MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 11/1975

Erweiterte Fassung

Mathematische Stochastik

9.3. bis 15.3.1975

Ziel der Tagung über Mathematische Stochastik (Leitung: K. Krickeberg) war es, sich gegenseitig anhand von Übersichtsvorträgen über neuere Entwicklungen in der Vereinigung der Gebiete der Wahrscheinlichkeitstheorie und der Statistik zu orientieren und so das Gespräch zwischen beiden Disziplinen weiter zu fördern. Darüberhinaus wurde über eine Reihe interessanter Einzelergebnisse berichtet.

Tagungsteilnehmer

Ahlswede, R., Göttingen
Bartoszynski, R., Warschau
Bauer, H., Erlangen
Baumann, V., Bochum
Bierlein, D., Regensburg
Dacunha-Castelle, Orsay
Davies, P.L., Münster
Drygas, H., Frankfurt
Fieger, W., Karlsruhe
Gänßler, P., Bochum
Galambos, J., Frankfurt
Gasser, Th., Zürich
Georgii, H.-O., Heidelberg
Hafner, Dortmund
Hampel, F., Zürich
Helwig, M., Frankfurt
Höglund, Th., Stockholm
Huber, C., Paris
Huber, P.J., Zürich
Janßen, K., Erlangen
Katona, G., Budapest
Kendall, D.G., Cambridge
Kersting, G., Göttingen
Komlos, J., Budapest

Krengel, U., Göttingen

Krickeberg, K., Paris

Lakeit, H.-J., München

Mandl, P., Prag

Morgenstern, D., Hannover

Müller, D.W., Heidelberg

Mürmann, M., Heidelberg

Nguyen X., L., Erlangen

Nguyen X., X., Bielefeld

Plachky, D., Münster

Richter, H., München

Ripley, B.O., Cambridge

Runnenburg, J.Th., Muiderberg

Schäl, M., Bonn

Schmitz, N., Münster

Schürger, K., Heidelberg

Schuh, H.-J., Erlangen

Sen, P.K., Freiburg

Siegmund, D., Zürich

Smith, W.L., Chapel Hill

Streit, F., Bern

Tautu, P., Heidelberg

v. Weizsäcker, H., München

Ziezold, H., Bielefeld

van Zwet, W.R., Leiden

Vortragsauszüge

AHLSWEDE, R.: Bounds on conditional probabilities with applications in multi-user communication.

We are given finite sets $\mathcal{X}, \mathcal{Y}$ and the transition probabilities $w(y|x)$ for $x \in \mathcal{X}$, $y \in \mathcal{Y}$. For the n-th cartesian power of $\mathcal{X}$ and $\mathcal{Y}$ we define $w^n(\underline{y}|\underline{x}) = \prod_{i=1}^{n} w(y_i|x_i)$, where $\underline{x} = x_1 \ldots x_n$ and $\underline{y} = y_1 \ldots y_n$. The set $B \subset \mathcal{Y}^n$ ε-decodes $\underline{x} \in \mathcal{X}^n$ if $w^n(B|\underline{x}) \geq 1 - \varepsilon$. We set $\psi_\varepsilon(B) \subset \mathcal{X}^n$ for the set of all the $\underline{x}$'s which are ε-decoded by B. We derive asymptotically optimal bounds on the maximum 'size' of $\psi_\varepsilon(B)$ if the 'size' of B is given. The result has far reaching applications in Information Theory. The work was done jointly with P. Gács and J. Körner.

KATONA, G.: Most stable sets in a Hamming space.

