diffeomorphism $F: \mathbb{R}^n \to \mathbb{R}^n$, making commutative this diagram:

$$\begin{array}{ccc} \mathbb{R}^n & \xrightarrow{F} & \mathbb{R}^n \\ \downarrow & & \downarrow \\ \mathbb{R}^n/G & \xrightarrow{f} & \mathbb{R}^n/\tilde{G} \end{array}$$

and with $F G F^{-1} = \tilde{G}$.

Proof (main ideas):
1) f analytic; existence of F is shown by complexification.
2) f smooth: f difeotop to ~~an~~ ~~ayl~~ analytic diffeomorphism f_0, existence of F then with homotopie-lifting-theorem of Bierstone.

KLAUS REICHARD (BOCHUM)

Moduli Spaces of Linear Dynamical Systems

Quite a few typical problems in linear system theory deal with the intersections of certain algebraic or semialgebraic subvarieties of the space $\Sigma_{n,m}$ of controllable systems resp. of $\mathrm{Rat}_{n,m,p}$ of linear systems of degree n. These spaces are defined as follows:
Let $\Sigma_{n,m} := \{ (A,B) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n \times m} \mid (A,B) \text{ controllable} \}$
and $\widetilde{\mathrm{Rat}}_{n,m,p} := \{ (A,B,C) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m} \times \mathbb{R}^{p\times n} \mid (A,B) \text{ controllable and observable} \}$.

These are open subsets of affine space and $GL_n(\mathbb{R})$ acts freely on these spaces by similarity:

$$(A,B) \mapsto (SAS^{-1}, SB) \quad , \quad S \in GL_n(\mathbb{R}) \, ,$$

resp.

$$(A,B,C) \mapsto (SAS^{-1}, SB, CS^{-1}) \quad , \quad S \in GL_n(\mathbb{R}) .$$

The orbit spaces $\Sigma_{n,m} := \tilde{\Sigma}_{n,m} / GL_n(\mathbb{R})$ resp. $\mathrm{Rat}_{n,m,p} := \widetilde{\mathrm{Rat}}_{n,m,p} / GL_n(\mathbb{R})$ are algebraic manifolds. Both spaces are homotopy equivalent up to a certain degree and $\mathrm{Rat}_{n,m,p}$ is known to be important in (2 dimensional) Yang Mills Theory.

We report on the cohomology of $\Sigma_{n,m}$:

Thm 1: The mod 2 homology groups of $\Sigma_{n,m}$ are isomorphic to the mod 2 homology groups of the Grassmann manifold $G_{n,n+m-1}(\mathbb{R})$:

$$H_*(\Sigma_{n,m}; \mathbb{Z}_2) \cong H_*(G_{n,n+m-1}(\mathbb{R}); \mathbb{Z}_2) .$$

It is shown, that $H_*(\Sigma_{n,m}; \mathbb{Z}_2)$ is totally algebraic with a basis given by algebraic "Kronecker cycles".

Thm 2: The mod 2 cohomology rings $H^*_*(\Sigma_{n,m}; \mathbb{Z}_2)$ is isomorphic to the invariant cohomology ring

$$H^*(\mathbb{P}_{m-1}(\mathbb{R}) \times \ldots \times \mathbb{P}_{m-1}(\mathbb{R}); \mathbb{Z}_2)^{S_n}$$

Using this, one may describe a complete analogon to the classical Schubert calculus for the Grassmannian, i.e. a Pieri type formula, formula of Giambelli etc for $\Sigma_{n,m}$. This part is joint work with C. I. Byrnes (Cambridge / Arizona).

Uwe Helmke (Bremen → Regensburg)

Analytic Right-Inverses to Quadratic Forms over Number Fields

Let F be a ~~quadratic~~ countable subfield of $\mathbb{C}$ (e.g. a ~~#~~ number field). Let $a_1, \cdots, a_m \in F^\times$ & $f(x_1, \cdots, x_m) = a_1 x_1^2 + \cdots + a_m x_m^2 = \langle a_1, \cdots, a_m \rangle$ be a quadratic form over F. Write $f(F^m)$ or $D_F(f)$ for $\{a_1 x_1^2 + \cdots + a_m x_m^2 \mid x_i \in F\}$

<u>Thm</u> There exist m functions $g_1, \cdots, g_m$ analytic in $\mathbb{C}$ minus the negative real axis satisfying $a_1 g_1(z)^2 + \cdots + a_m g_m(z)^2 = z$, & such that $g_i(z) \in F$ $\forall z \in f(F^m)$ not on the negative real axis.

Here we may replace "the negative real axis" with any branch cut for $z^{1/2}$. The theorem says that f has an analytic right-inverse $g = (g_1, \cdots, g_m)$, $(f \circ g)(z) = z$, s.t. $g(f(F^m)) \subseteq F^m$. Even for $m > 1$ the g_i cannot all be rational ($\in F(z)$) or even algebraic over $F(z)$, by Hilbert's Irreducibility theorem.

The theorem gives analytic versions of some arithmetic results about algebraic number fields F embedded in $\mathbb{C}$: (1) Let $f = \langle 1,1,1,1 \rangle$. Siegel proved (1921) that $f(F^4) = \{$sums of squares in $F\}$. For $F = \mathbb{Q}$ Heilbronn (1964) had already constructed an analytic version of Lagrange's thm., thereby answering a question of Kreisel.
(2) Estes, Hurrelbrink, & Perlis (1984) showed $\{$sums of squares in $F\}$ $= \bigcup_{q=1}^{\infty} D_F(\langle 1, q \rangle)$, i.e. every sum of squares is a square plus a sum of equal squares, $x^2 + y^2 + \cdots + y^2 = x^2 + q y^2$, some $q \in \mathbb{N}$. This answered a question of P. Conner. The theorem gives pairs of analytic functions g_q, h_q s.t. $z = g_q(z)^2 + q h_q(z)^2$, with $g_q(z), h_q(z) \in F$ $\forall z \in D_F(\langle 1, q \rangle)$ not on neg. real axis.

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Tameness of semi-algebraic sets

Let K be a real closed extension of $\mathbb{R}$, $\bar{\ }: K \to \mathbb{R} \cup \{\pm\infty\}$ the (place) map assigning to each $a \in K$ the unique real number infinitely close to it (or $\pm\infty$). (Note: $\bar{r} = r$ for $r \in \mathbb{R}$.)

Then the following are true:

(1) If $X \subset K^n$ is s.a./K, then $X \cap \mathbb{R}^n$ is s.a./$\mathbb{R}$

(2) If $X \subset K^n$ and $f: X \to K$ is s.a./K, then the function $f_{\mathbb{R}}: (X \cap \mathbb{R}^n) \to \mathbb{R} \cup \{\pm\infty\}$, defined by $f_{\mathbb{R}}(x) = \overline{f(x)}$, is s.a./$\mathbb{R}$.

(Note that (2) $\Rightarrow$ (1) by taking for f the characteristic function of X; "s.a." means "semi-algebraic", and for functions is not meant to imply continuity.)

These results can be interpreted in $\mathbb{R}$ as follows:

Let $S \subset \mathbb{R}^{m+n}$ be s.a. (over $\mathbb{R}$). For each $x \in \mathbb{R}^m$, put $S_x = \{y \in \mathbb{R}^n \mid (x,y) \in S\}$. We call $\{S_x \mid x \in \mathbb{R}^m\}$ a s.a. collection of sets. Now statement (1) implies:

(1*) Each subset of $\mathbb{R}^n$ which is the union or intersection of a chain of S_x's is semi-algebraic. (Chain: collection of sets totally ordered by inclusion.)

<u>More generally</u>: If $A \subset \mathbb{R}^n$ has the property that $\forall y_1, \dots, y_k \in \mathbb{R}^n \; \exists x \in \mathbb{R}^m . \; A \cap \{y_1, \dots, y_k\} = S_x \cap \{y_1, \dots, y_k\}$, then A is s.a.. (Let's call a set A with this property a <u>limit</u> set of $\{S_x \mid x \in \mathbb{R}^m\}$.

The "real" interpretation of (2) is as follows.

Let $g: \mathbb{R}^{m+n} \to \mathbb{R}$ be s.a. For each $x \in \mathbb{R}^m$, define $g_x: \mathbb{R}^n \to \mathbb{R}$ by $g_x(y) = g(x,y)$. This collection of functions

$\{g_x \mid x \in \mathbb{R}^m\}$ has the following property:

(2*) If a function $\phi : \mathbb{R}^n \to \mathbb{R} \cup \{\pm\infty\}$ is the pointwise limit of a sequence of g_x's, then ϕ is s.a.
(Moreover, each sequence of g_x's has a pointwise converging subsequence.)

Moregenerally: If $\phi : \mathbb{R}^n \to \mathbb{R} \cup \{\pm\infty\}$ is such that for all $y_1, \dots, y_k \in \mathbb{R}^n$ and neighborhoods $U_1 \ni \phi(y_1), \dots, U_k \ni \phi(y_k)$ there is $x \in \mathbb{R}^m$ with $U_1 \ni \phi_x(y_1), \dots, U_k \ni \phi_x(y_k)$, then ϕ is s.a..

Open Problem. Is the collection of limit sets of $\{S_x \mid x \in \mathbb{R}^m\}$ a semi-algebraic collection?

The results above, for semi-algebraic sets $/\mathbb{R}$, do not generalize to semi-algebraic sets over other real closed fields.

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Cycles under blowings up

Let $A_*(X)$ denote the Chow group of a smooth projective $\mathbb{R}$-variety X. A prime cycle z is called real if $z_{reg}(\mathbb{R}) = \emptyset$. Let $B_*(X)$ be the subgroup generated by the classes of non-real cycles and $A_*^{\mathbb{R}}(X)$ the group $A_*(X)/B_*(X)$. There is a canonical homomorphism $\lambda: A_*^{\mathbb{R}}(X) \to H_*(X, \mathbb{Z}_2)$. The graded group $T_*(X) = A^{\mathbb{R}}(X)/H_*(X(\mathbb{R}), \mathbb{Z}_2)$ is studied under birational transformations. For a blowing up $\tilde{X} \to X$ in a regular center Z of codimension d it is shown that $T_k(\tilde{X}) = T_k(X) \oplus \bigoplus_{i=0}^{d-2} T_{k-d+1+i}(Z)$, where $n = \dim X$.

Heinz-Werner Schülting, Dortmund.

Complex Nash functions

Let $U \subset \mathbb{C}^n$ be an open connected domain. We say that an holomorphic function f on U is a Nash function if ($f \in N(U)$) if it satisfies a polynomial relation. When U is semialgebraic, this is equivalent to have semialgebraic graph and it is also equivalent to have real part and imaginary part real Nash functions. Since, for a polycilinder U, $H^1(U, N) \neq 0$, we look for global properties. Let $F = \mathbb{C}^n - U$. If for every algebraic hypersurface Y, top.dim.$(Y \cap F) < 2n-2$, then $N(U) = \mathbb{C}[x_1 - x_n]$. In particular we obtain examples of Stein subsets $U \subsetneq \mathbb{C}^2$ with $N(U) = \mathbb{C}[X, Y]$. Finally, let $K \subset U$ be a semialgebraic, ~~[illegible]~~ rationally convex compact subset, then the ring $W(K)$ of germs of Nash functions defined in some neighborhood of K is noetherian; and if $U \subset \mathbb{C}^n$ is semialgebraic, relatively compact, with the property that for every $x \notin U$ there exist a polynomial P with $P(x) \notin \overline{P(U)}$, then $N(U)$ also is noetherian.

[illegible] (Genova)

Canonical stratifications of real algebraic varieties

Let S be a stratified set which is also subanalytic (in particular it might be real algebraic). The theorem of Hironaka and Hardt that a Whitney stratification of S exists is a statement about the C^1 differentiable structure of S. We make more precise the class of subanalytic sets in the C^1 DIFF category. Teissier (Arcata, 1974) defined a notion of E^*-regularity on a stratification for E an arbitrary equisingularity condition: this says that for each stratum Y of the stratified set S, and for each point $y \in Y$, there is (an open dense set of planes in $T_y S$ defining) a generic class of i-dimensional submanifolds W containing Y such that $(S \cap W, Y)$ is E-regular. For complex analytic varieties $(Wh) \Rightarrow (Wh)^*$ where (Wh) denotes Whitney regularity (Teissier, Navarro Aznar, Henry-Merle 1982). It is unknown for subanalytic sets or real algebraic varieties when $\dim(\operatorname{Sing} S) > 1$; when $\dim \operatorname{Sing} S \leq 1$ it is a result of Navarro Aznar & myself. However, ~~subanalytic~~ subanalytic sets admit $(Wh)^*$ stratifications (Navarro Aznar-Trotman; Orro). As a useful consequence there is the density of Morse-Lazzeri functions on ~~[illegible]~~ subanalytic sets since Orro (Orsay thesis, 1984) showed the density of Morse-Lazzeri functions on $(Wh)^*$-stratified sets.

Finally we mention examples of real algebraic varieties (of dimension 3 in $\mathbb{R}^4$) such that their canonical Whitney stratification is not defined by a filtration by Zariski closed subsets; the bad set for Whitney regularity may be $\dashv$ which is not even an Euler space.

David Trotman (Orsay)

Milnor Fibrations for real-analytic functions -

Let $f: \mathbb{R}^m \to \mathbb{R}^k$ an analytic function with $0 \in \mathbb{R}^m$ as isolated singularity. There exists ε_0 st. for any $\varepsilon < \varepsilon_0$ there exists η_ε s.t $\eta < \eta_\varepsilon \Rightarrow$ the restriction of f to $f^{-1}(S_\eta^{k-1}) \cap B_\varepsilon^m$ is a locally trivial fibration on S_η^{k-1}. Further it is possible to push this fibration on the sphere, so $S_\varepsilon^{m-1} - f^{-1}(0)$ is fibered onto S_η^{k-1}. But this fibration is not explicitly known.

Now, consider the projection $\pi: \mathbb{R}^k \to \mathbb{R}^{k-1}$ on the first $k-1$ factors and the composite $\tilde{f} = \pi \circ f$. $\tilde{f}$ has also $0 \in \mathbb{R}^m$ as isolated singularity and so fibers. If we denote F and $\tilde{F}$ the fibers of f and $\tilde{f}$ we obtain:

Thm: $\tilde{F}$ is homeomorphic to $F \times [0,1]$

Assume now that $k=2$ and that $f = (P,Q)$ has the following properties

A) on a neighborhood V of $0 \in \mathbb{R}^m$ there exists $\varepsilon > 0$ s.t

$$\forall x \in V - \{0\} \quad \frac{|\langle \nabla_x P, \nabla_x Q \rangle|}{\|\nabla_x P\| \, \|\nabla_x Q\|} < 1 - \varepsilon$$

B) the integral closures of $\left(\frac{\partial P}{\partial x_i}\right)_{1 \le i \le m}$ and $\left(\frac{\partial Q}{\partial x_i}\right)_{1 \le i \le m}$ are equal.

Then we can explicit the fibration on the sphere:

Thm: $\frac{(P,Q)}{(P^2+Q^2)^{1/2}} : S_\varepsilon^{m-1} - f^{-1}(0) \to S^1$ is a C^∞ locally trivial fibration (ε suff. small)

(this thm generalizes Milnor's one in the cplx case)

We are now interested in computing the Euler-Characteristic of the fiber. We can consider the more general case of an analytic function $f: \mathbb{R}^m \to \mathbb{R}^2$ s.t. $\Sigma_P = \Sigma_Q = \Sigma_f$ is compact (give by an equation $\varphi = 0$) and $\Sigma_f \subset f^{-1}(0)$. Denote by $V_\varepsilon = \{ x \,/\, |\varphi(x)| \le \varepsilon \}$. We can prove

Thm: $\partial V_\varepsilon - f^{-1}(0)$ is fibered onto S^1 and the Euler-Characteristic of the fiber F is given by $\chi(F) = \chi(\Sigma_f) - \deg_{\Sigma_f}(\nabla P)$.

Let us give some explainations about this notion of degree around the singular locus: orient ∂V_ε (for ε small) and consider on each connected component W_i of ∂V_ε $\omega_i^* = \left(\frac{\nabla_x P}{\|\nabla_x P\|}\right)_* : \underset{\simeq \mathbb{Z}}{H_{m-1}(W_i)} \to \underset{\simeq \mathbb{Z}}{H_{m-1}(S^{m-1})}$

We call degree of P $\deg_{\Sigma_f}(\nabla P) = \sum_i \omega_i^*(1_{W_i})$. (the same number occurs if we take Q).

Alain Jacquemard DIJON

Higher signatures of function fields over $\mathbb{R}$

Eberhard Becker (Dortmund)

Any character $\chi : K^\times \to \mu(2n)$ is called a signature of level n if then $\ker\chi$ is additively closed where K is a field, $\mu(2n)$ the group of $2n$-th roots of unity. It is talked about a ~~geo~~ geometric description of signatures in function field over $\mathbb{R}$. Furthermore a geometric description of the sums of $2n$-th powers in such fields are given. The relation between such sums and signatures is as follows: $(\sum K^{2n})^* = \bigcap \ker\chi$, χ ranging over all signatures of level n. The following results are mainly due to H.-W. Schülting. Let $F|\mathbb{R}$ be a formally real function fields, X a regular projective model of F, $x \in X(\mathbb{R})$ and $\mathrm{Sgn}_x(F) = \{\chi \text{ signature of level } n, \text{ center of } A(\chi) = x\}$ where $A(\chi)$ is the valuation ring attached to χ. Further choose an ordering P_0 with $A(P_0) = A(\chi)$. Then consider the map

$$\Phi : \mathrm{Sgn}_x(F) \longrightarrow \left(\mathbb{R}\text{-}\mathrm{Div}_x(X)\right)^*, \ \chi \mapsto \chi(\pi)\,\mathrm{sgn}_{P_0}(\pi) \quad \text{where}$$

$\mathbb{R}\text{-}\mathrm{Div}_x(X)$ is the real local divisor group, π a real prime element of $\mathcal{O}_x$.

<u>Thm 1</u> Φ is injective.

The intersection of all $\ker\eta$, $\eta \in \mathrm{Im}(\Phi)$ is computed for surfaces.
For the next theorem we also assume $\dim(F|\mathbb{R}) = 2$. Let $f \in F$,

$$\mathrm{div}(f) = \sum_{V_j \text{ real}} r_j \cdot V_j + \sum_{\substack{W_i \text{ not} \\ \text{real}}} s_i \cdot W_i \ , \quad D := \sum s_i \cdot W_i \quad \text{then}$$

<u>Thm 1'</u> $f \in \sum F^{2n} \iff f \in \sum F^2$, $2n \mid r_j$ for all j's,

$2n \mid m(\tilde{D}, \tilde{x})$ for all real infinitely near points of x

Reduced Witt rings of surfaces

Louis Mahé

The result shown in this talk is that any strictly positive polynomial function over a real surface S divides a term as $1+\sum_{i=1}^{6} x_i^{\,i}$.

As a consequence, using the work of L. Bröcker about the bounds for the number of inequalities involved in the description of an open semi-algebraic set, we are able to show that for any $\mathbb{Z}$-valued function f on a surface S, $2^{10} f$ is the signature of some quadratic form on that surface. In other words the reduced Witt ring $\hat{W}(R[S])$ contains $\mathbb{Z}\cdot 1 + 2^{10}\,\mathbb{Z}^{s}$ (s is the number of connected components).

L. M. (Rennes)

Louis MAHÉ

Spaces of orderings and semialgebraic sets by Ludwig Bröcker

For a basic open semialgebraic set $S \subset V(R)$, V an affine real algebraic variety over a real closed field R, we define $s(S) := \min\{s \mid S$ of the form $\{f_1>0,\dots,f_s>0\}\}$ where $f_i \in R[V]$. Define $s(V) = \max\{s(S) \mid S$ basic open in $V\}$ and $s(n) = \max\{s(V) \mid \dim V = n\}$.

For an open semialgebraic set $S \subset V$ define $t(S) := \min\{t \mid S = S_1 \cup \dots \cup S_t,\ S_i$ basic open$\}$. Define correspondingly $t(V)$ and $t(n)$ and also $\bar{s}(S)$, $\bar{s}(V)$, $\bar{s}(n)$ for the representation of basic closed sets and $\bar{t}(S)$, $\bar{t}(V)$, $\bar{t}(n)$ for the representation as unions of basic closed sets. It is shown, that the values $s(n)$, $\bar{s}(n)$, $t(n)$, $\bar{t}(n)$ are $< \infty$ for $n \in \mathbb{N}$. Good bounds and exact values are obtained for small n. The proof use essentially the theory of the spaces of orderings. The methods where also applied for the question, to what extend semialgebraic sets can be separated by polynomials.

L. Bröcker (Münster)

The p-adic inequality of Łojasiewicz by Johann-Heinrich Schinke

If (k, v) is a formally p-adic closed field of p-rank d, we have – as in the real case – semialgebraic sets in a k-variety V. If V is affine, we construct these from the coarse open subbasics $S(f,g) := \{x \in V(k) \mid v(f(x)) > v(g(x))\}$ resp. the fine open subbasics $P_n(f) := \{x \in V(k) \mid f(x) \in k^{*n}\}$, where $f, g \in k[V]$, $n \in \mathbb{N}$. Because of the elimination of quantifiers due to MacIntyre, the fine are the right ones. For such a bounded, closed semialgebraic set $M \subset k^n$ the following inequality of Łojasiewicz is valid: Let f, g be polynomials in $k[X_1, \dots, X_n]$ with $g(m) = 0 \Rightarrow f(m) = 0$ for all $m \in M$. Then there exist $c \in k^*$ and $N \in \mathbb{N}$ such that for all $m \in M$ we have: $v(g(m)) \leq v(c \cdot f^N(m))$. The proof was done a) in a neighborhood of 0 with the Hensel-lemma and the fact, that $|k^*/k^{*n}| < \infty$ for all $n \in \mathbb{N}$ and b) outside of the neighborhood of 0 by the ultrafilter-theorem for p-adic s.a. sets and an suitable convergence-property of so called proper ultrafilters.
We have similar conclusions as in the real case, for example, the finiteness-theorem of semialgebraic sets.

Joh.-H. Schinke (Münster)

Really real ideals

Let (X, x) be a real analytic germ, $I = (g_1, \dots, g_k)$ an ideal in $\mathcal{O}_{X,x}$, $f \in \mathcal{O}_{X,x}$. Then the Łojasiewicz exponent is $\theta(f, I) = \operatorname{Inf}\{\alpha \in \mathbb{R}_+ \mid \exists c > 0, \exists U \text{ open s.t. } |f(x)|^\alpha \leq C \sup(g_1(x), \dots, g_k(x))\}$, $x \in U$.
$\tilde{\theta}(f, I)$ has the same definition but with x complex.
It is shown that $\theta(f, I)$ can be computed by restriction to analytic curve germ, and that $\theta(f, I)$ is a rational number.

A really real ideal is an ideal such that $\theta(f, I) = \tilde{\theta}(f, I)$. Various conditions and conjectures are done about these ideals.

Jean Jacques RISLER (Paris)

On the semi- and sub-analytic geometry

S. Łojasiewicz

A survey of some basic fact on the semi- and sub-analitic sets (normal stratifiactions, regular separation property, proper images; elementary approach to subanalytic sets; Gabrielov complement theorem, Tamm-Kurdyka theorem on the set of smooth points, Pawlucki theorem on the set of semi-analytic points of a subanalytic set).

S. Łojasiewicz (KRAKÓW).

ROBUST STATISTICS

September 9 - 15, 1984

A Robust Straitjacket (Zwangsjacke).

There is a danger that the excitement of a (comparatively) new statistial technique will cause statisticians to neglect many of their earlier procedures. Robust statistics is no exception in that the emphasis has tended to be on point estimation. Now that we know how to do the calculations for robust regression (Rousseauw, Leroy, Yohai) it is time to consider other aspects such as model building. The history of robust regression is illustrated through published analyses of Brownlee's Stack Loss Data. These can be contrasted with a diagnostic analysis based on least squares which yields an extended model and so avoids the straitjacket of a simple model vindicated by outlier rejection. The importance of graphical interpretations of robust regression was stressed.

A. C. Atkinson, Imperial College.

M- Estimators: Differentiability and Probabilistic Convergence Arguments.

The M-functional/estimator of a vector valued parameter can be described as a solution of equations, ~~perhaps~~ (multivariate,) and defined through a psi function. The latter can be arrived at through minimizing suitable distances or loss functions, or alternatively, obtained via optimizing machinery, for instance where one obtains maximum efficiency subject to a bound on the gross error sensitivity (Hampel 1968). Study of uniform convergence and classical separability requirements can be used to give existence of a unique consistent root to the equations, and this root is selected by any "selection functional" satisfying properties analogous to those of classical distances. Using the theories of nonsmooth analysis it can be shown that the estimating functional is Fréchet differentiable with respect to Kolmogorov, Lévy and Prokhorov distances, thus giving credence to the gross error sensitivity as a measure of robustness. An overview of results is given

Brenton R. Clarke, Murdoch University

Precision of one-step approximations of estimators.

Many statistical estimators are defined implicitly as a solution of an equation or of a system of equations. This is the case of maximum likelihood estimators, M-estimators, R-estimators, among others. The solution of these equations can be technically difficult or, if there are more possible roots, we do not know whether that we have found is the consistent estimator. Other estimators, like the Pitman one, could be explicitly expressed but it is difficult to calculate their numerical value. Therefore a standard technique is to look for an iterative solution or a one-step version of the estimator which, being properly defined, leads to a consistent estimator asymptotically equivalent to the consistent version of the estimator under consideration.

Regularity conditions are given which enable to construct one-step version T_n^* of the estimator T_n such that $T_n^* - T_n = O_p(n^{-1})$, as $n \to \infty$. This covers the M-estimators of location, regression and ~~reg~~ general parameters, MLE, R-estimators of location and Pitman's estimator of location.

11/9/84 Jana Jurečková, Charles University, Prague

Asymptotic Behavior of M-Estimators of p Regression Parameters and of the Empiric c.d.f. of Residuals when p is Large.

Consider the model $Y_i = x_i'\beta + R_i$, $i = 1, \ldots, n$ with $\beta \in R^p$, $x_i \in R^p$, and $\{R_i\}$ i.i.d. when $p \to \infty$ so that p/n is small but p^2/n is moderate. Let $\hat\beta$ satisfy $0 = \sum x_i \Psi(Y_i - x_i'\hat\beta)$. The following results are given:

I. In ANOVA cases where $Y_{ij} = \beta_j + R_{ij}$, $j = 1, \ldots, n/p$, classical results show that $\|\hat\beta - \beta\|^2 = O_p(p^2/n)$. However, one can obtain the results that $\max_j (\hat\beta_j - \beta_j)^2 = O_p(p \ln n/n)$ and (if $\|a\|=1$) $a'(\hat\beta - \beta)$ is asymptotically normal if $p \ln^2 n/n \to 0$.

II. In regression cases where $\{x_i\}$ behave like a random sample in R^p, $\|\hat\beta - \beta\|^2 = O_p(p/n)$. Also one can obtain a uniform normal approximation for the distribution of $(X'X)^{1/2}(\hat\beta - \beta)$ in R^p if $p^{3/2} \ln n/n \to 0$ (under conditions which are artificial but hold in probability if $\{x_i\}$ form a sample from a mixed multivariate normal). There is also an expansion for $x_i'(\hat\beta - \beta)$ (uniform in $i = 1, \dots, n$) as a sum of five terms (involving sums of $\Psi(R_i)$, $(\Psi'(R_i) \cdot d)$, and $\Psi''(R_i)$) with error $O_p(p^{11/8} \ln n/n)^2$; so that $\sqrt{n}\,(x_i'(\hat\beta - \beta))$ has order $o_p(1)$ for the error term.

III. Let $\hat F_n(x)$ be the empiric c.d.f. of the residuals and let $\hat F_n^*(x)$ be the empiric c.d.f. of the errors, $\{R_i\}$. Then

$$\sqrt{n}\left(\hat F_n(x) - \hat F_n^*(x) - H_n(x)\right) - g(x) \cdot \frac{p}{2\sqrt{n}} \xrightarrow{P} 0$$

where $H_n(x) = \frac{1}{dn} f(x) \sum \Psi(R_i)$ (if the design has a constant term and H_n vanishes otherwise) and $g(x) = \frac{\sigma^2}{d^2} f'(x) + \frac{1}{d} f(x) \Psi(x)$. Tightness results provide asymptotic distribution theory: $\sqrt{n}\,(\hat F_n(x) - F(x))$ converges to the usual limiting process only if $H_n(x)$ and $g(x)$ can be estimated to permit appropriate adjustment, but converges to a different Gaussian process otherwise.

10/9/84 Stephen Portnoy, Univ. of Illinois, Urbana

Robustness and adaptation (with C. Klaassen (Leiden))

We review the requirements of robustness (in the context of the symmetric location model): local, infinitesimal, and minimax, and see to what extent they can be reconciled to full adaptation. We show that despite lack of uniformity in the convergence of adaptive estimates they can satisfy Huber's asymptotic minimax property as well as render the Huber estimate inadmissible. On the other hand, infinitesimal robustness can only be coupled with partial adaptation. We construct such estimates and pose some questions on the existence of scale equivariant adaptive estimates having these properties and on the role of robustness in other semiparametric models.

Peter J Bickel (Berkeley)

Small Sample Asymptotics and Robust Statistics

We discuss applications of small sample asymptotics (SSA) in robust statistics. We focus on

i) better numerical approximations to the distribution of robust estimators and tests;

ii) applications of SSA in the theory of robust statistics that improve "asymptotic" optimality results;

iii) connections of SSA with the bootstrap.

September 11, 1984 Elvezio Ronchetti, Princeton University

How to select variables that should be entered in a Robust Regression Procedure?

Let $Y = \sum_{j=1}^{\infty} x_j \beta_j + e$ be an infinite linear model and let $\hat\beta(p)$ be a robust M-estimate of the regression parameters $(\beta_{j_1}, \beta_{j_2}, \dots, \beta_{j_{r(p)}})$ with $p = (j_1, j_2, \dots, j_{r(p)})$ denoting a finite dimensional model. Given a collection of models $\mathcal{P}_n$ we ask how to find an asymptotically efficient model $\hat{p} \in \mathcal{P}_n$ that is computed from the data, can be found. We derive, based on a linearization argument, a model selection procedure and show asymptotic efficiency of that procedure.

Wolfgang Härdle, Frankfurt

Robust parameter estimation for ARMA processes

Supported by the LAN-expansion of the likelihood ratio for an ARMA process a broad class of regular estimators is defined locally at a point in the parameter space. Regular estimators allow for an asymptotic expansion as a sum of stationary and ergodic martingale differences. Contamination models of the ε-contamination type are introduced for the transition probabilities. The maximal risk of regular estimators over certain (contiguity-) submodels for the contamination is evaluated. Estimators which minimize the maximal risk are determined. This also gives the optimality of some known estimators, as the Hampel-Krasker-Welsch estimator.

H. Staab (Universität Bayreuth)

Some aspects of robustness in the presence of long-range correlation

Correlation is quite common even in series of observations where one might be tempted to assume independence. Moreover, in such series correlation often decays very slowly. The consequences of ignoring lack of independence for confidence intervals and related quantities can be drastic. It is proposed to model the main features of long-range correlation with the aid of self-similar processes. A method for estimating and testing the correlation parameter of self-similar processes is presented and applied to some observed time series. In addition to the robustness problem of outliers (in a broad sense), there arises the conceptually different problem that the model correlation structure might hold only approximately, even if the effects of outliers are left out of account. Specifically, we are prepared for short-term, high-frequency deviations from the model correlation structure. The proposed method of estimation can cope with these.

Hanspeter Graf, Basel

Confidence Sets and Monte Carlo

This paper describes three roles for Monte Carlo methods in the construction and assessment of confidence sets: (a) constructing critical values for ~~a~~ confidence sets by bootstrapping the root quantity; possible uses of Monte Carlo techniques to calculate the root quantity itself and to determine which elements of the parameter space lie in the confidence set; (b) reducing the discrepancy between nominal and actual level of a confidence set by prepivoting the root quantity before bootstrapping; (c) estimating the accuracy of a confidence set, viewed as a set estimate of the parameter of interest. Examples and asymptotic theory for the proposed procedures are given.

Rudolf Beran, Berkeley

PROGRES: A Program for Robust Regression

The least median of squares (LMS) method for high-breakdown regression is discussed, and some variants are mentioned (least trimmed squares, S-estimators). Methods like this are useful in the case of leverage points, that is, observations with outlying x. Their ultimate purpose is the same as that of regression diagnostics, but they use another approach to achieve this goal. The algorithm in program PROGRES involves the evaluation of an objective function over a sufficiently large collection of random subsamples. An example with six input variables is presented, and partial results of a simulation study ~~are~~ shown. Program PROGRES can be obtained by sending a 9-track magnetic tape to the author.

Peter Rousseeuw, Delft

Asymptotic stability of confidence sets

Let ξ be a statistical functional with values in a Banach space B. Introduce a decision space consisting of all balls $C(z,r)$ in B with centre z radius r. If Q^n is the data distribution at time n, define the loss function for a procedure $C(\hat{z},\hat{r})$ by $\sup_{y \in C(\hat{z},\hat{r})} |y - \xi(Q^n)|$. All procedures are subject to the $1-\alpha$ condition: $Q^n\{C(\hat{z},\hat{r}) \ni \xi(Q^n)\} \geq 1-\alpha$. In this framework a locally asymptotic minimax result is derived having the usual robustness interpretations. This is applied to the problems of giving a $1-\alpha$ confidence band for a) a measure indexed by a V-Č class and b) a minimum distance functional.

P. Warwick Millar,

Berkeley, CA

Fitting Heteroscedastic Regression Models

These models occur in a wide variety of situations. There are two basic models: (1) when the variance is a function of the mean and (2) when the variance depends on a single predictor. The major difficulty in building case deletion diagnostics is that estimation is nonlinear; that there is no standard method of estimation also poses problems. In my experience, case deletion diagnostics are easy to design, easy to implement, often used and often reliable. Interestingly enough, in the example considered here it turns out that the case deletion approach is rather unsatisfactory, while bounded influence does indicate a severe model failure. More examples need to be published comparing approaches to finding interesting points.

Raymond J. Carroll

Chapel Hill, NC

Oberwolfach 13 Sept 1984

Robustness of One and Two Sample t Statistics

Data are often over optimistically assumed to be realizations of independent and identically distributed (i.i.d.) Gaussian random variables. The two sample problem usually assumes two such sets of i.i.d. random variables, independent from each other. However in many real situations, such assumptions are inappropriate, although it still makes sense to make inferences about the location parameter of the "sample(s)".

This talk examines the one and two sample t statistics in the presence of heterogeneous variances, from the point of view of maintenance of Type I error and power. Robust versions of the t tests are also considered. Both are shown to maintain Type I error, but ~~that~~ the robust versions lose less power, should misweighting occur.

Noel Cressie
Iowa State University

Influence, Asymptotic Variance and Jackknife for Time Series

We show that the infinitesimal asymptotic bias of an estimator depending on the m-dimensional empirical marginal distribution can be written as $\int f(x_1, \dots, x_m)\, d\lambda$ where f depends on the estimator and the model while λ is determined by the contamination and also by the model. We then discuss various types of gross-error-sensitivity. The asymptotic variance can also be expressed with the same function f, but it is an unstable functional of the true ~~model~~ distribution. and should therefore be estimated by a non-parametric method. We propose to take out a block of k consecutive observations and then to move this block over the sample. in order to get a jackknife-estimate of variance. Trade-off between bias and asymptotic variance of this estimate leads to the rate $k = n^{1/3}$ (n = sample size)

Hansruedi Künsch
ETH Zürich

Minimax-robust filtering

We discuss the problem of filtering a time series which is a noisy version of signal which we want to estimate linearly. We assume that signal and noise are uncorrelated weakly stationary stochastic processes with spectral densities f_S and f_N. In contrast to the classical approach of Wiener and Kolmogorov, we do not assume f_S and f_N to be given. Instead, we start from some partial knowledge about the pair (f_S, f_N), summarized in the requirement $(f_S, f_N) \in \mathcal{D}$. We discuss the existence of minimax-robust linear filters with respect to the spectral information, i.e. of filters which achieve the minimal upper bound on the mean-square error if the pair (f_S, f_N) ranges over $\mathcal{D}$. We also indicate how to construct minimax-robust filters given a particular filtering problem and a particular spectral information set $\mathcal{D}$. We illustrate this method with an example from prediction theory.

Jürgen Franke (Universität Frankfurt)

Checking the adequacy of models in human neurophysiology

We consider experiments in neurophysiology where stimuli are repeatedly presented in order to obtain the response of the brain. When averaging across stimuli, one implicitly assumes that the responses to single stimuli remain invariant. Since this model is in doubt, but difficult to check, an alternative estimate was derived which should be "robust" against deviations from the model. Furthermore, three test statistics were constructed for checking the following alternatives: 1. amplitude variation across single signals, 2. some type of slow variation, 3. latency jitter. (valid asymptotically)

Theo Gasser, Mannheim / Heidelberg

Small sample asymptotics – Theory of Wiener germs.

Wiener germs are sequences $\{\mathcal{L}(X_n): n\to\infty\}$ or families $\{\mathcal{L}(X_t): t\to 0\}$ of distributions which tend to a point mass δ_μ in a particular fashion.

Def: $K(x)$ convex on $I \subseteq \mathbb{R}^d$. $K(\mu)=0$, $\operatorname{grad} K(\mu)=0$

$K''(x)$ positive definite

$H(x)$ smooth.

If $\{X_t: t\to 0\}$ is a family of random vectors with values in I, the ~~{L(~~ family of the distributions $\{\mathcal{L}(X_t): t\to 0\}$ is called a Wiener germ with entropy function $K(x)$ and modulating function $H(x)$ if

1) $\Pr(\{|X_t-\mu|\le\varepsilon\}) = 1-o(t^m)$ for all $\varepsilon>0$, m.

2) $\mathcal{L}(X_t)$ has a density of the form

$$f_t(x)\,dx = (2\pi t)^{-d/2}\cdot\exp\left(-\frac{1}{t}K(x)\right)\sqrt{|K''(x)|}\cdot H(x)\exp\left(t\cdot S(t,x)\right)dx$$

where the „correcting" term $S(t,x(t))$ converges if $x(t)\to\tilde{x}\in I$.

Wiener germs seem to arise in many places. The following examples were mentioned: sums of i.i.d. variables satisfying Cramér's condition (Daniels 1954), theory of estimating equations (Hampel & Field, Daniels, Biometrica 1982/3)

$\left(\sum_{j=1}^{n}\psi(X_j,\vartheta^{(n)})=0\right.$ for $\left.\psi(x,t) \text{ as usual}\right)$,

the beta distributions for parameters $(\alpha n, (1-\alpha)n)$ (α fixed) (compare D. Alfers and H. Dinges, Zeitschr. f. Wth. 1984). Also certain curved boundary problems seem to lead to Wiener germs (H. D. Klein, forthcoming)

There was not time to explain in detail the behaviour of the tailprobabilities. It was only indicated how the behaviour for small deviations is related to „robust" concepts of skewness, kurtosis, unbiasedness etc. For moderately large deviations one arrives at improvements of Cramér's theorem as presented in Petrov's book. Also to the theory of ~~for~~ very large deviations the concept of Wiener germ can contribute new aspects.

H. Dinges (Frankfurt)

Nonparametric Spectral Analysis with Missing Observations

The estimation of the spectral density of a discrete stationary process $X(t), t \in \mathbb{Z}$ is considered in the case where only an amplitude modulated version $Y(t) = m(t)X(t)$ is observed. An important special case is that where the modulating sequence $m(t)$ takes the value 1 when $X(t)$ is observed and 0 when $X(t)$ is missing. A spectral estimate which may be calculated using the Fast Fourier transform is considered and its properties relative to the structure of the modulating sequence are investigated. A necessary and sufficient condition on the structure of $m(t)$ is given for the spectral estimate to be asymptotically unbiased. Furthermore the asymptotic normality of a window type estimate is proved. Finally different estimates are compared in a Monte Carlo study.

Rainer Dahlhaus, Essen.

Asymptotic Power Comparison of One-sided Tests

(joint work with W. Ehm, W. Sauermann, S. Luckhaus, E. Mammen)
The problem of comparing nonparametric tests of no effect versus positive effect of treatment is considered. An interaction between treatment effect and shape of distribution has to be taken into account. For this problem, an intermediate approach between purely nonparametric criteria and parametric comparison is developed: it relies on the concept of a bundle of "regular" families of distributions. As a "standard of comparison", the "funnel" tests ~~of~~ dimension k (related to some k-dimensional approximation of the likelihood functions) are proposed. Asymptotic as well as Monte Carlo results show that sizeable improvements over asymptotically linear tests are possible. Several nonlinear

one-sided tests (f.i. the one-sided Anderson-Darling test) are compared with the funnel tests.

D.W. Müller, Heidelberg

Robust Statistics in the Analysis of Geochemical Variables

A preliminary casestudy of the analysis of geochemical variables is reported. Some ideas of resistant analysis and robust statistics are incorporated in certain statistical methods employed in geoscience: Descriptive statistics, principal components analysis, grouping of data, twodimensional presentation, multiple regression analysis, canonical correlation, outlier detection.
The main objective in this study is the outlier-resistant analysis of the data and the report of outliers which are interesting for exploration. This is also illustrated in geographical maps.

R. Dutter, ~~W~~ Vienna.

Cox's model for survival data: omission of a covariate

C. Huber-Carol, Paris, France

(joint work with J. Bretagnolle and S. Gross)

Omission of a covariate in Cox's model results generally in a bias for estimating the effect of covariates inclu-ded in the model. In a special case it can be proved that it results in an underestimation of this effect. When possible, omitted covariates may be dealt with by doing a matched pair experiments. Asymptotic normality and consistency are proved using point processes theory as in (Andersen et Gill (1982) for Cox estimate in that case, while M-L-E is not even consistent.

Importance Sampling for Robust Nonparametric Bootstrap Confidence Intervals

The problem of constructing robust confidence intervals for location by bootstrap resampling methods is discussed. An importance sampling principle is introduced which distorts the bootstrap distribution of a robust estimator so as to permit the use of a much smaller bootstrap replication size than would necessary for the usual bootstrap confidence interval method. A naive, easily implemented method, and a more sophisticated method involving nearly optimal exponential tilting to generate the importance sampling distribution are considered. Some simulation results for sample size = 20 using a particular one-step (from the median) m-estimator are presented for the normal and slash distributions. These results suggest that the bootstrap replication size may be reduced by as much as a factor of 10 or more when importance sampling is used. This may have significant implications for more complex confidence interval problems.

M. V. Johns (Stanford University)

Robust statistical computing with ROBETH

In the past two decades, a considerable theoretical effort has been made to develop robust statistical methods. Yet, except for the case of a location parameter, ~~little has been~~ much work remain to be done for studying the numerical-quantitative properties of the procedures and to acquire practical experience in using them with real data. This gap is probably due to the high amount of programming work which is required. Two projects are now trying to fill this gap: 1) Development of the subroutine library ROBETH. This library includes the most recent methods based on M-estimators and is permanently updated. It is mainly meant for programming experienced statisticians and should be particularly suitable for numerical studies. 2. Development of an "easy to use" package for robust statistical analysis intended for the real data applications of a broader user population.

A. Marazzi [Lausanne]

High breakdown point and high efficiency estimates for regression

Several estimates with high breakdown point are presented. We show that properly defined M-estimates corresponding to simultaneous estimation of the regression coefficients and scale may have at the same time high breakdown point and high efficiency under a central model, i.e., the Gaussian model. These estimates are qualitatively robust. We obtain asymptotic bounds for the bias for different level of contamination. These bounds are compared with those corresponding to optimal bounded influence estimators. Finally we propose a new family of estimates based on the minimization of a new scale parameter. These family contains estimates which have at the same time high breakdown point and high efficiency. The estimators in this class are asymptotically equivalent to an M estimator with psi-function a weighted average of 2 psi-functions: one highly efficient and the other with high breakdown point. The corresponding weights are automatically adapted to the sample.

V. J. Yohai (University of Buenos Aires and CEMA)

On construction of minimum bias-variance estimates for parametric models with shrinking contamination.

Let $\{P_\Theta : \Theta \in \Theta\}$ be a smooth parametric model with Θ an open subset of the real line. If this model is extended to a shrinking supermodel $\{\mathcal{P}_{\Theta,n} : \Theta \in \Theta\}$, where $\mathcal{P}_{\Theta,n}$ is a set of p.m. generated by capacities $v_{\Theta,n} = [P_\Theta + (1/\sqrt{n}) f \circ P_\Theta] \wedge 1$, one can ask about regular estimators which minimize convex combinations of asymptotic bias and variance under a family of sequences $\langle Q_n \rangle$, $Q_n \in \mathcal{P}_{\theta,n}^{\otimes n}$ (n - number of independent observations). It turns out then, that for the optimal estimates we must have

$$\sqrt{n}(\hat{\theta}_n - \theta) - (1/\sqrt{n}) \sum_{i=1}^{n} h(\Delta)_i \longrightarrow 0 \text{ in } P_\theta^{\otimes n}$$

with h proportional to $f'[F(\Delta^*)] - f'[1-F(\Delta^*)] + 2\tau\Delta^*$, where $F(x) = P_\theta(\Delta \le x)$, $\Delta^* = d_0 \vee \Delta \wedge d_1$, Δ is the log-derivative of the likelihood ratio at θ and τ a constant related to some robust asymptotic test problem. Thus a simple truncation of Δ would, in this framework, appear only if f is a linear function. It is also shown that the family of optimal solutions "stays" optimal for risks corresponding to convex combinations of increasing transformations of asymptotic bias and variance.

T. Bednarski (Polish Ac. of Sciences, Wrocław)

Some recent robustness work at Princeton University.

Configural polysampling allows us to work with samples of configurations -- for a chosen sample size -- with two or more sets of weights, configurations that can be regarded, alternatively, as a sample of configurations from each of two or more distributions, the distribution depending upon which set of weights is chosen.

Since (a) equivariant estimates are fixed by one number at each configuration and (b) we know our configurations in practice, this (among all equivariant estimates) allows us to do such things as finding the (one-parameter family of) bioptimal estimates -- those that jointly minimize the two variances of estimate for two distributions of situations, ~~thus~~ generalizing Pitman estimates. In one approach, we do this by combining numerical integration over each configuration in such a polysample with weighted random sampling of configurations. This tool, in the hands of Daryl Pregibon, Katherine Bell Krystinik, Michael Cohen and Stephan Morgenthaler, has taught us much about robust location. In another approach, where we sum over all $\binom{n}{2}$ assignments of "wild values" to the n observations of a sample, Fanny Zambato O'Brien has begun to teach us more about simple linear regression.

Using related techniques, Stefan Morgenthaler has studied many aspects of robust confidence limits, both for location and for scale. He argues for the use of strong confidence intervals, where only confidence is required to reach the nominal level, conditional on both configuration and underlying distribution. In particular, sign-test and Wilcoxon confidence intervals -- originally set up to provide only unconditional confidence, ~~and~~ averaged over all configurations -- are quite unsatisfactory as strong confidence intervals. However, ~~an~~ intermediate compromises are more satisfactory.

(Allen Schoolt and I have been working on procedures for robust factorial analysis of variance which are planned to be (a) relatively efficient and (b) generalizable to k-way patterns. Our present procedure involves three stages: decomposition with many zero residuals; comparison of ordered absolute residuals with typical values for unit Gaussian order statistics, and elimination ("assassination") of cells with unduly large absolute residuals; fitting constants to the (initial values for the) remaining cells by least squares. In determining "unduly large" we have been greatly helped by simulations performed by Edward Fowlkes and Jean McRae at Murray Hill. [This portion not presented orally.])

John W. Tukey (Princeton University and AT&T Bell Laboratories)

On the robustness of the Cox estimator

In survival data analysis the data are often in the form (t, δ, z): t the minimum of survival time^s and censor time$_c$, $\delta = 1_{\{s \le c\}}$, z covariable vector. In the proportional hazard model it is assumed that the hazard function is given by $\lambda_z(t) = \lambda_0(t) \exp(z\beta)$.

The Cox estimator for β is given by

$$\sum_{i=1}^{n} \delta_i \left(z_i - \Big(\sum_{t_j \ge t_i} z_j \exp(z_j\hat{\beta})\Big) \Big/ \Big(\sum_{t_j \ge t_i} \exp(z_j\hat{\beta})\Big) \right) = 0$$

This may be generalized to $\int \psi(x, \beta_n, F_n)\, dF_n(x) = 0$.

For general ψ the influence curve and regularity conditions for consistency and asymptotic normality were given.

It turns out that the influence curve for Cox' estimator is bounded in t, but unbounded in z.

There fore an idea of ~~Hpre~~ robustifying Cox' estimator was presented. It consists of the following: At first the estimate

$$\frac{\sum_{t_j \ge t} z_j \exp(z_j\beta)}{\sum_{t_j \ge t} \exp(z_j\beta)} = \theta_m$$

is robustified ~~estimated~~ for fixed t, β. by huberizing. Then ~~this~~ estimate ~~is~~ $\tilde{\theta}_n$ replaces θ_n and $\sum_{i=1}^{n} h_c[\delta_i(z_i - \tilde{\theta}(t_i, \hat{\beta}))] = 0$ is solved, there $h_c(x) = x\, 1_{[x<c]} + \operatorname{sign} x \cdot c \cdot 1_{[x \geq c]}$.

Then $\hat{\hat{\beta}}$ has a bounded influence function. The problem of consistency was discussed.

C. Baldauf, ETH Zürich

A Robustification of the Sign Test under a Mixing Condition

A robustified version of the two sample sign test is defined which is insensitive to certain deviations from the assumption of the independence of the observations. These deviations are described in terms of a mixing condition.
The asymptotic value of the power function of this robustified sign test is computed on contiguous alternatives possessing the same dependence structure. This entails the calculation of its asymptotic relative efficiencies with respect to some tests which are optimal on these alternatives in the independent case.
It becomes apparent that in general the relative performance of two tests heavily depends on the structure of dependence of the observations, i.e. it may either increase or decrease.

Wolfgang Kohne, Siegen

On the behaviour of ML-estimators in high-dimensional log-linear models

Classical asymptotic normality results for standardized contrasts of log cell means can be seriously violated if the number of unknown parameters is large. A uniform asymptotic normality result is presented which essentially requires p^2/N to be small, improving upon the known results by a factor p. If $\frac{p^2}{N}$ is large, the ml estimator gets a bias. A simple bias-correction device is proposed, which generalizes the well-known "add $\frac{1}{2}$"-rule.

