MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 6 / 1975

Einhüllende Algebren von Lie-Algebren

2.2. bis 8.2. 1975

Die Tagung über Einhüllende Algebren von Lie-Algebren war die zweite Tagung dieser Art, nachdem die erste im Jahr 1973 ebenfalls in Oberwolfach stattgefunden hatte . Das Gebiet, das sich in rascher Entwicklung befindet, steht im Schnittpunkt von nichtkommutativer Algebra, der Theorie der Operation algebraischer Gruppen und der unitären Darstellungstheorie Liescher Gruppen. Nachdem zu Beginn sich diese Theorie mit einhüllenden Algebren nilpotenter Lie-Algebren und sodann auflösbarer Lie-Algebren befaßte, liegen seit 1974 erste detaillierte Resultate im halbeinfachen Fall vor. Diese neue Entwicklung wird auch daran sichtbar, daß sich diesmal die Hälfte der Vorträge mit dem halbeinfachen Fall befaßten.
In der Zeit zwischen der Tagung und der Fertigstellung dieses Berichts hat der halbeinfache Fall eine stürmische Entwicklung genommen.
Die Tagung war international zusammengesetzt und ermöglichte viele ergiebige Gespräche und Diskussionen. Die Tagung wurde geleitet von P. Gabriel (Zürich) und R. Rentschler (Paris).

Teilnehmer

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Bamba, S., Paris
Borho, W., Bonn
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Vergne, M., Paris
Wallach, N. R., New Brunswick

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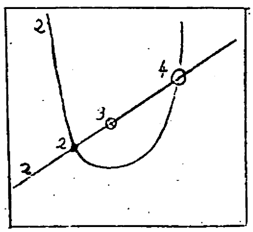
Vortragsauszüge

W. BORHO : Primitive completely prime ideals in the envelope of $\underline{so}(5,\mathbb{C})$

Let $\underline{g} = \underline{so}(5,\mathbb{C})$ be a complex Lie algebra of type B_2 . Let X denote the primitive spectrum of its enveloping algebra U (i.e. the set of primitive ideals, endowed with Jacobson's topology). Let $X^{r,d} = \{J \in X \mid \mathrm{rk}\, J = r \,,\, \mathrm{Dim}\, J = d\}$ for $r = 1, 2, \ldots$ and $d = 0, 1, 2, \ldots$, where $r = \mathrm{rk}\, J$ is the Goldie-rank, defined by $\mathrm{Quot}(U/J) \cong M_r(K)$, K skew field, and $\mathrm{Dim}\, J$ is the Gelfand-Kirillow-Dimension ("transcendence-degree") of U/J . Then $X^{1,d} = 0$ for $d \neq 0, 4, 6, 8$; $X^{1,0}$ = a single point (the augmentation ideal) ; $X^{1,4}$ = a single point (Joseph's ideal) ; $X^{1,6}$ = the union of two one-parameter families with 1 point in common , $X^{1,8} \cong$ affine complex plane. There is a canonical projection $p: X \twoheadrightarrow X^{1,8}$ = plane, defined by Duflo by $p(J) = (J \cap Z)U$ (Z the center of U), such that $p^{-1}(I) = \{I\}$ for all $I \in X^{1,8}$. In particular, p is the identity on $X^{1,8}$. The picture indicates, how the space $\bigcup_d X^{1,d}$ of all completely prime primitive ideals and its projection onto the plane look like. There are 4 points in the fibre of the augmentation ideal, 3 in the fibre of Joseph's ideal, 2 in the other fibres containing some $J \in X^{1,6}$, and only 1 in the "general" fibres. Moreover, Joseph's ideal turns out to be the first member of a sequence $J_1, J_2, \ldots$ with $J_r \in X^{r,4}$.