A Hamming space is simply the set of 0,1 sequences of length n with the Hamming metric $d(x,y)$ (= the number of different digits in x and y). Let A be a subset in H_n, choose one element of A randomly with uniform distribution. Suppose every digit can change its value with probability p. What is then the probability that the randomly chosen element leaves A? It is

$$\frac{1}{|A|} \sum_{x \in A} \sum_{y \in \bar{A}} p^{d(x,y)} (1-p)^{n-d(x,y)} .$$

We want to minimize this expression, if n,p and $|A|$ are fixed. Our expression can be written in the form $\sum_i k_i f(i)$, where k_i is the number of pairs (x,y) with $x \in A, y \in \bar{A}$ and $d(x,y) = i$. First we minimize the numbers k_i. The statement that we proved with R. Ahlswede is that the optimal A consists of the first $|A|$ sequences in the lexicographic order. As this is independent of i, this gives the optimal solution for $\sum k_i f(i)$ if $f(i)$ is nonnegative The number k_1 can be considered as the number of neighbouring elements of A with multiplicity. Thus, let g_1 be the number of element y of $\bar{A}$ for which there exists an element x of A with $d(x,y) = 1$. Supposing again that $|A|$ is fixed, g_1 will be minimum for a sphere in the Hamming space (all sequences with n,n-1,...,k one and some of the lexicographically first sequences with k-1 ones, for some k). So, the optimal systems for these two different but similar problems are completely different.

HÖGLUND, TH.: A refined saddle point approximation.

Let f^{n*} stand for the n:th convolution of a function $f: Z \to [0,\infty)$ of finite support. We give an approximation of $f^{n*}(x)$ for large n which is sharp for all $x \in Z$.

KERSTING, G.: Approximation of the Robbins-Monroe-Process by independent random variables.

Let (X_n) be a Robbins-Monroe-Process : $X_{n+1} = X_n - \frac{c}{n} Y_n$ belonging to some regression function $M(x)$ and converging to the solution θ of the equation $M(x) = 0$. Then under wide assumptions the following is true: If $\alpha = M'(\theta) > 0$ exists and $\alpha c > \frac{1}{2}$, then there is an $\varepsilon > 0$ such that

$$\sqrt{n}\,(X_n - \theta) + \frac{c}{\sqrt{n}} \sum_{k=1}^{n} \left(\frac{k}{n}\right)^A V_K = O(n^{-\varepsilon})$$

almost surely on a certain probability space, where V_K are i.i.d.r.v. and $A = \alpha c - 1$. A law of iterated logarithm and a weak invariance principle follow from this approximation.

SMITH, W.L.: Density versions of the central limit theorem.

Let $\{X_n\}$ be an infinite sequence of independent and identically distributed random variables such that $\mathcal{E}X_n = 0$ and $\mathcal{E}X_n^2 = 1$, and write $Z_n = (X_1+\ldots+X_n)/\sqrt{n}$, $g(x) = \exp(-x^2/2)/\sqrt{2\pi}$. Under suitable conditions Z_n has a probability density function $f_n(x)$ and, in the years since 1952, many authors have proved theorems concerned with the convergence of $f_n(x)$ to $g(x)$, and especially with the rapidity of this convergence. After some historical discussion the following result was announced. Suppose, for $x \geq 0$, $M(x)\uparrow$ and $M(x)/x\downarrow$ and, for integer $k \geq 2$, $\mathcal{E}|X|^k M(|X|) < \infty$. There are polynomials $q_{3\nu}(x)$ such that, if $f_n(x)$ is bounded for some n, then for all larger n,

$$f_n(x) = \left\{1 + \sum_{\nu=1}^{k-2} \frac{q_{3\nu}(x)}{n^{\frac{1}{2}\nu}}\right\} g(x) + \frac{\varepsilon_n(x)}{\{1 + |x|^k M(|x|)\}\, n^{\frac{1}{2}k-2}}$$

where: (A) $n\varepsilon_n(x) \longrightarrow 0$ as $n \longrightarrow \infty$, uniformly in x;

(B) $\sum_1^\infty \sup_x |\varepsilon_n(x)| < \infty$.

However, if $M(x)$ grows too slowly or too quickly (this made precise in the talk) the conditions for (B) need slight change.

v.WEIZSÄCKER, H.: On the application of Choquet's theorem in mathematical stochastics.