Werner Ehm, Heidelberg

Bounded-Influence Regression and Stable Inference

In this paper we introduced a new form of the Krasker-Welsch (JASA, 1982) bounded-influence regression estimator derived by bounding the influence function relative to the approximate variance at the contaminated model, $V_0 + \varepsilon CVF(x,y)$, where CVF is the change of variance function and ε is a specified fraction of ~~the~~ contamination. The resulting estimator is based on the function $\psi(x,r) = [C/d(x)] \tanh(r/d(x))$ where $d(x)$ is a robust distance. This family of estimators has bounded local shift sensitivity, robust asymptotic variance, and bounds the CVF from above and below.

We also discussed ways to find a central model for the x-data in regression by using a robust or high breakdown distance estimator to trim a fraction, ε, of the x-data with the largest distance. The empirical distribution of the remaining data forms the central model for x. Finally we showed how various tuning constants could be chosen by minimizing the supremum of the appropriate mean-square error in a specified ε-neighborhood of the central probability model, F_0, for y and $\tilde{x}$.

Roy E. Welsch, M.I.T., Cambridge.

(A) High Breakdown made simple; (B) Minimum Distance made simple.

Subtalk (A). The high-breakdown estimators described at this meeting by Rousseuw and Yohai raise certain questions. What is it about these estimators that gives them high breakdown? Do I need to use such estimators (which appear to be difficult to compute) to get high breakdown?

In this subtalk (Joint work with Peter Rousseuw), I introduce the "exact-fit property" for G-equivariant T - essentially an assertion that whenever a fraction δ^* of the data lie near a single G-orbit (e.g. hyperplane), the estimate T behaves as if all the data lie near that orbit. As it turns out $\delta^* = 1-\varepsilon^*$, where ε^* is the breakdown point.

In regression, the implication of this result is that high-breakdown estimators identify regression planes fitting a high proportion of the data, when such exist. Also note that downweighting ~~estim~~ points based on outlying x-values does not help the breakdown point. In fact, breakdown and efficiency are compatible in the sense that both react unfavorably under downweighting of extreme x-values.

Subtalk (B). I described some general quantitative robustness properties of minimum distance estimators. MD estimators in smooth families have the smallest bias sensitivity to distortions which are small in the metric being used for estimation. Also, they have generally good (small) bias over finite neighborhoods and the best possible breakdown over large neighborhoods, again when the contamination neighborhoods are those defined by the estimator. The variance counterparts to these good bias properties are not generally ~~present~~ obtained by minimum distance estimators. They can have high variance at the central model and low "variance breakdown pt" caused by nonuniqueness of the minimum distance projections over moderate neighborhoods of the normal model. However, for the Hellinger contamination model, the minimum Hellinger distance estimator can have good variance properties at the central model, as Beran (1977) showed. Results on the MHDE over Hellinger neighborhoods, obtained jointly with Richard Liu, show that it is very close to bias- and variance-minimax not just infinitesimally but also over quite large ~~finite~~ ($\varepsilon \leq .3$) neighborhoods. Similar results hold for the Beran (1978) MHDE estimator in the adaptive symmetric location model. Results on the robustness over finite ε Huber neighborhoods were also presented.

DAVID L. DONOHO
UNIVERSITY OF CALIFORNIA, BERKELEY

On median-stable laws

Let $X_1, X_2, \ldots$ be an iid-sequence of nondegenerate real-valued variables with unique median $\mu = 0$. Then

X_1 obeys a median stable law, if there is a sequence $(\alpha_n)_{n\in\mathbb{N}}$ of positive numbers, such that median$(\alpha_n X_1, \dots, \alpha_n X_n) \xrightarrow{\mathcal{D}} X_1$, whereas for a stable law in the common (strict) sense $\frac{\sum_{i=1}^{n} \alpha_n X_i}{n} \xrightarrow{\mathcal{D}} X_1$ must hold. The peculiarities of the usual stable laws for characteristic exponent $\alpha \in (0,1]$ are not shared by median-stable laws, which have multipliers $\alpha_n^{(\alpha)} \sim \left(\frac{2n}{\pi}\right)^{1-1/\alpha}$, $\alpha \in \mathbb{R} \setminus [0,1]$. Nevertheless median-stable laws have other peculiarities, which reflect the lack of differentiability of the absolute-error-loss at $x=0$, e.g. there are singular continuous median-stable laws.

Ferdinand Österreicher, Salzburg

Robust properties of minimum distance estimators

Let X_i, $1 \le i \le n$ be i.i.d. r.v's with a c.d.f. F and let G be a smooth d.f. It is shown that the statistic $\sqrt{n}\,(\varrho_a(F_n, G) - \varrho_a(F,G))$ admits a limit distribution, where F_n is empirical d.f. and ϱ_a is a class of metrics including the Kolmogorov and Levy one. If, moreover, F is continuous and $\{G_\theta = G(\cdot - \theta) : \theta \in \mathbb{R}^1\}$ is a family of probability measures on R^1 with a translation parameter θ, we obtain the limit distribution of $\sqrt{n}\,(\varrho_a(F_n, G_{\theta_n}) - \varrho_a(F, G_{\bar\theta}))$, $\sqrt{n}\,(\theta_n - \bar\theta)$ and $\sqrt{n}\,(G_{\theta_n}, G_{\bar\theta})$, where θ_n and $\bar\theta$ denote the minimum distance translation parameters for F_n and F, respectively.

The functional $\theta(F)$ defined by

$$\varrho_a(F, G_{\theta(F)}) = \inf_{\theta \in R} \varrho_a(F, G_\theta)$$

is not differentiable in any sense but it admits directional derivatives and its influence curve exists. We show that it is bounded and continuous under some regularity conditions and calculate the gross error sensitivity at uniform, double exponential and normal distributions. The gross error sensitivity of $\theta(F) = \frac{1}{2}$. M-functionals with influence curve proportional to the influence curve of $\theta(F)$ at the standard normal distribution have asymptotic variance 1.02 for $a = 0$ and 1.024 for $a = 1$.

Andrzej Kozek

ROBUST ESTIMATION OF FUNCTIONALS given a (smooth) parametric model $\{P_\theta : \theta \in \mathbb{R}^d\}$ which is "approximately" true we suggest to estimate the value of locally defined functionals

$$T_\psi(Q;\theta) = \theta + \int \psi_\theta \, dQ \quad ,$$

where ψ_θ has the properties of an influence curve, and for which, under the assumption of independent and identically distributed observations, the following local asymptotic minimax (LAM) bound can be formulated:

$$\lim_{c} \liminf_{n} \inf_{T_n} \sup_{Q \in U(\theta,c,n)} \int l(n \,|T_n - T_\psi(Q;\theta)|^2) \, dQ^n$$

$$\geq \int l(z' \left(\int \psi_\theta \psi_\theta' dP_\theta\right) z) \; N(0, 1_{d\times d})(dz) \quad .$$

The investigation of the shrinking balls $U(\theta,c,n)$ and the construction of LAM estimators is a first instance where robustness arguments are relevant, in a qualitative way.

The second instance of robustness considerations, of a quantitative kind, occurs after the proposal:

Estimate that functional which has lowest LAM-risk and satisfies certain robustness ~~properties~~ requirements.

These may be incorporated by passing to the maximum LAM-risk incurred by the estimation of an M-functional of location over a (symmetric) neighborhood of fixed size, which results in a pure variance theory. Bias can be brought in by separate consideration of the (standardized) oscillation of a general functional over shrinking balls of various types (ε-contamination, total variation, Hellinger, $L^2(\mu)$). This way we obtain abstract versions of the classical Huber (1964), Hampel (1968) - theory which now cover arbitrary estimators, a variety of infinitesimal balls, and concern the estimation of functionals extending the parameter θ outside of the given parametric model. These results are now comparable with results of Beran (1981, 1982) and Millar (1981), whose minimum-Hellinger-distance functionals are distinguished by minimizing the LAM-bound and the bias, and whose minimum-$L^2(\mu)$-distance functionals is distinguished by minimizing the bias, over corresponding balls.

Helmut Rieder

(Bayreuth)

RISK THEORY

September 16 - 22, 1984

On convex premium calculation principles

A principle of premium calculation is a functional that assigns a number (= the premium) to any insurable risk (a random variable). Convexity seems to be a very desirable property, because it implies several other desirable properties (invariance under translation, loading, superproportionality). The Fréchet gradient is a natural tool to discuss problems such as the minimization of the total premium and the search of an optimal insurance contract.

O. Deprez & H. Gerber (University of Lausanne)

A characterization of the standard deviation principle

In practice and theory there are a lot of methods (principles) for calculating risk premiums. Practicioners use the expected value, the variance or the standard deviation principle according to how they choose the safety loading. Beyond this there are several other methods such as the exponential, mean value, zero utility, Swiss, Orlicz and Esscher principles. It will be proved that the standard deviation principle is the only one of all the methods above, which fulfills extremely weak versions of homogeneity (a fundamental property for proportional reinsurance) and sub-additivity (fundamental for general insurance business).

It is well known that the standard deviation principle is positively homogeneous, sub-additive and translation invariant. On the other hand one has for all other principles above

Thm. Given $\alpha_0 \neq 1$, $c_0 \neq 0$. If

$$(1) \qquad \pi(\alpha_0 X) = \alpha_0 \pi(X), \quad \pi(X + c_0) = \pi(X) + c_0$$

hold for all 2-point distribution X, then $\pi(X) = E[X]$ for all X, i.e. π is the net premium.

In the case of the zero utility principle one has to substitute equation (1) by a similar one.

Axel Reich (Cologne, Die Kölnische Rück)

Premium Rating

From a purely semantical point of view, there is little difference between premium calculation and premium rating. And, by the way, premium calculation and premium rating are concerned with the same ultimate problem: the amount of money to be collected from a policyholder. These two subjects, however, although they should be closely related to each other, are seldom brought together in actuarial literature. Premium calculation problems constitute a highly theoretical, elegant and mathematically well-developed chapter of actuarial mathematics – whereas premium rating has remained, quite surprisingly, somewhat untouched by theoretical investigation: there exists indeed no unified and coherent theoretical approach to the tariff-construction problem, and the concept of a "good" tariff itself remains quite undefined. Our purpose here is to make this notion conceptually clearer by means of a few basic, intuitively appealing, tariff principles. These principles result in a definition of a balanced tariff. An algorithmic method for obtaining a balanced tariff has been proposed in a previous talk.

Marc Hallin – } Institut de Statistique –
Jean-François Ingenbleek – } Université Libre de Bruxelles.

MARTINGALES IN LIFE INSURANCE

A general probabilistic model for a life insurance portfolio, including ~~dis~~ stochastic discounting, was presented. Random variables corresponding to the total discounted loss, annual loss and prospective reserve were defined. The loss process ~~process~~ was shown to be a martingale and this property was used to derive three upper bounds for the probability of ruin for the portfolio. These upper bounds were illustrated in two numerical examples.

Howard Waters (Edinburgh)

A short remark on the existence of a non-trivial zero of the cumulant generating function

Let X be a random variable with $EX \neq 0$. Define

$\psi(t) := \log(E(e^{tX})) \leq \infty$, $t \in \mathbb{R}$ the cumulant generating function of X

and $I := \{t \in \mathbb{R} : \psi(t) < \infty\}$ the domain of its existence.

A. WALD has shown the following

Theorem: $\left.\begin{array}{l} I = \mathbb{R} \\ P(X>0)>0,\ P(X<0)>0 \end{array}\right\} \Rightarrow$ there exists $t_0 \neq 0 : \psi(t_0) = 0$ (E)

We will see that this is not true if $I \neq \mathbb{R}$ and find necessary and sufficient conditions for (E) to hold in this case.

V. Mammitzsch (Marburg)

Bivariate Time Series Models for Total Fertility Rate and Mean Age of Childbearing

Annual distributions of age-specific fertility rates are normalized ~~&~~ and their means calculated. The normalizing constants are called total fertility rates (TFR's), and the means are called mean ages of childbearing (MACB's). Demographers keep track of these statistics, use them to help form population projections, and form hypotheses about their behavior. The study presented here was motivated by a demographer's hypothesis that annual changes in the TFR's and MACB's tended to be contemporaneously negatively correlated. A bivariate time series model was constructed for TFR's and MACB's that adjusted for the impact of World War II. The model confirmed the demographer's hypothesis, but it also revealed a dynamic structure that implied (a) feedforward from TFR to MACB but no feedback from MACB to TFR and (b) long-run adjustment of MACB in the <u>same</u> direction as a change in TFR, despite the initial, contemporaneous change in the opposite direction.

R. B. Miller (Madison)

I. RECURSIVE CALCULATION OF RUIN PROBABILITIES

II. " " " AGGREGATE CLAIMS FOR THE INDIVIDUAL CLAIMS RISK MODEL

I. For a compound Poisson process, the complement of the ultimate ruin time probability can be written as the distribution function of a compound geometric random variable. This fact is exploited to develop a recursive formula for the ultimate ruin probabilities when certain discretizations are used.

II. A recursive formula is given for developing the distribution of total claims for the life insurance portfolio with possible accidental death benefits. The logarithm of the probability generating function is expanded and truncated to give the approximations necessary to apply the recursive formula.

H. H. PANJER (WATERLOO)

The new Belgian bonus-malus system

Jean Lemaire
Université Libre de Bruxelles

The statutory tariff in motorcar third party liability insurance in Belgium was introduced in 1971. The lack of balance of the bonus-malus system has created severe financial problems to the insurance companies and prompted them to appoint a study group, whose main task is to recommend a new tariff structure to the control authorities. Using, as main tools,
- the stationary premium income, evaluated by a simulation program
- the fairness of the tariff to the policyholders, measured by the efficiency of the bonus-malus system,

and - the magnitude of the hunger for bonus,

it is shown how a multicriteria analysis has enabled the study group to recommend a new bonus-malus system.

Choice of Statistic in Linear Bayes Estimation.

Walther Neuhaus
Storebrand - Norden Ins. Co.
Oslo - Norway.

The question on what statistic to base a linear Bayes (Credibility) estimator is discussed in a general setting. Decisions are based on the relative risk reduction obtained by a secondary statistic in the presence of a primary statistic. The ~~E~~ empirical procedures are justified by asymptotic results.

Premium Calculation for the simple risk model

Let the number of claims up to time t be denoted by $N(t)$ and generated by a renewal process; the claims come independently from a common distribution F.
Let $G_t(x)$ be the probability that the total claim amount at time t does not overshoot the level x. For a number of explicit choices of $\{N(t), t \geq 0\}$ and F we find asymptotic expressions or upper bounds for $1-G_t(x)$ when t is fixed but x is large. These bounds can then be used to find a function $P_\varepsilon(t)$ for which $1-G_t(u+P_\varepsilon(t)) = \varepsilon$ which can be interpreted as an appropriate premium scheme.

Jef L Teugels

A characterization of the class of credibility matrices corresponding to a certain class of discrete distributions.

M. Goovaerts - F. De Vylder

Recently (De Vylder & Goovaerts IME $\underline{3}$, 3, pp 179-188) so-called credibility matrices have been introduced and studied in the framework of general properties of matrices such as nonnegativity, total positivity etc. In the present note we characterize a class of credibility matrices generated by the normed sequence of functions $(p_1, p_2, \ldots, p_n)$ on $K = [a, b]$ where $p_i(\theta) = f(i) g(\theta)^i h(\theta)$ $i = 0, \ldots, n$, $\theta \in K$ and where f g h are nonnegative (eventually depending on n, n may be finite or infinite). For simplicity we suppose h to be monotonic and continuous.

M. J. Goovaerts

The structure of the distribution of a couple of observable random variables in credibility theory.
DE VYLDER & GOOVAERTS.

We consider Bühlmann's classical model in credibility theory and we assume that the set of possible values of the observable random variables $X_1, X_2, \ldots$ is finite, say, with n elements. Then the distribution of a couple (X_r, X_s) $(r \neq s)$ amounts to a square real matrix of order n, that we call a credibility matrix. In order to estimate credibility matrices or to adjust roughly estimated credibility matrices, we study the set of all credibility matrices and some subsets of it.

Finite-time ruin models with discounting and experience rating perturbed by a Brownian motion
ABIKHALIL F.
University libre de Bruxelles.

We consider two risk models:

i) A discounting one, when the income process (i.e. premium - claims) is a Brownian motion with drift.

ii) A generalization of a risk process under experience rating when the aggregation of claims up to time t is a diffusion process.

We prove, for both models, that the distribution of ruin time before t is equivalent to the distribution of the first passage time of Brownian motion for parabolic boundaries.

Using Wald's identity for Brownian motion, (pointed out by Shepp 1967) we give an explicit formula for the density of the finite-time ruin

Entropy estimates for ruin probabilities.

Anders Martin-Löf, Folksam ins. co. Stockholm Sweden

The classical problem of risk theory is reviewed in the light of "large deviation" theory. I.e. a random walk $\{S_n\}$ with negative drift is considered, and one wants to study the distribution of the first passage time, $N(c)$, to a high level $c>0$. It is shown that the well known limit theorems saying that $P(N(c)<\infty) \approx K e^{-Rc}$, and $P\big((N(c)/T - c)/\sqrt{c} \le x\big) \approx K e^{-Rc}\,\Phi(x/\sigma)$ as $c \to \infty$ can be derived using large deviation estimates in path space. These are based on the fact that there is an asymptotic "density" in path space of the form: $P(S_{ct}/c \approx x(t),\ 0 \le t \le T) \approx \exp \int_0^1 h(x'(t))\,dt$, where $h(x')$ is the entropy function of the distribution of S_1. This has as a consequence that R and T are defined by the maximum problem:
$-R = \max_t t\,h(1/t) = T\,h(1/T)$, and that the distribution of $N(c)/c$ is concentrated near T, and the above results can be derived using estimates based on these ideas. AM

This method allows direct generalization to a time dependent barrier, and state dependent drift of the random walk.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Inversed martingales in risk theory (F. Delbaen and J. Haezendonck)
presented by J. Haezendonck
University of Antwerp. Belgium

Let S_t $(t \in \mathbb{R}_+)$ be a classical risk process, i.e. $S_t = X_1 + \dots + X_{N_t}$ where N_t $(t \in \mathbb{R}_+)$ is an homogeneous Poisson process, and let $k + pt - S_t$ be the surplus process with $k \geq 0$. Using an inversed martingale argument we extend a result on ruin probabilities due to H. Cramer. More precisely we find an expression for the probability of no ruin before time t when the initial reserve $k \geq 0$, and we deduce several bounds on this probability.

Efficient estimation in the presence of incidental parameters
Christian Hipp, University of Cologne

For observations X_{ij}, $1 \leq i \leq N$, $1 \leq j \leq n$, the multiplicative model $X_{ij} = u_i v_j + \varepsilon_{ij}$ is used, where u_i, v_j are positive and the errors are iid standard normal. Keeping n fixed and letting N tend to infinity, the maximum likelihood estimator for $v_2, \dots, v_n$ is asymptotically efficient in the sense of Hajek's asymptotic local minimax property. These estimators are used to get reasonable estimators also in the IBNR observation scheme under the same model assumptions.

Christian Hipp

Duality results for positive and negative capital risk models

Jacques JANSSEN
Université Libre de Bruxelles

With a slight modification of the definitions of the ruin event, it is shown that any result for a positive capital risk model concerning the probability of ruin immediately gives a similar result concerning the associated negative capital risk model and vice-versa.

A multistep procedure to approximate the aggregate claim density

B. Ströter
Kölnische Rückversicherungsgesellschaft.

It is shown how recursive standard methods for solving Volterra Integralequations numerically apply to gain approximate values of aggregate density function. It is proven that recursion schemes are stable against local perturbation.

Maximum likelihood estimation in parametric counting process models with application to insurance.

It was described how the set-up for life data with left truncation and right censoring can be reformulated in a counting process framework. A brief outline was given on how counting processes, martingales and stochastic integrals can be used to derive the properties of the ML-estimator. Extension to more complex life history data was pointed out, as was extensions to regression to regression type models. The relevance of these results to insurance applications was indicated

Ørnulf Borgan (Oslo)

Finite formulae for the premium of the general reinsurance treaty based on ordered claims.

The calculation of a risk adequate premium is one of the main problems in nonproportional reinsurance. In the past 20 years several actuaries developed methods for rating. Only recently the author reexamined the problem of calculating the premiums for the largest claims reinsurance cover and the ECOMOR treaty and derived asymptotic formulae for the net premiums. In an additional paper the asymptotic results were generalized to a very large class of reinsurance treaties containing the LCR and ECOMOR covers.

Because of it's asymptotic nature the resulting formula approaches the correct premium only for large collectives. As a consequence the question arises, whether it is possible to give a general premium formula for the general treaty, which is exact for each finite collective. In the present paper such finite formulae are presented. The proofs are based on well-known formulae for the densities of the distributions of couples of order statistics.

Erhard Kremer (Hamburg)

ROBUST CREDIBILITY

STUART KLUGMAN, UNIVERSITY OF IOWA

The basic credibility model assumes that data is collected from k populations where the population means may be considered as a random sample from some probability distribution. Suppose this distribution is heavy-tailed to the point that the grand mean of the k sample means is not an efficient estimator of the prior mean. In that case a robust estimator of this mean would be preferred. In particular, it should not be sensitive to the form of the distribution so that restrictive assumptions need not be made. This paper investigates the use of M-estimators as introduced by Huber (1964). The net effect is likely to be an increase in the credibility factor and a decrease in the weight given the estimate of the prior mean. Simulation studies are recommended to determine the degree of reduction in mean squared error.

Hitting distributions of Brownian motion and random walks on curved boundaries

The lecture gives a survey about boundary crossing results for Brownian motion and random walks. The emphasis lies on the tangent approximation. Let $\psi(t)$ be a given curved boundary and let $T = \inf\{t > 0 \mid W(t) \geq \psi(t)\}$. The density f of the distribution of T can be approximated by the density at the accompanying tangent of ψ at t. It is given by $\frac{\Lambda(t)}{t^{3/2}} \varphi\left(\frac{\psi(t)}{\sqrt{t}}\right)$ with $\Lambda(t) = \psi(t) - t\psi'(t)$. Refinements of the tangent approximation lead to very exact approximations of f. The relation of our approach to classical fluctuation theory of Brownian motion is explained.

Hans Rudolf Lerche (Heidelberg)

An empirical study of claims arising from water damages in a portfolio of comprehensive policies for single family houses and

In a portfolio of comprehensive property insurance policies for single family houses and blocks of flats, an analysis of the losses due to claims arising from water damages has been carried out. In a model perscribing Poisson-distributed claim numbers, the claim frequency is found to be dependant on the size of the building, measured by floor area, in a log-log-linear manner. This dependency is the same for the blocks of flats as for the single family houses, whereas for the latter type there is a further dependency on the number of floors. Additionally it is shown, that the introduction of an extended cover for a new claim type, damage to pipes, has resulted in a change in the level of the claim frequency of water damages, such that the frequency is dependant on the coverage of the policies also.

Frank Cederbye (Copenhagen)

A Case Study about Subjective Probabilities and Moral Hazard.

The concept of subjective probabilities and the problem of moral hazard are investigated for the case of a consumer who uses a car during one year. Especially the following question is treated: Is it possible to avoid completely the moral hazard effect, and how could this be achieved?

Fritz Riebesel, Räterschen (Switzerland)

Repeated measures: its applications to insurance

Whenever an insurance company has only data on a short period and it observes a change in the tendency, it has interest to take into account similar data of other companies to detect if the change is significant. We indicate how two models of repeated measure design completed by adequate post-hoc comparisons can give adequate information.

José PARIS (Louvain-la-Neuve)

The Power-Ratio-Gamma Distribution

A flexible Pareto-tailed distribution is analysed and put forward as a tool to graduate claim size data, especially when exceptional claims are present. The density can be derived by considering a power transformation of the ratio of two independently distributed Gamma random variables. This distribution should provide an input for a coherent framework for stable estimation of the population mean, econometric trend analysis as well as equalization of results.

Peter ter Berg (Amsterdam)

Observation - Dependent Credibility Weights

Within the framework of credibility theory we propose several different models which can cope with outliers and misspecification of the prior mean. These models lead to a posterior mean which is a weighted average of the observations and of the prior mean, the weights (credibility factors) now depending upon the observations.
The talk is based on a common work with William S. Jewell

René Schnieper, Winterthur

Investment Policies and Reinsurance for Pension Funds

A pension fund typically faces two types of risk. In addition to the actuarial risk, there is an investment risk stemming from the stochastic nature of the rate of return on reserves. These risks depend on the level of reinsurance and the investment policy chosen by the pension fund. The application of Borch's theorem and a result on "mutual funds" make it possible for the optimal level of reinsurance and the optimal investment policy to be determined simultaneously. In particular, it turns out that a low level of reinsurance should never be combined with a cautious investment policy. In addition the paper shows how elements of capital and risk-theory can be combined in one model.

Heinz Müller (Zürich)

Transformations of distributions

Well-known transforms of life time or claim size distributions (Lorenz curve, Total time on test transform) are introduced and used to give connections between reliability theory and risk theory. An ~~example~~ application to exposure rating is given.

Wolf-Rüdiger Heilmann, Universität Hamburg

Premium Calculation

Consider the recursive equation of surplus R_t

$$R_t = (1+i_t)R_{t-1} + \Pi_t - S_t \qquad t = 1,2,3,\dots$$

where Π_t ~ annual net premium
$S_t\,(\geq 0)$ ~ annual claims

These quantities must be understood for the whole insurance company

without interest on surplus the postulate $R_t \geq 0$ for all t leads to the exponential principle (approximately the variance principle) for the determination of Π. The resulting formulae can be used for the individual risks as well because these two principles are additive

If one requires that total premiums also should finance dividends to the lenders of initial capital the total premium follows the standard deviation principle. This can however not be used to distribute total premiums among the individual risks.

It is remarkable that in a model where interest to surplus is also considered and where $R_t \geq 0$ $t = 1, 2, \dots$ is replaced by $R_t \geq -\frac{\Pi}{i}$; $t = 0, 1, 2, \dots$ (Latinamerican Ruin) the structure of premium rating formulae remains the same

Hans Bühlmann, Zürich

Approximate evaluations of the d.f. of aggregate claims

The idea of the approximation is first to transform the target d.f. $F(X)$ by means of some suitable function $y = v(X)$ in a shape

$$\bar{F}(y) = F(X)$$

which is (at least approximately) symmetric. Then it can be successfully approximated by the normal d.f.

$$\bar{F}(y) \approx N(y)$$

A polynomial (NP-approximation = Cornish Fisher) and a power expression suggested by Haldane (1938)

$$y = (X/m_X)^h$$

were examined and tested. The results (presented by tables and graphs) proved promising and ~~the~~ Haldane transformations seems to have some merits compared with traditional approaches. It is best for large collective, where the exact formulae (recursive, FFT) are becoming inconvenient.

1984-09-21, Teivo Pentikäinen, Helsinki

Recursive formulas for compound difference distributions

Since the generating function of $S = X_1 + \cdots + X_N$ is the composition of those of N and of X,

$$G_S(t) = G_N(G_X(t)),$$

the task of finding G_S becomes the task of executing G_N of a power series. When $N \sim \text{Poisson}(\lambda)$, computing $d \ln G_S(t)/dt$ leads to

$$G'_S(t) = \lambda\, G'_X(t)\, G_S(t)$$

the t^n coefficients of which declare

$$(n+1) f_{n+1} = \lambda \sum_{k=0}^{n} p_{k+1} f_{n-k}$$

There are other N's (e.g., binomial and negative binomial) for which G_N of a power series is easy to execute. When G_N is not in closed form but $n_k = \Pr\{N = k\}$ satisfies a difference equation (e.g., generalized Waring), the difference equation can be lifted to a difference equations on $f_k = \Pr\{S = k\}$. This presentation is derived from a paper to appear in *Transactions, Society of Actuaries.*

Beda Chan, London, Canada

Combinatorial Geometry

A. Dress (Bielefeld) 23.-29.9.84 J. M. Wills (Siegen)

24. bis 28. 9. 1984

STRUCTURAL TOPOLOGY / TOPOLOGICAL GEOMETRY

In scene analysis (the reconstruction of solid objects, given one single plane projection) and in mechanics (the rigidity of bar-and-joint structures, panel-and-hinge structures, etc.) we find problems which lead to a topological theory of objects which are more rigid, less pliable, than the "rubber sheets" studied by the topology ~~sto~~ of Henri Poincaré and his school. We present the first theorems of this new area of study.

In particular, we show how to extend the geometric rank function r, initially defined only for subsets $A \subseteq P$ of a set P of points in a geometric configuration G, to a <u>valuation</u> χ on the distributive lattice D of antichains of 2^P ($\simeq$ lattice of order ideals of 2^P $\simeq$ lattice of simplicial complexes on P). To carry out this extension, we use the <u>Möbius function</u> μ_E on 2^P <u>relative to an antichain</u> E, defined recursively by

$$\sum_{A \subseteq B} \mu_E(B) = \zeta_E(A) = \begin{cases} 1 & \text{if } A \subseteq C \text{ for some } C \in E \\ 0 & \text{otherwise} \end{cases}$$

Note that $\mu_E(A) = 0$ unless A is an intersection of sets in the antichain E. Then the <u>characteristic</u> χ of the geometry G has the value on each antichain E given by the formula

$$\chi(E) = \sum_{A \in L_E} \mu_E(A)\, r(A).$$

where L_E is the semilattice of intersections of parts of the antichain E.

The characteristic χ of a geometry generalizes the classical Euler characteristic: simply take G to be the rank 1 geometry where all points in P are coordinatized by scalar multiples of a single vector v, and the resulting valuation is generated by a function on sets which has value 1 on nonempty sets, value 0 on the empty set.

Associated with this more general characteristic is a homology theory capable of measuring phenomena such as geometric special position, degrees of freedom of linkages, etc.

Henry Crapo
INRIA, Rocquencourt, France

Polyhedral 2-manifolds in E^3 with few vertices

The minimal number f_0 of vertices of a polyhedral triangulation (i.e. one made up of plane triangles) $M \subset E^3$ of the closed orientable surface of genus g is only known for $g \leq 3$. Of special interest are those values of g for which a minimal polyhedral triangulation with complete 1-skeleton might exist, the next open case being $g = 6$, $f_0 = 12$. One method for realizing these cases in 3-space is to embed M ~~as~~ a subcomplex into the boundary complex of some neighborly 4-polytope P, and then project it down into a Schlegel-diagram of P. Strong necessary conditions for P to admit $M \subset skel_2 P$ are given

Christoph Schulz, Hagen / Siegen

Platonic manifolds

We call a compact polyhedral 2-manifold in euclidean E^3 a Platonic manifold if it has a Platonic rotationgroup, if its faces are (not necessarily convex) p-gons, if its vertices are q-valent and if a group isomorphic to the complete Platonic symmetry group acts transitively on its vertices or faces. Notation: $\{p, q; g\}$, g: genus.
We show the 12 known examples by Grünbaum, Shephard, McMullen, Schulz, Wills. These are perhaps the only ones, each three of genus $g = 3, 5, 7, 11$.
We also show 6 combinatorially regular polyhedra in E^3 with nontrivial symmetry groups which occured in papers of Bode-Stott, Coxeter, McMullen-Schulz-Wills (and other papers).

J. M. Wills (Siegen)

Triangulating the d-cube

Let $I = [0,1]$ be the closed unit interval, and consider the unit d-cube $I^d \subseteq \mathbb{R}^d$. Suppose you have a triangulation of I^d into a simplicial d-complex, without introducing any new vertices. What is the minimum possible number, $T(d)$, of d-simplices that is achievable? We outline what is presently known about this problem, sketch a new proof that $T(4) = 16$, and discuss some general methods of triangulating convex d-polytopes, and their implications for this problem.

Carl W. Lee, Bochum

Grassmann-Plücker-Relationen 4. Grades am Beispiel kombinatorischer Sphären.

Eine algorithmische Bestimmung aller kombinatorischen Typen von konvexen d-Polytopen fester Eckenanzahl läßt sich auf das Problem zurückführen, bei vorgegebener komb. Sphäre algorithmisch zu entscheiden, ob sie polytopal ist. Ein Algorithmusansatz wird am Beispiel der Altshulersphäre M^{10}_{416} diskutiert. Dadurch wird ein Problem von M.A. Perles (Oberwolfach, Juli 1984) positiv beantwortet.

Satz (J.B.) Es gibt ein 2-benachbartes 4-Polytop, das keine universelle Kante besitzt. (M^{10}_{416} ist polytopal)

Damit ist das Shemer'sche Näherverfahren nicht immer anwendbar.

Satz (J.B./K. Garms) M^{10}_{425} (gleiche Eckenfigur) ist nicht polytopal.

Literatur: Altshuler, Can. J. Math. 29, 2 (1977) 400-420

Jürgen Bokowski, Darmstadt

Die quasireguläre Polyeder zweiten Geschlechtes der Stufe größer als zwei

Wenn in dem Eckenzyklus $(\overset{1}{m,n}, \overset{2}{m,n}, \ldots, \overset{s}{m,n})$ eines quasiregulären Polyeders s alternierende Paare von zwei verschiedenen natürlichen Zahlen vorkommen, nennen wir diese Anzahl s die „Stufe" des betreffenden quasiregulären Polyeders. Die beiden klassischen elementaren quasiregulären Polyeder sind also zweiter Stufe. Von verallgemeinerten (topologischen und affinen) Polyeder des zweiten Geschlechtes sind alle quasiregulären Polyeder zweiter Stufe bekannt. Es werden konstruiert fünf weitere quasireguläre Polyeder zweiten Geschlechtes von welchen vier von dritter Stufe sind und eines von vierter Stufe. Damit sind alle quasiregulären Polyeder zweiten Geschlechtes gegeben.

Stanko Bilinski (Zagreb)

Triangulated tori, the expanded simplex, and crystallographic groups

We call a combinatorial manifold 2-neighborly if for any pair of vertices the edge joining them belongs to the triangulation.

THEOREM: For any dimension d there exists a 2-neighborly combinatorial d-torus ($\simeq \mathbb{R}^n/\mathbb{Z}^n$) with $n := 2^{d+1}-1$ vertices. Its automorphism group of order $2\cdot(d+1)(2^{d+1}-1)$ acts transitively on the set of vertices.

Its universal covering is a subdivision of the tessellation of d-space by translated duals of the expanded simplex $e\alpha_d$ (Coxeter's notation), and the automorphism group appears as the quotient of two crystallographic groups preserving this tessellation. With a certain $\mathbb{Z}_n$-labeling of the n vertices of the ~~quotient~~ d-torus this group is isomorphic to the group of affine $\mathbb{Z}_n$-transformations generated by $x \mapsto x+1$, $x \mapsto -x$, $x \mapsto 2\cdot x$.

For $d=2$ we just get the 7-vertex torus with its automorphism group of order 42.

Wolfgang Kühnel (TU Berlin)

Distance Geometry & Macromolecular Structure

A new method for determining the structure of biological macromolecules in solution is described. The experimental basis of the method is a spectroscopic technique known as nuclear magnetic resonance, which yields a large number of short, imprecise distances between protons, together with estimates of selected dihedral angles. From these imprecise measurements of local geometric parameters a complete molecular spatial structure must be constructed. Our computational methods for doing this a combinatorial in nature. First, a complete set of lower & upper bounds on all distances must be extrapolated from those distance limits which were measured directly. This is done with a shortest-path type calculation. Next, a random metric space is chosen within these bounds, and coordinates (3D, Cartesian) obtained from these by a projection method. Finally, these coordinates are refined by a nonlinear optimization vs a penalty function of their deviations from the distance & angle bounds. Finally, some problems arising in the description of nonrigid molecules are given.

Tim Havel
Universität Bielefeld

Greedy Sequences and Slope Critical Sequences

Robert E. Jamison, Clemson Univ., SC (z. Z. Freiburg)

A slope-critical configuration is a set of n pts in the plane $\mathbb{R}^2$ which determines only n-1 slopes – i.e., directions of connecting lines. (P. Ungar [JCT, 1982] has shown that n-1 is the minimum.) For some time I have been interested in classifying and, where possible, in finding limitations to the structure of such configuration. This talk reports on this project with several such limitations discussed. General constructions show that these limitations do not hold in the weaker setting of oriented matroids of rank 3.

Motions in Frameworks

Joint work with Walter Whiteley on infinitesimal motions in bar-and-body frameworks is first surveyed. Then recent progress on hinge-and-body frameworks is described. In both cases, rigidity or non-rigidity of such frameworks is determined in terms of Cayley (or Grassmann) algebra and determinantal algebra. Nicer results, both combinatorial and algebraic, are obtained for these types of frameworks in all dimensions than are possible for the more widely studied bar-and-joint frameworks. A digression is then pursued on the topic of matroids satisfying certain counting conditions, of which the structural matroid for bar-and-body frameworks is an example. In these matroids defined on the edges of hypergraphs we state a "pruning theorem" which allows us to replace large edges in an independent set by small ones. This theorem is a generalization of well-known matching theorems, and we give an application to frameworks.

Neil L. White, University of Florida

On geometrical presentation of discrete transformation groups in the sense of Poincaré

Emil Molnár, Eötvös L. University, Budapest

Poincaré proposed a method for describing isometry groups acting discontinuously on spaces of constant curvature. Such a group G is given by a fundamental polyhedron F whose faces are identified by isometries generating the group G. The so-called edge condition for the equivalence classes of segments guarantee the discontinuous action of G on the space M and they serve a complete set of relations for the presentation of G. We can applicate the method in various directions, e.g. the minimal presentation of (euclidean, hyperbolic, spherical) space groups by fundamental topological polyhedra (with minimal number of curved faces) can be of interest. There are intensive investigation in the hyperbolic space to guarantee the free action of G on M by criteria for F because then the factor space M/G will be a hyperbolic space form. Some examples will be presented, connections with well-known groups, e.g. Coxeter groups will be mentioned. 27.09.1984. Molnár

NEW REGULAR POLYTOPES AND APEIROTOPES

The symmetry group of a regular polytope or apeirotope (regular, that is, in the general sense proposed by Grünbaum) in some euclidean space is generated by reflexions (involutions) $R_0, R_1, \ldots, R_{n-1}$, say, where R_i and R_j commute if $|j-i| \geq 2$. Conversely, given such a group of isometries, one may be able to construct from it a regular polytope or apeirotope. Various methods, some old and some new, were described, which enable one to construct new regular polytopes or apeirotopes from old ones. In particular, any two (or more) may be combined by mixing; for example, any regular polygon is a mixture of planar polygons. While this leads to many examples, it also suggests that one should search among the regular polytopes and apeirotopes of a given isomorphism class for those that are unmixed —

these are the primitive realizations of that class. Among more specific examples, the description of the apeirotopes of rank 4 in $\mathbb{E}^3$ completes the classification of all the possible regular polytopes and apeirotopes in ordinary space.

Peter McMullen, London.

A GEOMETRIC CURIOSITY CONCERNING POLYHEDRAL 2-MANIFOLDS

Let $\mathcal{P}$ be a polyhedral 2-manifold in E^3, i.e. a geometric cell-complex whose underlying point-set set $\mathcal{P}$ is a closed connected 2-manifold in E^3. A vertex v of $\mathcal{P}$ is called convex if, and only if, at least one of the two components into which set $\mathcal{P}$ divides a sufficiently small ball centred at v is convex. Let $n(g)$ denote the minimal number of non-convex vertices of polyhedral 2-manifolds in E^3. Solving a problem of Barnette we have $n(g) = 5$ $(g \geq 1)$. This result is a joint work with U. Betke.

Peter Gritzmann, Siegen

Problems in Discrete Geometry. At the Discrete Geometry week, Oberwolfach July 1977 I presented 14 unsolved problems posed by my brother Leo Moser (1921-1970). Since then this collection has grown substantially and has been periodically revised and widely distributed: the 1981 edition contained 68 problems and was mailed to 500 mathematicians. Now I present (in written form) problems 1-50 of the 1984 edition. Copies will be mailed to all participants of this meeting; copies of problems 51-100 will be mailed later this year. I welcome information on these problems, new problems and requests for copies of the 1984 edition.

William Moser (Montreal) Sept. 27, 1984

POINT SETS WITH MANY UNIT CIRCLES

Consider n points in the plane. Each triple of points may determine a circle. What is the maximum number $K(n)$ of congruent circles, say unit circles? For this problem of P. Erdős it is known $cn \leq K(n) \leq \frac{n(n-1)}{3}$. The exact values $K(n) = 1, 4, 4, 8, 11$ for $n = 3, 4, 5, 6, 7$ are determined, which is tedious for $n=5$ and $n=7$.

Heiko Harborth, Braunschweig

Über einige reguläre Inzidenzpolytope, deren Automorphismengruppen die $PSL(3,2)$ als Normalteiler enthalten

Es wird das reguläre Inzidenzpolytop $P := \{3,8\}_8 \in \begin{pmatrix} 2 & 2 & 2 \\ & 6 & 16 \\ & & 672 \end{pmatrix}$ näher untersucht. Für die Automorphismengruppe $A(P)$ gilt nach COXETER (Trans. AMS 45 (1939))

$$A(P) \cong C_2 \times PGL(2,7), \text{ also auch}$$
$$\cong C_2 \times (C_2 \cdot PSL(3,2)) \cong V_4 \cdot PSL(3,2).$$

Durch Färbung der Dreiecke von P (56 weiße, sowie je 8 farbige in 7 Farben) wird ein Isomorphismus zwischen der Kollineationsgruppe der FANO-Ebene und dem o.g. Normalteiler von $A(P)$ hergestellt. – Die Dreiecke jeder einzelnen Farbe bilden dieselbe Konfiguration wie die acht Dreiecke des Kuboktaeders. Demgemäß wird ein Modell von P in $\mathbb{E}^3$ konstruiert, das unter der Würfelgruppe invariant ist. Es besteht aus einem archimedischen und sechs weiteren, untereinander kongruenten, konvexen Kuboktaedern, die zum ersten allerdings nur kombinatorisch isomorph sind.

Dualisierung, Anwendung der PETRIE-Operation und „Facetting" liefern eine Reihe von mit

P verwandten Inzidenzpolytopen.

L. Danzer, Dortmund

SIMPLICIAL SCHEMES. A simplicial scheme is a certain structure defined on graphs. The purpose of this concept is a graph-theoretical description of simplicial complexes. Graphs and simplicial schemes give rise precisely to simplicial pseudocomplexes which are homogenous, every simplex of codimension 1 is contained in 1 or 2 top-dimensional simplices and the open star of each simplex is strongly connected. There is a close relation between graph covering maps and nonsingular simplicial maps between corresponding simplicial complexes. This gives us some nice applications.

B. Mohar, Ljubljana.

Tiling the Torus and other space forms. Let Γ be a normal isohedral, isogonal or isotoxal tiling of E^2 with maximal symmetry for its graph. Since we can identify its symmetry group with the automorphism group of the graph, Aut Γ is a 2-dimensional crystallographic group. ~~~~ Let F be a fixed-point-free subgroup of $E(2)$. If $F \subseteq \mathrm{Aut}\,\Gamma$ then $\gamma = \Gamma/F$ is a graph on the space form E^2/F which has the same local properties as Γ but may not be transitive: Aut γ acts transitively on γ iff the normalizer N of F in Aut Γ acts transitively on Γ. An algorithm for computing subgroups of crystallographic groups enables us to determine N, F, and N/F, and thus classify and enumerate transitive graphs on E^2/F. Applications to combinatorial problems in 2 dimensions, and extensions to higher dimensions, are discussed.

M. Senechal, Northampton Mass.

POINT SETS WITH INTEGRAL DISTANCES

A point set of cardinality n in the euclidean plane is said to be an integral n-gon if the points of the set have mutual integral distances. The diameter of the n-gon is defined as the maximum distance.
For any n you can find integral n-gons not all points in line, but there does not exist in the plane an infinite set of noncollinear points with all mutual distances integral (Anning, Erdős 1945).
Some assertions about minimal diameters of integral n-gons with certain properties are created. E.g., 73 is the minimal diameter of a 5-gon with the properties that no three points are collinear and no four are on a circle.

Arnfried Kemnitz, Braunschweig

EQUIVARIANT TESSELLATIONS OF $\mathbb{E}^n$ ($n \leq 4$) IN SOME QUANTUM MECHANICAL PROBLEMS. We start from the equivariant spectral problem for the Hodge Laplacian in the L^2-space of differential forms over $\mathbb{E}^n$, with the group of affine orthogonal transformations as symmetry group. We approximate the problem by one in the l^2-space defined over a tessellation of $\mathbb{E}^n$. The possible choices will differ in the discrete symmetry left over, and in the number of degrees of freedom compared to the dimension $D = 2^n$ of the exterior algebra. We may ask the tessellation to admit as symmetry group the symmorphic space group corresponding to a maximal finite subgroup of $Gl(n, \mathbb{Z})$, in order to keep "as much as possible" of the original continuous symmetry. The degrees of freedom for the "discrete" Hodge Laplacian are counted by the dimension of its space of forms, i.e. by the orbits of k-faces ($k = 0, 1, \ldots, n$) of the equivariant tessellation under lattice translation. Their number D_Γ depends on the group Γ and on the tessellation. Since $D_\Gamma \geq 2^n$, we would like to take, for a given Γ, those Γ-equivariant tessellations which give the smallest value of D_Γ. We discuss the combinatorics of the construction and spectral analysis of discrete Hodge Laplacians which meet the above requirements

Hans Rummler, Bern

Designing and Describing a Spherical Polyhedron in 3-space.
We present the equivalent problems of (i) making a realizable sequence of choices in the design of a spatial polyhedron and (ii) giving a minimal set of measurements of a polyhedron to uniquely determine the object, at least locally. A number of classical theorems give very partial results: Steinitz' Theorem (the realizability of 3-connected planar graphs), theorems of Cauchy and Alexandrov (the local uniqueness of 3-connected spherical polyhedra with a set of edges triangulating all faces) etc.. Some other old and new results about the statics and mechanics of frameworks can also be applied: Maxwell's theorem connecting plane stresses and projected polyhedra gives results on the dependence of dihedral angles, the rigidity of 4-connected spheres with one edge replaced by a dihedral angle, etc. We survey these results, and describe some

recent algorithms on bar and joint frameworks in space which throw ~~insight~~ light on these problems.

Walter Whiteley, Montreal Canada.

A computer-implemented Method for the description of Molecules and the Perception of their Symmetry

Molecular structures can be described in the form of finite relational systems. The atoms of a molecule are represented by number labels and the structural relationships among the atoms by the tuples of several relations on the set of labels. These relations are collected in blocks, called Identification, Connectivity, Angularity, Dihedrality and Orientivity.
A canonization procedure (renumbering and ordering of the relational system) eliminates all arbitrariness of the description; the resulting unique description is taken as the structural name of the molecule. The canonization algorithm also finds classes of equivalent numberings (automorphisms and enantiomorphisms) which characterize a symmetry group of the described molecule.
This method has been implemented in a computer program called ONOMA. The program leads to unique descriptions of molecules and to the perception of their symmetry.

Philipp Flersheim, Zürich.

Hyperbolic Coxeter Groups

The study of hyperbolic Coxeter groups is equivalent with the study of hyperbolic polyhedra the dihedral angles of which are natural, i.e., of the form $\frac{\pi}{p}$, $p \in \mathbb{N}$, $p \geq 2$. We present several constructions of such polyhedra in hyperbolic 3-space. They all can be described in projective space (equipped with a suitable quadric) as configurations of 6 planes $H_0, \dots, H_5$ such that $H_i \perp H_j$ for $j \neq i-1, i, i+1$ (indices mod 6). The set of all these configurations with natural dihedral angles can be classified: There exist infinite families in dimension 3, but only finitely many analogous configurations in higher dimensional hyperbolic spaces.

Hans-Christoph Im Hof, Basel

Über die Symmetrien der Packungen und der Überdeckungen

Ein altes Problem der Überdeckungen war **die dünnste** Überdeckung eines Kreises **durch** fünf (bzw. sechs) kongruente Kreise. Im ersten Teil des Vortrags wir geben diese Extremalkonfiguration an, und betrachten noch auch hier einige ähnliche Probleme für das ~~gleichsei~~ reguläre Dreieck und das Quadrat.

Ein öfter behandeltes Problem der Geometrie der **Zahlen** und der diskreten Geometrie ist die Angabe **der** dichtesten Kugelpackungen in konvexen Körpern im $\mathbb{R}^d$. Hier beschäftigen wir uns mit diesem Problem und suchen im Tetraeder, Oktaeder und Würfel ($\subseteq \mathbb{R}^3$) dichteste Kugelpackungen.

Im dritten Teil des Vortrags ist die Rede über ein Problem von L. Fejes Tóth. Wir karakterisieren eine Men-

ge von Schnittpunkten der gegebenen kongruenten Kreisen, die entweder regulär oder überall dicht ist (in der Ebene).
Zum Schluss betrachten wir eine "reguläre" Aufgabe, wobei die Hauptrolle das reguläre Dreieck spielt.

Károly Bezdek, Budapest

UBIQUITOUS DISTANCES, angles and areas.

Let $x_1, x_2, \ldots, x_n \in E_d$. The distance $d = \overline{x_i x_j}$ is not ubiquitous if it occurs only $o(n^3)$ times. DISTANCES ARE ubiquitous in E_6 but not E_5, an old result of Paul Erdös and areas are ubiquitous in E_6, but not E_5, ~~also~~ an old result of mine. A new result is that acute angles are ubiquitous in E_6, but not E_5, and right angles are not ubiquitous in E_k for any k. We discuss some generalizations to solid angles, giving some results and conjectures. For example solid angles are not ubiquitous in E_3 but are in E_8. Presumably they are not ubiquitous in E_7.

George Purdy, College Station
Texas.

Some applications of generalized Schläfli-symbols

As one application of the general theory of „chamber systems of tesselations", outlined by A. Dress in his talk, the following is proved.

Theorem. There exist precisely 37 combinatorial types of equivariant tilings (T, Γ) of the Euclidean plane such that the group Γ has exactly 2 orbits on the vertices, edges and tiles, respectively.

A drawing of each of these tilings is presented.

R. Scharlau

Polystroma of type {6,3,3}

We identify vertices of a hyperbolic honeycomb {6,3,3} so as to obtain a "naturally generate" 3-polystroma (a partially ordered set with least element 0 and chains of length at most 3) $\{\{6,3\}_{bc}, \{3,3\}\}$ with toroidal facets of type $\{6,3\}_{bc}$ and tetrahedral vertex figures. Naturally generated means that the polystroma is build step-by-step from disjoint copies of facets identifying elements only as directed by the vertex figures. We discuss the symmetry group of the polystroma and give the list of all known polystroma of this kind.

