MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 7/1975

Kombinatorik

9.2. bis 15.2.1975

Die diesjährige Tagung über Kombinatorik stand unter der Leitung von D. Foata (Strasbourg) und A. Kerber (Aachen). Es war das Ziel, Mathematiker zusammenzubringen, die auf den folgenden Spezialgebieten arbeiten:

- Darstellungstheorie symmetrischer Gruppen,
- symmetrische Funktionen,
- Partitionen,
- Abzählungstheorie.

Neben Vorträgen über diese Spezialgebiete sollten insbesondere die natürlichen Verbindungen zwischen der Darstellungstheorie der symmetrischen Gruppen und gewissen Problemen in der Theorie der Partitionen und der Abzählungstheorie im Vordergrund stehen.

Dieses Ziel wurde erreicht, zudem ergaben sich interessante Ausblicke auf Anwendungen u.a. in algebraischer Geometrie, Geometrie und Invariantentheorie, modularer Darstellungstheorie symmetrischer Gruppen.

Die Diskussionen verliefen sehr anregend und in angenehmer Atmosphäre. Dazu trug auch die ausgezeichnete Organisation bei, für die Tagungsleiter und -teilnehmer Herrn Prof. Dr. M. Barner, dem Direktor des Instituts, und seinen Mitarbeitern danken. Ein übriges taten das frühlingshafte Wetter und die hervorragenden Darbietungen klassischer Klaviermusik (zu zwei und vier Händen), mit denen uns Frau Comtet und Herr Boerner begeisterten.

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Vortragsauszüge

J. André: Neuere Ergebnisse aus der kombinatorischen Geometrie

Bericht über eine Vorlesung über kombinatorische Geometrie, die ich im Wintersemester 1974/75 an der Universität Saarbrücken hielt. (Vorlesungsausarbeitung ist in Vorbereitung.)

Insbesondere zwei Aspekte wurden hervorgehoben: (1) Eingrenzung des Gebiets "Kombinatorische Geometrie". (2) Genauerer Bericht über das letzte Kapitel der Vorlesung, das sich mit (speziell nichtkommutativen) Verbindungsräumen beschäftigte.

G.E. Andrews: Recent results in the theory of partitions

This talk discussed some of the recently discovered partition identities (including the analytic generalization of the Rogers-Ramanujan identities which provides a simple expansion for $\Pi(1-q^n)^{-1}$ where n runs over those positive integers $\not\equiv 0, \pm i$ (mod 2k+1)). Also some of the possibilities for further progress in the subject were broached.

Particular emphasis was given to the Rogers-Ramanujan identities, Gordon's generalization, and Schur's theorem. These results motivate the concept of "linked partition ideal" which seems to be related to a majority of the partition identities of interest. The concept of partition ideal was introduced in Advances in Math. 9 (1972), 10-51, and linked partition ideals were treated in Bull. Amer. Math. Soc. 80 (1974), 1033-1052.

H. Brown: Automated chemical structure elucidation: An application of combinatorial computing

During the past several years, a package of computer programs designed to aid the analytic chemist in determining the topological structure of an unknown organic compound has been under development by the DENDRAL poject at Stanford University. At the core of this package is a program which accepts as input a list of atoms, a "good list" of required substructural fragments and features and a "bad list" of excluded substructural fragments and features and which outputs all distinct, connected multigraphs based on the atom list and satisfying the "good list" and "bad list" conditions. This program is based on a blending of combinatorics, group theory and computing techniques.

The chemical origins and some of the mathematical aspects of this program will be described. Also, a number of combinatorial problems which arise in chemical structure elucidation questions and for which, at best, only partial solutions are known will be presented.

L. Comtet: Trigonometric numbers

Let us call "cosine" numbers the numbers v(n,k) generated by

$$\sum_{n\geq k} v(n,k)t^n = \frac{t^k}{(1-t)(1-2^2t)\dots(1-k^2t)} .$$

They are often called "central divided differences of zero" but the reason of my terminology will appear easily during the talk. I intend to give some (perhaps?) new results on these numbers, essentially related to their combinatorial significance and their global behaviour.

H.K. Farahat: Combinatorics and the modular representations of the symmetric groups

The non-modular representation theory of the symmetric groups admits of an elegant and complete analysis by means of the combinatorial notions of partition and diagram. The purpose of this lecture is to survey the present state of affairs in the attempt to provide a similar analysis of the modular theory. The main emphasis will be on the so-called natural representation modules, their homomorphisms and their relation to the Specht modules. Some purely combinatorial problems will arise from these considerations.