Eine Version des Satzes von Choquet für nicht kompakte Mengen von straffen Maßen über beliebigen topologischen Räumen wurde angegeben Eine Reihe von Anwendungen auf konvexe Mengen von Prozessen und von randomisierten Entscheidungen wurde diskutiert.

SCHUH,H.-J.: Eine Bedingung für das Aussterben eines Verzweigungsprozesses mit absorbierender unterer Schranke

Das klassische Beispiel eines Verzweigungsprozesses, der Galton-Watson-Prozess mit diskreter Zeit (Interpretation: z.B. W-theoretisches Modell für das zahlenmäßige Wachstum einer Population), wird insoweit modifiziert, als zusätzlich eine (zeitlich variable) untere Schranke g für die Populationsgröße vorgegeben wird.
Ist die Anzahl der Individuen der n-ten Generation, die aus der (n-1)-ten Generation nach den Regeln des Galton-Watson-Prozesses entsteht (n N), nicht größer als die Schranke g(n), so gehen alle Individuen zugrunde, und die Populationsgröße aller nachfolgenden Generationen ist deshalb gliech Null. ("Nur hinreichend große Populationen überleben").
Es soll eine notwendige und hinreichende Bedingung dafür angegeben werden, dass dieser Prozess mit Wahrscheinlichkeit 1 ausstirbt, d.h. P(U_n = 0 für ein n aus N) = 1; dabei ist P das W-Gesetz von U_r.

BARTOSZYNSKI.R.: A scheme of subjective classification.

The lecture concerns conditions for the existence of schemes of subjective classifications based on sequences of 'standards', under the asumption that the choice probabilities vary among the subjects.

SIEGMUND,D.: Importance sampling in Monte Carlo estimation of small probabilities.

The Monte Carlo technique of 'importance sampling' is discussed with reference to estimating the probability of gambler's ruin and the tail probability of stochastically monotone Markov processes. Applications are given to sequential analysis and queuing theory. It is shown that from a Monte Carlo viewpoint Wald's method for approximating the gambler's ruin probability has a precise optimality property.

H.v.Weizsäcker (München)

MÜLLER,D.W.: Conditions for asymptotic normality of statistical experiments.

Let $\mathcal{E} = (\mathfrak{X},\mathfrak{A};\ P_\theta : \theta\in\Theta)$ be an experiment, $\Theta\subset R^k$ open such that $0\in\Theta$. Conditions on $\mathcal{E}$ and $\mathcal{E}_n = (\mathfrak{X}^n,\mathfrak{A}^n;\ P_{t/\sqrt{n}}: |t|\leq c)$ are investigated, in particular:

$$(R)\qquad P_c(A) \longrightarrow 0 \quad ==> \quad \limsup_{\theta\longrightarrow 0} \int_A X_\theta^2\, dP_o \longrightarrow 0,$$

where $X_\theta = \frac{1}{\theta}\left(\sqrt{\frac{dP_\theta}{dP_o}} - 1\right)$. (R) ist almost equivalent to saying that small events do not contain much information, i.e. disregarding the observation falling into small sets does not change the separation properties of $\mathcal{E}_n$ too much. (R) is related to the normality of the limit points of the distributions of the likelihood function. Moreover, if $\mathcal{E}_n$ has a limit $\mathcal{G}$ (w.r. to the distance of deficiency Δ), regularity conditions on $\mathcal{G}$ may force it to be Gaussian . For instance (R): $\mathcal{G}$ has to be Gaussian w.r. to its null distribution, as far as its finite dimensional subexperiments are concerned; if $\Theta = (-\infty,+\infty)$ and $\mathcal{G}$ is a linearly indexed exponential family, $\mathcal{G}$ is already Gaussian.

GALAMBOS,J.: Asymptotic distribution of extremes of order statistics.