Asia Ivić Weiss, Toronto

Tiling space by isomorphic polytopes

We discuss the following problem of Ludwig Danzer: given a convex 3-polytope P, is there a locally finite (face-to-face) tiling of 3-space by convex polytopes all isomorphic to P?

Egon Schulte, Dortmund

On weakly neighborly maps

Def.: A weakly neighborly polyhedral map (wnp map) is a 2-dimensional topological cell complex which decomposes a compact connected 2-manifold without boundary such that the 2-cells are closed topological discs and the meet of any two cells is connected and for any two vertices there is a 2-cell containing them.

We give a complete list of all (combinatorial types of) wnp maps on 2-manifolds (orientable or non-orientable) of Eulercharacteristic χ with $|\chi| \leq 2$ and investigate which of these wnp maps can be realized in E^3 with convex planar facets.

We show that for each 2-manifold except the sphere there are only finitely many wnp maps and give an upper bound $\bar{V}(\chi)$ with $\lim_{|\chi|\to\infty} \frac{\bar{V}(\chi)}{|2\chi|^{2/3}} = 1$ for the number of vertices of wnp maps of Eulercharacteristic χ.

Most of the mentioned results are contained in joint works of A. Altshuler and myself.

Ulrich Brehm, Honolulu

Amalgamations of tilings

Let T be a tiling of $\mathbb{E}^2$. If $X \subseteq T$ (the set of tiles) then we denote by $[X]$ the region of $\mathbb{E}^2$ which is the union of the tiles of X: if $[X]$ is a topological disc then it is a general tile of T.

An amalgamation of T is a tiling all of whose tiles are general tiles of T, and an amalgamator of T is an isomorphism from T to an amalgamation of T (where isomorphisms are considered to act on the set of general tiles).

We describe some elementary properties of amalgamations, and present a method of generating all possible "~~str~~ strongly isohedral" ~~[illegible]~~ amalgamations of an isohedral tiling

W. J. G. Wingate
The Open University, Milton Keynes, U.K.

The Geometry of Tiling Hierarchies

Let α be an amalgamator [see Mr. Wingate's abstract on page 91] on a tiling T. The tiling hierarchy $J(T,\alpha)$ is the sequence $\{T, \alpha(T), \alpha^2(T), \ldots\}$. We consider how the symmetry properties change as we go up in the hierarchy, starting from an isohedral tiling T; in particular, for each type of isohedral tiling T having the maximum symmetry for its topological type, we show how much of the symmetry can be preserved by a strict amalgamator α (i.e., the tiles of $\alpha(T)$ are each composed of more than one tile of T).

FG Holroyd

The Open University, Milton Keynes, U.K.

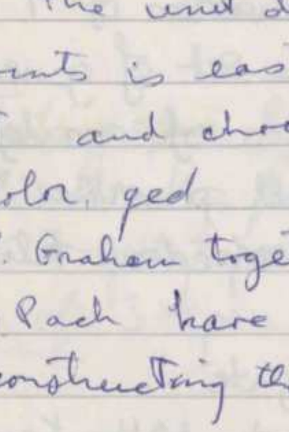
Increasing the Minimum Distance Among a Set of Points

P. Erdös has proposed the following problem. Let $f(d,n)$ be the largest number such that among any n points in E^d with minimum distance 1 there exist $f(d,n)$ points with minimum distance >1. He asked for a proof or disproof of $f(d,n) > \frac{cn}{d}$. Clearly $f(d,n) \le \frac{n}{d+1}$ by considering $\frac{n}{d+1}$ widely spaced d-simplices. Moreover he had a proof that $f(2,n) \ge \frac{n}{5}$ and suggested that $f(2,n) \ge \frac{n}{3}$ might be easy.

Th $f(2,n) \ge \frac{n}{4}$

Pf the unit distance graph on the given set of points is easily seen to be planar. 4-color it and choose the points with the most popular color. qed.

R. Graham together with F. Chung and independently J. Pach have observed that $f(2,n) \le \frac{6}{19} n$ by constructing the following diagram.



(all edges have length 1)

R. Pollack N.Y.C.

Arbeitsgemeinschaft über 4-dimensionale Mannigfaltigkeiten

7. – 13. 10. 1984

Differenzierbare Strukturen auf höherdimensionalen Mannigfaltigkeiten

In diesem 1. Vortrag der AG ging es darum, über höher dimensionale Mannigfaltigkeiten zu berichten (Definition: "höher" heißt: >4), um dadurch Problematik und Dramatik der Dimension 4 besser zu verstehen. Inhalt:

① Das h-Cobordismen-Theorem (hier geht es ein: $\dim > 4$) und die Poincaré-Vermutung in höheren Dimensionen.

② Milnors 1956-Beispiele von exotischen 7-Sphären; eine 8-dim. Mannigfaltigkeit ohne differenzierbare Struktur.

③ Klassifikationssatz: Die differenzierbaren Strukturen auf M (modulo "Concordanz") entsprechen bijektiv (wenn es überhaupt eine gibt), der Menge $[M, TOP/O]$. Die Zuordnung läßt sich leicht beschreiben; daß sie bijektiv ist, wurde nicht bewiesen. Hieraus (und für $n<4$ aus klassischen Sätzen): Der $\mathbb{R}^n$ besitzt für $n \neq 4$ genau eine differenzierbare Struktur (bis auf Diffeomorphie).

④ Existenzsatz: Sei M eine topologische Mannigfaltigkeit ($\partial M = \emptyset$) und τM das Tangential-Mikro-Bündel. Erster Satz: M sei höherdimensional. Dann: $\tau(M)$ ist stabil ein Vektorraumbündel $\Rightarrow$ M hat eine differenzierbare Struktur. Zweiter Satz: Sei M offen und $\dim M = 4$. Dann: $\tau(M)$ ist ein Vektorraumbündel $\Rightarrow$ M hat differenzierbare Struktur. Mit mehr Bündeltheorie kann man die Frage, ob τM (stabil) ein Vektorraumbündel ist, in ein Liftungsproblem

$$\begin{tikzcd} & BO \arrow[d] \\ M \arrow[r] \arrow[ur, dashed] & BTOP \end{tikzcd} \quad \text{bzw.} \quad \begin{tikzcd} & BO_4 \arrow[d] \\ M \arrow[r] \arrow[ur, dashed] & BTOP_4 \end{tikzcd}$$

übersetzen und erhält eine Hindernistheorie.

Ralph Stöcker, Bochum

Henkelzerlegung und der h-Kobordismussatz in höheren dimensionen

In diesen zweiten Vortrag der AG wurden die Grundlegende Begriffe über Henkelkörper eingeführt und die wichtigsten Schritte des Beweises des h-Kobordismussatzes skizziert. Auch wurde erläutert daß alle Schritte des Beweises in Milnor's Buch auch für Dimension 4 funktionieren, außer dem Whitney Tric. In Dimension 4 kann man eine Whitney-Scheibe im algemeinen noch differenzierbar immersieren (mit normale Kreuzungen) doch nicht differenzierbar einbetten. Es wurde angegeben welche Information man noch weiter benötigen würde um ~~das~~ den Beweis auch in Dimension 4 durch zu führen. Dirk Siersma, Utrecht.

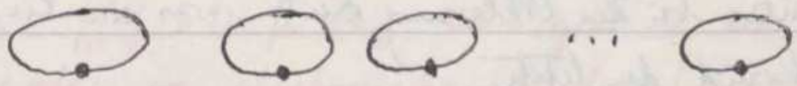
Ältere Resultate über 4-Mannigfaltigkeiten und quadratische Formen.

Es wurde zunächst über die Klassifikation orientierter geschlossener einfach-zusammenhängender 4-dimensionaler Mannigfaltigkeiten berichtet. Nach Resultaten von Whitehead 1949 / Milnor 1956 ist eine solche Mannigfaltigkeit durch ihre Schnittform q_M auf der 2-dimensionalen Homologie bis auf Homotopie und, wie 1964 von Wall bzw. Novikov gezeigt wurde, sogar bis auf h-Kobordismus festgelegt. Nach einer Skizze der Beweise dieser Sätze wurden die indefiniten unimodularen quadratischen Formen über $\mathbb{Z}$ klassifiziert und ihre Realisierbarkeit als Schnittform diskutiert. Abschließend wurde der Satz von Rohlin (1952) bewiesen, demzufolge die Signatur der Schnittform einer 4-dimensionalen geschlossenen differenzierbaren Spin-Mannigfaltigkeit durch 16 teilbar ist. Dies schließt zum Beispiel die gerade Form E_8 als Schnittform differenzierbarer ~~zusammen~~ geschlossener 4-Mannigfaltigkeiten aus.

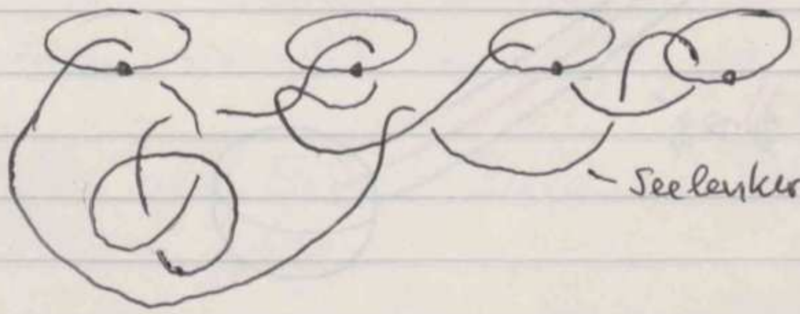
Norbert Klingen, Köln.

Henkelkörperzerlegung von 4-Mannigfaltigkeiten.

Wir betrachten zusammenhängende kompakte differenzierbare 4-Mannigfaltigkeiten (mit Rand). Die Morsetheorie lehrt, daß eine solche M eine Zerlegung
$M = D^4 \cup \{1\text{-Henkel}\} \cup \{2\text{-Henkel}\} \cup \{3\text{-Henkel}\} \cup$ ein 4-Henkel
besitzt. Der 1-Henkelkörper ist isomorph zur randverbundenen Summe $\natural_k S^1 \times D^3$. Diese entsteht auch durch Ausbohren von k 2-Henkeln (ε-Umgebungen disjunkt eingebetteter Standardscheiben $(D^2, \partial D^2) \to (D^4, \partial D^4)$) aus D^4. Die ausgebohrten Henkel treffen $S^3 = \partial D^4$ in k unverknoteten unverketteten Volltori, deren Lage wir durch ihre Seelenkreise (gepunktet) bezeichnen:



Der Rand $\#_k S^1 \times S^2$ entsteht, indem man in S^4 um diese Kreise Volltori ausbohrt und die komplementären Volltori einsetzt. Die Zweihenkel $\alpha = D^2 \times D^2$ werden durch Einbettungen $\varphi : S^1 \times D^2 \hookrightarrow \#_k S^1 \times S^2$ angeheftet, sodaß $\varphi(S^1 \times D^2)$ in (S^3 ohne die ausgebohrten Volltoren der 1-Henkel) liegt. Die Seelenkurven $\varphi(S^1 \times 0)$ bilden zusammen mit obigen Kurven eine Verkettung, das Verkettungsdiagramm von M:

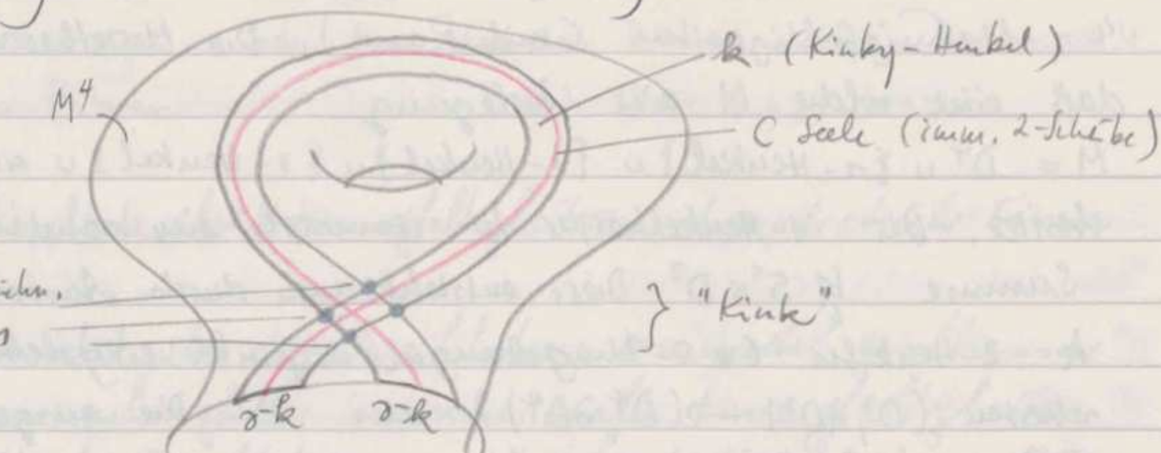


Jede Seelenkurve ist mit einer Rahmungszahl $n \in \mathbb{Z}$ versehen, die Windungszahl von $\varphi | S^1 \times \varepsilon$ um $\varphi | S^1 \times 0$ für $\varepsilon \neq 0$. Das so bewertete Verkettungsdiagramm bestimmt den 2-Henkelkörper und auch M. Die Kürzungsregel wird formuliert und bewiesen.

Theodor Bröcker

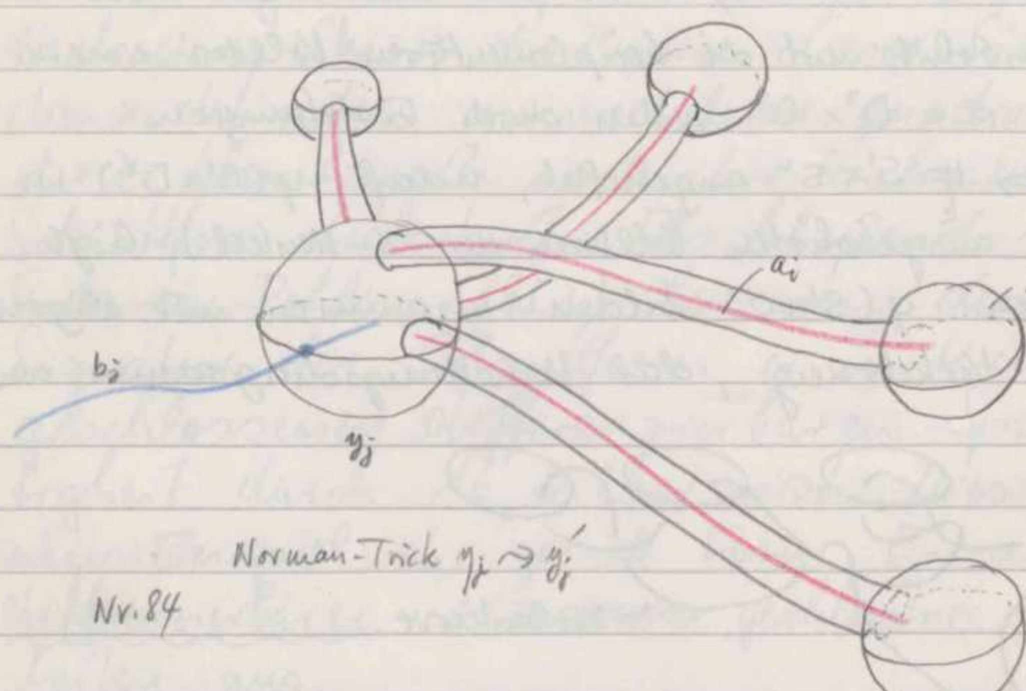
Kinky- und Casson-Henkel. Cassons Finger-Lemma

Vorführung einer Serie von vierundachtzig Bildchen, wovon das erste



Nr. 1

als Gedächtnis-Stütze bei der Erklärung diente, was ein Kinky-Henkel sei, während das letzte



Nr. 84

den letzten Schritt des Beweises des "Simultan-Finger-Lemmas" illustrierte, einen mehrfachen Norman-Trick zur Verbesserung gewisser geometrischer Duale. – Vorbereitet nach Freedman's Arbeit im J. diff. Geom. 1982 unter Benutzung der ersten 25 Seiten von Freedman u. Quinn's "Topology of 4-manifolds", preprint Januar 1983, und gelegentlichen Blicken in die Guillou-Notes (1980) der Casson-Lectures (1973-76).

Klaus Jänich, Regensburg

Existenz von Casson-Henkeln in 4-Mannigfaltigkeiten

Satz von Casson: M 1-zshgd. 4-Mannigf'kt.; $f_i : (D^2, \partial D^2) \to (M, \partial M)$ eine Familie normaler Immersionen, welche am Rand disjunkt eingebettet ist. Ist $f_i \cdot f_j = 0$ für $i \neq j$, und gibt es eine Familie $x_i \in H_2(M)$ gerader ge. algebraische Duale (d.h. $x_i \cdot x_i$ = gerade und $x_i \cdot f_j = \delta_{ij}$), so ist die f_i homotop zu den Seelen der ersten Stufen disjunkt eingebettete Casson-Henkel.

Der Beweis des Satzes erfolgt durch sukzessiven Aufbau der Stufen der Türme der Casson-Henkel unter wiederholter Anwendung von Cassons Finger-Lemma, des "Norman-Tricks" und 'Twisting'.

M. Rost, Regensburg.

(Sehen-See 104.)

Anwendungen des Satzes von Freedman:

Eine wichtige Konsequenz des Satzes von Freedman ist der folgende Satz:

h-Kobordismus Satz

Es sei $(W; M_1, M_2)$ ein glatter, eigentlicher, 1-zusammenhängender 5-dimensionaler h-Kobordismus. Ferner sei W einfach zusammenhängend im Unendlichen. Dann ist W homöomorph zu $M_1 \times [0,1]$.

Mithilfe des Quinn'schen Resultates, dass eine ~~4~~ geschlossene topologische 4-Mannigfaltigkeit im Komplement eines Punktes glättbar ist, ergibt aus obigem Satz und ~~Surgerytechniken~~ der exakten Surgerysequenz der folgende

Klassifikationssatz

Es seien M_1, M_2 zwei geschlossene, topologische 1-zusammenhängende 4-Mannigfaltigkeiten. Dann gilt

M_1 homöomorph zu $M_2 \Leftrightarrow S(M_1) \cong S(M_2)$ und $K(M_1) = K(M_2)$.

Hierbei ist $S(M_i)$ die Schnittform von M_i und $K(M_i) \in \mathbb{Z}/2$ die Kirby-Siebenmann-Invariante, die das erste Hindernis darstellt einen Lift $M \dashrightarrow BO$, $M \longrightarrow B\mathrm{Top}$ zu finden.

Ebenfalls mithilfe des Freedman Satzes (und anderer Resultaten) folgt:

Realisierungssatz:

Es sei S eine symmetrische, unimodulare Form und $K \in \mathbb{Z}/2$ mit $\frac{1}{8} \operatorname{sign}(S) \equiv K \bmod 2$, falls S gerade.
Dann gibt es eine geschlossene, topologische, 1-zusammenhängende 4-Mannigfaltigkeit M mit $S(M) \cong S$ und $K(M) = K$.

Stephan Stolz, Mainz

Selbst-Dualität in der Riemannschen Geometrie

Sei $P \xrightarrow{\pi} X$ ein G-Prinzipalbündel auf d. diffb. Mfk. X. Es werden die folgenden fundamentalen Begriffe behandelt: Zusammenhänge auf P, zug. Zsm. 1-Formen, Krümmung, Kovariante äußere Ableitung. Ist nun A ein Zusammenhang auf P mit Krümmung $F(A) \in \Omega^2(\mathfrak{g}) := \mathrm{Hom}(\Lambda^2 TX, P \times^{Ad} \mathfrak{g})$, so zerlegt sich diese in $F(A) = F(A)_+ + F(A)_-$ gemäß der Eigenraum-Zerlegung $\Omega^2(\mathfrak{g}) = \Omega^2(\mathfrak{g})_+ \oplus \Omega^2(\mathfrak{g})_-$ zum Hodge $*$ Op. $* : \Omega^2(\mathfrak{g}) \to \Omega^2(\mathfrak{g})$, $*^2 = 1$. Hier ist X als orientierte 4-dim. Riemannsche Mfk. vorausgesetzt. Der Zusammenhang A heißt selbst-dual, wenn $F(A)_- = 0$, d.h. $* F(A) = F(A)$. Die selbst-dualen Zushg. bilden einen affinen Raum $\mathcal{A}^+$ auf dem die Eichtransformationsgruppe $\mathcal{G}$ von P operiert: $\varphi \in \mathcal{G}$: φ ist ein G-inv. Bündelaut. von P, der auf X triv. op. Eine entscheidende Rolle beim Beweis des Satzes von Donaldson spielt der Modulraum $\mathcal{M} = \mathcal{A}^+ / \mathcal{G}$.

<u>Satz (Donaldson):</u> Sei X eine einf. zshgd., geschl., diffb. 4-Mfk. mit pos. def. Schnittform Q auf $H^2(X, \mathbb{Z})$. Dann ist $Q \sim$ Standardform $_{\mathrm{I}}$

Bew. Eine modifizierte Version des Modulraumes dient zur Konstr. eines Cobordismus zwischen X und $n(Q) = \frac{1}{2} \# \{\alpha \mid Q(\alpha) = 1\}$ Exemplaren $P^2\mathbb{C}$. Man erh. $\mathrm{Rg}\, Q = \mathrm{Sign}\, X = \sum_1^{n(Q)} \mathrm{Sign}\, P^2\mathbb{C} = \sum_1^{n(Q)} \pm 1 \leq n(Q)$. Hieraus folgt algebraisch: $\mathrm{Rg}\, Q = n(Q)$, $Q \sim$ Standardform.

Für $U(1)$-Bündel P über einf.-zshg. 4-Mfk. wurde gezeigt (u.a.): Ist Q pos. def., so ex. genau ein (bis auf Eichäquiv.) Zshg. A auf P ~~mit~~, der selbst-dual ist.
Ist $P \to S^4$ das eind. (bis auf Diffeo.) best. $SU(2)$-Bdl. mit $c_2(P) = -1$, so haben Atiyah, Hitchin, Drinfeld, Manin alle selbst-dualen Zshg. auf P bestimmt: $\mathcal{M} = \mathcal{A}^+/\mathcal{G} \approx SO(5,1)^+ / SO(5) \approx \mathring{B}^5$. Bei geeigneter Kompaktifizierung von $\mathcal{M}$ ist also $\bar{\mathcal{M}} \approx B^5$, folglich $\partial \bar{\mathcal{M}} = S^4$, die Ausgangs-4-Mfk.. Verfolgt man diese Diffeo. explizit so ergibt sich folgende intrinsische Char. des Randes der richtigen

Kompaktifizierung von M: Sei $x \in S^4$ und $[A_i] \in M$ mit $[A_i] \to x$, ($[A_i] \in \mathring{B}^5$ aufgefaßt), dann konvergieren die Krümmungen $F(A_i)$ gegen die „Diracsche"-Delta-Funktion in x und umgekehrt. Dies dient als Leitmotiv bei der Donaldsonschen Kompaktifizierung des Modulraumes M bei bel. diffb. 1-zshgd. geschl. 4-Mfk. X.

C. Deninger, Regensburg

Der Modulraum der selbstdualen Zusammenhänge

Sei X Riemannsche Mfkt, kompakt, orientierbar, der Dimension 4, mit positiver Schnittform sowie einfach zusammenhängend. Betrachtet werden der Raum aller Zusammenhänge $\mathfrak{A}$ auf einem fixierten hermiteschen Vektorbündel ~~mit~~ E mit $c_1(E)=0$, $c_2(E)=-1$ modulo der Operation der Eichgruppe. ~~Es wird~~ Auf dem Quotienten $\mathfrak{A}/\mathcal{G}$ kann man die Struktur einer Banach-mannigfaltigkeit etablieren. ~~Darin~~ (jedenfalls wenn man die reduziblen Zshänge herausnimmt) Darin enthalten ist der Teilraum der Klassen ~~der~~ ~~selbe~~, die durch selbstduale Zshänge repräsentiert werden. Die Dimension und lokale Struktur dieses Modulraumes $\mathfrak{M}$ wurde unter der Zusatzannahme $H^2_A = 0$ (Kohomologie des elliptischen Komplexes $\underline{g} \to \Omega^1_X(\underline{g}) \to \Omega^2_X(\underline{g})^-$) berechnet. Es ergab sich: Ist $H^0_A = 0$, so ~~ergibt sich~~ ist die Umgebung von A in $\mathfrak{M}$ glatt und 5-dimensional. Ist H^0_A 1-dimensional, so ergibt sich eine Singularität vom Typ $\mathbb{C}^3/S^1$, der Kegel über $\mathbb{P}^2(\mathbb{C})$.

U. Stuhler, Wuppertal

Der gestörte Modulraum und seine Orientierbarkeit

Die definierende Differentialgleichung für den Modulraum läßt sich als Schnitt in einem geeigneten Bündel interpretieren. Wird dieser Schnitt transversal zum Nullschnitt gemacht, so ist das Urbild eine Mannigfaltigkeit. Es wurde erläutert, wie in der vorliegenden Situation eine Reduktion auf Transversalität im Endlich-Dimensionalen möglich gemacht werden kann.

Die Orientierbarkeit des Modulraumes läßt sich durch Verwendung des Index-Bündels für den Operator $d_A^* + d_A$ darauf zurückführen, daß der einfache Zusammenhang des Raumes $\mathcal{B}^*$ der Äquivalenzklassen irreduzibler Zusammenhänge gezeigt wird. Dieses führt vermöge exakter Sequenzen auf die Gruppe der Zusammenhangskomponenten der Eichgruppe, ~~was~~ die mit klassischer Homotopie Theorie bestimmt werden kann.

T. tom Dieck (Göttingen)

Die Kompaktifizierung des Modulraums.

Mit Hilfe von Ergebnissen von Karen Uhlenbeck wurde gezeigt, daß für eine Folge A_i im Modulraum, die keine bis auf Eichung konvergente Teilfolge besitzt, nach Übergang zu einer Teilfolge genau ein Pkt. x_0 der Mannigf. existiert, so daß sich für $i \to \infty$ die Krümmung von A_i in x_0 mehr und mehr konzentriert. Dies legt eine Abbildung des Endes des Modulraums auf ein Produkt der Basismannigfaltigkeit M mit $\mathbb{R}$ nahe. Mit Hilfe des Taubesschen Satzes über die Existenz selbstdualer Zusammenhänge weist man nach, daß diese Abbildung ein Diffeomorphismus ist. Der Modulraum läßt sich somit durch M kompaktifizieren.

Elmar Vogt (Berlin)

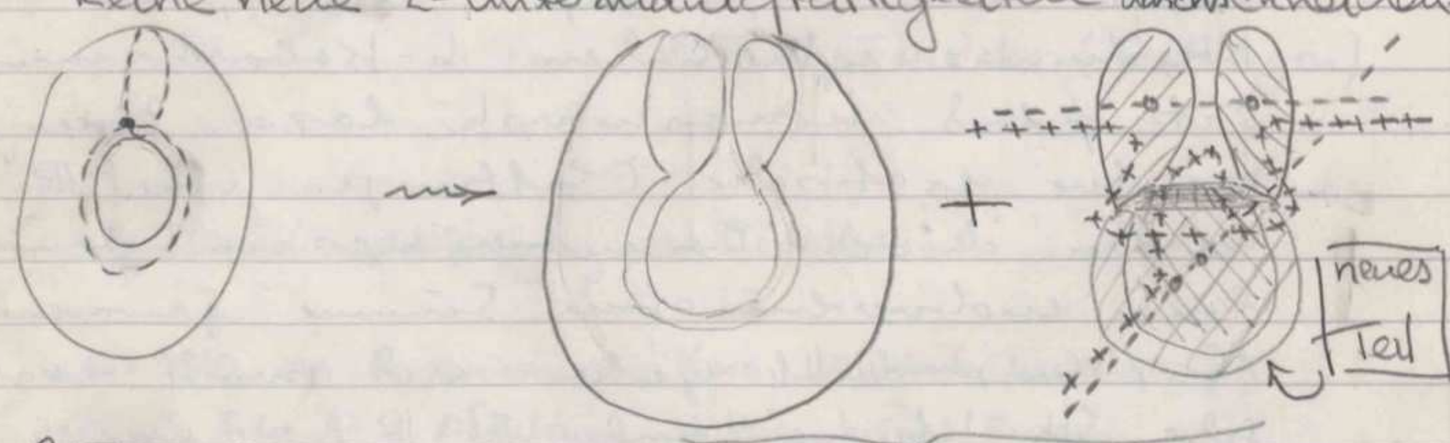
Anwendung der Resultate von Freedman und Donaldson auf exotische $\mathbb{R}^4$'s

Die algebraische Fläche X entstehe aus $\mathbb{C}P^2$ durch neun σ-Prozesse. Die Schnittform von X läßt sich in $\langle 1\rangle \oplus -E_8 \oplus \langle -1\rangle$ zerlegen, und nach den Sätzen von Casson und Freedman läßt sich der erste Summand durch eine zu $\mathbb{C}P^2 \setminus \{*\}$ homöomorphe offene Teilmenge W' repräsentieren. W' läßt sich auch differenzierbar als $W \subset \mathbb{C}P^2$ einbetten. Einer Geraden in $\mathbb{C}P^2 \setminus \{*\}$ entspreche die topologische Sphäre $\Sigma \subset W$. Donaldson's Satz zeigt, daß die beiden Enden von $W \setminus \Sigma \approx S^3 \times \mathbb{R}$ nicht durch eine differenzierbare Sphäre getrennt werden können. $R := \mathbb{C}P^2 \setminus \Sigma$ ist deshalb nicht diffeomorph zu $\mathbb{R}^4$, nach Freedman's eigentlichem h-Kobordismussatz jedoch homöomorph dazu. Zwei weitere exotische Glättungen von $\mathbb{R}^4$ werden durch Orientierungsumkehrung bzw. endverbundene Summe gewonnen. Als Nebenprodukt ergeben sich zwölf exotische Strukturen auf $S^3 \times \mathbb{R}$.

Klaus Wirthmüller
(Regensburg)

Jeder Casson-Henkel ist topologisch standard

Dieser Satz von M.H. Freedman 1981 ist die Achse der topologischen Klassification der einfach zusammenhängenden geschlossenen 4-Mannigfaltigkeiten. Ich habe den Beweis skizziert. Seit 1981 haben M.H. Freedman, F. Quinn, und R.D. Edwards die Kinky-henkel methoden von Casson weit ausgedehnt und verbessert: vgl. Warshau IMK Nachricht von Freedman, Artikel in Durham Conf. Proc. 1982 (Springer 1984?) von Edwards, Buch von Quinn-Freedman (manuscript 1983, Bladesburg Va).
Insbesonders, hat eine schöne neue elementare Processe aufgetaucht. Sie wandelt gewisse eingebettete 2-Toren innerhalb jeder einfach zusammenhängenden differenzierbaren 4-mannigfaltigkeit in immersierten 2-Sphären die, auf Kosten einigen Fingerprocessen, keine neue 2-Untermannigfaltigkeiten durchschneiden.



2-Torus

2-Sphäre

Das neue Teil kommt von 2 immersierten Scheiben, die Meridian und Longitude anhängen. Die Fingerprozessen machen Unterlungen um dieses Teil zu vermeiden [vgl Umwandlung ‐ ‐ ‐ ‐ ⟼ + + + + + + + an der Skizze]

Damit vereinfachert sich das Kinky-henkel Teil des grossen Satzes von Freedman

Larry Siebenmann
ORSAY

ANALYTISCHE ZAHLENTHEORIE
14.10. — 20.10.1984

Some asymptotic formulas involving
the largest prime factor of an integer

The distribution of prime factors of an integer is reflected in the asymptotic behavior of sums with the functions $P(n)$, $\omega(n)$ and $\Omega(n)$. In the usual notation these functions denote the largest prime factor of $n \geq 2$, the number of distinct prime factors of n and the total number of prime factors of n, respectively. A survey of recent results in this field is presented. These include the asymptotic formulas (obtained jointly with P. Erdős and C. Pomerance)

$$\sum_{2 \leq n \leq x} \frac{1}{(P(n))^{\omega(n)}} = \exp\left\{(4+o(1))\frac{\log^{1/2} x}{\log\log x}\right\},$$

$$\sum_{2 \leq n \leq x} \frac{1}{(P(n))^{\Omega(n)}} = \log\log x + D + O\left(\frac{1}{\log x}\right), \quad (D>0)$$

$$\sum_{2 \leq n \leq x} \frac{\Omega(n)-\omega(n)}{P(n)} = (C+o(1))\sum_{2 \leq n \leq x}\frac{1}{P(n)},$$

$$\sum_{2 \leq n \leq x} \frac{1}{P(n)} = x\,\delta(x)\left(1+O\left(\left(\frac{\log\log x}{\log x}\right)^{1/2}\right)\right).$$

Here $C>0$ is an absolute constant, and $\delta(x)$ is a precisely defined function for which we have, as $x \to \infty$,

$$\delta(x) = \exp\left\{-(2\log x\log\log x)^{1/2}\left(1+O\left(\frac{\log\log\log x}{\log x}\right)\right)\right\}.$$

Aleksandar Ivić
(BELGRADE)

On the Sieving Limit of the Rosser - Iwaniec Sieve

Let $\kappa > \frac{1}{2}$ and $q(s)$ a (special) solution of the difference - differential equation

$$(sq(s))' = \kappa q(s) + \kappa q(s+1).$$

If $\zeta = \beta - 1$ is the largest zero of $q(s)$, then β is the sieving limit of the Rosser - Iwaniec - Sieve with dimension κ (cf. Iwaniec, Rosser's Sieve, AA 36 (1980)).

A good lower bound estimate for this largest zero ζ is of importance for an improvement of Iwaniec's result. Iwaniec proved

$$\zeta > \kappa + \kappa\sqrt{1-\frac{1}{\kappa}}\ , \quad \kappa > \frac{3}{2}.$$

For $\kappa \geq 2$ better results can be proved, for example

$$\zeta > (2 + \log 2)\kappa + \sqrt{\frac{\kappa}{2}} - 3.5\ , \quad \kappa > 3,$$

holds true.

[illegible] Grupp

(Ulm)

On Waring's problem for smaller exponents

Let $R(N)$ denote the number of natural numbers not exceeding N which are the sum of three cubes of natural numbers, let $G(k)$ denote the smallest s such that ~~for~~ every sufficiently large natural number is the sum of at most s kth powers (of natural numbers) and let $G_1(4)$ denote the smallest s such that whenever $1 \leq r \leq s$ every sufficiently large ~~numb~~ natural number n with $n \equiv r \pmod{16}$ is the sum of at most s biquadrates. Then Davenport ~~(1939)~~ showed that $R(N) \gg N^{13/15-\varepsilon}$ (1939), $R(N) \gg N^{47/54-\varepsilon}$ (1951), $G_1(4) \leq 14$ (1939), $G(5) \leq 23$, $G(6) \leq 36$ (1942). His methods can be used to show that $G(7) \leq 53$, $G(8) \leq 73$, and Thanigasalam (1977) has shown that $G(9) \leq 90$. Very recently Thanigasalam has shown that $G(5) \leq 22$, $G(6) \leq 34$, $G(7) \leq 50$, $G(8) \leq 68$, $G(9) \leq 87$.

We introduce a new method which enables us to establish the following theorems.

Theorem 1 $R(N) \gg N^{\frac{8}{9}-\varepsilon}$.

Theorem 2. $G_1(4) \leq 13$.

Theorem 3. $G(5) \leq 21$, $G(6) \leq 32$, $G(7) \leq 45$, $G(8) \leq 62$, $G(9) \leq 82$.

R. C. Vaughan.

The Weighted Linear Sieve and Selberg's λ^2-method

As usual in the theory of the linear sieve, write $\#\{a \in \mathcal{A}_x, a \equiv 0 \bmod d\} = \frac{x}{d}\, g(d) + R(\mathcal{A}, d)$ where g is multiplicative and $-L < \sum_{w \leq p < z} \frac{g(p)-1}{p} \log p < A_1$; $\frac{g(p)}{p} < 1 - \frac{1}{A_2}$. Introduce a "level of distribution" $y = y(x)$ (e.g. such that $\sum_{d < y} 3^{\omega(d)} |R(\mathcal{A}, d)| < x/\log^2 y$ $(x > x_0)$) and a degree g such that $a \in \mathcal{A}_x \Rightarrow a < y^g$. We aim for results like: if $g < R - \delta_R = \Lambda_R$ then some a in $\mathcal{A}_x$ has $\leq R$ prime factors. Numerical work based on the theorem below leads to the following improved values: $\delta_2 = 0.047$, $\delta_3 = 0.076$, $\delta_4 < 0.105$, although improvements on earlier results do not appear to follow for large R.

In the theorem there is a weight function $w(p) = W\left(\frac{\log p}{\log y}\right)$. Let $V < 1/2 < U < 1$ with $V + RU \geq g$. We require $W(1) = U - V$ and

$0 \le W(t) \le t - v$ if $v \le t \le u$ and $t > 0$ (1)

$0 \le W(t) \le t - \max\{v, \frac{1-u}{2}\}$ if $\max\{v, \frac{1-u}{2}\} < t < \frac{1}{3}$ (2)

$W(t) \le 9(u - \frac{1}{3}) t^2$ if $0 < t < u$. (3)

Let $p(a)$ denote the least prime factor of a.

Theorem. Under the stated conditions

$$\sum_{\substack{a \in \mathcal{A}_x \\ \omega(a) \le R}} w(p(a)) \ge 2 \times e^{\gamma} \prod_{p < y}\left(1 - \frac{g(p)}{p}\right)\left\{\mathfrak{M}(w) + O\left(\frac{L}{\log^{1/3} y}\right)\right\}$$

$$+ O\left\{\sum_{d < y} 3^{\omega(d)} |R(\mathcal{A}, d)|\right\},$$

where

$$\mathfrak{M}(w) = -\int_{\frac{1}{2}}^{1} \frac{W(1) - W(t)}{1-t}\frac{dt}{t} + \int_0^{\frac{1}{2}} \frac{W(t)}{t}\left\{\frac{1}{1-t} - h(t)\right\} dt,$$

for a certain function $h(t)$ satisfying $h(t) < 1/5$.

[We also suppose $1 < L < \log^{1/3} y$.]

The function h is smaller than that in the author's article in Acta Arith. 40, but the requirements (2), (3) impose additional restrictions on the function w.

The proof depends on a use of $\sum_{d|a} \kappa(d)\{W(1) - \sum_{p|d} \omega(p)\}$ where the function $\kappa(d)$ is that implicit in the paper by Jurkat and Richert in Acta Arithmetica in 1965.

George Greaves

On a special partition function connected with the number of finite abelian groups of order n.

In order to study the distribution of values of the function $a(n)$ (the number of non-isomorphic abelian groups of order n), A. Ivić (J. Nb. Theory 16 (1983), 119–137) defined the function $E(x) = \sum_{n \le x} b(n)$, where $b(n)$ denotes the number of essentially different solutions of the equation $n = a(s)$ in square-full s. $E(x)$ may be interpreted as a special partition function, and by applying a general result on partitions (due to the speaker, Crelle 1968) it is proved that

$$\log E(x) = B\cdot \log^{2/3} x + B^{*}\cdot \log^{1/3} x \cdot \log\log x + O\left(\log^{1/3} x\cdot\sqrt{\log\log x}\right)$$

(joint with J. Herzog).
The same method is applied to prove the Tauberian part of a result of J.-L. Geluk (Proc. AMS 82 (1981), 571–575).
Finally J. Herzog showed, again applying a Tauberian result of the speaker, that there is an asymptotic formula for $E(x)$ itself.

Wolfgang Schwarz.

Oscillatory properties of arithmetical functions

Let $f(x)$ be a real function and suppose $F(s)=\int_0^\infty f(x)x^{-s-1}dx$ is regular for $\sigma>\Theta$, but not for any $\sigma>\Theta-\varepsilon$. Let $V(f,Y)$ denote the number of sign changes of the function $f(x)$ in the interval $[0,Y]$. Landau proved that if $F(s)$ is regular for $s=\Theta$ then $V(f,Y)\to\infty$. Let $\gamma=\{\inf t\geq 0;\ F(s)$ is not regular in $s=\Theta+it\}$ and $\gamma=\infty$ if $F(s)$ is regular for $\sigma=\Theta$. Pólya showed that if $F(s)$ is meromorphic in some halfplane $\sigma>\Theta-c_0$ then $\limsup_{Y\to\infty}\frac{V(f,Y)}{\log Y}\geq\gamma/\pi$. Grosswald generalized it if the function may have singularities $\{\rho_\nu\}$ with principal part $P_\nu(s-\rho_\nu)\log(s-\rho_\nu)+F_\nu(s)$ where P_ν are polynomials with $\sup_\nu \deg P_\nu<\infty$ and F_ν meromorphic at $s=\rho_\nu$. Using an idea of J. Kaczorowski, the speaker and Kaczorowski showed that if $F(s)$ have a denumerable set of singularities of the form $F(s)=g_\nu(s)(s-\rho_\nu)^{a_\nu}\log^{k_\nu}(s-\rho_\nu)+F_\nu(s)$, where $g_\nu(s)$ is regular, $F_\nu(s)$ is meromorphic at $s=\rho_\nu$, a_ν is an arbitrary complex number, $k_\nu = 0, 1, 2, \dots$ then $\liminf\limits_{Y\to\infty} \frac{V(f, Y)}{\log Y} \geq \gamma/\pi$. The assumptions of the theorem are fulfilled if:

a) $f(x) = \sum\limits_{n\le x} \Lambda(n) - x$ or $\sum\limits_{n\le x} \frac{\Lambda(n)}{\log n} - \mathrm{li}\, x$.

b) $M(x) = \sum\limits_{n\le x} \mu(n)$ c) $R_k(x) = \sum\limits_{\substack{n\le x \\ p|n \to p^k \nmid n}} 1 - \frac{x}{\zeta(k)}$

d) $\psi(x,q,l_1) - \psi(x,q,l_2) = \sum\limits_{\substack{n\le x \\ n\equiv l_1 (q)}} \Lambda(n) - \sum\limits_{\substack{n\le x \\ n\equiv l_2 (q)}} \Lambda(n)$ $(l_1,q)=(l_2,q)=1$, $l_1 \not\equiv l_2 (q)$

e) $\pi(x,q,l_1) - \pi(x,q,l_2)$ if both l_1 and l_2 are quadratic non-residues or both are quadratic residues, ~~(l,q)~~ $l_1 \not\equiv l_2 (q)$, if in cases d) and e) we assume that L-functions have no positive real zeros.

Concerning the average size of oscillations (the speaker) ~~one~~ has shown that if $\int_1^\infty \Delta(x) x^{-s-1}\, dx = F(s)/G(s)$, where

(i) $F(s), G(s)$ are for $\sigma \geq -1$ regular and $|F(s)|, |G(s)| < \cancel{C|s+2|}$ $C|s+2|^{K-2}$

(ii) ~~If~~ if there exists a ~~zero~~ $\rho_0 = \beta_0 + i\gamma_0$ $(\beta_0 \geq 0)$ zero of order ν of $G(s)$ and $F(s) = a_0(s-\rho_0)^{\nu-l} + a_1(s-\rho_0)^{\nu-l+1} + \dots$ $(l>0)$ then $Y^{-1}\int_1^Y |\Delta(x)|\, dx > \frac{|a_0|}{|\rho_0+2|^K C (l-1)!} Y^{\beta_0} \log^{l-1} Y$ for $Y > c_0$.

This gives a lower estimate of type $c\sqrt{Y}$ for the functions $\psi(x) - x$, $M(x)$, $\psi(x,q,l_1) - \psi(x,q,l_2)$ and $c_k Y^{1/2k}$ for $R_k(x)$. Apart the value of the constant this is optimal in case of $\psi(x) - x$ for every $Y > c$ if the Riemann hypothesis holds.

Remark: The examples $f(x) = x^{\beta+i\gamma} + x^{\beta-i\gamma}$ and $f(x) = x^{1/2}$ show that the two theorems are essentially best possible.

János Pintz

Distribution of q-additive functions on the set of primes

Let $q \geq 2$, $q \in \mathbb{N}$, $\mathcal{A}_q = \{0, 1, \dots, q-1\}$. Then $\forall n \in \mathbb{N}$ can be written as

$$n = \sum_{j=0}^{\infty} a_j(n) q^j \quad , a_j(n) \in \mathcal{A}_q .$$

Let $N = N(x) \in \mathbb{N}$ be defined by $q^N \leq x < q^{N+1}$.

Let $(0 \leq) j_1 < \dots < j_r \in \mathbb{N}$, $b_1, \dots, b_r \in \mathcal{A}_q$; $\mathcal{J} = \begin{bmatrix} j_1, \dots, j_r \\ b_1, \dots, b_r \end{bmatrix}$;

$\bar{\mathcal{J}} = \left[\begin{smallmatrix} 0 \\ b \end{smallmatrix} \middle| \mathcal{J} \right]$.

Let $\quad A(x|\mathcal{J}) = \#\{(0 \leq) n < x \mid a_{j_\ell}(n) = b_\ell \ (\ell = 1, \dots, r)\}$

$\pi(x|\mathcal{J}) = \#\{p < x, \ p \text{ prime} \mid a_{j_\ell}(p) = b_\ell \ (\ell = 1, \dots, r)\}$.

The following assertions are proved:

<u>Theorem 1.</u> If $0 \leq r \leq \sqrt{N}$, $(b, q) = 1$, then

$$\pi(x|\bar{\mathcal{J}}) = \frac{\operatorname{li} x}{q^r \varphi(q)} + O\left(\frac{\operatorname{li} x}{q^r} e^{-dN^{1/2}}\right) + O\left(\frac{\operatorname{li} x}{q^r} N^3 \left(\frac{q^{j_r}}{x}\right)^{1/2}\right).$$

<u>Theorem 2.</u> Let $2^r < N^{1/5}$. Then

$$\frac{\pi(x|\bar{\mathcal{J}}) \log x}{A(x|\bar{\mathcal{J}})} = \frac{q}{\varphi(q)} + O\left((\log x)^{\frac{9}{20} - 1}\right).$$

<u>Def.</u> $f: \mathbb{N}_0 \to \mathbb{R}$ is q-additive, if $f(0) = 0$ and

$$f(n) = \sum_{j=0}^{\infty} f(a_j(n) q^j).$$

Let $\quad m_k = \frac{1}{q} \sum_{b \in \mathcal{A}_q} f(bq^k)$, $\quad \sigma_k^2 = \frac{1}{q} \sum f^2(bq^k) - m_k^2 \quad (k \geq 1)$

$$m_0 = \frac{1}{\varphi(q)} \sum_{(b,q)=1} f(b), \quad \sigma_0^2 = \frac{1}{\varphi(q)} \sum_{(b,q)=1} f^2(b) - m_0^2$$

$$M(x) = \sum_{k \leq N} m_k, \quad D^2(x) = \sum_{k \leq N} \sigma_k^2$$

By Theorems 1 and 2 the following theorems can be proved.

Theorem 3. $\sum_{p<x} (f(p) - M(x))^2 \leq c\pi(x) D^2(x)$.

Theorem 4. Let $f(bq^j) = O(1)$ as $bq^j \to \infty$, $b \in A_q$; $D(x) \to \infty$.

Let
$$F_x(y) := \frac{1}{\operatorname{li} x} \# \left\{ p < x \;\middle|\; \frac{f(p) - M(x)}{D(x)} < y \right\}.$$

Then, for a suitable sequence $x_\nu \to \infty$, $\lim F_{x_\nu}(y) = \Phi(y)$ (Φ = Gaussian law).

If, in additionaly, $D(x/\log_q x)/D(x) \to 1$ $(x \to \infty)$, then
$$\lim_{x\to\infty} F_x(y) = \Phi(y).$$

Theorem 5. Assume that $D(x)$ is bounded. Then
$$\frac{1}{\operatorname{li} x} \# \{ p < x \mid f(p) - M(x) < y \} \to F(y), \quad (p.p.)$$
F is a distribution function.

If, moreover, $M(x)$ is convergent, then

(*) $$\frac{1}{\operatorname{li} x} \# \{ p < x \mid f(p) < y \} \to G(y) \quad (p.p.)$$

G is a distribution function.

Theorem 6. Let $f(bq^j) = O(1)$, and assume that (*) holds. Then $M(x)$ has a limit as $x \to \infty$, $D(x)$ is bounded.

Imre Kátai (Budapest)

On integers free of large prime divisors

In 1938, in a well-known paper on large differences between consecutive primes, R. A. Rankin derived an upper bound for $\Psi(x,y)$, the number of positive integers $\leq x$ and free of prime factors $> y$, by means of a simple, but very effective trick: For every $\alpha > 0$,

$$\Psi(x,y) \leq \sum_{\substack{n \geq 1 \\ p|n \Rightarrow p \leq y}} \left(\frac{x}{n}\right)^{\alpha} = x^{\alpha} \prod_{p \leq y} \left(1 - \frac{1}{p^{\alpha}}\right)^{-1}.$$

The optimal value for α is given by the equation

$$(*) \quad \log x = \sum_{p \leq y} \frac{\log p}{p^{\alpha} - 1}.$$

Using analytic tools, Rankin's upper bound can be improved to a fairly sharp approximation for $\Psi(x,y)$:

Theorem: Uniformly for $x \geq y \geq 2$, we have

$$\Psi(x,y) = c(x,y)\, x^{\alpha} \prod_{p \leq y} \left(1 - \frac{1}{p^{\alpha}}\right)^{-1} \left(1 + O\left(\frac{\log y}{\log x}\right) + O\left(\frac{\log y}{y}\right)\right),$$

where $c(x,y) = \alpha^{-1} \left(2\pi \sum_{p \leq y} \frac{p^{\alpha} \log^2 p}{(p^{\alpha}-1)^2}\right)^{-1/2}$ and $\alpha = \alpha(x,y)$ is defined by $(*)$.

The Theorem gives an asymptotic formula for $\Psi(x,y)$, whenever y and $\log x/\log y$ tend to infinity. Previously, asymptotic formulae have been known only for the regions $\log x \leq \exp(\log y)^{3/5-\varepsilon}$ and $\log x \geq y^{4/3+\varepsilon}$. The factor $c(x,y)$, ~~is~~ measures the discrepancy between Rankin's bound and $\Psi(x,y)$, is easy to estimate. For example, it satisfies $(y/\log y)^{-1/2} \ll c(x,y) \ll (\log y)^{-1}$ uniformly for $x \geq y \geq 2$ and is asymptotically equal to $(2\pi y/\log y)^{-1/2}$, if y and $(\log x)/y$ tend to infinity.

Adolf Hildebrand
(joint work with Gérald Tenenbaum)

Recent advances in primality testing.