D. Foata: A symmetry property of the Genocchi numbers

The Genocchi numbers (G_{2n}), $n \geq 0$, may be defined by means of their generating function:

$$2u/(e^u + 1) = u + \sum_{n\geq 1} (u^{2n}/(2n)!)(-1)^n G_{2n}.$$

The recurrence relation $F_1(x,y,z) = 1$, and for $n \geq 2$:

$$F_n(x,y,z) = (x+y)(y+z)F_{n-1}(x,y,z+1) - z^2F_{n-1}(x,y,z)$$

provides with a refinement of the Genocchi numbers

$$F_n(1,1,1) = G_{2n+2}, \quad n > 0.$$

Moreover, the polynomial $F_n(x,y,z)$ is symmetric with respect to the set of the three variables x,y,z. Finally, a combinatorial interpretation for the polynomials $F_n(x,y,z)$ in terms of "exceedant surjections" of the interval $[1,2n]$ onto the subset of even integers $\{2,4,6,\dots,2n\}$.

F. Gaeta: <u>Geometrical interpretation of invariant theory in terms of classical algebraic varieties (Segre, Veronese, Grassmannians)</u>

A minimal projective embedding of the set $F(n_1,\dots,n_k;n)$ of flags of length k: $\mathbb{P}_{n_1} \supset \mathbb{P}_{n_2} \supset \dots \supset \mathbb{P}_{n_k}$ in $\mathbb{P}_n$ furnishes an irreducible representation attached to a column partition $n_1+1, n_2+1, \dots, n_k+1$. For k = 1 we obtain the usual embedding of a Grassmannian and then by the Segre product:

$\prod_{j=1}^{k} \mathbb{P}(V_j) = \mathbb{P}(\bigotimes_{j=1}^{k} V_j)$ applied to the ambient spaces $\bigwedge^{n_j+1} V_{n+1}$ of $G(n_j;n)$, we obtain the embedding of $F(n_1,\dots,n_k;n)$ as a subvariety. The operators Q,P of Young appear naturally (alternation by columns, symmetrization by rows).

The Littlewood theory can be paraphrased intrinsically without coordinates or indices. Every Littlewood operation has a projective interpretation in terms of Grassmannians $[1^{k+1}]$, Veronese varieties $[k]$ or arbitrary flag manifolds.

A separate collection of examples was given, related to the previous constructions and to the others arising in Schubert calculus.

L. Geissinger: <u>Permutation representations of the hyperoctrahedral groups</u>

We define a set of permutation characters for the hyperoctrahedral groups B_n analogous to the characters of Young subgroups of the symmetric groups. We compute their inner products with the irreducible characters, prove they form an integral basis for the representation ring, and derive a similar result for the D_n Coxeter (Weyl) groups. We show that certain inner products have combinatorial significance (weights for Z_2-sets, chains of partitions, etc.). We define an order on these permutation characters, which is the analogue of dominance for characters of S_n, and describe its structure, including two natural striations of it into sublattices. Finally we show how to make the correspondence between characters and polynomials symmetric in two sets of variables.

K.U. Gutschke: Einige Schranken zu einem Extremalproblem von P. Turân

Sei N eine endliche Menge ($|N| = n$) und sei $[N]^k := \{Y \subseteq N: |Y| = k\}$. Ein k-Graph G ist eine Teilmenge von $[N]^k$, seine Elemente werden k-Kanten genannt. Wir bezeichnen mit T(k,l,n) die Maximalzahl von k-Kanten, die ein k-Graph besitzen kann, ohne einen vollständigen Untergraphen auf l Punkten zu enthalten. Der Satz von Turân (1941) bestimmt diese Zahlen für k = 2.

Durch Definition geeigneter Strukturparameter eines k.Graphen und Herleitung von Gleichungen und Ungleichungen zwischen diesen Parametern werden folgende Schranken gewonnen:

$$T(k,l,n) \leq \binom{n}{k}\left[1 - \frac{n-l+1}{n-k+1}\binom{l-1}{k-1}^{-1}\right]$$

und

$$\lim_{n\to\infty} T(3,4,n)\binom{n}{3}^{-1} < 0{,}5952,$$

d.h. eine Menge von Dreiecken mit n gemeinsamen Eckpunkten, in der mindestens 59,52 % aller $\binom{n}{3}$ möglichen Dreiecke vertreten sind, enthält für hinreichend großes n ein Tetraeder.