The asymptotic distribution of extremes will be discussed with main emphasis on relaxing the assumption of independence of the observations. In particular, recent results of the speaker will be presented on the asymptotic theory of order statistics of multivariate observations (to appear in J. Amer. Stat. Ass.), for for independent samples with random sample size (to appear in Zeitschrift f. Wahrscheinlichkeitstheorie verw. Geb.) and for other general classes of dependent random variables which can be handled by different types of sieve arguments.

VAN ZWET,W.R.: Asymptotic expansions for linear combinations of order statistics.

A review is given of results obtained in the area of asymtotic expansions for the distributions of nonparametric test statistics and related estimators. Some recent results concerning linear combinations of order statistics are discussed.

SEN, P.K.: Statistical theory of the strength of a bundle of filaments.

The strength of a bundle of $n(\geq 1)$ parallel filaments is expressed as

$$B_n = \max\left\{ (n-r+1)X_{n,r} : 1 \leq r \leq n \right\},$$

where $X_{n,1} < \dots < X_{n,n}$ are the ordered random variables for the individual strength of the n filaments. Characterizing $Z_n = n^{-1}B_n = \sup\left\{ x(1-F_n(x)) \right\}$, where F_n is the empirical distribution, and using some asymptotic properties of the empirical process, limiting behavior of Z_n has been studied. Applications of these results to some sequential tests and estimates (for $\theta = \sup\{x(1-F(x))\}$) are also considered.

GEORGII, H.O.: An extension of de Finetti's theorem in the theory of Gibbs random fields.

We conceive the product space $\Omega = \{0,1\}^S$ as the space of configurations of particles on the d-dimensional integer lattice S. The particle number is affected by a numerical parameter φ and the interaction of distinct particles is described by a real function Φ on the finite subsets of S. These quantities determine the grand canonical Gibbs distributions in the finite subsets V of S. A shift invariant probability measure μ on Ω is called a Gibbs random field with respect to φ and Φ if the g.c.G.d. are versions of its conditional probabilities of configurations in V given the configuration in $S \setminus V$. Similarly, Φ defines the canonical G.d. in V with fixed particle number in V. We call μ a canonical GRF w.r. to Φ if for all finite V the c.G.d. in V are versions of its conditional probabilities of configurations in V given the particle number in V and the configuration in $S \setminus V$.

Theorem. μ is extremal in the set of all CGRFs w.r. to Φ if and only if it is extremal in the set of all GRFs w.r. to Φ and some φ. If $\Phi \equiv 0$ the theorem reduces to de Finetti's theorem on exchangeable 0-1 variables.

MÜRMANN,M.: Cluster Processes in Statistical Mechanics.

Let Φ be a stable potential with finite interaction radius R>0 in $\mathbb{R}^d$. A tuple $(x_1,\ldots,x_n)$ of points is called a (physical) cluster iff the graph obtained by combining all points with a distance $\leq$ R is connected. Let $\mathcal{C}$ be the set of all clusters and for $\Lambda \subset \mathbb{R}^d$ let $\Lambda_* \subset \mathcal{C}$ be the set of clusters with all points in Λ and $\Lambda^* \subset \mathcal{C}$ be the set of clusters with at least one point in Λ.
The grand canonical distribution of points in a Lebesgue measurable bounded set $\Lambda \subset \mathbb{R}^d$ leads to the distribution of the clusters in Λ_* given by

$$\frac{1}{\mathcal{Z}(\Lambda_*)} \frac{H(X_1,\ldots,X_k)}{k!} \, d\mu(X_1)\ldots d\mu(X_k)$$

with the measure

$$d\mu(X) = \frac{z^{|X|}}{|X|!} e^{-\beta V(X)} \, dX \qquad (z,\beta > 0;\ V \text{ physical energy})$$

on $\mathcal{C}$ and the indicator function H of non overlapping clusters. $\mathcal{Z}(\Lambda_*)$ corresponds to the partition function.
From a Gibbs measure on the space of all locally finite configurations in $\mathbb{R}^d$ one can derive a cluster process iff it has a.s. no infinite clusters. This is essentially true iff $\mu(\Lambda^*) < \infty$ which holds for sufficiently small z. In this case a Gibbs measure is equivalent to a cluster process with the property: for $A \subset \mathcal{C}$ with $\mu(A) < \infty$ the conditional distribution of the clusters in A given the clusters $(Y_i)_i$ in CA is given by