Most modern primality testing algorithms depend in some way or another on Fermat's theorem: n prime $\Rightarrow \forall a \in \mathbb{Z}: a^n \equiv a \bmod n$. Two difficulties arise if one tries to use this theorem to prove the primality of n. The first difficulty is that $\Leftarrow$ is wrong (e.g. $n = 1729 = 7 \cdot 13 \cdot 19$). It is overcome by strengthening Fermat's theorem. For example: n prime $\Leftrightarrow \forall a \in \mathbb{Z}, \gcd(a,n) = 1: a^{(n-1)/2} \equiv \left(\frac{a}{n}\right) \bmod n$ [here n is odd, $n > 1$]; or: n prime $\Leftrightarrow \forall$ commutative rings R: $\forall a, b \in R: (a+b)^n \equiv a^n + b^n \bmod nR$. The second difficulty is that it is not computationally feasible to check <u>all</u> $a \in \mathbb{Z}$ (even mod n), or <u>all</u> commutative rings R ... . It is overcome by trying only a small and well-chosen collection of tests of this type, with the property that <u>if</u> n passes all tests then all divisors of n "behave in a certain way" as powers of n. This information on the divisors is then hopefully sufficient to finish the primality proof. As an example, we have: <u>Theorem</u>. Suppose n is odd and $\exists a \in \mathbb{Z}: a^{(n-1)/2} \equiv -1 \bmod n$. Then: (a) $\forall r | n: \mathrm{ord}_2(r-1) \geq \mathrm{ord}_2(n-1)$ [so each $r|n$ is a power of n modulo 2^{k+1}, where $2^k \| n-1$]; (b) if $b \in \mathbb{Z}$, $b^{(n-1)/2} \equiv \pm 1 \bmod n$, then $b^{(n-1)/2} \equiv \left(\frac{b}{n}\right) \bmod n$ and $\forall r|n: \left(\frac{b}{r}\right) = \left(\frac{b}{n}\right)^{(r-1)/(n-1)}$ [where the exponent is in $\mathbb{Z}_2$, by (a)]; (c) if $2^{(n-1)/2} \equiv \pm 1 \bmod n$ and $3^{(n-1)/2} \equiv \pm 1 \bmod n$, then $\forall r|n: \exists i: r \equiv n^i \bmod 24$. The proof of this theorem is elementary. The conclusion of the theorem is too weak for practical purposes. Replacing the Jacobi symbol by characters of higher order, and expressions like $b^{(n-1)/2}$ by certain expressions in Gaussian sums, one can replace 24 by larger numbers. This gives rise to the primality test of Adleman et al. [Ann. Math. <u>117</u> (1983), 173–206; Math. Comp. <u>42</u> (1984), 297–330]. Older tests (Lucas, Pocklington, Lehmer, ...) can also be formulated as implying that the divisors of n "behave" like powers of n. This makes it possible to combine the two types of tests. In this context the language of Galois theory for rings is useful.

Hendrik Lenstra

Universiteit van Amsterdam.

On the measure of large trigonometric sums

Let K be set of k integers

$$K=\{a_0<a_1<\dots<a_{k-1}\},\ a_j\in\mathbb{Z},\ 0\le j\le k-1,$$

$$S_K(\alpha)=\sum_{j=0}^{k-1}e^{2\pi i\alpha a_j},$$

E_u - set of all those values of α for which

$$|S_K(\alpha)|\ge k-u,\quad 0\le\alpha<1$$

$$\mu_K(u)=\operatorname{mes}E_u$$

$$\mu^*(u)=\sup_{|K|=k}\mu_K(u)$$

Theorem I (Freiman 1968) Let $a_0=0$, $a_{k-1}<0{,}05k^{\frac{3}{2}}$, then

$$\mu^*(1)=\frac{2\sqrt{6}}{\pi}k^{-\frac{3}{2}}+O(k^{-2})$$

Theorem II (Yudin, 1968) If $u=o(k)$ then

$$\mu^*(u)=\frac{2\sqrt{6}}{\pi}\frac{1}{k}\left(\frac{u}{k}\right)^{\frac{1}{2}}(1+o(1))$$

Theorem III (Freiman, Lev (Besser,) 1984) $\exists\,\varepsilon>0$ such that for $u<\varepsilon k$

$$\mu^*(u)=\frac{2\sqrt{6}}{\pi}\frac{1}{k}\left(\frac{u}{k}\right)^{\frac{1}{2}}\Gamma(k,u),$$

$$\Gamma(k,u)=\frac{\alpha_0}{\frac{\sqrt{6}}{\pi}\frac{1}{k}\left(\frac{u}{k}\right)^{\frac{1}{2}}},$$

α_0 can be found from the equation

$$\frac{\sin\pi\alpha_0 k}{\sin\pi\alpha_0}=k-u$$ and this maximum is achieved by arithmetic progression

We have $\lim_{\frac{u}{k}\to 0_+} \Gamma(k,u) = 1$, $\Gamma(k,u) = 1 - \frac{3}{10}\frac{u}{k} + \dots$

We have

(1) $\mu_k(u)(k-u)^2 < \int_0^1 |S|^2 d\alpha = k \Longrightarrow$

$$\mu_k(u) < \frac{k}{(k-u)^2} \sim \frac{1}{k}\left(1 + 2\frac{u}{k}\right)$$

For arithmetic progression we have

$$|S| = \left|\frac{\sin \pi\alpha k}{\sin \pi\alpha}\right|$$

and it gives us $\mu_k^*(u) = 2\alpha_0$ - values of the theorem.

Suppose for some K

$$\mu_k(u) \geq \mu^*(u).$$

Using the

<u>Lemma</u> If $|S(\alpha_1)| \geq k-u$ and $|S(\alpha_2)| \geq k-u$ then $|S(\alpha_1+\alpha_2)| \geq k-4u$ (i.e. $2E_u \subset E_{4u}$)

~~From this~~s we receive for some $0<\delta<1$

$$\text{mes}\, 2E_u < (2+\delta)\,\text{mes}\, E_u$$

(perhaps not for E_u but for E_{4u} or E_{16u})

This condition gives us the structure of E_u: it is contained in the union of the segments with the centers $\frac{p}{q}$, $q \in \mathbb{N}$, $0 \leq p < q$ with lengths $\leq \frac{1+\delta}{q}\mu(E_u)$

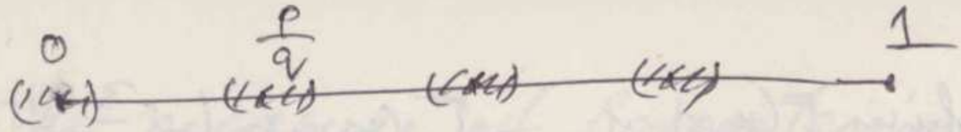


Fig. I

It gives us a preliminary description of the structure of K and we can show afterwards that compressing K to arithmetic progression we can only to increase the measure.

~~Re~~ From (1) in a case

$$|S| > \delta K, \quad \delta \text{ small}$$

we receive

$$(2) \qquad \mu^* \ell < \frac{1}{\delta^2 K}$$

Some remarks were made how to study Roth's problem on 3-arithmetical progressions in case of improving (2). After the lecture Dr Halasz pointed out to me that in general case (2) cannot be improved but possibility to study Roth's sets from this point of view still exist.

G Freiman

Arithmetic Functions and Integer Products.

If $r_1, r_2, \dots$ is a sequence of positive rational numbers let $\Gamma(r_i)$ denote the subgp of Q^* (the multiplicative gp. of positive rationals) generated by them. Define the quotient group $G = Q^*/\Gamma(r_i)$. This group reflects the extent to which an arbitrary rational s has a product representation

$$S = \prod_{i=1}^{k} r_{j_i}^{\varepsilon_i}, \quad \varepsilon_i = \pm 1$$

Let $R(x)$ be a rational fn. of x, say $P_1(x)/P_2(x)$, (†) $P_j(x) \in \mathbb{Z}[x]$, leading coefficients > 0. For $k > 0$ let r_i run through the values $R(n)$, $n > k$.

Conj: 1). G is independent of k

2) If R is an irreducible poly., or more generally, has no factor which is a power > 1 of a poly., then G is the direct sum of a free gp. and a finite gp.

Conj ~~…~~ 2) is true for ① $R(x) = \prod_{i \le i_0} (x - a_i)^{b_i}$, $a_i, b_i \in \mathbb{Z}$, $(b_1, \ldots, b_{i_0}) = 1$ ② $R(x) = x^2 + bx + c$, $b^2 \neq 4c$, ③ $R(x) = (ax+b)/(cx+d)$, $ad \neq bc$, ④ $R(x) = x(x^2+c)$, $c \neq 0$, ⑤ $R(x) = x^{-1}(bx^2+c)$, $cb \neq 0$.

Conj. 1) is true for cases ①, ④, ⑤.

There is a polynomial analogue. $Q(x)^*/\Delta(R)$ of G, $Q(x)$ = mult. gp gen. by $P_1(x)/P_2(x)$, $\Delta(R)$ is the subgp. gen. by $R(P(x))$, $P(x)$ of the above type (†)

In studying this gp. the notion of Persistence of form is useful, a polynomial $F(x)$ has persistence of form if $\exists\, P_i(x) \in \mathbb{Z}[x]$, leading coeff > 0, $+ d_i \in \mathbb{Z}$, not all zero, s.that

$$\prod_{i \le i_1} F(P_i)^{d_i} = \text{constant}.$$

e.g.: If $F = x^2 + bx + c$ then $F(F+x) = F(x)F(x+1)$, if $F = bx^2 + c$, $bc \neq 0$, $\exists$ ~~…~~ int. D_0, D_j $1 \le j \le 4$ s.that $F(D_0 F(D_4 x)) = \prod_{i \le 3} F(D_i x)$. As an example: $Q(x)^*/\Delta(z^{-1}(az^2+b))$ is trivial, $ab \neq 0$, and $\exists$ identity $x^3 = \prod_{i \le i_2} (P_i(P_i^2 + c))^{\varepsilon_i}$, $c \neq 0$. not finished, please leave to foot of page.

The gps. G can be studied by considering their homomorphisms into other gps, e.g. $(\mathbb{R}/\mathbb{Z}, +)$, $(\mathbb{R}, +)$, or G/pG (p prime) into finite field of p elements. E.g. If $\cdot f: G \to \mathbb{R}/\mathbb{Z}$ is trivial, then so is f. ~~An add~~ Such an f is an example of an additive function, and we need to characterize f: $f(mn) = f(m) + f(n)$, $\forall\, m,n \in \mathbb{Z}$, $m,n > 0$, which takes values in (say) $\mathbb{R}/\mathbb{Z}$, and satisfy $f(r_i) = 0$ on a given sequence of rationals r_i.

Thm. Let $a_1 < a_2 < \ldots$ be a seq. of integers of upper density $\limsup_{x \to \infty} x^{-1} \sum_{a_i \le x} 1 = d > 0$. Let f ~~…~~ satisfy $f(mn) = f(m) + f(n)$ for all positive integers m, n, and take values in $\mathbb{R}/\mathbb{Z}$. Suppose $f(a_j) = 0$ $\forall j$. Then $\exists$ integer m, $1 \le m \le d^{-1}$, s. that

$$\sum_{\|mf(p)\| \neq 0} \frac{1}{p} \le \text{const}(\text{dep. on } d) < \infty,$$

where $\|y\|$ = distance of y to a nearest integer.

Thm. In the circumstances of the previous theorem $\exists$ primes q_i, $\sum_{i=1}^{\infty} \frac{1}{q_i^2} \le c_1(d) < \infty$ s.th. those n not divisible by any q_i generate mod $\Gamma(a_n)$ a subgroup of G of finite order $\leq d^{-1}$.

Corollary. All such n have a rep. $n^s = \prod_{i \le i_3} a_{j_i}^{\varepsilon_i}$, $\varepsilon_i = \pm 1$, with $1 \leq s \leq \frac{1}{d}$.

In particular cases, e.g. $\mathbb{Q}^* / \Gamma(p_j + 1)$, p_j prime, one can "remove" the exceptional prime q_i say by a sieve, and prove $|G_1| \leq d^{-1}$.

Remark The condition on $R(x)$ given in (Conjecture 2) above differs from the condition given in the problem section of my book[⊕]: that $R \neq$ a power of another rat. function. (⊕ Springer, Grundlehren 272, to appear Nov/Dec '84), c.f. The following example of Lenstra and Schinzel: $R = x^2(x^2+1)$.

P. D. T. A. Elliott.

In complete convolutions of the Moebius-function

A fundamental property of the Moebiusfunction is that
$$\sum_{d \mid n} \mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0 & \text{if } n > 1. \end{cases}$$

We investigate the truncated sums, especially we are interested in $M(n) := \sup_z \left| \sum_{\substack{d \mid n \\ d \le z}} \mu(n) \right|$

An upper bound, valid for almost all integers, is obtained from a bound of Hooley's Δ-function: $\Delta(n) = \sup_z \left| \sum_{\substack{d \mid n \\ z \le d < ez}} 1 \right|$. In a joint paper with G. Tenenbaum the author obtained $\Delta(n) \ll \psi(n) \log\log n$ (p.p.)

Moments of additive functions.

Halberstam proved the following result:
Let f be a real-valued strongly additive function satisfying
$|f(p)| \leq M$ for every prime p and $\sum_p \frac{f(p)^2}{p} = \infty$.
Set $A(x) = \sum_{p \leq x} \frac{f(p)}{p}$ and $B(x) = \left(\sum_{p \leq x} \frac{f(p)^2}{p} \right)^{1/2}$.

Then, as x tends to infinity, for every positive integer q,
$\frac{1}{x} \sum_{n \leq x} \left(\frac{f(n) - A(x)}{B(x)} \right)^q$ tends to $\mu_q = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t^q e^{-t^2/2} dt$.

The proof involved complicated calculations.
We give a simple method to prove general results of the same kind. It is essentially based upon a lemma on strongly additive functions satisfying $f(p) = 0$ for all primes p which do not belong to a given finite set. The proof of that lemma is very simple. We apply it to the truncated function $f_y(n) = \sum_{\substack{p|n \\ p \leq y}} f(p)$.

Hubert Delange

where $\psi(x) \to \infty$ arbitrarily slowly. From this it follows that $M(n) \ll \psi(n) \log\log n$ (p.p.)

The main proportion of the lecture was devoted to the lower bound for $M(n)$. For this purpose a large block of consecutive divisors was constructed, for which the Moebiusfunction assumes the same value different from zero. This construction was carried out by a complicated induction proof. The partial products $n_{(k)} = \prod_{\substack{\log_2 p < \rho^k v(x) \\ p|n}} p$ were considered. It was concluded that if for most n, $n_{(k)}$ contained a block of 2^k divisors, then for most n, $n_{(k+1)}$ would contain a block of 2^{k+1} divisors. The induction proof uses Fourieranalysis and theory of Multiplicative functions.

Helmut Maier

p+a without large prime factors

Let $a \neq 0$ be a fixed integer, and

$$\pi(x,y) = \pi_a(x,y) = \sum_{\substack{p \leq x \\ P(p+a) \leq y}} 1 ,$$

where $P(u)$ denotes the greatest prime factor of $u \neq 1$. It is shown that

THEOREM: If $y \geq x^{0.35}$ then

$$\pi(x,y) \gg \frac{x}{\log^2 x} .$$

In other words, for a great many primes p all prime factors of $p+a$ are at most $p^{0.35}$.

The result has the following corollary

COROLLARY: For infinitely many n the number of solutions of the equation

$$\varphi(m) = n \qquad (\varphi \text{ is Euler's function})$$

is at least $n^{0.65}$.

Antal Balog

Solved and unsolved problems in combinatorial and analytic number theory.

Several of my old problems have been solved in the last 2-3 years. Mirsky and I conjectured that $d(n) = d(n+1)$ has infinitely many solutions. Using a previous weaker result of Claudia Spiro this was proved by Heath-Brown.

I conjectured and Montgomery and Vaughan proved that if $1 = a_1 < a_2 < \dots < a_{\varphi(n)} = n-1$ are the integers relatively prime to n then

$$\sum_{i=1}^{\varphi(n)-1} (a_{i+1} - a_i)^{2r} < c_r \frac{n^r}{(\varphi(n))^{r-1}}$$

I offered 500 dollars for this.

One of my oldest conjectures stated that ~~for~~ almost all integers n have two divisors $d_1 < d_2 < 2d_1$. This ~~wo~~ and much more was proved by Maier and Tenenbaum this cost me more than 500 dollars.

I conjectured that if $n_{k+1}/n_k > c > 1$ then $n_1 < n_2 < \dots$ can not be an essential component.

This and much more was very recently proved by Ruzsa,

Here are three very old conjectures:

Let $1 \le a_1 < a_2 < \dots < a_k \le n$, all sums $\sum_i \varepsilon_i a_i$ are distinct $\varepsilon_i = 0$ or 1. $\max k < \frac{\log n}{\log 2} + C$ (500 dollars).

Let $a_1 < \dots$ The number of solutions of $n = a_i + a_j$. Turán + I conjectured that if $f(n) > 0$ then $\limsup f(n) = \infty$ (500 dollars)

$a_i \pmod{n_i}$ $1 < n_1 < \dots < n_k$ is a covering congruence if every integer satisfies at least one of these congruences. Can n_1 be as large as we please? (1000 dollars). Selfridge + I believe that the fact that $n - 2^u - 2^v$ is not a prime for inf many n can not be proved by covering congruences i.e there is no finite set of primes $p_1, \dots p_k$ so that for infinitely many n, $n - 2^u - 2^v$ is always a multiple of one of these primes

Diophantine Problems in Many Variables.

Let F_i $(i=1,\dots,R)$ be R diagonal (= additive) forms of degree k in s variables $x_1,\dots,x_s$. Davenport and Lewis studied in depth the problem of non-trivial solvability in integers of the system of equations $F_i = 0$ when the forms F_i are integral. If the forms have real coefficients one seeks non-trivial solvability of the Diophantine inequalities $|F_i| < \varepsilon$, for arbitrarily small $\varepsilon > 0$. The talk covered work on this and related problems by Cook, Tolliver, Lloyd, Nadesalingam and the speaker.

In order to attack the problem by the Hardy-Littlewood method, one needs to consider a mixed system of t inequalities $|F_i| < 1$ and $R-t$ equations $F_i = 0$ with integral F_i's and to give bounds for the smallest non-trivial solution of such a system. This approach has been used for $R=2$ by the speaker and for $R \geqslant 2$ and ~~even~~ odd k by Dr Nadesalingam. Two of the main difficulties in extending this work to even k for $R>2$ have been the need for a suitable condition for the existence of a non-singular solution and the need for bounds on such a solution. Two lemmas overcoming these difficulties were described.

Jane Pitman.

On the distance between consecutive divisors of an integer.

Let $\omega(n)$ denote the number of distinct prime divisors of a positive integer n. Then we define $h: \mathbb{N} \to \mathbb{R}$ by $h(n) = 0$ if $\omega(n) \le 1$ and $h(n) = \sum_{i=2}^{r} 1/(q_i - q_{i-1})$ if $n = q_1^{\alpha_1} q_2^{\alpha_2} \cdots q_r^{\alpha_r}$, where $q_1 < q_2 < \dots < q_r$ are primes and $r \ge 2$. Similarly denote by $\tau(n)$ the number of divisors of n and let $H: \mathbb{N} \to \mathbb{R}$ be defined by $H(n) = \sum_{i=2}^{\tau(n)} 1/(d_i - d_{i-1})$, where $1 = d_1 < d_2 < \dots < d_{\tau(n)} = n$ are the divisors of n. We prove that there exist constants A and B such that $\sum_{n \le x} h(n) = Ax + O(x(\log\log x)/\log x)$ and $\sum_{n \le x} H(n) = Bx + O(x/(\log x)^{1/3})$.

(joint work with Aleksandar Ivić) J. M. De Koninck (QUÉBEC)

A $\mathcal{B}^q$ - criterion for exponentially multiplicative functions.

By means of exponentially multiplicative functions (Erdös and Rényi) simpler proofs for mean-value theorems of Delange, Elliott and Daboussi for multiplicative functions are given. As a special case of the theorem of Daboussi a $\mathcal{B}^2$-criterion for exponentially multiplicative functions is shown.

Karin Waibel (Freiburg)

Some properties of multiplicative functions

Let f be multiplicative and $\mathcal{L}^q := \{f: \mathbb{N} \to \mathbb{C}.\ \|f\|_q := \{\limsup x^{-1} \sum_{n \le x} |f(n)|^q\}^{1/q} < \infty\}$ $(q \ge 1)$. Then the following result holds: Let $f \in \mathcal{L}^q$, $q > 1$, $\varepsilon > 0$, and α be irrational. Then there exists a $x_0 = x_0(\|f\|_q, q, \alpha, \varepsilon)$ such that $|x^{-1} \sum_{n \le x} f(n) e(n\alpha)| \le \varepsilon$ for all $x \ge x_0$. Further, some results on the characterizations of multiplicative functions $f \in \mathcal{L}'$ are given. In the last part of the talk several theorems on ~~almost~~ uniformly summable multiplicative functions are formulated. As an application it is shown, that, if $f(n) = \tau^2(n) n^{-11}$, where τ denotes Ramanujan's τ-function, the mean-values of $|f|^{1-\varepsilon}$ exist and are zero for all $0<\varepsilon<1$, whereas the mean-value of f is different from zero (result of Rankin (19?)).

Karl-Heinz Indlekofer.

Some applications of zero density theorems for L-functions

By means of zero density results the following mean value theorem (which is similar to the Bombieri-Vinogradov theorem) is proved.

Thm. 1 Let $S(N,\alpha) = \sum_{n\le N} \Lambda(n) e(n\alpha)$ $(\alpha \in \mathbb{R})$,

$B>0$, $Q = N^{\frac{1}{3}-\frac{1}{100}}$, $\vartheta = Q^{-3}(\ln N)^{2(B+7)}$. Then

$$\sum_{q\le Q} \max_{(a,q)=1} \max_{y\le N} \max_{|\beta|\le\vartheta} \left| S\left(y, \frac{a}{q}+\beta\right) - \frac{\mu(q)}{\varphi(q)} \sum_{n\le y} e(n\beta) \right| \ll_B \frac{N}{\ln^B N}.$$

There are no immediate applications of this theorem, but by a slight modification of the proof one gets the following version of the Goldbach-Vinogradov theorem.

Thm 2. There is a sub-set $\mathbb{P}_1 \subseteq \mathbb{P}$ with the properties

a) $P_1(x) = |\{p_1 \le x,\ p_1 \in \mathbb{P}_1\}| \ll x^{9/10+\varepsilon}$

b) $\forall N \ge N_0$, $N \equiv 1(2)$: $N = p_1+p_2+p_3$, $p_\nu \in \mathbb{P}_1$.

c) $\mathbb{P}_1$ is the union of sets $\{p \equiv 1\,(q_\nu),$ q_ν prime, $M_\nu < p \le 2M_\nu\}$.

On the minor arcs a recent result of Balog and Perelli is used.

D. Wolke (Freiburg)

An analytical approach to the Prime Number Theorem of Pjatecki-Shapiro

1953 bewies Pjat.-Shap. für $\gamma \in (\frac{11}{12}, 1)$

$$\sum_{\substack{m \le x \\ [m^{1/\gamma}] \text{ prim}}} 1 \sim \gamma \cdot \frac{x}{\log x}, \qquad (*)$$

was in den nachfolgenden Jahren mehrfach verbessert wurde von Kolesnik, Graham und Leitmann (unabhängig), Heath-Brown und noch einmal Kolesnik: der Gültigkeitsbereich von (*) ist $(\frac{34}{39}, 1)$.

Alle genannten Arbeiten reduzieren das Problem auf die Abschätzung von Exponentialsummen über Primzahlen, der technische Aufwand ist verhältnismäßig groß.
Der analytische Zugang ist neu (in diesem Problem) und kommt völlig ohne Exponentialsummen aus; verwendet werden vielmehr Nullstellen der ζ-Funktion bzw. der Dirichlet'schen L-Reihen.
Das numerische Ergebnis ist noch schwach:
trotz Verwendung der Riemann'schen Vermutung kann (*) nur für $\gamma \in (\frac{21}{22}, 1)$, die Existenz ∞-vieler γ-Primzahlen für $\gamma \in (\frac{8}{9}, 1)$ gezeigt werden.

Gunter Dufner (Freiburg)

New methods and results in the study of some arithmetical concentration functions

The arithmetical function

$$\Delta(n) := \max_u \operatorname{card}\{d : d|n, e^u < d \le e^{u+1}\}$$

can be easily interpreted in terms of the concentration function of the random variable D_n taking the values $\log d$, as d runs through all divisors of n, with uniform probability

$1/\tau(n)$. The best known results on the normal and average orders of $\Delta(n)$ are

(i) $\quad (\log\log n)^{\gamma} < \Delta(n) < \Psi(n) \log\log n$

for almost all n, where γ is any constant $< -\log 2/\log(1-1/\log 3) = 0{,}28754$, and $\Psi(n)$ is any function tending to infinity

(ii) $\quad x \log\log x \ll \sum_{n \le x} \Delta(n) < x\,\mathcal{L}(\log x)$

where $\mathcal{L}(u)$ is the slowly increasing function defined by

$$\mathcal{L}(u) = \exp\{c\sqrt{\log u \cdot \log\log u}\}$$

for a suitable absolute constant c.

Result (i) is due to Maier and Tenenbaum (Invent. Math. 76 (1984), and J. London Math. Soc., to appear). In particular, the lower bound solves an ancient conjecture of Erdős (1934).

Result (ii) was proved in a joint work with R.R. Hall (J. London Math. Soc. (2) 25 (1982)) for the lower bound, and the upper bound, proved by the author, is submitted for publication. It improves previous works by Hooley, and Hall-Tenenbaum. As shown by Hooley, it has applications on different branches in Number Theory, as Diophantine approximation and Waring's Problem.

A probabilistic interpretation of the results above can also be given, which contrasts surprisingly with the classical theorems of Probability Theory, like the Kolmogorov-Rogozin Inequality.

Gérald Tenenbaum (Nancy)

Brun - Titchmarsh theorem - application to Fermat last theorem.

We prove new upper bounds for the function $\pi(x;q,a)$ valid for almost all q between $x^{1/2}$ and $x^{1-\varepsilon}$. As an application, we prove, that infinitely often the greatest prime factor of $p-1$ exceeds $p^{0,6687}$. This result, combined with a criterion due to Adleman and Heath-Brown, implies that the first case of Fermat Last Theorem is true for infinitely many primes exponents. The improvements are based on results coming from dispersion method and Kloosterman sums.

Etienne Fouvry (Bordeaux)

Applications of Voronoi's summation formula to Riemann's zeta-function

The following results can be obtained using the classical Voronoi summation formula, or some or other of its generalizations.

1) Transformation of Dirichlet polynomials.

Let

$$S = S(M_1, M_2) = \sum_{M_1 \le m \le M_2} d(m)\, m^{-\frac{1}{2}-it},$$

$r = \frac{h}{k}$ (rational number), $hk \ll t^{1-\varepsilon}$, $\frac{t}{4\pi r} \le M_1 < \frac{t}{2\pi r} < M_2 \le \frac{t}{\pi r}$,

$M_j = \frac{t}{2\pi r} + (-1)^j m_j$, $m_1 \asymp m_2$ (i.e. of the same order). Suppose that $m_j \gg t^{\varepsilon} \max(t^{\frac{1}{2}} r^{-1}, t^{\frac{2}{5}} r^{-\frac{3}{5}} k, hk)$. Define $n_j = m_j^2 h^2 M_j^{-1}$, $\varphi(x) = \operatorname{arsinh}(x^{\frac{1}{2}}) + (x+x^2)^{1/2}$.

Then

$$S = \Big\{ (hk)^{-\frac{1}{2}} \Big(\log\Big(\frac{t}{2\pi}\Big) + 2\gamma - \log hk\Big)$$

$$+ \pi^{\frac{1}{4}} (2hkt)^{-\frac{1}{4}} \sum_{j=1}^{2} \sum_{n \le n_j} d(n)\, e\Big(n\Big(\frac{\bar h}{k} - \frac{1}{2hk}\Big)\Big) n^{-\frac{1}{4}} \Big(1 + \frac{\pi n}{2hkt}\Big)^{-\frac{1}{4}} \times$$

$$\times \exp\Big(i(-1)^{j-1}\Big(2t\varphi\Big(\frac{\pi n}{2hkt}\Big) + \frac{\pi}{4}\Big)\Big)\Big\} \chi\Big(\tfrac{1}{2}+it\Big) \Big(\frac{h}{k}\Big)^{it}$$

$$+ O\big(r^{-\frac{3}{2}} m_1^{-1} t^{\frac{1}{2}+\varepsilon}\big) + O\big(h\, m_1^{\frac{1}{2}} t^{-\frac{1}{2}+\varepsilon}\big) + O\big(h^{-\frac{1}{4}} k^{\frac{3}{4}} m_1^{-\frac{1}{4}} t^{\varepsilon}\big),$$

where γ is Euler's constant and the function $\chi(s)$ is as in the functional equation $\zeta(s) = \chi(s)\zeta(1-s)$. Furthermore, $\bar{h}$ is defined by $h\bar{h} \equiv 1 \pmod{k}$.
– A similar formula but with a better error term can be obtained for a "smoothed" sum.

2) The approximate functional equation for $\zeta^2(s)$.
Let $0 < \sigma < 1$, $t \geq 2$, and consider the error term $R(x,s)$ in the approximate functional equation

$$\zeta^2(s) = \sum_{n \leq x} d(n) n^{-s} + \chi^2(s) \sum_{n \leq y} d(n) n^{s-1} + R(x,s), \quad xy = (t/2\pi)^2.$$

Titchmarsh proved in 1938 that $R(x,s) \ll x^{\frac{1}{2}-\sigma} \log t$. The following improved version can be proved by Voronoi's summation formula:

$$R(x,s) \ll x^{-\sigma} t^{\frac{1}{2}} \min(1, xt^{-1}) \log t \log(tx^{-1} + t^{-1}x)$$
$$+ x^{1-\sigma}(1 + tx^{-1}) \min(x^{\varepsilon} + \log t, y^{\varepsilon} + \log t).$$

For $x \asymp t$ this result, like that of Titchmarsh, is best possible, but in the special case $x = y = \frac{t}{2\pi}$ much more is true, e.g. that

$$R(x,s) \ll t^{\frac{1}{3}-\sigma+\varepsilon}.$$

Motohashi found more explicit formulae, e.g. one relating $R(\frac{t}{2\pi}, s)$ to $\Delta(\frac{t}{2\pi})$, where Δ stands for the error term in the divisor problem. This last mentioned result can probably be proved also by Voronoi's summation formula (Motohashi uses a different method which leads to very precise asymptotic formulae).

3) The twelfth moment of L-functions.
A recent result of Tom Meurman (Univ. of Turku,

Thesis (1984): $\sum_{\chi \bmod q} \int_0^T |L(\tfrac{1}{2}+it, \chi)|^{12} dt \ll T^{2+\varepsilon} q^{3+\varepsilon}$.

Matti Jutila (Turku)

Covering sets by subsets

Let M be a finite non empty set and a mapping $M \to$ collection of all subsets of M, $x \mapsto M(x)$ such that
$M(x) \neq \phi$ for all $x \in M$ and $x \in M(y) \Rightarrow y \in M(x)$ for all $x, y \in M$.
Let $\mathcal{X}$ be the collection of all subsets $X \subset M$ such that
$\bigcup_{x \in X} M(x) = M$. Since $M \in \mathcal{X}$ the collection $\mathcal{X}$ is $\neq \phi$.

Put $m := \min_{X \in \mathcal{X}} |X|$ and $h := \min_{x \in M} |M(x)|$

The following upper estimations of m are stated:

(1) $m \leq |M| \dfrac{1+\log h}{h}$

(2) $m \leq 1 + \left[\dfrac{\log |M|}{\log \frac{1}{1-\frac{h}{|M|}}} \right]$

Applications to Abbotts lattic point problem and additive situations in number theory are given

Richard Warlimont, Regensburg

The fundamental lemma of Brun's sieve in a new setting

The following theorem has been announced

Theorem Let A be a finite set and $\{T_p\}$ a family of sets indexed by primes from a certain set P. Assume that for a certain multiplicative function $\omega(d)$ defined on all squarefree positive integers d and suitable real numbers $X>0$, $A_1 \geq 1$, $A_2 \geq 1$, A_3, $k \geq 1$, κ we have

$$0 \leq \frac{\omega(p)}{p} \leq 1 - \frac{1}{A_1} \quad \text{for all primes } p,$$

$$\sum_{w \leq p < z} \frac{\omega(p)\log p}{p} \leq \kappa \log\frac{z}{w} + A_2 \quad \text{for all } w, z \text{ with } 2 \leq w < z,$$

$$\Big|\,\Big|A \cap \bigcap_{\substack{p \in P \\ p \mid d}} T_p\Big| - \frac{\omega(d)}{d} X\Big| \leq A_3 X^{1-\frac{1}{k}} d^{k-1} \omega(d).$$

Then for all $z \leq X$ the number

$$S(A;P,z) = \Big|A \setminus \bigcup_{\substack{p \in P \\ p < z}} T_p\Big|$$

satisfies the relation

$$S(A;P,z) = X \prod_{p<z}\left(1 - \frac{\omega(p)}{p}\right)\Big\{1 + O\Big(\exp -\frac{u}{k^2}\big(\log u$$
$$- \log\log 3u - \log \kappa - 2\big) + O\big(\exp(-k\sqrt{\log X}\,)\big)\Big).$$

This theorem generalizes Theorem 2.5 from the book of H. Halberstam and H.-E. Richert, The sieve methods, obtained for $k=1$. In the typical applications to sets of lattice points k is the dimension of the relevant Euclidean space

Andrzej Schinzel
Warszawa

Multiplicative functions with $\Delta f \to 0$

The following theorem is proved:
Let $\mathcal{G}^*$ be the set of nonzero Gauss-integers (or integers in any other imaginary quadratic field). If $f: \mathcal{G}^* \to \mathbb{R}/\mathbb{Z}$ is additive and $\Delta f \to 0$ in the canonical metric of this group, then

$$f(a) = \tau \log |a| / \mathbb{Z} + k \frac{\arg a}{2\pi}, \quad \tau \in \mathbb{R}, \; k \in \mathbb{Z}.$$

This sharpens a theorem of Kátai and Amer, where the condition
$$|\Delta f(a)| \leq \delta(|a|), \quad \sum_r \delta(2^r) < \infty$$
is needed.

It applies to multiplicative $F: \mathbb{N} \to \mathbb{C}$ as follows: If again $\Delta F(a) \to 0$ as $|a| \to \infty$, then
$$F(a) \to 0 \quad \text{or} \quad F(a) = |a|^{\sigma + i\tau} e^{ik \arg a}.$$

Concerning the analog theorem with $\mathbb{N}$ instead of $\mathcal{G}^*$ see the lecture during the meeting on Diophantine Approximation, this April.

E. Wirsing

Additive properties of sequences.

Let $\mathcal{A}: 1 \leq a_1 < \ldots$ be a sequence of positive integers, $R_1(n)$ be the number of solutions $a_i + a_j = n$, $R_2(n)$ resp $R_3(n)$ be the number of solutions $a_i + a_j = n$, $a_i < a_j$ resp $a_i \leq a_j$. Different behaviours of $R_\ell(n)$ are investigated (Erdős–Sárközi, Erdős–Sárközi–Sós) E.g. If $R_1(n)$ is mon. increasing for $n > n_0$, then $n \in \mathcal{A}$ for $n > n_1$, but $\exists\, \mathcal{A}$ s.t. $R_2(n)$ is mon. increasing and $\sum_{a_i \leq n} 1 < n - c n^{1/3}$. $\mathcal{A}$ is called a Sidon-sequence, if

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$R_3(n) \le 1$. Analogously we call $S \subseteq G$ a Sidon-set of the group G, if for $\forall\ x, y, z, w \in S$ of which at least three are different $xy \neq zw$ (resp. $xy^{-1} \neq zw^{-1}$).

Theorem: Every group G of order n contains a Sidon-set of size $> cn^{1/3}$. For some abelian groups this $n^{1/2}$ (as best possible result) can be proved. These and some further results have applications in comb. grouptheory. (Results with L. Babai (European Journ. of Comb.)

Vera T. Sós

The distribution of reduced residues (mod q)

In 1940, Erdős posed the problem of showing that

$$\sum_{i=1}^{\varphi(q)} (a_{i+1} - a_i)^2 \ll q^2/\varphi(q)$$

where $1 = a_1 < a_2 < \ldots$ are the numbers relatively prime to q. Recently R. C. Vaughan and I received \$250 apiece for establishing this estimate. In fact we have shown that if $\gamma > 0$ is a fixed real number then

$$\sum_{i=1}^{\varphi(q)} (a_{i+1} - a_i)^\gamma \ll \varphi(q) \left(q/\varphi(q)\right)^\gamma .$$

(C. Hooley, F.R.S., received no money for showing this for $\gamma < 2$.)

This follows easily from the following more fundamental result: Let k be a fixed positive integer. Then

$$\sum_{n=1}^{q} \Big(\sum_{\substack{m=1 \\ (m+n,\, q)=1}}^{h} 1 \; - Ph \Big)^{2k} \ll q (Ph)^k + q Ph$$

where $P = \varphi(q)/q$ is the 'probability' that a randomly chosen integer is coprime to q. This estimate can be seen to be sharp if

$$\prod_{\substack{p \mid q \\ p > h}} \left(1 - \frac{1}{p}\right) \le \frac{1}{10} .$$

Our proof depends on combining estimates obtained by means of the finite Fourier transform with estimates based on combinatorial considerations.

Hugh L. Montgomery

Some contributions to lattice point theory

At first we consider a compact domain $\mathcal{D} \subseteq \mathbb{R}^2$ the boundary $\partial\mathcal{D} = C$ of which is a Jordan curve defined by $\phi(u,v)=0$, where ϕ is analytic on C and grad $\phi \neq (0,0)$. For a large parameter t, we consider the "lattice rest" $P(t) = |\sqrt{t}\mathcal{D} \cap \mathbb{Z}^2| - Vt$ (V the area of $\mathcal{D}$). It is a classical result of Van der Corput that $P(t) \ll t^{\theta}$ for some $\theta < \frac{1}{3}$, provided that the curvature $\varkappa$ of C vanishes nowhere. Colin de Verdière proved in 1977 that $P(t) \ll t^{(1-1/n)/2}$, if $\varkappa$ has zeros of order $\leq n-2$. We obtain

Theorem 1: Suppose that the slope of C is rational in every point P_i where $\varkappa = 0$ and let $n_i - 2$ denote the order of the zero of $\varkappa$ in P_i. Then

$$P(t) = \sum_{P_i : \varkappa = 0} \sum_{j \geq 1} F_{j,n_i}(t)\, t^{(1-j/n_i)/2} + O(t^{\theta}) \qquad (\theta < \tfrac{1}{3})$$

where the functions $F_{j,n_i}(t)$ are both $O(1)$ and (in general) $\Omega_{\pm}(1)$ as $t \to \infty$.

Furthermore, we can give several Ω-estimates

Theorem 2: Let $\mathcal{D} \subset \mathbb{R}^2$ be compact, convex, $\partial\mathcal{D} \in C^{\infty}$, $0 \in \mathcal{D}^{\circ}$ and suppose that $\varkappa \neq 0$ throughout. Then $P(t) = \Omega_{-}(t^{1/4}(\log t)^{1/4})$.

Theorem 3: For $s \geq 2$, $\mathcal{D} \subset \mathbb{R}^s$ suppose the above assumptions to be satisfied. Then $P(t) = \Omega(t^{(s-1)/4}(\log t)^{1/4})$.

The proofs of these results are based on asymptotic formulas for Fourier integrals over convex bodies due to E. Hlawka. Moreover, we deal with the sphere problem in $\mathbb{R}^3$: after I.M. Vinogradov, G. Szegö and Chandrasekharan/Narasimhan it is known that $P(t) \ll t^{2/3}\log^6 t$, $P(t) = \Omega_{-}(t^{1/2}(\log t)^{1/2})$ and $\overline{\lim}_{t\to\infty} P(t)\, t^{-1/2} = +\infty$. We have

Theorem 4: $P(t) = \Omega_{+}(t^{1/2}(\log_2 t)^{1/2}(\log_3 t)^{-1/2})$

The proof is based on a method of Gangadharan and an explicit formula for $r_3(n)$.

W. G. Nowak (Wien)

Nonlinear Evolution Equations

21. 10. 84 – 27. 10. 84

Integrodifferential equations of Volterra type

Consider the initial-boundary value problem for the integrodifferential equation of Volterra type:

$$\frac{\partial u}{\partial t}(x,t) - \Delta u(x,t) = \int_0^t a(t-s) \sum_{i=1}^n \frac{\partial}{\partial x_i} \sigma_i(\nabla u(x,s))\, ds, \quad x \in \Omega, \quad 0<t<\infty,$$

$$u(x,t) = 0 \qquad x \in \partial\Omega, \quad 0<t<\infty,$$

$$u(x,0) = u_0(x) \qquad x \in \Omega,$$

where σ_i, $i=1,\dots,n$, are real valued functions with bounded continuous first order derivatives in $\mathbb{R}^n$. For any initial value $u_0 \in W_0^{1,p}(\Omega)$, $1<p<\infty$, an L^p solution of this problem exists. The uniqueness follows from the fact that if we consider the above problem in the space $W^{-1,p}(\Omega)$, then the nonlinear part satisfies a uniform Lipschitz condition. That Δ with the Dirichlet boundary condition generates an analytic semi group in $W^{-1,p}(\Omega)$ can be shown with the aid of R. Seeley's interpolation theorem. Professor H. Amann kindly informed me a proof of the last fact which is simpler and derives a better result than what I deviced.

Hiroki Tanabe (Osaka, Japan)

Maximal regularity and periodic solutions of parabolic equations (G. DA PRATO, PISA, ITALY)

Let X and D be Banach spaces with $D \subseteq X$. Let $\{A(t)\}_{t \in [0,T]}$ be a family of linear operators with domain D and such that:

i) $A(t)$ generates an analytic semigroup in X

ii) $A \in C^\alpha([0,2\pi]; \mathcal{L}(D; X))$, $\alpha \in]0,1[$

THEOREM 1 (G. DA PRATO, A. LUNARDI)

Assume that 1 belongs to the resolvent set of $G(2\pi)$ ($G(t)$ is the Green function relative to A). Then if $f \in C^\alpha([0,2\pi]; X)$, $f(0) = f(2\pi)$ and $A(0) = A(2\pi)$ there exists a unique strict solution of the problem

(1) $\quad u'(t) = A(t)u(t) + f(t) \quad , \quad u(0) = u(2\pi)$

This result is used to study, by linearization, the nonlinear problem:

(2) $\quad u'(t) = f(\lambda, t, u(t)) \quad , \quad u(0) = u(2\pi)$

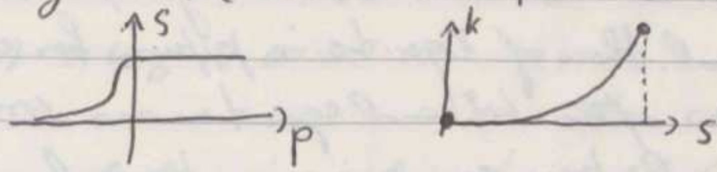
where $f: [0,1] \times [0,2\pi] \times D \to X$ is regular and such that $f_x(0,t,0) = A(t)$ verifies the hypotheses of Theorem 1.

Non-steady fluid flow through porous media

The flow of water through an earth dam is described by the elliptic-parabolic equation

$$\partial_t s(p) - \nabla \cdot k(s(p))(\nabla p + e_n) = 0 \quad \text{in } \Omega \subset \mathbb{R}^n,$$

where p is the pressure, and the saturation s and the conductivity k are nonlinear functions like



We impose Dirichlet, Neumann, and overflow condition on $\partial\Omega$, and initial condition on s. Under regularity assumptions on the data there is a unique solution.

For certain applications one is interested in the solution p_ε for for saturation functions $s_\varepsilon(\xi) := s(\xi/\varepsilon)$ as $\varepsilon \to 0$. In the limit one gets the well known elliptic-hyperbolic free boundary problem

$$\partial_t s - \nabla \cdot k(s)(\nabla p + e) = 0 \text{ in } \Omega \quad \text{with } s \in \chi_{\{p>0\}}.$$

We prove the strong convergence of a subsequence of p_ε and $s_\varepsilon(p_\varepsilon)$ to a solution p, s satisfying an entropy condition. (Joint work with S. Luckhaus)

H. W. Alt

Nonlinear Wave Equations with Scalar Nonlinearities

Let h be a separable Hilbert space with scalar product (u, v) and $A: D_A \to h$ a linear, self-adjoint operator with a purely point spectrum $Au_i = \lambda_i u_i$, $i = 1, 2, \ldots$ Let μ, δ, ς be constants and $f_1, f_2, f_3, \vec{\beta}$ real valued functions. D. Wenacht and I consider nonlinear equations:

$$\ddot{u} + \mu\dot{u} + \delta A^2 u + \varsigma A^2\dot{u} + f_1\{\tfrac{1}{2}(Au, u)\}Au + f_2\{\tfrac{1}{2}(Au, \dot{u})\}Au + f_3\{\tfrac{1}{2}(A\dot{u}, \dot{u})\}A\dot{u} = \vec{\beta}(t),$$

and ask for periodic solutions in "t". Writing $u = \sum_{i=1}^{\infty} \alpha_i(t) u_i$, we obtain an equivalent coupled system of nonlinear ordinary differential equations for $\vec{\alpha}(t)$ in l^2. The simplest, uncoupled motions of such equations are determined by the generalized Liénard equation

$$\ddot{\alpha} + \mu\dot{\alpha} + \gamma\lambda^2\alpha + \delta\lambda^2\dot{\alpha} + g_1\{\tfrac{1}{2}\lambda\alpha^2\}\lambda\alpha$$
$$+ g_2\{\tfrac{1}{2}\lambda\alpha\dot{\alpha}\}\lambda\alpha + g_3\{\tfrac{1}{2}\lambda\dot{\alpha}^2\}\lambda\dot{\alpha} = \beta(t),$$

assuming that only one component is forced. These equations correspond to the motions of extensible beams, panel flutter and flow of liquids in pipes, for example. Periodic solutions of the Liénard equation are complicated and are determined by an averaging procedure and a method of L. Brüll. The complexity of solutions possible for the given equation is enormous.

N. Bagley

Quasilinear Parabolic Systems

We consider general quasilinear parabolic systems of order 2m acting on N-vector valued functions under Dirichlet boundary conditions on a bounded domain. Given only regularity hypotheses, but neither compatibility nor growth conditions, we show that there exists a unique maximal classical solution depending continuously upon the initial data. The proof is based upon a general theorem about quasilinear parabolic evolution equations in Banach spaces.

Herbert Amann (Zürich)

Global boundedness for a differential-delay equation

It is shown that solutions of

$$\partial_t u - \Delta u = u(1 - u(\cdot, \cdot - \tau)) \qquad u = u^0 \geq 0 \text{ or } \partial_\nu u = 0 \text{ on } \partial\Omega$$

$$u(\cdot, 0) = u_0 \geq 0$$

are bounded with a bound (depending on u^0, Ω) independent of u_0 in the cases
$\Omega = (a, b)$, or $\tau < \tau(u)$ ($\tau(u, \mathrm{curv}\,\partial\Omega)$) in the case of Neumann conditions).

On the other hand for $\Omega = \mathbb{R}$ or $\dim(\Omega) > 1$, Ω large enough there are at least examples $u \geq 0$, $\mathrm{supp}(u) \subset\subset \Omega$, which are solutions of the inequality $\partial_t u - \Delta u \leq u(1 - u(\cdot, \cdot - \tau))$ and grow like $e^{\alpha t}$

Stephan Luckhaus

A non-stationary free boundary problem for the Navier-Stokes equations

It is shown that a classical solution to the following free boundary problem exists

$$v_t - \nu \Delta v + Dp + (v \cdot D) v = f_0 + h \quad \text{in } \cup \Omega(t)$$
$$D \cdot v = 0$$

$$v \cdot n = v_\Sigma, \quad t_k \cdot T \cdot n = 0, \quad n \cdot T \cdot n = 0 \quad \text{on } \cup \Sigma(t)$$
$$v(x, 0) = v_0(x) \quad \forall x \in \Omega(0).$$

This system describes the flow of a viscous incompressible fluid body under the influence of self attraction $f_0 =$

$= D \int_{\Omega(t)} |x-y|^{-1} dy$ and an additional force h that generates a flow.

The proof is based on a version of the hard implicit function theorem which is due to E. Zehnder and on transforming the implicit equation $n \cdot T \cdot n = 0$ into an integral equation; this was studied first by L. Lichtenstein.

The solution exists for a small intervall of time; the associated stationary problem admits a classical solution, too. The system (*) may be regarded as a generalization of the classical equilibrium figures of rotating liquids.

Josef Bemelmans (Saarbrücken)

Some new aspects of Hopf bifurcation for evolution equations.

We consider parameter-dependent evolution equations

$$\frac{du}{dt} + A(\lambda)u + F(\lambda,u) = 0$$

in some real Hilbert space E. The classical Hopf bifurcation theorem was considerably generalized by Alexander-Yorke, Ize, Chow-Mallet-Paret-Yorke. Their condition simply says that at a critical value λ_0 a nonzero number of eigenvalues of $A(\lambda)$ crosses the imaginary axis apart from zero. We think that this nonzero crossing number is not the essential condition for Hopf bifurcation. After fixing the phase and eliminating the unknown period we end up with a system of codimension 1. It is a nonzero crossing number of this reduced system which entails bifurcation. Our condition allows a zero crossing number of the original system as well as an eigenvalue zero of $A(\lambda_0)$.

Hansjörg Kielhöfer (Augsburg)

A SPECIAL CLASS OF QUASILINEAR EVOLUTION EQUATIONS

We consider the evolution equation on a Hilbert space $\mathcal{H}$ (real)

$$(*) \qquad \frac{du}{dt} + N(u) - \lambda u = h(t)$$

where the nonlinear term N is given by

$$N(u) = g\{\tfrac{1}{2}(Au,u)\}\, Au$$

and is a "scalar nonlinearity" introduced independently by MEDEIROS and by BAZLEY and KÜPPER. The operator A is assumed to be positive-definite and self-adjoint. For simplicity, consider the case when A has purely discrete spectrum with eigenvalues $\lambda_1 \le \lambda_2 \le \dots \le \lambda_n \le \to +\infty$ and a corresponding complete set of eigenfunctions $\{u_i\}$ which are orthonormal. The real valued function g is assumed positive and nondecreasing. The forcing term $h(t)$ is given, as is the real parameter λ.