Durch eine Variation des Ansatzes wird mit der gleichen Methode eine verbesserte obere Schranke für eine Ramsey-Zahl bewiesen: $R_3(4,4) \leq 15$.

H.-R. Halder: On the existance of a certain kind of (k,n)-arcs in finite projective planes.

A subset of a finite projective plane PG(2,q) is called an arc of type (m,s), if every line of the plane has exactly 0 or m or s common points with K and if the number of lines through x intersecting K in exactly m points is m for all $x \in K$. Only few such arcs are known. We constuct a series of new arcs in cyclic planes.

W. Hamernik: On the radical of the group ring over the symmetric group S_p.

Let p be an arbitrary (but fixed) prime and let F be any field of characteristic p . Denote by S_p the symmetric group on p letters. The principal block of S_p is the only non-simple one and contains p ordinary irreducible and p-1 Brauer characters. Using J.A. Green's result on the projective resolution of the trivial S_p-module one obtains by means of a detailed analysis of the lattice of submodules of the mutually non-isomorphic projective indecomposible FS_p-modules in terms of the Specht modules in characteristic p an explicit expression for an F-basis of a "multiplicity-free" part of the (Jacobson-) radical of the group ring FS_p.

W.R.Heise: On sharply k-ply transitive sets of permutations.

Let Γ be a sharply k-ply transitive set of permutations containing the identity and acting on a n-set . If $k = n - 2 \geq 3$ then $\Gamma \cong A_n$. If $k = 3$, $n \equiv 1 \bmod 2$, then $\Gamma \cong LF(2,n)$ and $n = 2^m$, $m \in \mathbb{N}$. There is no sharply quadruply transitive set of permutations of degree 10.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

C. Hoede: Eine generierende Funktion für einige Klassen von Graphen, die durch ein generatives Prinzip charakterisiert sind

Unter den Euler-Kreisen für den vollständigen Graphen mit 2n+1 Ecken gibt es einen, den man kanonisch nennen könnte, weil sämtliche Ecken in äquivalenter Weise n-mal in dem Kreis vorkommen. Dieser Kreis induziert eine Ordnung der Kennzahlen der Vektoren, die man üblicherweise den partiellen Graphen zuordnet, welche so ist, dass die Basisvektoren einiger Vektorräume von Graphen ein besonderes Regelmass aufweisen. Die Bildung von Elementen eines solchen Vektorraumes könnte man als generatives Prinzip bezeichnen. Man kann leicht eine zugehörige generierende Funktion in der Form einer Spur eines Produktes von Matrizen konstruieren, die das vorher genannte Regelmass ebenfalls enthält. Die resultierende Problemstellung ist die Berechnung einer solchen Spur. Diese ist für einige einfache Fälle schon möglich und für den allgemeinen Fall als Problem in der Darstellungstheorie zu interpretieren.

G.D. James: Representations of the Symmetric Groups over Finite Fields

It is not known how to find the decomposition matrices of the symmetric groups for the prime 2. The talk demonstrated how to construct all the irreducible representations of the symmetric groups over an arbitrary field, and showed how to work out part of the decomposition matrix for p=2 for all symmetric groups. The methods given can be used to find the 2-modular decomposition matrices of S_n for $n \leq 10$.

A. Kerber: Applications of representation theory to Redfield-Pólya-de Bruijn type enumeration theory

The Pólya problem (i.e. enumeration under E^H) was discussed, especially the methods used for solutions of the following cases: Number of classes, number of classes of given type, construction of representatives. For examples the permutation characters of the composition, the exponentiation and of two

kinds of matrix groups of two permutation groups of finite degree were given which yield the corresponding cycle-indices by Moebius-inversion.
After that the de Bruijn-type and the Redfield-type problems were mentioned als generalizations.

R.C. King: Symmetric functions, Young tableaux and their connection with representations of the classical groups

The use of partitions and their associated Young diagrams in the classification of irreducible representations of both the symmetric groups and the classical (unitary, orthogonal and symplectic) groups is discussed. The specification of the corresponding representation module by means of Young tableaux is given and some associated enumeration problems are mentioned. The connection with various symmetric functions including Schur functions are given and a number of group operations defined in terms of Schur functions are given a combinatorial interpretation.