$$\frac{1}{\mathcal{Z}(A\setminus I(Y_i))} \frac{H(X_1,\ldots,X_k)}{k!} \mathbb{1}_{[CI(Y_i)]^k} (X_1,\ldots,X_k) d\mu(X_1)\ldots d\mu(X_k)$$

with the 'forbidden' set $I(Y_i) = \bigcup_i \{X : H(X,Y_i) = 0\}$. Existence and uniqueness theorems can be derived.

TAUTU,P.: A configuration process on a lattice.

A random finite system of multitype B-objects in interaction is a a system of living cells of different types $K = \{1,\ldots,k\}$ having a life cycle $S = \{1,\ldots,s\}$. These B-objects occupy the sites of an r-dimensional lattice $\mathbb{Z}^r$ $(r \geq 1)$ and interact with their nearest neighbours. Let us call configuration a mapping $c: Z \to K \times S$ where Z is a subset of $\mathbb{Z}^r$. Let C denote the set of all configurations with the σ-algebra $\mathcal{C}$ on C. A stochastic configuration process $\{\gamma_t\}_{t\geq 0}$ is a Markovian jump process where $\gamma_t = (c_t, \zeta_t, P_c)$.

Let ϑ_m denote the first moment at which the process $\{\gamma_t\}_{t\geq 0}$ occupies a site of $\mathbb{Z}^r$ at graph theoretical distance m (≥ 1) from the origin, and let N_t denote the number of B-objects on the lattice at time t. The following conjecture is made:

$$P(\ N_t \sim \alpha t^r) = 1, \alpha > 0$$

as $t \longrightarrow \infty$. In order to get a bound for α, a subadditivity condition

$$E(\vartheta_{m+n}) \leq E\vartheta_m + E\vartheta_n + g(m+n) \qquad m,n \geq 1$$

is assumed, where g(n) is a function not increasing too quickly for $n \longrightarrow \infty$. An upper bound of α is deduced, $\alpha \leq \alpha_r / A^r$, where α_r and A are both positive constants.

SCHÜRGER,K.: Some results about random graphs.

We consider random graphs $\Gamma_{m^*,N}$ being defined as follows: The set of vertices of $\Gamma_{m^*,N}$ is a disjoint union of $p \geq 2$ sets containing $m_1,\ldots,m_p$ elements, respectively. We assume $m_i \sim c_i w$ where $w = w(n) \longrightarrow \infty$ as $n \longrightarrow \infty$ and $c_i > 0$, $1 \leq i \leq p$. Put $m^* = (m_1,\ldots,m_p)$. Allowing only (undirected) edges connecting points of differents sets, we have the number of all possible edges equal to $\sum_{1\leq i<j\leq p} m_i m_j =: a(m^*)$.
$\Gamma_{m^*,N}$ has $N = N(w)$ edges where $1 \leq N(w) \leq a(m^*)$ and $N(w) \longrightarrow \infty$, $w \longrightarrow \infty$. The random mechanism is as follows: Choose N edges out of all $a(m^*)$ edges such that all choices have the same probability (the case p = 1 was considered by Erdös and Rényi, Palásti did some work in the case p = 2). The most interesting result possibly is the following. Assume (for the sake of simplicity) $c_1 = .. = c_p = c$ and put $N = \frac{pcw}{2}\log(pcw) + \omega_n w$ where $\{\omega_n\}_{n\geq 1}$ is a sequence of real numbers such that $\inf \omega_n > -\infty$. Denoting by $\mathcal{T}$ the set of all (in some sense 'trivial') graphs having at most one connected component with ≥ 2 elements. Then $\lim_{n\to} P\{\Gamma_{m^*,N} \in \mathcal{T}\} = 1$. (A similar phenomenon occurs in cases p = 1,2).