The equation (*) can be resolved explicitly in terms of a solution of a scalar nonlinear integrodifferential equation of nonlocal type. We also consider equilibrium solutions of (*) and present results on their stability.

R J Weinacht (NEWARK)

Gradient Estimates for First Order Quasilinear Evolution Equations in Bounded Domains

For Hamilton-Jacobi equations

$$(*) \qquad \partial_t u + H(x, u, \nabla_x u) = 0$$

in bounded time-space cylinders $\Omega \times [0,T] \subset \mathbb{R}^{n+1}$

it was shown how to derive a priori estimates for $\|\nabla_x u\|_{L^\infty}$ and $\|\partial_t u\|_{L^\infty}$ for the "viscosity" solutions u that are obtained via parabolic regularization

$$\partial_t u^\varepsilon - \varepsilon L u^\varepsilon + H(x, u^\varepsilon, \nabla_x u^\varepsilon) = 0 .$$

Through suitable choices of the elliptic operator L, a pointwise condition linking the Hamiltonian H and the boundary data φ on $\partial\Omega \times [0,T]$ could be used (via Bernstein's technique) which allows the extension of the solution operator for (*) to spaces of continuous boundary data in a natural manner. The stationary equation corresponding to (*) was also discussed.

Hans Engler (Washington, USA)

Continuous Glimm functionals

Consider a system of conservation laws

$$u_t + f(u)_x = 0 \qquad t \geq 0,\ x \in \mathbb{R},\ u \in \mathcal{U} \subset \mathbb{R}^N$$

which is assumed to be strictly hyperbolic i.e. $Df(u)$ has N distinct real eigenvalues

$$\lambda_1(u) < \dots < \lambda_N(u),$$

with corresponding eigenvectors

$$Df(u)\, r_i(u) = \lambda_i(u)\, r_i(u)$$

and eigenlinear forms

$$l_i(u)\, Df(u) = \lambda_i(u)\, l_i(u).$$

There exists a functional $\mathcal{F}$ such that: If u is piecewise Lipschitz continuous, satisfies a suitable entropy condition, ~~then~~ and is locally self-similar at meeting points of singularities or points where singularities appear or disappear, and if the initial total variation of u is small enough, then $t \mapsto \mathcal{F}(u(t))$ deverses. The functional $\mathcal{F}$ satisfies the inequalities

$$C^{-1}\, TV(u) \leq \mathcal{F}(u) \leq C\, TV(u)$$

for all u with values in $\mathcal{U}_0$ small enough, and of bounded variation. When u is smooth,

$$\mathcal{F}(u) = \sum_i \int |l_i u_x|\,dx + M\Big\{ \sum_{i>j} \int_{x<y} |l_i u_x|(x)\,|l_j u_y|(y)\,dx\,dy$$

$$+ \Big\{ \sum_{i \in GNL} \int_{x<y} \Big(|l_i u_x|(x)\,|l_i u_y| - l_i u_x^+\, l_i u_y^+ \Big)\,dx\,dy \Big)$$

where M is a suitable constant.

The characteristic fields are either genuinely nonlinear: $D\lambda_i\, r_i \equiv 1$ or linearly degenerate $D\lambda_i\, r_i \equiv 0$.

Michelle Schatzman, Lyon, France.

Blow up for non-local evolution equations

Let $\omega = {}^t(\omega^1, \omega^2, \omega^3)$ be the curl of the solution to the incompressible Euler equations in $\mathbb{R}^3$. Then ω satisfies the Helmholtz equation

(H) $\quad \omega_t + u(\omega)\cdot\nabla\omega = \mathcal{U}(\omega)\,\omega \quad$ with $u(\omega)$, $\mathcal{U}(\omega)$

given by $(-\Delta)^{-1/2}(R\times\omega)$, $\mathcal{U}_{ij} = R_j (R\times\omega)_i$, R_j the Riesz transform

We prove that the first dimensional toy model (A, Majda)

$$\omega_t = \omega H \omega$$
$$\omega(0) = \omega_0$$

H = the Hilbert transform, is explicitly solvable and thus characterize the blow up conditions.

We also prove that the "fake Euler" system

$$\begin{cases} U_t + U^2 = -\mathcal{R}(\mathrm{Tr}\, U^2) \\ U(0) \text{ given} \end{cases} \qquad U = (U_{ij})_{i,j=1\cdots n}$$

$$\mathcal{R} = (R_i R_j)_{i,j}$$

blows up in finite time, for u_0 suitably chosen.

P. Constantin, Courant Institute NY.

Existence questions for the Navier-Stokes system.

We consider a fundamental question for the Navier-Stokes system. Of course we have a global weak solution which satisfies the energy inequality. If the space dimension is three because of the lack of apriori estimate we do not know the regularity. We try to clarify the difference between two dimensional case and three dimensional case. If we use the concept of scaling dim., our theorems can be read as follows. If a solution have a finite scaling dimension zero estimate, then solution should be regular. (This description is slightly philosophical.) In fact we present theorem which can be explained in this way. In two dimensional case energy estimate has scaling dim zero, so regularity follows of course. We also derive condition which guarantee the regularity. Mostly we consider only bdd domain.

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Non linear integrodifferential equations in Banach spaces

Eugenio SINESTRARI, Dipartimento di Matematica
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(ITALIEN)

Strict solutions are ~~sought~~ found for the equation

$$\begin{cases} u'(t) = f(t, u(t)) + \int_0^t g(t, s, u(s))\, ds, & t \geq 0 \\ u(0) = u_0 \end{cases}$$

where $f : R_+ \times D \to E$, $g : \{(t,s) \in R^2, 0 \leq s \leq t\} \times D \to E$ with $D \subseteq E$ Banach spaces under the assumption that there exists $f_u(0, u_0)$ and generates an analytic semigroup in E with domain D — The dependence of f ~~and~~ and g ~~from~~ on t is α-hölder and that of g on s is $L^{1/1-\alpha}$ while f and g depend on u in C^2 way — the method of solution is based on perturbation methods and maximal regularity results for the problem

$$\begin{cases} u'(t) = Au(t) + f(t) \\ u(0) = u_0 \end{cases}$$

with A generator of analytic semigroup and f hölder continuous —

In the autonomous and convolution case, conditions are given for the global existence of a solution and its Lyapunov stability for 'small' initial data —

Applications are given to classical solutions of the partial ~~p.d. equa~~ integro differential equation

$$\begin{cases} u_t(t,x) = f(\Delta u(t,x)) + \int_0^t g(t, s, \Delta u(s,x))\, ds + \varphi(t) & (t,x) \in [0,T] \times \overline{\Omega} \\ u(t,x) = 0 & (t,x) \in [0,T] \times \partial\Omega \\ u(0,x) = u_0(x) & x \in \overline{\Omega} \end{cases}$$

Wellposedness of some quasilinear equations from population biology

Let $\Omega \subset \mathbb{R}^N$ be a bounded and smooth domain and consider the problem

$$(1)\quad \begin{cases} u_t + u_a + \delta u = \mathrm{div}_x \left[u \int_0^\infty K(a,a',x,u) \nabla_x u(t,a',x)\, da' \right], & t,a \geq 0,\ x \in \Omega \\ u \cdot \int_0^\infty K(a,a',x,u) \frac{\partial u}{\partial n}(t,a',x)\, da' = 0, & t,a \geq 0,\ x \in \partial\Omega \\ u(t,0,x) = \int_0^\infty \beta(a,x,u) u(t,a,x)\, da & t \geq 0,\ x \in \bar{\Omega} \\ u(0,a,x) = u_0(a,x) & a \geq 0,\ x \in \Omega, \end{cases}$$

where δ, β, K, u_0 are given functions and $n(x)$ denotes the outer normal at $x \in \partial\Omega$. We showed how this problem arises in population biology and presented several wellposedness results in $L^1_+(\mathbb{R}_+ \times \Omega)$, the natural space for this problem. In particular, we proved the existence of a nonlinear semigroup $T(t)$ in $L^1_+(\mathbb{R}_+ \times \Omega)$ associated with (1), assuming $\beta = \beta(a,x)$, $\delta = \delta(a,x)$ and $K = K(a,x)\delta_0(a-a')$, i.e. for the case when the right hand side of eq. (1) is a local operator also with respect to a.

Jan Prüß

Scattering theory in the energy space for the nonlinear Schrödinger equation (NLS). (joint work with G. Velo)

We present a general theory of scattering for the NLS equation $i\varphi_t = -(1/2)\Delta\varphi + f(\varphi)$ for general initial data and asymptotic states in the energy space H^1. Here φ is a complex function defined in space time $\mathbb{R}^{n+1}$ and f is a nonlinear interaction, typically

$$f(\varphi) = \left(\lambda_1 |\varphi|^{p_1 - 1} + \lambda_2 |\varphi|^{p_2 - 1}\right)\varphi \qquad (*)$$

with $1 \leq p_1 \leq p_2 < \infty$. The existence of the wave operators is proved by solving the Cauchy problem at infinity by a contraction method in a space of functions satisfying suitable space-time integrability properties. That argument yields asymptotic completeness for small data in H^1 as a by product. Finally, for $n \geqslant 3$, asymptotic completeness in H^1 for repulsive interactions is proved by an extension of the method of Lin and Strauss. In particular, all solutions with initial data in H^1 are proved to satisfy suitable space time integrability properties under assumptions on f that reduce to $\lambda_i > 0$ and $1 + 4/n < p_1 \leq p_2 < 1 + 4/(n-2)$ in the special case $(*)$.

Jean Ginibre

Periodic solutions to some nonlinear evolution equations

The problem (P) given by

$$(P)\begin{cases} c\,(-1)^{m+1}\dfrac{\partial^{2m}}{\partial t^{2m}}u(t,x) + (-1)^{n}\dfrac{\partial^{2n}u}{\partial x^{2n}}(t,x) = F(u(t,x)), & 0<x<\pi, \\ \dfrac{\partial^{2j}u}{\partial x^{2j}}(t,0) = \dfrac{\partial^{2j}}{\partial x^{2j}}(t,\pi) = 0, & t \in R,\ j = 0,\dots,n-1, \\ u(t+2\pi\cancel{\phi},x) = u(t,x) = 0, & t \in R,\ 0<x<\pi \end{cases}$$

is considered, where $m, n \in N$, $c > 0$ and F is an odd sublinear or superlinear function. The assumptions on c, m, n and F are stated under which the problem (P) has a sequence of generalized nonzero solutions, whose (suitable) norms converge to $\cancel{\infty}$ 0 (if F is sublinear) or to ∞ (if F is superlinear)

Vladimír Lovicar

On Blow-up for a class of Nonlinear Schrödinger equations.

We are looking for nonexistence results for quasilinear Schrödinger equations of the form (for the one-dimensional case)

$$(1)\begin{cases} iu_t = -u_{xx} + f(|u|^2)u + \varkappa\, \partial_x^2 h(|u|^2)\cdot h'(|u|^2)\cdot u \\ u(x,0) = u_0(x) \end{cases}$$

with given functions f and h, $\varkappa = \text{const}$. If certain assumptions are satisfied on f, h and the initial data, namely

(2) $f(s)s \le c\, g(s) \quad (\forall s \ge 0), \quad g(s) = \int_0^s f(t)\,dt;$

(3) $c \ge 3 \ (\varkappa = 0), \quad 3 \le c \le 4 \ (\varkappa > 0), \quad c \ge 4 \ (\varkappa < 0)$

(4) $\varkappa \dfrac{d}{ds} h'^2(s) \ge 0 \quad (\forall s \ge 0)$

(5) <u>either</u> $E_1(0) < 0$

<u>or</u> $E_1(0) \ge 0, \ 2G(0)^2 > (c-1)E_1(0)F(0)$ *)

(where $E_1(0)$ is defined by

$$E_1(t) = \int_{\mathbb{R}} \left\{ |\partial_x u(x,t)|^2 + g(|u(x,t)|^2) - \frac{\varkappa}{2}\left[\partial_x h(|u(x,t)|^2)\right]^2 \right\} dx \,),$$

we can prove, that any classical solution $u(x,t)$ (with the right decay properties) has a finite lifetime T^* and there is a $T \in (0, T^*]$ such that

$$\lim_{t\to T} \int |\partial_x u(x,t)|^2 dx = \infty, \quad \lim_{t\to T} \max_x |u(x,t)| = \infty, \quad \lim_{t\to T} \int |u(x,t)|^p dx = \begin{cases} \infty & p > 2 \\ 0 & 1 \le p < 2 \end{cases}$$

*) $G(0) = \operatorname{Im} \int x u_0^* \partial_x u_0\, dx, \quad F(0) = \int x^2 |u_0|^2 dx$

Horst Lange

Regularity problems for the equations of Navier Stokes

We prove the regularity of weak solutions of the equations of Navier Stokes if the space variables are sufficiently large; it holds the following

Theorem: Let $\Omega \subseteq \mathbb{R}^3$ be a smooth (C^∞) exterior domain, $u_0 \in \mathring{H}^{1/2}(\Omega) \cap H^{3/2}(\Omega) \cap L^q(\Omega)$ with $\frac{10}{7} \le q < \frac{3}{2}$, and let u be a weak solution of the Navier Stokes equations

$$u_t - \Delta u + (u \cdot \nabla) u + \nabla p = 0, \quad \operatorname{div} u = 0, \quad u|_{\partial\Omega} = 0, \quad u(0) = u_0$$

such that additionally the generalized energy inequality holds. Then there exist constants $K > 0$, $L > 0$ such that

$$|u(x,t)| \le K \quad \text{a.e.}$$

holds for $|x| \ge L$, $t \ge 0$. It follows the C^∞-regularity of u and p in space for large $|x|$ and $t > 0$.

Hermann Sohr

The Cauchy problem in the energy space for the non linear Schrödinger equation and for the non linear Klein–Gordon equation (joint work with J. Ginibre)

We review the existence and uniqueness results for the solutions of the Cauchy problem for the non linear Schrödinger equation (NLS) $i\dot{\varphi} = -\frac{1}{2}\Delta\varphi + f(\varphi)$ and for the non linear Klein–Gordon equation (NLKG) $\Box\varphi + f(\varphi) = 0$ for general initial data in the energy space. A typical f is

$$f(\varphi) = \left(\lambda_1 |\varphi|^{p_1 - 1} + \lambda_2 |\varphi|^{p_2 - 1}\right)\varphi$$

with $1 \le p_1 < p_2$ and $\lambda_2 > 0$. We compare the results obtained by the method of contraction for the local Cauchy problem and a priori estimates for the global problem with the method of compactness for the existence of weak global solutions and partial contraction for uniqueness. The best results are obtained by the second method, whereby global existence and uniqueness are proved, in the case of the previously written f, for $1 \le p_1 \le p_2 < 1 + \frac{4}{n-2}$ for the NLKG equation, and under a similar, but slightly stronger condition for the NLS equation.

Giorgio Velo

Uniform estimates for solutions of some semilinear evolution equations.

We consider semilinear Klein-Gordon equations of the form

(K.G) $u_{tt} - \Delta u + m^2 u = g(u)$ in $\mathbb{R}^N$

and semilinear heat equations of the form

(H) $u_t - \Delta u = g(u)$ in $\Omega \subset \mathbb{R}^N$, $u_{|\partial\Omega} = 0$

On applying methods based upon differential inequalities we prove that if g satisfies some growth and some superlinearity conditions, then any global solution (that is any solution that exist for all $t \geq 0$) to (K.G) or (H) is uniformly bounded (with respect to t) in some appropriate norm.

Thierry Cazenave.

Everywhere defined scattering operators for nonlinear Klein-Gordon equations.

The proof of the existence of everywhere defined scattering-operators for certain nonlinear Klein-Gordon equations. We derive slight extensions of the $L_r(\mathbb{R}; L_q^s(\mathbb{R}^n))$-estimates due to Segal, Strichartz, Marshall (and Pecher for the wave equation), and apply previous results which relate space-time means of solutions to the linear and the nonlinear Klein-Gordon equation. This allows the extension of some of the $L_r(L_q^s)$-estimates for the linear equation to the corresponding nonlinear equation. Applying an argument due to W. Strauss in the small-data-case the existence of a Scattering operator defined on all of the energy-space for nonlinearities of the form $|u|^{\rho-1}u$, where $1+4/n < \rho \leq 1+4/(n-1)$, $n \geq 3$.

Philip Brenner

Low Energy Scattering for Nonlinear Klein-Gordon Equations

We consider the pair of equations

$u_{tt} + Au + f(u) = 0$ (NLKG)

$u_{tt} + Au = 0$ (KG),

where $u: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$, $A = -\Delta + m^2$ $(m \neq 0)$, $f \in C^1(\mathbb{R}, \mathbb{R})$, $f(0) = 0$, $|f'(s)| \leq c\,|s|^{\rho-1}$ $\forall s \in \mathbb{R}$.

The following THEOREM holds:

If $1 + \frac{4}{n} \leq \rho < 1 + \frac{4}{n-2}$, $\rho < \infty$, $n \geq 2$ arbitrary, then there exists the scattering operator in the sense of energy norms for the pair of equations above in a whole neighbourhood of the origin in energy space. The result also holds in the case $\rho = 1 + \frac{4}{n-2}$, provided $3 \leq n \leq 5$. This improves previous work of W. Strauss and M. Tsutsumi - N. Hayashi. The main ingredients of the proof are space-time estimates for (KG) and Schauder's fixed point theorem.

Hartmut Pecher (Wuppertal)

Formation of Singularities in Three Dimensional Compressible Fluids

The compressible Euler equations of gas dynamics

$$\rho_t + \nabla\cdot(\rho u) = 0$$
$$\rho(u_t + u\cdot\nabla u) + \nabla p = 0 \,, \quad p = \rho^\gamma e^S \quad (\gamma > 1)$$
$$S_t + u\cdot\nabla S = 0$$

with smooth initial data

$$\rho(x,0) = \rho_0(x) > 0, \quad u(x,0) = u_0(x), \quad S(x,0) = S_0(x)$$

have a C^1 local solution defined on $\mathbb{R}^3 \times [0,T)$ for some $T > 0$. The maximal value for T is called the life span of the solution. If the gas is initially localized and, on average, compressed and outgoing, then singularities develop; that is, the life span T is finite. To be precise:

Th$^{\underline{m}}$: If there exist positive constants $R_1 < R_2$ such that

(i) $(\rho_0(x), u_0(x), S_0(x)) \equiv (\bar\rho, \bar u, \bar S) = \text{Const.}$, for $|x| \geq R_2$,

(ii) $\displaystyle\int_{|x|>r} |x|^{-1} (|x|-r)^2 (\rho_0 - \bar\rho)\, dx > 0$, $R_1 < r < R_2$, and

(iii) $\displaystyle\int_{|x|>r} |x|^{-3} (|x|^2 - r^2)\, \rho_0 \,[x\cdot(u_0 - \bar u)]\, dx > 0$, $R_1 < r < R_2$,

then the life span T is finite.

– T. Sideris

Regularity results for Boltzmann and Vlasov Equation.

These equation deal with functions $f(x,v,t)$ defined in the (x,v) phase space. In this talk we emphasize the similarity beetween the regularity results for these equations and the one concerning the "macroscopic equations" like the ~~wav~~ non linear wave equation.

We show that the function

$\int f(x,v,t)dv$ may eventually decay

uniformly for $t \longrightarrow \infty$ (for conveniently small enough and localised initial data). ~~and in~~ This is a "dispersion" property and it is used to prove existence in the large for small initial data and large space di-
-mension ($d \geq 3$).

Claude Bardos.

Kato's Inequality: A Characterization of Generators of Positive Semigroups.

Let A be the generator of a (linear) C_0-semi-group on $X = L^p(\Omega, \Sigma, \mu)$, where $1 \leq p < \infty$ and (X, Σ, μ) is σ-finite. Then the semigroup consists of positive (= positivity preserving) operators iff A satisfies the following abstract version of Kato's inequality

$$\langle \operatorname{sign} f\, Af, \varphi \rangle \leq \langle |f|, A'\varphi \rangle$$
$$f \in D(A), \quad 0 \leq \varphi \in D(A')$$

and there exists $\varphi \in D(A')$ such that $\varphi(x) > 0$ a.e and $A'\varphi \leq \lambda\varphi$ for some $\lambda \in \mathbb{R}$ (where A' denotes the adjoint of A).

Wolfgang Arendt

Single Point Blow-up for a Semilinear Heat Equation

We consider the finite time blow-up behavior of solutions to the semilinear heat equation:

$$(*) \quad \begin{cases} u_t(t,x) = \Delta u(t,x) + F(u(t,x)) & , t>0, \ x \in \Omega \\ u(t,y) = 0 & t>0, \ y \in \partial\Omega \\ u(0,x) = f(x) \geq 0, & x \in \overline{\Omega} \end{cases}$$

which is formally equivalent to the integral equation

$$(**) \quad u(t) = e^{t\Delta} f + \int_0^t e^{(t-s)\Delta} F(u(s))\, ds.$$

<u>Theorem</u>. Suppose $\Omega = \{x \in \mathbb{R}^n : |x| < R\}$ or $\mathbb{R}^n$. Assume $f \geq 0$ in $C_0(\Omega)$ is radially symmetric and radially non-increasing. Suppose further that $F: [0,\infty) \to [0,\infty)$ is convex, $F(0) = 0$, $F(x) > 0$ for $x > 0$, and $\liminf_{x\to\infty} xF'(x)/F(x) > 1$. Let $u(t) = u(t,\cdot)$ be the corresponding maximal solution of $(**)$ and $(*)$, and suppose its existence time ~~$\emptyset$~~ T is finite. Then for all $x \neq 0$ in Ω:

$$\limsup_{t\to T} u(t,x) < \infty.$$

Fred B. Weissler
10/26/84

Local classical solvability of an initial-boundary-value problem for a quasilinear hyperbolic equation with the third boundary condition.

The problem

$$u_{tt}(t,x) - \partial_i\big(a_{ij}(t,x,u(t,x))\,\partial_j u(t,x)\big) = f(t,x,u(t,x),\nabla_x u(t,x)) \quad , x \in \Omega \subset \mathbb{R}^3$$

bd, C^∞-bdr.

$$a_{ij}(t,x,u(t,x))\cdot \nu_i(x)\,\partial_j u(t,x) + \sigma(t,x)u(t,x) = 0 \quad , x \in \partial\Omega$$

$\nu :=$ outer Normal to Ω

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x).$$

is solved in the space $u \in C^0([0,T[, H^4(\Omega)) \cap C^1([0,T[, H^3(\Omega)) \cdots \cap C^4([0,T[, L^2(\Omega))$ (which is embedded into $C^2([0,T]\times\overline{\Omega})$), provided that:

(1) $u_0 \in H^4(\Omega)$, $u_1 \in H^3(\Omega)$, and u_0, u_1 satisfy some compatibility conditions

(2) a_{ij}, f, σ are sufficiently smooth functions on all their arguments.

This extends a result by IKAWA, who treated the linear case. (J. Math. Soc. Jp. Vol. 20, 1968). In the quasilinear case a different regularity method is necessary.

Peter Weidemaier

Global existence of classical solutions to the initial boundary value problem for the NLS and the NLKG equations in an exterior domain.

We consider the problem

$$\begin{cases} i u_t = Lu + f(u) & \Omega \times [0,\infty) \\ u|_{\partial\Omega} = 0 \\ u(x,0) = u_0(x) \end{cases}$$

where Ω is an exterior domain with smooth bdy and

$$Lu = \sum_{i,j=1}^{n} \partial_i(a_{ij}\partial_j u) + q u.$$

Under the some assumptions on the coefficients of L and the nonlinear term $f(u)$, we can prove the global existence of classical solutions provided the initial data are sufficiently small. Morawetz type estimation is found to be useful to prove the space-time estimates for solutions of the corresponding linear problem, which is played an important role to establish the global existence.

Masayoshi Tsutsumi
Waseda University, Tokyo Japan

Let us consider the problem:

$$\begin{cases} u_{tt}(x,t) = (u(x,t))^m_{xx} & a < x < b \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) & \\ u(a,t) = u(b,t) = 0 & (t \geq 0) \end{cases}$$

where m is an ~~odd~~ integer with $m \geq 5$.

We establish the local existence of classical solutions giving suitable initial data and prove the uniqueness of solutions in the class.

Yukiyoshi Ebihara
Fukuoka University, Japan

GEOMETRIE

28.10 – 3.11.84

Über die Totale Absolutkrümmung geschlossener Kurven im sphärischen Raum

Für die totale Absolutkrümmung TAK $:= \frac{1}{\pi}\int |\varkappa| ds$ ($\varkappa$ = Kurvenkrümmung, ds = Bogenelement) geschlossener Kurven des Euklidischen Raumes gelten die Fenchel'sche Ungleichung und speziell für verknotete Kurven die Ungleichung von Fary – Milnor – Fox. Wir beweisen für geschlossene Kurven des sphärischen Raumes S^n die folgenden Ungleichungen:

$$2\left(1 - \frac{L}{2\pi}\right) \leq \text{TAK} \quad , L = \text{Länge der Kurve}$$

und speziell für verknotete Kurven:

$$2\left(g - \frac{L}{2\pi}\right) \leq \text{TAK} \quad , g := \text{minimale Anzahl Erzeugender, um die Knotengruppe zu erzeugen.}$$

Außerdem wird eine der ersten Ungleichung entsprechende für Flächen in S^n angegeben.

Eberhard Teufel.

Die zweifachen Blutelschen Kegelschnittflächen

Es werden im projektiven Raum $P^3(\mathbb{R})$ konjugierte Netze aus Kegelschnitten, derart daß die Trägerfläche bezüglich der 1. Schar eine BLUTELsche Kegelschnittfläche ist, betrachtet. Es zeigt sich, daß dies dann auch für die 2. Schar gilt und daß die Trägerebenen beider Kegelschnittscharen je einem Büschel angehören. Die Ableitungsgleichungen lassen sich vollständig integrieren und führen

a) im Falle sich schneidender Büschelachsen auf zweifache Translationsflächen, wobei ein Kegelschnitt längs eines anderen verschoben wird, und

b) im allgemeinen Falle auf eine explizite Darstellung der Art

$$\underline{x}(s,t) = \frac{1}{f(s)}\left(a(s)\,\underline{p}_1 + b(s)\,\underline{p}_2\right) + \frac{1}{g(t)}\left(c(t)\,\underline{p}_3 + d(t)\,\underline{p}_4\right),$$

wobei a, b, f lin. unabh. quadrat. Formen in s (und analog c, d, g in t) sind. Bis auf ~~drei~~ Ausartungsfälle handelt es sich um allgemeine Zykliden (alg. Flächen 4.O. mit einem nicht-entarteten Kegelschnitt als Doppelkurve); diese lassen sich komplex-projektiv auf DUPINsche Zykliden abbilden. Einer der Ausnahmefälle führt auf Quadriken bzw. auf Flächen 3.O., die zu parabolischen DUPINschen Zykliden äquivalent sind.

W. Degen (Stuttgart)

„Über eine kinematische Eigenschaft der Exponentialabbildung eines linearen Zusammenhangs und JACOBI-Felder k-ter Ordnung."

Sei M eine n-dim C^∞-Mannigf., ∇ eine kovariante Ableitung für M. Sei E die maximale Umgebung des Nullschnitts in TM, auf der die Exponentialabb. $\exp: E \to M$ von ∇ definiert ist. Sei J ein Intervall von $\mathbb{R}$ und $Z: J \to E\ (\subset TM)$ ein C^∞ Weg mit der Fusspunktskurve $\gamma: J \to M$; (o.B.d.A. $0 \in J$). Wir berechnen für den Zeitpunkt $0 \in J$ den Geschwindigkeits- und Beschleunigungsvektor (sowie in Spezialfällen auch Beschleunigungsvektoren höherer Ordnung) des C^∞-Weges $\exp\circ Z: J \to M$ und zwar mittels Jacobi-Feldern 1-ter, 2-ter, ... Ordnung längs der ∇-Geodätischen $c: [0,1] \to M$ mit den Anfangsbedingungen $c(0) = \gamma(0)$, $\dot{c}(0) = Z(0)$. Die lin.-inhomogene DGl. 2-ter Ordnung für die Jacobi-Felder 2-ter Ordnung längs c wurden für torsionsfreies ∇ explizit angegeben, für die 3-ter Ordnung nur im Spezialfall $\dot{Z}(0) = 0$. Diese Berechnungsmöglichkeit für die Geschwindigkeits- bzw. Beschleunigungs-Verteilung längs bewegter geodätischer „Stäbe" in Mannigfaltigkeiten wurde interpretiert als eine „Nahewirkungs-" oder „Feld"-Geometrie. Wie der geodätische Spray in diesem Sinne das „Führungs"-Feld für die geodätischen Wege in M repräsentiert, so tut dies ein (im Vortrag eingeführter) „JACOBI-Spray 1-ter Ordnung", ein gewisses Vektorfeld $\overset{(1)}{S}$ auf TTM, für die Geschwindigkeitsverteilung längs c. [Die Integralkurven von $\overset{(1)}{S}$ sind genau die Phasen-Bahnen $\dot{Y}: I \to TTM$ von JACOBI-Feldern $Y: I \to TM$ von ∇.]

30. 10. 1984 P. Dombrowski (Köln)

The k^{th} fundamental forms of immersions of Riemannian manifolds.

Let M be an m-dimensional C^∞-Riemannian manifold, let $p_0 \in M$. The exponential map $\exp_{p_0}$ sends a neighbourhood V of the origin of TM_{p_0} diffeomorphically onto a neighbourhood U of M containing p_0. The inverse of this map sends a point $p \in U$ to a vector $v \in V$, and the pull-back of the euclidean metric of V to p provides two symmetric bilinear forms at p, namely that given by the pulled back metric and that of the original metric at p. Let σ_k denote the elementary symmetric function of the (real) eigenvalues of the pulled back metric with respect to the original metric. If $\sigma_k(v) = f_k(\|v\|)$ for all $v \in V$, we say that the metric is k-harmonic at p_0. If this property holds for all $p_0 \in M$, we say that M is k-harmonic.

Conjecture. M k-harmonic for some $k \in \mathbb{Z}^+ \Rightarrow$ M k-harmonic for all k.

The conjecture is true if M is locally symmetric. Moreover we know that m-harmonic $\Rightarrow$ 1-harmonic, and, due to Vanhecke, 1-harmonic $\Rightarrow$ m harmonic.

In a lecture given recently, James Eells examined the more general case $(M^m, g) \xrightarrow{\phi} (N^n, h)$, introducing the first and second fundamental forms of ϕ, and deriving geometrically significant expressions in the form of integrands depending on the first and second forms $\alpha_1(\phi)$, $\alpha_2(\phi)$. We use the ideas of Eells to construct corresponding expressions for the special case mentioned in the first paragraph. It is hoped that in this way more light will be shed on how to resolve the conjecture, though it seems that fundamental forms $\alpha_3(\phi)$, $\alpha_4(\phi)$, .. may be necessary. These forms may be relevant to the theory of Jacobi fields of higher order described by P. Dombrowski on page 161 of this book. Certainly they give rise to the study of maps which are not classically harmonic but can be described as harmonic of order k.

T. J. Willmore (Durham)

31. 10. 84.

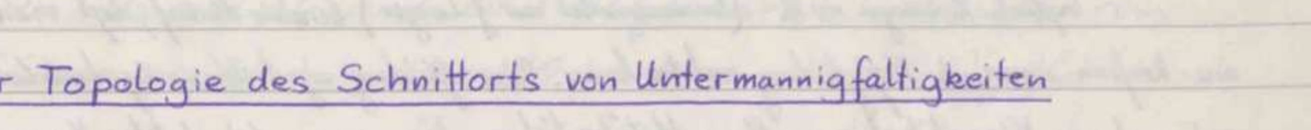
Brennfläche und Krümmungsliniennetz zu Nabelpunkten

Gegenstand des Vortrags ist eine Methode zum Feststellen geometrischer Abhängigkeiten, dargestellt am Beispiel der Brennfläche und des Krümmungsliniennetzes zu einem Nabelpunkt einer Fläche im R^3.
Dort gibt es im Regelfall 3 mögliche Typen von Krümmungsliniennetzen und 2 Typen von Brennflächen. Diese werden durch den Index des Netzes und die Anzahl der Richtungen, in die Krümmungslinien in den Nabelpunkt einlaufen, bzw. durch die Anzahl der in den entsprechenden Brennflächenpunkt einlaufenden Gratlinien gekennzeichnet.
Mittels (koordinatenfrei gebildeter) geometrischer Invarianten lassen sich die geometrischen Abhängigkeiten durch systematische Methoden der Invariantentheorie auf Abhängigkeiten dieser Invarianten zurückführen. So kommt man zu Aussagen über die Vereinbarkeit verschiedener Typen und z.B. Winkel zwischen einlaufenden Krümmungslinien.

Martin Lang (Darmstadt)

Zur Topologie des Schnittorts von Untermannigfaltigkeiten

Der Schnittort C_A einer abgeschlossenen Untermannigfaltigkeit A einer vollständigen Riemannschen Mannigfaltigkeit M hat analoge topologische Eigenschaften wie der Schnittort eines Punktes, wenn A mindestens zweimal stetig differenzierbar eingebettet ist.
Ist A reell-analytisch, so ist C_A triangulierbar.
Ist A eine kompakte Untermannigfaltigkeit von R^n oder S^n $(n \leq 4)$, so kann für eine generische Einbettung von A die lokale topologische Struktur von C_A durch endlich viele Modellräume beschrieben werden: Für $n = 3$ sind dies die folgenden:



Anwendung für Knoten in der S^3:

Außer der Kleeblattschlinge haben alle echten Knoten in generischer Lage mindestens einen "Verzweigungspunkt" im Schnittort. Die Kleeblattschlinge kann so gelegt werden, daß ihr Schnittort keinen Verzweigungspunkt enthält.

Elkehart Kaufmann (Bonn)

Verkürzen von Kurven:

Das Krümmungsnormalenfeld $k \cdot N = \frac{1}{|c'|} \cdot \nabla_{c'}\left(\frac{c'}{|c'|}\right)$ längs einer regulären geschlossenen Kurve in einer Riemann'schen Mannigfaltigkeit (M, g) ist: $-L^2$-grad (Bogenlängenfunktional). Für genügend kleine $t \geq 0$ existiert der zugehörige Fluß, also eine Familie von Kurven $t \mapsto (c_t : \mathbb{R}/\mathbb{Z} \to M)$ mit:

$$(*) \qquad \frac{d}{dt} c_t = k \cdot N$$

Über das globale Verhalten des Flusses ist nur wenig bekannt. Selbst die Vermutung, daß einfache geschlossene Kurven in $\mathbb{R}^2$ eingebettet bleiben und kreisförmig werden, wenn ihre Länge gegen 0 schrumpft, ist erst für konvexe Kurven bewiesen (Gage, Hamilton).

Explizite Rechnungen in $\mathbb{R}^2$ (Zusammenarbeit mit J. Langer) beruhen darauf, daß man $(*)$ durch eine konform invariante Flußgleichung ersetzen kann. Diese Gleichung eignet sich zu Regularitätsuntersuchungen und auch zur Klassifikation aller selbstähnlichen Lösungen von $(*)$. Letztere Kurven entwickeln sich in t via Homothetien. Sie haben periodische Krümmungsfunktion $\kappa = \lambda \cdot \exp(\frac{1}{2}B)$, wobei B (bei Parametrisierung nach Bogenlänge) der Oszillatorgleichung

$$B'' + 2\lambda^2 (e^B - 1) = 0$$

genügt. Der Parameter λ bestimmt die Länge, während die Gestalt der Kurve durch den Parameter $\eta \geq 0$ im ersten Integral $\frac{1}{2}B'^2 + 2\lambda^2 \cdot (e^B - B - 1) = 2\lambda^2 \cdot \eta$ bestimmt wird. Die hierdurch definierten Kurven sind rotationssymmetrisch; ihr Tangentialvektor dreht sich pro Periode der Krümmungsfunktion um den Winkel

$$\theta(\eta) = \int_{B_-}^{B_+} \frac{dB}{\sqrt{e^{-B}[\eta - (e^B - B - 1)]}} \qquad ; \eta > 0 \quad .$$

Wenn $\eta > 0$, so schließen sich die Kurven genau dann, wenn $\theta(\eta) = \frac{m}{n} \cdot 2\pi$. Weil θ monoton fallend mit Werten in $(\pi, \pi \cdot \sqrt{2})$ ist, sind die homothetischen Lösungen mit Ausnahme des Kreises $(\eta = 0,\ \theta = \pi \cdot \sqrt{2})$ durch die Windungszahl m und die Anzahl n der Perioden der Krümmungsfunktion klassifiziert; Bedingung: $\frac{1}{2} < \frac{m}{n} < \frac{1}{\sqrt{2}}$. ($\rightsquigarrow$ Selbstschnitte!) Krümmung und Abstand zum Symmetriezentrum sind verknüpft durch

$$x = x_{min} \cdot \exp\left(\frac{1}{2} \frac{x_{min}}{r_{min}} \cdot \left(r^2 - r_{min}^2\right)\right)$$

Folglich sind alle nicht-kreisförmigen homothetischen Lösungen transzendente Kurven.

Uwe Abresch (Bonn)

Gromov's Beweis des Sphärensatzes

Nach Ideen von Gromov wurde über einen neuen Beweis des Sphärensatzes berichtet:

Sphärensatz (Berger, Klingenberg 1961)
M kompakt, einfach zusammenhängend, $0 < \delta \leq K \leq \Delta$ mit $\Delta/\delta < 4$ $\Rightarrow$ M homöomorph zur Sphäre, genauer: M diffeomorph zu $S_\varphi := D^n \cup_\varphi D^n$ für einen Diffeomorphismus $\varphi: \partial D \to \partial D$.

Dieser Satz wurde auf eine allgemeiner Tatsache in Räumen nicht-negativer Krümmung zurückgeführt:

Satz (Gromov 1982) M vollstd., $K \geq 0$, $\dim M = n \geq 3$
S zush., kompakt, $\dim S = n-1$
$y: S \to M$ ε-konvexe Immersion
$\Rightarrow$ Es gibt Diffeomorphismus $\varphi: \partial D \to S$ und Immersion $\hat{y}: D \to M$ mit $\hat{y}|_{\partial D} = y \circ \varphi$ so daß die mittlere Krümmungsnormale von $y(S)$ nach innen (zu $\hat{y}(D)$) weist.

Dabei heißt eine (immersierte) Hyperfläche ε-konvex, wenn alle Hauptkrümmungen gleiches Vorzeichen und Absolutwert $\geq \varepsilon$ haben, für einen Wert $\varepsilon > 0$.

Elemente des Beweises des Satzes von Gromov

sind: – Abstandsfunktion und ihre Stützfunktionen
– Glättungen
– Konvexitäts-Betrachtungen

Jost Eschenburg, Münster

Untermannigfaltigkeiten des Basisraumes einer Riemannschen Submersion.

Es sei $\pi : N \to B$ eine Riemannsche Submersion und M eine horizontale Untermannigfaltigkeit von N. Dann ist $f := \pi|M$ eine isometrische Immersion, und zwischen den zweiten Fundamentalformen von f und M besteht ein enger Zusammenhang. Z.B. ist f genau dann total geodätisch, minimal oder nabelsch, wenn dasselbe für M gilt. Da in vielen Fällen die Geometrie von N besser bekannt ist als die von B, ist es daher von Interesse, zu untersuchen, ob eine gegebene Untermannigfaltigkeit von B einen "horizontalen Lift" besitzt. Dazu führe ich die sog. horizontale Integrabilitätsstruktur von π ein, das ist eine Familie $\mathcal{J} = (\mathcal{J}_b)_{b\in B}$ von Teilmengen $\mathcal{J}_b \subset \mathrm{End}(T_bB)$. Im Falle, daß π lokal homogene Fasern hat, ist $\mathcal{J}$ eine Familie von Vektorräumen, die lokal durch C^∞-Tensorfelder erzeugt wird. In wichtigen konkreten Fällen ist $\mathcal{J}$ ein wohlbekanntes Vektorbündel, z.B. eine komplexe bzw. quaternionale Struktur. In letzteren Fällen ist eine Untermannigfaltigkeit $M \subset B$ genau dann lokal Bild horizontaler Untermannigfaltigkeiten $\subset N$, wenn M total reell (= anti-invariant) ist. Auf diese Weise gewinnt man eine enge Verbindung zwischen anti-invarianten Untermannigfaltigkeiten von Sasaki- und Kähler-Mannigfaltigkeiten.

H. Reckziegel, Köln

Eine differentialgeometrische Kennzeichnung der euklidischen Schraubungen

Durch Zurückführung auf mehrere Differentialgleichungen (partielle u. gewöhnliche) und deren Integration wird gezeigt: Unter den einparametrigen Bewegungsvorgängen des 3-dimensionalen euklidischen Raumes werden die Schraubenbewegungen dadurch gekennzeichnet, daß die Bahnkurve jedes bewegten Punktes X des Gangraumes Böschungslinie gegenüber einer festen Richtung des Rastraumes ist. Der zugehörige Neigungswinkel hängt von der Lage des Punktes X im bewegten Raum ab.

H. R. Müller (Braunschweig)

Fortsetzungseigenschaften von Funktionen auf $\mathbb{R}^n \setminus \{0\}$, die nur in Polarkoordinaten gegeben sind.

Hat man eine C^∞-Abbildung $f: \mathbb{R} \times S^n \times M \to L$ in eine affine C^∞-Mannigfaltigkeit (L, ∇), so kann man die Abbildung $h: \mathbb{R}^{n+1} \setminus \{0\} \times M \to L$, $(b, p) \mapsto f(\|b\|, \frac{b}{\|b\|}, p)$ daraufhin untersuchen, ob sie zu einer C^k-Abbildung ($k \in \mathbb{N} \cup \{\infty\}$) auf $\mathbb{R}^{n+1} \times M$ fortgesetzt werden kann, ob also die Singularitäten $(0, p)$ ($p \in M$) C^k-gehoben werden können. Es gilt: Ist h eine C^{k-1}-Abbildung, so ist h genau dann eine C^k-Abbildung, wenn für alle $p \in M$ die Abbildung

$$\left(\nabla^{k-1}_{\frac{\partial}{\partial t}} f_* \frac{\partial}{\partial t}\right)(0, \ldots, p) : S^n \to T_{h(0)} L$$

Einschränkung einer homogenen ganzrationalen Funktion $\lambda: \mathbb{R}^{n+1} \to T_{h(0)} L$ vom Grad k auf S^n ist.

Damit gelingt es in der Radargeometrie eines beschleunigten Beobachters in der allgemeinen Relativitätstheorie, Sätze über die Differenzierbarkeit der Zeitfunktion eines Beobachters zu beweisen.

U. Proff (Köln)

Geodätische auf 2-dimensionalen Sphären

Anhand von Resultaten von Birkhoff und Lusternik-Schnirelmann und einiger eigenen Ideen kann man Aussagen über Geodätische, vorallem über geschlossene Geodätische, auf 2-dimensionalen Riemannschen Sphären gewinnen. Insbesondere wird gezeigt, daß für eine Menge von Metriken, die C^2-offen und C^∞-dicht in der Menge aller Riemannschen Metriken ist, stets unendlich viele geschlossene Geodätische existieren.

V. Bangert (Bonn)

Einige Bemerkungen über isoparametrische Untermannigfaltigkeiten

Faßt man die r-te mittlere Krümmung h_r einer isometrischen Immersion $F: M^m \to \tilde{M}^{m+k}$ zwischen riemannschen Mannigfaltigkeiten als symmetrische r-Form auf dem Normalenbündel von F auf, so sind die Untermannigfaltigkeiten ausgezeichnet, deren sämtliche mittleren Krümmungen (kovariant) konstant sind. Solche Untermannigfaltigkeiten seien - wie im Hyperflächenfall - isoparametrisch genannt. Zu ihnen gehören z.B. die symmetrischen Untermannigfaltigkeiten und die Fokalflächen isoparametrischer Hyperflächen in S^{m+1}.

Satz: Eine isoparametrische Untermannigfaltigkeit mit flachem Normalenbündel ist genau dann symmetrisch, wenn die simultanen Eigenbündel aller zweiten Fundamentaltensoren parallel sind (dies ist insbes. der Fall, wenn für deren Anzahl g gilt: $g \leq 2$).

Ferner wird gezeigt: (i) Die isoparametrischen Flächen in den Standardraumformen sind genau die Flächen mit paralleler zweiter Fundamentalform. (ii) (Unter Verwendung eines Resultats von B. SMYTH) Eine isoparametrische kählersche Hyperfläche einer Kählermannigfaltigkeit konstanter holomorpher Schnittkrümmung $\tilde{c}$ ist totalgeodätisch (und von konst. holom. Schnittkrümmung $\tilde{c}$) oder lokal holomorph isometrisch zur Hyperquadrik Q^n in $\mathbb{P}^{n+1}(\mathbb{C})$, letzteres tritt nur auf, falls $\tilde{c} > 0$.

W. Strübing (Dortmund)

Geradenkongruenzen mit ausgezeichneter Mittelfläche.

Im euklidischen Raum $\mathbb{R}^3$ sei Φ eine reguläre Zeitfläche einer regulären C^3-Geradenkongruenz Σ. Σ sei nicht die Normalenkongruenz von Σ. Dann sind jene Kurven in Φ, welche die Erzeugenden von Σ jeweils orthogonal schneiden („Σ-Querlinien von Φ") und ihre Orthogonaltrajektorien („Σ-Spurlinien von Φ") eindeutig definiert. Ferner bezeichne φ den Winkel einer allgemeinen Kongruenzgeraden e gegen die Tangentialebene an Φ im Punkt $e \cap \Phi$. Dann gilt z.B.:

SATZ 1: Sei Φ eine reguläre Zeitfläche einer regulären Geradenkongruenz Σ mit $0 < \varphi < \frac{\pi}{2}$; durch Reflexion der Kongruenzgeraden von Σ an Φ entstehe wieder (vermöge $\varphi \to -\varphi$) eine reguläre Geradenkongruenz Σ^*. Mit (1)–(4) seien folgende Eigenschaften bezeichnet:

(1) :⇔ Φ ist die Mittelfläche von Σ und Σ^*.

(2) :⇔ Die Σ-Spurlinien von Φ (= die Σ^*-Spurlinien von Φ) sind Zeitkurven von Hauptdrallflächen [SANNIA-Hauptflächen] von Σ und Σ^*.

(3) :⇔ Die Σ-Querlinien von Φ (= die Σ^*-Querlinien von Φ) sind Zeitkurven von Hauptdrallflächen von Σ und Σ^*.

(4) :⇔ Φ ist eine Minimalregelfläche (d.h. Φ liegt in einer allgemeinen Wendelfläche oder einer Ebene), die geradlinigen Erzeugenden von Φ sind die Σ-Querlinien von Φ, $\varphi = \text{const}$ längs jeder Orthogonaltrajektorie der geradlinigen Erzeugenden von Φ.

Dann gilt: $(i) \wedge (j) \Leftrightarrow (4)$ für $1 \leqq i < j \leqq 3$.

SATZ 2: Sei Σ eine isotrope Geradenkongruenz mit regulärer Mittelfläche Φ. Dann sind die Eigenschaften (1)–(5) äquivalent:

(1) Φ ist eine Minimalfläche (d.h. mittlere Krümmung $H \equiv 0$).

(2) Die Σ-Spurlinien von Φ sind Schmiegelinien von Φ.

(3) Die Σ-Querlinien von Φ sind Schmiegelinien von Φ.

(4) Der Winkel φ der Erzeugenden von Σ gegen Φ ist längs jeder Σ-Spurlinie von Φ konstant.

(5) Φ ist eine Minimalregelfläche (d.h. Φ liegt in einer allgemeinen Wendelfläche oder Φ ist eben).

[ANMERKUNG: Die Aussage (1) $\Rightarrow$ (5) bewies bereits RIBAUCOUR (1882)].

SATZ 3: Sei Σ eine isotrope Geradenkongruenz mit regulärer Mittelfläche ϕ. Dann sind die Eigenschaften (1)–(4) äquivalent:

(1) ϕ ist eine Torse (d.h. Gaußsche Krümmung $K \equiv 0$).

(2) Die Σ-Spurlinien von ϕ sind Krümmungslinien von ϕ.

(3) Die Σ-Querlinien von ϕ sind Krümmungslinien von ϕ.

(4) ϕ ist eben.

1. 11. 84 R. Koch (TU München)

Hyperflächen konstanter mittlerer Krümmung im hyperbolischen Räumen (Bericht über eine Arbeit mit M.P. do Carmo und J. de M. Gomes)

Es sei $\overline{H}^{n+1} = H^{n+1} \cup S^n(\infty)$ die Kompaktifizierung des hyp. Raumes H^{n+1}. Der asymptotische Rand $\partial_\infty A$ einer Teilmenge $A \subset H^{n+1}$ ist $\overline{A} \cap S^n(\infty)$, wobei $\overline{A}$ die abgeschl. Hülle von A in $\overline{H}^{n+1}$ bezeichnet. Es gilt dann, daß eine vollst., eingebettete Hyperfläche $M^n \subset H^{n+1}$ mit konstanter mittlerer Krümmung $H \in [0,1)$ keine isolierten Pkte im asymptotischen Rand haben. Dieses Ergebnis ist optimal, da der asymptotische Rand ~~v~~ einer Horosphäre ($H \equiv 1$) aus einem einzigen Punkt besteht. Tuben um Geodätische ($H > 1$) haben zwei Punkte im asymptotischen Rand. Es gilt auch, daß eine vollst. eingebettete Hyperfl. konst. mittlerer Krümmung $H \in [0,1)$ die Eigenschaft hat, daß die Komponenten des as. Randes bezüglich einer „Metrik" auf den Teilmengen in $S^n(\infty)$ nicht zu weit auseinander liegen können.

G. Thorbergsson (Bonn)

Taute Untermannigfaltigkeiten sind algebraisch

Eine kompakte Untermannigfaltigkeit M^n in $\mathbb{R}^m$ heißt taut, wenn für jeden abgeschlossenen Ball $B \subset \mathbb{R}^m$, dessen Rand ∂B die Mannigfaltigkeit ∂B transversal schneidet, der Inklusionshomomorphismus $H_*(M^n \cap B, \mathbb{Z}_2) \to H_*(M^n, \mathbb{Z}_2)$ injektiv ist.

Eine Untermannigfaltigkeit $M^n \subset \mathbb{R}^m$ heißt Dupin'sch, wenn auf den Tuben um M^n alle Krümmungslinien Kreise sind. In dem Vortrag wurden zwei Sätze besprochen:

Satz 1: Taute Untermannigfaltigkeiten sind Dupin'sch.

Satz 2: Dupin'sche Untermannigfaltigkeiten sind (semi-) algebraisch.