M. Klemm: Charaktere der symmetrischen und mehrfach transitiven Gruppen

Das folgende Resultat wird besprochen:
G sei eine k-fach transitive Permutationsgruppe auf Ω vom Grad $n \geq 2(k+1)$, r, s und i seien ganze Zahlen mit $r \geq 0$, $1 \leq i \leq s-1$ und $r+s = k+1$. Genau dann ist der Mengenstabilisator jeder $(k+1)$-elementigen Teilmenge Σ von Ω auf Σ $(r+i)$-fach transitiv, wenn $(\chi_\lambda, \chi_\mu)_G = (\chi_\lambda, \chi_\mu)_S$ für alle irreduziblen Charaktere χ_λ, χ_μ der symmetrischen Gruppe S_n mit dim $\lambda=r$, dim $\mu=s$, $\mu=(\mu_1,\mu_2,\ldots) \neq (n-s,1)$ und $\mu_2 \geq s-i$ gilt.

R. Knörr: Uniserial blocks of the S_n

Let F be a field with charF = p, G a finite group. A block B of the group algebra FG is called uniserial, if all of its indecomposable projective modules have exactly one composition series. The question, for which p and n the group algebra FS_n of the symmetric group S_n over F contains an uniserial block

is answered by the result:
FS_n contains a non-simple uniserial p-block, if and only if either $p = 2$ and $n \in 2 + \{\frac{1}{2}(i+1)i \mid i = 0,1, ..\}$ or $p = 3$ and $n \in 3 + \{(k+i)(k+i+1)-ki \mid k,i = 0,1, ..\}$.
The proof uses the fact that a non-simple block has a cyclic defect group only if it is of weight 1; then the decomposition numbers of B are known to be

$$D = \begin{pmatrix} 1 & 0 \\ 1 & \ddots \\ & \ddots & 1 \\ 0 & & 1 \end{pmatrix} \quad \text{(p rows, p-1 columns)} .$$

Since the Cartan-matrix of an uniserial block is of the form

$$C = \begin{pmatrix} r+1 & & \\ r & \ddots & r \\ & & r+1 \end{pmatrix}$$

for some $r \in \mathbb{N}$, the relation ${}^tDD = C$ implies $p = 2$ or $p = 3$.

A. Lascoux: <u>Applications des fonctions de Schur à la geometrie algebrique</u>

Soit K un λ-anneau, i.e. muni d'une famille d'endomorphismes possédant certaines propriétés de linearité. Alors pour toute partition I, on définit les endomorphismes de Schur t_I dans K (les λ^i correspondant aux partitions colonnes). Le cas le plus simple se présente avec les variétés "grassmanniennes": leur anneau de Grothendieck des fibrés vectoriels a pour base, comme module libre, les $t_I(Q)$. Q étant le fibré "tautologique", pour toutes les partitions contenues dans un rectangle fixe. De même, l'anneau de Chow des sous-variétés algébriques d'une grassmannienne admet une base de sous-variétés de "Schubert", indexées par les mêmes partitions, dont les propriétés (intersection, degré, postulation) sont liées respectivement à la multiplication des fonctions de Schur, au nombre de tableaux standards et de partitions planes de forme donnée.

W. Lehmann: <u>Representation theory and enumeration under group action</u>

Let $D: A \to Aut_K(V)$ be a reducible representation of the finite group A over a field K : $D = \dot{+} D_i$, $V = \oplus V_i$, $D_i = D|V_i$ not neccesary irreducible. W: $\{V_1, .. ,V_r\} \to K$ is an arbitrary function. Define the weighted symmetry operator

$$T_D^W := |A|^{-1} \sum_{a\in A} D_W(a)$$

with $D_W(a) := \dotplus W(V_i) D_i(a)$.

Theorem: $\sum_i W(V_i)s_i$ = trace T_D^W, where s_i is the multiplicity of the 1-representation in D_i.

This theorem is the representation theoretical background of the lemmata of Redfield and Burnside and of results of N.G. de Bruijn and S.G. Williamson in the theory of enumeration under group action.

E.K. Lloyd: Pólya-de Bruijn Type Enumeration

If a group G acts on a finite set D then there is a natural action of G on a set of mappings E^D. Pólya's theorem concerns the number of orbits of E^D/G. De Bruijn generalized the problem by allowing a group H to act on E as well, and sought the number of orbits of $E^D/G\times H$. In addition, he considered the set $E^{(D)}$ of one-one mappings. This leads to the problem: what other subsets of E^D can be dealt with? The problem may be formulated in the language of graph theory and solved in some simple cases. Some problems in chemical enumeration studied by Balaban may be solved by these methods.