RIPLEY, B.D.: Point Processes and Stochastic Geometry.

The principle of stochastic geometry is to represent random collections of geometrical objects as point processes on homogeneous spaces. These spaces can naturally be given a σ-field and a class of subsets on which the point process is to be finite. We show how a point process may be defined on a set with only these structures, and that the same process can be constructed from the random set theories of Kendall and Matheron.

ZIEZOLD, H.: On expected figures and a strong law of large numbers for random elements in quasi-metric spaces.

M. Fréchet's definition of an expected element of a random element in a metric space is introduced and a strong law of large numbers is announced. As figures are considered equivalence classes of elements of $(R^N)^n$ with respect to a closed subgroup of the group of all motions in R^N and a subgroup of the group S_n of all permutations of $\{1,\dots,n\}$. The results will appear in the Transactions of the Seventh Prague Conference 1974.

NGUYEN XUAN, X.: r-dimensionale Ergodensätze für zufällige Maße.

Es seien $X = R^r$, $\mathcal{M}$ die Menge der positiven Radonschen Maße in X und T_x, $x \in X$, die durch die gewöhnlichen Translationen induzierten Transformationen von X.
Sei P ein stationäres zufälliges Maß in X, d.h. ein W-Maß auf $\mathcal{M}$. Unter der Bedingung

(1) $$E_P(\Pi_{F_0}^p) < +\infty \quad \text{für ein } p \geq 1,$$

wobei $F_0 := \{(x_1,\dots,x_r) : -\frac{1}{2} \leq x_i \leq \frac{1}{2} \quad i = 1,\dots,r\}$ und $\Pi_G(\mu) := \mu(G)$ ist, existiert für jede meßbare, beschränkte, reellwertige Funktion g auf $\mathcal{M}$ der Limes

(2) $$Y(\ ) := \lim_{G \to \infty} \frac{1}{\lambda(G)} \int_G g(T_x\mu)\ \mu(dx)$$

sowohl P-f.s. als auch in $L_p(P)$, wenn das Gebiet G in geeigneter Weise unendlich groß wird. λ sei dabei das Lebesguesche Maß in X. Für p = 1 gilt die Konvergenz (2) ohne die Bedingung (1) für alle g aus $L_1(P^o)$, wobei P^o das zu P gehörige Palmsche Maß ist. Einige Anwendungen wurden angegeben.

HAMPEL,F.R.: Some topics in robust estimation.

The concepts and methods for robust estimation of a simple variance are 'classical' by now, except for the analogues of the hyperbolic tangent estimators. Covariance matrices pose new problems. One approach is via robust correlations, another one uses affine invariant M-estimators. Most 'obvious' estimators of the latter type have a very low breakdown point in high dimensions, but recently a very robust estimator has been found.

STREIT,F.: Bemerkungen zu statistischen Tests für Punktprozesse.

In diesem Vortrag werden zwei spezielle Resultate über statistische Tests für einfache Punktprozesse angegeben und es wird gezeigt, daß diese sich mit Hilfe des Fundamentallemmas von Neyman und Pearson beweisen lassen.

Das erste Ergebnis bezieht sich auf einen von D.R. Cox vorgeschlagenen Test zum Vergleich der Nullhypothese, welche besagt, dass ein homogener Poissonpunktprozess vorliegt, mit Tren ternativen. Es werden Optimalitätseigenschaften hergeleitet, insbesondere auch für den Fall, daß keine einschränkende Bedingung für die Anzahl der Ereignisse in der Beobachtungsstrecke vorgegeben wird.

Das zweite Resultat enthält Angaben über beste Tests zum Prüfen von homogenen Poissonpunktprozessen gegen Punktprozesse, bei denen die Folge der ereignisfreien Zwischenräume ein Erneuerungsprozess mit einseitig gestutzter und absolut stetiger Verteilung der Zwischenraumslängen ist.

RUNNENBURG,J.Th.: A simple model in neuron firing.