U. Pinkall (Bonn)

Über eine Formel von Blaschke für Affinoberfläche.

Von W. Blaschke 1923 stammt die folgende (äquiaffin invariante) Darstellung für die Affinoberfläche $O_{aff}(F)$ einer analytischen Eifläche F des dreidimensionalen euklidischen Raums R_3:

$$O_{aff}(F) = \lim_{\delta \to 0} \sqrt{\pi}\, \frac{V(K) - V(K_{[\delta]})}{\sqrt{\delta}} .$$

Hierin bedeutet $V(K)$ bzw. $V(K_{[\delta]})$ das Volumen des von F umschlossenen konvexen Körpers K bzw. das Volumen desjenigen Körpers $K_{[\delta]}$, der aus K durch Weglassen aller ebenen Segmente vom Volumen $\delta > 0$ entsteht.

Es wird gezeigt, daß sich obige Formel auf alle Eihyperflächen des R_n der Klasse C_2 ausdehnen läßt (Satz 1.). Der Beweis liest beruht u.a. auf der schwachen Stetigkeit der Alexandrovschen Oberflächenfunktionen und einer lokalen Version des Blaschkeschen Rollungssatzes für Eihyperflächen. Desgleichen wird noch eine Lokalisierung von Satz 1 angegeben (Satz 2.).

K. Leichtweiß (Stuttgart)

Ein geometrischer Zugang zur Schnitt-Theorie.

Das Ziel des Vortrages bestand darin, einen geometrischen Zugang zur Schnitt-Theorie zu beschreiben. Wir haben heutzutage die bemerkenswerte Theorie von Fulton und MacPherson über Schnitt-Theorie (vgl. W. Fulton, Intersection theory, Springer-Verlag, 1984). Unser Gegenstand besteht darin, eine additive Zerlegung der Bezoutschen Zahl Grad(X)·Grad(Y) herzuleiten, wobei X und Y beliebige projektive rein-dimensionale Unterschemata des $\mathbb{P}_K^n$ sind und K ein algebraisch abgeschlossener Körper ist. Wir benutzen im Gegensatz zu Fulton und MacPherson eine alte geometrische Idee, indem wir die Geometrie der „join" Konstruktion im $\mathbb{P}^{2n+1}$ über einer geeigneten Körpererweiterung von K verfeinern. Beide Theorien liefern auch in der Tat unterschiedliche „Zerlegungen". Durch Anwendung unserer Methode können wir ein Ergebnis von Jacobi verbessern, das er bereits 1836 im Crelle J. publizierte. Wir wollen jedoch betonen, daß Jacobi's Untersuchungen auf eine Idee von Euler zurückgehen, die er 1748 veröffentlichte.

Literaturhinweis: W. Vogel: Lectures on results on Bezout's theorem. Lecture notes, Tata Institute of Fundamental of Bombay, No. 74. (Notes by D. P. Patil). Springer-Verlag Berlin-Heidelberg-New York-Tokyo, 1984.

Wolfgang Vogel (Halle).

Über die Geometrie von R-Räumen

Sei G Liegruppe mit halbeinfacher Lie-Algebra $\mathfrak{g}$, $<,>$ die Killing-Form, $K \subset G$ eine maximale Untergruppe von G mit Liealgebra $\mathfrak{k}$. Dann $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$, $\mathfrak{p}$ mit $<,>$ euklidischer Vektorraum.
Sei $Z \in \mathfrak{p}$ und $K(Z) \subset K$ definiert durch $Ad(k)Z = Z$ f.a. $k \in K$.
$K/K(Z)$ heißt R-Raum und $\varphi : K/K' \to \mathfrak{p}$, $[k] \mapsto Ad(k)Z$ die Standard-Einbettung von $K/K(Z)$. $K/K(Z)$ heißt regulär, wenn $\dim K/K(Z) = \max\limits_{W \in \mathfrak{p}} \dim(K/K(W))$.

Neben der Vorstellung von Beispielen und Bemerkungen über einige Resultate im Zusammenhang mit R-Räumen wird der Satz bewiesen: Das Normalenbündel von regulären R-Räumen ist trivial (d.h. besitzt eine Basis aus parallelen globalen Normalenschnitten).

Literatur: Y. OHNITA The Degrees of the Standard Imbeddings of R-Spaces Tôhoku Math. J. 35 (1983) 499 – 502

Stephan Schirmacher

Symmetrische Bilinearformen auf der p-ten äußeren Potenz eines n-dimensionalen Vektorraums.

Wir betrachten folgende Fragestellung: Gegeben sei ein Variationsproblem in Parameterform mit einem p-fachen Integral $\int\cdots\int F(x, X_1, \dots, X_p)\, du^1 \wedge \dots \wedge du^p$, $x := (x^1, \dots, x^n)$, $X_i := \partial x / \partial u^i$, $1 \le i \le p$, $1 \le p \le n-1$; wann ist dann F^2 – der Integrand des zugehörigen „Energieintegrals" – auf ganz $\mathbb{R}^{n+pn}$ von der Klasse C^2?

Indem man F^2 nach Taylor entwickelt und die positive Homogenität vom Grade 2 von F^2 berücksichtigt, erhält man

$$F^2(x, X_1, \dots, X_p) = \sum a_{i_1 j_1 \dots i_p j_p}(x)\, X_1^{i_1} X_1^{j_1} \cdots X_p^{i_p} X_p^{j_p}$$

und die oben gestellte Frage reduziert sich auf das folgende algebraische Problem: Welche „p-quadratischen" Formen ϕ über einem K-Vektorraum ($\mathrm{char}(K) = 0$) haben die Eigenschaft, daß ihr Wert $\phi(X_1, \dots, X_p)$ nur vom äußeren Produkt $X_1 \wedge \dots \wedge X_p$ der Argumente abhängt? Es zeigt sich, daß der Raum $Q_p^*(V)$ aller dieser ϕ unter $GL(V)$ irreduzibel und in natürlicher Weise äquivalent zu jener irred. Komponente $\mathcal{L}_p(V)$ der symmetrischen Bilinearformen auf $\Lambda^p V$ ist, welche dem Young-Diagramm

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline \vdots & \vdots \\ \hline 2p-1 & 2p \\ \hline \end{array}$$

entspricht; insbesondere ergibt sich, daß sich jedes $\phi \in Q_p^*(V)$ eindeutig schreiben läßt als

$$\phi(X_1, \dots, X_p) = R(X_1 \wedge \dots \wedge X_p,\ X_1 \wedge \dots \wedge X_p) \text{ mit } R \in \mathcal{L}_p(V).$$

Siegfried Steiner (Stuttgart)

Geometrische Abschätzungen kleiner Eigenwerte des Laplaceoperators

Satz:

Sei (M, g) eine 2-dimensionale, geschlossene, orientierte und zusammenhängende Mannigfaltigkeit mit der Gauß-Krümmung K. Gelten weiterhin $6K_1 < 12K_0$ und

$$\max_M \|\operatorname{grad} K\|^2 < \frac{4}{27}\left[10K_1^3 + 18K_0K_1^2 - 216K_0^2(K_1 - K_0) + (7K_1^2 - 24K_1K_0 + 36K_0^2)^{3/2}\right].$$

mit $K_1 := \max_M K$, $K_0 := \min_M K$,

so hat das Polynom

$$P(\lambda) = \frac{1}{4}(\lambda - 12K_0)(\lambda - 6K_1)(\lambda - 2K_1) + \max_M \|\operatorname{grad} K\|^2$$

zwei Nullstellen a, b mit $6K_1 \leq a < b \leq 12K_0$ und kein Eigenwert des Laplaceoperators liegt im Intervall (a, b).

Aus meiner Dissertation an der TU-Berlin (1984)

Targo Pavlista

Die Oktavenebene für Differentialgeometer

Die Größen der inneren und äußeren Geometrie von $\mathbb{O}P^2$ im (euklidischen) Raum der hermiteschen 3×3-Matrizen über $\mathbb{O}$ sollen möglichst elementar berechnet werden. Dazu dient eine Kombination von Jordanalgebra-Methoden (wie bei U. Hirzebruch, Math. Z. 1965) und direkter Matrizenrechnung im Spezialfall.

D. Ferus (TU Berlin)

Sunadas Beispiele isospektraler Mannigfaltigkeiten

Zwei kompakte Riemannsche Mannigfaltigkeiten heissen isospektral, wenn sie dasselbe Eigenwertspektrum des Laplace-Beltrami Operators besitzen. Beispiele isospektraler aber nicht isometrischer Mannigfaltigkeiten gibt es von Milnor, Ikeda, Urakawa, Vignéras u.a. 1983 hat Sunada eine allgemeine Konstruktion angegeben, bei welcher er unter anderem isospektrale Mannigfaltigkeiten mit verschiedener Fundamentalgruppe erhält. Im Vortrag wurde über eine Modifizierung berichtet, die gestattet, nicht isometrische isospektrale Riemannsche Flächen (konstanter negativer Krümmung) vom Geschlecht g für $g = 5$ und für $g \geq 7$ zu konstruieren. Dieselbe Methode liefert auch Beispiele von im $\mathbb{R}^3$ eingebetteten Flächen.

J. P. B

Der Kompaktheitssatz von Gromov

Der Kompaktheitssatz (Gromov 1981) besagt, daß in der Klasse $\mathfrak{M}(n, \Lambda, D, \varepsilon)$ der kompakten Riemannschen Mannigfaltigkeiten der Dimension n mit Schnittkrümmung $|K| \leq \Lambda^2$, Durchmesser $d(M) \leq D$ und Injektivitätsradius $i(M) \geq \varepsilon$ jede Folge eine Teilfolge hat, die bezüglich des Lipschitz-Abstandes konvergiert gegen eine n-dimensionale Mannigfaltigkeit, deren Metrik aber i.a. nur stetig ist. Ein wesentliches Hilfsmittel ist der Satz, daß in $\mathfrak{M}$ die Lipschitz- mit der Hausdorff-Topologie übereinstimmt. Dieser Satz, in dessen Beweis bei Gromov einige Details offen blieben, wird hier mit anderen Methoden bewiesen. Der Beweis ähnelt dem des Vortragenden für Cheegers Endlichkeitssatz (1984). Im weiteren wurde gezeigt, daß man durch die Verwendung harmonischer statt Normalkoordinaten bessere Regularitätseigenschaften für die Grenzmetrik im Kompaktheitssatz erhält. Erreichbar ist $C^{1+\alpha}$, $\alpha < 1$, was fast optimal ist, da Beispiele zeigen, daß man mehr als $C^{1,1}$ i.a. nicht erreichen kann.

Stefan Peters (Dortmund)

Obere Schranken für λ_1

Es wurden die Abschätzungen von Cheng (1975) und Bleecker-Weiner (1976) diskutiert und teilweise verallgemeinert. Beispiele:

Satz: M kompakt, $K_M \geq 0$

(i) Enthält M zwei kompakte minimale Untermgf. der Kodimension n' im Abstand d, so

$$\lambda_1(M) \leq \frac{4\, j^2_{\frac{n'}{2}-1}}{d^2},$$

wobei j_k die 1. positive Nullstelle der Besselfunktion vom Index k bezeichnet.

(ii) $Y \subset M$ kompakt minimal, $n' = \operatorname{codim} Y$, so

$$\lambda_1(M) \leq c(k^n, n') \left(\frac{\operatorname{vol} Y}{\operatorname{vol} M}\right)^{2/n'}, \quad n = \dim M.$$

Satz: $M \subset \bar{M}$, $\dim M = n$, M kompakt, $\bar{M}$ vollständig, einfach zshgd., $K_{\bar{M}} \leq 0$. Dann gilt

$$\lambda_1(M) \leq \frac{n}{\operatorname{vol} M} \int_M H^2,$$

H die mittlere Krümmung der Immersion.

E. Heintze (Münster)

Stochastische Analysis

4.11. – 10.11.84

The Smoluchowski limit for a simple mechanical model

The motion of a brownian particle in a potential $U^A(x) = U(\frac{1}{\sqrt{A}}x)$, which varies on a macroscopic scale ($\sqrt{A}$) may be described on a macroscopic scale by a diffusion process for the position $X(t)$ of the molecule at time t. This diffusion is given by the Smoluchowski equation

$$(1) \quad dX(t) = -\frac{\nabla U}{\gamma}\,dt + \frac{\sigma}{\gamma}\,dW_t$$

For a stationary system the stationary solution for the position is the Gibbsstate $\sim e^{-U/kT}$, k Boltzmann constant, T absolute Temperature. Therefore in (1) $\frac{2\gamma}{\sigma^2} = \frac{1}{kT}$, which is the famous Einstein relation. We wish to derive (1) from "first principles" starting from a deterministic mechanical system, where the stochasticity comes only from the random intial configurations. A system, which may be treated, is a one dimensional ideal gas with an identical tagged particle serving as the molecule, on which a force is acting. The motion of the molecule is then determined by elastic collisions with the gas particles and by the newtonian motion in the force field. The ideal gas is described by a Poissonpoint process in position-velocity (q-v) space, where $f(v)$ is the density of the velocity distribution. $X(t)$ and $V(t)$ are position and velocity of the molecule and $V(0)$ is distributed also by $f(v)$. We considered the simplest case $f(v) = \frac{1}{2}\delta_1(v) + \frac{1}{2}\delta_{-1}(v)$. The force field is $\frac{1}{\sqrt{A}} F\ (= \nabla U^A(x))$ and we consider $X_A(t) = \frac{1}{\sqrt{A}} X(At)$. We obtain the surprising result that $X_A(t) \Rightarrow W(t)$, i.e. there is no drift in the limit

This fact is due to recollisions and a typical one dimensional phenomenon. In a two dimensional model, where the molecule is a stick moving along the x-axis without rotation and $f(v)$ is supported equally by velocities on the diagonals we obtain an equation like (1) in the scaling limit.
The reason that the Einstein relation is violated comes from the fact, that our velocity distribution is not the stationary - maxwellian - one. In fact, we conjecture that if $f(v)$ has a density at zero equation (1) comes out and γ and σ are related by $\frac{2\gamma}{\sigma^2} = \frac{1}{\langle v^2 \rangle}$. It should be noted that the derivation of (1) in more than one dimensions, with the maxwellian distribution is out of reach at the moment, even the free case i.e. $U=0$ has not been treated until now.

D. Dürr (Bures sur Yvette)

We investigate the qualitative behaviour of flows defined by SDE associated with isotropic homogeneous velocity fields.
Our results include the computation of Lyapunov exponents and the proof of existence, in the unstable case, of a non trivial statistical equilibrium. The proof involves a detailed study of isotropic B. M on matrices.

Y. Le Jan (Paris)

Wiener's Chaos Revisited

Let $(H, \Theta, \mathcal{W})$ be an abstract Wiener space and define D^m to be the Sobolev extention of m^{th} order differentiation in directions of H. Let ∂^m be the adjoint of D^m. Then, Wiener's decomposition of a function into states of homogenious chaos is:

$$\Phi = \sum_0^\infty \frac{1}{m!} \partial^m E^{\mathcal{W}}\left[D^m \Phi\right].$$

D. Stroock

Malliavin Operator for Poisson space

Let $(\Omega, \mathcal{F}, P)$ be the canonical space of Poisson measure (μ) with intensity $\nu(ds, dy) = ds \otimes dy$, on $[0,T) \times E$ (E = open subset of $\mathbb{R}^m$). We construct a Malliavin Operator (L, R) on this space, starting with "regular functionals", and such that the coordinates of the solution of the equation

$$dX_t = a(X_t)dt + [b(X_t)\,dW_t] + c(X_{t-}, y)(d\mu - d\nu)$$

be in the domain. We can add a Wiener process as above, and take the "product" Malliavin operator, to deal with the ~~operator~~ above equation.

This allows to study the smoothness (and existence) of a density for the r.v.'s X_t.

Jean Jacod (Paris)

Propagation of chaos for diffusion processes with mean-field singular interaction

We consider a system ~~of~~ consisting of n-white diffusing particles distributed left to ~~[illegible]~~ n-red diffusing particles. Reds and whites interact in a repulsive way so that the group of reds are ~~kept~~ kept segregated from the group of whites. (Take e.g. $f(x) = 1/x^4$ as an interaction). It is assumed that the absolute value of the interaction is a decreasing function of the distance of a pair of white and red particles. We prove first of all the existence of a unique solution for a system of equations of $2n$-particles and also for the corresponding one of (non-linear) diffusion processes in the mean-field limit. Then we prove the propagation of chaos, i.e. the convergence of the $2n$-particles, as $n \to \infty$, to the infinite independent copies of the mean-field diffusions. This is a report of what has been done by H. Tanaka and myself. This gives a mathematical justification of "statistical quantization" which goes one step deeper than "stoch. quantization".

M. Nagasawa (Zürich)

Asymptotic results for the density of states and Lyapunov exponent for the one-dimensional Schrödinger operator

Given the Schrödinger operator

$$Hy = -\ddot{y} + \sigma\, \xi(t)\, y$$

in $L^2(\mathbb{R}, dt)$, where $\xi(t)$ is a nice diffusion on an interval $(\alpha, \beta) \subset \mathbb{R}$ with invariant distribution ν and $\langle \xi(t) \rangle = 0$.
Let $\lambda_\sigma(E)$ be its Lyapunov number and $N_\sigma(E)$ its integrated density of states.

Theorem: Let $\sigma \to \infty$, $E = E_1 \sigma$, $E_1 \in \mathbb{R}$.
Then

$$\lambda_\sigma(E_1\sigma) = \sqrt{\sigma} \left\langle \sqrt{(\xi - E_1)^+} \right\rangle + O(1) \quad \text{for } E_1 < \beta$$

$$= \frac{1}{32} \left\langle \frac{b^2(\xi)}{(\xi - E_1)^2} \right\rangle + O\left(\frac{1}{\sqrt{\sigma}}\right) \quad \text{for } E_1 > \beta$$

(b = diffusion coeff. of ξ)

$$\pi N_\sigma(E_1\sigma) \equiv 0 \quad \text{for } E_1 < \alpha$$

$$= \sqrt{\sigma} \left\langle \sqrt{(E_1 - \xi)^+} \right\rangle + O\left(\frac{1}{\sqrt{\sigma}}\right), \quad E_1 > \alpha$$

Ludwig Arnold (Bremen)

Hypercontractivité de semigroupes de diffusion

(Travail commun avec Bakry)

Si (X_t) est une diffusion markovienne continue, stationnaire, réversible ~~...~~, de loi μ sur un espace d'états E, une condition suffisante pour que X soit hypercontractif et que soit satisfaite l'inégalité de Sobolev logarithmique de Gross est

$$\forall x\ \forall f \qquad \Gamma_2(f,f)(x) \geq c\, \Gamma(f,f)(x),$$

où, désignant par L le générateur de X, on a posé

$$\Gamma(f,g) = \frac{1}{2}\left[L(fg) - fLg - gLf\right]$$
$$\Gamma_2(f,g) = \frac{1}{2}\left[L\Gamma(f,g) - \Gamma(f,Lg) - \Gamma(Lf,g)\right].$$

M. Emery (Strasbourg)

Riesz Transforms for some symmetric semigroups

X_t being a Markov process, symmetric with respect to μ, with semigroup P_t and generator L, such that there is a algebra of functions on the state space, D, stable by L and by P_t, define on D Γ and Γ_2 like in Emery's abstract. Put $C = -(-L)^{1/2}$, and suppose $\Gamma_2 \geq 0$, then you can compare the norms of $\Gamma(f,f)^{1/2}$ and Cf in $L^p(\mu)$ in the following way.

. $p \geq 2$ $\quad \|\Gamma(f,f)^{1/2}\|_p \leq c_p \|Cf\|_p$; $1 < p \leq 2$ $\quad \|Cf\|_p \leq C_p \|\Gamma(f,f)^{1/2}\|_p$

Furthermore, if L is a diffusion, then

$$\|Cf\|_p \leq c_p \|\Gamma(f,f)^{1/2}\|_p, \quad \forall\ 1 < p < +\infty$$

D. BAKRY (Strasbourg)

Diffusions on the Wiener space

We consider a diffusion on the abstract Wiener space (B, H, μ) generated by the operator of the form $A = \frac{1}{2}L + b$, L being the Ornstein-Uhlenbeck operator and b being an H-valued function on B, which we regard as a vector field on B. We discuss about invariants measures of this diffusion and obtained that there exist an invariant measure which is absolutely continuous with respect to μ. Moreover, the uniqueness holds if we restrict ourselves to measures which are of finite total variation and are absolutely continuous with respect to μ.

We prove the existence by two steps. First we show it in finite dimensional case and secondly in infinite dimensional case by the limitting procedure. In this procedure, the Gross logarithmic Sobolev inequality is crucial.

I. Shigekawa (Osaka)

Probabilistic proof of some quasi-classical expansions.

For $\varepsilon \neq 0$ consider the solution of:

$$\begin{cases} \frac{\partial \psi}{\partial t}(t,x) = \mathcal{L}_\varepsilon \psi(t,x) = \frac{\varepsilon^2}{2} \sum_{j=1}^{2} A_j^2 \psi(t,x) + A_0 \psi(t,x) + \frac{1}{\varepsilon^2} V(x)\psi(t,x) \\ \psi(0,x) = f(x) \exp\left(-\frac{S(x)}{\varepsilon^2}\right), \quad t \in [0,T], \quad x \in M \end{cases}$$

where M is a smooth manifold, $A_0, A_1, \ldots, A_n$ are vector fields on M, V, f, S regular functions from M to $\mathbb{C}$. We give, when the second order differential operator $\mathcal{L}_\varepsilon$ is smooth but eventually completely degenerated, an asymptotic expansion with respect to ε ($\varepsilon \to 0$) for the solution of (1). The estimates obtained in the real case allow one to study, under some conditions, the SChrödinger equation with complex analytic coefficients and also some situation where $\mathcal{L}_\varepsilon$ is hyperbolic.

H. Doss (Paris).

Equilibrium- and Non-equilibrium Theory of a magnetic model (Summary of joint works with R. Ellis, F. Comets, M. Schatzman)

A magnetic model with a long-range interaction is discussed. It has paramagnetic, ferromagnetic and antiferromagnetic phases. In both its equilibrium and non-equilibrium theory, we prove the thermodynamic limit, resp. the law of large numbers, and we show the fluctuation theorems. The equilibrium fluctuation fields represent stationary distributions of the nonequilibrium fluctuation processes. This holds true also in the critical bordurline cases of second-order phase transitions, where the usual Gaussian fields break down. Here, we get degenerate non-Gaussian fields with densities $\exp(-t^4 c)$.

Th. Eisele

Markov processes and field theory.

With every symmetric Markov process X a Gaussian random field φ is associated with the Hamiltonian $H(\varphi) = \frac{1}{2}\mathcal{E}(\varphi,\varphi)$ where $\mathcal{E}$ is the Dirichlet form of X. It turns out that the path of X can be used for investigating not only properties of φ but also for investigating non-Gaussian fields with the Hamiltonians $H(\varphi) = \frac{1}{2}\mathcal{E}(\varphi,\varphi) + V(\xi)$ where V is ~~the~~ a functional of the field $\xi_x = \frac{1}{2}\varphi_x^2$. In particular, the specification corresponding to H can be expressed in terms of the occupation field or of the hitting field for the Poisson "gas" in the space of paths and loops.

The program outlined in the talk can be viewed as an interpretation, from a point of view of a probabilist, of Symanzik's ideas in quantum field theory.

E Dynkin

Joint Regularity of solutions to SDE driven by Poisson measures

This continues Jacod's exposition (p180). Under suitable conditions on the coefficients a, b, c, the transition kernel $P_t(x,dy)$ of the solution X_t has a density $p_t(x,y)$ regular in (t,x,y).

Klaus Bichteler

Two-sided stochastic integral and stochastic calculus

Consider a forward diffusion X_t, solution of:

$$X_t = x_0 + \int_0^t b(s, X_s)\, ds + \int_0^t \sigma(s, X_s)\, dW_s , \quad 0 \le t \le 1$$

and a backward diffusion Y^t, solution of:

$$Y^t + \int_t^1 c(s, Y^s)\, ds + \int_t^1 \gamma(s, Y^s)\, dW_s = y , \quad 0 \le t \le 1$$

A stochastic integral of the type $\int_0^1 \Phi(X_t, Y^t) (d) W_t$ is defined. This permits to write down the differential of the process of the type $\Psi(X_t, Y^t)$. Both Ito and Stratonovitch calculus are considered.

E. Pardoux (Marseille) [joint work with Ph. Protter]

Linear Parabolic Differential Equations as Limits of Space-Time Jump Markov Processes

A class of linear parabolic differential equations on a bounded domain in $\mathbb{R}^n$ is obtained as the class of deterministic limits of space-time jump Markov processes X^N. These X^N describe particle systems which are spatially inhomogeneous due to diffusion and random change in the number of particles. The deviation of X^N from its deterministic limit is a distribution valued generalized Ornstein-Uhlenbeck process and can be represented as the mild solution of a stochastic partial differential equation, whose driving term is the sum of two independent Gaussian martingales arising from diffusion and change in the number of particles, respectively.

P. Kotelenez (Bremen)

Gauge Fields as generalized Markov processes.

To every Markov process on a compact Lie group G, which is stationary and left as well as right invariant there exist a Markov stochastic gauge field in two dimensions such that the curvature is a homogeneous chaos with values in the Lie-algebra of $\mathfrak{g}$. This homogeneous chaos is the same as the one ~~of~~ which governs the increments of the Markov process. These continous Gauge theories are exactely the theoris which can be described as limits of lattice Gauge theories as the lattice converge to zero. The Gauge Field of physics is obtained by starting with the solution of the Markov process which is the solution of the heat equation. This gives the white noise Gaussian homogeneous chaos $\xi(x)$, and the corresponding Gauge field is constructed by solving the non linear stochastic differential equation $D\omega = \xi$ for the conection form ω where $D\omega$ is the covariant exterior derivative given by the conectionform ω.

Raphael Haegh Krohn

Characteristic exponents and invariant subbundles for random diffeomorphisms

Let $f_1, f_2, \ldots$ be a sequence of i.i.d. random diffeomorphisms of a compact Riemannian manifold M. Then under natural conditions for any x outside of some exceptional set and each ξ from the tangent space $T_x M$ at x with probability one the limit $\lim_{n\to\infty} \frac{1}{n} \log \| Df_n \circ \dots \circ Df_1 \| = \beta(\xi)$ exists and it is non-random. Moreover for these x there exists a filtration of (non-random) subspaces $L_x^{r(x)} \subset \dots \subset L_x^1 \subset L_x^0 = T_x M$ and (non-random) numbers $\beta_x^{r(x)} < \dots < \beta_x^0$ such that $\beta(\xi) = \beta_x^i$ for any $\xi \in L_x^i \setminus L_x^{i+1}$.

Yu. Kifer (Jerusalem)

Double ~~p~~ points of Brownian motion in $\mathbb{R}^d$ ($d=2,3$) and related stochastic calculus.

Simpler proofs of Tanaka-Rosen formulae for the local times of intersection of complex, or 3-dimensional Brownian motion, are given, taking advantage of Hardy's L^2 inequality, which is closely related to the second order equation : $\frac{1}{r} g'(r) + g''(r) = h(r)$. In dimension 2, Varadhan's renormalisation appears as simple consequence of the new Tanaka-Rosen formula thus obtained. In dimension 3, a new convergence in distribution for the renormalised local time of intersection is obtained - However, the relationship which may exist between this limit in distribution and Westwater's renormalisation is not understood-

Marc Yor

Nelson's stochastic mechanics

A survey is given of some recent results of E. Carlen (Comm. Math. Phys. 1984) and W. A. Zheng (Ann. Inst. H. Poincaré 1985) showing how to ~~[illegible]~~ construct ~~[illegible]~~ the diffusions ~~[illegible]~~ with singular drifts needed to develop Stochastic Mechanics

P. A. Meyer

Regularity of jump process in the ~~case of~~ degenerate case

We give examples of self-interactions in a jump process which allow to the process to have a density. We determine a type of jump process which is always on a submanifold of $\mathbb{R}^d$. And we determine the regularity of a jump process whose Levy measure is fixed and supported by a smooth curved.

We use technicals of the calculus of variation of Bismut

Léandre.

Non-linear filtering – the degenerate cases

Control of diffusions with partial information leads to the problem: find the conditional density of X_t given $\{Y_s : s \leq t\}$ where

$$dX_t = b(t, X_t, Y)\,dt + \sigma(t, X_t, Y)\,dw$$
$$dY_t = h(t, X_t)\,dt + d\tilde{w},$$

where one may not assume any regularity of b, σ with respect to (t, Y), e.g. no continuity in t. It is shown, under suitable hypotheses, that even when $\sigma\sigma^T$ is degenerate a conditional density exists and it's unnormalized version satisfies the pathwise (non-stochastic) p.d.e. usually associated with ~~related to~~ the Zakai equation.

Ulrich Haussmann

Lyapunov exponents for stochastic flows

Let $\{F_t\}_{t\geq 0}$ be the flow of the S.D.E. $dx_t = \sum X^i(x_t)\circ dB^i_t + A(x_t)\,dt$ on a compact Riemannian manifold M, and let $\mathcal{A}$ be its differential generator. Results of Carverhill have extended Ruelle's ergodic theory of diffeomorphisms to show that Lyapunov exponents can be defined for F_t which describe how sample solutions from different points of M behave in relationship to one another in the long term. Simple examples for $M = S^1$ with $\mathcal{A} = \frac{1}{2}\Delta$ show that these exponents are not determined by $\mathcal{A}$. However computations with M. Chappell show that the sum of the exponents, given by,

$$\lambda_{\Sigma 1} = \lim_{t\to\infty} \frac{1}{t} \log \det T_{x_0}F_t$$

is always non positive for $\mathcal{A} = \frac{1}{2}\Delta$ and is dominated by (the principal eigenvalue of $\frac{1}{2}\Delta$)$\times \dim M$ if $\mathcal{A} = \frac{1}{2}\Delta$ and each X^i is a gradient. Estimates for the canonical flow on the frame bundle of M have been obtained by Carverhill and Elworthy.

David Elworthy

The Atiyah-Singer theorem: a probabilistic approach.

The proof of the index theorem given in JFA 57, 56-99 (1984) has been presented. The case of the Dirac operator on the spin complex has been completely treated.

Jean-Michel Bismut

Where do random fields come from in quantum field theory?

The notion of ground state representation can be used to explain easily how random fields enter into the construction of quantum fields. This is an entirely expository lecture, which takes advantage of the familiarity of the audience with Markov processes, infinite dimensional integration theory, and the Ornstein-Uhlenbeck process to motivate, starting with quantum mechanics, the definition of the free Euclidean Markov field and the study of the additive functional $\int_\Lambda :\varphi^4: dx$, $\Lambda \subset R^2$.

Leonard Gross

Quasi-isometry of Riemmannian Manifolds and Discrete Approximation

If a property of a manifold can be shown to be dependant on a network having a similar property then that property is likely to be fairly robust and preserved under Quasi-isometry. Many such properties have been so treated (particularly transience). However an example was outlined showing how the existence of non constant positive ~~harm~~ or bounded harmonic functions (or a non trivial shift invariant tail σ-field) is not preserved under quasi-isometry. Moser's Harnack theorem shows an important subclass of manifolds where these properties are preserved.

Terry Lyons

Non-degeneracy of the Malliavin Covariance Matrix

Let φ_t denote the flow associated to the s.d.e

$$dx_t = X_0(x_t)\,dt + X_i(x_t)\,dw_t^i$$

(with coefficients $X_0, X_1, \ldots, X_m \in C_0^\infty(\mathbb{R}^d, \mathbb{R}^d)$ and w_t a $BM(\mathbb{R}^m)$).
The Malliavin Covariance Matrix for the solution $x_t \equiv \varphi_t(x)$ starting from x is given by

$$C_t = \int_0^t (\varphi_s^{*-1} X_i)(x) \otimes (\varphi_s^{*-1} X_i)(x)\,ds .$$

Malliavin, Bismut, Stroock, etc have shown that provided $C_t^{-1} \in L^p(\mathbb{P})$ $\forall p<\infty$, x_t will have a smooth density.
It was further shown by Stroock that if

H_1: $X_1, \ldots, X_m; [X_i, X_j]_{i,j=0}^m; [X_i, [X_j, X_k]]_{i,j,k=0}^m; \ldots$ etc evaluated at x span $\mathbb{R}^d$

then indeed $C_t^{-1} \in L^p(\mathbb{P})$ $\forall p<\infty$ (see Springer LNM 976). The following lemma was presented which affords a simplification of Stroocks proof

Let T be a bounded stopping time.
Let $Y_t = y + \int_0^t a_s\,ds + \int_0^t u_s^i\,dw_s^i$

where $y \in \mathbb{R}$, and $a_t, u_t^1, \dots, u_t^m$ all have the form

$$\alpha + \int_0^t \beta_s \, ds + \int_0^t \gamma_s^i \, dw_s^i$$

with $\alpha \in \mathbb{R}$, and $\beta_s, \gamma_s^1, \dots, \gamma_s^m$ previsible with

$$\sup_{s \le T} |\beta_s| \, , \, \sup_{s \le T} |\gamma_s| \in L^p(\mathbb{P}) \quad \forall p < \infty$$

Then $\mathbb{P}\left\{ \int_0^T Y_t^2 \, dt < \varepsilon^q \text{ and } \int_0^T (|a_t|^2 + |u_t|^2) \, dt \ge \varepsilon \right\}$

$$= O(\varepsilon^p) \text{ for all } p < \infty \text{, for each } q > 36 \quad \square$$

James Norris

Stochastic Quantization: construction of renormalized diffusion processes

(Joint work with P. K. Mitter)

Using techniques from constructive field theory we prove the existence of weak solutions for the stochastic differential equation

$$d\varphi_t = -\frac{1}{2}(\varphi_t + \lambda C * :\varphi_t^3:) \, dt + dW_t$$

where $\varphi_t(x)$ is a random field on $\mathbb{R}^2$, W_t is a brownian motion with covariance

$$E(W_t W_{t'}) = \min(t, t') \, C(x, x')$$

and $C(x, x') = (-\Delta + 1)^{-1}$ - Δ is the Laplacian in two dimensions - This process is ergodic for x restricted to a finite domain and it has as stationary measure the Euclidean φ_2^4 measure in finite volume -

G. Jona-Lasinio

Some recent work on Dirichlet forms and quantum theory

We review some recent work on Dirichlet forms over $\mathbb{R}^n$ and over infinite dimensional spaces, with particular attention to problems which also have a counterpart in quantum mechanics and quantum field theory. In both cases we consider in particular the existence problem (closability) (new sufficient condition are mentioned) and the uniqueness problem. The condition of N. Wielens in finite dimensions and the ones of Takeda and Kusuoka ~~on~~ in infinite dimensions are also discussed. The significance of uniqueness is stressed, especially in connection with problems of quantum field theory. In finite dimensions the problems of unattainability of zeros of the wave function, the ergodicity of the process and the quantum mechanical tunneling are also discussed, as well as the convergence of Dirichlet forms and associated processes.

Sergio Albeverio.

Gauge fields

A survey of the global aspects of non-abelian gauge fields was given.

PK Mitter

Time reversal of infinite dimensional diffusions

(joint work with H. Föllmer)

We consider the problem under which conditions the "time reversal" $\hat{X}_t = X_{1-t}$ of an infinite dimensional diffusion $X = (X_t^i)$, $i \in I$ countable, $dX_t^i = b^i(X_t, t)dt + dW_i$ (W^i independent Wiener processes) is again "of the same type, with forward drift (b^i) and backward drift $(\hat{b}^i)$ being related by an analog of Kolmogorov's classical relation, namely

$$b^i((x^i,\xi),t) + \hat{b}^i((x^i,\xi),1-t) = \frac{\partial}{\partial x^i} \log \rho^i_t(x^i|\xi)$$

where $\rho^i_t(\cdot|\xi)$ is the conditional density of X^i_t, given $(X^j_t)_{j\neq i} = \xi$

Thm: This holds true if for all $i \in I$, the relative entropy of the law P of X with respect to P_i is finite (where P_i is the law of the process which arises from X by replacing X^i by a Wiener process independent of $(X^j)_{j\neq i}$.

Moreover, conditions on the drift (b^i) are given which guarantee this finite entropy condition, and which are not far from the usual conditions ensuring existence and uniqueness of the strong solution of an infinite dimensional stochastic differential equation.

Anton Wakolbinger

Stochastic Quantization: A Remark on Un~~stable~~ Actions.

(Joint work in progress with Ph. Blanchard and R. Sénéor).
It was explained how the procedure of stochastic quantization can possibly used to give a meaning to the formal perturbation expansion of Euclidean quantum field theories with unstable action functionals.

Jürgen Potthoff

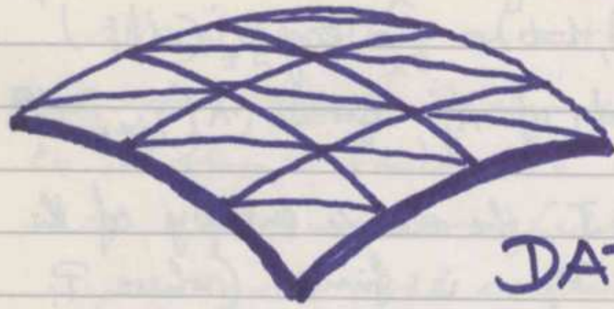
A survey on Propagation of chaos

A survey concerning the recent results on propagation of chaos, that link certain particle models with certain non linear evolution equations (spatially homogeneous Boltzmann equation, Vlasov equation ...) and certain non linear Markov processes was given. Some results concerning fluctuations and large deviation theorems one can get were explained.

A.S. Sznitman

1.

12. – 17. NOV. 1984



FLÄCHEN IN DER GEOMETRISCHEN DATENVERARBEITUNG

For curves or surfaces respectively undesired domains of curvature or Gaussian curvature respectively can be detected by K-orthotomic – curves or surfaces. The K-orthotomics have singularities if a curve has inflection points or a surface has parabolic points. If such undesired domains are detected they can be smoothed interactively or by an algebraic procedure using Bezout Eliminating Method. As smoothing criteria are used points with stationary curvature or stationary Gaussian curvature. For surfaces some numerical problems may arise therefore the method is changed to "Smoothing along parametric curves".

13.11.84 Josef Hoschek

Thin Plate Splines with Tension

The deflection of a thin plate subjected to point loads and mid-plane (membrane) forces is developed. This function is then used to interpolate scattered data in two dimensions. Application of "moderate" ~~too~~ tension controls overshoot in the vicinity of steep gradients. Examples are given which demonstrate this behavior.

Richard Franke, Monterey, 12 Nov. 19

Convergence of Control Polygons:

It is well known that a curve which is parametrically represented using Bernstein polynomials has a sequence of control polygons which converges to the curve as the degree tends to infinity. We show the same result holds for splines where either the degree increases to infinity or knots are inserted repeatedly. The method involves using certain quasiinterpolants, and delivers error bounds (rates of convergence) which are optimal order (linear for degree raising, quadratic for knot insertion). The analogous results hold for tensor-product surfaces.

Larry Schumaker Nov. 14, 1984.
Texas A+M Univ.

Transfinite Interpolation and Shape Control

The desire to control the shape of parametric spline curves in computer aided geometric design has inspired a variety of shape-preserving fitting methods. Some are relatively modest, aiming only to reproduce straight line segments, while others deal with general convex regions. These nonlinear methods can be used to generate blending rules for TFI surface fitting schemes, but strong shape control over surfaces probably cannot be obtained in this way. As an illustration of what appears to be a more fruitful approach, a simple form of shape control appropriate to lofted surfaces is defined, and a technique is described which can be used to guarantee this shape control.

Alan K. Jones 15.11.84
BOEING, SEATTLE, USA

BIVARIATE SPLINE ALGORITHMS

Triangular patches were first considered by de Casteljau in 1963. B-splines over triangular grids were first constructed by Sabin in 1977. In the early '80's de Boor et al. pointed out that Sabin's B-splines are bivariate box splines. – In my talk I gave three simple algorithms for calculating bivariate box splines and their linear combinations:

(1) a Mansfield-like recursion formula,
(2) refinement of control net together with the constructing of ~~the~~ discrete box splines, and
(3) construction of the Bézier net from the box spline control net together with the construction of a single Sabin spline.

The indicated proofs are all very geometric.

Wolfgang Böhm
TU-Braunschweig 12.11.84

Piecewise Algebraic Surfaces for Solid Modelling

A technique is presented for modelling with algebraic surfaces. The surface is defined as the intersection of the hypersurface $w = f(x, y, z)$ with the hyperplane $w = 0$. The hypersurface is expressed as a trivariate Bernstein polynomial defined by a tetrahedral lattice of Bernstein Bézier control points. This scheme lends itself to solid modelling because a "free-form" volume is naturally defined by the intersection of the half space $f(x, y, z) \leq 0$ with the defining tetrahedron. It is noted that these surfaces can be pieced together with derivative continuity; they are easily translated, rotated or scaled; and that intuitively meaningful techniques exist for forcing the surface to interpolate a point and to have a specified normal at that point.

Thomas W. Sederberg
Brigham Young University
13.11.84

Some Thoughts on the Smoothing of Parametric Curves and Surfaces

Classical smoothing theory is not ideally suited for application to parametric curves and surfaces. Some experiences are described and some suggestions for practical algorithms outlined. The paper will, on the whole, raise more questions than it answers.

Michael J. Pratt,
Cranfield Institute of Technology, England.
14 November 1984.

2 Clough-Tocher Interpolants

The standard Clough-Tocher interpolant is developed in terms of Bernstein-Bézier triangular patches. An alternative to the standard method of condensation of parameters is proposed (minimize C^2 discontinuities between adjacent patches). Reflection lines are used to compare both interpolants.
A trivariate Clough-Tocher interpolant is also proposed: a tetrahedral domain is split into 12 subtetrahedra. The interpolation conditions are incorporated in Bernstein-Bézier terms.

Gerald Farin
U. o. Utah
13 Nov 84

Surface Design with Curve Networks

We present a technique for surface design based upon a network of curves. Some new patch segments required for these networks are discussed. Examples which show the value of tension parameters are presented.

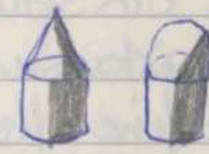
Gregory M. Nielson
November 14, 1984

A region-oriented analytical visibility method for tensor product surfaces

An analytical method to determine the visible parts of tensor product surfaces is presented. The boundaries of visible regions are computed in the (u, v)-plane. This offers many advantages over approximating algorithms in image-space: the result is device-~~in~~dependent, calculates only the visible parts and is invariant under image-transformations.

Wolfgang Straßer 14/11/84
TH Darmstadt

Detecting surface irregularities using isophotes



In car industry we need methods to control the smoothness of a surface. One way to do this is to display isophotes, i.e. lines of equal light intensity of a surface, that reflect C^1 or C^2 continuity of the surface by C^0 or C^1 continuity. The light point can be chosen arbitrarily. The advantage of the isophotes is, that they can be computed easily.

Thomas Pöschl 15/11/84
AUDI-NSU Ingolstadt

Subdivision algorithms for the generation of box spline surfaces (W. Dahmen, C.A. Micchelli)

The notion of box spline allows for a unified treatment of multivariate splines on various types of uniform grids. Generating box spline surfaces or finding intersections of such surfaces is facilitated by a simple but efficient subdivision algorithm which essentially consists of successively forming line averages of the original control points. Interpreting these line averages as quadrature formulas or, alternatively, employing a multivariate analog of Schoenberg's variation diminishing operator allows to prove quadratic convergence (with respect to the level of refinement) of the refined control nets to the spline surface.

Wolfgang Dahmen 15/11/84
Universität Bielefeld

Generation of Box Spline Surfaces

A box spline surface is related to a control net. There exists subdivision algorithms for box spline surfaces that refine the corresponding control net so that the refined control net approaches the spline.
In my talk subdivision algorithms were regarded as a special case of numerical integration. Thus the method of subdivision was generalized and new algorithms for generating a box spline surface were presented.
Using the existing error analysis for numerical integration it is easy to show how rapidly the refined control net approaches the spline surface.
Some error estimates for the given generation methods were presented which are better than the known one for the subdivision algorithm

Hartmut Prautzsch
TU Braunschweig
14. November 1984

Surfaces in Computer Aided Geometric Design: A Survey with New Results

"Surfaces in CAGD" focuses on the representation and design of surfaces in a computer graphics environment. It is a new area having the dual attraction of interesting research problems and important applications. The subject can be approached from two points of view. The Design of Surfaces includes the interactive modification of geometric information while the Representation of Surfaces implies that the geometric information is fixed. Design takes place in 3-space and Representation can be higher dimensional. Surfaces in CAGD can be traced from its inception as Coons patches and Bezier patches to triangular patches which are current research topics. Triangular patches interpolate and approximate to arbitrarily located data and require the preprocessing steps of triangulation and of derivative estimation. New contouring algorithms have also resulted from this research. Finally, multidimensional interpolation is discussed and surfaces in 4-space are illustrated by means of color computer graphics.

Robert E. Barnhill
University of Utah – Math
12 November 1984

Multistage Surface Representation Methods for the Graphical Display of Arbitrarily-Spaced Data

In many applications at NASA Ames, the scientist has a set of discrete points at which some parameters, such as pressure or velocity, are given. To graphically display this data (as contours of constant pressure, for example), the discrete data must be represented by a continuous function. This continuous "surface" can then be evaluated at any point as required for graphical display. The arbitrarily-spaced point data in these applications may or may not have some inherent structure or connectivity. Multistage algorithms for representing arbitrarily-spaced data are presented in this report. These algorithms are applied to data which occur along lines in three-space (so there is some inherent structure in the data). This type of data commonly occurs in wind tunnel tests where data are recorded at taps on the aircraft body. Some geophysical and geological applications which have this type of data are also discussed.

Sarah E. Stead
NASA Ames Research Center
November 15, 1984

Implementation of a Divide-and-Conquer Method for Intersection of Parametric Surfaces

Use of parametric forms allows development of a single surface/surface intersection algorithm dependent only on availability of parametric evaluators for the surfaces. The algorithm here comprises four steps. First, the surfaces are subdivided using an iterative process. The second step is intersection of the pairs of subpieces which may possibly interact. The third step sorts the intersection segments into connected curves. Finally, the fourth step is refinement of the points of intersection. Design and implementation considerations are discussed and some results are shown.

Elizabeth G. Houghton
McDonnell Aircraft Co.
November 16, 1984

~~Surfaces~~
Computer Aided Design on the set of Bézier curves and Bézier surfaces

H. Frank , R. Arnold

In engineering practice even exactly performable objects in the sence of mathmatics are merely approximatively treated. Therefore it seems to be reasonable to consider all objects of a manufacturing process in the same class of approximation. The solution of a problem also has to belong to this approximation class.
Under this point of view we consider the problems of the intersection of surfaces and the offset curves on the approximation class of the cubic Bézier curves and the bicubic Bézier surfaces. Some solutions of these problems are given on subclasses.

R. Arnold
Universität Dortmund
15. 11. 84

Approximate Conversions of Surface Representations with Polynomial Bases.

Lothar Dannenberg, Horst Nowacki

A majority of current CAD surface modelling systems have surface representations with parametric polynomial bases, however, with differing maximum degrees. In transferring data from one system to another, in order to maintain sufficient shape fidelity, the need for approximate conversion in degrees and mesh spacing arises. The authors have developed schemes for converting high order, coarse mesh representations on regular rasters into low order, fine mesh approximations and vice versa. The methods are based on least squares positional error criteria with constraints on the original boundary curves and their derivatives. The original mesh is subdivided based on a "uniform error" strategy derived from de Boor.

Horst Nowacki
TU Berlin
15.11.84

Boolean Hermite trigonometric interpolation

Boolean interpolation is used to construct bivariate Hermite polynomial interpolation schemes of local type. These schemes produce piecewise polynomial interpolants on a rectangular. Univariate trigonometric interpolation on a uniform mesh is a good global approximation method for 2π-periodic analytic functions. Boolean interpolation is applied to Hermite trigonometric interpolation to generate a global approximation method for bivariate 2π-periodic analytic functions by complete Hermite interpolation on a bivariate uniform mesh.

F. J. Delvos
(Siegen)

Problems using Splines and Spline-surfaces in Car Body Design

In the view of the CAD-System SYRKO devellopped for Car body design and construction at Daimler Benz there are treated solved and unsolved problems which occured in practice. The deal with approximation of geometric curves by points and splines as well as with the fitting problems of surfaces. Another point are offset splines and surfaces. The aim is to ask questions in order to get answers.

15.11.84 R. Klass (Sindelfingen)

GEOMETRY CELLS AND SURFACE DEFINITION BY FINITE ELEMENTS

The approximation of the geometry as applied in the finite element method can be used to derive line, surface and solid cells for computer-aided design. These cells can be used for the representation of one, two- and three-dimensional objects. Typical cell forms are curve, triangle, quadrilateral, tetrahedron, pentahedron and hexahedron. This elementwise representation can be handled interactively on a display console and is very well suited if the designed object will be later analysed with with finite element method and an automatic mesh generation program. Furthermore the surface definition problem is presented with finite elements. The surface is defined as a thin plate in bending. Special regard is given to the interpolation in an triangular element based on a complete interpolation of the fifth order.

15. Nov. 1984

Ingolf Grieger
Universität Stuttgart

Application of parametric surfaces in industry

A standard model to interface geometry between design and manufacturing is proposed. This standard should combine the benefits of low storage requirements of simple surfaces and the necessity of higher level parametric geometry (B-splines, Bezier ...). By means of standardized evaluation all kind of parametric curves and surfaces can be included. Some comments on the geometry of existing geometry modellers are given.

15. Nov. 1984 Harald Eckert
MBB

Extended Application of a Simple Data Interface for Free Form Geometry (VDAFS) in Automotive Design

A summary of the CAD/CAM activities in automotive design at BMW and the functions and limitations of the VDAFS data interface used for communication between different CAD systems are discussed. A proposal of a model description founded on the idea of basis entities, interpretation entities, relation entities and composition geometry is outlined. The VDAFS-entities are considered as a subset of this standard. Required mathematical algorithms and their relationship to VDAFS datastructures are specified. The usage and industrial context of these algorithms are discussed.

15. November 1984. Werner Dankwort
BMW, München

Can the Oslo Algorithm be made more efficient?

The Oslo Algorithm is a general method for adding knots to a B-spline curve or tensor product surface. The method provides a framework in CAGD for both manipulating and rendering of spline curves and surfaces, and is derived from properties of discrete B-splines. An improved version of the Oslo Algorithm was presented and compared with other methods for adding knotlines to a tensor product B-spline surface.