R. W. CARTER : On certain commutator relations in enveloping algebras of semisimple Lie algebras

Let $\underline{g}$ be a simple Lie algebra of type A_ℓ over $\mathbb{C}$ with root system $\{r_{ij} \quad i \neq j\} = \Phi$, positive system $\Phi^+ = \{r_{ij} \quad i < j\}$ and fundamental system $\Pi = \{r_{i\ i+1}\}$. Let U be the enveloping algebra of $\underline{g}$ with generators $e_r, f_r, h_r, \; r \in \Phi^+$ where $[e_r, f_r] = h_r$. Let $\underline{g} = \underline{h} \oplus \underline{n}_+ \oplus \underline{n}_-$ and $\underline{b}_- = \underline{h} \oplus \underline{n}_-$.
Let $S_{r_{ij}} \in U(\underline{b}_-)$ be defined by

$$S_{r_{ij}} = \left| \begin{array}{cccc} f_{i\ i+1} & h_{i+1\ i} - 1 & 0 & 0 \\ f_{i\ i+2} & f_{i+1\ i+2} & h_{i+2\ i} - 2 & \\ & \ddots & \ddots & 0 \\ & & & h_{j-1\ i} - (j-1-i) \\ f_{i\ j} & f_{i+1\ j} & \cdots & f_{j-1\ j} \end{array} \right|$$

The element $S_{r_{ij}}$ lies in the $-r_{ij}$ weight space of $U(\underline{b}_-)$. Then for each $r \in \Phi^+$, $s \in \pi$ and all $k, d \in \mathbb{Z}$ with $k > 0$, $d > 0$ we have

$$(e_s^k/k!)(S_r^d/d!) - (S_r^d/d!)(e_s^k/k!) \in U\underline{n}_+ + U(h_r + \rho(h_r) - d) .$$

Applications of these commutator relations to the modular representation theory were discussed. Let V_λ be a finite dimensional irreducible representation of U, let $V_{\lambda,\mathbb{Z}} = U_{\mathbb{Z}}\Phi_\lambda$ where Φ_λ is the extreme generator of V_λ and $U_{\mathbb{Z}}$ is the Kostant subring of U. Let K be an algebraically closed field of caracteristic p, let $\bar{U} = U_{\mathbb{Z}} \otimes_{\mathbb{Z}} K$, $\bar{V}_\lambda = V_{\lambda,\mathbb{Z}} \otimes_{\mathbb{Z}} K$. Then $\bar{V}_\lambda$ is a finite dimensional $\bar{U}$ - module, called a Weyl module, and we consider $\mathrm{Hom}_{\bar{U}}(\bar{V}_\mu, \bar{V}_\lambda)$.

Let $L_{r,k} = \{\lambda ; (\lambda+\rho)(h_r) = kp\}$, $k \in \mathbb{Z}$, and let $w_{r,k}$ be the reflection in $L_{r,k}$. Suppose λ, μ are dominant integral such that $\mu = w_{r,k}(\lambda)$, $\mu < \lambda$ and $0 < d < p$ where $d = (\lambda+\rho)(h_r) - kp$ represents the distance of λ from the reflecting hyperplane $L_{r,k}$. Then it may be shown by considering the above elements in U and other related elements that $\dim \mathrm{Hom}_{\bar{U}}(\bar{V}_\mu, \bar{V}_\lambda) = 1$.

References :

R.W. Carter and G. Lusztig : Math. Zeit. 136 (1974) 193 - 242 .

N.N. Shapovalov : J. Funk. An. e Priloz. 6 (1972) 65 - 70 .

M.T.J. Payne : Ph.D thesis, University of Warwick, to appear .