A.O. Morris: Combinatorial Aspects of Projective Representations of Finite Groups

If $\lambda = (\lambda_1,\ldots,\lambda_m)$ is a partition of n such that $\lambda_1>\lambda_2>\ldots>\lambda_m>0$, the shifting diagram corresponding to λ has λ_1 nodes in the 1st row, λ_2 nodes in the 2nd row,... but the 1st node of the (i+1)th row is placed under the 2nd node of the ith row ($i\geq 1$). The degree of an irreducible projective character of S_n corresponding to λ can be given in terms of (i) standard Young tableau (Y.T.) (ii) hook lengths associated with the above shifting diagram. If $Q_\lambda(-1)$ is the "Schur" function and k_μ, where $\mu\vdash n$, are monomial symmetric functions, then $Q_\lambda(-1) = \sum_{\mu\vdash n} L_{\lambda\mu}k_\mu$ and $L_{\lambda\mu}$ is obtained as follows: A "generalized" Y.T. of shape λ is obtained by replacing the nodes of the shifting diagram λ by positive integers such that (1) the numbers in each row must be in non-decreasing order from left to right (2) the numbers in each column must be in non-decreasing order from top to bottom (3) the numbers in each diagonal from left to right must be in strictly increasing order. Consrtuct all the "generalized" Y.T. of shape λ that can be formed from 1's, 2's, ... Then, for each such tableaux D of shape λ put $L^D_{\lambda\mu} = 2^k$, where k = # of continuous bands of integers in this construction. Then $L_{\lambda\mu} = \sum_D L^D_{\lambda\mu}$, where the sum is over all generalized Y.T. of shape λ which can be formed as above.

P.A. Morris: Combinatorics and Algorithms for Plethysm

It is well known that some enumeration problems concerning finite graphs can be cast in terms of S-functions operations. Here this way is gone backward: S-function theorems, mainly concerning the plethysm of S-functions, are obtained by discussing relations among certain types of graphs, especially bipartite graphs. Emphasis is laid on deriving theorems and recursion formulae which are suited to computer programming.

H. Pahlings: Darstellungsgrade von Kranzprodukten

For a finite group G let $m_p(G)$ denote the number of in-equivalent irreducible $\mathbb{C}$-representations of G with dimension coprime to p . We compute $m_p(G \wr S_n)$, where $G \wr S_n$ is the wreath product of G with the symmetric group S_n . If $n = a_o + a_1 p + \ldots + a_t p^t$ $(0 \leq a_i < p)$ and if

$P(x) = \prod_{i=1}^{\infty} (1-x^i)^{-1}$, then $m_p(G \wr S_n) = \prod_{i=1}^{t} [P(x)^{rp^i}]_{x^{a_i}}$, where

$[Q(x)]_{x^j}$ is the coefficient of x^j in $Q(x)$ and $r = m_p(G)$.

For $|G| \leq 2$ this is due to Macdonald. For p = 2 one gets as a special case: If $n = 2^{k_1} + \ldots + 2^{k_s}$ $(k_1 < \ldots < k_s)$ then $m_2(G \wr S_n) = 2^{k_1 + \ldots + k_s} (m_2(G))^s$.

M. H. Peel: Specht Modules

Following the approaches of W. Specht and H. Garnir, we construct a collection of representation modules (called Specht modules) of the symmetric group S_n over an integral domain K. Each tableau of a given partition of n determines a polynomial, and the corresponding module is generated over K by these polynomials. The standard tableaux determine a K-basis for the module. The annihilator ideals of the polynomials are determined in terms of certain relations between the polynomials obtained by Garnir. Over a field of characteristic zero, the modules constitute a full set of inequivalent, absolutely irreducible representation modules of S_n. Over a field of characteristic $p \neq 0$, composition series of the modules determine decomposition numbers of S_n.
The modules corresponding to partitios $(n-r,1^r)$ can be analysed for characteristic $p \neq 2$. The modules corresponding to partitions $(n-r,r)$ can be analysed for characteristic $p > r$ using general results on the decomposition numbers of S_n.

G. de B. Robinson: Representations of representation of a finite group

The significance of representation theory for modern theoretical physics led the Author to consider the algebras of representations and classes of a finite group G with the result that a representation theory of representation of G emerged[1] which led naturally to a reinterpretation of Frobenius' Reciprocity Theorem[2]. In this form the Theorem was interpretable also for the classes of G.
In the present paper the processes of restricting and inducing on representation of G and a subgroup $\hat{G}$ are reexamined and their inverses defined. These inverses are easily expressible for the symmetric group S_n in terms of Young's raising operators and the condition for their existence in the general case is given[3].