15. November 1984 Tom Lyche
Univ. of Oslo, Norway.

1. MONOTONICITY PRESERVING BICUBIC INTERPOLATION

Given monotonic data on a rectangular mesh, the problem is to determine a monotonic interpolant. (Here, we say $f(x,y)$ is monotonic iff $f(\cdot,y)$ and $f(x,\cdot)$ are monotonic univariate functions.) We use piecewise cubic Hermite interpolation, and describe a five-step algorithm that produces Hermite derivatives $f_x(x_i,y_j)$, $f_y(x_i,y_j)$, $f_{xy}(x_i,y_j)$ that guarantee a $C^{1,1}$ monotonic piecewise bicubic interpolant if the data $f(x_i,y_j)$ are monotonic.

2. USE OF FOWLER-WILSON SPLINES FOR SURFACES OF REVOLUTION OR TRANSLATION

The F-W spline is a C^1 interpolant to planar data determined by local coordinate systems on each segment. This is used for the APT TABCYL, and has recently been added to the TIPS solid modeler at LLNL. Relations to standard spline entities were described and examples from TIPS shown. [signature]
Lawrence Livermore Nat'l. Lab., Livermore, California, USA

Bernstein - Bézier methods for bivariate spline approximation

The Bernstein - Bézier method of representation of polynomials over triangles can be very useful for the explicit construction of different kinds of bivariate spline approximants (in the sense of smooth piecewise polynomial functions). Three different examples illustrate this assertion:

1) quasi-interpolants of high degree of accuracy on a regular 3-direction mesh
2) composite finite elements for Hermite interpolation at arbitrary points of the plane
3) quadratic splines for solving some Lagrange interpolation problems on criss-cross triangulations.

15 Nov. 1984. Paul Sablonnière
University of Lille (France)

Families of adjoint patches for a Bézier triangular surface

For any point $P \in T$, the domain triangle of the nth Bézier triangular surface, families of triangular patches B_P^m $(m = 1, 2, \cdots, n)$ associated with the surface at P are defined through the de Casteljau algorithm. The following two theorems are proved.

1. The original Bézier surface is the envelope of each family of the adjoint patches B_P^m, where $m = 1, 2, \cdots, n-1$.

2. If the Bézier net is convex over T, then so is B_P^m, $m = 1, 2, \cdots, n$ and $P \in T$. Furthermore we have that $B_P^i \geq B_P^j$ holds in the domain

of B_p^j for $1 \le j < i \le n$.

A general formula for degree elevations and its applications ~~are~~ also mentioned.

Geng-zhe Chang
12, Nov. 1984
University of Science and Technology of China,
Hefei, Anhui
The People's Republic of China

Bernstein-Bézier representation of volumes

Theme of the lecture is the tensor product description of spatial domains by the method Bernstein-Bézier. 2 problems are discussed: The construction of a volume point and of all derivatives in this point and the interpolation problem.
First problem: A de Casteljau scheme for volumes is be given, by which point and derivatives are constructed. The question for the number of possible de Casteljau algorithms could be answered and the question for the best of these too. The results are specialized to Bézier surfaces.
Second problem: By the question follows a system of $(l+1)(m+1)(n+1)$ linear equations for the Bézier points. The splitting of the coefficient matrix leads to 3 smaller problems of degree $l+1$, of degree $m+1$ and of degree $(m+1)(n+1)$. In the special case of an equidistant parametrization for Splines of degree (l,m,n) with $l=m=n$ additional simplifications follow. Example: For $l=m=n=4$ only 8 elements of a 5×5 matrix have to be calculated instead of inverting an unfavourable filled 125×125 matrix.

Dieter Lasser 16.11.84
D-61 Darmstadt
TH-Darmstadt
Mathematik - AG 3

Continuous fitting of Bézier's surface patches

The algorithms of C^2 arranging $n \times m$ Bézier patches to interpolate a spatial field of vertices and the given first interior row of Bézier control points along the boundary were recalled and mixed for the closed and non closed case. Based on the notion of "contact of order r" (abrev.: $\bar{C}^r$) for two differentiable manifolds immersed in some affine space, the analogous $\bar{C}^2$ interpolation problem for Bézier patches of degree 3 was posed. A solution was given in the case of linear parameter transformations preserving parameter lines.

W. B.

Surfaces with geometric continuity

Barnhill, Birkhoff and Gordon initiated "triangular Coons patches" in 1973, using parallel projectors. A triangular interpolant with radial projectors was developed by Nielson. This "side vertex scheme" can be generalized to a geometric triangular patch:

A geometric Hermite operator is defined by

$$HO(g) := He(t)\,g(e) + \overline{He}(t)\,g'(e) + \frac{\overline{\overline{He}}}{2\|g'(e)\|}\Big[\big[g'(e), g''(e)\big], g'(e)\Big]; \; e = 0,1$$

$He, \overline{He}, \overline{\overline{He}}$ is the quintic Hermite base and

$$\big[[g'(e), g''(e)], g'(e)\big] = \|g'\|^4 \cdot \varkappa \cdot e_2 = \|g'\|^4 (\varkappa_n N + \varkappa_g [N, g'])$$

where $\varkappa$ = curvature of the surface curve

e_2 = principal normal vector

N = normal vector of the surface

$\varkappa_N$ = normal section curvature

$\varkappa_g$ = geodesic curvature

Applying the geometric Hermite operator to

$$R_i(t) := F(t S_i + (1-t) V_i) \,; \quad t = 1 - b_i$$

we define

$$P_i[F] = H_e(t) R_i(t) + \bar{H}_e(t) R_i'(t) + \frac{\bar{\bar{H}}_e(t)}{2\|R_i'(t)\|} \left[[R_i'(t), R_i''(t)], R_i'(t) \right]$$

using convex combinations we get the interpolation scheme

$$P[F] := \frac{b_2^3 b_3^3 P_1[F] + b_1^3 b_3^3 P_2[F] + b_1^3 b_2^3 P_3[F]}{b_2^3 b_3^3 + b_1^3 b_3^3 + b_1^3 b_2^3}$$

Hans Hagen

A B-spline based sculptured surface modeller developed in the German / Norwegian CAD/CAM project APS

The geometry can be defined by different methods, curves as straight lines, quadrics (conics), interpolating splines and intersections between surfaces. Surfaces as ruled, lofted, rectangular curve mesh surfaces, rotational and offset surface. The modeller uses B-splines as the common storage format for geometry. Intersection functions are developed for the following geometric entities: two B-spline represented curves, a B-spline represented curve and a conic curves/surfaces, B-spline represented curve and B-spline represented surface, and conic surfaces and B-spline represented surfaces.

Tor Dokken

INTERPOLATION TO BOUNDARY DATA ON SIMPLICES

The problem of multi-dimensional interpolation to boundary data on a simplex is considered. An explicit formulation of a C^0 scheme which matches a finite set of function and derivative values is described. The scheme is then used in the development of a C^N transfinite interpolant which matches data given everywhere on the boundary of the simplex.

John A. Gregory
Brunel University (England)

Integral parameters from patch modeled volumes

Practical application of bicubic Coons patches is considered for calculation of integral parameters of floating bodies, such as volume, moments of volume, area of intersection with water surface, first and second moments of that area. Integration is performed using divergence theorem and Gaussian approximation. Deviations between true surface, modeled surface and Gaussian approximation are discussed.

Uwe Rabien
Germ. Lloyd Hamburg

Predicting the Shape of Bernstein-Bézier Curves

The definitions of convex points, inflection points, and cusps for planar curves are reviewed. These definitions are applied to cubics (Bernstein-Bézier) and several interpolation problems are posed. The problem of finding a Bernstein-Bézier cubic with initial and final positions specified, as well as initial and final tangent directions and a tangent direction at a cusp, is investigated. Necessary and sufficient conditions are found for existence of such an interpolant.

Harry W. McLaughlin
Rensselaer Polytechnic Inst.

Fortbildungstagung für Mathematiklehrer 18.-23.11.84 (4.)

Methoden der Informatik und Erfahrungen mit dem Computer im Mathematikunterricht

Statistische Simulationen und deren Graphische Darstellung auf dem Bildschirm zu den Themen Qualitätskontrolle und Testtheorie wurden vorgeführt und besprochen.

Die Bedeutung des Monotoniesatzes wurde illustriert an einem Beweis der Transzendenz von e. Ausgehend von Wachstumsraten, über Eigenschaften der Exponentialfunktion bis zu numerischen Lösungen von $f' = f$ wurden die heuristischen Ideen entwickelt, die der Approximation von exp zugrunde liegen, die dann im Transzendenzbeweis wichtig sind.
"Vom Monotoniesatz zur Transzendenz von e" erläutert in den Semesterber.
An Irrationalitätsbeweisen, Tangenten- und Flächendiskussionen an Parabel und Kreis wurde exemplarisch erläutert, wo tragende Ideen des Analysisunterrichts zum ersten Mal, und vor den endgültigen Definitionen auftreten.

Um die rasche Konvergenz von $x_{n+1} = x_n + \sin x_n$ gegen π zu erläutern wurde das Newtonverfahren vom Standpunkt der Umkehrfunktion g zu f diskutiert. Die überdurchschnittliche Approximation einer Wendetangente erklärt dann die rasche Konvergenz von $x_{n+1} \to \pi$. Dabei wurde auch eine rasche Berechnung von sin mit Hilfe des Monotoniesatzes begründet.

23.11.84 H. Karcher

Es werden fünf größere Unterrichtseinheiten skizziert (Projekte), in denen gleichzeitig grundlegende Konzepte der Informatik und der diskreten Mathematik entwickelt werden können.

1. Lösung des Traveling-Salesman-Problems (mit Dreiecksungleichung): „Brutaler" Algorithmus, naive Heuristik, Konstruktion des minimal spannenden Baumes nach Kruskal. Herleitung des worst-case Gütefaktors 2. Hierbei kommen u.a. vor: Grundkonzepte über (bewertete) gewöhnliche Graphen, freie Bäume, Zentrum und Bizentrum. Sortieralgorithmen, Aufbau von linearen Listen über Pointer
2. Elementare Zahlentheorie und Public-key Kryptosysteme. Primzahlen, Primzahltest, Primzahlzerlegung. Erstellung einer Primzahl-Liste. Sieb des Eratosthenes. Prime Restklassen. Eulersche φ-Funktion. Zerlegung einer Zahl in zwei große Primzahlen. Lösung der Gleichung $ed \equiv 1 \bmod \varphi(n)$. Numerisches Codieren u. Decodieren.
3. Dateiverarbeitung. Am Beispiel einer Schüler-Kurs-Datei werden grundlegende Anfrage-Operationen behandelt.
4. Der Greedy-Algorithmus als heuristisches Prinzip. Matroide. Scheduling-Probleme
5. Wurzelbäume als Suchbäume. Erstellung eines optimalen Präfix-Codes. Huffman-Algorithmus.

Zu den Projekten 1 bis 3 sind Pascal-Programme erstellt worden, die auf einem Rechner Apple IIe demonstriert wurden (gemeinsam mit H. Handels)

24.11.84 W. Oberschelp (Aachen)

Im ersten Themenbereich "Eigenschaften von Programmiersprachen" wurden Anforderungen an Programmiersprachen behandelt: Modularität (insbesondere Funktionalität), Interaktivität, Erweiterbarkeit. Eine rein funktionale Lösung des "minimal spanning tree" Algorithmus wurde in einer listenverarbeitenden Sprache (Logo) gegeben. Die Erweiterung von Programmiersprachen im Bereiche der Kontrollstrukturen wurde behandelt.

Im zweiten Themenbereiche "Differenzengleichungen und ihre Anwendungen" wurde auf folgende Aspekte eingegangen: Elementarisierung durch Diskretisierung, graphische Darstellungsmöglichkeiten, geschlossene Lösungen, Stellenwert und Beziehungshaltigkeit des Themas im Mathematikunterricht. 24.11.84 J. Ziegenbalg (Reutlingen)

«Effiziente Algorithmen»

5, 1.12.84

26.11 — 30.11.84

Fault Tolerant Distributed Computing

The general problem considered is that of increasing common knowledge among a number of processors participating in a point to point communication network, in the presence of faults. We study a family of algorithms for reaching agreement in this context. With respect to termination time, the family is provably optimal in a simple model of communication. Here we study the effect on optimality of moving to a more realistic intermediate model on the way to an implementation.

Ray Strong (IBM - San Jose)

Polymorphic Arrays: A Novel VLSI Layout for Systolic Computers

We present a novel architecture for massively parallel systolic computers, which is based on results from lattice theory. In the proposed architecture, each processor is connected to four other processors via constant-length wires in a regular borderless pattern. The mapping of processes to processors is continuous, and the architecture guarantees exceptional load uniformity for rectangular process arrays of arbitrary sizes. In addition, no timesharing is ever required when the ratio of processes to processors is smaller than $1/\sqrt{5}$.

Adi Shamir

How to Share Memory in a Distributed System

We study the power of shared-memory in models of parallel computation. We describe a novel distributed data structure that eliminates the need for shared memory without significantly increasing the run time of the parallel computation. More specifically we show how a complete network of processors can deterministicly simulate one PRAM step in $O[(\log n)^2 (\log\log n)]$ time, when both models use n processors, and the size of the PRAM's shared memory is polynomial in n. We also establish that this upper bounds are nearly optimal. We prove that an on-line simulation of T PRAM steps by a complete network of processors requires $\Omega(T \frac{\log n}{\log\log n})$ time.

A simple consequence of the upper bound is that an Ultracomputer (the only current feasible general purpose parallel machine), can simulate one step of a PRAM (the most convenient model of parallel computation), in $O[(\log n)^2 \log\log n]$ steps.

אלי אפפל   Eli Upfal   26/11/84

Efficient Parallel Solution of Linear Systems

We describe a quadratically convergent method for matrix inversion that requires $O(\log n)^2$ time using $M(n)$ processors (where $M(n)$ is the number of processors required to multiply two $n \times n$ matrices in $O(\log n)$ time). This is an optimum processor bound and a $\sqrt{n}$ improvement of known bounds. It is the first known polylog time matrix inversion algorithm that is numerically stable.

Also, we give a direct method for exact solution of a sparse $n \times n$ Hermitian linear system $Ax = b$. If the graph $G(A)$ (which has n vertices and an edge for each nonzero entry) is $s(n)$-separable, then our algorithm requires $O(\log n (\log s(n))^2)$ time and $|E| + M(s(n))\sqrt{s(n)}$ processors. The algorithm computes a recursive factorization of A so that to solve another linear system $Ax = b'$ with the same A requires only $O(\log n \log s(n))$ time and $|E| + s(n)^2$ processors.

(This work was done with Victor Pan)

John H Reif 27/1/84
(MIT and Harvard)

OPTIMAL SOLUTIONS FOR A CLASS OF POINT RETRIEVAL PROBLEMS

Let P be a set of n points in the Euclidean plane and let C be a convex figure. We study the problem of preprocessing P so that for any query point q, the points of P in $C+q$ can be enumerated efficiently. If constant time suffices for deciding the inclusion of a point in C, we then demonstrate the existence of an optimal solution: the algorithm requires $O(n)$ space and $O(k+\log n)$ time for a query with output size k. If C is a disk, the problem becomes the well-known FIXED-RADIUS NEIGHBOR problem, to which we thus provide the first known optimal solution.

BERNARD CHAZELLE (i.V) and Herbert Edelsbrunner

On the single-operation worst-case time complexity of the disjoint set union problem.

Let $S_1, S_2, \ldots, S_n$ be n pairwise disjoint sets each of size 1. We consider operations of the following type:
FIND (x): determine the set containing the element x.
UNION(A,B,C): combine the two disjoint sets A and B into a new set named C.
A sequence of FIND- and UNION-operations which is performed on-line is called the <u>disjoint set union problem</u>.
We give an algorithm, which has single-operation time complexity $O(\frac{\log n}{\log\log n})$. Also we define a class $\mathcal{B}$ of algorithms, containing the class of algorithms, defined by Tarjan and prove: Every algorithm from the class $\mathcal{B}$ has single-operation time complexity $\Omega(\frac{\log n}{\log\log n})$ in the worst-case.

Norbert Blum (Saarbrücken)

Additive Weights of Trees

We consider a general additive weight of trees which depends on the structure of the subtrees and on a weight function defined on the number of internal and external nodes and on the degrees of the nodes. Choosing particular weight functions, the corresponding weights are identical to well-known parameters appearing in the analysis of some algorithms (e.g. internal and external path length, internal and external degree path length, number of internal or external nodes, number of nodes of degree $s \in \mathbb{N}_0$ etc).

For a simply generated family of rooted planar trees $\mathcal{F}$ (eg. all trees defined by restrictions on the set of allowed node degrees), we derive a general approach to the computation of the average weight of a tree $T \in \mathcal{F}$ with n nodes and m leaves for an arbitrary weight function. This general result implies exact and asymptotic formulas for the average weight of a tree $T \in \mathcal{F}$ with n nodes if the weight function is a polynomial in the number of nodes and leaves with coefficients depending on the node degrees.

Finally, we apply the above results to three types of "tree search cost measures" for a tree $T \in \mathcal{F}$ recently introduced in connection with the representation of trees in major-minor loop configuration of bubble memories.

(R. Kemp, Frankfurt)

Topological methods in computation theory.

There is a general agreement that problems which are highly complex in any naive sense are also difficult from the computational point of view. It is therefore of interest to find (in analogy to the Galois group of an algebraic equation) invariants and invariant structures which measure in some respect the complexity of a given problem.

The questions which we are going to consider are classification problems which arise in a natural way in connection with non-uniform computations. The computations are described by questionnaire- or branching programs. The 'complexity' of the problem is measured by classical topological invariant – Betti numbers and Euler-Poincaré-characteristic χ – of topological structures (simplicial complexes, topological spaces) which can be defined in a natural way to any classification problem C. These topological spaces Pure K and Mix K are complementary subspaces of the space Cond S which can be proved to be homotopic to a bouquet of spheres. In case of boolean functions this is exactly one sphere and yields therefore the Lefschetz-Alexander-duality between Pure K and Mix K: $H_i(\mathrm{Pure}(K); \mathbb{Z}) \cong H_{N-i}(\mathrm{Cond}\, S, \mathrm{Mix}\, K; \mathbb{Z})$. It is shown that $h_0(\mathrm{Pure}\, K)$ and $h_{N-1}(\mathrm{Mix}\, K)$ are lower bounds for the size, i.e. the number of nodes, in an optimal decision tree for K. This yields sharp lower bounds for different concrete problems.

L. Budach (Humboldt-Univ. Berlin)

Hierarchical Graph Algorithms

By using hierarchical definitions of graphs one can define very large graphs with short descriptions. The blow-up from the length of the description to the size of the graph can be as large as exponential. Thus even the most efficient graph algorithms become hopelessly inefficient when their complexity is measured in terms of the length of the hierarchical description. Practical running times of such algorithms are increased substantially by the amount of paging that is necessary to process a graph that does not fit in main memory because of its large size. We discuss how the structure implicit in the hierarchical description of the graph can be exploited to increase the efficiency of algorithms processing the graph. We mention efficient solutions of graph problems such as connectivity properties, minimum spanning forest, planarity, path problems and others.

T. Lengauer (Paderborn)

On the power of two-way random generators

We consider space bounded machines with an additional two-way (read-only) random tape and show that these machines use precisely logarithmically less space than the machines with one-way random tape (i.e. probabilistic machines in the usual sense). This answers the question of Borodin, Cook, and Pippenger, whether their deterministic squared space simulation of probabilistic machines works also for the case of two-way random tape.
Even Las Vegas (i.e. error free) machines without any work space can simulate any deterministic linear bounded automaton; with log n work space they accept exactly the sets in PSPACE. Also for time bounded machines the space bound can be logarithmically reduced using a two-way random tape; e.g. there is a probabilistic algorithm which recognizes in polynomial time and log n space (using a two-way random tape of polynomial length).
This work was done with Marek Karpinski, Dortmund.

Rutger Verbeek, Bonn

The complexity of embedding graphs into binary trees

We consider the problem of embedding graphs into binary trees. As the cost of such an embedding we define the maximum distance in the binary trees between the images of nodes which are adjacent in G. Three results are presented.

(1) The problem of embedding graphs into binary trees is NP complete even when the class

of inputs is restricted to the class of trees of height 3.

(2) Every outerplanar graph G can be embedded into a binary tree with edge length $\lceil \log_2 d \rceil + \lceil \log_2 \log_2 d \rceil + 5$, where $d = 2\,\mathrm{maxdegree}(G) - 2$.

(3) Preorder trees, Inordertrees and outerplanar graphs of maximal degree 3 can be embedded into binary trees with edge length 3.

B. Monien (Paderborn)

Abstract semantics of a "single assignment" language for nonsequential algorithms

Single assignment languages provide means to formulate algorithms without imposing a total order of execution on their statements. Thus the order of execution depends "data availability" only. The single assignment rule which is to be satisfied guarantees determinacy of the result no matter in which order statements are executed. This situation gives rise to parallel execution of statements.

The talk discusses the different motivations for designing single assignment languages, outlines the features of the language DONALD developed by the authors and studies semantics and determinacy of DONALD-programs in an abstract framework which models parallel execution of statements.

Bernd Mahr (TU Berlin)

Simulation of Large Networks on Small(er) Networks.

Parallel algorithms are normally designed for execution on networks of N processors, with N depending on the problem size. In practice networks have a fixed (and smaller) size. This has motivated the study of suitable, structure preserving mappings (called emulations) of networks onto smaller networks. Load-balancing is enforced by requiring that every node of the smaller graph emulates an equal number of nodes of the larger network. We present a detailed analysis and complete characterisation by intricate combinatorial means of the emulations for common classes of interconnection networks such as the shuffle-exchange network and the hypercube. The presentation reports on ongoing work jointly with H.L. Bodlaender.

Jan van Leeuwen (Utrecht)

On the complexity of slice functions

The monotone representation of a Boolean function $f:\{0,1\}^n \to \{0,1\}$ is given by $(f_1,\dots,f_n)$ where $f_k(x) = (f(x) \wedge T_k(x)) \vee T_{k+1}(x)$ where T_k are the threshold functions. The Boolean network complexity of f and $(f_1,\dots,f_n)$ is nearly the same. For the slices f_k of f the network complexity over the monotone basis $\{\wedge,\vee\}$ is only by an additive term $O(n \min(k, n-k, \log^2 n))$ larger than its network complexity over complete bases. Here we may simulate negations by monotone subcircuits. This is a new approach for the proof of lower bounds on Boolean network complexity. We show some more results on the complexity of the pseudo-negations used in this approach and give a geometrical representation of Boolean networks. Using this representation we present for some slices of the Boolean convolution or the clique function efficient algorithms.

Ingo Wegener (Frankfurt a. M.)

Fast algorithms for n-dimensional restrictions of hard problems

Let M be a parallel RAM with p processors (with indirect addressing, arithmetic +, −) recognizing $L \subset \mathbb{N}^n$ in t steps. Then L can be recognized by a linear search algorithm (LSA) in time $O(n^4(\log(n)+\log(p)+t))$. This result generalizes a previously known LSA for the knapsack problem (see Tagung "Complexity Theory" Oberwolfach, fall 83) and destroys the hope to prove nonpolynomial lower bounds in the model of LSA's for NP-complete problems as Binary Programming, version of Integer Programming, Travelling Salesman Problem and even the Δ_2^P-complete Unique Optimal Travelling Salesman problem.

Friedhelm Meyer auf der Heide (San Jose)

On clock synchronization

A simple and efficient distributed algorithm for synchronizing clocks will be presented. The algorithm tolerates both link and node failures in an arbitrary network, but requires the existence of an authentication scheme. We will later prove that if we restrict clocks to running within some linear function of real-time, the clock synchronization is impossible (when no authentication is used) when one-third of the processors are faulty.

Danny Dolev (Jerusalem)

Optimum Scan-Width Selection Under Containment Constraints

We consider the following algorithmic problem, which arises in connection with optimally choosing beam widths and positions for electron exposure of integrated circuit wafers. Let $H>0$ be a fixed real number, and let c be a fixed, positive valued, non-decreasing cost function defined on $(0,H]$. An instance of the problem consists of a given range, $R=[a,b]$, and n given intervals $I_i=[a_i,b_i]$, $1\le i\le n$, each contained in R and of length not exceeding H. A solution for such an instance is a collection of segments, $S_1, S_2, \ldots, S_k$, each of length $\le H$ and contained in R, s.t. each given interval is contained in at least one segment and the union of all the segments is R. The goal is to find an optimal solution with respect to c, i.e. a solution for which the sum of the costs of the individual segment lengths is as small as possible. Using dynamic programming techniques, we give efficient algorithms and data structures for solving this problem for several natural classes of cost functions, the most general of which is the class of all concave increasing functions, solved by an algorithm that runs in time $O(n^2)$. If the cost function for a segment is simply its length, then the time complexity is linear (which is optimal), and when the cost is linear in the length (incl. a free term) the complexity is $O(n \log n)$.

Ron Pinter, IBM Israel
(joint work with
Michael R. Garey,
AT&T Bell Labs)

Layouts with Wires of Balanced Length

Layout systems for VLSI-circuits have to place the predesigned pieces of the circuit on the chip in such a way, that the wires interconnecting corresponding ports can be routed according to given electrical constraints and design rules. F.e.: the designer wants to arrange modules, that fit together and have interconnections as short as possible, since long wires result in large signal delays. This gives motivation for the following graph theoretical questions: Consider a graph with fixed boundary. Does there exist a layout L of G such that the maximum distance of any node to its neighbors is minimal? How can this layout be constructed?

We show the following:
For any graph (with fixed boundary) there exists a layout, which minimizes the maximum distance of any node to its neighbors. This layout balances the length of the wires (and is therefore called (length-)balanced layout.
Furthermore the existence of a unique 'optimal' balanced layout L with the following properties is proved:
- L is the minimal element of an ordering defined on the set of layouts of a given graph G
- L is the limit of the lp-optimal layouts of G.
- If G is planar with fixed boundary, L is 'quasi-planar'.

Bernd Becker, Saarbrücken
(joint work with H. G. Osthof,
Saarbrücken)

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Lower Bounds for very Fast Parallel Computations

For the two models of parallel random access machines, the WRAM which may perform simultaneous writes and the PRAM which may not one can establish lower bounds on the computation time due to information theoretic reasons. For the PRAM the lower bound is $\log_\alpha c(f)$ where $\alpha = \frac{1}{4}(1+\sqrt{5})^2$ and $c(f)$ is the critical index of the function f to be computed. This index is defined as $\max_{x_1 \ldots x_n \in D^n} \#\{i \mid \exists y_i \in D, f(x_1 \ldots x_i \ldots x_n) \neq f(x_1 \ldots y_i \ldots x_n)\}$ where D^n denotes the domain of f. An n-ary function for which the critical index takes the maximal value n is called critical. A simple function with this property is the boolean "or". This $\Omega(\log n)$ lower bound for the PRAM contrasts to the 2 steps in which every boolean function can be computed by a WRAM, thus simultaneous writes may help a lot. It can be shown that this phenomenon does not occur in general, there exist simple arithmetic functions $f: \mathbb{N}^n \to \mathbb{N}$ for which a $1 + \log_2 n$ lower time bound holds. An example is the sum of n numbers. The result follows from a general lower bound for functions f fulfilling the stronger condition of being super-critical. It requires that for all $x_1 \ldots x_n \in \mathbb{N}^n$ and all $y_i \neq x_i$ $f(x_1 \ldots x_i \ldots x_n)$ is different from $f(x_1 \ldots y_i \ldots x_n)$.

Rüdiger Reischuk
Universität Bielefeld

Software for geometric computation

We describe experimental implementations of geometric algorithms aimed at identifying suitable components of a library of subroutines for geometric computation. We aim at simple techniques with a stable behavior over a wide range of applications.

1. Processing large configurations of geometric objects: A plane sweep skeleton has been implemented which can be adapted with little effort to solve problems such as scan conversion or region identification.
 - Run time 3 msec (n+s) log n on a 16 bit PC (Lilith)
 - If objects are defined on an M×M grid you need 6M+4 bits in the mantissa to maintain consistent ordering
2. Storing spatial objects: The grid file is a dynamic multi-key access data structure that answers any point query in 2 disk accesses and uses disk accesses sparingly for complex region queries. Intersection queries on spatial objects are reduced to cone-shaped region queries on points in a higher-dim space.
3. Describing and constructing geometric objects from constraints. A Prolog interpreter interfaced to Modula-2 is used to construct objects defined by constraints. E.g. 10 Prolog rules suffice to construct a polyhedron given by a sufficient number of coordinates of vertices, lengths and slopes of lines.

Jurg Nievergelt, Informatik ETH Zurich

Efficient diophantine approximation

Given $a_1, \dots, a_n \in \mathbb{R}^d$ with $d < n$, and $\varepsilon > 0$, how can we find a non-trivial $x = (x_1, \dots, x_n) \in \mathbb{Z}^n$ of minimal norm ν such that $|x_1 a_1 + \dots + x_n a_n| < \varepsilon$ holds? – A weaker version of this classical task has a rather fast solution: If ε may be exceeded by a factor 2^{cn}, then some $x \neq 0$ with $|x| < \nu \cdot 2^{c'n}$ can be found in time $O\left(n^2 \left(d \frac{n}{n-d} \lg\left(\frac{1}{\varepsilon}\right)\right)^{2+o(1)}\right)$. The main tool is an improved basis reduction algorithm for integer lattices.

A. Schönhage

Tight Bounds for Maximum Upward-Right Matching

The maximum upward-right matching problem is rapidly emerging as one of the fundamental problems associated with the average case analysis of algorithms. The problem was originally identified by Karp, Luby and Marchetti-Spaccamela in association with algorithms for 2-dimensional bin packing. More recently, the problem has arisen in the average case analysis of algorithms for 1-dimensional bin packing and dynamic allocation. In each instance, the average case behavior of the algorithm under consideration can be evaluated in terms of the expected number of unmatched points in a random upward-right matching problem. Previously, the expected number of unmatched points for a random N-point maximum upward-right matching problem was shown to be $O(\sqrt{N}\log N)$ by Karp, Luby and Marchetti-Spaccamela, and $\Omega(\sqrt{N}\log^{3/4} N)$ by Shor. In this paper, we show that with very high probability, at most $O(\sqrt{N}\log^{3/4} N)$ points remain unmatched in a random N-point maximum upward-right matching, thus achieving Shor's lower bound. As a direct result, we obtain improved bounds on the average case behavior of the best algorithms known for 2-dimensional bin packing, 1-dimensional ~~bin~~ on-line bin packing and dynamic allocation.

Tom Leighton
MIT

On approximate string-matching

Here are presented recent results by A.G. Ivanov published in Izvestia Acad. Sci. USSR, ser. Math., 1984, N=3, p. 520-568 (in Russian). We consider string-matching with a fixed number of faults. Given an input $U\#V$, find all V_1 s.t. $U=U_1V_1U_2$ for some U_1 and U_2 and V_1 differs from V only in a number of places not greater than the fixed constant. This problem is real-time solvable by multi-tape Turing machine if U and V go to input simultaneously.

For l^∞ metric the problem of approximate string-matching is real-time equivalent to string-matching with don't-cares.

A. Slisenko, Leningrad.

Special cases of the Hidden-Line-Elimination Problem

Hidden Line Elimination is one of the very basic problems in computer graphics. We study three special cases of increasing difficulty of this problem: In the first problem A only rectilinear faces, all parallel to the projection plane are allowed. In problem B the faces may be c-oriented but still all parallel to the projection plane. Finally, in problem C we are given a set of c-oriented solids in 3-dim space. We describe plane sweep solutions for all three problems. The obtained solutions are more efficient than the ones obtained for the general hidden line elim. problem. But they still depend on the number of intersections in the projected scene. By a completely different approach called dynamic contour maintenance we obtain solutions for problem A and B whose time and space complexity depend only on the size of the input and the size of the output i.e. of the number of visible edges but not on the number of intersections in the projection. The results are obtained in collaboration with H. R. Güting from the Univ. of Dortmund

Thomas Ottmann
Karlsruhe

GEOMETRIC DATA STRUCTURES IN COMPUTER GRAPHICS

A well-known problem in Computational Geometry is the range searching problem: Given a set of points in the plane, store them in such a way that, given a "range", i.e., an axis-parallel rectangle, the points inside the rectangle can be reported efficiently. The problem has been studied by many people. It has many applications in such areas as Database systems and Computer Graphics. In Computer Graphics, the problem is the so-called windowing problem: Given a picture, determine which part lies in a given window (= range). Hence, it is the range searching problem, but the set does not consists of points but of line segments. It is shown that the range searching problem for a set of non-intersecting line segments can be solved with a structure using $O(n \log n)$ storage with a query time of $O(\log^2 n + k)$, where n is the number of line segments and k the number of reported line segments. The structure is dynamic in the sense that line segments (that do not intersect any of the existing line segments) can be inserted or deleted in $O(\log n)$ time. The structure is related to ~~the~~ known solutions to geometric problems.

Mark H. Overmars
Utrecht (the Netherlands)

Finding the Next Smaller Element.

It is shown that $2n-3$ comparisons are necessary and sufficient in the worst case to find the largest element smaller than a specified element in a set of size n. $n + \min(k, n-k) + o(n)$ comparisons are shown to be necessary and sufficient on the average if the specified element turns out to be of rank k.

Ian Munro

Fractional Cascading: A Data Structuring Technique with Geometric Applications

Bernard Chazelle & Leo Guibas

This talk introduces a new data structuring technique for improving existing solutions to retrieval problems. Many such problems require for their solution an efficient method for multiple lookups: repeatedly looking up a key x in several sorted lists $L_1, L_2, \ldots L_n$. If all the lists have length c, then this can be done in time $O(n \log c)$ by independent binary searches. Using fractional cascading we show how, under certain conditions, the multiple look ups can be done in time $O(n + \log c)$, while keeping the overall storage linear. These conditions are as follows: the set of possible lists we may want to search must be put in a 1-1 correspondence with a graph of "bounded degree". Multiple lookups are only allowed on collections of lists corresponding to connected subgraphs. Using fractional cascading we are able to improve the running time of dozens of geometric algorithms by a log factor.

Dynamic Interpolation Search

We present a new data structure called Interpolation Search Tree (IST) which supports interpolation search and insertions and deletions.
Insertions and Deletions have expected amortized cost $O(\log\log n)$ and worst case amortized cost $O(\log n)$.
The worst case search time is $O((\log n)^2)$ and the expected search time is $O(\log\log n)$. This is not only true for the uniform distribution but for a wide class of density functions, the so-called smooth density functions.

Athanasios Tsakalidis (Saarbrücken)
(joint work with Kurt Mehlhorn)

Singular Differential Systems + Cauchy = multi-dimensional Search

Philippe FLAJOLET, INRIA Rocquencourt (France)

Multidimensional tree structures first proposed in the early 70's by Bentley and others make it possible to represent dynamically growing sets of points in multi-dimensional space. The results concern quad-trees and k-d-trees. Let k be the dimension of the universe of records, n the file size and s the number of specified attributes in query. Then the following results are obtained in the average case:

	$s = k$	$s < k$
quadtrees	$\frac{2}{k} \log n + O(1)$ $k=2 \Rightarrow$ $\left(1+\frac{1}{3n}\right) H_n - \frac{n+1}{6n}$	$\gamma_1 n^{1-s/k+\theta_1(s/k)}$
k-d-trees	$2 \log n + O(1)$	$\gamma_2 n^{1-s/k+\theta_2(s/k)}$.

The proof consist in setting up integrodifferential that transform into linear differential systems over generating functions. From there, a singularity analysis combined with a suitable application of Cauchy's formula for coefficients of analytic functions leads to the result.

Cases where the floor function does not help

Clemens Lautemann, University of Edinburgh

For a number of problems, the floor function ($\lfloor x \rfloor := \max\{z \in \mathbb{Z} \mid z \leq x\}$) has been used in algorithms with surprisingly low cost. When trying to prove lower bounds on the height of computation trees, one discovers that the known topological methods do not apply directly, if the floor function is contained in the set of basic operations. In an attempt to extend these techniques it is shown that, in certain situations, floor steps can be replaced by comparisons or subtractions, leaving the computation correct for a subset of inputs.
Application of this yields the desired extension of lower bounds for a variety of problems. For sorting n integers, for instance, it is shown that over $\{+,-,*,\lfloor\cdot\rfloor\}$ and rational constants, $\Omega(n \log n)$ comparisons are necessary.
These bounds hold for computation trees as well as for RAM's.

GROPING SEARCH OF CIRCULAR RANGES - Circular range search is a considerably more difficult problem than other range searches (such as, orthogonal range searches). Combining the principles of "filtering search" with the effectiveness of a structure closely related to higher-order Voronoi diagrams, we present a technique that executes a search in time $\Theta(\log n + k)$ using $O(n(\log n \log\log n)^2)$ space, where n is the universe size and k is the size of the reported set. Efficiency is obtained by a visit of a primary search structure, where the visit is extended only in "directions" providing an adequate payoff: hence the qualifier "groping"

(Joint work with B. Chazelle, R. Cole, and C. Yap)

Franco P. Preparata

Karmarkar's Linear Programming algorithm

Narendra Karmarkar has discovered a new polynomial time algorithm for linear programming, and has made extravagant claims about its performance. We describe projective transformations and the details of computing the steps taken by the algorithm. The issue of weather the algorithm will be useful in practice is unresolved.

Daniel Sleator

White Pebbles Help

Pebbling games on directed acyclic graphs can be used to model the space complexity of evaluating straight-line programs. Black pebbles may be placed on nodes when all immediate predecessors are pebbled. They may be removed at any time. Black pebbles correspond to registers that contain values in deterministic computations. White pepples correspond to non-deterministic "guesses" of values. A white pebble may be placed on any node at any time, but it may be removed only when all of its predecessors have been pebbled, i.e., only when its value has been "verified". Bob Wilber has just shown that there are straight-line programs that can be evaluated nondeterministically using asymptotically less space.

Merrick Furst
Carnegie-Mellon University

MATHEMATISCHE MODELLE IN DER BIOLOGIE

2.12. – 8.12.84

Networks of Neuron Analog Circuits

A new circuit analog of a nerve cell was described. This is based on modulation of a voltage controlled oscillator by signals entering through a circuit analog of a chemical synapse. Phase-locking of frequency encoded information was described for the von Euler mechanism of respiration control. This demonstrates synchronization of breathing with stride by runners.

Frank Hoppensteadt
University of Utah

A mathematical model for the larch - larch bud moth hypothesis

The larch - larch bud moth hypothesis states that the oscillations of the abundance of the larch bud moth originate from a physiological change of the larch needles as a reaction of the larch to the defoliation by larval feeding. A discrete 2x2 recursion is analysed where one variable is the size of the bud moth population in the egg stage and the other measures the physiological state of the larch. The model differs from previous models by Van den Bos & Rabbinge, Fischlin & Baltensweiler, and Fischlin in the functional relationships between the physiological state of the larch, the defoliation, the food consumption, and the starvation mortality. It can be shown that the appearance of undamped oscillations follows from biologically reasonable assumptions on these functions. Numerical simulations exhibit a good agreement of the modelled oscillations with the observed oscillations in the upper Engadine valley.

D. Schumacher, Heidelberg

Pulse-like solutions of spatially aggregating population models

By the motivation of aggregation phenomenon, we consider a non-local reaction-diffusion system in one dimensional space.

$$\begin{cases} u_t = [du_x - \chi S_x u]_x + f(u) & x \in \mathbb{R},\ t > 0 \\ S_t = k * u - S. \end{cases}$$

We show the existence of two different type of pulse-like stationary solutions by using "singular perturbation method", It is numerically investigated that one is stable and the other is unstable. The kernel $k(x)$ used here is $k(x) = \frac{1}{\beta^2} \times (1+\beta|x|)\, e^{-\beta|x|}$.

Mayan MIMURA
Hiroshima University.

A multigroup model for diseases with non-symptomatic infectives

In some diseases, a large proportion of infected individuals are infectious to others but may have no symptoms or only mild symptoms. Together with Dr. José M. Ferreira, we have formulated and analyzed a model for an endemic disease in a population with two groups, in each of which there are both symptomatic and non-symptomatic infectives. The basic reproductive rate, R, is expressed in terms of contact and recovery rates. It is proved that if R is greater than 1, there is a globally stable positive equilibrium state, but if R is less than 1, the only stable equilibrium state has zero infectives. The model is applied to a study of gonorrhea incidence and comparisons are made with available data.

Kenneth L. Cooke
Pomona College, Claremont, California

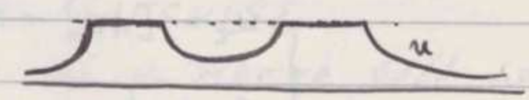
AGGREGATION MODELS WITH FREE BOUNDARY

Density dependent diffusion equations $u_t = \{\mu(u) u_x - uV\}$ with aggregative drift determined by a "pseudo-steady" elliptic equation of the form (VISCOUS FLUID ANALOGON OR CHEMOTAXIS)

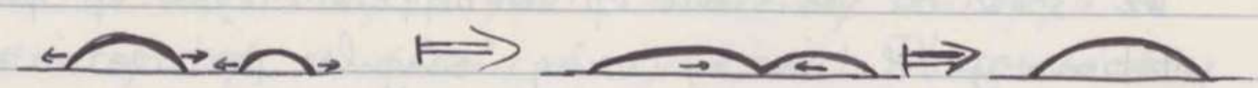
$$0 = \{\eta(u) V_x + g(u)\}_x - \varphi V$$

can model different types of biological aggregation as

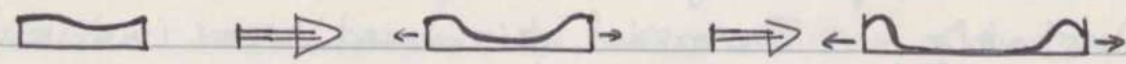
1. AGGREGATE FORMATION (slime molds)



2. APPROXIMATION OF DIFFUSIVE SWARMS (slime bacteria)



3. SEPERATION AND RECOLLECTION OF "ORDERED" SWARMS (birds)



Whereas 1 and 2 are degenerate parabolic problems, 3 is given by a hyperbolic equation ($\mu = 0$) with functional drift, where the additional free boundary condition is a pressure balance

$$\eta(u) V_x + g(u) = K(u)$$

between attraction from the interior and an outward "pressure" $K(u)$ for individuals at the edge of the swarm.

Wolfgang ALT (Univ. Heidelberg)

TIME PATTERNS IN GLYCOLYSIS; MODEL AND EXPERIMENTS

A model of sugar metabolism (glycolysis) is analyzed using enzymic rate laws that have been obtained in great detail from kinetic measurements. A large variety of periodic, quasiperiodic and chaotic solutions are obtained for periodic substrate input flux. At sinusoidal input flux, conditions exist at which oscillations having a period in the range of minutes are modulated with a period in the range of ~~ten~~ hours. The degree of randomness of the chaotic oscillations, as indicated by the Liapunov dimension, can be considerably higher under amplitude or frequency modulated input than under sinusoidal input. Furthermore, it is shown that up to four attractors can coexist in phase space under the same set of bifurcation parameters, and that the dynamics

of the system follows complicated hysteresis loops.
Fluorescence measurements in yeast extracts under periodic glucose input show responses with periods 1, 2, 3, 4, 5, 7 and 9 times the input period, quasiperiodic oscillations and chaos in the predicted range of control parameters.
In whole yeast cells, synchronization of oscillating glycolysis and oscillating membrane potential is observed.

Mario Markus (Max-Planck-Inst., Dortmund)

WHAT AND HOW DOES A FROG SEE?

Seeing in general is not to be confused with "taking a photograph". Seeing always means interpretation and evaluation of an image. Frogs like all other amphibians have the problem to distinguish between prey, enemy, and other objects. In my lecture I discussed how neural networks in the retina and in the brain, in particular in the tectum opticum, of amphibians participate in the analysis of the visual scene. The operations of these networks are described and modeled by systems of time- and space-dependent differential- and integral-equations. These networks are able to separate moving from non-moving objects, to discriminate various sizes and shapes of moving objects, and to give different responses to different orientations of the same object with respect to its direction of movement.
The electrophysiological activity of a great number of different types of nerve cells observed experimentally in the retina and optic tectum of frogs, toads, and salamanders can be understood and predicted

from the interactions between nerve cells in the model network. Astonishingly, variation of a few of the network parameters is sufficient to generate many of the response types observed in these animals.

References. U. an der Heiden & G. Roth: Cooperative Neural Processes in Amphibian Visual Prey Recognition. In: "Synergetics of the Brain" (E. Basar et al., eds.), Springer-Verlag, 1983.

Uwe an der Heiden (Univ. of Bremen)

Persistence in Ecology Models

A system of ODE's, $x' = f(x)$, $x \in R^n$, $x(0) \in R_3^+$ said to be persistent if $\liminf_{t \to \infty} x_i(t) > 0$ where x_i denotes a component and uniformly persistent if $\liminf x_i(t) \geq \alpha > 0$. The system of interest is of the form

$$
\begin{aligned}
u' &= u\, f(u,v,w) \\
(*) \quad v' &= v\, g(u,v,w) \\
w' &= w\, h(u,v,w) \\
u(0)>0,\ & v(0)>0,\ w(0)>0
\end{aligned}
$$

where u models a prey population and w a predator population (and v may be either). Sufficient conditions are given on ∂R_3^+ for $(*)$ to be persistent.

Paul Waltman
Emory University.

Periods and Thermodynamics of the Volterra-Lotka System

The classical predator-prey system of Volterra and Lotka can be converted to a Hamiltonian system. The canonical partition function is explicitely $Z(\beta,\lambda,\mu)=z(\beta\lambda)z(\beta\mu)$ with $z(\gamma)=e^{\gamma(1-\log\gamma)}\Gamma(\gamma)$. Because this is the Laplace transform of the energy-period-function, one gets asymptotic expansions for the periods of small- and large orbits. One the other hand, the period is a convolution integral which is shown to satisfy a convexity in appropriate logarithmic ~~parameters~~ scales. This implies that the periods of oscillations are increasing with their amplitude and allows to compare periods for different parameters.

Franz Rothe

Coevolution

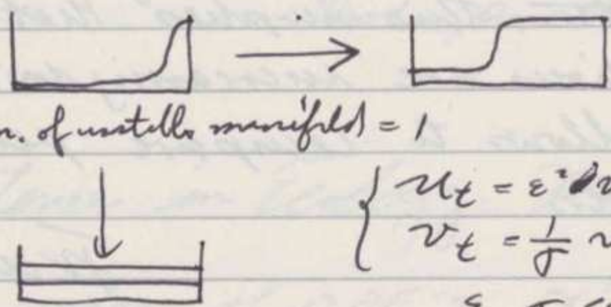
A review is given of the modelling of evolutionary problems involving interacting populations. These range from the "gene-for-gene" systems of cereal plants and their fungal parasites, in which the genetic basis of resistance and virulence is well understood and encoded at reciprocal loci in parasite and host, to diffuse coevolution in which many species interact. A detailed discussion is given of the evolution of reduced levels of virulence in the myxoma virus ~~introduced~~ to control European rabbits in Australia. Analytic results are given for simplified models of the S-I-R type, and more detailed discussions are given for computer simulations of the spatio-temporal dynamics

Simon Levin
Cornell U.

Stability of layer type solutions of Reaction-Diffusion system and their orbital connection

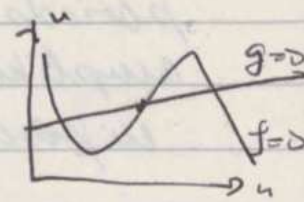
In the process of pattern formation, "front wave" plays an important role. Travelling wave on the infinite line is a mathematical idealization which describe the transition from one constant state to another constant state.

We consider the stability (or instability) of inner layer (or boundary layer) solution, respectively, and the connection orbit between these solutions for the following system



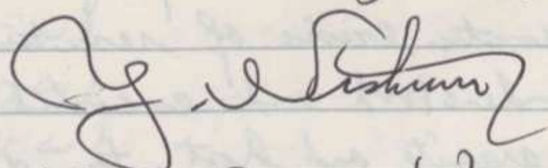
$$\begin{cases} u_t = \varepsilon^2 u_{xx} + f(u,v) \\ v_t = \frac{1}{\sigma} v_{xx} + g(u,v), \end{cases}$$

$\varepsilon, \sigma \ll 1$



where f has a sigmoidal shape and g intersects with f three times like

This study might give insight to clarify the global dynamical behavior of the reaction-diffusion system for $\varepsilon, \sigma \ll 1$.



Kyoto Sangyo University.

Chaos in Simple Autonomous C^1 Systems

After introducing two open problems (linear chaos; transfinite iteration), the Mackey-Glass functional differential equation is used as a motivation to consider the piecewise linear O.D.E.,

$$\begin{cases} \dot{x}_1 = -x_1 + f(x_n) \\ \dot{x}_2 = x_1 - x_2 \\ \vdots \\ \dot{x}_n = x_{n-1} - x_n \end{cases} \qquad (1)$$

for both n very large and n minimal for chaos to occur. The conjecture proposed is that the order of the chaos must be $n-2$. That is, that there are maximally $n-2$ positive Lyapunov characteristic exponents in Eq. (1). The $n=3$ analysis is carried out in some detail (joint work with B. Uehleke).

An analogous equation was found for a traveling wave in an excitable system boundary value problem (with C. Kahlert).

Otto Rössler, Tübingen

The Selection Mutation Equation

Fisher's „Fundamental Theorem of Natural Selection" is generalized to the selection - mutation equation with special mutation rates ε_{ij} depending only on the target gene ($\varepsilon_{ij} = \varepsilon_i$), by taking $V(x) = \bar{w}^{1-\varepsilon} \prod_{i=1}^{n} x_i^{2\varepsilon_i}$ ($\varepsilon = \sum \varepsilon_i$) as generalized mean fitness function. The selection - mutation equation is then the gradient of V if the probability simplex is equipped with Shahshahani's metric. For other mutation rates this is not true, and a theorem of Akin implies the existence of periodic orbits for suitable chosen selection part. In a particular 3-allelic example with cyclically symmetric mutation rates stable limit cycles are found.

Josef Hofbauer, University of Vienna (Austria)

Spatial diffusion and delays in models of genetic control by repression

A class of models based on the Jacob and Monod theory of genetic repression of biosynthetic pathways in cells is considered. Both spatial diffusion and time delays are taken into account. A method is developed for representing the effects of spatial diffusion as distributed delay terms. This method is applied to two specific models and the interaction between diffusion and delay terms is studied. The destabilization of the steady-state and the bifurcation of oscillatory solutions are studied as functions of the diffusivities and the delays. The limits of very small and very large diffusivities are analyzed and comparisons with well-mixed compartment models are made

Stavros N. Busenberg,
Harvey Mudd College, Claremont
U.S.A.