J. DIXMIER : On primitive completely prime ideals in the enveloping algebra of $\underline{sl}(n, \mathbb{C})$

This first part of the talk concerns partial generalisations for $\underline{sl}(n, \mathbb{C})$ of the known results concerning $\underline{sl}(3, \mathbb{C})$. The 2nd part is largely conjectural. Let $\underline{g}$ be a simple complex Lie algebra, G the adjoint group, $\underline{h}$ a Cartan subalgebra, Δ the root system. For $\Gamma \subset \Delta$, let $\underline{h}_\Gamma$ be the set of $H \in \underline{h}$ with $\alpha(H) = 0$ for $\alpha \in \Gamma$, $\alpha(H) \neq 0$ for $\alpha \in \Delta - \Gamma$. Let $\underline{g}_\Gamma = G(\underline{h}_\Gamma)$. Distinct sets $\underline{g}_{\Gamma_1}, \dots, \underline{g}_{\Gamma_p}$ are disjoint. Let $\underline{g}_\Gamma^{\sim}$ be the closure of $\underline{g}_\Gamma$ inside the set of elements of $\underline{g}$ for which the centralizer has the same dimension than along $\underline{g}_\Gamma$. Then $\underline{g}' = \underline{g}_{\Gamma_1}^{\sim} \cup \dots \cup \underline{g}_{\Gamma_p}^{\sim}$ is the set of elements in $\underline{g}$ which admit polarisations. Let $\underline{g}'' = \underline{g}_{\Gamma_1}^{\sim} \amalg \dots \amalg \underline{g}_{\Gamma_p}^{\sim}$. For $x \in \underline{g}_{\Gamma_i}^{\sim}$ let $\underline{p}_1, \dots, \underline{p}_k$ the polarizations of x which are obtained as limits of polarizations of elements in $\underline{g}_{\Gamma_i}$. Let $I_1, \dots, I_k$ the ideals deduced from x by the twisted induction process, using $\underline{p}_1, \dots, \underline{p}_k$. Conjecture : $I_1 = \dots = I_k$. This would define a map of $\underline{g}''/G$ onto the non - mysterious part P of Primc $U(\underline{g})$. Conjecture : this map defines a bijection from $\underline{g}''/G$ onto P. . Theorem : for $\underline{g} = \underline{sl}(n, \mathbb{C})$, $\underline{g}$ is the disjoint union of $\underline{g}^{\sim}_{r_1}, \ldots, \underline{g}^{\sim}_{r_p}$, so that $\underline{g}''$ can be identified with $\underline{g}$. That should be the reason why $\underline{sl}(\underline{n}, \mathbb{C})$ is better than other simple Lie algebras for the study of Primc $U(\underline{g})$. Note : The conjecture was immediately disproved by Borho - Rentschler (case of B_2) .

C. GODFREY : Ideals of coadjoint orbits of nilpotent Lie algebras

Let $\underline{g}$ be a finite dimensional nilpotent Lie algebra over a field of characteristic 0 . For each $f \in g^*$ Dixmier has constructed a rational ideal $I(f)$ in $U(\underline{g})$ and showed that $I(f) = I(\underline{h})$ iff f and h are in the same coadjoint orbit in $\underline{g}^*$. Let $J(f)$ be the polynomials in $S(\underline{g})$ vanishing on the orbit through f .
A Kirillov - type inductive process gives a formula for constructing from any basis $\{h_1, \ldots, h_r\}$ for the isotropy algebra $\underline{g}^f$, polynomials $P_1, \ldots, P_r$ generating $J(f)$ such that $T(P_1), \ldots, T(P_r)$ generate $I(f)$, where T is the canonical symmetrization map of $S(\underline{g})$ into $U(\underline{g})$.

A. VAN DEN HOMBERGH : On step algebras

Let $\underline{g}$ be a finite dimensional Lie algebra, $\underline{k}$ a reductive subalgebra of $\underline{g}$ with Cartan subalgebra $\underline{h}$ and $\underline{g}'$ a $\underline{k}$ - invariant complement in $\underline{g}$ of $\underline{k}$. One has : $\underline{k} = \underline{k}_- \oplus \underline{h} \oplus \underline{k}_+$. Define $S'(\underline{g}, \underline{k}) = \{ u \in U(\underline{g}) \mid \underline{k}_+ u \subseteq U(\underline{g})\underline{k}_+ \}$ and $S(\underline{g}, \underline{k}) = S'(\underline{g}, \underline{k}) / U(\underline{g})\underline{k}_+$, $S_0(\underline{g}, \underline{k})$ is the subalgebra of $S(\underline{g}, \underline{k})$ generated by $S(\underline{g}, \underline{k}) \cap (U(\underline{k})\underline{g}' + U(\underline{g})\underline{k}_+ / U(\underline{g})\underline{k}_+)$ and $\bar{\underline{h}} = \underline{h} + U(\underline{g})\underline{k}_+ / U(\underline{g})\underline{k}_+$. If λ is a highest weight of $\underline{k}$, then J_λ is the left ideal of $U(\underline{k})$ that kills the highest weigt vector of a simple finite dimensional $\underline{k}$ - module with highest weight λ . Call an $S_0(\underline{g}, \underline{k})$ - module <u>dominant</u> if it is $\bar{\underline{h}}$ - diagonisable and its weights are $\underline{k}$ - dominant and <u>extreme</u> if it is dominant and the elements of weight λ are killed by $S'_0(\underline{g}, \underline{k}) \cap U(\underline{g})J_\lambda$.
<u>Theorem</u>. There is a 1 - 1 correspondence between $\underline{k}$ - finite finitely generated $\underline{g}$ - modules and finitely generated extreme $S_0(\underline{g}, \underline{k})$ - modules.
It turns out that the structure of $S_0(\underline{g}, \underline{k})$ is very nice in the cases that $\underline{g}$ is of type $\underline{so}(n, 1)$ or $\underline{su}(n, 1)$ and $\underline{k}$ is the compact subalgebra of $\underline{g}$. In these cases one obtains at least all $\underline{k}$ - finite irreducible $\underline{g}$ - modules .