In Råde's book Thinning of Renewal Point Processes (Göteborg 1972) the interaction of two renewal processes is studied. If one considers in particular the case where these processes are obtained through cointossing, one can (following Råde) easily find a number of interesting generating functions by using the method of collective marks (due to van Dantzig: A generating function is interpreted as the probability that a catastrophe does not occur). The structure of the whole process can be analysed in terms of a semi-Markov chain (finitely many states, discrete time), from which states have to be eliminated, lumped and weakly lumped (Kemeny + Snell) to obtain the generating functions one is interested in.

FIEGER,W.: On the mean number of maxima of stochastic processes.

Ist $\{X(t) : t \in R^1\}$ ein stochastischer Prozeß mit stetigen Pfaden, und bezeichnet $m(a,b;\omega)$ die Anzahl der lokalen Maximalstellen von $X(t,\omega)$ in dem Intervall (a,b), so gilt

$$E\, m(a,b;\omega) = \sup\left\{ \sum_{\varrho=1}^{n-1} p(S_\varrho(Z)) \;:\; Z:\; a = t_0 < \dots < t_n = b \right\}$$

mit $S_\varrho(Z) = \{\omega : X(t_{\varrho-1},\omega) \leq X(t_\varrho,\omega),\; X(t_{\varrho+1},\omega) \leq X(t_\varrho,\omega)\}$.

Ausgehend von dieser Beziehung läßt sich für jeden Gaußschen Prozeß $\{X(t): t \in R^1\}$ mit $E\,X(t) \equiv 0$, $\mathrm{var}\,X(t) \equiv 1$ zeigen: Ist dieser Prozeß stationär, so ist $E\, m(a,b;\omega) < +\infty$ äquivalent mit der Existenz der vierten Ableitung der Kovarianzfunktion $r(t) = EX(0)EX(t)$ im Punkte $t = 0$; ohne die Voraussetzung der Sationarität kann man beweisen, daß $E\, m(a,b;\omega) < +\infty$ ist, falls alle vierten Ableitungen der Kovarianzfunktion $r(s,t) = E\,X(t)X(s)$ existieren und stetig sind.

KOMLOS,J.: Weak convergence and embeddings (joint work with P. Major, G. Tusnady).

A new embedding is introduced: conditional quantile process on a dyadic scheme. Typical results are:

$$S_n = X_1 + \dots + X_n\ , \quad T_n = Y_1 + \dots + Y_n$$

the X's are i.i.d. distributed according to F, $\int x\, F(dx) = 0$, $\int x^2\, F(dx) = 1$, and the Y's are i.i.d. standard normal. If $\int \exp(tx)\, F(dx) < \infty$ for $|t| < t_0$, then we construct (S_n, T_n) such that

$$P\Big(\sup_{1 \leq k < \infty} \big(|S_k - T_k| - C\log k \big) > x \Big) < K\, e^{-\lambda x} \quad \text{for all } x > 0.$$

Consequently $|S_n - T_n| = O(\log n)$ with probability 1.

If $\int |x|^r\, F(dx) < \infty$, $r > 2$, is assumed, we get $|S_n - T_n| = o(n^{1/r})$.

For the Prohorov distance of the corresponding processes σ_n, w_n we obtain

$$\frac{c \log n}{\sqrt{n}} < \pi(\sigma_n, w_n) < \frac{C \log n}{\sqrt{n}}$$

in the first case and in the second

$$\pi(\sigma_n, w_n) = o(n^{\frac{r-2}{2(r+1)}}).$$

For the empirical distribution function $F_n(x)$ for samples uniformly distributed in (0,1) one gets a similar result

$$P\left(\sup_{0 \le x \le 1} \left| \sqrt{n}\,(F_n(x)-x) - B(x) \right| > \frac{\log n + y}{\sqrt{n}} \right) < K e^{-\lambda y}.$$

All results are shown to be best possible. Two dimensional generalizations are given.