Convolution models in neurophysiology.

~~We give~~ This is a report on a joint work with E. Bienenstock and Boris Moore. We have made up a tool box of phenomenological models of selectivity, using the ideas of cooperation, embodied by the convolution with a given even ~~kernel~~ kernel w, and competition represented by a non linear term, which will generally be taken to be global. More precisely, we will consider the solution $u(x,t)$ of a non-linear evolution equation of the following form

$$u(x,t) = \begin{cases} F(u)(x,t) & \text{if } u(x,t) > 0 \text{ or if } F(u)(x,t) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where

$$F(u) = \{ w * u - G(u) \} 1_\Omega \, ;$$

Ω is a bounded ~~subset~~ of $X = \mathbb{R}^N$ or $\mathbb{T}^N$, and 1_Ω its characteristic function. The coordinate x may be ~~the~~ spatial, or describe a family of stimuli, or both (if it is multidimensional). Numerical experiments, ~~or~~ and mathematical analysis give different asymptotic states for different nonlinear terms G. If

$$G_1(u) = \left(\int u\,dx\right)^2, \quad w(0) > w(x),\ x \neq 0, \quad \int w\,dx > 0,$$

the stable stationary solutions are Dirac masses, and this describes orientation selectivity in a given neuron. If

$$G_2(u) = \left(\int u(x,y)\,dx\right)^2, \quad w(0,y) > w(x,y),\ x \neq 0 \ \& \text{ other}$$

conditions, the stable solutions are Dirac ~~densities~~ masses along the graph of a mapping y to x ~~If~~ and this describes the spatial organization of orientation selectivity. If

$$G_3(u) = \left(\int u(x,y)\,dx\right)\left(\int u(x,y)\,dy\right)$$

the ~~stable~~ solutions are Dirac ~~densities~~ masses along the graph of a bijection $x \leftrightarrow y$.

Michelle Schatzman
Université Claude Bernard
69622 Villeurbanne Cedex - FRANCE

Worm's sexuality from a mathematical point of view.

The work presented here is a joint work with W. Hirsch and H. Hanisch. Its goal is the study of the possible relation between the sexual behavior of a parasitic worm and the statics and dynamics of its transmission. Very little is kown today about the intimate life of parasitic worms and we think that a mathematical approach, based even on oversimplified models, can be helpful to shed some light on the underlying mechanisms of worms reproduction. It suggests, for example, that the usual classification of worms into hermaphroditic and dioecious organisms might be insufficient to describe their transmission.

JP Gabriel Fribourg University
Switzerland

Optimal Ages of Vaccination for Measles

The great diversity throughout the world in the recommended ages of vaccination for measles indicates that there is not general agreement on the best vaccination strategies. A modeling approach can be used to determine ages of vaccination which minimize the lifetime expected risk due to measles in a population. Although no two-dose strategy is theoretically optimal, there can be practical reasons for using two doses in some countries. However, in developing countries where there are limited resources for measles vaccination, the calculations using an age dependent model show that vaccination of a large fraction at one optimal age is much better than vaccination of half as many children at two ages. Optimal ages of vaccination are calculated from approx. seroconversion curves and estimated parameter values for Kenya, South America and the USA.

Herb Hethcote
Dec 6, 1984

A remark on 3 dim. models of the B.Z.-Reaction.

Three models of the B.Z.-reaction have been studied: for them a method of finding appropriate parameters has been pointed out and that the resulting equations exhibit the dynamics to be modeled. On this basis a chaotic dynamic was explained as an affair of switching between two different limit cycles.

Erich Boll, Konstanz Univ.

Existence and Uniqueness of Solutions to the "Dutch" Daphnia Model

Proving existence and uniqueness of solutions to the Daphnia model presented by J.A.J. Metz in his talk is not straightforward due to a discontinuity of the birth rate excluding an easy application of Banach's fixed point theorem. This difficulty can be overcome by establishing the relation $n(t,a) \leq \text{const}\ \partial_a l(t,a)$ between the age distribution $n(t,\cdot)$ of the Daphnia population and the age distribution $l(t,\cdot)$ of the length of individual daphniae at time t. This requires an assumption involving an actual biological restriction, namely the relation $\kappa l_j \geq l_b$ between the lengths l_b, l_j of individuals at birth and the beginning of reproduction respectively and the fraction κ of assimilated food put into metabolic maintenance and growth.

Horst Thieme, Amsterdam & Heidelberg

A model for size and age dependent population processes in Daphnia magna and other simple ectotherms

Starting from simple energetic considerations on the individual level a model is built for the dynamics of populations of simple ectotherms. Individual growth is modelled by a von Bertalanffy growth equation coupled to a Holling type functional response. Birth rate is modelled by assuming that a fixed fraction of ingested energy is channelled to reproduction as soon as length exceeds a fixed value. All these assumptions are borne out by experiments on individual _Daphnia magna_. When food becomes scarce maintenance is assumed to take priority. When total intake cannot keep pace with maintenance the animal dies. Apart from this death is only dependent on age. The population state is given by the age distribution and the instantaneous age-length relation. The development of the population process is generated by a set of coupled first order partial differential equations together with a boundary condition representing births.

If food availability is coupled dynamically to population size we get a model for population regulation. If the trivial equilibrium is unstable there is a unique internal equilibrium. Assuming chemostat food dynamics the stability boundaries in the random death-feeding rate plane (the two variables which can be changed experimentally) can be calculated numerically from the characteristic equation. Inside the unstable region the Hopf oscillations go through period doubling and all other nonlinear oddities, due to an interference of the minimum age with the naturally generated "generation" cycles.

Hans Metz ITB Leiden, Strathclyde Glasgow

A stochastic age-dependent epidemic model

A marked point process approach is presented here to model the evolution of an age-dependent epidemic system. Based on the martingale properties of the associated counting processes, estimators of the parameters of the system are generated following Aalen's methods. Asymptotic properties of the estimators are also derived.

Vincenzo Capasso
Università di Bari
(Italia)

Persistent regularity in the chaotic dynamical behaviour in some discrete biological models.

We consider a general discrete growth model for a single species population with nonoverlapping generations. We write N_t for the number of individuals per unit area in generation t. Consider the model $N_{t+1} = F(N_t)$ where F is some nonlinear differentiable function from the set of nonnegative real numbers into itself such that F has one or more critical points (that are points at which the derivative vanishes), and F vanishes at zero. We assume that F is a chaotic Axiom A map.

The following results have been obtained:

1) There is no chaos, in other words, any arbitrarily chosen initial value that will lead to aperiodic (chaotic) behaviour has probability zero.
2) The periodic points for F with period n can be computed for each positive integer n.
3) One can associate a nonnegative real number with the model measuring the complexity of the dynamical behaviour.
4) F is C^1-omega-stable, i.e. the structure of the nonwandering set of F doesn't change under small smooth C^1-perturbations.
5) F is C^2-structurally stable, provided that F satisfies some reasonable conditions.

Remark 1. Assume that $f \in C^3(X,X)$ has the following properties: (i) f has a negative Schwarzian derivative, (ii) the set of critical points of f are contained in the domains of attraction, (iii) f is contracting on the set of asymptotically stable periodic points. Then f is an Axiom A map.

Remark 2. The standard examples $N_{t+1} = N_t\{1+2(1-\frac{N_t}{K})\}$ and $N_{t+1} = N_t \exp[r(1-\frac{N_t}{K})]$ will be discussed.

Leny Nusse State University of Groningen

8 dec 1984

Reference Thesis R.U. Utrecht may 1983

Stage-Structure Models: Theory and an Application to Zooplankton Dynamics

Gurney, Blythe and I have recently studied the systematic formulation of "stage-structure" models in which complex life histories are reviewed in ways that yield the population dynamics in terms of delay differential equations. I review the "tool-kit" of available techniques with particular emphasis on models of marine copepods where the processes of growth and development are partially decoupled.

Roger Nisbet - Dept. of Applied Physics, Univ. Strathclyde, Glasgow, U.K.

Nonlinear interaction during very short life stages.

A complicated prey-age dependent prey-predator model, incorporating a saturating functional response, is simplified by letting the age specific attack rate converge to a multiple of the delta "function" at prey age zero.
Both for egg predation and for egg cannibalism the stability boundary (corresponding to the unique nontrivial steady state) in a two-dimensional parameter space is determined. Upon crossing of the boundary a Hopf bifurcation occurs.

Odo Diekmann, CWI, Amsterdam

An age-structured model for recurrent epidemics

The question is raised, whether recurrent epidemics of infections like measles are linked to the school year. A new model is presented, which divides the child population into "grades" and which assumes a higher contact rate among school children. Then, with the beginning of each new school year a whole cohort of children becomes exposed to a high intra-school infection rate. This creates an annually recurring "shove" on infection transmission, giving rise to coexistent periodic incidence patterns with periods of one, two and several years. Model simulation results conform to differing observations from countries where school years start in fall or spring, i.e. after or before summer vacation. The model especially describes to some detail the measles data from England since 1950.

J. Schenzle

Chaotic Cardiac Dynamics

Nonperiodic cardiac rhythms are often seen clinically on the electrocardiogram. Several different irregular rhythms can be induced in a population of spontaneously beating cardiac cells by stimulation with a periodic train of current pulses. Consideration of the phase-resetting response of the mutually entrained population to an isolated current-pulse stimulus leads to a formulation of the response to periodic stimulation in terms of a one-dimensional finite-difference equation. Analysis of this equation results in the identification of one particular experimentally observed nonperiodic rhythm as a manifestation of "chaotic" dynamics. This behaviour is found only at an intermediate level of the stimulus amplitude; it does not occur at higher or lower stimulus levels. The implications of this work for other forced biological oscillators is discussed.

Michael R. Guevara, Dept. of Physiology,
Univ. of Amsterdam

A birth and death process with killing and application to parasite infection.

A birth and death process with killing and reestablishment of the population can be described by a degenerate first order system of partial differential equations, which can be reduced to a single renewal equation for one function (the probability of 'no population'). The population can be interpreted as the parasite population within one host. With a transmission law, i.e. a nonlinear function coupling the immigration rate of parasites into hosts to the average parasite load, one arrives at a simplified version of an epidemic model introduced earlier by H. and Dietz.

K. P. Hadeler, Tübingen.

A size structured model for the growth, budding and scar class distribution of Saccharomyces cerevisiae

A size structured model for populations of S. cerevisiae is discussed. It is shown that under special conditions the system can be reduced to a scalar renewal equation. Using standard renewal theorems one can determine the asymptotic behaviour of solutions: All solutions converge (in a damped oscillatory manner) towards a steady state. In the general case, when one cannot use renewal theory, the same result is obtained by spectral theory of C_0-semigroups

Mats Gyllenberg, Mathematisch Centrum, Amsterdam

What Clocks the Cell Cycle?

It is demonstrated that the probabilistic assumption concerning the onset of mitosis in a simple mitogen model of the cell cycle is equivalent to the assumption that the crossing of a threshold by a "chaotic" intracellular oscillator triggers mitosis. Under the assumption that successive maxima of this "chaotic" oscillator are related to one another by a Rènyi transformation, a straightforward application of a result of Lasota and Yorke [Rend. Sem. Math. Univ. Padova (1981), 64, 141] allows the results of Lasota and Mackey [J. Math. Biology (1984), 19, 43] to be reinterpreted in a strictly deterministic framework. This model, with two available parameters, shows excellent agreement with published $\alpha(t)$ and $\beta(t)$ data from a variety of pro- and eukaryotic cell types. Further, the assumption that such intracellular oscillators exist gives an explanation for the occurence of polyploidy and/or reduction divisions -- phenomena that conventional "sequential" cell cycle models cannot account for.

Michael C. Mackey
Department of Physiology
McGill / Montreal Canada

Multigrid Methods

9.12. – 15.12.84

Application of a multigrid method to a fluid dynamics problem

Based upon a multigrid method, a standard subroutine for solving linear algebraic structures has been constructed, for discretizations of elliptic partial differential equations. The user provides only the matrix and the right-hand-side. There are no parameters to be chosen, and the ingredients of the multigrid methods are chosen once and for all.

In cooperation with Zenon Nowak the method was applied to the solution of the transonic full potential equation for the flow around an airfoil. The equations resulting from finite volume discretization are Newton-linearized. The performance of the standard subroutine mentioned above on the resulting linear systems was very satisfactory. But Newton iteration was found to have only a small region of attraction. It is conjectured that the transonic flow equations are not (very) well posed.

Pieter Wesseling, Delft University of Technology.

Calculation of three-dimensional, inviscid, rotational flows in turbomachines

A finite element method has been developed to calculate three-dimensional, inviscid, rotational flows in turbomachines. The method is an extension of the potential flow model, since the velocity vector is described by two unknown functions. This approach is general for flows in a stator. In a rotor, an additional condition must be fulfilled at entrance.

The method leads to a system of three partial differential equations, which are discretized using the finite element method. The resulting systems of equations are solved using a direct method or a multigrid method. In the multigrid approach a special choice of coarse grid weightfunctions is adopted, such that the space of coarse grid functions is always an inclusion of the space of fine grid functions. This leads to a consistent definition of restriction and prolongation operators.

Applications of the multigrid method are shown for potential flows as well as for the more general rotational flow.

Chris Lacor, Department of Fluid Mechanics (Prof. Ch. Hirsch),
Vrije Universiteit Brussel

Multigrid methods for the calculation of Eigenfrequencies of resonant cavities.

The construction of the high-energie particle accelerators planned in many sites the accelerating fields have to be known very accurately. Therefore fast and reliable methods to calculate the eigenfrequencies of resonators are necessary. A carefully optimised combination of subspace-iteration for eigenvalues com~~bined~~ with multigrid methods for Maxwells equation promises to be sufficiently fast and accurate. Some of the problems arising in this context regarding Maxwells equation, multigrid methods for eigenvalue problems and implementation of such a method are addressed.

B. Steffen, Zentralinstitut für angewandte Mathematik,
Kernforschungsanlage Jülich.

The Fast Adaptive Composite Grid Method (FAC) for $\mathcal{L}\mathcal{U}=\lambda\mathcal{F}\mathcal{U}$

Several authors have considered generally two categories of multigrid methods for solving the generalized eigenproblem $\mathcal{L}\mathcal{U}=\lambda\mathcal{F}\mathcal{U}$ for elliptic equations. One uses a linear multigrid solver for the inner loop of a linearized (e.g., by inverse iteration) problem; the other integrates multigrid into the nonlinear problem and may be called FAS-type. We, however, present what might be a simpler but somewhat more effective approach that involves minimization of the Rayleigh quotient by, say, coordinate relaxation and a variationally formulated coarse grid correction.

We develop the algorithm, establish a simple V-cycle theory, and exhibit numerical results for a single group neutron diffusion model. We show (both theoretically and numerically) how this method can be used in a local grid context by developing an FAC version of it.

Steve McCormick, University of Colorado at Denver

Multigrid methods in a variational framework: Formalization, basic concepts, estimation of convergence factors for some smoothers -

To solve the symmetric variational problem: $u \in H$, $a(u,v) = l(v)$, $\forall v \in H$, multigrid methods are constructed using a sequence of nested subspaces of H. Bounds for the convergence rate in the "energy-norm" are obtained in two ways. The first gives a result which depends on the integer μ characterizing the cycle type and does not include the V-cycle case.

The second gives a bound, not depending on μ, and proves the convergence of the V-cycle. The convergence factors are made more precise for different classes of smoothers: S.O.R., p steps of Richardson, p steps of two-block G.Seidel ... ~~All~~ the factors are computed for the monodimensional Laplacian on a regular mesh and the Richardson smoother, showing the sharpness of the convergence bounds in the case of such a model problem.

J.F. MAITRE, F. MUSY, Ecole Centrale de Lyon (France)

A new Multigrid Method for the Euler equations

For the solution of the Euler-equations for inviscid flow a new Multigrid Method was described. This method is fully conservative and uses the Finite Volume technique on all levels of discretization. Hence the selection of a proper prolongation and restriction follows from the weighted residual principle an a good sequence of nested discretizations is obtained. In this sequence all coarse discretizations are Galerkin approximations of finer ones (which makes the coarse grid corrections work properly). Further, an Osher type of approximate Riemann-solver is used for flux-splitting. This leads to a discretization that is ① monotone ② entropy condition satisfying and ③ allows for a completely consistent treatment of the boundary conditions. (① and ② make a local relaxation method work). Using the FAS multigrid scheme with (e.g.) Symmetric Gauss Seidel as a relaxation method, an iterative method is →

obtained which – for a transonic standard problem – has a convergence factor 0.25 for each FAS iteration step. Starting with initial estimates obtained by FMG, the transonic problem is solved up to truncation error within two V-cycles, independently of the meshsize on which the final solution is required.

P.W. Hemker, Centrum voor Wiskunde en Informatica, Amsterdam

Multi-Level Continuation Techniques for parameter-dependent nonlinear elliptic boundary value problems

A technique is presented to solve parameter-dependent nonlinear elliptic boundary value problems. Several new ideas have been combined with known methods to yield a very efficient and reliable algorithm for continuation along solution branches. Some of the features of the program in which this algorithm is implemented are: continuation on the coarsest mesh, adaptive local mesh refinement in the multi-grid iteration, it allows to hit target points, runs in interactive mode, locates singular points and switches branches. Important further applications of the program are linear eigenvalue problems and homotopy continuation. Numerical results are presented for several problems including one with a tertiary symmetry-breaking bifurcation point. This paper represents joint work with R. Bank and T. Chan.

H.D. Mittelmann, Dept. Math., Arizona State University, Tempe

O. Axelsson, Catholic University, Nimwegen

An efficient finite element method for nonlinear diffusion problems.

A mixed variable f. e. method is used for the derivation of an efficient iterative method for diffusion problems. The formulation has two advantages as compared to classical finite element methods:
(i) updating of the material coefficients is simplified
(ii) the discrete approximation is much more accurate for problems with (almost) discontinuous coefficients, where the discontinuity occurs in the interior of the elements.

An iterative method based on preconditioning by the lower order (piecewise linear b.f.) is used for the solution of the higher order (piecewise quadratic b.f.) approximations. The solution of the linear b.f. equations on the form $B M(u)^{-1} B^T \alpha = F$ is done efficiently by use of the generalized inverses of B and B^T. This means that two Poisson solvers are applied at every nonlinear iteration. A similar application on the Stokes problem is also discussed.

O Axelsson

A nonstandard multi-level method not depending on regularity:

In this talk the use of hierarchical bases in finite element computations has been discussed. It has been shown that for plane elliptic boundary value problems the use of hierarchical bases reduces the exponential growth of the ~~discretisation matrices when using hierarchical bases~~ total!!

condition numbers of the discretization matrices when using ~~hierarchical~~ modal bases to a quadratic growth in the number of refinement levels. In combination with simple but very fast algorithms for handling hierarchical bases this leads to iterative schemes which are of nearly optimal computational complexity and are very easy to program.

Harry Yserentant Institut für Geometrie und Praktische Mathematik der RWTH Aachen

On some aspects of homogenisation and aggregation in the multi-level context.

A general two-level algorithm of solving linear equations of the type $u = Au + b$ in Banach space is shown to be locally convergent. As particular cases of this algorithm one can consider the aggregation method usually used in algebraic problems of economical sciences, the classical homogenisation method of nuclear reactor physics on the one hand, and some new techniques of reducing the dimensionality of some models of mathematical physics on the other hand. A multi-level version of the algorithm is also discussed.

Ivo Marek, Charles University
Prague, Czechoslovakia

Simultaneous eigenvalue calculation

To compute k eigenvalues and eigenvectors of a general unsymmetric matrix L, one can determine k vectors u_i $(i=1,\dots,k)$ collected in U and an upper triangular matrix Λ of size $k \times k$ such that $LU - U\Lambda = 0$, $U^T U = I$. The Newton method applied to this problem leads to linear equations of the form $L\,\delta U - \delta U\,\Lambda - U\,\delta\Lambda = F$, $\delta U^T U + U^T \delta U = G$ (F, G given). It is shown that these equations form a staggered system. According to this structure, a multi-grid algorithm is constructed.

W. Hackbusch, Institut f. Informatik, Universität Kiel

On the Numerical Solution of the Biharmonic Equation

The mixed formulation of the biharmonic equation $\Delta^2 u = f$ in Ω, $u = \partial u/\partial n$ on $\partial\Omega$, leads to a linear system of the form $\begin{pmatrix} M & B^T \\ B & 0 \end{pmatrix}\begin{pmatrix} w \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix}$
We dicuss the preconditioned cg-method for the reduced system $BM^{-1}B^T u = f$. The first step for preconditioning is the modification of the boundary conditions to get 2 Poisson equations. This gives rise to a condition number $O(h^{-1})$. The second step consists in applying the incomplete Choleski decomposition. The standard way leads to a condition number $O(h^{-3})$, i.e. not to a squaring of the usual $O(h^{-1})$ result for the Poisson equation. It is shown for which norms multigrid procedures are assumed to have good convergence factors such that they give rise to good preconditioners.

Dietrich Braess, Bochum

The Use of Accelerated Smoothing Procedures in Multi-grid Iterations

We consider the use of acceleration procedures (e.g. Chebyshev and conjugate gradient) for enhancing the effectiveness of the basic smoother in the multi-grid iteration. For a fixed smoothing procedure, the use of acceleration can increase the convergence rate from $O(\frac{1}{m})$ to $O(\frac{1}{m^2})$, where m is the number of smoothing iterations. It is shown that the minimum residual version of the conjugate gradient algorithm computes an optimal sequence of acceleration parameters.

Randolph E Bank, University of California at San Diego

Iterative solution of mixed finite element approximation of the Stokes problem

We present a preconditioned conjugate residual algorithm for mixed finite element approximations of the Stokes problem. The preconditioning consists in replacing the H^1-scalar product for the velocity by a mesh dependent scalar product which is spectrally equivalent up to a factor $O(|\log h|)$. Using hierarchical basis functions the new scalar product has a diagonal stiffness matrix except a very small diagonal block. Hence it can easily be inverted. The cost of the preconditioning roughly corresponds to the calculation of 3 scalar products. The algorithm has the quasi optimal convergence rate $1 - O(|\log h|)$ We give some numerical results for Stokes and Navier-Stokes problems and compare them to results ~~to~~ obtained by other iterative methods

Rüdiger Verfürth Ruhr-Universität Bochum

UZAWA smoother for saddle point problems

We consider a class of multigrid methods for the numerical solution of saddle point problems. These methods are constructed from a sequence of nested subspace of both Hilbert spaces on which the variational problem to solve is defined. The subspaces have to fit together such that an inf sup condition is satisfied at each level with a constant independant of the level. The Uzawa method is studied as smoothing process. For the two-level convergence factor, we establish a bound which proves the multigrid convergence for the W-cycle scheme. The 2-dimensional Stokes equation with appropriate discretizations is considered as an application. We report some numerical experiments where the Uzawa smoother compare favourably with the distributive relaxation.

François MUSY. Ecole centrale de LYON.

MUTIGRID TREATMENT OF STOKESIAN BOUNDARY CONDITIONS.

In order to treat a steady two-dimensional Stokes problem as coupled stream function and vorticity Dirichlet systems, the method of Thom is used to obtain a discrete boundary condition on vorticity. Such a boundary condition requires special considerations when a multigrid solver employing pointwise relaxation is used. We demonstrate some boundary treatments that exhibit ideal global smoothing rates, when lexicographic collective Gauss-Seidel relaxations with full weighting and W-cycles are used. For one particular treatment, ideal rates are also obtained with injection.

H. Holstein and G. Papamanolis
U.C.W. Aberystwyth, U.K. SY23 3BZ.

Multigrid Algorithms for the Dam Problem

We deal with the application of multigrid techniques to the dam problem. Esspecially H. W. Alt's variational inequality formulation of the problem is considered, which works in a quite general situation. Two multilevel algorithms have been developed. The first one consists of relaxation steps and in solving auxiliary problems of obstacle type in the saturated part of the dam. These auxiliary problems only have the discrete pressure as unknown. They are solved approximatively by a twogrid method. The second method is a pure multigrid algorithm of FAS-type. The natural f.e. residual weighting operator has to be modified in boundary-nodes, in order to handle a condition of complementarity in the right way. Solving several test-problems our first algorithm turned out to be faster than the relaxation method of H. W. Alt, however the pure multigrid method is significantly the fastest. The speed of convergence is nearly independent of meshsize and not sensitive to changes in the permeability

Christoph Bollrath Ruhr-Universität Bochum

Solving elliptic eigenvalue problems by multigrid methods

Analogously to the SOR-Algorithm for eigenvalue problems one can find a multigrid method. This method involves projections on the orthogonal space of the eigenspaces to solve the singular coarse-grid equations. In this talk estimates are given, which show the effect of dropping this projections and of errors in the coarse-grid eigenvalues to the convergence of this multigrid method.

Götz Hofmann, Christian Albrechts Universität Kiel

On the multigrid solution of the Stokes equations

The bilinear/constant finite element method for the Stokes problem on a rectangular domain yields a simple finite difference method which works well when combined with direct solvers, despite the known lack of stability of the method. We show by numerical experiments that the lack of stability causes severe difficulties in the multigrid solution of the system. However, the convergence of the multigrid method can be recovered by adding an appropriate pressure smoothing step after each relaxation step. The added step can also be interpreted as stabilization of the finite element method. A similar strategy is shown to work also in connection with some other simple but unstable methods for the Stokes problem in two or three dimensions.

Juhani Pitkäranta, Helsinki University of Technology

Convergence rates of the Multigrid Method for a Threedimensional Model Problems

For the Poisson-equation on a subspace of $\mathbb{R}^3$ we study a finite element discretization by cubical elements and an alternating multilevel algorithm. We compute exact convergence rates via a strengthened Cauchy inequality by the application of an algebraic method. A comparison of the these rates with some rates we derive by the method of Fourier-analysis for finite difference discretizations shows that they are too pessimistic for many cases. However we have to take into account that they also hold for none convex domains.

Johannes Kordelsch Ruhr Universität Bochum

A Multilevel Algorithm for the Biharmonic Problem.
We consider a finite element discretization of the mixed variable formulation of the biharmonic equation. For the numerical solution of the discrete equations a multilevel algorithm is applied. Convergence is proved under the assumption of H^3-regularity. This assumption includes domains which are convex polygons.
Peter Peisker, Ruhr-Uni-Bochum.

Some aspects of the application of multigrid in oil reservoir simulation

An up-to-date reservoir simulator should be able to cope with a variety of oil/gas flow problems without loosing its reliability. Simulators therefore consist of huge program packages, which are usually very expensive in their set-up. A problem in reservoir simulation is the large amount ~~loosing its reliabilit~~ of computing time per simulation. Frequently, much of this time is spent in the solution of sparse linear algebraic systems.

In 1980, multigrid (MG) had proved to be a very efficient solution method for large linear systems arising from discretisations of simple cases of partial differential equations.

Hence, it was decided to start a four-year project to make MG a fast and reliable sparse system solver in reservoir simulation, though it was understood that MG might have further possibilities as well. Investigation of the different types of discretisations taught us the necessity of extending MG to e.g. non-rectangular regions, strong inhomogeneities, strong anisotropies, convection-dominated equations, systems of partial differential equations and 3D. This involved careful choices of the MG components. The result is a fast and reliable sparse system solver in reservoir simulation, but the implementation into the existing software turned out to be rather involved.
Rob. Kettler, Delft

A Navier Stokes problem in stream-function – vorticity formulation

In this talk a multigrid method for the computation of stationary flow fields in the gap between two coaxially rotating spheres is presented. The question of introducing artificial viscosity in order to keep the discrete equations h-elliptic is discussed as well as the question how to treat the no-slip boundary conditions of the stream function correctly in MG-methods. This is studied in some detail for the model case of the biharmonic equation. An efficient multigrid solver for this 4th order bvp is presented.
Finally numerical results are shown for the flow in the gap where the outer sphere is rotating and the inner one is at rest.

Johannes Linden, Universität Essen

Mesh Independence Principle for Newton Methods and Applications.

Let $Fz=0$ be an operator equation which is discretized into $F^h z^h = 0$. We compute z and z^h, resp., by Newtons method

$$F'(z_\nu)(z_{\nu+1} - z_\nu) = -Fz_\nu \ , \quad F^{h\prime}(z^h_\nu)(z^h_{\nu+1} - z^h_\nu) = -F^h z^h_\nu \text{ with } I^h z_0 =: z^h_0$$

where I^h represents a restriction operator and where we assume the usual Newton-Kantorovitch conditions which guarantee the quadratic convergence of Newton method for $Fz=0$. Under regularity assumptions for z and the iterates z_0, z_ν (which are available) and the usual stability and consistency conditions (of order p) for F, F^h, and (unusually) for F' and $F^{h\prime}$, we have

$$z^h_\nu - I^h z_\nu = O(h^p),$$
$$F^h z^h_\nu - \hat{I}^h F z_\nu = O(h^p).$$

Furthermore, the number of iterations to obtain a certain tolerance τ starting with $\bar{I}^h z_0$ is independent of the stepsize h for h small enough. This result is used to formulate efficient strategies to solve nonlinear systems of equations arising in the discretization of operator equations. It is possible to obtain z^h, essentially independent of the starting value $\bar{I}^{h_0} z_0$ on a coarse grid, in the equivalence of 2-3 Newton iterations on the final grid

Klaus Böhmer, Universität Marburg.

Basic smoothing methods for anisotropic 3D operators; some remarks about the SUPRENUM project

In this talk the "3D-activities" of the GMD-MG group are surveyed. Systematical investigations for the anisotropic operator $au_{xx} + bu_{yy} + cu_{zz}$ have been made by model problem analysis. It is shown that in certain cases plane-relaxation is needed (if standard coarsening is used). Contrary to the expectations of many experts, suitable plane-relaxation algorithms (very crude 2D-MG cycles) turn out to be fully ~~sufficient~~ efficient, very cheap and easily implementable. In all cases FMG solutions (up to the level of truncation error) can be obtained. The SUPRENUM project (Numerischer Superrechner) and its connection to non-adaptive and adaptive 3D-MG methods

is described
Ulrich Trottenberg (Universität Essen, GMD Bonn)
Clemens Thole (GMD-Bonn)

Survey on several MG activities in the GMD

Four activities of the GMD-MG group are surveyed.
1) A MG code for the 2D-full potential equation describing the flow around an airfoil has been developed (Klaus Becker). The approach is characterized by the following difficulties:
(1) use of Cartesian coordinates,
(2) Neumann boundary condition on the airfoil,
(3) the far field boundary condition at infinity,
(4) the Kutta-Joukowski-condition.
By proper MG treatment one obtains convergence factors of 0.1 in 3-4 work-units. A special feature of the code is the adaptive local refinement of the grids (massive concentration of gridpoints) near the profile.
2) Advanced refinement techniques (with local relaxation and λ-FMG) have been systematically studied for the Poisson equation in 2D-regions with reentrant corners (Ritzdorf).
3) The "NUSIMOT" project is described. Here a MG code for the simulation of flow in the combustion chamber of Otto engines is under development (B. Ruttmann, K. Solchenbach).
4) Guided waves in optical components can be described by an unsymmetric eigenvalue problem for two coupled 2D elliptic boundary value problems,

derived from Maxwells equations. A MG code for this problem is described and first results are shown.

Kristian Witsch (Universität Düsseldorf / GMD-Bonn)
Ulrich Trottenberg (Univ. Essen / GMD Bonn)

Accuracy and fast solution of non-elliptic boundary value problems.

For general linear algebraic systems and appropriate relaxation schemes, a general property of slow-to-converge errors is formulated. This property makes the error approximable on much lower dimensional spaces. In case of elliptic operators this property is equivalent to smoothness. Scale-dependent measures of elliptucity are defined for differential and discrete operators. Types of non-ellipticity and corresponding types of multigrid solvers. They typically solve to truncation level in one-cycle FMG, and their solutions can in fact be better approximations than the exact discrete solutions. Theoretical and practical tools used to predict and design the multigrid efficiency are modified for non-elliptic cases. Examples include non-staggered approximations to Stokes equations and convection-dominated boundary-value problems.

Achi Brandt, Weizmann Institute, Rehovot, Israel

On The Connection Between Multigrid And Cyclic Reduction

A technique is shown whereby it is possible to relate a particular multigrid process to cyclic reduction using purely mathematical arguments. This technique suggests methods for solving Poisson's equation in 1-, 2-, or 3 dimensions with Dirichlet or Neumann boundary conditions, In one dimension the method is exact and, in fact reduces to cyclic reduction. This provides a valuable reference point for understanding multigrid techniques The particular multigrid process analyzed is referred to here as Approximate Cyclic Reduction (ACR) and is one of a class known as Multigrid Reduction methods in the literature. It involves one approximation with a known error term. It is possible to relate the error term in this approximation to certain eigenvector components of the error. These are sharply reduced in amplitude by certain classical relaxation techniques. The approximation can thus be made a very good one.

Marshal L. Merriam, Ames Research Center, Moffet Field, California, USA

Domain Decomposition Methods for the Stokes Problem Application to the NAVIER STOKES EQUATIONS

The main goal of this paper is to present iterative and direct methods, using DOMAIN DECOMPOSITION, for solving the Stokes problem.

The idea is to decompose Ω in SUBDOMAINS with or without overlapping, then solve LOCAL Stokes problems and recouple these LOCAL solutions in order to obtain a method for solving the global problem. Schwarz methods can be used but also more sophisticated iterative or direct methods MATCHING the velocity and pressure on the interfaces of adjacent subdomains. The treatment of the incompressibility introduces extra difficulty, compared for example to the solution of a Standard Poisson problem. This is particularly true for the finite element approximation of the Stoke problem, specially if $\underset{\sim}{\nabla} \cdot \underset{\sim}{u} = 0$ is approximatively WEAKLY satisfied.

This methodology is then introduced in the solution of the incompressible NAVIER-STOKES equations, which formulation is founded on time discretization by operator splitting methods. These methods provide an efficient ~~bad~~ way to decouple the two main difficulties of the problem, i.e. the incompressibility (local Quasi Stokes problems) and the non linearity (local non linear problems solved by least squares).

These methods are well suited for a solution by multiprocessor machines, and numerical results obtained using such computer systems are presented.

J. PERIAUX, AMD/BA, 78 Quai Carnot, St Cloud. 92214 FRANCE

Algebraic Multigrid methods applied to systems of PDE's.

Algebraic multigrid is a method of applying multigrid ideas to the solution of a matrix equation without explicit use of the geometry or origin of the original problem. All information needed for the choice of the coarse-grid and grid transfer and coarse-grid operators is taken from the matrix itself. This method works well for a number of scalar problems (those for which all unknowns represent the same quantity) including finite difference and finite element discretizations of anisotropic problems, diffusion problems with discontinuous coefficients, and convection-diffusion problems.

This talk briefly explains the ideas involved in AMG and its relation to multigrid methods. Also, some of the problems which arise when attempting to apply AMG to discretizations of systems of PDE's. More information about the original problem must be provided, in particular, which unknowns correspond to the same quantity in the continuous problem. Using this information it is shown how AMG can be extended to cover 2-d linear elasticity problems. In addition, further modifications to relaxation and interpolation allow the handling of more complicated systems, such as Stokes equations.

Also discussed are several alternate approaches to systems, each of which has some advantages in different cases

John W. Ruge · University of Colorado at Denver

Algebraic analysis of multigrid method

Multigrid methods often run with one or few smoothings steps, although the theory guarantees convergence only if the number of smoothing steps is large enough (except V-cycle proofs for the H^2-regular case). We prove W-cycle convergence for one smoothing step only assuming a discrete analogue of $H^{1+\alpha}$ regularity for symmetric, positive definite problems.

Jan Mandel, Computing Centre,
Charles University, Malostranské n. 25,
ČSSR

FUNCTIONAL EQUATIONS

16.12. – 22.12.1984

A Contribution to a Problem of I. S. Fenyö

Fenyö (1982) asks for the general solution of

$$(*)\quad f(x f(y) + f(x) y - xy) = f(x) f(y) .$$

Volkmann, Weigel (1984) determine the general continuous solution $f : \mathbb{R} \to \mathbb{R}$ of $(*)$, a class which contains $\aleph_0$ many functions. We show that there are $2^{\aleph}$ many discontinuous solutions $f : \mathbb{R} \to \mathbb{R}$ of $(*)$. We present classes of solutions, which are nowhere continuous, which are continuous in only one point of $\mathbb{R}$, which are discontinuous in one, in two points of $\mathbb{R}$. The construction methods come via multiplicatively closed curves (Dhombres (1982)) and from semi group theory which apply also to more general cases.

W. Benz (Hamburg)

Some properties of the Jacobian elliptic functions

Using some theorems of functional equation we could find some properties of the Jacobian $sn(z,k)$ function which seems to be new

J. Fenyö (Budapest)

Funktionalgleichungen für stetige, nirgends differenzierbare Funktionen

Es sei p eine feste Primzahl, $[p] := \{p^n \mid n \in \mathbb{N}_0\}$. Durch $S_p(x) := \sum_{n=0}^{\infty} \frac{1}{p^n} \sin 2\pi p^n x$ wird eine reelle Funktion erklärt, die stetig, 1-periodisch und ungerade ist.
S_p gehört zu einer von Weierstraß eingeführten Klasse nirgends differenzierbarer Funktionen.
Weiter gilt $S_p(x) = \sum_{m=0}^{k-1} S_p\left(\frac{x+m}{k}\right)$ für $k \in [p]$ und $\sum_{m=0}^{k-1} S_p\left(\frac{x+m}{k}\right) = 0$ für $k \in \mathbb{N} \setminus [p]$.
Wir untersuchen, inwiefern S_p (und damit die Eigenschaft "nirgends differenzierbar") durch die o.a. Funktionalgleichungen charakterisiert ist.

SATZ 1. $f: \mathbb{R} \to \mathbb{R}$ sei stetig, 1-periodisch und ungerade. Ferner gelte $\sum_{m=0}^{p-1} f\left(\frac{x+m}{p}\right) = f(x)$ sowie $\sum_{m=0}^{q-1} f\left(\frac{x+m}{q}\right) = 0$ für alle von p verschiedenen Primzahlen q. Ist dann $f \neq 0$, so ist f nirgends differenzierbar.
Ist $f\left(\frac{1}{2p}\right) = \sin \frac{\pi}{p}$, so gilt $f = S_p$.

SATZ 2. $f: \mathbb{R} \to \mathbb{R}$ sei stetig, 1-periodisch und ungerade. Ferner gelte $\sum_{m=0}^{p-1} f\left(\frac{x+m}{p}\right) = f(x)$ sowie $f(x) - \sum_{m=0}^{k-1} f\left(\frac{x+m}{k}\right) = \Phi_k(x)$ für alle $k \in \mathbb{N} \setminus [p]$. Es folgt

a) Für die Fourierreihe $S(f)$ von f gilt:

$$S(f)(x) = b_1(f)\, S_p(x) + \sum_{k \in \mathbb{N} \setminus [p]} \frac{b_1(f) - b_1(\Phi_k)}{k} \sin 2\pi k x$$

b) $f = S_p \iff b_1(\Phi_k) = 1$ für alle $k \in \mathbb{N} \setminus [p]$

K.H. Kairies (Clausthal)

Bellman's Functional Equation and the Optimal Investment Ratio of an Economy

Problem: Given (the production structure of) an economy, its initial capital stock K_0 [= amount of capital goods (buildings machines) involved in the production process] at the beginning of the year 1, and a time horizont T (≥ 3, finite or infinite). Find a vector $(u_1^*, u_2^*, \ldots, u_T^*)$ of investment ratios

$$u_t = \frac{\text{Gross domestic investment during the year } t}{\text{Gross domestic product during the year } t}$$

which maximizes the macroeconomic consumption during the

years from year 1 to year T.
The problem is solved within the framework of a model of an aggregated economy by means of Bellman's principle of backward dynamic programming applied to Bellman's functional equation(s). ~~The functional equation(s) depend(s) on the is (are) closely related to the~~ The model yields the functional equation(s).

Wolfgang Eichhorn, Karlsruhe

On completely additive functions

A strictly decreasing sequence of positive real numbers (λ_n) is an interval filling sequence if $\sum_{n=1}^{\infty} \lambda_n = L \in \mathbb{R}$ and for any $x \in [0,L]$ there exists a sequence $(\varepsilon_n): \mathbb{N} \to \{0,1\}$ such that $x = \sum_{n=1}^{\infty} \varepsilon_n \lambda_n$. The function $F: [0,L] \to \mathbb{R}$ is completely additive (with respect to the interval filling sequence (λ_n)) if $F(\sum_{n=1}^{\infty} \varepsilon_n \lambda_n) = \sum_{n=1}^{\infty} \varepsilon_n F(\lambda_n)$ for all $(\varepsilon_n): \mathbb{N} \to \{0,1\}$. Professors Daróczy, Járai and Kátai, under various further assumptions on the interval filling sequence, have determined the all completely additive functions by showing that they are linear. In this talk we suppose nothing on the interval filling sequence and we prove the same for those completely additive functions which are nonnegative or differentiable at a point.

Gyula Maksa, Debrecen

On a difference-functional equation.

In 1980 C. Borelli Forti and I. Fenyö considered the difference equation

$$(\Delta^m f)(x; y_1, \dots, y_m) = d\,(x; y_1, \dots, y_m)$$

where $f: X \to E$, $d: X^m \times X \to E$ (X is an abelian group, E is a Banach space), d is a given bounded function, and gave the explicit expression of the general solution.

The previous result is used for solving an equation of the form

(1) $$(\Delta^m f)(x; y_1, \dots, y_m) = \Phi\{x; y_1, \dots, y_m; (\Delta^i f)(x; y_{k_1}, \dots, y_{k_i}); f(x)\}$$

$i = 1, \dots, m$ (we have indicated only one of the positions of the variables depending on i and $(k_1, \dots, k_i)$, actually they are $2^m - 1$), when Φ satisfies a Lipschitz condition and a condition of boundedness. More precisely we prove that there exists at most one bounded solution of (1) and it is the uniform limit of a sequence of functions explicitely described.

Gian Luigi Forti, Milano

On the functional equations: $f[x f(y) + y f(x)] = \alpha f(x) f(y)$ and $f[x f(y) + y f(x)] = \alpha f(xy)$

Consider the two following functional equations:

(1) $f[x f(y) + y f(x)] = \alpha f(x) f(y)$ where α is a non-negative

(2) $f[x f(y) + y f(x)] = \alpha f(xy)$ real number

We have the following results:

<u>Theorem 1</u> Let E be a real topological vector space. When $\alpha = 0$, the unique continuous solution $f: E \to \mathbb{R}$ of (1) and (2) is $f \equiv 0$

<u>Theorem 2</u> Let E be a real locally convex topological vector space. When α is a strictly positive real number, all continuous solutions $f: E \to \mathbb{R}$ of (1) are given by: (i) $f \equiv 0$ (ii) $f \equiv \frac{1}{\alpha}$

and, in the case $\alpha = 2$ only:

(iii) $f(x) = \begin{cases} \langle x^*, x \rangle & \text{for } x \in \overline{K} \\ 0 & \text{for } x \notin \overline{K} \end{cases}$ (iv) $f(x) = \begin{cases} \langle x^*, x \rangle & \text{for } x \in \overline{K \cup (-K)} \\ 0 & \text{for } x \notin \overline{K \cup (-K)} \end{cases}$

where $x^* \in E^* \setminus \{0\}$ and K is a non empty open convex cone with vertex 0 contained in $\{x \in E \mid \langle x^*, x \rangle > 0\}$

Theorem 3 When $\alpha = 1$, all continuous solutions $f: \mathbb{R} \to \mathbb{R}$ of (2) are given by: (i) $f \equiv a$, $a \in \mathbb{R}$ (ii) $f(x) = \frac{1}{2} x$
(iii) $f(x) = \mathrm{Inf}(x, 0)$ (iv) $f(x) = \mathrm{Inf}(-x, \frac{x}{2})$

Nicole Belluot (Brillouët) - Nantes - France

On the stability of a functional equation for homogenous functions

Consider an abelian group G, a Banach space H and consider furthermore for $\alpha \in \mathbb{Z}^n$, $x = (x_1, \dots, x_n) \in G$, $f: G \to H$ the expression

$$L_{n,\alpha} f(x) := f\left(\sum_{i=1}^{n} \alpha_i x_i\right) - \sum_{i=1}^{n} \alpha_i^{\,n} f(x_i) - \sum_{s \in S} \frac{\alpha^s}{s!} K_n f(x^s),$$

where $S = \{ s = (s_1, \dots, s_n) \mid s_i \geq 1, \sum_{i=1}^{n} s_i = n, s_i \in \mathbb{N}_0 \}$, $s! = \prod s_i!$, $\alpha^s = \prod \alpha_i^{s_i}$, $x^s = (\underbrace{x_1, \dots, x_1}_{s_1\text{-times}}, \dots, \underbrace{x_n \dots x_n}_{s_n\text{-times}})$ and $K_n f(x_1, \dots, x_n) := \Delta_{x_1 \dots x_n} f(0) + (-1)^{n+1} f(0)$

$\left(\Delta_{x_1 \dots x_n} = \Delta_{x_1} \circ \dots \circ \Delta_{x_n}, \ \Delta_x f(y) := f(x+y) - f(y) \right)$.

In this context K. J. Heuvers showed in 1980, that $L_{n,\alpha} f \equiv 0$ for all $\alpha \in \mathbb{Z}^n$ is equivalent to the fact, that f is a homogenous polynomial of degree n.

Now the following 'Hyers-type' stability theorem holds

Theorem. Let $f: G \to H$ be a mapping, such that for all $\alpha \in \mathbb{Z}^n$ there exists some $\delta(\alpha) \geq 0$ with $|L_{n,\alpha} f(x)| \leq \delta(\alpha)$ for all $x \in G$.
Then there exists one and only one homogeneous polynomial g of degree n, such that $f-g$ is bounded. Furthermore the best possible 'universal' bound for $|f-g|$ is $\delta(\alpha^0)\cdot(2^n-1)^{-1}$, where $\alpha^0 = (1,0\ldots,0)$.

Jens Schwaiger, Graz

Some Recent Results about Inset Entropies
on Open Domains

Several problems regarding inset entropies on open domains (i.e. without empty sets and 0 probabilities) have been solved recently. Among these are determinations of (i) all semi-symmetric, β-recursive entropies (including the weakly regular ones), (ii) all additive inset entropies with measurable sum property, and (iii) all semisymmetric entropies which are recursive of multiplicative type. Some of these results will be discussed.

Bruce Ebanks

Functional equations and characterization of inner product spaces

Let ABC be a triangle in the euclidean space. It is well known that it is possible to compute the length of any median from the lengths of the three sides. In a real normed space, this mere possibility (median property) implies that the norm derives from an inner product. The original proof of Lorch depended on an earlier result of Ficken where at least some lines of computation seem missing. Other works use smoothness property of the unit sphere, an approach which does not seem to be neither direct nor simple.

Our purpose is to give a self-contained proof of Lorch's theorem, second to systematically use functional equations and more precisely conditional functional equations to perform the proof, third to generalize Lorch's result by showing the role played by the field underlying the normed space. This raises interesting (and some open) questions for functional equations

Jean Dhombres (Nantes France).

Iterative roots of Laguerre polynomials.

A joint paper by Marek Kuczma (Katowice, Poland) has been reported. The problem indicated in the title of the talk leads to the system of functional equation

$$(*)\quad \begin{cases} H^n(t) = \dfrac{t}{t-1} \\ \prod_{k=0}^{n-1} G(H^k(t)) = (1-t)^{-\alpha-1}, \quad \alpha > -1 \end{cases}$$

for power series H and G generating roots of the sequence of Laguerre polynomials. The series generating the latter sequence occur on the right-hand sides of $(*)$. General solution of $(*)$ has been obtained.

Bogdan Choczewski (Kraków, Poland)

On an inhomogeneous Cauchy's equation connected with the Jacobi's elliptic function.

Consider the following functional equation in the complex domain $\mathbb{C}$:

(1) $g(z_1+z_2) - g(z_1) - g(z_2) = f(z_1)f(z_2)f(z_1+z_2)$

The following theorem holds (this is a common result with prof. Fenyő):

Theorem. Let (f, g) be a pair of analytic functions, defined in a neighborhood of the origin and solutions of (1).

(i) If $f(0) \neq 0$, then there exist $\alpha, \gamma \in \mathbb{C}$ such that

$f(z) = \alpha \qquad g(z) = -\alpha^3 + \gamma z$

(ii) If $f(0) = 0$ and $f'(z) = \alpha$, then there exists $\gamma \in \mathbb{C}$ such that

$$f(z) = \alpha z \qquad g(z) = -\alpha^3 \frac{z^3}{3} + \gamma z$$

(iii) If $f(0) = 0$ and f' is not constant, then there exist $\alpha, \beta, \gamma, \kappa \in \mathbb{C}$ such that:

$$f(z) = \alpha\, \mathrm{sn}(\beta z, \kappa) \qquad g(z) = \alpha^3 \int_0^{\beta z} \mathrm{sn}^2(t, \kappa)\, dt$$

(here $\mathrm{sn}(z, \kappa)$ denotes the classical Jacobi's elliptic function).

Ludwig Reich

<u>Non-additive information measures</u>

Let $I, G, L : [0,1] \to \mathbb{R}$ and let

$\Delta_n = \{ P = (p_1, \dots, p_n) \mid p_i \geq 0, \sum_{i=1}^{n} p_i = 1 \}$, $n \in \mathbb{N}$.

The functional equation

$$(1) \quad \sum_{i=1}^{n} \sum_{j=1}^{m} I(p_i q_j) = \sum_{i=1}^{n} \sum_{j=1}^{m} G(p_i) I(q_j) + \sum_{i=1}^{n} \sum_{j=1}^{m} L(q_j) I(p_i), \quad P \in \Delta_n, Q \in \Delta_m$$

is of interest in information theory, since the special cases

$$G(p) = p + \lambda I(p), \quad L(p) = p, \quad \lambda \in \mathbb{R}$$

respectively

$$G(p) = p^\alpha, \quad L(p) = p^\beta, \quad \alpha, \beta \in \mathbb{R}$$

play important roles in the characterization of the entropies of degree α and of the entropies of degree (α, β).

We determine all measurable triples (I, G, L) satisfying (1) when $G(0) = L(0) = 0$ and holding for some fixed pair (n, m), $n \geq 3$, $m \geq 3$.

Especially we get all measurable functions I satisfying (1) with

$$G(p) = p^\alpha + \lambda I(p), \quad L(p) = p^\beta, \quad \alpha, \beta, \lambda \in \mathbb{R}$$

and we get a new characterization of the entropies of degree (α, β).

Our results are extensions of some recent results about this topic.

Wolfgang Sander