R.L. HUDSON : Casimir elements for U(n)

We construct Casimir elements H_r^n, $r = 0, 1, \ldots, n$, for $U(n)$, whose eigenvalues in the irreducible representation whose highest weight has components $l_1 - n + 1$, $l_2 - n + 2$, ..., l_n, are the elementary symmetric polynomials σ_r^n, $r = 0, 1, \ldots, n$. We consider the Casimir element

$$G_n^n = \sum_{\alpha,\beta\in S_n} \operatorname{sign}\alpha\beta\, A_{1\beta}^{1\alpha}\ldots A_{n\beta}^{n\alpha} \quad , \quad \text{where} \quad [A_j^i, A_l^k] = \delta_j^k A_l^i - \delta_l^i A_j^k \; ,$$

and Casimir elements G_r^n, $r = 0, 1, 2, \ldots, n-1$, obtained from coefficients of the polynomial resulting from the substitution $A_j^i \longmapsto A_j^i + \theta\delta_j^i$ in the expression for G_n^n. The eigenvalues of G_r^n are linear combinations of $\sigma_0^n, \sigma_1^n, \ldots, \sigma_r^n$, whence H_r^n is a linear combination of $G_0^n, G_1^n, \ldots, G_r^n$. By comparing the behaviour of G_r^n under the transformation $A_j^i \longmapsto A_j^i + \theta\delta_j^i$ and under the summation process of the Weyl branching formula with corresponding behaviour of the σ_r^n this combination is found to be

$$H_r^n = \sum_{s=0}^{r} \frac{(n-s)!(n-r+s)!}{n!(n-r)!\, s!}\, \sigma_{r-s}^n(0,1,\ldots,n-1)G_s^n \quad .$$

J.C. JANTZEN : On modular representations of Chevalley groups

Let $\underline{g}$ be a simple complex Lie-algebra, $U_{\mathbb{Z}}$ the Kostant - $\mathbb{Z}$ - form of its enveloping algebra, R the roots, R^+ the positive roots, $\rho = \frac{1}{2}\sum_{\alpha\in R^+}\alpha$, W the Weyl group, v a highest weight vector in a finite dimensional, irreducible representation $V(\lambda)$ of $\underline{g}$ with highest weight λ, $V(\lambda)_{\mathbb{Z}} = U_{\mathbb{Z}}v$, $D_\lambda(\mu)$ the determinant of the contravariant form on $V(\lambda)$ (normalized by $(v,v) = 1$) restricted to the weight space $V(\lambda)_{\mathbb{Z}}^{\mu}$, p a prime number, ν_p the p - adic valuation ($\nu_p(p^n) = n$), $\nu_p(D_\lambda(\mu))\, e(\mu)$, where $(e(\mu))_\mu$ is the canonical basis of the group ring of the group of weights,

$$\chi(\lambda) = \sum_{w\in W} \det(w)\, e(w(\lambda+\rho)) \Big/ \sum_{w\in W} \det(w)\, e(w(\rho)) \quad \text{for all weights} .$$

Theorem : For all dominant weights λ we have

$$\nu_p(D\ ) = \sum_{\alpha\in R^+}\ \sum_{0<r<(\lambda+\rho)(H_\alpha)} \nu_p(\ (\lambda+\rho)(H_\alpha) - r\ )\,\chi(\lambda - r\alpha)$$

if a) R is of type A_n, B_2, B_3, C_3, D_4, G_2 and p arbitrary

or b) R is not of type E_6, E_7, E_8, F_4 and p > Coxeter number of R

This theorem has applications for representation theory in caracteristic p. It can for example be used to prove an analogon of the Bernstein - Gel'fand - Gel'fand theorem on Verma modiles.