MANDL,P.: Diffusion approximations in controlled Markov chains.

Im Vortrag werden steuerbare Markovsche Ketten mit Übergangswahrscheinlichkeiten $p_{ij}(z)$, i,j I (I die Menge der Zustände) betrachtet. z ist der Steuerungsparameter. Man bezeichnet mit $\{X_n,\ n = 0,1,..\}$, $\{Z_n,\ n = 0,1,...\}$ die durch die nacheinanderfolgenden Zustände bzw. Steuerungsparameterwerte gebildeten Pfade. Man führt eine additive Funktion dieser Pfade

$$C_n = \sum_{m=0}^{n-1} c(X_m, X_{m+1}; Z_m) \qquad n = 1,2,...$$

ein. Besteht die Beziehung $Z_n = z(x_n)$, n = o,1,... , so spricht man von stationärer Steuerung. In Problemen, wo C_n eine Grenze nicht unterschreiten darf, sind Steuerungen der Form $Z_n = z(X_n, C_n)$, n = 0,1,... wichtig. Für solche Steuerungen lassen sich die Optimierungsaufgaben unter gewissen Voraussetzungen in Annäherung auf Steuerungsprobleme für eindimensionale Diffusionsprozesse überführen. Dabei bilden die stationären Steuerungen den Parameterraum.

KRENGEL,U.: On the stochastic ordering of random vectors.

Let E be a Polish space with a closed partial order relation $\le$. A partial order on the space of probability measures on E is given by $P \prec Q$ iff $\int f\,dP \le \int f\,dQ$ for all bounded increasing realvalued functions f on E. The basic theorem states that $P \prec Q$ iff there exists a measure ν on ExE with support in $\{(x,y) : x \leq y\}$ which has P for its first marginal measure and Q for its second marginal measure. (This is due to P. Major in this generality and independently for countable E, for $E = R^n$, for $E = C[0,1]$ and for $E = D[0,1]$ to Kamae, myself, O'Brien). Related results that are weaker but hold for certain spaces that are not Polish are due to Nachbin and Hommel. Equivalent formulations: (1) $P \prec Q$ iff there exist random variables $X \leq Y$ with P = distr(X) and Q = distr(Y). (2) $P \prec Q$ iff there exists a kernel k(.,.) which transports P into Q and has the property $k(x,\{y:y \geq x\}) = 1$ for all x in E. In the case $E = R^1$ Lakeit has imposed further conditions on k to ensure uniqueness. For a sequence $P_1 \prec P_2 \prec \ldots$ one gets: there exists a sequence of randomvariables $X_1 \leq X_2 \leq \ldots$ with $P_i = \text{distr}(X_i)$. X_i converges stochastically iff $\{P_i\}$ is tight and this happens iff $\{P_i\}$ converges weakly. Also a.e. convergence is equivalent iff E has the additional property $x_n \uparrow x$ and $x_n \leq y_n \leq x$ imply $y_n \longrightarrow x$.

HELWIG,M.: Wiener-Hopf factorisation of infinitesimal generators of semigroups.

Let $(P_t)_{t \geq 0}$ be a Feller semigroup of substochastic transition functions in $(R^k, \mathcal{B}^k)$; A its infinitesimal generator. We consider a transitive and reflexive relation "$\leq$" in R^k and prove the following under some additional regularity assumptions

<u>Theorem</u>: There exist operators A^-, A^+ such that $A^-f(x) = A^-g(x)$ if $f(y) = g(y)$ for $y \leq x$, $A^+f(x) = A^+g(x)$ if $f(y) = g(y)$ for $y \geq x$ and $f \in \mathcal{D}(A) \Longrightarrow A^-A^+f = Af$.

In some sense this factorisation is unique.

This is the generalisation of the Wiener-Hopf Factorisation of a probability measure and the factorisation of the infinitesimal generator of convolution semigroups (Prabhu 1973), following the ideas of H. Dinges, who proved the theorem above in the discrete case (H. Dinges 1969).

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