1. Tensor product Representation, J. Algebra 20 (1972), 118-123
2. The dual of Frobenius' Reciprocity Theorem, Can. J. Math. 25 (1973), 1051-59
3. Restricting and inducing on inner products of representation of finite group, (to appear in Can. J. Math.).

M. Schützenberger: Une q-serie associeè aux permutationes alternantes

Une permutation $\sigma:[n]\longrightarrow[n]$ est dite alternante ssi l'on a $(2i-1)\sigma > (2i)\sigma < (2i+1)\sigma$ pour tout $i \leq {}^{n}/_{2}$. L'ênumêration de ces objects est fournie par les fonctions tg t et $(\cos t)^{-1}$, d'aprês un resultat classique. On montre que l'ênumêration de nombre des inversions sur les permutations alternantes est fournie par la q-gênêralisation des fonctions prêcêdentes.

J. Tappe: Ein neuer Beweis der Vermutung von Nakayama

Es wird folgender Satz bewiesen: Zwei gewöhnliche irreduzible Charaktere der symmetrischen Gruppe S_n gehören genau dann zu demselben p-Block, wenn die zugehörigen Partitionen der Zahl n denselben p-Kern haben.
Der Beweis wird mit Hilfe des zweiten Hauptsatzes von R. Brauer, eines Satzes von M. Osima und der Rekursionsformel von Murnaghan und Nakayama geführt.

G. P. Thomas: The constructions of Schensted, Schützenberger and Robinson

The first topic of this talk will be concerned with Schensted's construction. Some new results will be given and a generalization of Schensted's construction will also be described. From these results , a combinatorial proof of the Littlewood-Richardson rule for the multiplication of Schur functions will be outlined.

Part two of the talk will be concerned with a construction first described by Schützenberger in the "Permutations" colloquium in Paris in 1972. This construction will be shown to be related to Schensted's construction and a proof of the uniqueness of Schützenberger's construction will be given.

Part three will describe a generalization of a construction first used by Robinson in his proof of the Littlewood-Richardson rule. Connections between this construction and the previous constructions will be given.

D.Wille: Zur Abzählung selbstkomplementärer Strukturen

Gegeben seien endliche Mengen D und $R_k := \{0,1,\ldots,k\}$. Auf R_k^D wird vermöge einer auf D operierenden Permutationsgruppe G eine Äquivalenzrelation $\sim$ definiert durch
$f_1 \sim f_2 :\Leftrightarrow \exists \pi \in G \quad \forall d \in D \quad f_2(d) = f_1(\pi^{-1}(d))$. Zu $f \in R_k^D$ heißt $\overline{f}$ mit $\overline{f}(d) := k - f(d)$ die zu f komplementäre Funktion. f heißt selbstkomplementär, falls $f \sim \overline{f}$.
Die Anzahl der nicht-äquivalenten (nicht-isomorphen) Funktionen aus R_k^D erhält man mittels der Sätze von Pólya und de Bruijn, indem man im Zykelindex von G für Variablen mit geradem Index k+1 substituiert und für Variablen mit ungeradem Index 0, falls k ungerade ist, und 1, falls k gerade ist.
Durch Spezialisierung von D und G ergeben sich daraus Formeln für die Anzahlen $g_k(n)$ der nicht-isomorphen selbstkomplementären Graphen über n Punkten, bei denen bis zu k Kanten zwischen je zwei Punkten erlaubt sind, sowie Formeln für die Anzahlen $r_m(n)$ der nicht-isomorphen selbstkomplementären m-stelligen Relationen über n Elementen. Durch Vergleich der Formeln erhält man das überraschende Ergebnis $r_2(2n) = g_1(4n+1)$, ähnlich der von R.C.Read gefundenen Korrespondenz $d(2n) = g_1(4n)$. ($d(2n) :=$ Anzahl der nicht-isomorphen selbstkomplementären Digraphen über 2n Punkten).
Die noch recht unhandlichen Formeln lassen sich zur Gewinnung beliebig genauer Approximationen auswerten. So erhält man z.B. in erster Näherung

$$r_m(2n) \sim \frac{2^{\frac{1}{2}(2n)^m}}{2^n \cdot n!}$$

A. Kerber (Aachen)