Besides the main results of my paper in Math. Z. 140 (1974), 127 - 149 were reported.

A. JOSEPH : The minimal orbit of a simple Lie algebra and its associated maximal ideal

Let $\underline{g}$ be a simple Lie algebra over the complex field $\mathbb{C}$ not isomorphic to $\underline{sl}(n)$: n = 1, 2, ... Identify $\underline{g}$ with its dual $\underline{g}^*$ through the Killing form and fix a Cartan subalgebra $\underline{h}$. Let $\Delta = \Delta(\underline{g}, \underline{h})$ be the set of non-zero roots and for each $\alpha \in \Delta$, let $\underline{g}^{\alpha}$ be the root subspace, E_{α} a non-zero vector in $\underline{g}^{\alpha}$ and $H_{\alpha} = [E_{\alpha}, E_{-\alpha}]$. Fix an orderimg in Δ and let β be the highest root. It is known that $\underline{g}^*$ admits a single non-trivial orbit $\mathcal{O}_0$ of minimal dimension. Furthermore $\mathcal{O}_0$ consists of nilpotent elements, contains $\underline{g}^{-\beta} - \{0\}$, and is not polarizable . Set $\Gamma = \{\gamma \in \Delta : (\beta, \gamma) > 0\}$ and $\underline{g}^{\Gamma} =$ lin span $\{\underline{g}^{\beta} : \gamma \in \Gamma\}$. Then $\underline{g}^{\Gamma}$ is a Heisenberg algebra with centre $\underline{g}^{\beta}$. Furthermore $\underline{r} = \underline{g}^{\Gamma} \oplus \mathbb{C} H_{\beta}$ is a subalgebra of $\underline{g}$ which identifies with the tangent space to $E_{-\beta}$. It is shown that $\underline{g}$ admits a unique embedding i in the localization of $U(\underline{r})$ at E such that $i|_{\underline{r}} = \mathrm{Id}$. Let J_0 be the two sided ideal of $U(\underline{g})$ defined by this embedding. Then J_0 is completely prime and it is shown to be the only completely prime ideal in $U(\underline{g})$ whose caracteristic variety coincides with $\mathcal{O}_0 \cup \{0\}$. It is further shown that J_0 is a maximal ideal and that it cannot be induced from any proper subalgebra of $\underline{g}$. The ideal J_0 is given explicitly and its central character computed. For $\underline{sp}(2n)$: n = 1, 2, ... , J_0 coincides with the kernel of the Weil representation of $U(\underline{sp}(2n))$.

A. JOSEPH : Second commutant theorems in the enveloping algebra of a Lie algebra

Let $\underline{g}$ be a split semisimple Lie algebra over a field k of characteristic zero. Let $\underline{h}$ be a Cartan subalgebra and $\underline{n}$ the subalgebra spanned by the positive root eigenvectors. Given $A \subset U(\underline{g})$, let A'' denote its second commutant in $U(\underline{g})$. Let $Z(\underline{g})$ denote the centre of $U(\underline{g})$.

Theorem : 1) $U(\underline{h})'' = U(\underline{h})\, Z(\underline{g})$ 2) $U(\underline{n})'' = U(\underline{n})\, Z(\underline{g})$

1) answers positively a question raised by Dixmier. Both generalize to $U/J_{min}(\lambda)$, $\lambda \in \underline{h}^*$, where $J_{min}(\lambda)$ is minimal primitive. In this case 2) becomes a result of Dixmier. 2) generalizes in the following manner :

Let A be an integral domain over k , $\underline{n}$ a finite dimensional Lie algebra embedded in A and suppose that each derivation $\mathrm{ad}_A X$, $X \in \underline{n}$, is locally nilpotent. Let U be the subalgebra of A generated by $\underline{n}$, and U', U'' the first and second commutants of U in A . Set $Y = U \cap U'$; $Z = U' \cap U''$.

Let Dim_k denote Gelfand-Kirillov dimension.

Theorem : I) There exists $z \in Z$ such that $(U'')_z = (U\,Z)_z$

2) $\mathrm{Dim}_k U'' - \mathrm{Dim}_k Z = \mathrm{Dim}_k U - \mathrm{Dim}_k Y$

3) If A is the subalgebra of an enveloping algebra, then $\mathrm{Dim}_k A - \mathrm{Dim}_k U' \leq \mathrm{Dim}_k U$.

J. LEPOWSKI : Conical vectors in induced modules

Let $\underline{g} = \underline{k} \oplus \underline{a} \oplus \underline{n}$ be an Iwasawa decomposition of a semisimple symmetric Lie algebra over a field of characteristic zero, with $\underline{a}$ a splitting Cartan subspace. Let $\underline{m}$ be the centralizer of $\underline{a}$ in $\underline{k}$. Given a linear functional ν on $\underline{a}$, extend ν to a character of $\underline{m} \oplus \underline{a} \oplus \underline{n}$ which is trivial on $\underline{m} \oplus \underline{n}$, and let X^{ν} be the corresponding algebraically induced $\underline{g}$ - module. Certain conical vectors (nonzero $\underline{m} \oplus \underline{n}$ - invariant vectors) are shown to exist in the X^{ν} , and it is conjectured that this are all, once a condition involving a certain finite group of Lie algebra automorphismes of $\underline{g}$ is imposed. The results generalize the study of Verma modules, and are also related to representations of semisimple Lie groups.

Paul MALLIAVIN : Poisson formulae in the Siegel half plane plane of rank 2

(Report on a joint work with A. Korcenyi to appear in Acta Mathematica)

Statement : $H = \{W \in \mathcal{M}_3^{\mathrm{sym}}(\mathbb{C})\ ;\ \mathrm{Im}\, W = V \gg 0$, $\partial_W = \begin{pmatrix} 2^{1/2}\partial_{w_1} & \partial_{w_2} \\ \partial_{w_3} & 2^{1/2}\partial_{w_4} \end{pmatrix}$

$\Delta_2 = \mathrm{trace}\,(\bar{\partial}_W \cdot V \cdot \partial_W)$, Δ_1 = Laplacian for the Riemannian metric .

Let h be a bounded function on H . Then h is a poisson integral on the Shilov boundary iff $\Delta_1 h = \Delta_2 h = 0$

Method of proof : (i) Let $Z^{(i)}$ the diffusion processes associated to Δ_i $(i = 1, 2)$; deterministic aspects of the trajectories of $Z^{(i)}$ are obtained via ordinary differential equation of comparison

(ii) Realization of a compound process mixing $Z^{(1)}$ and $Z^{(2)}$

(iii) Fatou theorem along the the trajectories of the compound process

J. MICKELSSON : Step algrbras and $\underline{gl}(2) \oplus \underline{gl}(2)$ - finite $\underline{gl}(4)$ - modules

Let G be a complex Lie algebra and K a semi-simple subalgebra of G . We study irreducible K - finite G - modules using the step algebra $S_0(G, K)$

of the pair (G, K) . If V is a G - module , we denote by V_α the sum of all irreducible finite dimensional K - submodules of V with maximal weight . We define $A_{\beta,\alpha}$ as the subalgebra of the enveloping algebra $U(G)$ of G such that $A_{\beta,\alpha} V_\alpha^+ \subset V_\beta^+$ for any G - module V ; here V_α^+ consists of maximal vectors in V_α . Let $M_\alpha = \sum_{\beta < \alpha} A_{\beta,\alpha}$. It is shown that the equivalence classes $[V]$ of irreducible G - modules V , such that $V_\alpha \neq 0$ and $V_\beta = 0$ for $\beta < \alpha$, are in natural $1 - 1$ correspondence with the equivalence classes of irreducible $D/(D \cap U(G) M_\alpha)$ - modules , D is the zero-step algebra , $D V_\alpha^+ \subset V_\alpha^+$ for any V and any α . Using this result, all irreducible $\underline{gl}(2) \oplus \underline{gl}(2)$ - finite $\underline{gl}(4)$ modules are explicitly described .

J.T. STAFFORD : Number of generators of right ideals in simple rings

It is proved that if R is a simple right Noetherian ring (with an identity) of Krull dimension n , then any right ideal I of R can be generated by $n + 1$ elements , one of which may be chosen to be any element which generates an essential submodule of I. In particular, this theorem applies to the n^{th} Weyl Algebra A_n over a field of characteristic zero.
This result is obtained as a consequence of the following module theoretic result. Let M be a completely faithful Noeterian right R - module of Krull dimension n over any ring R of Krull dimension greater than n (or indeed such that R does not have a Krull dimension). Then M can be generated by n elements.
Finally this result is extended to two further cases. In particular the first result still holds when R is a polynomial extension in one variable over a Weyl Algebra .

P. TAUVEL : Sur les représentations irréductibles des algèbres de Lie nilpotentes

Soient $\underline{g}$ une algèbre de Lie nilpotente sur un corps k algébriquement clos de caractéristique 0 et ρ une représentation irréductible de $\underline{g}$ dans un espace vectoriel V. Soit I le noyau de ρ dans l'algèbre enveloppante $U(\underline{g})$ de $\underline{g}$. On définit une représentation σ de $\underline{g}$ dans $\mathrm{End}_k V$ par

$$\sigma(x).f = \rho(x) \circ f - f \circ \rho(x) , \quad x \in \underline{g} , \quad f \in \mathrm{End}_k V .$$

On montre que les conditions suivantes sont equivalentes :

(i) $f \in \rho(U(\underline{g}))$.

(ii) Pour tout $x \in \underline{g}$, il existe $n \in \mathbb{N}$ tel que $\sigma(x)^n.f = 0$.

La démonstration se fait en deux étapes. On prouve d'abord le résultat pour une algèbre de Weyl ou une algèbre de Heisenberg. On passe ensuite au cas général en utilisant le fait que $U(\underline{g})/I$ est une algèbre de Weyl.

M. VERGNE : Orbites de la représentation coadjointe

Soit $\underline{g}$ une algèbre de Lie de dimension finie sur un corps k algébriquement clos de caractéristique zero . Il est conjecturé que si $\underline{g}$ est unimodulaire, les orbites $\Gamma . f$ d'un point générique f de $\underline{g}^*$ sous le groupe adjoint algébrique sont fermées. Les résultats suivants sont obtenus (Dixmier - Duflo - Vergne) :

Soit $\Gamma_0 = \bigcap \operatorname{Ker} \chi$ (χ caractères rationnels de Γ)

et soit $\Pi(\underline{g}) = \{\chi \mid \chi$ caractère rationnel de G , tels qu'il existe une fonction rationnelle $R \neq 0$ sur $\underline{g}^*$ avec $\gamma . R = \chi(\gamma) R\}$.

Alors : a) quelque soit $\underline{g}$, $\det \gamma \in \Pi(\underline{g})$

b) il existe un ouvert U de Zariski tel que $\Gamma_1 = \Gamma(f)\Gamma_0$ est indépendant de f (où $\Gamma(f) = \{\gamma \in \Gamma \mid \gamma f = f\}$)

c) $\Pi(\underline{g}) = \{\chi$ tel que $\chi | \Gamma_1 = \mathrm{id}\}$

d) si $\Gamma_1 = \Gamma$, alors l'orbite $\Gamma . f$ pour f générique est fermée

e) si $\underline{g}'$ est un idéal de $\underline{g}$, tel que $\underline{g}/\underline{g}'$ soit nilpotent, alors $\Pi(\underline{g}) = \Pi(\underline{g}')$

N.R. WALLACH : On a complex analogue of a p-adic result of Casselman

Let G be a semisimple Lie group . Fix P_0 a minimal parabolic subgroup of G . Let (π, V) be an irreducible finite dimensional representation of G . To each P a parabolic $P \supset P_0$ we can associate in a canonical way a representation $I_{P,V}$ of G so that if $P_1 \supset P_2$, $I_{P_2,V} \supset I_{P_1,V}$. We set

$$\pi_V = I_{V,P_0} / \sum_{P \supsetneq P_0} I_{V,P} .$$

Theorem : If G is complex and if (π, V) is equivalent to its complex conjugate dual representation then π_V is infinitessimally equivalent with a unitary principal series representation.

Theorem : If $G = SL(n, \mathbb{R})$ and (π, V) is self dual then π_V is unitarizable with commutant of dimension at least $2^{[n/2]}$.

These representations are related to the cohomology of discrete uniform subgroups.

R. Rentschler (Orsay)

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